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EFFECTIVE FIELD THEORY AND FUNDAMENTAL INTERACTIONS

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Effective field theory and fundamental interactions

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Abstract. There are many gravitational applications of **effective approach** to quantum field theory. We consider **cosmological constant** problem, **inflation** driven by vacuum quantum effects and imposing constraints on a **non-metric gravity** in effective framework.

The effective approach implies low-energy phenomena being independent on (sometimes unknown) fundamental physics.

One example is low-energy QCD, where the Chiral Perturbation Theory helps to achieve results fitting both lattice simulations and experiment, in a situation when usual perturbative methods are not applicable.

Sometimes, in the effective approach, one can reduce the requirements to a theory, e.g. extract relevant low-energy information even from the non-renormalizable theories.

Important aspects of effective approach are renormalization group and decoupling.

At classical level decoupling means that a heavy field doesn't propagate at low energies

$$\frac{1}{k^2+M^2} \approx \frac{1}{M^2} + \mathcal{O}\left(\frac{k^2}{M^4}\right), \qquad k^2 \ll M^2 .$$

Decoupling theorem explains similar phenomenon at quantum level.

QED example (flat space):

The 1-loop vacuum polarization is

$$-\frac{e^2\,\theta_{\mu\nu}}{2\pi^2}\int_0^1 dx\,x(1-x)\,\ln\frac{m_e^2+p^2x(1-x)}{4\pi\mu^2},$$

where $\theta_{\mu\nu}=(p_\mu p_\nu-p^2g_{\mu\nu})$, μ is the parameter of dimensional regularization.

$\beta^{\overline{\mathrm{MS}}}$ is $\frac{e}{2}\mu\frac{d}{d\mu}$ acting on the formfactor of $\theta_{\mu\nu}$

$$\beta_e^{\overline{\mathrm{MS}}} = \frac{e^3}{12\pi^2}.$$

β_e in the physical **mass-dependent scheme:** subtract at $p^2=M^2$ and take $\frac{e}{2}M\frac{d}{dM}$.

The UV limit ($M\gg m_e$): $\quad \beta_e = \beta_e^{\overline{\mathrm{MS}}}$.

The IR limit ($M\ll m_e$):

$$\beta_e = \frac{e^3}{60\pi^2}\cdot\frac{M^2}{m_e^2} + \mathcal{O}\left(\frac{M^4}{m^4}\right).$$

Appelquist & Carazzone, (1975)

Compared to $\beta_e^{UV} = \beta_e^{\overline{\mathrm{MS}}}$, in the IR there is a **suppression (decoupling)** $\sim p^2/m_e^2$.

Our first interest is investigating decoupling law for quantum matter on curved background.

Why this is **interesting and important**?

Sh., Solà, España-Bonet, Ruiz-Lapuente, Ph.Let.*B574*(2003), also NPhB(PS,2004)*71*; Babic,Guberina,Horvat,Stefancic, PRD65(2002).

Assume the AC-like quadratic decoupling holds for a cosmological constant. Associate the scale $\mu \equiv H$ (Hubble parameter).

Remember that high-energy $\beta_\Lambda \sim m^4$, m being mass of a quantum matter field.

Then the AC suppressed, low-energy

$$\beta_\Lambda = \sum_i c_i \frac{H^2}{m_i^2} \times m_i^4 = \frac{\sigma}{(4\pi)^2} M^2 H^2 \,,$$

where M is unknown **mass parameter** and $\sigma = \pm 1$ depending on whether fermions or bosons dominate **at the highest energies.**

$$M^2 = M_P^2 \quad \Longrightarrow \quad |\beta_\Lambda| \sim 10^{-47}\, GeV^4 \,,$$

close to the SN & CMB data on DE.

Cosmological model with running CC.

Sh., Solà, España-Bonet, Ruiz-Lapuente,
Phys.Let. *B574* (2003); JCAP *02* (2004)

For simplicity $k = 0$ case.

