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CONFERENCE ON FUNDAMENTAL SYMMETRIES AND FUNDAMENTAL CONSTANTS

15 - 18 September 2004

EFFECTS OF VARIATION OF FUNDAMENTAL CONSTANTS FROM BIG BANG TO ATOMIC CLOCKS

V. Flambaum
New South Wales University, Australia

Effects of variation of fundamental constants from Big Bang to atomic clocks

Theories unifying gravity with other interactions suggest temporal and spatial variation of the fundamental "constants" in expanding Universe. I discuss effects of variation of the fine structure constant $\alpha = e^2/\hbar c$, strong interaction, quark mass and gravitational constant. The measurements of these variations cover lifespan of the Universe from few minutes after Big Bang to the present time and give controversial results. There are some hints for the variation in Big Bang nucleosynthesis, quasar absorption spectra and Oklo natural nuclear reactor data.

A very promising method to search for the variation of the fundamental constants consists in comparison of different atomic clocks. A billion times enhancement of the variation effects happens in transition between accidentally degenerate atomic energy levels.

Do fundamental constants of Nature vary with time and distance?

Were the laws of Nature the same 10 billion light years from us?

Theory: V.A.Dzuba,¹⁾ V.V.Flambaum,¹⁾ M. Marchenko¹⁾, M.G. Kozlov²⁾
new: E. Anstamann¹⁾, J. Berengut¹⁾, V. Dmitriev,³⁾
E. Shuryak⁴⁾, D. Leinweber, A. Thomas, R. Young.

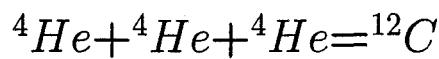
Data: J. Webb, M. Murphy, M. Drinkwater,¹⁾ W. Walsh

Quasar observations: C. Churchill, J. Prochazka,⁶⁾ A. Wolfe⁷⁾, W. Sargent⁸⁾, R. Simcoe⁸⁾

- 1) UNSW, Sydney
- 2) St. Petersburg Institute for Nuclear Physics
- 3) Novosibirsk Institute for Nuclear Physics
- 4) State University of New York
- 5) Penn. State
- 6) Carnegie
- 7) University of California, San Diego
- 8) Cal. Tech.
- 9) J. Barrow DAMPT, Cambridge UK

Some special “tuning” of fundamental constants is needed for humans to exist.

Example: low-energy resonance in carbon production reaction in stars:



Different coupling constants \rightarrow no low-energy resonance \rightarrow no carbon \rightarrow no life.

Variation of coupling constants in space could provide a natural explanation of “fine tuning”: we appeared in area of the Universe where values of fundamental constants are consistent with our existence.

Search for the variation of the constants

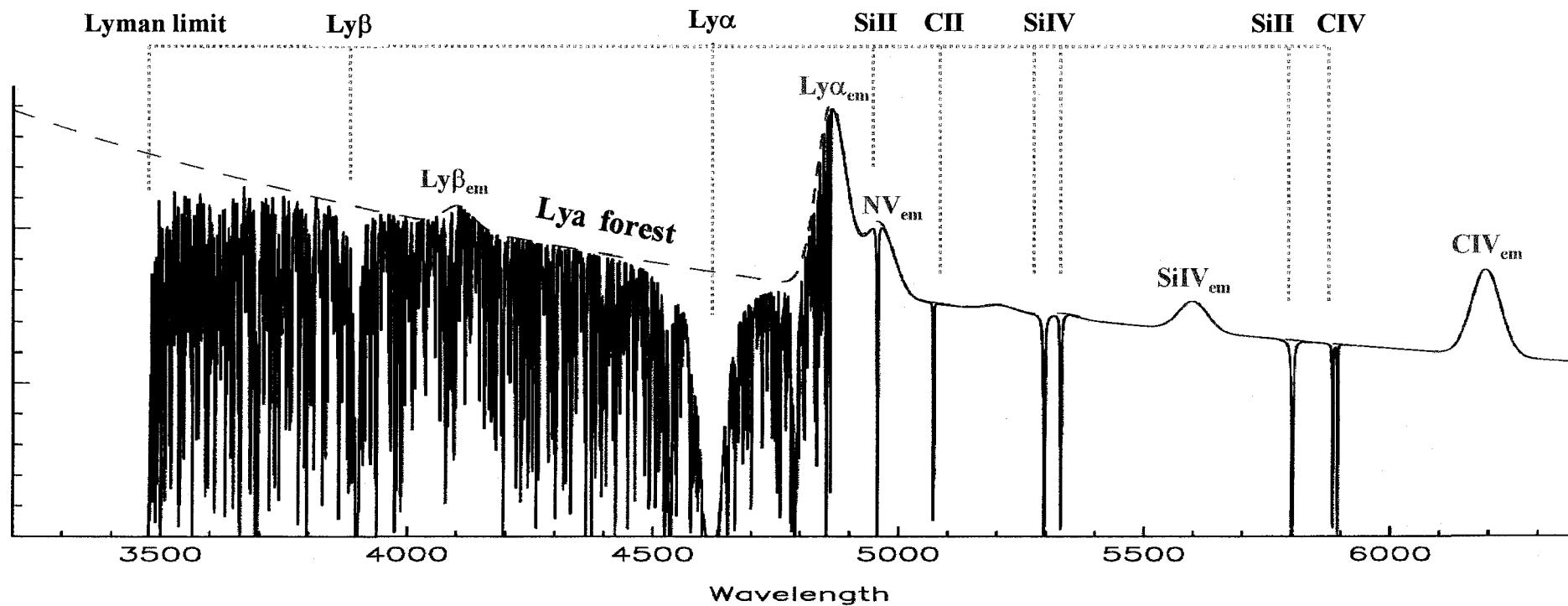
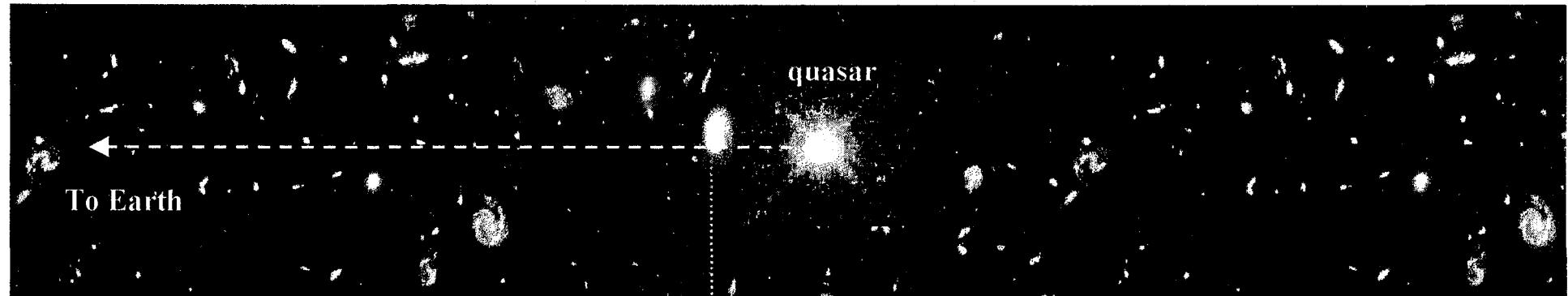
The possibility for the fundamental constants to vary is predicted by theories unifying gravity with other interactions.

The search goes in

Quasar absorption spectra	α		$\Delta \neq 0 ! ?$
Big Bang Nucleosynthesis	$\alpha, \frac{m_q}{\Lambda_{QCD}}, \frac{\Lambda_{QCD}}{M_{Plank}}$		$\Delta \neq 0 ! ?$
Oklo natural nuclear reactor	$\alpha, \frac{m_q}{\Lambda_{QCD}}$		$\Delta \neq 0 ! ?$
Atomic clocks	$\alpha, \frac{m_{q,e}}{\Lambda_{QCD}}$		$\Delta < \delta$



4.2 Astrophysical constraints: Quasars - probing the universe back to much earlier times



The alkali doublet (AD) method:

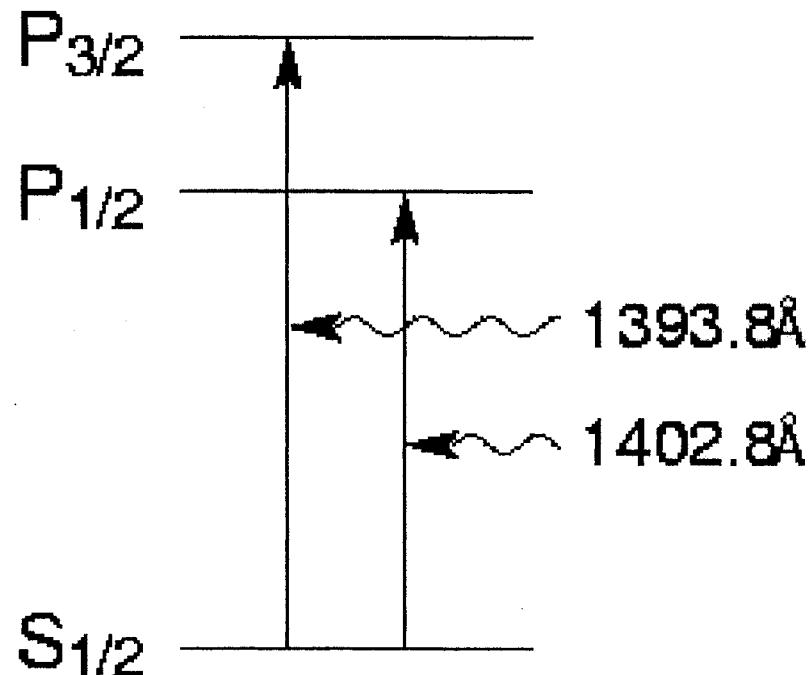
$$\frac{\omega_1 - \omega_2}{\bar{\omega}} \sim \lambda^2$$

