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**abdus salam**  
international centre for theoretical physics

40<sup>th</sup> anniversary  
1964  
2004

**SMR.1580 - 26**

**CONFERENCE ON FUNDAMENTAL SYMMETRIES  
AND FUNDAMENTAL CONSTANTS**

**15 - 18 September 2004**

**DARK ENERGY ARISING IN STRING THEORY**

**E. Copeland  
Sussex U., UK**

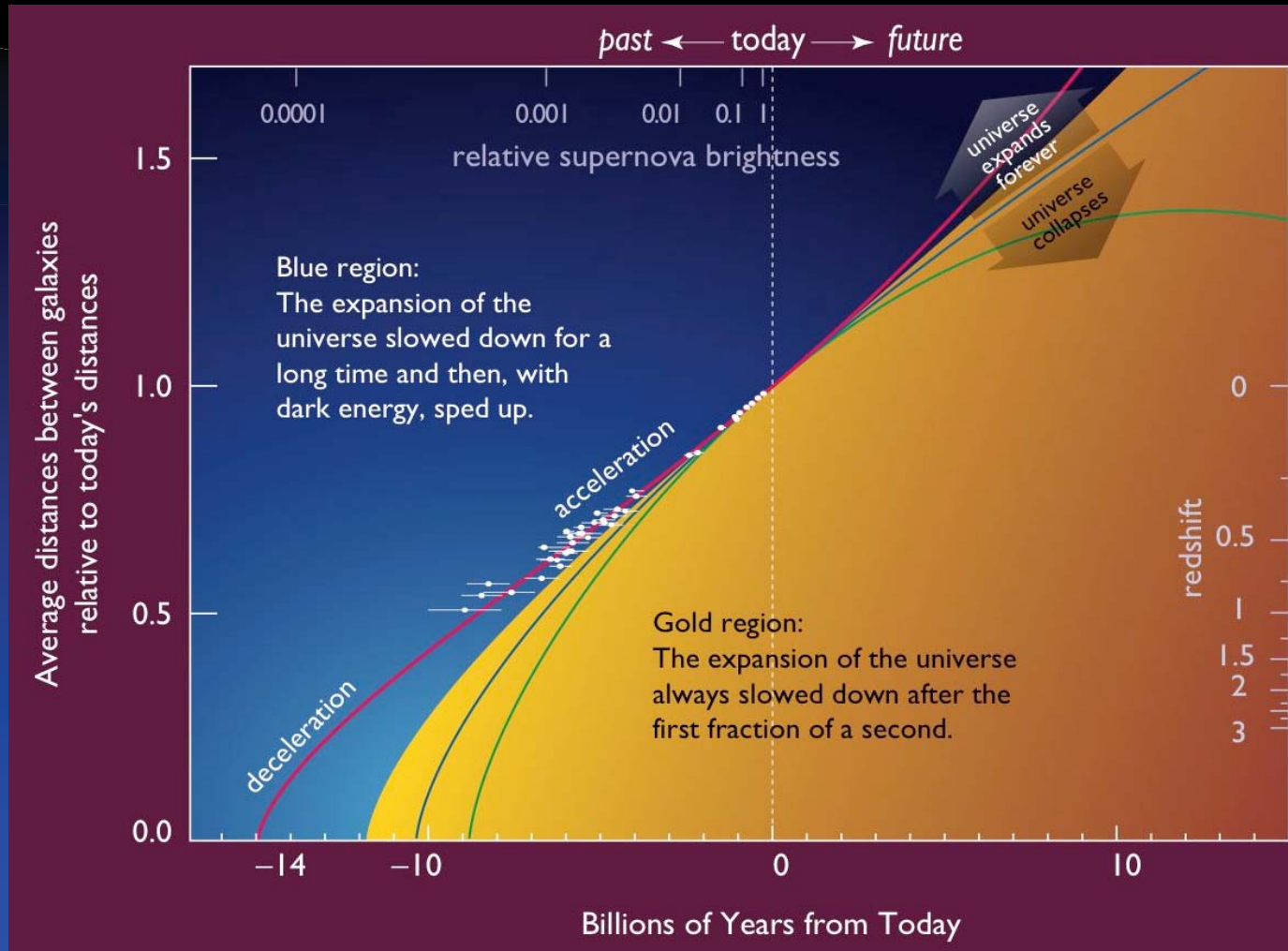
# Attempts to explain the nature of dark energy -- How desperate can we get?

**Ed Copeland    University of Sussex**

1. Quintessence – tracking solutions.
2. Models (some inspired by particle physics).
3. K-essence v Quintessence
4. Evidence for evolving dark energy (enter WMAP) ?
5. Problems facing models of dark energy.

**Trieste, Sept 17th, 2004**

# Evidence for Acceleration



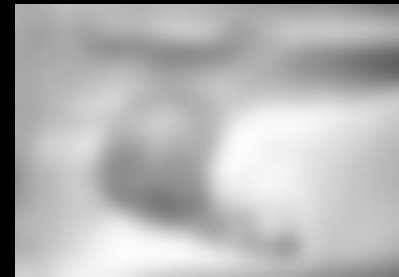
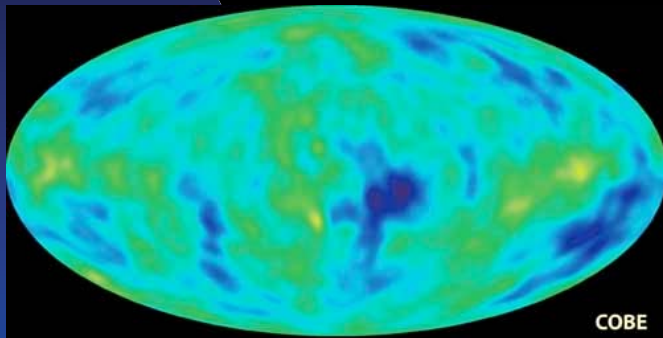
data from Supernova Cosmology Project (LBL)

graphic by Barnett, Linder, Perlmutter & Smoot (for OSTP)

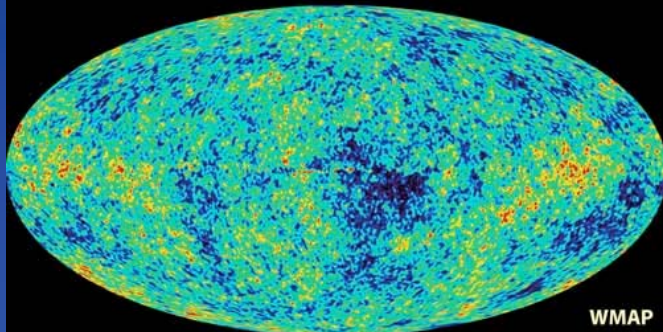
**Exploding stars – supernovae – bright beacons that allow us to measure the expansion over the last 10 billion years.**

# Looking for those primordial fluctuations. Ripples in the smooth cosmic microwave background radiation.

1992



2003



9/24/04

# Evidence for Dark Energy?

Enter CMBR:

Provides clue. 1<sup>st</sup> angular peak  
in power spectrum.

$$3. \Omega_0 = \Omega_m + \Omega_\Lambda$$

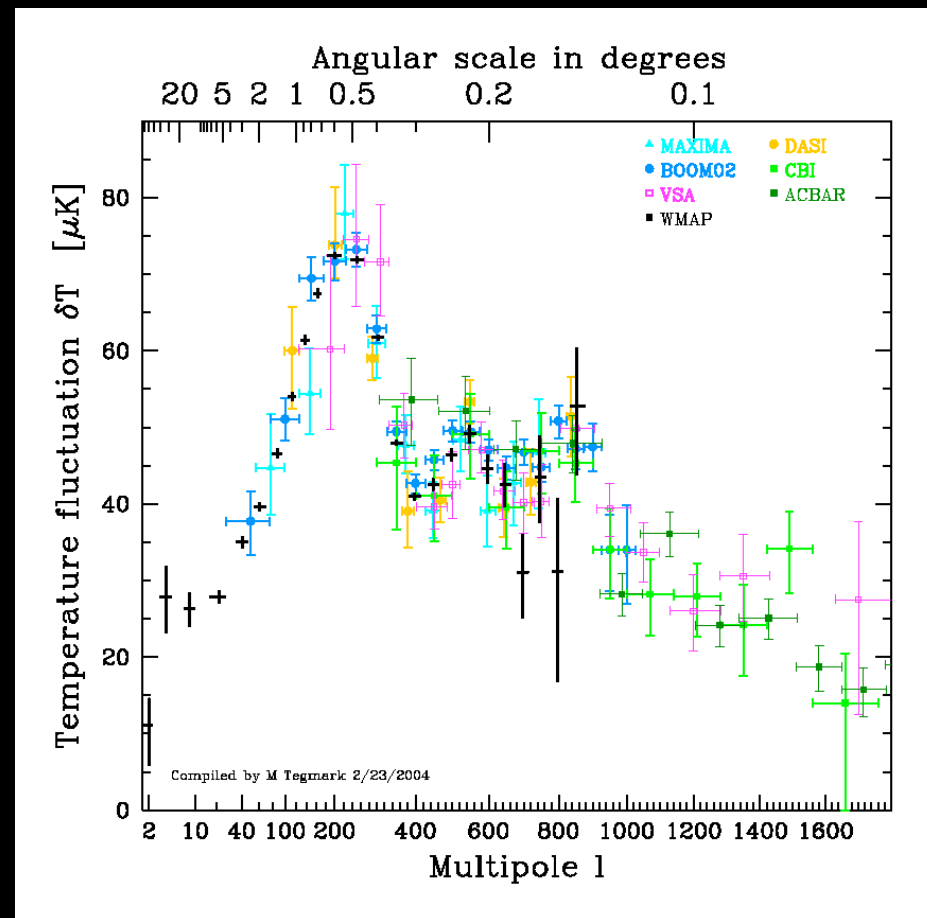
$$l_{\text{peak}} \approx \frac{220}{\sqrt{\Omega_0}}$$

$$\Omega_0 = 1.095^{+0.094}_{-0.144}$$

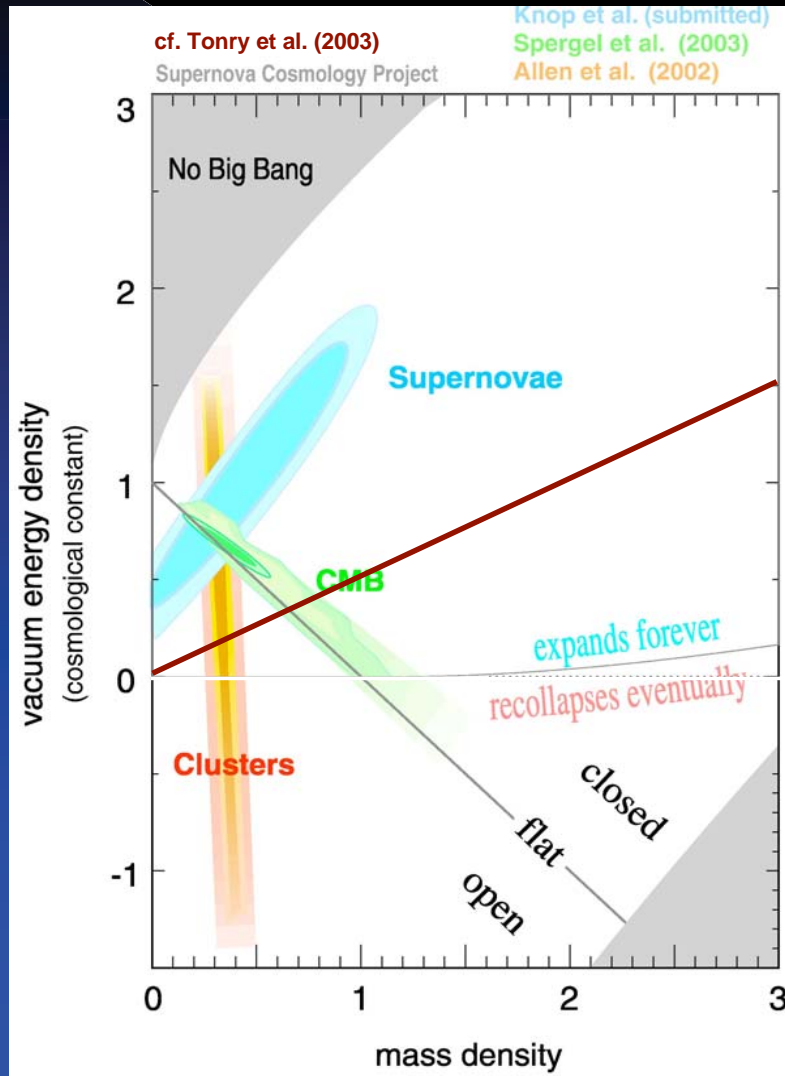
WMAP-Depends  
on assumed priors

Tegmark et al 2003

9/24/04



# Cosmic Concordance



- **Supernovae alone**  
⇒ Accelerating expansion  
⇒  $\Lambda > 0$
- **CMB (plus LSS)**  
⇒ Flat universe  
⇒  $\Lambda > 0$
- **Any two of SN, CMB, LSS**  
⇒ Dark energy  $\sim 75\%$