Along with RG, there is Friedmann equation

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\,(\rho + \Lambda)\ ,$$

$\rho = \rho_R + \rho_M$, and the conservation law

$$\frac{d\Lambda}{dt} + \frac{d\rho}{dt} + 3H\,(\rho + p) = 0\,.$$

$\forall \quad p = \alpha\rho$ the solution is analytical.
In terms of the red-shift $z = a_0/a - 1$

$$\rho(z;\nu) = \rho_0\,(1+z)^r \qquad (1)$$

and

$$\Lambda(z;\nu) = \Lambda_0 + \frac{\nu}{1-\nu}\,[\,\rho(z;\nu) - \rho_0\,]\ , \qquad (2)$$

where ρ_0, Λ_0 are present day values, and

$\nu = \dfrac{\sigma\, M^2}{12\pi M_P^2}$ and $r = 3\,(1-\nu)\,(\alpha+1)\,.$

At $\nu \to 0$ we recover the standard result for $\Lambda = const.$

Test: **nucleosynthesis epoch** $\rho_{\rm rad} \gg \rho_{\rm mat}$.

$$\rho_{rad}(T) = \frac{\pi^2 g_*}{30} T^4 \left(\frac{T_0}{T}\right)^{4\nu}$$

$T_0 \simeq 2.75\,K$ is present CMB temperature.

Clearly, ν gets restricted, because for $\nu \geq 1$ the $\rho_{rad}(T)$ would be constant.
In order not to be ruled out by the nucleosynthesis:

$$|\,\Lambda_R\,/\,\rho_R\,| \simeq |\,\nu\,/\,(1-\nu)\,| \simeq |\nu| \ll 1\,.$$

A nontrivial range is, e.g., $0 < |\nu| \leq 0.1$.
Both signs of ν are allowed.

The "canonical" choice $M^2 = M_P^2$ gives

$$|\nu| = \frac{1}{12\,\pi} \simeq 2.6 \times 10^{-2}\,.$$

The nucleosynthesis constraint is consistent with the effective approach.

Zero CC in the remote future $\sim \nu \approx 0.7$

Marginal value for the nucleosynthesis!

Whether the permitted values $\nu \ll 1$ may lead to **observable consequences?**

The remarkable answer is: **yes.**

Consider "recent" Universe $0 < z \leq 2$.

We can evaluate cosmological parameters which can be improved by the future observations, e.g. the SNAP project.

Example: Relative deviation

$$\delta_\Lambda(z;\nu) = \frac{\Lambda(z;\nu) - \Lambda_0}{\Lambda_0}$$

The $\exists$ SN data correspond to $z = z_0$.
In $\mathcal{O}(\nu)$ approximation

$$\delta_\Lambda(z;\nu) = \frac{\nu\,\Omega_M^0}{\Omega_\Lambda^0}\left[(1+z)^3 - (1+z_0)^3\right].$$

Taking $z_0 \simeq 0.5$ with $\Omega_M^0 = 0.3$ and $\Omega_\Lambda^0 = 0.7$, and $\nu = \nu_0$, we find

$$\delta_\Lambda(z = 1.5;\nu_0) \;\approx\; 14\%\,,$$

that would be a measurable effect.

Observations:

1) ν is unique arbitrary parameter of this model.
Cubic z-dependence should manifest itself in the future **SNAP** experiments,
where the range $z \leq 2$ will be tested.

2) The AC quadratic decoupling for the CC **is compatible with covariance**.

Behind the RG there is a well-defined object called Effective Action

$$e^{i\Gamma[g_{\mu\nu}]} = \int \mathcal{D}\Phi \, e^{iS[\Phi\,,\,g_{\mu\nu}]}$$

$\Gamma[g_{\mu\nu}]$ is complicated non-local functional.
But **due to covariance it is even in** $\partial_\lambda g_{\mu\nu}$.

Cosmology: $\mathcal{O}(H)$ quantum corrections (e.g. from QCD) are **completely ruled out.**
Minimal possible quantum effect on CC is

$$M^2 H^2 \implies$$

only Planck scale physics can be relevant!

First works on **Decoupling in Gravity**
Ed.Gorbar,I.Sh. JHEP 02,06(2003); 02(2004).