- 1976: Wolfe, Brown & Roberts first applied the AD method to intervening Mg II absorption lines.
- 2000: Varshalovich et al. obtained $\Delta\alpha/\alpha = (-4.6 \pm 4.3 \pm 1.4) \times 10^{-5}$ using the AD method with 16 Si IV absorption systems ($z_{\text{avg}} = 2.6$).
- 2001: We have used improved lab wavelengths and new data from Keck to find $\Delta\alpha/\alpha = (-0.5 \pm 1.3) \times 10^{-5}$ ($z_{\text{avg}} = 2.8$).
- 2003: Bahcall, Steinhhardt & Schlegel used [O III] emission lines in 44 SDSS QSOs and found $\Delta\alpha/\alpha = (-2 \pm 1.2) \times 10^{-4}$ ($z_{\text{avg}} = 0.38$).
 $(0.7 \pm 1.4) \times 10^{-4}$

The alkali doublet (AD) method:

- The AD method is simple ... but inefficient.
- The common S ground state in ADs has maximal relativistic corrections!

(a) SiIV alkali doublet



A new method was proposed in

V. A. Dzuba, V.V. Flambaum, J. K. Webb , Phys. Rev. Lett. 82, 888-891, 1999.

The effect is 10 times larger and statistics is 10 times larger.

Necessary atomic calculations: V.A. Dzuba, V.V. Flambaum, and J.K. Webb. Phys. Rev. A59, 230 -237, 1999. V.A. Dzuba, V.V. Flambaum, M.T. Murphy and J.K. Webb. Phys. Rev. A. 63, 042509-1 - 5 , (2001). *V. A. Dzuba, V. Flambaum, M. Marchenko, M. Kozlov. Phys. Rev. A (2002)*

Measurements - 3 independent samples of optical data, 140 quasar

absorption systems, spread from 2 to 9 billion years after Big Bang:

J. K. Webb , V.V. Flambaum, C.W. Churchill, M.J. Drinkwater, and J.D. Barrow, Phys. Rev. Lett., 82, 884-887, 1999. J.K. Webb, M.T. Murphy, V.V. Flambaum, V.A. Dzuba, J.D. Barrow, C.W. Churchill, J.X. Prochaska, and A.M. Wolfe, Phys. Rev. Lett. 87, 091301 -1-4 (2001). M. T. Murphy, J. K. Webb, V. V. Flambaum, V. A. Dzuba, C. W. Churchill, J. X. Prochaska, J. D. Barrow and A. M. Wolfe. MNRAS 327, 1208, 2001. M. T. Murphy, J. K. Webb, V. V. Flambaum, J. X. Prochaska and A. M. Wolfe, MNRAS 327, 1237, 2001. *Murphy, Webb, Flambaum (2003)*

Systematic errors: M. T. Murphy, J. K. Webb, V. V. Flambaum, C. W. Churchill and J. X. Prochaska. MNRAS 327, 1223, 2001.

Radio samples: M.J. Drinkwater, J.K. Webb, J.D. Barrow, V.V. Flambaum. MNRAS 295, 457, 1998. M. T. Murphy, J. K. Webb, V. V. Flambaum, M. J. Drinkwater, F. Combes and T. Wiklind. MNRAS 327, 1244, 2001.

Review: V.V. Flambaum. Atomic Physics 17, AIP conference proceedings, V. 551, pp.86-99, Editors: Arimondo, De Natale, Inguscio, 2001.

New accurate laboratory wavelength measurements in Imperial College (London), Lund University and NIST

A new method

- Relativistic corrections are large when the electron is near the nucleus:

$$\frac{mv^2}{2} - \frac{Ze^2}{r} = E, \text{ so } \frac{v^2}{c^2} \propto \frac{1}{r}$$

- S-electron ($l=0$) has maximal probability to be at small distances

→ maximal relativistic corrections.

However, S- electron has no spin-orbit splitting, $\therefore \underset{\sim}{L} \cdot \underset{\sim}{S} = 0 !$

- New method - compare spectra of different atoms → more than 10 X increase in sensitivity.

$$\frac{\text{relativistic correction}}{\text{energy of electron}} \approx (Z\alpha)^2 \left[\frac{1}{j+1/2} - C \right] \frac{1}{v}$$

Z - nuclear charge

j = $\underset{\sim}{l} + \underset{\sim}{s}$ - total electron angular momentum (s-electron $j = 1/2$)

C ≈ 0.6 - contribution of the many - body effect

v - effective principal quantum number

Probing the variability of α with QSO absorption lines

Procedure:

1. Compare heavy ($Z \sim 30$) and light ($Z < 10$) atoms, OR
2. Compare $s \rightarrow p$ and $d \rightarrow p$ transitions in heavy atoms.

Shifts can be of opposite sign.

Basic formula:

$$E_z = E_{z=0} + q \left[\left(\frac{\alpha_z}{\alpha_0} \right)^2 - 1 \right]$$

$E_{z=0}$ is the laboratory frequency. 2nd term is non-zero only if α has changed. q is derived from atomic calculations.

Relativistic shift of the central line in the multiplet

$$q = Q + K(L.S)$$

Numerical examples: (units = cm⁻¹)

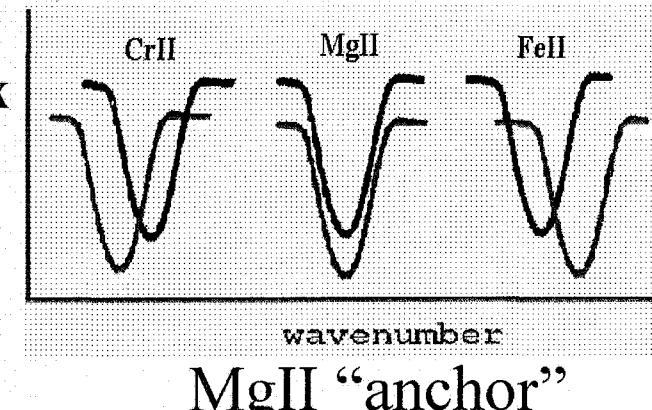
$Z=26$ ($s \rightarrow p$) FeII 2383A: $\omega_0 = 38458.987(2) + 1449x$

$Z=12$ ($s \rightarrow p$) MgII 2796A: $\omega_0 = 35669.298(2) + 120x$

$Z=24$ ($d \rightarrow p$) CrII 2066A: $\omega_0 = 48398.666(2) - 1267x$

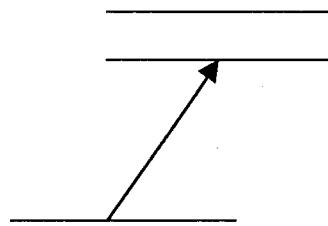
$$x = (\alpha_z/\alpha_0)^2 - 1$$

K is the spin-orbit splitting parameter. $Q \sim 10K$



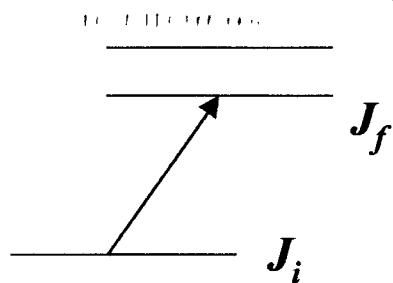
Advantages of the new method

1. Includes the total relativistic shift of frequencies (e.g. for s-electron)
2. Includes relativistic shift in the ground state $\sim |E|^{3/2}$



(Spin-orbit method: splitting in excited state - relativistic correction is smaller, since excited electron is far from the nucleus)

3. Can include many lines in many multiplets



(Spin-orbit method: comparison of 2-3 lines of 1 multiplet due to selection rule for E1 $|J_i - J_f| \leq 1$ transitions - cannot explore the full multiplet splitting)

4. Very large statistics - all ions and atoms, different frequencies, different redshifts (epochs/distances)
5. Opposite signs of relativistic shifts helps to cancel some systematics.

Probing the variability of α with QSO absorption lines

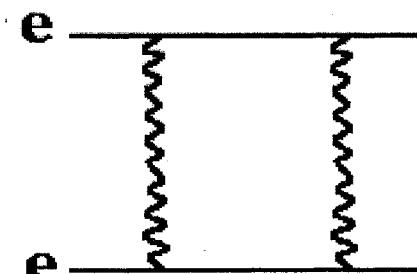
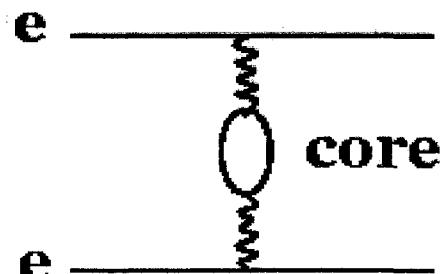
To find dependence of atomic transition frequencies on α we have performed calculations of atomic transition frequencies for different values of α .