# Different approaches to Dark Energy include amongst many:

- A true cosmological constant -- if so, why this value?
- Many possible cosmological constants (false vacua)
- A time-dependent cosmological constant.
- Solid –dark energy such as arising from frustrated network of domain walls.
- Time dependent solutions arising out of evolving scalar fields -- Quintessence/K-essence.
- Modifications of Einstein gravity leading to acceleration today.

Over 350 papers on archives since 1998 with dark energy in title.

# Key equations :

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{3}\rho - \frac{k}{a^2} + \Lambda$$

Friedmann eqn

$$\dot{\rho} + 3H(\rho + p) = 0$$

Fluid eqn.

$$\frac{\ddot{a}}{a} = -\frac{\kappa^2}{6}(\rho + 3p) + \frac{\Lambda}{3}$$

Acceleration eqn

Note:

where

$$\kappa^2 \equiv 8\pi G$$

$$\frac{\ddot{a}}{a} \geq 0 < - > = (\rho + 3p) \leq 0$$



## Coincidence problem – why now?

$$\frac{\ddot{a}}{a} \geq 0 \iff (\rho + 3p) \leq 0$$

If:  $\rho_x = \rho_x^0 a^{-3(1+w_x)}$

Universe dom by  
Quintessence at:

$$z_x = \left( \frac{\Omega_x}{\Omega_m} \right)^{\frac{1}{3w_x}} - 1$$

$$\left( \frac{\Omega_x}{\Omega_m} \right) = \frac{7}{3} \rightarrow z_x = 0.5, 0.3 \text{ for } w_x = -\frac{2}{3}, -1$$

Univ  
accelerates at:

$$z_a = \left( - (1 + 3w_x) \frac{\Omega_x}{\Omega_m} \right)^{\frac{-1}{3w_x}} - 1$$

$$z_a = 0.7, 0.5 \text{ for } w_x = -\frac{2}{3}, -1$$

# Tracker solutions

Wetterich; EC, Liddle and Wands

Peebles and Ratra,

Zlatev, Wang and Steinhardt

Scalar field:

$$\phi: \rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi); \quad p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)$$

EoM:

$$H = -\frac{\kappa^2}{2} (\dot{\phi}^2 + \gamma \rho_B)$$

$$\rho_B = -3\gamma H \dot{\phi}$$

$$\ddot{\phi} = -3H \dot{\phi} - \frac{dV}{d\phi}$$

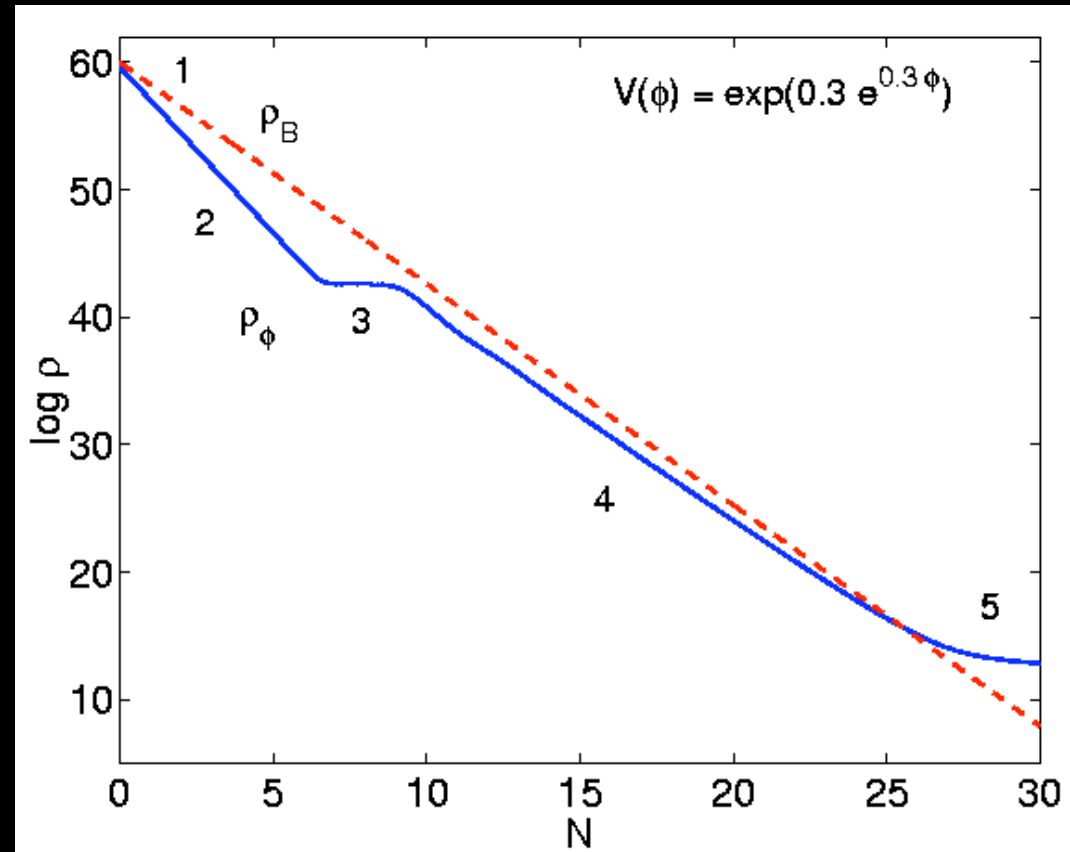
+ constraint:

$$H^2 = \frac{\kappa^2}{3} (\rho_\phi + \rho_B)$$

# Generic behaviour

Ng, Nunes and  
Rosati

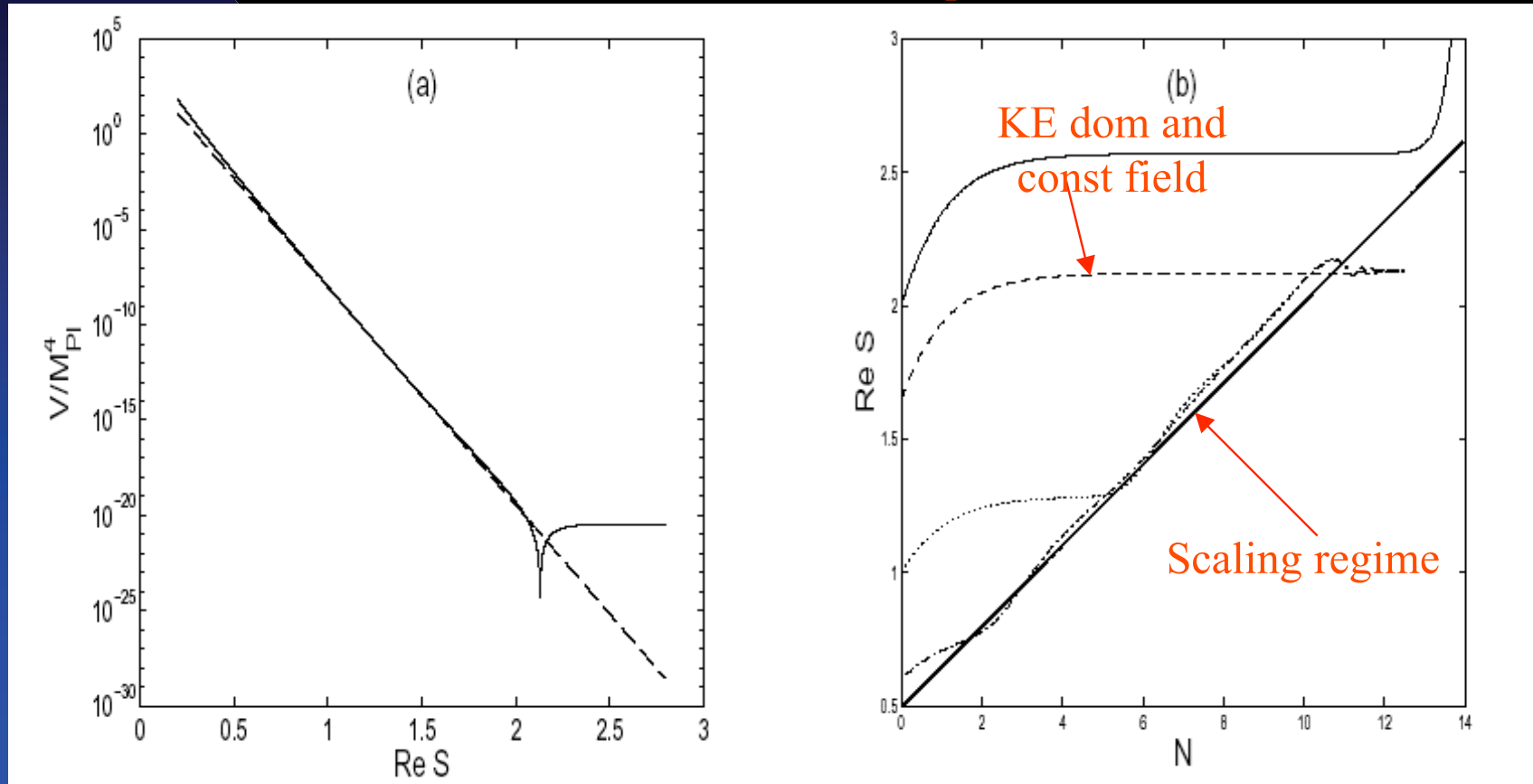
1. PE  $\rightarrow$  KE
2. KE dom scalar field energy den.
3. Const field.
4. Attractor solution: almost const ratio KE/PE.
5. PE dom.