Consider massive scalar field:

$$S_s = \frac{1}{2}\int d^4x\, g^{1/2}\,\left\{(\nabla\varphi)^2 + m^2\varphi^2 + \xi R\varphi^2\right\}\,.$$

Euclidean Effective Action is

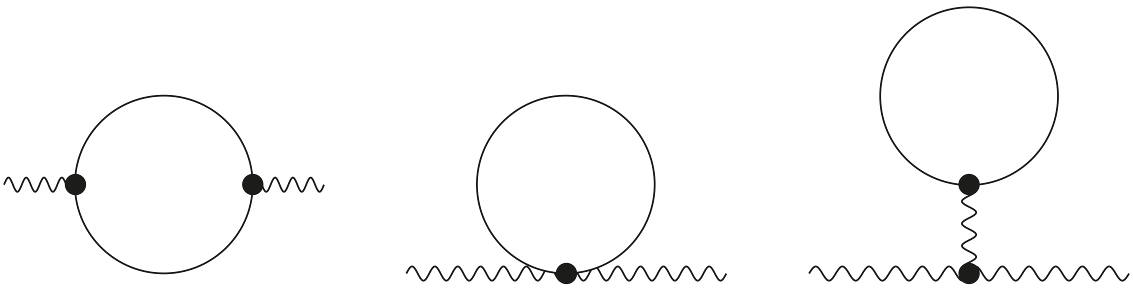
$$\Gamma[g_{\mu\nu}] = -\frac{1}{2}\,\mathrm{Tr}\,\ln\left(-\nabla^2 + m^2 + \xi R\right).$$

Weak point: **No** covariant version of a mass-dependent renormalization scheme.

We can perform calculations only for the **linearized** gravity on the flat background

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\,.$$

Corrections to the graviton propagator:



The polarization operator must be compared to the tensor structure of the Lagrangians

$$L_{HE} = -\frac{1}{16\pi G}(R + 2\Lambda) \qquad \text{and}$$

$$L_{HD} = a_1 C^2 + a_2 E + a_3 \Box R + a_4 R^2 \,.$$

For the formfactors we find, e.g.

$$k_\Lambda = \frac{3m^4}{8\,(4\pi)^2}\,, \qquad k_R = \frac{m^2}{2\,(4\pi)^2}\,\tilde{\xi}\,,$$

$$k_1(a) = \frac{8A}{15\,a^4} + \frac{2}{45\,a^2} + \frac{1}{150}\,,$$

where $\tilde{\xi} = \xi - 1/6$,

$$A = 1 + \frac{1}{a}\ln\left|\frac{2-a}{2+a}\right|\,, \qquad a^2 = \frac{4\Box}{4m^2 - \Box}.$$

Result confirmed using covariant $\mathcal{O}(R^2)$
heat kernel solution
*Avramidi, Sov.J.Nucl.Phys.*49 (1989);
*Barvinsky, Vilkovisky, Nucl.Ph.*B282 (1990),

(properly generalized for a massive case).

Similar expressions were obtained for massive **fermions and vectors.** (JHEP 06-2003).

How do we define RG in curved space?

Remember we are dealing with the theory of $h_{\mu\nu}$ **in flat space.**

Then RG scaling is the momentum scaling

$$p^2 \to e^{2t} p^2 .$$

In the **mass-dependent scheme**

$$\beta_\lambda = -2p^2 \frac{\partial \lambda}{\partial p^2} ,$$

where we identify $p^2 = -\Box$. For Weyl term:

$$\beta_1 = -\frac{1}{(4\pi)^2} \left(\frac{1}{18a^2} - \frac{1}{180} - \frac{a^2-4}{6a^4} A \right) .$$

Then

$$\beta_1^{UV} = -\frac{1}{(4\pi)^2} \frac{1}{120} + \mathcal{O}\left(\frac{m^2}{p^2}\right) = \beta_1^{\overline{MS}} + \mathcal{O}\left(\frac{m^2}{p^2}\right),$$

$$\beta_1^{IR} = -\frac{1}{1680\,(4\pi)^2} \cdot \frac{p^2}{m^2} + \mathcal{O}\left(\frac{p^4}{m^4}\right) ,$$

Appelquist & Carazzone Th. for **gravity!**

An expansion $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ works well for higher derivative terms, but not for Λ and G.

Why did we obtain $\beta_\Lambda = \beta_{1/G} \equiv 0$?

