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CONFERENCE ON FUNDAMENTAL SYMMETRIES AND FUNDAMENTAL CONSTANTS

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RELATIVISTIC SYMMETRY AS A LONG DISTANCE EFFECT

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Relativistic Symmetry as a Long Distance Effect.

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- Concentrate on Yang-Mills x matter

- General Idea

- Underlying low energy quantum f. Theories is a lattice system with some (conceptually) simple physical structure on it (e.g. Ising)

- All parameters should be determined by dynamics. This is difficult because we have to generate many length scales which differ widely

- In this talk we shall make some assumptions about the basic couplings and hope that the inclusion of gravity in a complete form will justify these assumptions.

For each A introduce ψ_A and ψ_A^+ s.t.

$$\{\psi_A^+, \psi_B\} = \delta_{AB}, \qquad \{\psi_A, \psi_B\} = 0 = \{\psi_A^+, \psi_B^+\}$$

Then

$$Z = \mathrm{Tr}\left(e^{hF} \; e^{-\frac{1}{2}\sum_{A,B} \psi_A^+\psi_A \, J_{AB} \, \psi_B^+\psi_B}\right)$$

$$= \int_{\psi(\beta) = -e^h \psi(0)} (D\bar\psi(t) D\psi(t)) \exp\left[-\int_0^\beta dt \sum_A \bar\psi_A(t)\partial_t \psi_A(t) + \right.$$

$$\left. + \frac{1}{2\beta}\sum_{A,B} \bar\psi_A(t)\psi_A(t) J_{AB} \bar\psi_B(t)\psi_B(t)\right]$$

i) Choose $h = i\pi \Rightarrow \psi(\beta) = \psi(0)$

ii) Let $\beta \to 0 \Rightarrow$ Only $\partial_t \psi_A(t) = 0$ contribute

iii) Assume $A = (n, i)$ $\quad n \in F_4, \quad i = 1, \dots, N$

$J_{AB} = J(n-m)$ indep. of i.

The array of sites A, decompose into cells with N-points in each cell. The interaction is between cells, rather than sites.

- Starting Point

- $$Z = \int \prod_n d\bar{\Psi}(n) d\Psi(n)\, e^{-\frac{1}{2}\sum_{n,m} \bar{\Psi}(n)\Psi(n)\, J(n-m)\, \bar{\Psi}(m)\Psi(m)}$$

$$\psi(n) = \begin{pmatrix} \psi^1(n) \\ \vdots \\ \psi^N(n) \end{pmatrix} \qquad \bar{\psi}(n) = \left(\bar{\psi}_1(n) \; \cdots \; \bar{\psi}_N(n)\right)$$

$n \in F_4 \equiv$ a 4-dim'al Lattice

- Symmetries of Z

Local $GL(N, \mathbb{C})$

$$\psi'(n) = a(n)\,\psi(n)$$

$$\bar{\psi}'(n) = \bar{\Psi}(n)\, a(n)^{-1} \qquad a(n) \in GL(N, \mathbb{C})$$

$$\prod_n d\bar{\Psi}(n)\, d\Psi(n) = \text{inv.}$$

- Possible Ising type Origin for Z

$$Z = \sum_{\{S_A\}} e^{-\frac{1}{2}\sum_{A,B} S_A J_{AB} S_B + h \sum S_A}$$

A = an arry of sites $\qquad S_A = 0, 1$

- Gap Eqn and the Ground State

- H-S transfn (c.f. superconductivity)

Introduce an auxiliary field $\chi_i^j(n,m)$

$$Z = \int \prod_n d\bar{\psi}(n)\, d\psi(n)\, d\chi(n,m) \cdot$$
$$\exp\left[-\sum_{n,m}\left(\bar{\psi}(n)\, \chi(n,m)\, \psi(m) + \frac{\mathrm{tr}\left(\chi(n,m)\,\chi(m,n)\right)}{2\,J(n-m)}\right)\right]$$

- If you integrate over χ, you obtain the original 4-Fermi theory.
- Integrate over $\bar{\psi}$, ψ

$$Z = \int \prod d\chi \;\; e^{-S_{\mathrm{eff}}(\chi)}$$

$$S_{\mathrm{eff}} = -\ln \mathrm{Det}\, \chi + \sum_{n,m} \frac{\mathrm{tr}\left(\chi(n,m)\,\chi(m,n)\right)}{2\,J(n-m)}$$