# Useful way of stabilising moduli in string cosmology. Sources provide extra friction when potentials steep.

Barreiro, de Carlos and EC : hep/th-9805005

Brustein, Alwis and Martins : hep-th/0408160



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Two condensate model with  $V \sim e^{-a \text{Re } S}$  as approach minima

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# A few models

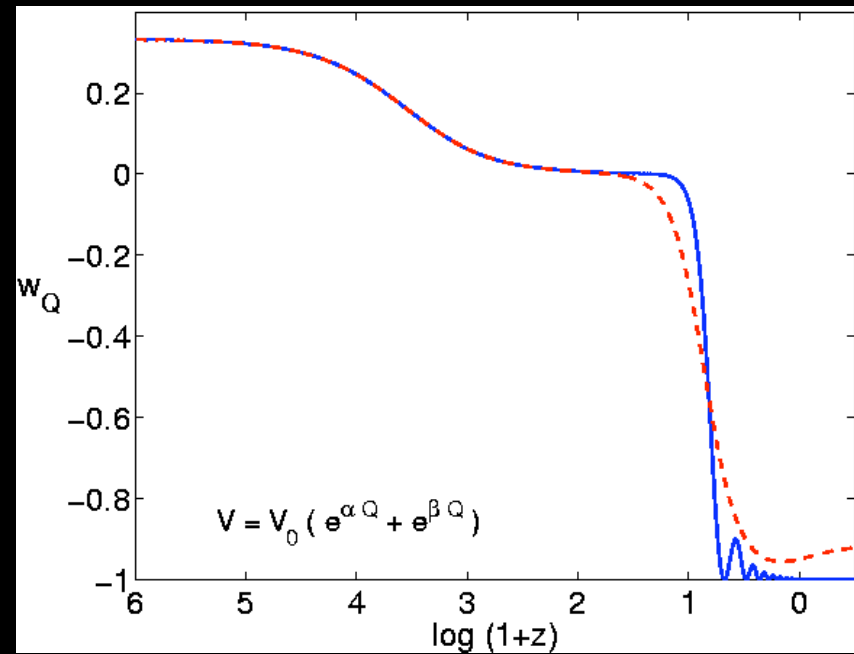
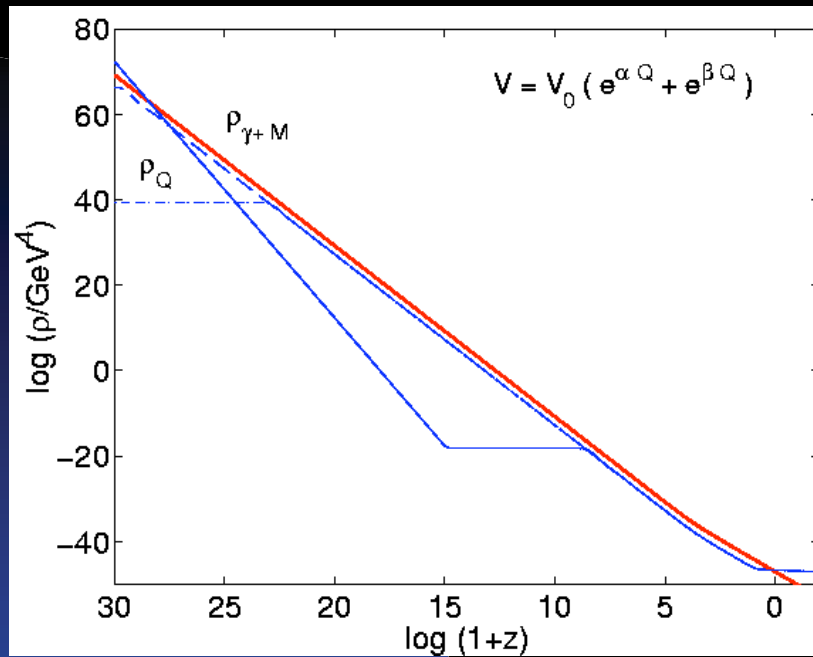
1. Inverse polynomial – found in SUSY QCD - Binetruy
2. Multiple exponential potentials – SUGRA and String compactification.

$$\begin{aligned} V(\phi) &= V_1 + V_2 \\ &= V_{01} e^{-\kappa\lambda_1\phi} + V_{02} e^{-\kappa\lambda_2\phi} \end{aligned}$$

Barreiro, EC,  
Nunes

**Enters two scaling regimes depends on lambda, one tracking radiation and matter, second one dominating at end. Must ensure do not violate nucleosynthesis constraints.**

$$\alpha = 20; \beta = 0.5$$



Scaling for wide range of i.c.

**Fine tuning:**  $V_0 \approx \rho_\phi \approx 10^{-47} \text{ GeV}^4 \approx (10^{-3} \text{ eV})^4$

**Mass:**

$$m \approx \sqrt{\frac{V_0}{M_{\text{pl}}^2}} \approx 10^{-33} \text{ eV}$$

**Fifth force !**

## Quintessential Axion -- Kim and Nilles

Linear combination of two axions together through hidden sector supergravity breaking.

**Light CDM axion (solve strong CP problem) with decay const through hidden sector squark condensation:**

$$F_a \sim 10^{12} \text{GeV}$$

**Quintaxion (dark energy) with decay const as expected for model independent axion of string theory:**

$$F_q \sim 10^{18} \text{GeV}$$

**Model works because of similarities in mass scales:**

**Scale of susy breaking and scale of QCD axion.**

**Scale of vacuum energy and mass of QCD axion.**  $\lambda^4 \sim (0.003 \text{eV})^4$

**Potential for quintaxion remains very flat, because of smallness of hidden sector quark masses, ideal for Quintessence. Quintessence mass protected through existence of global symmetry associated with pseudo Nambu-Goldstone boson.**

# Acceleration from new Gravitational Physics? Starobinski 1980, Carroll et al 2003

$$S = \frac{M_{\text{P}}^2}{2} \int d^4x \sqrt{-g} \left( R - \frac{\mu^4}{R} \right) + \int d^4x \sqrt{-g} \mathcal{L}_M$$

Modify Einstein

Const curv vac  
solutions:

$$\nabla_{\mu} R = 0, \rightarrow R = \pm \sqrt{3} \mu^2$$

de Sitter or Anti  
de Sitter

Transform to EH  
action:

$$\tilde{g}_{\mu\nu} = p(\phi) g_{\mu\nu}, \quad p \equiv \exp \left( \sqrt{\frac{2}{3}} \frac{\phi}{M_{\text{P}}} \right) \equiv 1 + \frac{\mu^4}{R^2}$$

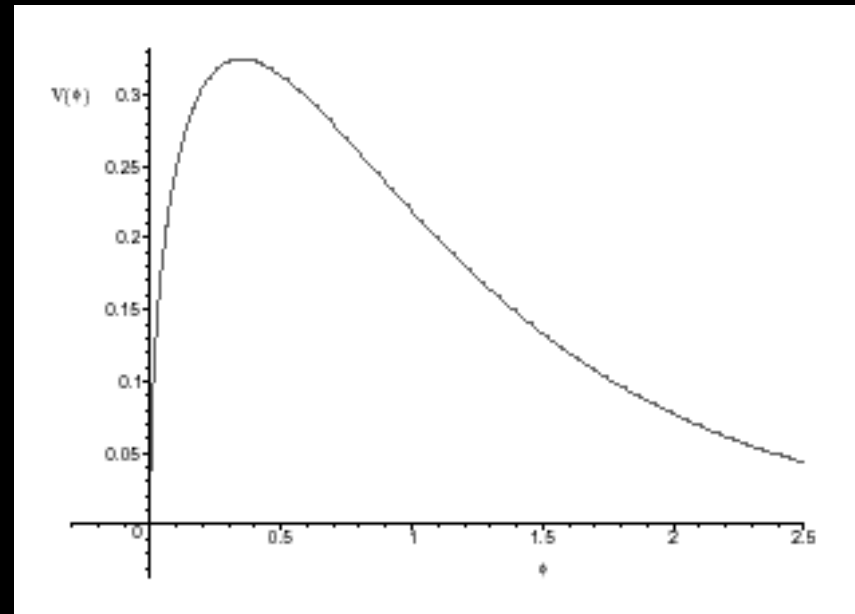
Scalar field min coupled to gravity and non minimally  
coupled to matter fields with potential:

$$V(\phi) = \mu^2 M_{\text{P}}^2 \frac{\sqrt{p-1}}{p^2}$$



## Cosmological solutions:

1. **Eternal de Sitter** -  $\phi$  just reaches  $V_{\max}$  and stays there. Fine tuned and unstable.
2. **Power law inflation** --  $\phi$  overshoots  $V_{\max}$ , universe asymptotes with  $w_{\text{DE}} = -2/3$ .
3. **Future singularity**--  $\phi$  doesn't reach  $V_{\max}$ , and evolves back towards  $\phi=0$ .



Fine tuning needed so acceleration only recently:  
 $\mu \sim 10^{-33} \text{eV}$

Also, any modification of Einstein-Hilbert action needs to be consistent with classic solar system tests of gravity. Not obvious these models are!

## Quintessence and M-theory -- where are the realistic models?

'No go' theorem: forbids cosmic acceleration in cosmological solutions arising from compactification of pure SUGR models where internal space is time-independent, non-singular compact manifold without boundary --[Gibbons]

Why? : 1. acceleration requires violation of strong energy condition.

i.e  $R_{00} \leq 0$

2. Strong energy condition not violated by either 11D SUGR or any of the 10D SUGR theories
3. For any compactification described above, if higher dim stress tensor satisfies SEC then so does the lower dimensional stress tensor.

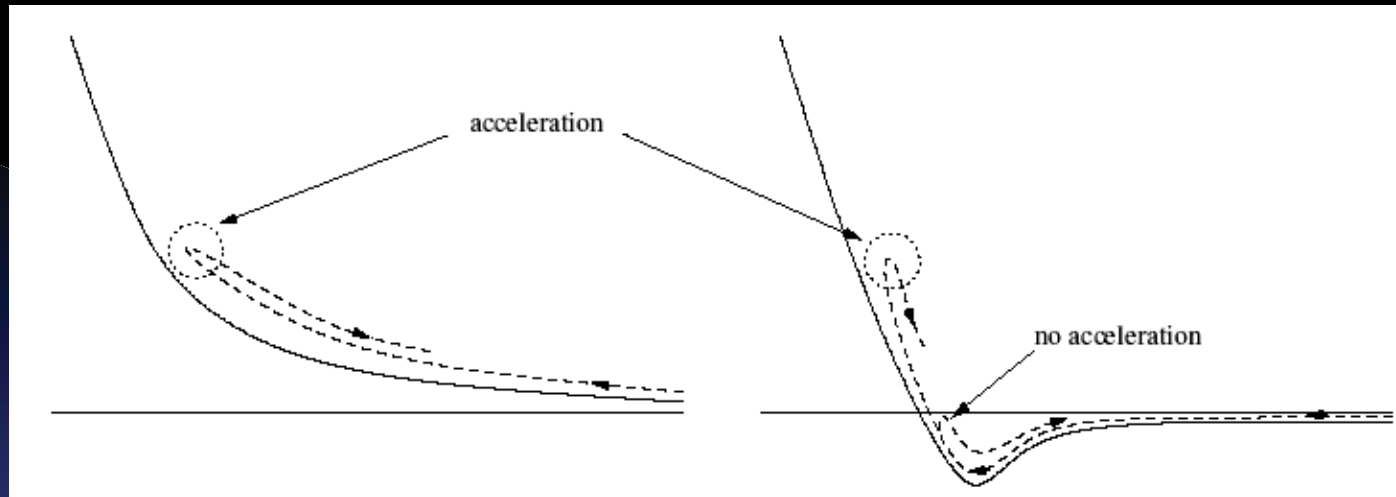
Must avoid no-go theorem by relaxing conditions of the theorem.

1. Drop condition that internal space is compact,  
but not so realistic -- Townsend

2. Allow internal space to be time-dependent,  
analogue of time-dependent scalar fields -- Lukas  
et al, Kaloper et al, Townsend & Wohlfarth, Emparan & Garriga.

Compactified spaces are hyperbolic and lead to cosmologies with transient accelerating phase. Four dimensional picture, solutions correspond to bouncing the radion field off its exponential potential.

Acceleration occurs at the turning point where the radion stops and potential energy momentarily dominates.



- Field starts at large positive values, with large kinetic energy.
- At turning point, energy is pot dominated and acceleration.
- Left picture, two positive potentials, right picture, sum of positive and negative potentials.