1. Zero Approximation – Relativistic Hartree-Fock method:
energies, wave functions, Green's functions

2. Many-body perturbation theory to calculate effective Hamiltonian for valence electrons including self-energy operator and screening; perturbation

$$\longrightarrow V = H - H_{HF}$$

$$\Sigma(\mathbf{r}, \mathbf{r}', E)$$

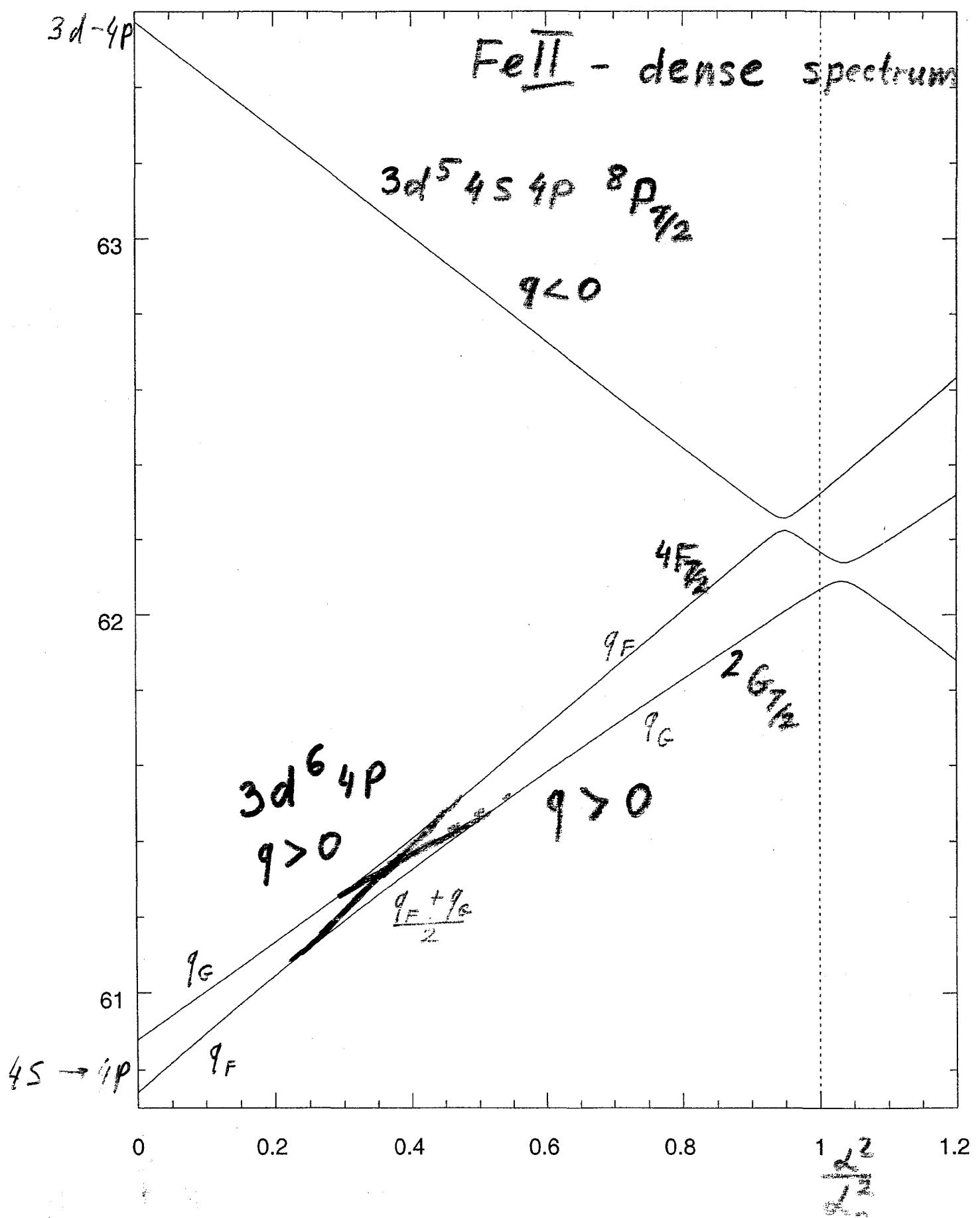


+ ...