- Analyse S_{eff} in Semi-classical approximation

Semi-classical gr. st. should be a solution of

$$\frac{\delta S_{\mathrm{eff}}}{\delta \chi} = 0$$

- The Lattice F_4

- We shall make use of the symmetries of F_4
- Consider an orthonormal basis $\underline{e}_\mu$, of $\mathbb{R}^4$

$$\underline{e}_\mu \cdot \underline{e}_\nu = \delta_{\mu\nu} \qquad \mu, \nu = 1, \dots, 4$$

Define $\underline{b}_\mu$

$$\underline{b}_1 = \underline{e}_1 - \underline{e}_2 \qquad \underline{b}_2 = \underline{e}_2 - \underline{e}_3 \qquad b_3 = e_3 - e_4 \qquad b_4 = 2e_4$$

F_4 is generated by $\underline{b}_\mu$

$$F_4 = \left\{ n^\mu b_\mu \;\middle|\; n^\mu \in \mathbb{Z} \quad \mu = 1, \dots, 4 \right\}$$

- F_4 is a crystal lattice, i.e. $\exists\, G \subset O(4)$ s.t.

  any $g \in G$ defines a $g : F_4 \to F_4$

  and g is an integral matrix w.r.t. $\{\underline{b}_\mu\}$.

- G has 1152 elements (including the improper transformations $\det g = -1$) and is the largest crystollographic subgroup of $O(4)$ (Hurely 1950)

- We have constructed all the irreducible repns of G. They are as follows

$1^2,\ 2^4,\ 3^4,\ 4^2,\ 6^2,\ 8^4,\ 9^2$ (tensorial)

$2^4,\ 4^8,\ 6^8,\ 12^2$ (spinorial)

- The action of G decomposes F_4 into orbits around $\underline{n}=0$

- $(\text{length})^2$ of points belonging to an orbit characterizes the orbit uniquely.

- Examples

1) $(\text{length})^2 = 0$ $\quad \underline{n}=0$ $\quad 1$

2) $(\text{Length})^2 = 2$ $\quad \pm e_\mu \pm e_\nu \quad \mu \neq \nu$ $\quad 24$

3) $(\text{length})^2 = 4$ $\quad \pm 2e_\mu$

$\quad \pm e_1 \pm e_2 \pm e_3 \pm e_4$ $\quad 24$

4) $(\text{length})^2 = 6$ $\quad \pm 2e_\mu \pm e_\nu \pm e_\lambda \quad \mu \neq \nu \neq \lambda$ $\quad 96$

$\vdots$

There are orbits up to 576 points (if we consider only the proper subgroup of G

- G-Invariant Ansatz

We have to $\frac{\delta S_{eff}}{\delta \chi(n,m)} = 0$ or

$$\chi_0(n,m) = J(n-m)\, G(n,m)$$

$$G(n,m) = \chi_0^{-1}(n,m) \equiv \text{Fermion Propag.}$$

- Assume χ_0 has all the symmetries of F_4

i) Translational inv. $\Rightarrow$ $\chi_0(n,m) = \chi_0(n-m)$

ii) G-inv. $\Rightarrow$

$$\chi_0(gs) = a(g)\,\chi_0(s)\,a(g)^{-1} \qquad g \in G$$

$a : G \to GL(N, \mathbb{C})$ is a homomorphism

Since $\chi_0 = \langle \Omega | \psi \bar{\psi} | \Omega \rangle$

$\Rightarrow$

$$U(g,b)\,|\Omega\rangle = |\Omega\rangle$$

- The gap eqns are G-covariant

$$\chi_0(s) = J(s)\, G(s)$$

- The couplings $J(gs) = J(s)$ are defined on the orbits

- Fourier transform

$$G(s) = \int \frac{d^4k}{(2\pi)^4} \, \tilde{G}(k) \, e^{ik\cdot s}$$

The integration is on a Brillion Zone of vol $= (2\pi)^4$.

$$\tilde{G}(k) = \sum_s G(s) \, e^{-ik\cdot s}$$

periodic in the reciprocal lattice

We have

$$\begin{cases} \tilde{G}^{-1}(k) = \sum_s \chi_0(s) \, e^{-ik\cdot s} \\ \chi_0(s) = J(s) \int \frac{d^4k}{(2\pi)^4} \, \tilde{G}(k) \, e^{ik\cdot s} \end{cases}$$

- $\chi_0 \neq 0$ only for those orbits for which $J \neq 0$

e.g. If $J(0) \neq 0$ and $J(s) = 0 \quad s \neq 0$

$$\chi_0(s) = \text{const.} \; 1_{N \times N}$$

- We do not have any non-trivial solutions yet!