### **Problems:**

Difficult to obtain sustained period of inflation.

Current realistic potentials are too steep

These models have kinetic domination, not matter domination before entering accelerated phase.

However progress is being made to obtain inflation in string theory:

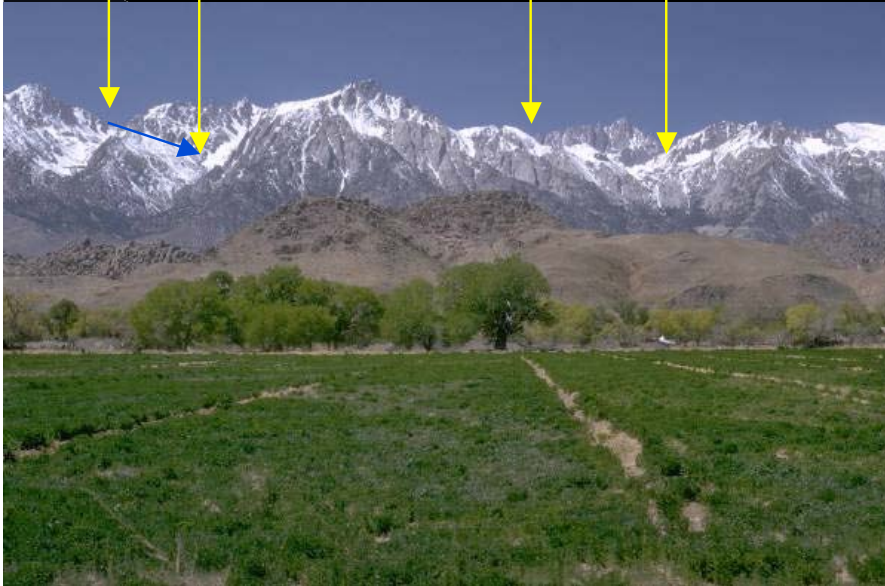
Metastable de Sitter string vacua in Type IIB string theory, based on stable highly warped IIB compactifications with NS and RR three-form fluxes.

[Kachru et al 2003]

There remain fine tuning issues in these brane models concerning the method of volume stabilisation, the warping of the internal space and the source of the inflationary energy scale.

Still early days for inflation in string/M-theory.

## The String Landscape approach



Type IIB String theory compactified from 10 dimensions to 4.

Internal dimensions stabilised by fluxes.

Many many vacua  $\sim 10^{500}$  !

Typical separation  $\sim 10^{-500} \Lambda_{pl}$

Assume randomly distributed, tunnelling allowed between vacua --> separate universes .

Anthropic : Galaxies require vacua  $< 10^{-118} \Lambda_{pl}$  [Weinberg]

Most likely to find values not equal to zero!

## Modifications of Friedmann equation in 4D:

Write:

$$H^2 = \frac{8\pi}{3m_4^2} \rho L^2(\rho)$$

$$L(\rho) = 1$$

Standard Friedmann

$$L(\rho) = \sqrt{1 + \frac{\rho}{2\sigma}}; \quad \sigma^{1/4} > 2.0 \text{ MeV}$$

Randall-Sundrum II: co-dimension one brane, embedded in 5D AdS space.

$$L(\rho) = \sqrt{1 - \frac{\rho}{2|\sigma|}}; \quad \sigma < 0$$

Shtanov-Sahni: co-dimension one brane, negative tension embedded in 5D conformally flat Einstein space where signature of 5th dim is timelike

$$L(\rho) = \sqrt{1 + A\rho^n}; \quad n < -1/3$$

Cardassian: only matter present --> late time acceleration. Freese & Lewis

$$L = \frac{1}{\sqrt{B\rho}} \left[ \mp 1 + \sqrt{1 + B\rho} \right]; B \equiv \frac{8\pi m_4^2}{3m_5^6}$$

Dvali-Gabadadze-Porrati: 3-brane embedded in flat 5D Minkowski with Ricci scalar term included in brane action. Bulk empty.

**Next feature: can relate solutions through dualities -- EC, Lidsey, Lee & Mizuno**

DGP model: 
$$H^2 \pm \frac{H}{r_0} = \frac{8\pi}{3m_4^2} \rho; \quad r_0 \equiv m_4^2 / (2m_5^3)$$

Gravity like 4D gravity on short scales, but propagates into bulk on large scales. Induces corrections to Friedmann eqn, characterised by length  $r_0$ .

Two ways of embedding brane in bulk given by  $\pm$

**-sign** --> self accelerating phase (deS) for any decreasing energy density -- ( $w \rightarrow -1$ )

**+sign** --> Minkowski phase. Brane extrinsically curved so that for  $H \sim r_0^{-1}$  gravity screens the effects of the brane energy momentum

**Consider our univ (brane) with homogeneous dust and lambda:**

$$H^2 + \frac{H}{r_0} = \frac{8\pi}{3m_4^2} \rho_M(t) + \lambda$$



Infer effective dark energy :

$$\frac{8\pi}{3m_4^2}\rho_{DE}^{eff}(t) = \lambda - \frac{H}{r_0}$$

Lue & Starkman  
astro-ph/0408246

H decreases with time, effective dark energy increases! For DE domination  $w_{eff} < -1$  (mimics effect of phantom energy).

As universe evolves, screening term becomes weaker and eff dark energy density appears to increase

Degree of growth modulated by  $r_0$ . As  $r_0 \rightarrow \infty$  recover standard  $\Lambda$ CDM.

For any cut off  $r_0$ ,  $w_{eff} \rightarrow -1$  with time and pure  $\Lambda$  cosmology recovered in future.

Possible concern over entering strong coupling regime for large distances.

## K-essence v Quintessence

**K-essence** -- scalar fields with non-canonical kinetic terms. Advantage over Quintessence through solving the coincidence model? -- Armendariz-Picon, Mukhanov, Steinhardt

Long period of perfect tracking, followed by domination of dark energy triggered by transition to matter domination -- an epoch during which structures can form.

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{16\pi G} R + K(\phi) \tilde{p}(X) \right]$$

$$K(\phi) > 0, \quad X = \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi$$

Eqn of state

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$$w_k = \frac{\tilde{p}(X)}{\tilde{\epsilon}(X)} = \frac{\tilde{p}(X)}{2X\tilde{p}'(X) - \tilde{p}(X)}$$

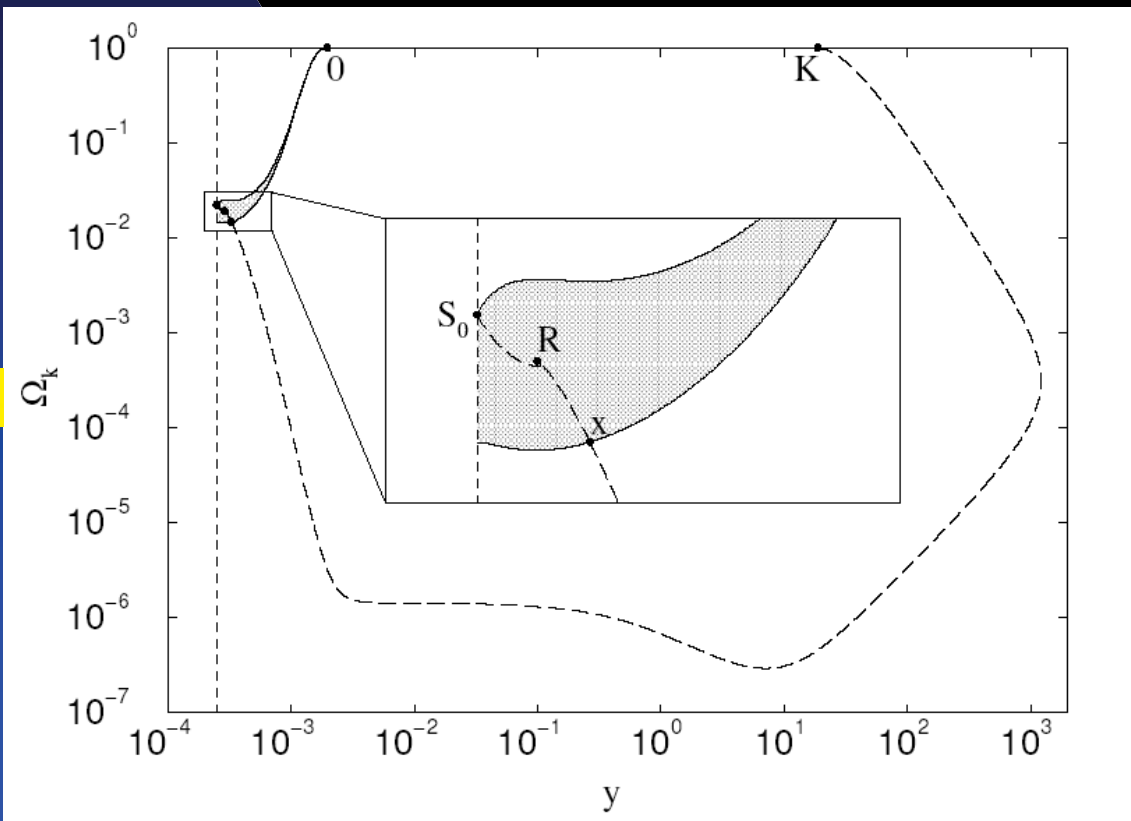
can be  $< -1$

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## Fine tuning in K-essence as well: -- Malquarti, EJC, Liddle

Not so clear that K-essence solves the coincidence problem. The basin of attraction into the regime of tracker solutions is small compared to those where it immediately goes into K-essence domination.

$\Omega_k$



Shaded region is basin of attraction for stable tracker solution at point R. All other trajectories go to K-essence dom at point K.

Based on K-essence model astro-ph/0004134, Armendariz-Picon et al.

# Concordat between SN1a and CMBR

SN1a measures

$$(4/3) \Omega_m - \Omega_\Lambda$$

CMB measures

$$\Omega_0 = \Omega_m + \Omega_\Lambda$$

Almost  
orthogonal

$$\Omega_0 \cong 1 \quad ; \quad \Omega_m \cong 1/3 \quad ; \quad \Omega_\Lambda \cong 2/3$$

Consistent with 2df LSS survey and  
clusters.

Efstathiou et al, astro-ph/0109152

# Evidence for dynamical dark energy ?

Ideally look for evidence in evolution of equation of state as go back in time.