3. Diagonalization of the effective Hamiltonian

Test: Energy levels in Mg II to 0.2% accuracy

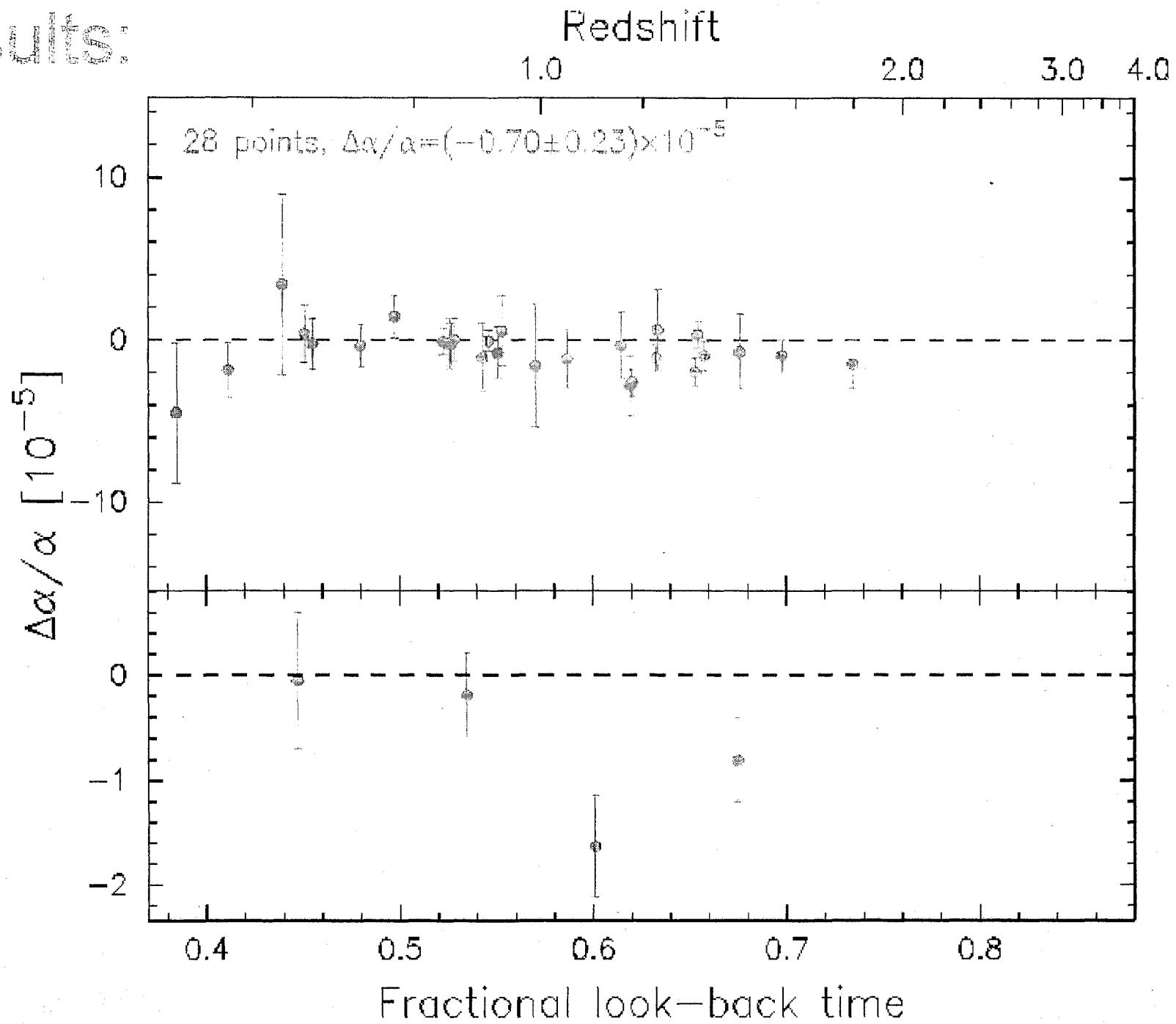
Problem : level pseudo-crossing



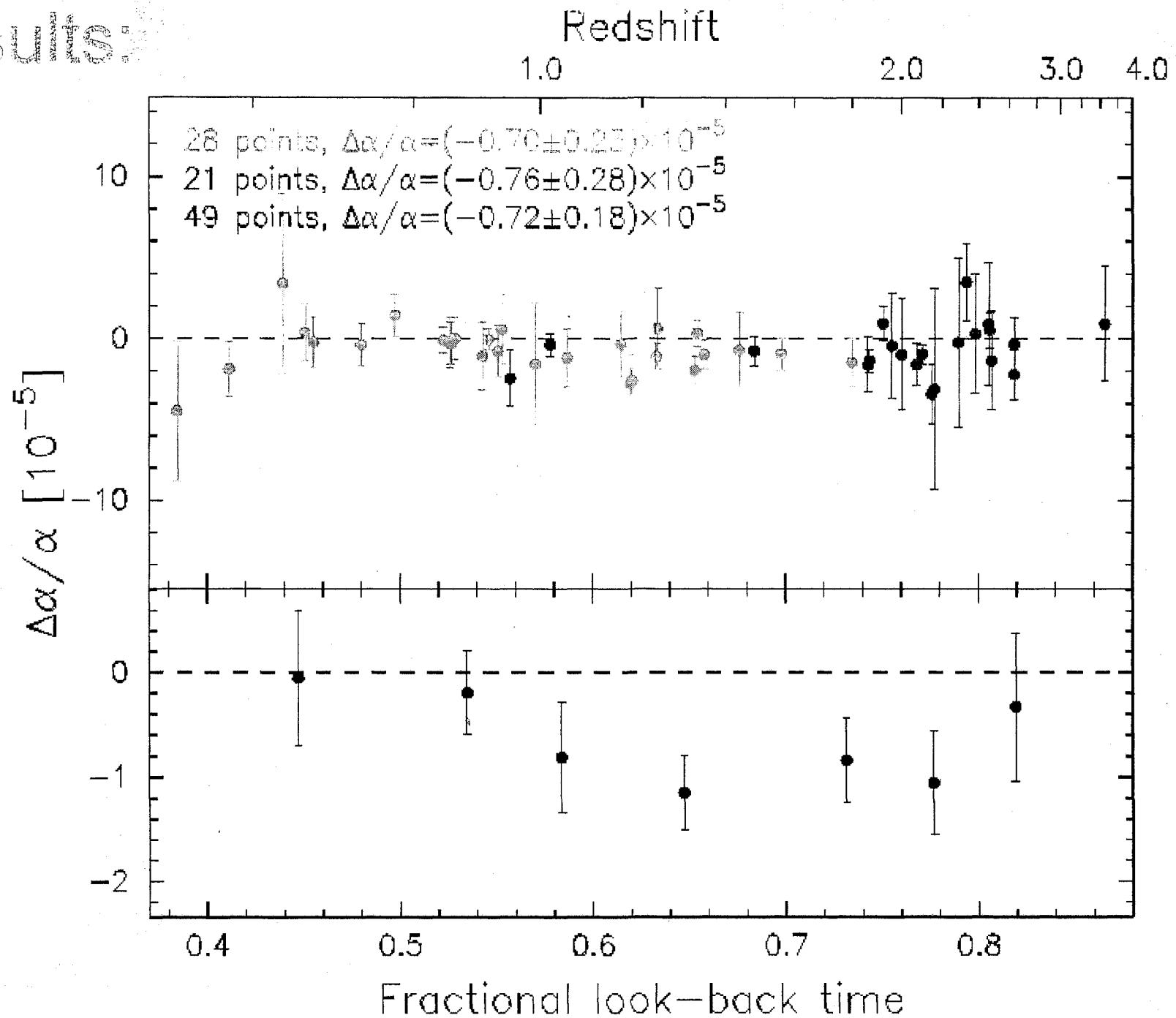
Lines used in the analysis (cm⁻¹)

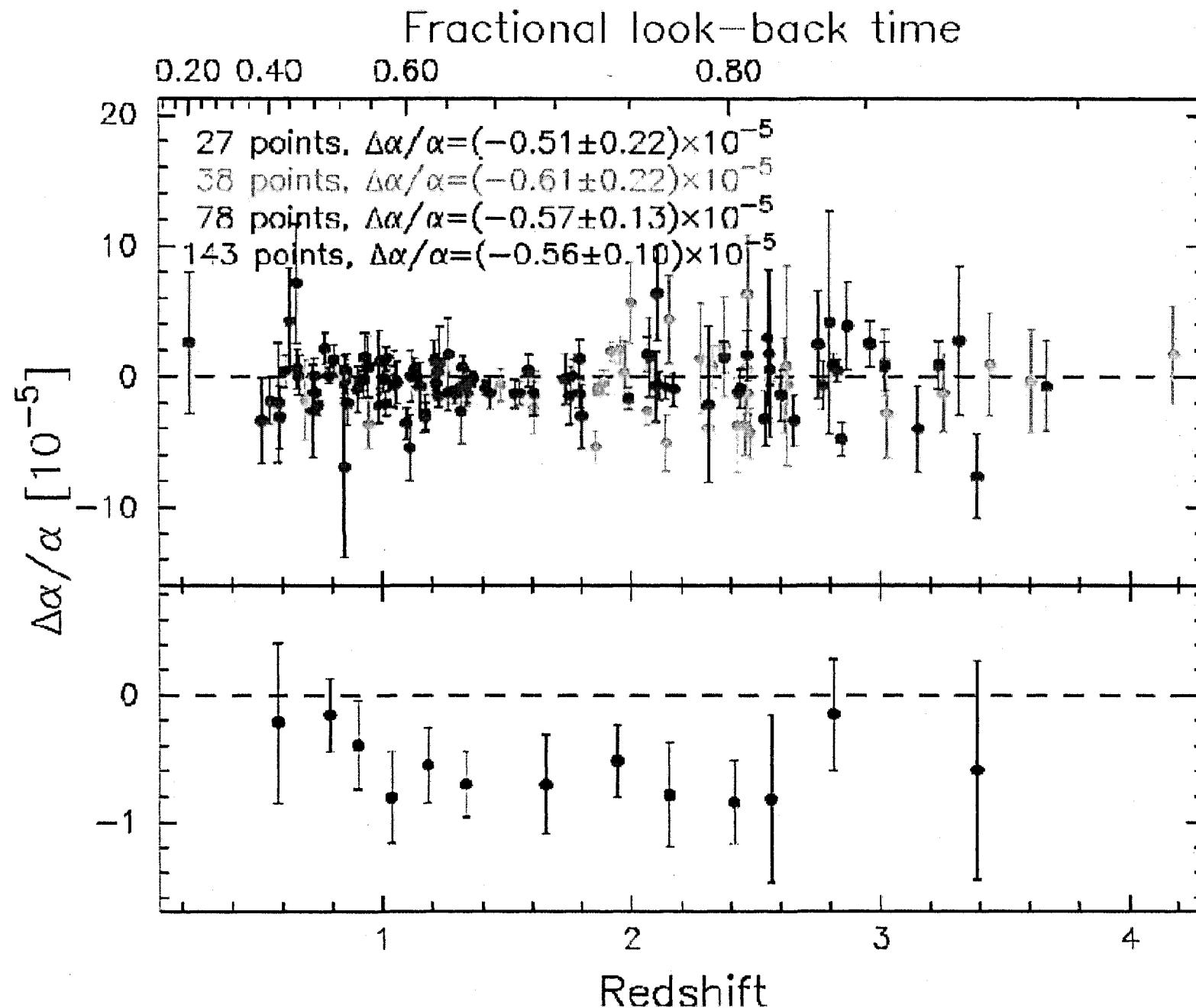
Anchor lines				Negative shifters			
MgI	35051.277(1)	+	86x	NiII	57420.013(4)	-	1400x
MgII	35760.848(2)	+	211x	NiII	57080.373(4)	-	700x
MgII	35669.298(2)	+	120x	CrII	48632.055(2)	-	1110x
SiII	55309.3365(4)	+	520x	CrII	48491.053(2)	-	1280x
SiII	65500.4492(7)	+	50x	CrII	48398.868(2)	-	1360x
AlIII	59851.924(4)	+	270x	Fell	62171.625(4)	-	1300x
AlIII	53916.540(1)	+	464x	Positive shifters			
AlIII	53682.880(2)	+	216x	Fell	62065.528(3)	+	1100x
NiII	58493.071(4)	-	20x	Fell	42658.2404(2)	+	1210x
$\omega = \omega_{Lab} + qx$				Fell	42114.8329(2)	+	1590x
$x = \frac{\alpha^2}{\alpha_{Lab}^2} - 1$				Fell	41968.0642(2)	+	1460x
				Fell	38660.0494(2)	+	1490x
				Fell	38458.9871(2)	+	1330x
				ZnII	49355.002(2)	+	2490x
				ZnII	48481.077(2)	+	1584x

Results:

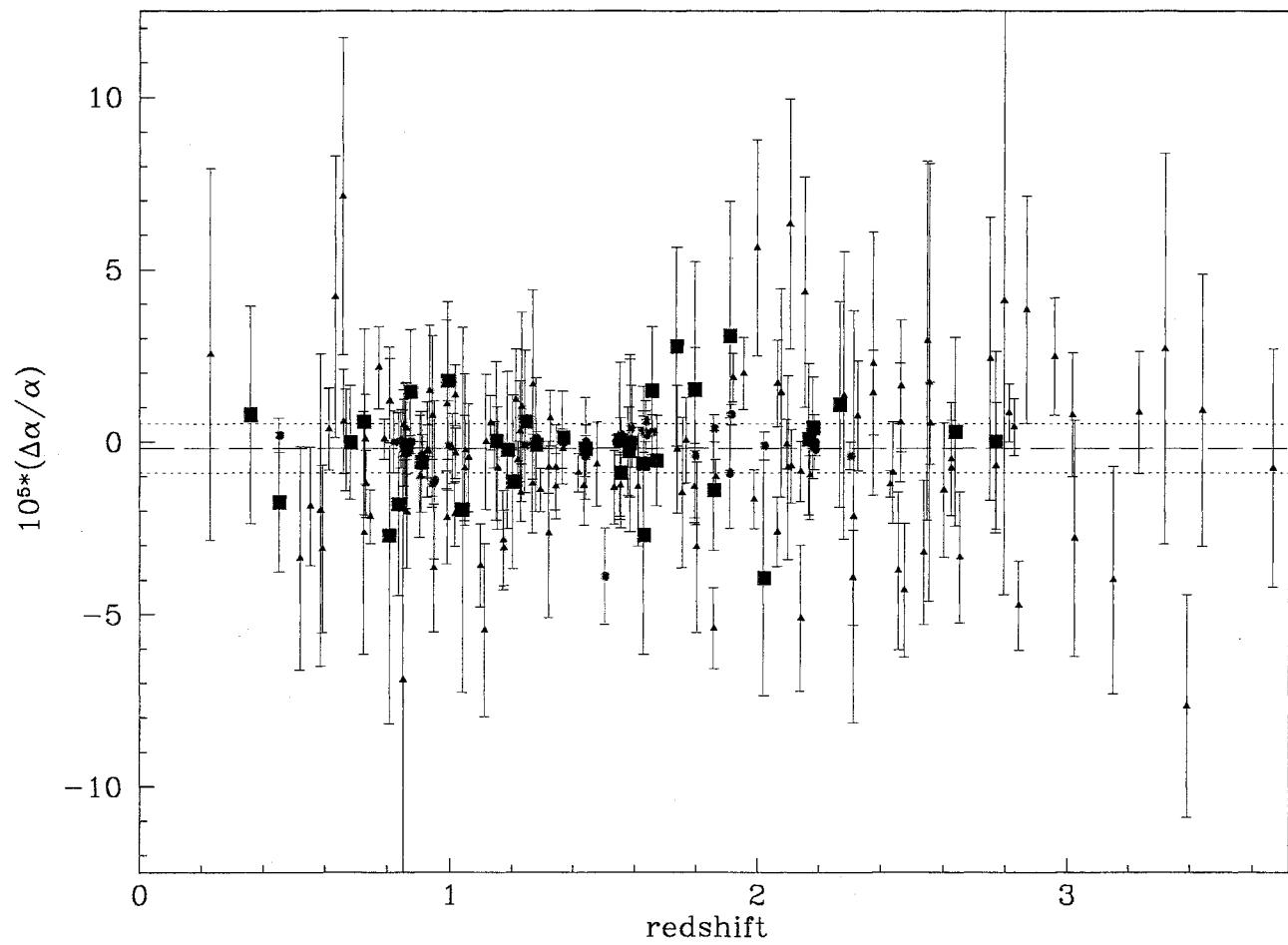


Results





Summary of QSO absorption system results



Results:

1998-2003, Keck telescope, Hawaii, red shift $0.2 < z < 4.3$,
143 absorption systems, 23 transitions,
3 independent samples:

$$\frac{\delta\alpha}{\alpha} = (-0.543 \pm 0.116) \cdot 10^{-5}$$

Statistical significance 4.7σ from zero.

2004, VLT-UVES, Chile (different hemisphere), red shift
 $0.4 < z < 2.8$

full sample, 74 systems $\frac{\delta\alpha}{\alpha} = (-0.020 \pm 0.092) \cdot 10^{-5}$

clean sample, 52 systems $\frac{\delta\alpha}{\alpha} = (-0.004 \pm 0.098) \cdot 10^{-5}$

Strianand et al sample, 23 systems $\frac{\delta\alpha}{\alpha} = (-0.061 \pm 0.126) \cdot 10^{-5}$

VLT: $|\frac{\delta\alpha}{\alpha}| < 0.1 \cdot 10^{-5}$. Zero!

Too large scatter, more realistic preliminary result

$$\frac{\delta\alpha}{\alpha} = (-0.05 \pm 0.29) \cdot 10^{-5}$$

Other groups results from VLT-UVES:

Strianand, Chand, Petitjean, Aracil (2004), 23 systems, 12
transitions, $0.4 < z < 2.3$:

$$\frac{\delta\alpha}{\alpha} = (-0.06 \pm 0.06) \cdot 10^{-5}$$

Quast, Reimer, Levshakov (2004): 1 system, Fe II only, 6
transitions, $z = 1.15$:

$$\frac{\delta\alpha}{\alpha} = (-0.04 \pm 0.19 \pm 0.27_{syst}) \cdot 10^{-5}$$

Difference between Keck and VLT data:

Undiscovered systematic effect?

Spatial variation of α ?

C. L. Steinhardt, hep-ph/0308253, 2004:

It might be spatial variation,because *Chand et al* use data from Southern Hemisphere only

$$\left(\frac{\Delta\alpha}{\alpha}\right)_{\text{South}} = (-0.06 \pm 0.06) \times 10^{-5}$$

while *Murphy et al* use both:

$$\left(\frac{\Delta\alpha}{\alpha}\right)_{\text{North}} = (-0.66 \pm 0.12) \times 10^{-5}$$

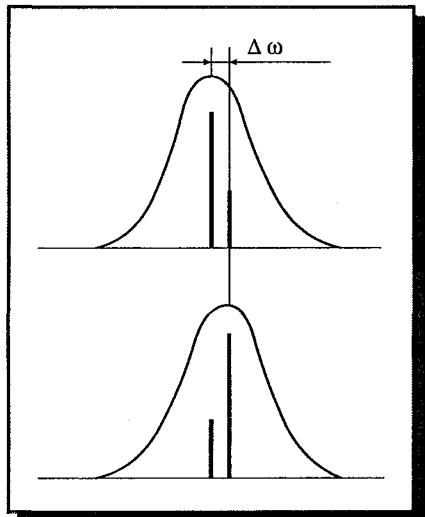
$$\left(\frac{\Delta\alpha}{\alpha}\right)_{\text{South}} = (-0.36 \pm 0.19) \times 10^{-5}$$

Potential systematic effects:

- ☺ Wavelength calibration errors
- ☺ Laboratory wavelength errors
- ☺ Heliocentric velocity variation
- ☺ Temperature changes during observations
- ☺ Line blending
- ☺ Differential Isotopic saturation
- ☺ Hyperfine structure effects
- ☺ Instrumental profile variations
- ☺ ... and of course, Magnetic fields
- ~~☺ Atmospheric dispersion effects~~
- ☹ Isotopic ratio evolution

Isotopic abundance variation

Observed frequencies are averaged over different isotopes.



Any change in isotope abundances would lead to a frequency shift.

We have calculated isotopic shift for MgII, SiII, SiIV, ZnII.

For other atoms (CrII, FeII, NiII, etc.) the work is in progress.

Independent calculations or/and measurements would be very useful!

Checks on general, unknown systematics:

- Line removal: In each system, remove each transition and iterate to find $\Delta\alpha/\alpha$ again. Compare the $\Delta\alpha/\alpha$'s before and after line removal. We have done this for all species and see no inconsistencies. Tests for: Lab wavelength errors, line blending, isotopic ratio and hyperfine structure variation.
- Positive-negative shifter test: Find the subset of systems that contain an anchor line, a positive shifter AND a negative shifter. Remove each type of line collectively and recalculate $\Delta\alpha/\alpha$.

Results: subset contains 12 systems (only in high z sample)

No lines removed: $\Delta\alpha/\alpha = (-1.31 \pm 0.39) \times 10^{-5}$

Anchors removed: $\Delta\alpha/\alpha = (-1.49 \pm 0.44) \times 10^{-5}$

+ve-shifters removed: $\Delta\alpha/\alpha = (-1.54 \pm 1.03) \times 10^{-5}$

-ve-shifters removed: $\Delta\alpha/\alpha = (-1.41 \pm 0.65) \times 10^{-5}$

Procedure:

1. Compare heavy ($Z \sim 30$) and light ($Z < 10$) atoms, OR
2. Compare $s \rightarrow p$ and $d \rightarrow p$ transitions in heavy atoms.

Shifts can be of opposite sign.

Basic formula:

$$E_z = E_{z=0} + q \left[\left(\frac{\alpha_z}{\alpha_0} \right)^2 - 1 \right]$$

$E_{z=0}$ is the laboratory frequency. 2nd term is non-zero only if α has changed. q is derived from atomic calculations. (Method: frequencies of different lines are computed for different values of α).