Running $\sim$ $f(\Box) = \ln(\Box/\mu^2)$ formfactor.

$$-\frac{e^2}{4} F_{\mu\nu} F^{\mu\nu} + \frac{e^4}{3(4\pi)^2} F_{\mu\nu} \ln\left(-\frac{\Box}{\mu^2}\right) F^{\mu\nu} .$$

Similarly in gravity it is possible to insert

$$C_{\mu\nu\alpha\beta} \, f(\Box) \, C^{\mu\nu\alpha\beta} \quad \text{or} \quad R \, f(\Box) \, R \, .$$

But, **no insertion is possible** for Λ and $1/G$, since $\Box\Lambda = 0$ and $\Box R$ is a total derivative.

Is it true that β_Λ and $\beta_{1/G}$ **equal zero?**

Definitely not! The unknown "correct" β_Λ and $\beta_{1/G}$ should tend to the corresponding $\beta^{\overline{MS}}$-functions in the UV limit.

Perhaps **calculations on a flat background are not appropriate for the RG equations for** Λ and $1/G$.

Consider the vacuum sector of a theory with the Spontaneous Symmetry Breaking **(SSB).**

In the matter fields sector SSB theory provides an exception from the AC theorem.

The theory of charged scalar φ coupled to the Abelian gauge vector A_μ:

$$S = \int d^4x\sqrt{g}\,\Big\{|(\partial_\mu + ieA_\mu)\varphi|^2 + \mu_0^2\,\varphi^*\varphi$$

$$-\lambda(\varphi^*\varphi)^2 + \xi\,R\,\varphi^*\varphi - 1/4\,F_{\mu\nu}F^{\mu\nu}\Big\}\,.$$

At the classical level the VEV for φ is v:

$$-\Box v + \mu_0^2 v + \xi R v - \;2\lambda v^3 = 0.$$

For $\xi = 0$ the vacuum solution is constant

$$v_0^2 \,=\, \mu_0^2/2\,\lambda\,.$$

For $\xi \neq 0$ the derivatives can not be ignored.

For the general case of a non-constant scalar curvature the solution can be presented in the form of the power series in curvature (or equivalently in ξ)

$$v(x) \,=\, v_0 \,+\, v_1(x) \,+\, v_2(x) \,+\, \dots\,.$$

For the first order term $v_1(x)$

$$-\Box v_1 + \mu^2 v_1 + \xi R\, v_0 - 6\lambda v_0^2\, v_1 = 0\,,$$

and the solution is

$$v_1 = \frac{\xi v_0}{\Box + 4\lambda v_0^2} R\,.$$

In a similar way, we find

$$v_2 = \frac{\xi^2\, v_0}{\Box + 4\lambda v_0^2}\, R\, \frac{1}{\Box + 4\lambda v_0^2}\, R$$

$$- \frac{6\,\lambda\,\xi^2\, v_0^3}{\Box + 4\lambda v_0^2} \left(\frac{1}{\Box + 4\,\lambda v_0^2}\, R \right)^2\,, \qquad \text{etc.}$$

One can continue the expansion of v to any order. The induced vacuum action is

$$S_{ind} = \int d^4x\sqrt{g}\{\,(\nabla v)^2 + (\mu_0^2 + \xi R)\, v^2 - \lambda v^4\}$$

or in details $\quad S_{ind} =$

$$= \int d^4x\sqrt{g}\left\{\lambda v_0^4 + \xi R v_0^2 + R\frac{\xi^2 v_0^2}{\Box + 4\lambda v_0^2} R + \ldots\right\}.$$

The first terms are induced CC and Einstein-Hilbert action. One has to some them with the corresponding vacuum terms.

Other (∞ many) terms are quasi-local in IR.

Quantum effects in the SSB theory.

Detailed analysis: the SSB theory

is renormalizable,

but to achieve this one has to include **non-local terms** (typical for induced action) into the **classical action of vacuum**.

At quantum level we meet renormalization of these non-local terms.

Renormalization group and decoupling in a mass-dependent scheme are perfectly seen for all higher derivative structures (including non-local),

but not for cosmological, Einstein-Hilbert terms and their non-local generalizations.