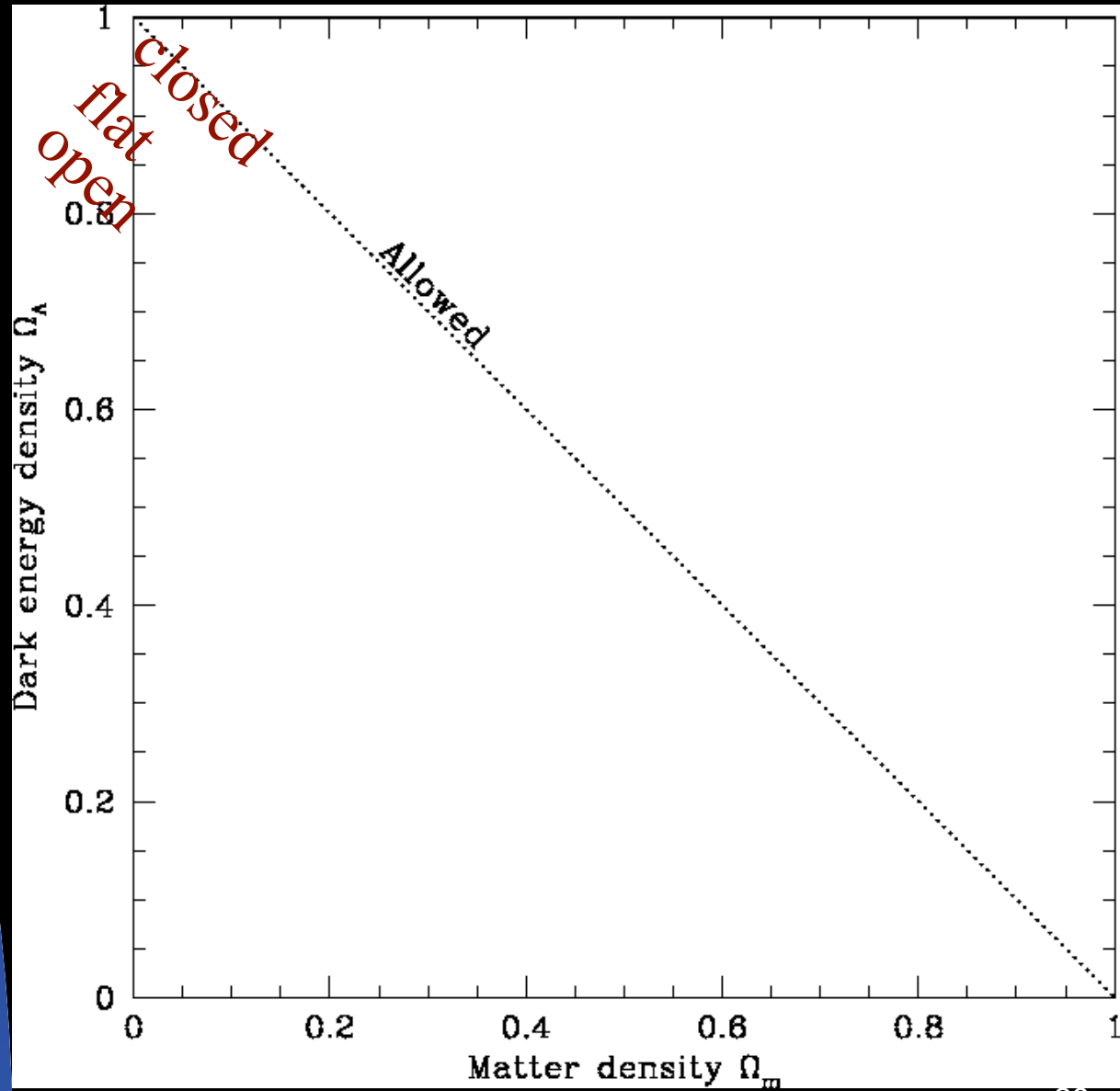
1. Precision CMB anisotropies – lots of models currently compatible.
2. Combined LSS , SN1a and CMB data – tend to give  $w_Q < -0.85 \rightarrow$  difficult to tell from cosmological constant.
3. Look for more SN1a – SNAP will find over 2000 – can then start to constrain eqn of state.
4. Constraining eqn of state with SZ cluster surveys – compute number of clusters for given set of cosm parameters.
5. Probing the Dark Energy with Quasar clustering – redshift distortions constrain cosm parameters – sensitive to matter-lambda combination.
6. Reconstruct eqn of state from observation – offers hope of method indep of potentials – example is **Statefinder** method.
7. Look for evidence in variation of fine structure constant.

# How much dark energy is there?

Friedmann eqn:

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1$$

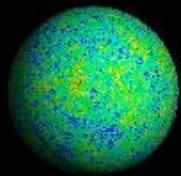
Tegmark et al.  
astro-ph/0310723



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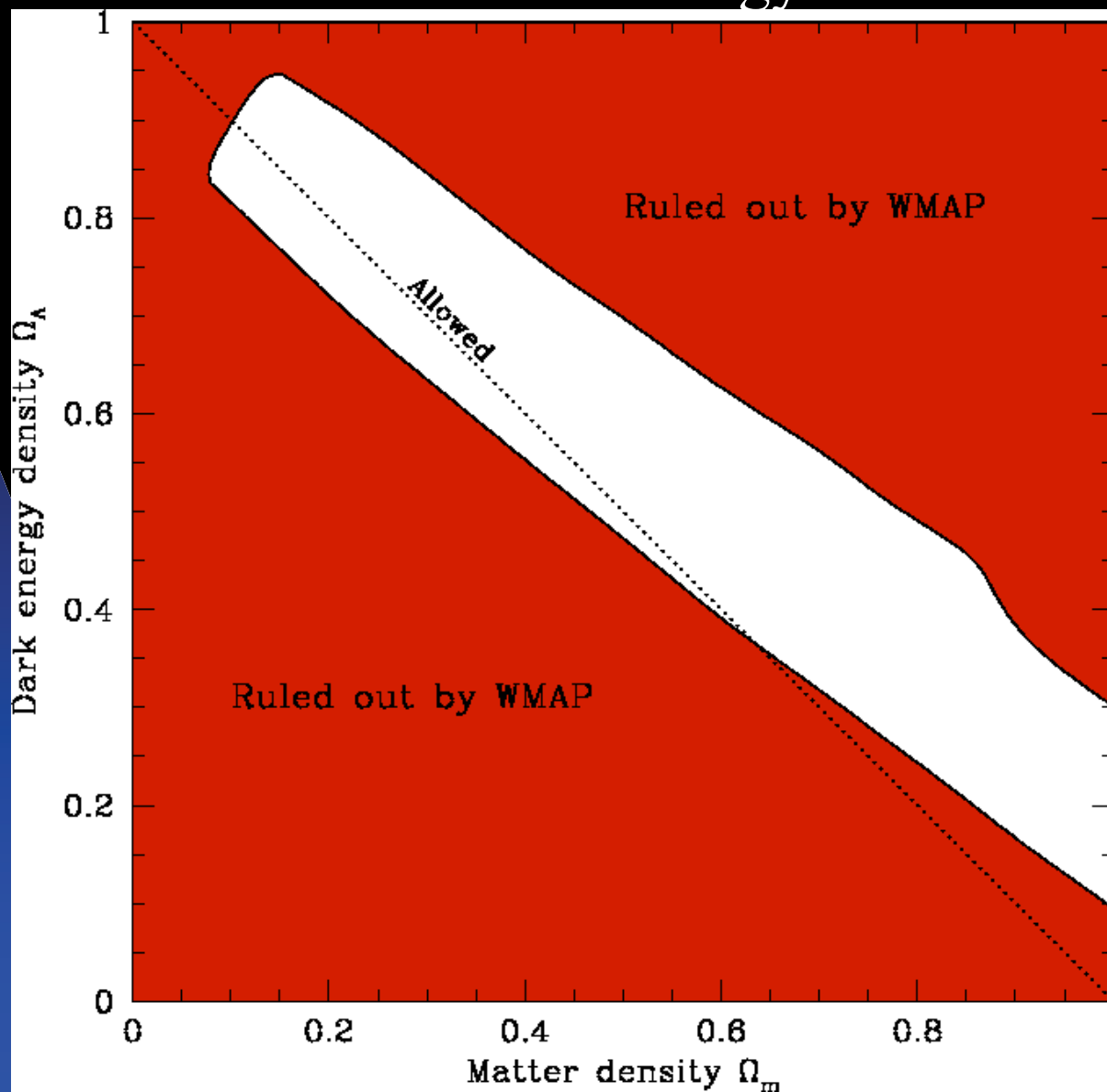
WMAP + SDSS: lots

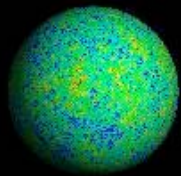
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CMB