Relativistic shift of the central line in the multiplet

$$q = Q + K(L.S)$$

K is the spin-orbit splitting parameter. $Q \sim 10K$

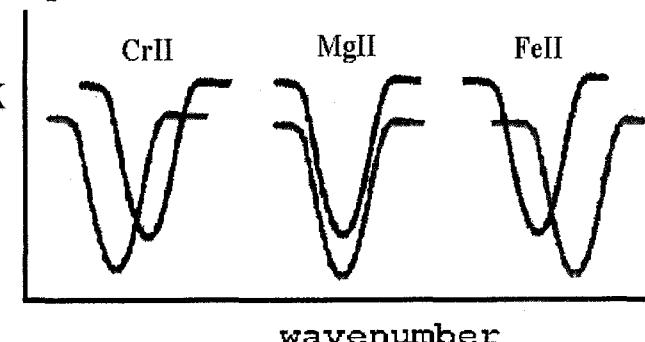
Numerical examples: (units = cm^{-1})

$Z=26$ ($s \rightarrow p$) FeII 2383A: $\omega_0 = 38458.987(2) + 1449x$

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$Z=24$ ($d \rightarrow p$) CrII 2066A: $\omega_0 = 48398.666(2) - 1267x$

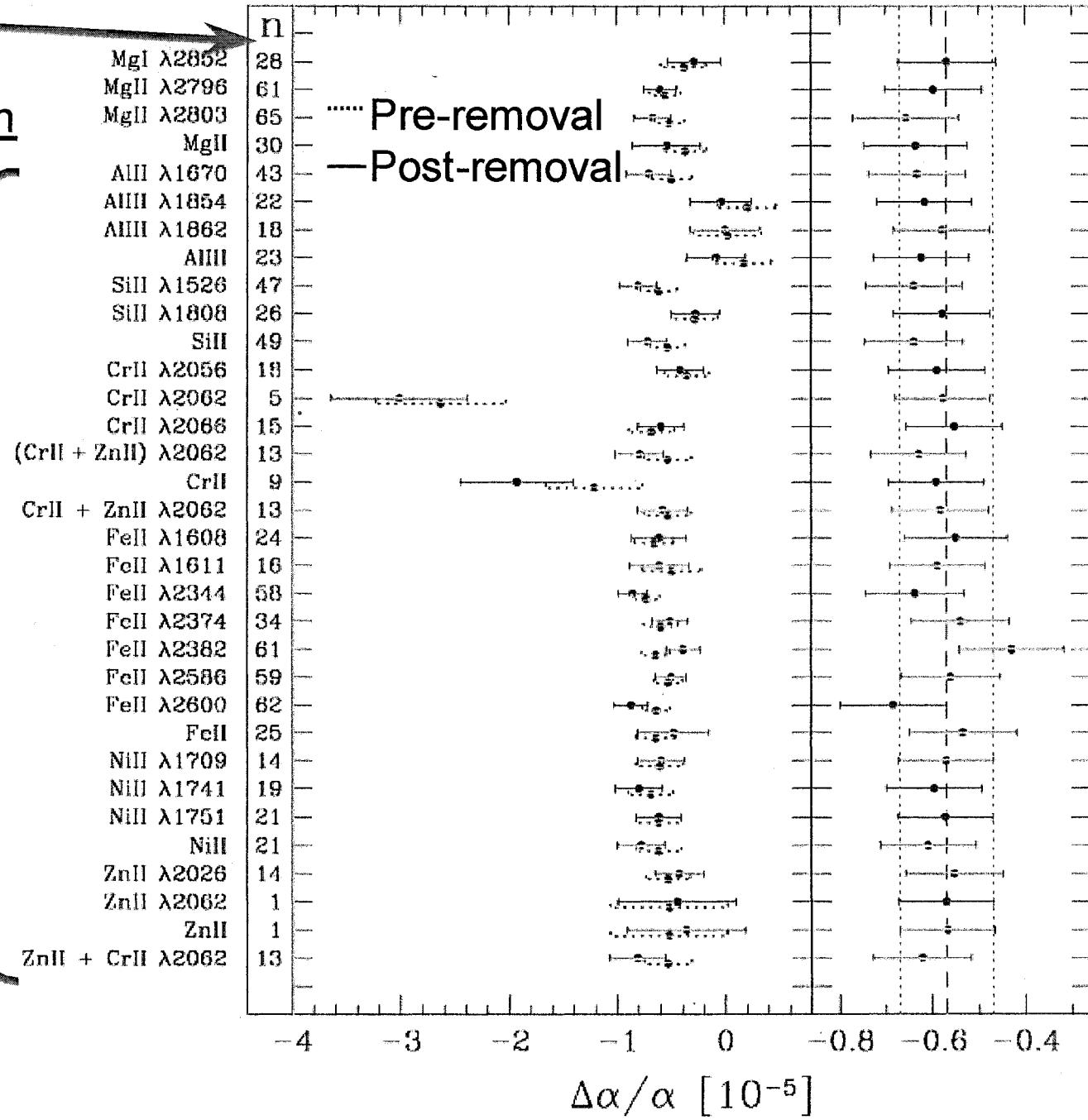
$$x = (\alpha_z/\alpha_0)^2 - 1$$



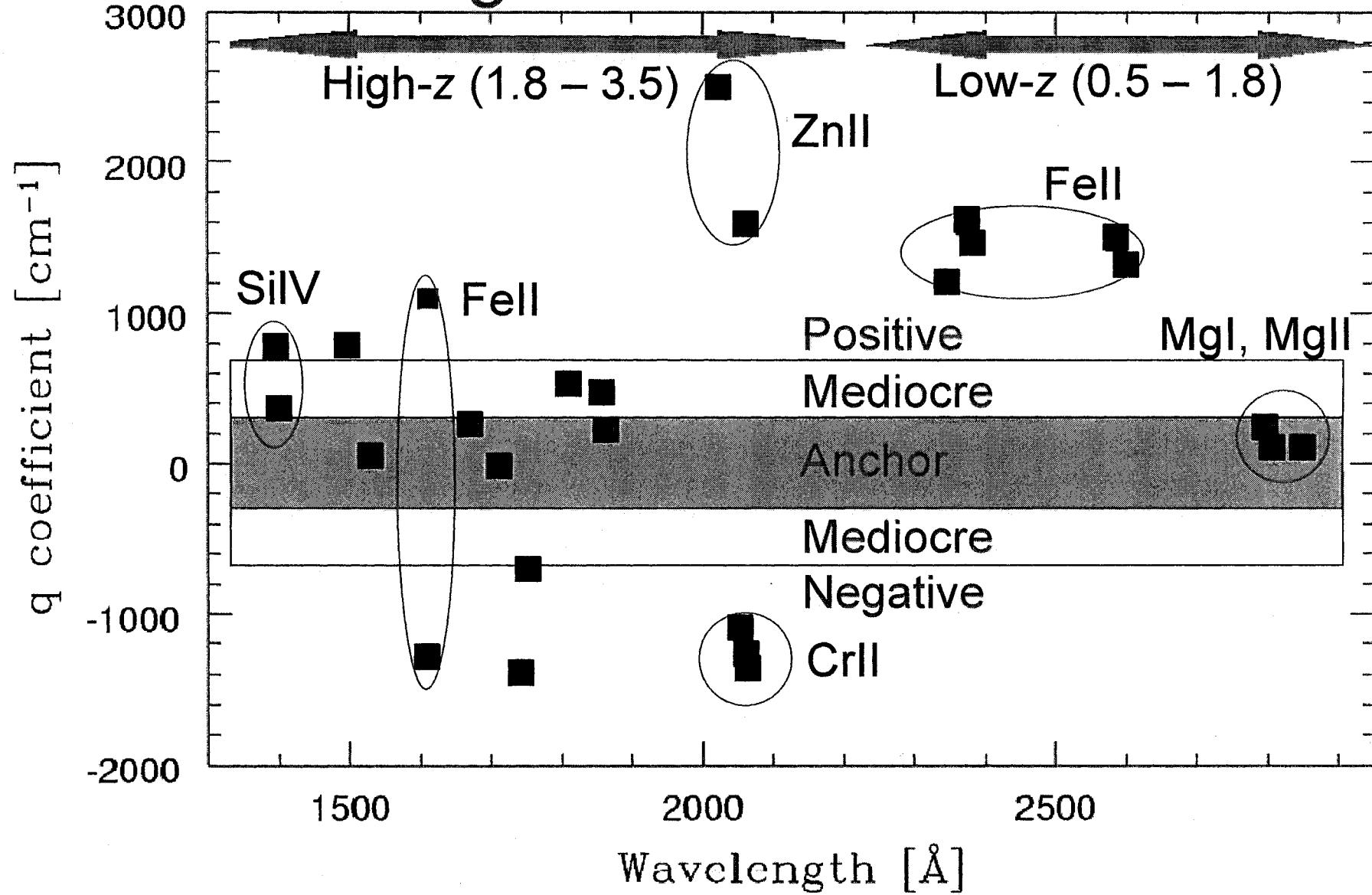
MgII “anchor”

Number of systems where transition(s) can be removed

Transition(s) removed



Low-z vs. High-z constraints:



Two samples of line pairs :

1. $\Delta d < 0$ can be imitated by compression of spectrum
2. $\Delta d < 0$ can be imitated by expansion of spectrum

Both samples give $\Delta d < 0$!