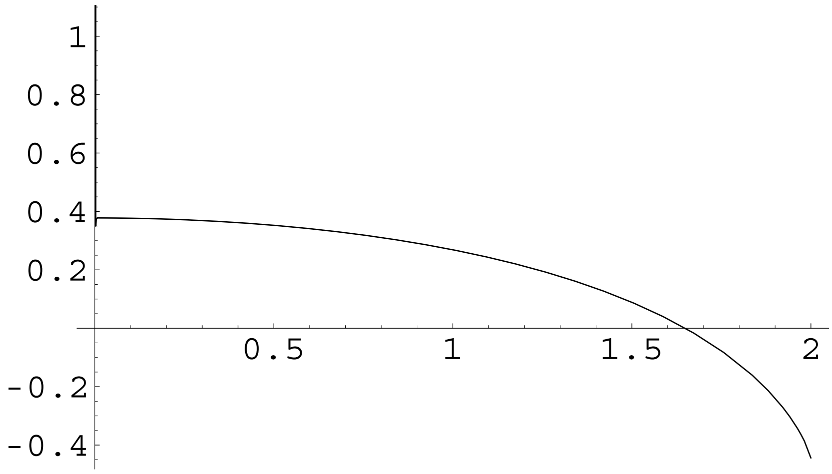
For coefficient of the induced $\int R^2$ - term

$$\beta_3^{UV} = -\frac{1}{180(4\pi)^2} + \mathcal{O}\left(\frac{m^2}{p^2}\right),$$

IR limit shows AC-like decoupling

$$\beta_3^{IR} = -\frac{1}{1260\,(4\pi)^2}\,\frac{p^2}{m^2} + \mathcal{O}\left(\frac{p^4}{m^4}\right).$$

Broken **SUSY:** β_3 **changes sign** between **UV** and **IR**, because sparticles decouple.



Key fact for the **anomaly-induced inflation** (modified Starobinsky model), providing interface between stable inflation at UV and the FRW-like regime at IR.

Consider Conformal and Free (or AF) Fields.

N_0 *scalars*, $N_{1/2}$ *fermions*, N_1 *vectors*

Notice: Vacuum quantum effects come from **virtual** particles. $N_{0,\,1/2,\,1}$ have **no relation to the real matter** in the Universe.

Classical vacuum action of conformal theory

$$S_{vac} = \int d^4x\sqrt{-g}\,\left\{l_1 C^2 + l_2 E + l_3 \Box R\right\}\,.$$

$C = C_{\mu\nu\alpha\beta}$ is Weyl tensor,

$E = R^2_{\mu\nu\alpha\beta} - 4R^2_{\mu\nu} + R^2$ is Gauss-Bonnet term.

S_{vac} does not affect cosmological solution.

Quantum correction: **Conformal Anomaly**

$$T = < T^\mu_\mu > = -\,(wC^2 + bE + c\,\Box R)\,,$$

$w,\, b,\, c$ are β-functions for $l_1,\, l_2,\, l_3$

$$\begin{pmatrix} w \\ -\,b \\ c \end{pmatrix} = \frac{1}{360(4\pi)^2}\begin{pmatrix} 3N_0 + 18N_{1/2} + 36N_1 \\ N_0 + 11N_{1/2} + 62N_1 \\ 2N_0 + 12N_{1/2} - 36N_1 \end{pmatrix}$$

Remark: $b < 0$ and **alternating sign** for c.

Recent investigation of $\Box R$-type ambiguity:
Asorey, Gorbar & Sh. Clas.Q.Gr. *21* (2003).

Anomaly-Induced Effective Action (EA)

$$-\frac{2}{\sqrt{-g}}\,g_{\mu\nu}\,\frac{\delta\bar{\Gamma}_{ind}}{\delta g_{\mu\nu}} = T\,.$$

(Reigert, Fradkin & Tseytlin, 84)

The EA **is exact** solution for FRW metric

$$\bar{\Gamma}_{ind} = S_c[\bar{g}_{\mu\nu}] + \int d^4x\sqrt{\bar{g}}\,\{w\sigma\bar{C}^2 + b\sigma(\bar{E} - \frac{2}{3}\Box\bar{R})$$

$$+2b\sigma\bar{\Delta}\sigma\,\} - \frac{3c-2b}{36}\int d^4x\sqrt{g}\,R^2,$$

where $g_{\mu\nu} = a^2(x)\bar{g}_{\mu\nu}, \quad a^2(x) = e^{2\sigma(x)}$,
$S_c[g_{\mu\nu}]$ an arbitrary conformal functional,
$\Delta = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\Box + \frac{1}{3}(\nabla^\mu R)\nabla_\mu.$

Local covariant solution via auxiliary fields
(A.Jacksenaev & I.Sh., Phys.Lett.B, 1994)

$$\Gamma_{ind} = S_c - \frac{3c-2b}{36}\int_x R^2 + \frac{1}{2}\int_x \{\varphi\Delta\varphi - \psi\Delta\psi$$

$$+\varphi\left[\sqrt{-b}(E - \frac{2}{3}\Box R) - \frac{w}{\sqrt{-b}}C^2\right] + \frac{w}{\sqrt{-b}}\psi C^2\}\,.$$

The most useful form of the vacuum EA for the conformal matter fields.