- Desirable Solution

Assume N = multiple of 4 and look for solutions of the form

$$\chi_0(s) = c(s) \cdot \mathbb{1}$$

$$c(s) = 4 \times 4 \qquad \mathbb{1} = 1_{N/4 \times N/4}$$

- Inv. condition $\rightarrow$

$$c(gs) = a(g)\, c(s)\, a(g)$$

- We have parameterized this ansatz for the orbits $s^2 = 2, 4, 6$. The general form of $c(s)$ is

$$c(s) = \underbrace{c_1(s) + c_2(s)}_{\text{orbit functions}} s^\mu \gamma_\mu$$

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu} \qquad \mu, \nu = 1, \dots, 4$$

- $c_1(s)$ and $c_2(s)$ should be determined from the gap eqn.

- Dirac Propagator

$$\tilde{G}^{-1}(k) = \sum_s \chi_0(s)\, e^{-iks}$$

$$= \sum_s c(s)\, e^{-iks}$$

$$= \sum_s \left(c_1(s) + c_2(s)\, s_\mu \gamma^\mu \right) e^{-iks}$$

$$= \sum_{s^2=0,2,\dots} \left[c_1(s) + c_2(s)\, i \gamma^\mu \frac{\partial}{\partial k^\mu} \right] \sum_{s \in \text{orbit}} e^{-iks}$$

$$= M(k) + i \gamma^\mu Q_\mu(k)$$

$$M(k) = \sum_{s^2=0,2,\dots} c_1(s)\, F(k, s^2)$$

$$Q_\mu(k) = \frac{\partial}{\partial k^\mu} \sum_{s^2=0,2,\dots} c_2(s)\, F(k, s^2)$$

$$F(k, s^2) = \sum_{s \in \text{orbit}} e^{-iks}$$

- $F(k, s^2)$

$$F(k, 0) = 1$$

$$F(k,2) = 4 \big(\cos k_1 \cos k_2 + \cos k_1 \cos k_3 + \cos k_1 \cos k_4 + \cos k_2 \cos k_3 + \cos k_2 \cos k_4 + \cos k_3 \cos k_4 \big)$$

$$F(k,4) = 2 \left(\cos 2k_1 + \cos 2k_2 + \cos 2k_3 + \cos 2k_4 \right) + 16 \cos k_1 \cos k_2 \cos k_3 \cos k_4$$

Long wave length limit

As $k \to 0$

$$M(0) = c_1(0) + 24 c_1(2) + 24 c_1(4) + 96 c_1(6) + \cdots$$

$$\equiv m Z_2^{-1}$$

$$Q_\mu(k) = k_\mu Z_2^{-1}$$

$$Z_2^{-1} = -12 C_2(2) - 24 C_2(4) - 14 C_2(6) - \cdots$$

$\Rightarrow$

$$m = \frac{C_1(0) + 24 C_1(2) + 24 C_1(4) + \cdots}{-12 C_2(2) - 24 C_2(4) - \cdots}$$

where c_1 and c_2 should be determined from

$$c_1(s^2) + s^\mu \gamma_\mu C_2(s) = J(s) \int \frac{d^4k}{(2\pi)^4} \frac{1}{M(k) + i \not{Q}(k)} e^{ik\cdot s}$$

$$\tilde{G}^{-1}(k) = Z_2^{-1} (m + i \not{k})$$

- Fermion Action

$$\sum_{n,m} \bar{\psi}(n)\, \chi(n,m)\, \psi(m) =$$

$$= \sum_{n,m} \bar{\psi}(n)\, \chi_0(n-m)\, \psi(m)$$

$$= \sum_{n,s} \bar{\psi}(n)\, c(s)\, \psi(n-s)$$

$$= \int \frac{d^4k}{(2\pi)^4} \frac{d^4k'}{(2\pi)^4}\, \tilde{\bar{\psi}}(k') \sum_{s,n} e^{-ik'n+ik(n-s)}\, c(s)\, \tilde{\psi}(k)$$

$$= \int \frac{d^4k}{(2\pi)^4}\, \tilde{\bar{\psi}}(k) \sum_s c(s)\, e^{-iks}\, \tilde{\psi}(k)$$

$$= \int \frac{d^4k}{(2\pi)^4}\, \tilde{\bar{\psi}}(k)\, \tilde{G}^{-1}(k)\, \tilde{\psi}(k)$$

$$= \int \frac{d^4k}{(2\pi)^4}\, \tilde{\bar{\psi}}(k) \left(M(k) + i\gamma^\mu Q_\mu(k) \right) \tilde{\psi}(k)$$