Radio constraints:

- Hydrogen hyperfine transition at $\lambda_H = 21\text{cm}$.
- Molecular rotational transitions CO, HCO⁺, HCN, HNC, CN, CS ...
- $\omega_H/\omega_M \propto \alpha^2 g_P$ where g_P is the proton magnetic g -factor.

$$g_p = g_p \left(\frac{m_q}{\Lambda_{QCD}} \right)$$

Our Measurements

$$\frac{\left(\frac{g}{g_p}\right)}{\left(\frac{g}{g_p}\right)_0} = \frac{\delta X}{X} = (-0.16 \pm 0.54) \cdot 10^{-5}$$

$Z \approx 0.7$, 6 billion years ago

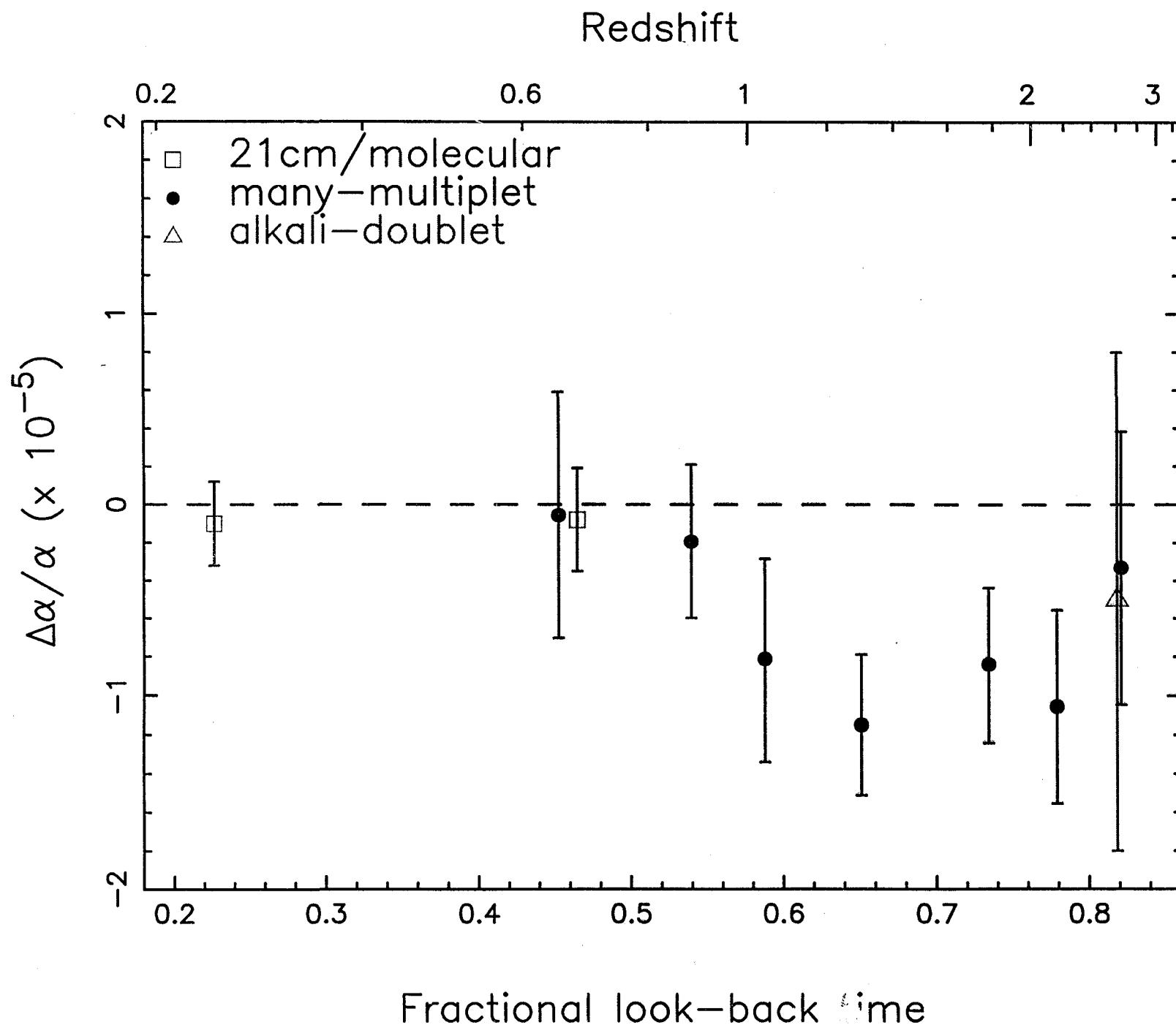
$$X = \alpha^2 \left(\frac{m_q}{\Lambda_{QCD}} \right)^{-0.09}$$

GUT models :

Calmet, Fritzsch; Langecker, Segre, Strassler

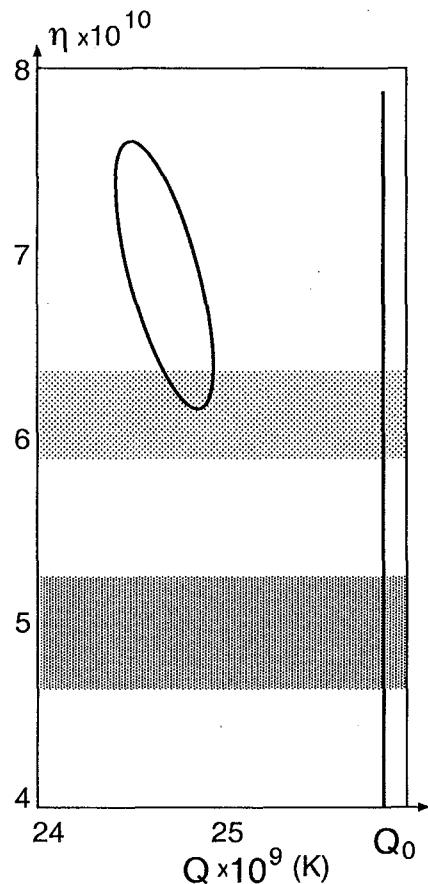
$$\frac{\delta(m/\Lambda_{QCD})}{m/\Lambda_{QCD}} \sim 35 \frac{\delta \alpha}{\alpha}$$

Weak / strong variation
may be more important !



Big Bang Nucleosynthesis

(Dmitriev, Flambaum, Webb)



Productions of D, ${}^4\text{He}$, ${}^7\text{Li}$
are exponentially sensitive to
deuteron binding energy E_d

$$\sim e^{-\frac{E_d}{T_f}}$$

- η from cosmic microwave background fluctuations (η - barion to photon ratio).
- η from BBN for present value of Q ($Q = |E_d|$)

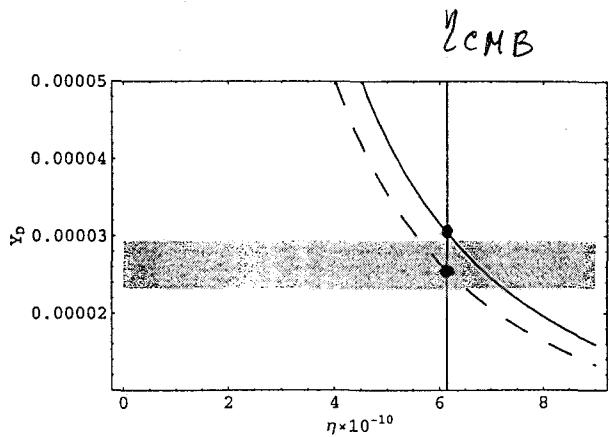


FIG. 10: The deuteron abundances as a functions of η at two different deuteron binding energies. The solid line corresponds to $Q_{BBN} = 24.87$ K. The dashed line corresponds to the modern value $Q = 25.82$ k.

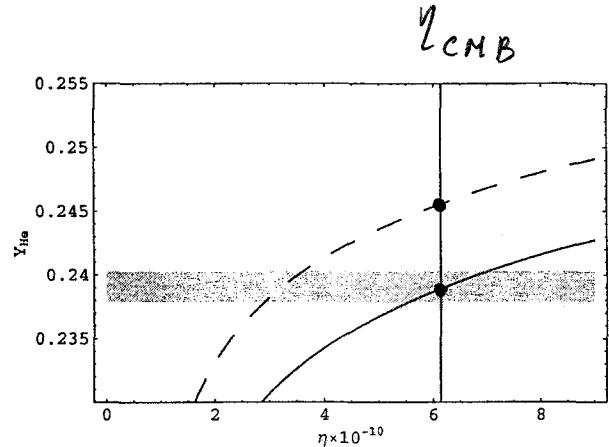


FIG. 11: The helium abundances as a functions of η at two different deuteron binding energies. The solid line corresponds to $Q_{BBN} = 24.87$ K. The dashed line corresponds to the modern value $Q = 25.82$ k.