Cosmological Model based on the action

$$S_{total} = -\frac{M_P^2}{16\pi}\int d^4x\sqrt{-g}\,(R+2\Lambda)$$

$$+S_{matter} + S_{vac} + \bar{\Gamma}_{ind}\,.$$

Equation of motion for $a(t)$, $\quad dt = a\,(\eta)\,d\eta$

$$\frac{\ddddot{a}}{a} + \frac{3\dot{a}\dddot{a}}{a^2} + \frac{\ddot{a}^2}{a^2} - \left(5+\frac{4b}{c}\right)\frac{\ddot{a}\dot{a}^2}{a^3} - 2k\left(1+\frac{2b}{c}\right)\frac{\ddot{a}}{a^3}$$

$$-\frac{M_P^2}{8\pi c}\left(\frac{\ddot{a}}{a} + \frac{\ddot{a}^2}{a^2} + \frac{k}{a^2} - \frac{2\Lambda}{3}\right) = 0\,,$$

$k = 0, \pm 1.$

Particular solutions (Starobinsky, PhLB-1980)

$$a(t) = a_0\begin{pmatrix} \exp[Ht]\,, & k=0 \\ \cosh[Ht]\,, & k=1 \\ \sinh[Ht]\,, & k=-1 \end{pmatrix},$$

where Hubble parameter $H = \dot{a}/a$ is

$$H = \frac{M_P}{\sqrt{-32\pi b}}\left(1 \pm \sqrt{1 + \frac{64\pi b}{3}\,\frac{\Lambda}{M_P^2}}\right)^{1/2}.$$

For $0 < \Lambda \ll M_P^2$ there are two solutions:

$$H \approx \sqrt{\Lambda/3}\,; \qquad\qquad (IR)$$

$$H \approx \sqrt{-\frac{M_P^2}{16\pi b} - \frac{\Lambda}{3}} \approx \frac{M_P}{\sqrt{-16\pi b}}\,. \qquad\qquad (UV)$$

Perturbations of the conformal factor

$$\sigma(t) \to \sigma(t) + y(t).$$

The criterion for a **stable** (UV) **inflation**

$$c > 0 \quad \Longleftrightarrow \quad N_1 < \frac{1}{3} N_{1/2} + \frac{1}{18} N_0\,,$$

in agreement with Starobinsky (1980).

The original Starobinsky model
is **based on the unstable case**
and involves **heavy fine-tunings.**

Our purpose is to avoid fine-tunings at all.

Suppose at **UV** $(H \gg M_F)$ there is **SUSY**, e.g. **MSSM** with a particle content

$$N_1 = 12\,, \qquad N_{1/2} = 32\,, \qquad N_0 = 104\,.$$

This provides **stable inflation**, because $c > 0$

$$N_1 < \frac{1}{3} N_{1/2} + \frac{1}{18} N_0\,.$$

Similar for any realistic SUSY model.
Inflation is **independent on initial data**.

Fine!

But why should inflation end?

Already for MSM $(N_{1,\,1/2,\,0} = 12, 24, 4)$
inflation is unstable, $c < 0$.

Natural interpretation
I.Sh. Int.J.Mod.Ph. *11D* (2002)

All **sparticles** are heavy $\Rightarrow$ **decouple**, when H becomes smaller than their masses.

According to calculations (JHEP,2003) the transition $c > 0 \Longrightarrow c < 0$ is smooth, giving a hope for a graceful exit.

Simple test of the model. Late Universe, $k = 0$, $H_0 = \sqrt{\Lambda/3}$. Only photon is active

$$N_0 = 0\,, \qquad N_{1/2} = 0\,, \qquad N_1 = 1\,.$$

Graviton typical energy is $H_0 \approx 10^{-42}\,GeV$,
$\Longrightarrow$ all massive particles (even neutrino) $m_\nu \geq 10^{-12}\,GeV$ **decouple** from gravity.