For those $\tilde{\psi}(k)$ with support at $k \approx 0$

this goes over to

$$\int \frac{d^4k}{(2\pi)^4}\, \tilde{\bar{\psi}}(k)\, Z_2^{-1} \left(m + i\not{k} \right) \tilde{\psi}(k)$$

– Yang - Mills

- S_{eff} is inv. under

$$\chi'(n,m) = e^{i\theta(n)} \, \chi(n,m) \, e^{-i\theta(m)}$$

$$\theta(n) = N \times N \quad \text{complex}$$

- This symmetry is broken by ground state, which is inv. only under those transfns which leave χ_0 inv.

$$\chi_0(n-m) = e^{i\theta(n)} \, \chi_0(n-m) \, e^{-i\theta(m)}$$

- From
$$\chi_0^{\dagger}(s) = \chi_0(s) \implies \theta(n)^{\dagger} = \theta(n)$$

$\Rightarrow$ For the Ansatz $\quad \chi_0(s) = C(s)_{4\times4} \cdot 1_{N/4\times N/4}$

$\theta(n)$ is a hermitian $N/4 \times N/4$ matrix.

- We would like to express this local inv. in terms of local gauge fields.
- We have to fish out the Yang - Mills part from $\chi(n,m)$

- Write

$$\chi(n,m) = c(s) + \phi(n,m)$$

expand S_{eff} in powers of ϕ

$$S_{eff}(c+\phi) = S_{eff}(c) + \sum_{n,m} \frac{tr\left(\phi(n,m)\,\phi(m,n)\right)}{2J(n-m)}$$

$$+ \frac{1}{2} tr\left(G\phi G\phi\right) + \cdots$$

- This is inv. under

$$\chi'(n,m) = c(n-m) + \phi'(n,m)$$

$$= e^{i\theta(n)}\left(c(n-m) + \phi(n,m)\right) e^{-i\theta(m)}$$

- Consider ∞-mal $\theta(n)$

$$\delta\phi(n-m) = i\,c(n-m)\left(\theta(n) - \theta(m)\right) +$$

$$+ i\theta(n)\phi(n,m) - i\phi(n,m)\,\theta(m)$$

- The inhomogenous part in $\delta\phi$ is characteristic of local gauge transfns.

$$\delta A_\mu = \partial_\mu \theta + \cdots$$

- Leading order Ansatz
- Fourier transform

Set $n - m = s$

$$\phi(n, n-s) = \int \frac{d^4k}{(2\pi)^4}\, \tilde{\phi}(k,s)\, e^{ikn}$$

- $$\delta\tilde{\phi}(k,s) = i\,c(s)\,\tilde{\theta}(k)\left(1 - e^{-iks}\right) + \text{homog. terms}$$

At $k=0$ the inhomog. terms vanish and we recover the homogeneous global symmetries.

- Consider the Ansatz

$$\tilde{\phi}(k,s) = i\,c(s)\,\frac{1 - e^{-iks}}{iks}\, s_\mu \tilde{A}_\mu(k) + \cdots$$

- As $k \to 0$, we shall have

$$\delta\tilde{A}_\mu(k) = i k_\mu\, \tilde{\theta}(k) + \cdots$$

This is a good indication for a correct ansatz

But not sufficient!

- We have to check the Ward identities.

