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TOPOLOGICAL DEFECTS IN THEORIES WITH LORENTZ VIOLATION

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Topological Defects In Theories With Lorentz Violation.

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Plan of the talk

- *Solitons in N.C. theories.*
  - *Number.*
  - *Characteristics.*
- *Global defects in the J.C model.*
  - *The Jacobson-Corley model.*
  - *Asymptotic behavior.*
  - *Energy density.*
  - *Cosmological incidences.*

- Conclusions ans Questions.

References.

- R. Gopakumar, S. Minwalla, A. Strominger JHEP 0005:020,2000.
- S. Corley, T. Jacobson , Phys.Rev.D54:1568-1586,1996.
- M.Lubo, in progress.

Solitons in Non Commutative Theories.

The model.

$2+1$ space time.

Commutation relation

$$[x,y]=i\theta$$

$$E=\frac{1}{g^2}\int d^2z(\partial_z\phi\star\partial_{\bar{z}}\phi+\theta V(\phi))$$

$$(A\star B)(z,\bar{z})=\left[\exp\left(\frac{1}{2}(\partial_z\partial_{\bar{z}'}-\partial_{z'}\partial_{\bar{z}})\right)A(z,\bar{z})B(z',\bar{z}')\right]_{z'=z}$$

Configurations of finite energy.

Big θ: neglect the kinetic term.

$$\frac{\partial V}{\partial\phi}=m^2\phi+b_3\phi\star\phi+b_4\phi\star\phi\star\phi=0$$

Commutative case: only the configurations $\phi=\lambda_i$ where the λ_i are extrema of V.

Non commutativity: infinite number of solutions

$$\phi = \sum c_n \phi_n(r^2) \quad , \quad \phi_n(r^2) = 2(-1)e^{-r^2} L_n(r^2)$$

Laguerre polynomials, c_n in the λ_i.

$$E \sim \frac{2\pi\theta}{g^2} f(c_1, \cdots, c_n)$$

Size $\sim \sqrt{n}$, undergoes n oscillations.

- More than one solution.
- E is a growing function of θ.
- ϕ_n oscillatory.

Global Defects in the Jacobsion-Corley model.

The Jacobson-Corley Model.

Possible effects of quantum gravity: change in the dispersion relations for the other fields.

Jacobson-Corley dispersion relation(transplanckian effect for black hole, inflation: translation, rotations)

$$p_0^2 = \vec{p}^2 + \mu(\vec{p}^2)^2 \quad , \quad \mu > 0 \quad .$$

The domain wall.

$$\mathcal{L} = \frac{1}{2}\eta^{\rho\tau}\partial_\rho\phi\partial_\tau\phi - \frac{\mu}{2}(\Delta\phi)^2 - \frac{\lambda}{4}(\phi^2 - v^2)^2 \quad .$$

$$x = z/\sqrt{\mu} \quad , \quad \phi = v f(x)$$

$$f^{(4)}(x) - f^{(2)}(x) + \alpha f(x)(f^2(x) - 1) = 0 \quad \text{where} \quad \alpha = \lambda\mu v^2 \quad .$$

$$x \longrightarrow \infty \quad , \quad f(x) = 1 + g(x) \quad ,$$

$$
\begin{aligned}
g(x) &= \sum_{k=1}^{4} C_k \exp\left(\beta_k x\right) \quad \text{with} \\
\beta_1 &= -\beta_2 = \frac{1}{\sqrt{2}}\sqrt{1-\sqrt{1-8\alpha}} \quad , \\
\beta_3 &= -\beta_4 = \frac{1}{\sqrt{2}}\sqrt{1+\sqrt{1-8\alpha}} \quad . \quad .
\end{aligned}
$$

Small values of α.

$$
g(x) = C_1 \exp\left(-\sqrt{2\alpha}\, x\right) + C_2 \exp\left(-(1-\alpha)x\right) \quad .
$$

$$
\begin{aligned}
f(x) &\rightarrow -1 + C_1 \ \exp\left(\sqrt{2\alpha}\, x\right) \text{ as } x \rightarrow -\infty \text{ and} \\
f(x) &\rightarrow 1 - C_1 \ \exp\left(-\sqrt{2\alpha}\, x\right) \text{ as } x \rightarrow \infty \quad .
\end{aligned}
$$

$$
\begin{aligned}
f(x) &\rightarrow -1 + C_3 \ \exp\left((1-\alpha)x\right) \text{ as } x \rightarrow \infty \text{ and} \\
f(x) &\rightarrow 1 - C_3 \ \exp\left(-(1-\alpha)x\right) \text{ as } x \rightarrow -\infty .
\end{aligned}
$$

$$
\sigma = \int dz T_0^0 = \kappa(\alpha)\frac{v^2}{\sqrt{\mu}}
$$

$$\lambda \sim 1 \quad , \quad \alpha = \left(\frac{M_{GUT}}{M_{pl}}\right)^2 = 10^{-6} \quad : \quad \kappa(\alpha) \sim 100$$

Usual walls

$$\sigma = \sqrt{\lambda} v^3 \quad .$$

Time of the begining of domain wall domination much earlier in the Jacobson-Corley model.

$$t_d = \frac{1}{8\pi G \sigma} \quad ;$$

$$\sigma_{jc} \sim 10^3 \sigma_{usual} \quad .$$

Bigger values: $\alpha \geq 1/8$; no solution $\leftrightarrow \theta \sim \infty$

$$\begin{aligned} \alpha &= 1 \\ \beta_1 &= 0.813442 - 0.813135i \\ \beta_2 &= 0.978318 + 0.676097i \end{aligned}$$

$$C_1 \exp\left(-\beta_1 x\right) + C_2 \exp\left(-\beta_2 x\right) \quad \text{notreal.}$$

The cosmic string.

$$\Phi = v f(r/\sqrt{\mu}) \exp{(i\theta)} \quad .$$

$$f^{(4)}(x) \quad + \quad \frac{2}{x} f^{(3)}(x) - \left(1 + \frac{3}{x^2}\right) f^{(2)}(x) + \left(\frac{3}{x^3} - \frac{1}{x}\right) f^{'}(x)$$
$$+\alpha f^3(x) \quad + \quad \left(-\frac{3}{x^3} + \frac{1}{x^2} - \alpha\right) f(x) \quad .$$

$$x \sim 0 \quad : \quad f(x) \sim x^3 \quad .$$

$$x \to \infty \quad , \quad f(x) \sim 1 + C_2 \exp{(-\sqrt{2\alpha}\, x)} + C_4 \exp{(-(1-\alpha)x)}.$$

$$E = \int dx dy T_0^0 = 2\pi v^2 \kappa(\alpha) \quad .$$

milder dependence on the new scale since otherwise

$$E \sim v^2$$

The Monopole.

$$\Phi_1 = vf(r/\sqrt{\mu})\sin\theta\cos\phi \quad , \quad \Phi_2 = vf(r/\sqrt{\mu})\sin\theta\sin\phi \quad ,$$
$$\Phi_3 = vf(r/\sqrt{\mu})\cos\theta \quad .$$

initial and final behavior: same as for the string.

mass of the monopole

$$M = 4\pi v^2 \sqrt{\mu}\kappa(\alpha) \quad ; \quad .$$

smaller than for the usual case(local)

$$M = 4\pi \frac{M_X}{g^2} \quad .$$

The constraints are

$$\Omega_M \leq 10^{-18} \left(\frac{m_M}{GeV}\right) h_0^{-2} \quad .$$

Conclusions and Questions.

- Stability: Minimal Energy
- Restauration of symmetry at high temperature?
- Local defects, cut-off.