$c < 0 \quad \Longrightarrow \quad$ **today inflation is unstable.**

Stability for the small $H = H_0$ case:

$H \to H_0 + const \cdot e^{\lambda t} \quad \Longrightarrow$

$$\lambda^3 + 7H_0\lambda^2 + \left[\frac{(3c-b)4{H_0}^2}{c} - \frac{M_P^2}{8\pi c}\right]\lambda$$

$$- \frac{32\pi b {H_0}^3 + M_P^2 H_0}{2\pi c} = 0\,.$$

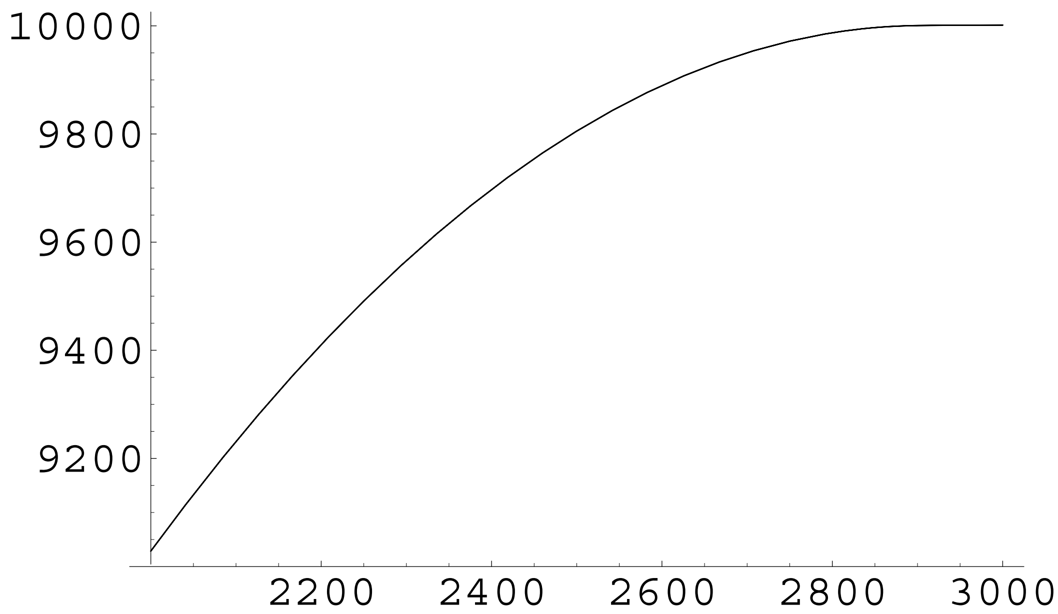
The solutions for λ are

$$\lambda_1 = -4H_0\,, \qquad \lambda_{2/3} = -\frac{3}{2}H_0 \pm \frac{M_P}{\sqrt{8\pi|c|}}\,i\,.$$

$\Lambda > 0$ **protects our world from quantum corrections!**

Anomaly-induced inflation slows down if taking masses of quantum fields into account.

Sh., Sola, Phys.Lett. *530B*(2002);
Pelinson,Sh.,Takakura, Nucl.Ph.*648B*(2003).



$$\log\,[a(t)] \approx H_0\,t - \frac{H_0^2}{4}\,\tilde{f}\,t^2\,, \qquad H_0 \propto M_P$$

The total amount of e-folds may be as large as 10^{32}, **but only** 65 **last ones**, where

$$H \propto M_*$$

(SUSY breaking scale) **are relevant.**

In the last 65 e-folds the production of gravitational waves is restricted

$$H(t) \ll 10^{-5} M_P.$$

Furthermore, once created, in this model **gravitational waves do not amplify.**

Fabris,Pelinson,Sh.,Nucl.Phys. *597B*(2001); Pelinson,Sh.,Takakura, Nucl.Phys. *648B* (2003); Fabris,Pelinson,Sh,Takakura,NPB(PS)*127*(2004).

All in all, modified Starobinsky model is a **promising candidate** to describe inflation in a natural way.

However, **small information is available about intermediate stage of inflation.**

In order to obtain this information one needs further development of QFT in curved space-time. This represents a strong motivation for the future work.