- Substitute the ansatz in the bilinear part of S_{eff} and write it as

$$S_2 = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} A^\alpha_\mu(k)\, \tilde{\Delta}^{-1}_{\mu\nu}(k)\, A^\alpha_\nu(-k)$$

$$\tilde{\Delta}^{-1}_{\mu\nu}(k) =$$

$$= \sum_{s,s'} \frac{1-e^{iks}}{ks}\, s_\mu \left[\frac{\delta_{s+s',0}}{J(s)} \operatorname{tr}\left(c(s)\, c(s')\right) + \int \frac{d^4q}{(2\pi)^4} \operatorname{tr}\left(c(s)\, \tilde{G}(q)\, c(s')\, \tilde{G}(q-k)\, e^{-iq\cdot(s+s')}\right)\right] \frac{1-e^{iks'}}{ks'}\, s'_\nu$$

- If our Ansatz is correct $\tilde{\Delta}^{-1}_{\mu\nu}(k)$ should satisfy

$$k^\mu\, \tilde{\Delta}^{-1}_{\mu\nu}(k) = 0$$

- This can be shown to be true

- The non-linear parts should take care of themselves by gauge inv.

- Yang-Mills coupling constant

As $k \to 0$ we must have

$$\tilde{\Delta}^{-1}_{\mu\nu}(k) = \frac{1}{g^2}\left(k_\mu k_\nu - k^2 \delta_{\mu\nu}\right) + \cdots$$

- g^2 can be expressed in terms of $c(s)$ and ultimately in terms of $J(s)$.

- $$\frac{i}{g^2} = \frac{1}{24} \sum_{s,s'} s \cdot s' \left[\frac{1}{3}\left(s^2 + \frac{3}{2}\, s \cdot s' + s'^2\right) W(s,s') + (s+s')_\lambda W_\lambda(s,s') - W_{\lambda\lambda}(s,s') \right]$$

where W, W_λ and $W_{\lambda\rho}$ are defined by

$$W(s,s') + i k_\lambda W_\lambda(s,s') + \frac{1}{2} k_\lambda k_\rho W_{\lambda\rho}(s,s') + \cdots$$

$$= \frac{\delta_{s+s',0}}{J(s)} \operatorname{tr}\left(c(s)\, c(s')\right) +$$

$$+ \int \frac{d^4 q}{(2\pi)^4} \operatorname{tr}\left(c(s)\, \tilde{G}(q)\, c(s')\, \tilde{G}(q-k)\right) e^{-iq(s\cdot}$$

- Fermion Couplings to Yang-Mills and Gravity
- Speculations on Gravity
- $$S_{eff} = -\ln \mathrm{Det}\, \chi + \sum_{n,m} \frac{\mathrm{tr}\, \chi(n,m)\, \chi(m,n)}{2\, J(n,m)}$$

is invariant under

$$\chi'(n',m') = \chi(n,m)$$

$$J'(n',m') = J(n,m)$$

$n' = f(n) \Rightarrow S_\infty$

- J should be dynamical, i.e. there has to be a term $S(J)$ and $S_{eff} \to S_{tot} = S_{eff} + S(J)$

The hope is that the extended eqns.

$$\left.\frac{\delta S_{tot}}{\delta \chi}\right|_{\chi_0, J_0} = 0$$

$$\left.\frac{\delta S_{tot}}{\delta J}\right|_{\chi_0, J_0} = 0$$

will produce the structure we have been assuming

$$J_0(n,m) = J(n-m)$$

$$\chi_0(n,m) = \chi_0(n-m) = C(s) \cdot 1_{N/4 \times N/4}$$

s.t.

$$J(gs) = J(s)$$

$$X_0(gs) = a(g)\, X_0(s)\, a(g)^{-1} \qquad g \in G$$

G will play the role of the Lorentz group in G.R.

- More ambitious programme

  The splitting of the site index A on ψ_A or S_A, into $A = (n, i)$ $\quad i = 1, \dots, N$, should emerge as a solution.

- At the moment we are treating $J(n,m)$ as a background object, like the metric tensor in

$$S = \int d^4x \sqrt{g} \left(\nabla_\mu \phi \nabla^\mu \phi - \frac{m^2}{2} \phi^2 + \dots \right)$$

$\therefore$ We should expect to obtain actions of this type for Fermions

- Fermion coupling

Introduce interpolating fields $\chi(x, x-s)$

$$\chi(x, x-s) = E(x,s)\, T e^{i\int_{x-s}^{x} \Omega}$$

$\Omega \in$ algebra of $O(4) \times U(N/4) \subset GL(N, \mathbb{C})$

$E(x,s) = 4 \times 4$ matrix

- Under

$$\chi'(x, x-s) = e^{i\theta(x)}\, \chi(x, x-s)\, e^{-i\theta(x-s)}$$

demand a homog. transfn for $E(x,s)$

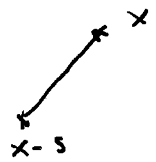
$$E'(x,s) = e^{i\theta(x)}\, E(x,s)\, e^{-i\theta(x)}$$

$$\Rightarrow \quad T e^{i\int_{x-s}^{x} \Omega} \longrightarrow e^{i\theta(x)}\, e^{i\int_{x-s}^{x} \Omega}\, e^{-i\theta(x-s)}$$