# How much dark energy is there?





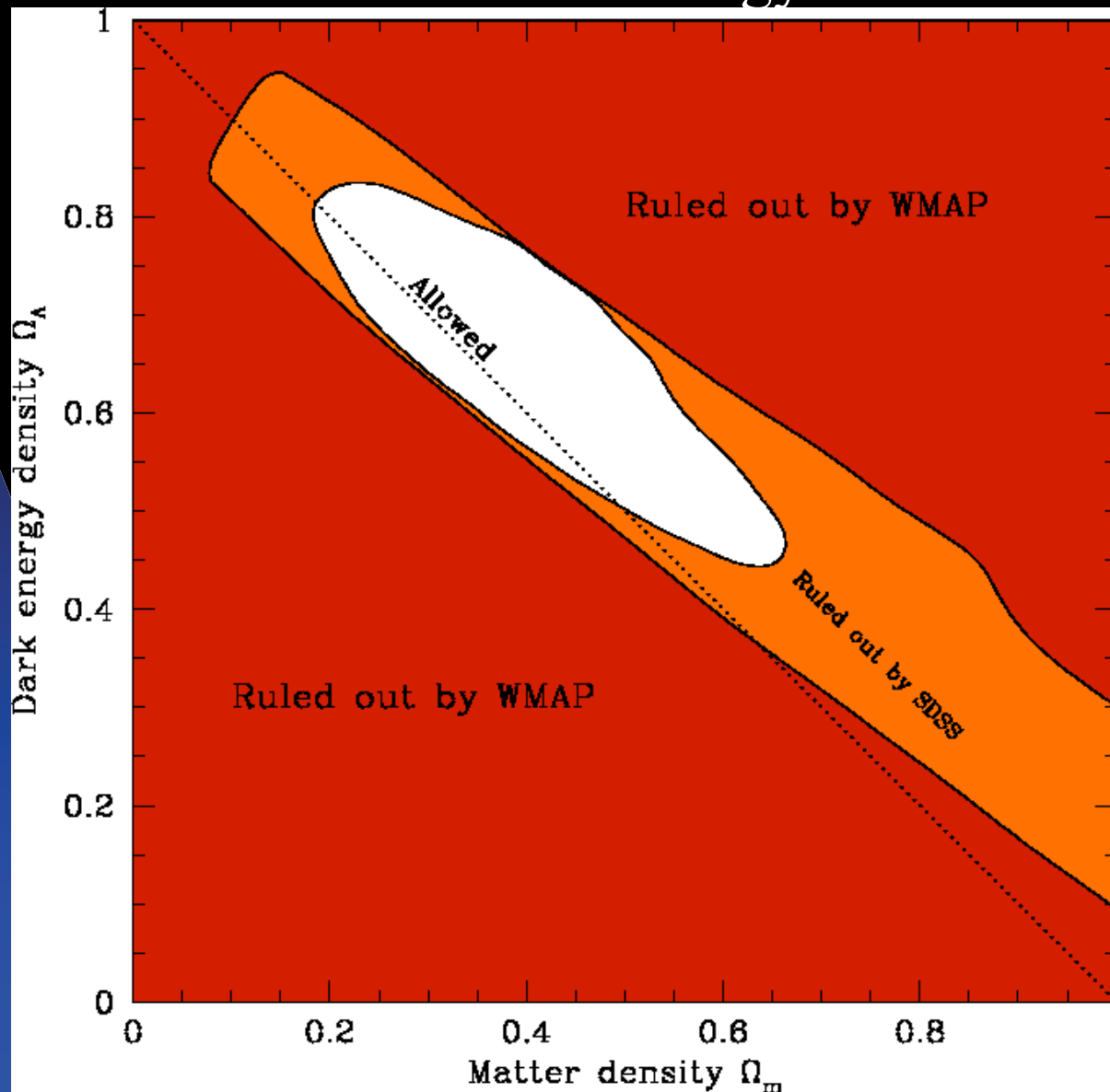
CMB

+



LSS

# How much dark energy is there?

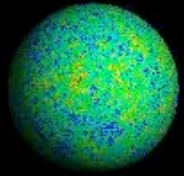


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WMAP + SDSS: lots

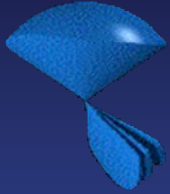


# How much dark energy is there?

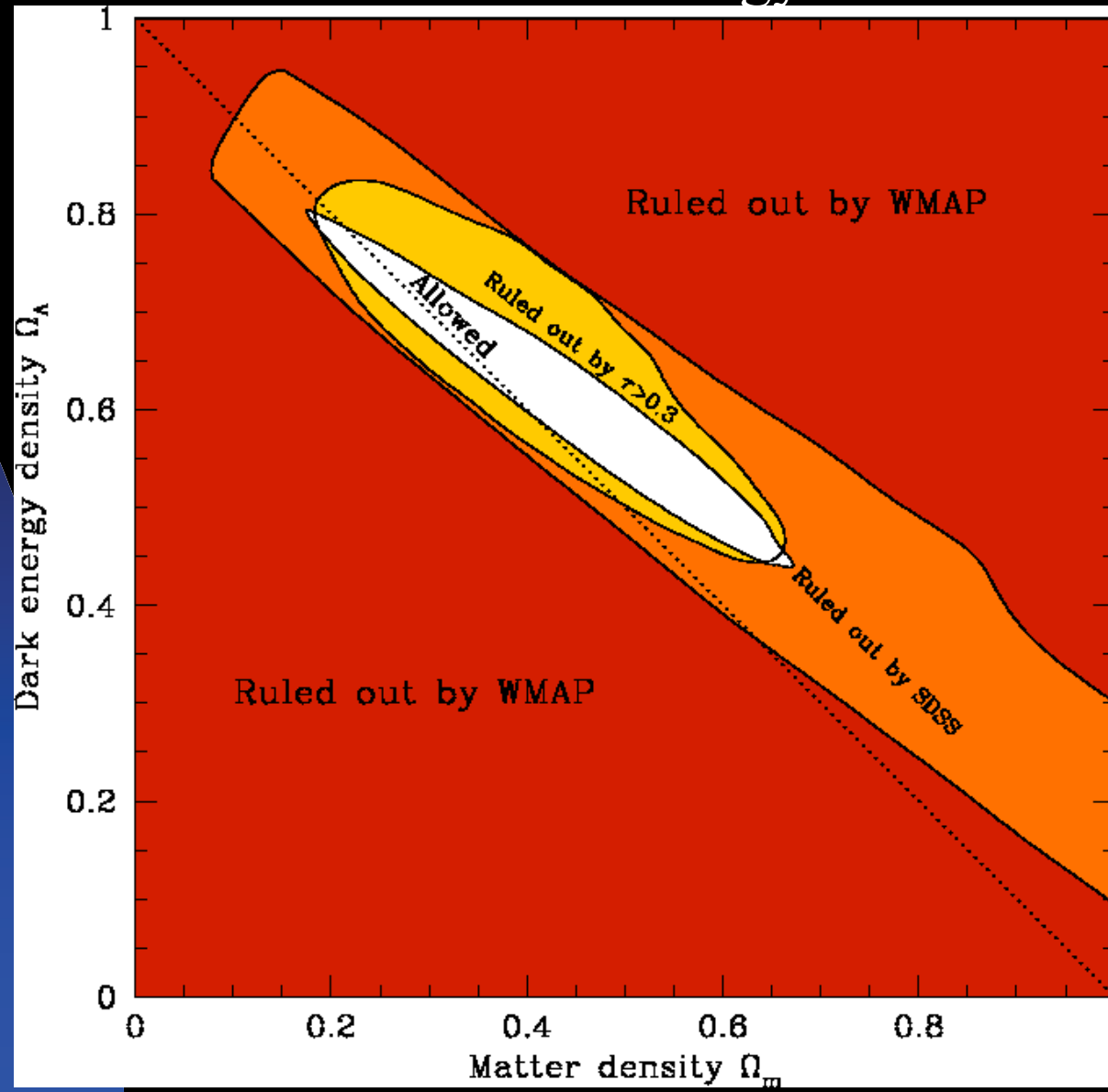


CMB

+

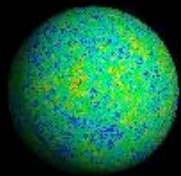


LSS



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CMB

+

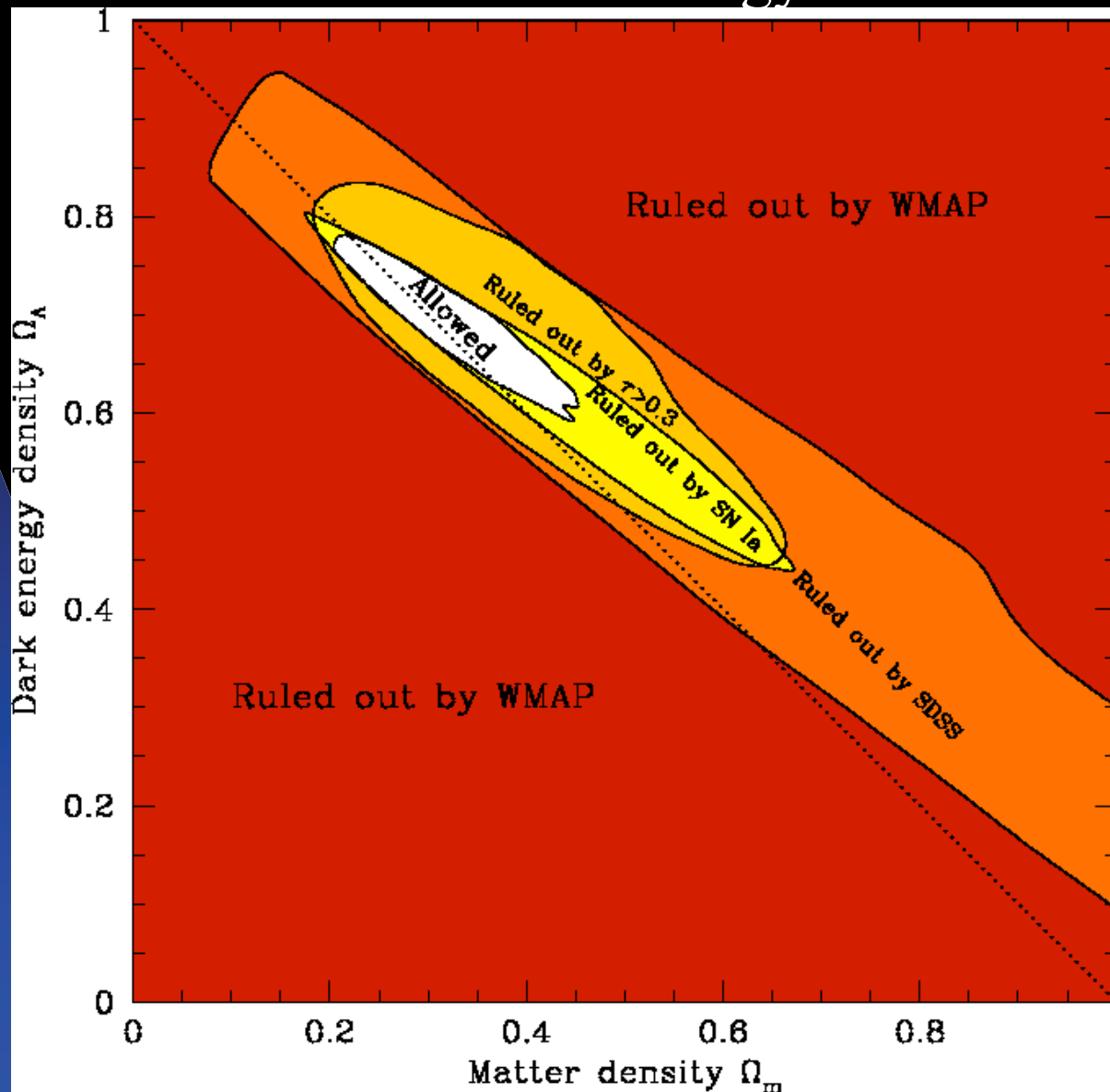


LSS

Note  
impact of  
SN 1a -- if  
excluded  
parameter  
space  
opens up

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# How much dark energy is there?



# Dynamical evolution of $w$ ?

Weller and Albrecht; Kujat et al; Maor et al;  
Gerke and Efstathiou, Kratochvil et al

SNAP as a  
discriminator

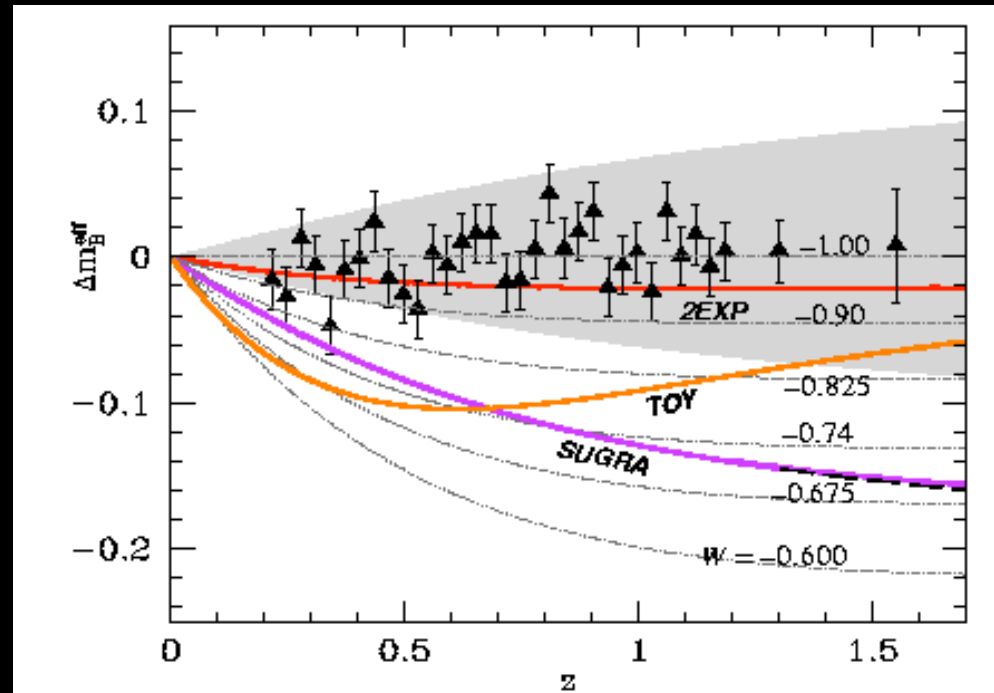
Write:

$$w(z) = \sum_{i=0}^N w_i z^i$$

or:

$$w(z) = \sum_{i=0}^N w_i \ln(1+z)^i$$

Evaluate magnitude difference for each model  
and compare with Monte Carlo simulated data  
sets.