Retrictions on Space-Time Geometry from Quantum Field Theory

Consider how one can impose the restrictions on a Space-Time Geometry using effective approach to Quantum Field Theory.

A.Belyaev & I.Sh., Phys.Lett.*425B* (1998)
B.-Peixoto,Helayel-Neto,Sh.,*JHEP*02(2000).
I.Sh., *Phys.Rep. 357*(2002).

It is quite popular to consider gravity described by metric and torsion.

$$\tilde{\Gamma}^{\alpha}_{\beta\gamma} - \tilde{\Gamma}^{\alpha}_{\gamma\beta} = T^{\alpha}_{\beta\gamma}\,, \qquad \tilde{\nabla}_{\mu} g_{\alpha\beta} = 0\,.$$

One can see that **existence of torsion $T^{\alpha}_{\beta\gamma}$ as independent propagating field meets serious obstacles.**

It is useful dividing torsion into irreducible components

$$T_{\alpha\beta\mu} = \frac{1}{3}\left(T_{\beta} g_{\alpha\mu} - T_{\mu} g_{\alpha\beta}\right) - \frac{1}{6}\varepsilon_{\alpha\beta\mu\nu}\, S^{\nu} + q_{\alpha\beta\mu}$$

Interaction to Dirac fermion

$$\mathcal{L} = i\bar{\psi}\gamma^\mu(\partial_\mu + i\eta_1\gamma^5 S_\mu + i\eta_2 T_\mu)\psi + m\bar{\psi}\psi\,.$$

η_1, η_2 are nonminimal parameters. Minimal case: $\eta_1 = 1/8$, $\eta_2 = 0$.

For simplicity consider $g_{\mu\nu} = T_\mu = 0$. Meet just a fermion coupled to an axial vector S_μ

$$S = i\int d^4x\,\bar{\psi}\,\gamma^\mu\,\Big(\partial_\mu + i\eta_1\gamma^5\,S_\mu - im\Big)\,\psi\,.$$

Known CPT & Lorentz violating term!

The question is: whether we can construct a quantum theory for ψ, S_μ which would be:
1) Unitary;
2) Renormalizable as effective field theory.

First step. Quantizing ψ we meet two types of divergences:

$$S_{\mu\nu}\,S^{\mu\nu} \qquad \text{and} \qquad m^2 S_\mu\,S^\mu\,,$$

where $S_{\mu\nu} = \partial_\mu S_\nu - \partial_\nu S_\mu$.

Unitarity forbids simultaneous $S_\mu^\perp$ and $S_\mu^{||}$. The unique possibility for dynamical torsion

$$S_{tor} = \int d^4x\,\Big\{-\frac{1}{4}\,S_{\mu\nu}^2 + M^2 S_\mu^2\Big\}\,.$$

Second step.
Is the effective quantum theory of fermion coupled to dynamical torsion consistent?

Involved calculation yields

$$\Gamma^{(1)}_{div} = -\frac{\mu^{n-4}}{\varepsilon}\int d^n x\,\left\{ ... + \frac{8\,\eta^4\,m^2}{M^4}\,(\bar{\psi}\psi)^2 \right\}.$$

Then at the two-loop level we meet a longitudinal $(\partial_\mu S^\mu)^2$-type divergence.

This means there is a **conflict between unitarity and renormalizability** in the low-energy corner of the theory.

One possible solution is taking

$$\frac{\eta^4\,m^2}{M^4} \ll 1\,. \qquad (*)$$

This means that either $M \gg m$ for **all** fermions, or that η is extremely small.

In both cases there is no chance to observe propagating torsion.

E.g. the lower bound for M from LEP is $M/\eta \leq 3\,TeV$, that does not fit with $(*)$.

Conclusions

1. The effective approach to quantum field theory in curved space-time may tell us a lot about gravitational physics, especially in cosmology.

2. The most interesting problem is the evaluation of vacuum effective action for massive quantum fields.

Working in this direction one may prove or disprove possibility of z-dependent cosmological constant.

Also it is important for the anomaly-induced inflation.

3. Even at the present state of knowledge we can learn something about the possible form of quantum corrections.

4. Surprisingly, one can exclude some options for the space-time geometry using quantum field theory methods, especifically an effective approach.