- $E(x,s)$ is a generalization of $c(s)$

$$E(x,s) = c_1(s) + c_2(s)\, \gamma^a\, \underset{\substack{\uparrow \\ \text{4- bein}}}{e_a^{\mu}(x)}\, s_\mu$$

- $$\int_{x-s}^{x}\Omega = \int_0^1 dt\, \frac{dy^\mu}{dt}\, \Omega_\mu\left(y(t)\right)$$



$$= \int_0^1 dt\, s^\mu \Omega_\mu\left(x^\mu + (t-1)s^\mu\right) \qquad y^\mu(t) = x^\mu + (t-1)s^\mu$$

$$= \int_0^1 dt\, s^\mu\, e^{(t-1)s.\partial}\, \Omega_\mu(x)$$

$$= \frac{1 - e^{-s.\partial}}{s.\partial}\, s.\Omega(x) \longrightarrow s.\Omega(x) + \cdots$$

- $$T\, e^{i\int_{x-s}^{x}\Omega} = 1 + i\int_0^1 dt_1\, \Omega(t_1) + \frac{i^2}{2}\int dt_1 dt_2\, T\big(\Omega(t_1)\Omega($$
$$+ \cdots$$

$$= 1 + i\,\frac{1 - e^{-s\partial}}{s\partial}\, s_\mu \Omega_\mu(x) + \cdots$$

$$= 1 + i\, s.\Omega(x) - \frac{1}{2}\left(s.\Omega(x)\right)^2 +$$

$$+ \frac{1}{2}\, s.\partial\left(s.\Omega(x)\right)^2 + \cdots$$

Minimal Coupling

$$\sum_{n,m} \bar\psi(n)\, \chi(n,m)\, \psi(m) =$$

$$\sum_n \to \int d^4x\, e(x) \qquad = \sum_n \sum_s \bar\psi(n)\, \chi(n, n-s)\, \psi(n-s)$$

$$\to \int d^4x\, e(x)\, \bar\psi(x) \sum_s E(x,s)\, T e^{i\int_{x-s}^{x} \Omega} \psi(x-s)$$

$$= \int d^4x\, e(x)\, \bar\psi(x) \sum_s \left(c_1(s^2) + c_2(s^2)\, \gamma^a e_a^{\mu}(x)\, s_\mu\right)\left(1 + i s.\Omega(x) + \cdots\right)\left(1 - s.\partial + \cdots\right)\psi(x)$$

$$= \int d^4x\, e(x)\, \bar\psi(x)\Big(\sum_s c_1(s) - \sum_s c_2(s)\, s_\mu s^\nu \gamma^a e_a^{\mu}(x)\left(\partial_\nu - i\Omega(x)\right) + \\ + \cdots\Big)\psi(x)$$

$$= \int d^4x\, e(x)\, Z_2^{-1}\, \bar\psi(x)\left(m + \not{D} + \cdots\right)\psi(x)$$

where :

$$Z_2^{-1}\, \delta_\mu^\nu = - \sum_s c_2(s)\, s_\mu s^\nu$$

$$Z_2^{-1}\, m = \sum_s c_1(s)$$

$$\not{D} = \gamma^a e_a^{\mu}\left(\partial_\mu - i\Omega_\mu(x)\right)$$

$$\frac{1}{2}\,\omega_{\mu[ab]}\,\sigma^{ab} + \text{Y. Mills}$$

Work to be done

- Find some non-trivial solutions of the gap Eqns.
- Understand the spectrum better
  (Higgs, higher rank tensor fields, ...)
- Include the dynamics of J's.
- Check the gravitational Ward identities
  $\Rightarrow$ Calculate G_{Newton}

Summary

- Starting from a fermionic lattice system we generated a low E Yang-Mills x Fermionic relativistic field theory.
- The fermion mass and the Yang-Mills coupling constant can be expressed in terms of the parameters of the original model.
- Gravity can be included in the system as a background field.