# Evolution of Fine Structure Constant

Olive and Pospelov

Non-trivial coupling to emg:  $L_m = -\frac{1}{4} B_F(\phi) F_{\mu\nu} F^{\mu\nu}$

Expand about current value of field:

$$B_F(\phi) = 1 + \zeta_F \phi + \frac{1}{2} \xi_F \phi^2$$

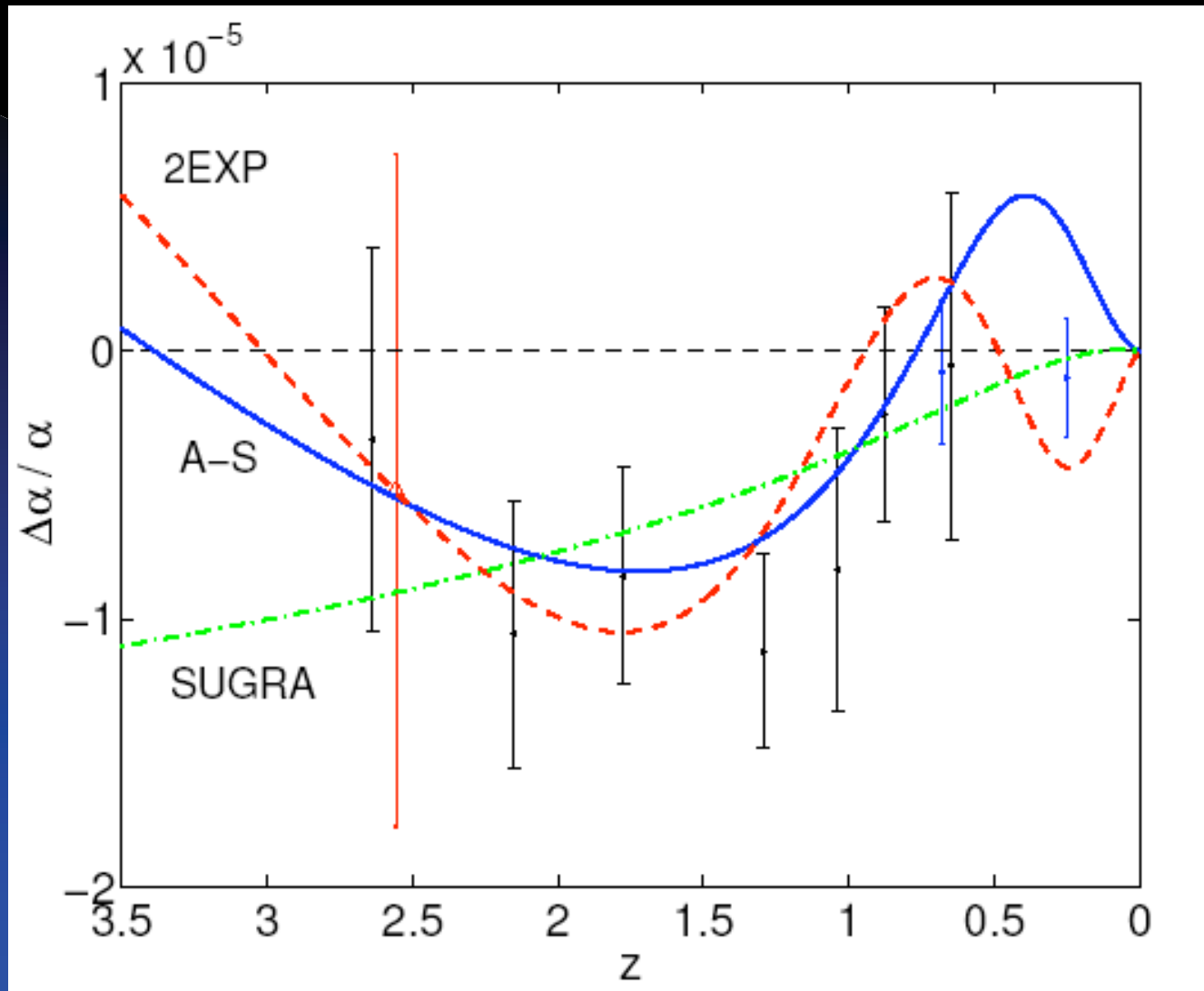
Eff fine structure const depends on value of field

$$\alpha(\phi) = \frac{e_0^2}{4\pi B_F(\phi)}$$
$$\frac{\Delta\alpha}{\alpha} = \zeta_F \phi + \frac{1}{2} (\xi_F - 2\zeta_F^2) \phi^2$$

Claim from analysing quasar absorption spectra:

$$\frac{\Delta\alpha}{\alpha} (z = 0.5 - 3.5) \approx 10^{-5}$$

Webb et al



EC, Nunes &  
Pospelov;  
Bento,  
Bertolami &  
Santos

A way of constraining the eqn of state?

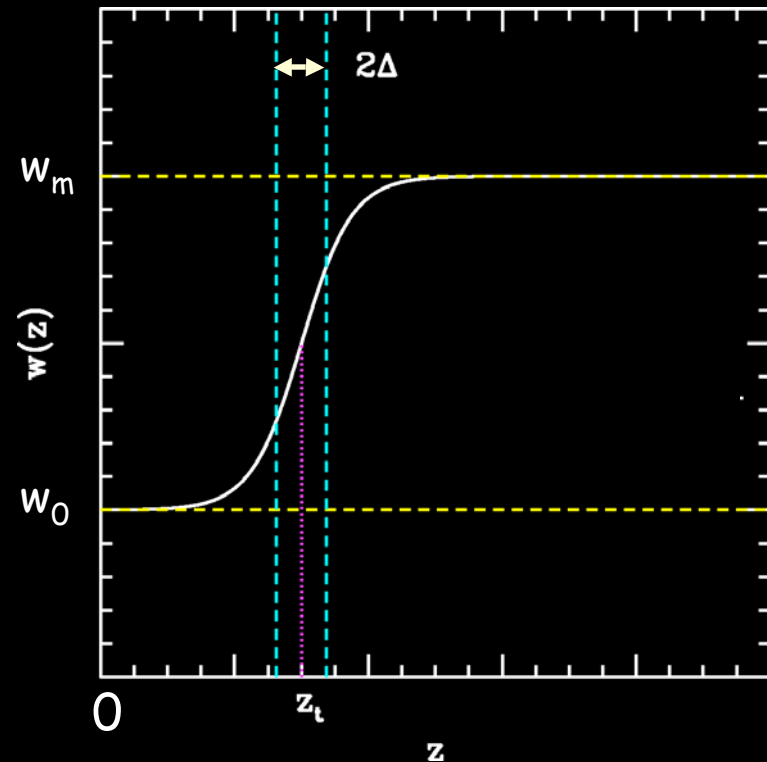
# Modelling quintessence Corasaniti & EC

Impose an equation of state  $w(z)$  which captures the essential features of quintessence.

*typical expectations:*

- recent acceleration  
→  $w_0 < -1/3$
- avoid fine tuning the initial energy density  
→  $w_m > -1/3$
- there is a **transition** at a given redshift  $z_+$  with a given width  $\Delta$ .
- $\Lambda$  corresponds to  $w_0 = -1$  and either  $w_m = -1$  or  $z_+ \gg 1$ .

9/24/04



## Strategy:

- compute predictions for many models with different parameters (ie  $H_0$ ,  $w_0$ ,  $w_m$ ,  $n_s$ ,  $t$  and the normalisation)
- compare with data sets (we use WMAP + SN-Ia)
- derive constraints on parameters (Markov-Chain Monte Carlo code with modified cmbfast)
- draw conclusions about the physical nature of the models.

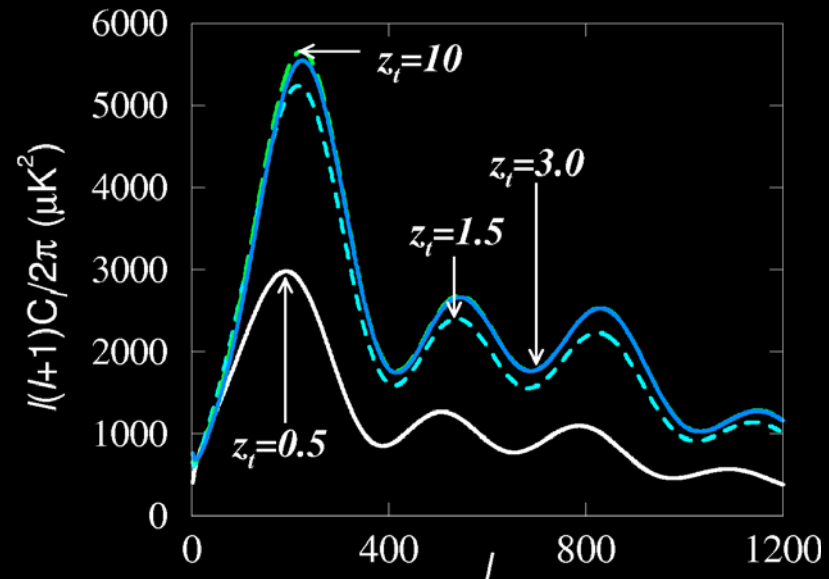
Kunz et al astro-ph/0307346; Corasaniti et al astro-ph/0406608

# $w(z)$ impact on the CMB through ISW

$$\left[ \frac{\delta T(e)}{T} \right]_{SW} = \frac{1}{3} \Phi(ex_{ls}) + 2 \int_{\tau_{ls}}^{\tau_0} \frac{\partial \Phi(ex, \tau)}{\partial \tau} d\tau$$

## rapid transition :

- late onset of expansion changes ISW effect which acts at large  $l$
- peak lower after COBE normalisation



- Cosmic variance makes the effect hard to observe, especially for models with slowly varying equation of state.
- A data set which connects large and small angular scales is crucial for a correct normalisation  $\Rightarrow$  WMAP.



# cosmological parameters

- limits slightly wider, but no clear difference
- **NO** new degeneracies!

$$\Omega_m = 0.29 \pm 0.04$$

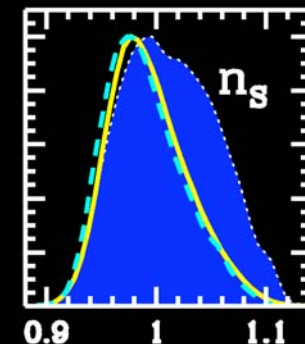
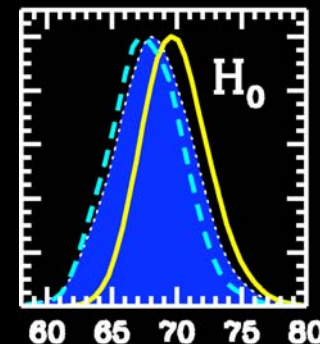
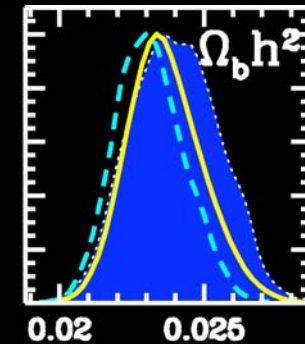
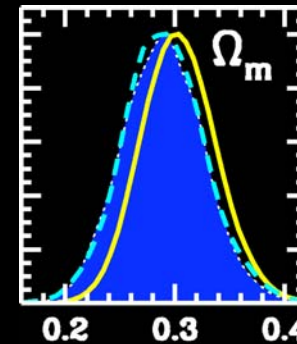
$$\Omega_b h^2 = 0.0240 \pm 0.0015$$

$$H_0 = 68 \pm 3$$

$$n_s = 1.01 \pm 0.04$$

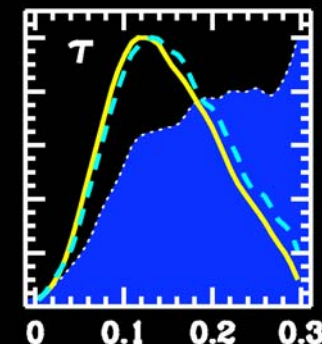
$$\tau = 0.19 \pm 0.07$$

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quintessence  
with  $\Omega_b$  prior

pure  $\Lambda$ CDM



# dark energy parameters

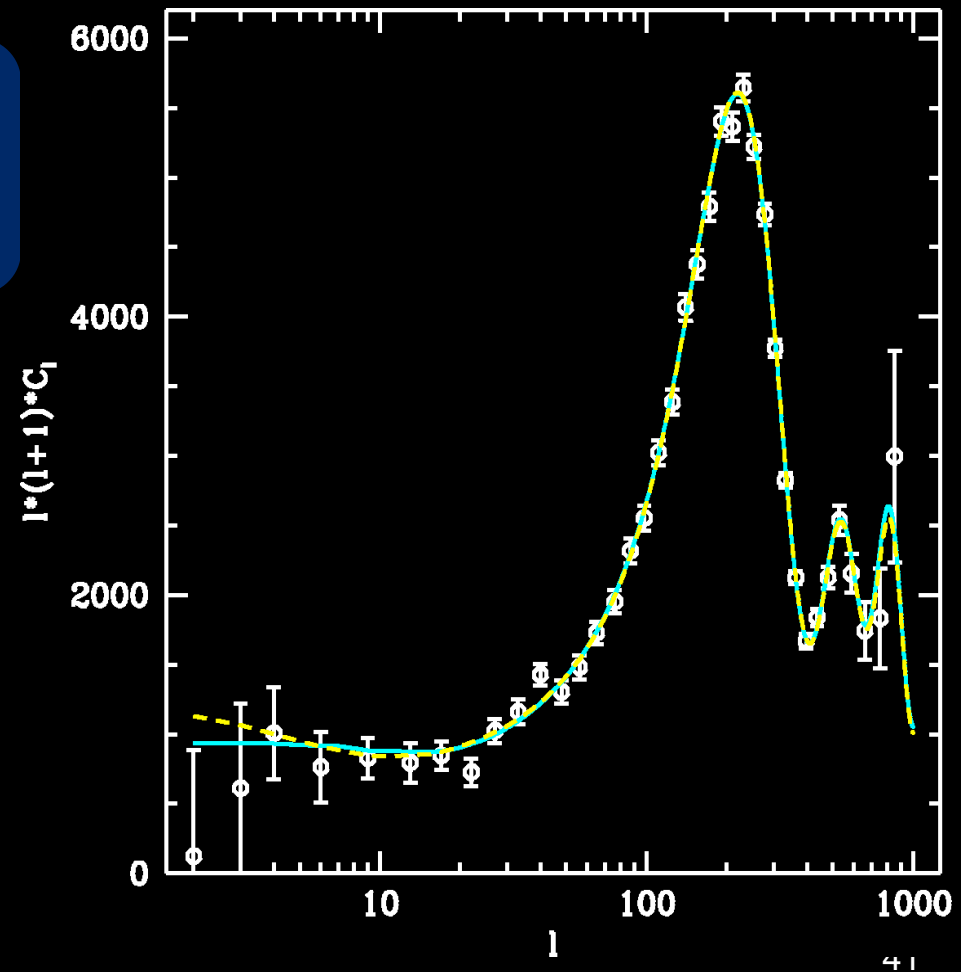
$w_0 < -0.80$  at 95% CL  
 $z_t > 0.6$  (fast transitions)

best-fit quintessence model:

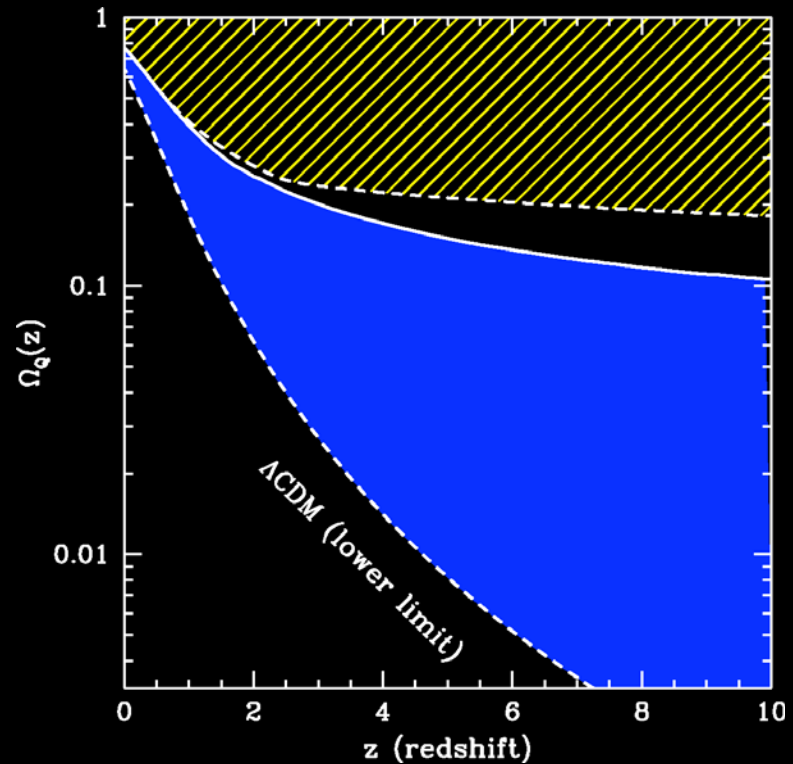
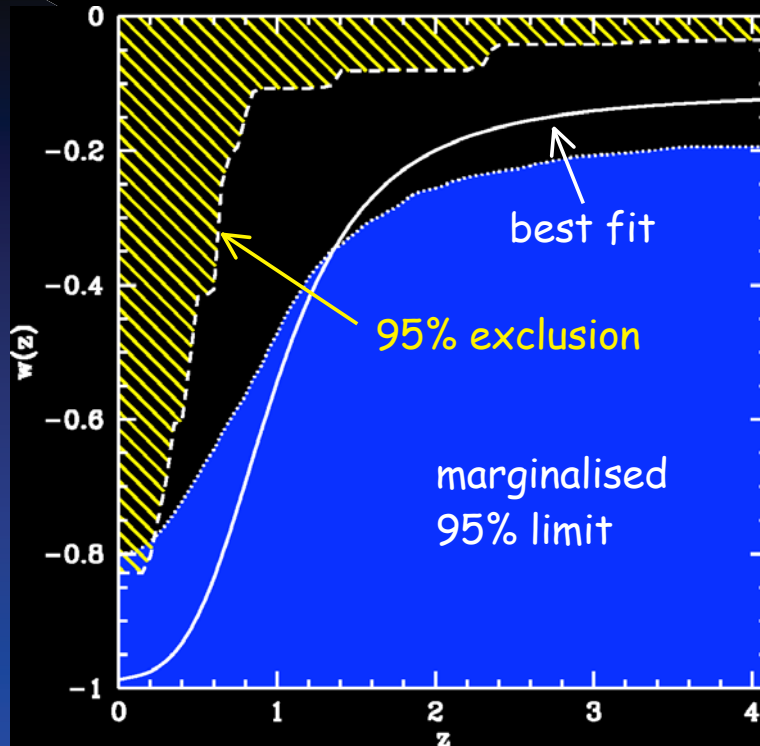
- $w_0 = -1$
- $w_m = -0.13$
- $a_t = 0.5$  ( $z_t = 1$ )
- effective  $\chi^2 = 1603$

best  $\Lambda$ CDM :  $\chi^2 = 1606$

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# time behaviour of the DE

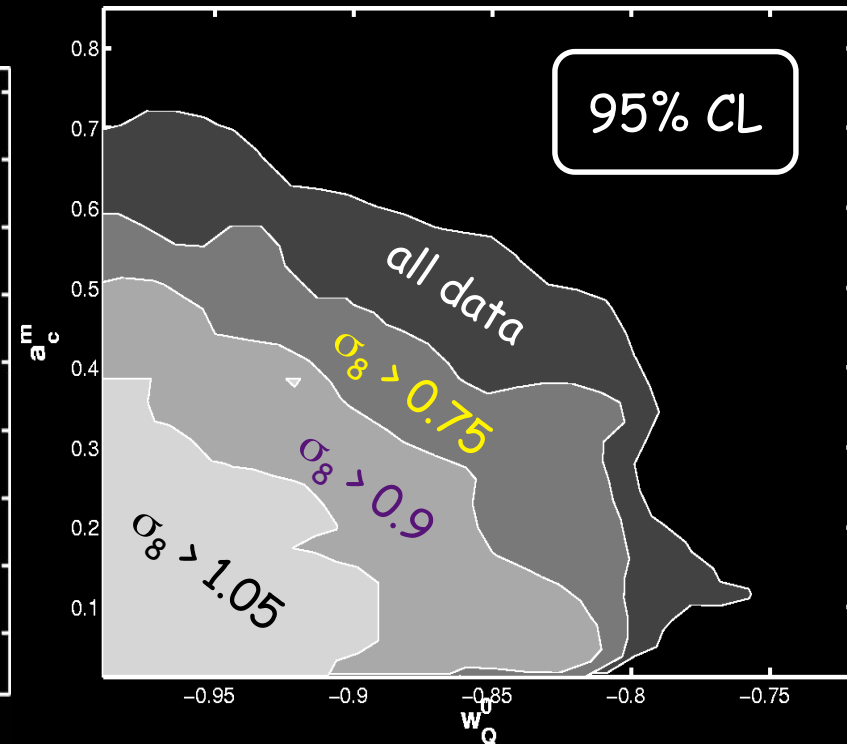
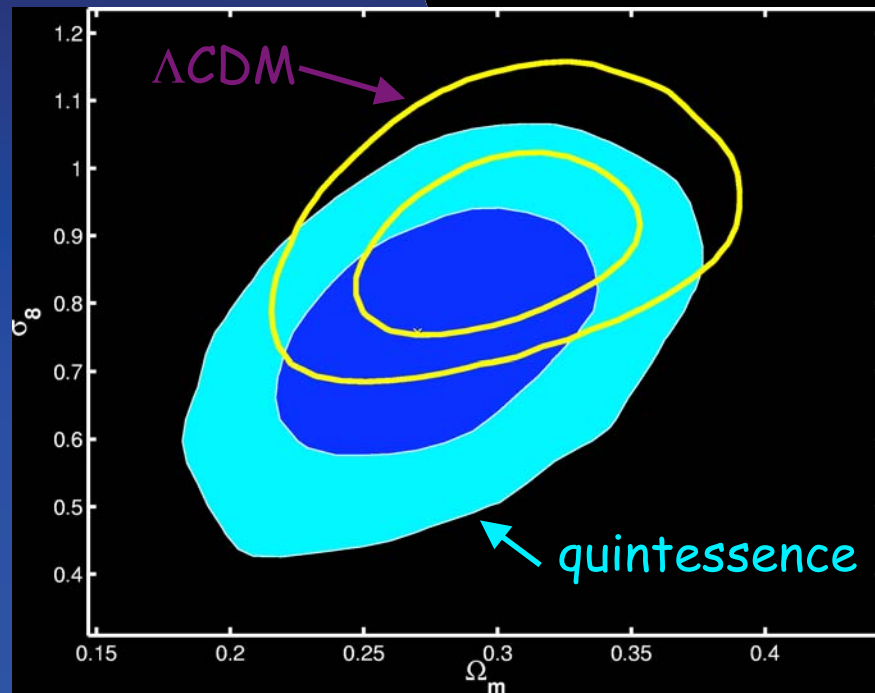


- really strong constraints on  $w$  only for  $z < 0.2$
- $w < 0$  for  $z < 5$  (matter e.o.s. / tracking) might spell trouble for exponential potentials...
- dark energy becomes quickly subdominant in the past
- Everything consistent with Lambda !

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# the effect on clustering

- the ISW changes the overall normalisation
  - this in turn changes the normalisation of the matter  $P(k)$
  - we can detect this if we know the amplitude of  $P(k)$  or  $\sigma_8$
  - **BUT:** we can only observe *galaxies*
- we don't know  $\sigma_8$  very well!



# Summary

- Observations transforming field, especially CMBR and LSS. Constraining the cosmological parameters. Theory can't really keep up.
- Why is the universe inflating today – can particle physics provide an answer? Can we relate early universe inflation and inflation today?
- Brane inspired cosmology – much going on in this area -- still early days.
- New Gravitational Physics -- possibly linked to Braneworlds.
- Is  $w_0 = -1$ ? Is it constant? Vital we find out.
- Finally -- could we all be wrong and we do not need a lambda term? -- [Break in primordial power spec -- Blanchard et al ]