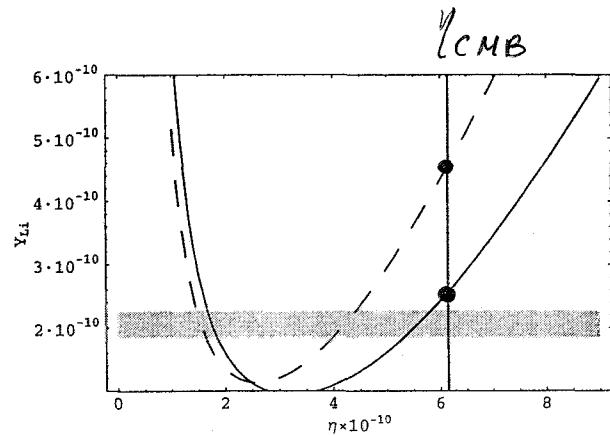
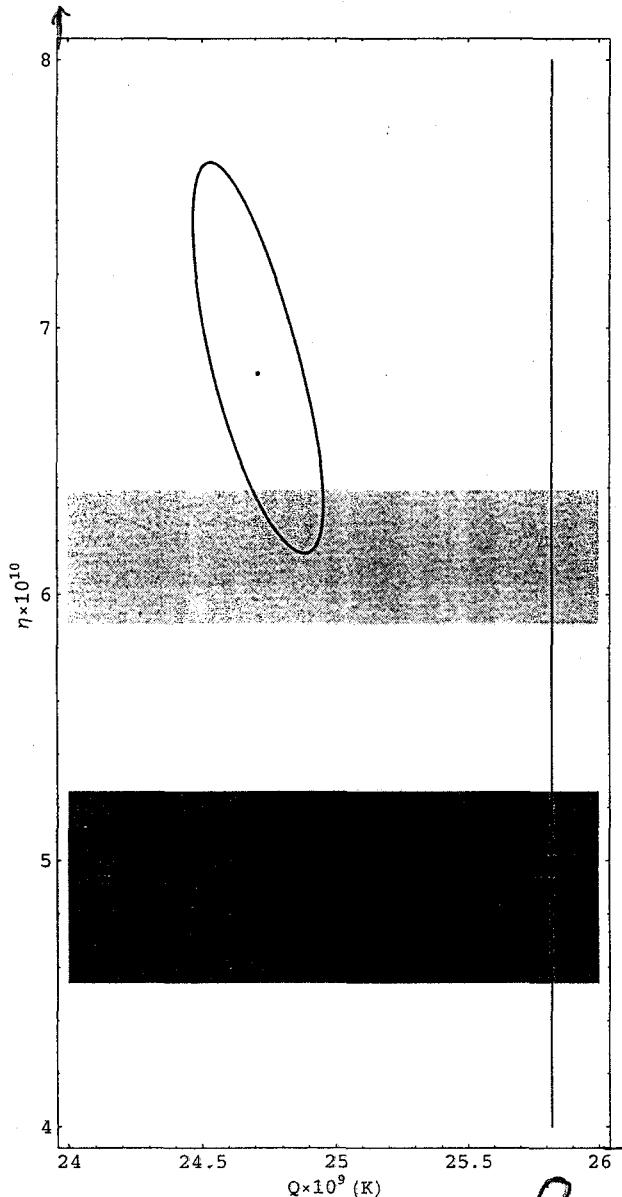


FIG. 12: The lithium abundances as a functions of η at two different deuteron binding energies. The solid line corresponds to $Q_{BBN} = 24.87$ K. The dashed line corresponds to the modern value $Q = 25.82$ k.

- Variation of deuteron binding energy
→ radical improvement for BBN and CMB

$$\eta = \frac{\text{barion number}}{\text{photon number}}$$



BBN results for
variable deuteron binding

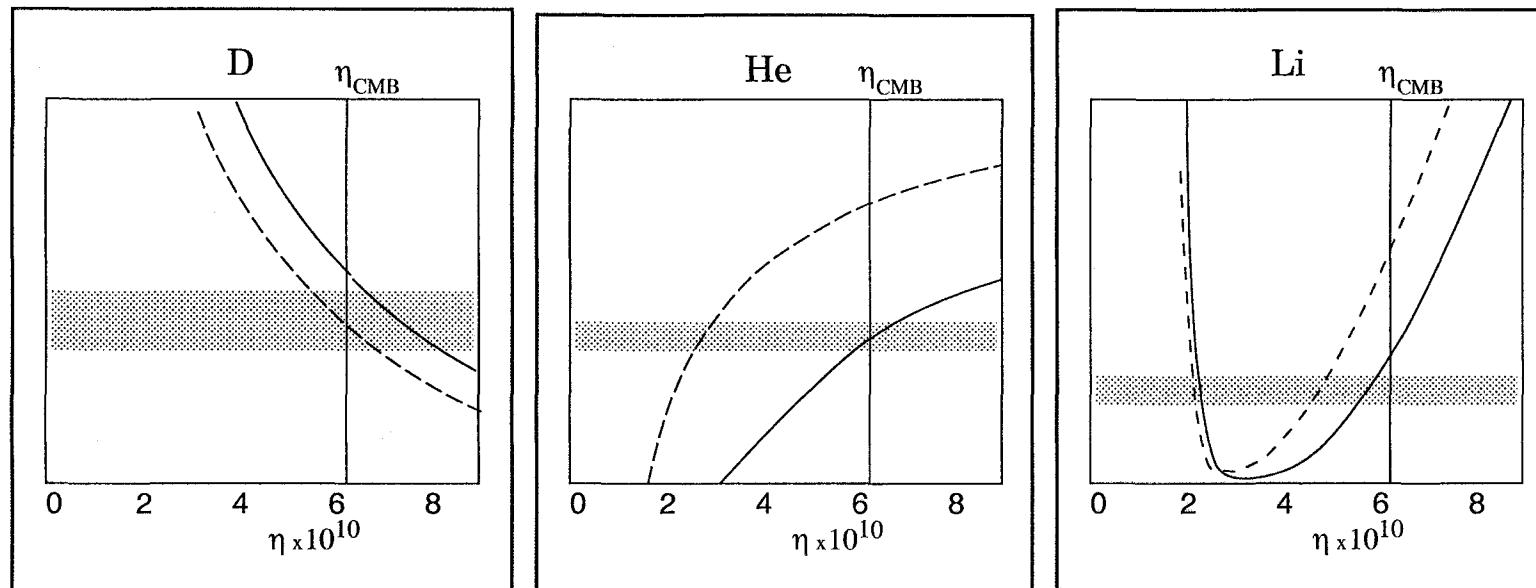
η from Cosmic Microwave
Background fluctuations

η from Big Bang

Nucleosynthesis for
present value of deuteron
binding

Deuteron binding energy
 $Q_{\text{present}} = |F_d|$

FIG. 9: The 1σ -range for the total likelihood function in $\eta - Q$ plane.



Comparison with observations gives

$$\frac{\delta E_d}{E_d} = -0.019 \pm 0.005$$

This also leads to agreement

$$\eta(\text{BBN}) \approx \eta(\text{CMB})$$

Previous limits on $\Delta\alpha/\alpha$: The Oklo bound

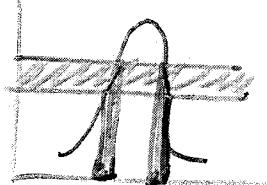
- Heavy nuclei have very sharp resonances in their neutron absorption cross-section.
- Thus, abundances of decay products of ^{235}U fission give constraints on variation in the nuclear energies and thus a constraint on $\Delta\alpha/\alpha$.
- 1976: Shylakter first analyzed Samarium abundances from Oklo to constrain $\Delta\alpha/\alpha$.
- 1996: Damour & Dyson re-analyzed the same data to obtain a stronger constraint: $\Delta\alpha/\alpha < 1 \times 10^{-7}$.
- 2000: Fujii et al. find $\Delta\alpha/\alpha = (-0.04 \pm 0.15) \times 10^{-7}$ from new data.

OKLO Natural Nuclear Reactor

- active $1.8 \cdot 10^9$ years ago

Measured $n + {}^{149}\text{Sm} \rightarrow {}^{150}\text{Sm}$
capture cross-section determined
by position of the low-lying
resonance

6'



Two solutions for
each resonance

E_r

$$E_r =$$

{ Our calculation: $\delta E_r = 1.7 \cdot 10^8 \text{ eV} \frac{\delta(m_s/N)}{(m_s/N)}$

{ Positions of resonance: Shlykhter;
Damour, Dyson; Fujii et. al.

$$\left| \frac{\delta(m_s/N)}{m_s/N} \right| < 1.2 \cdot 10^{-10}$$

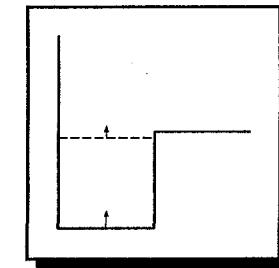
or $\frac{\delta(m_s/N)}{m_s/N} = (-0.56 \pm 0.05) \cdot 10^{-9}$

S. Lamoreaux: no zero solutions!

Flambaum, Shuryak: Deuteron Binding Energy is very sensitive to variation of *strange* quark mass (4 factors of enhancement):

1. Deuteron is a shallow bound level.

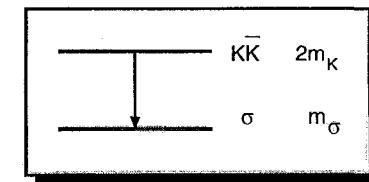
Virtual level in $n+p \rightarrow d+\gamma$ is even more sensitive to the variation of the potential.



2. Strong compensation between σ -meson and ω -meson exchange in potential (Walecka model): $4\pi rV = -g_s^2 e^{-m_\sigma r} + g_v^2 e^{-m_\omega r}$

$$3. \sigma = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}), \quad m_\sigma \approx m_s + 2\Lambda_{QCD}$$

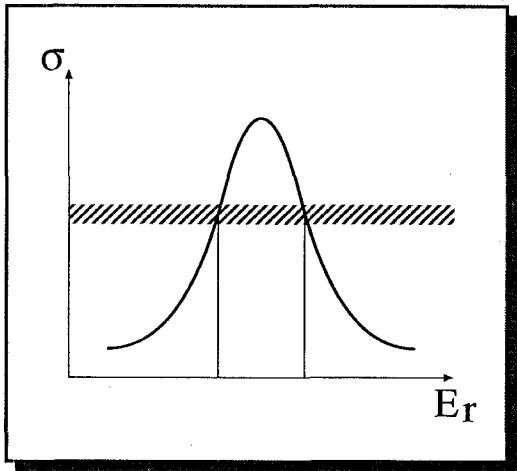
4. Repulsion of σ from $K\bar{K}$ threshold



$$\text{Total } \frac{\delta E_d}{E_d} \approx -17 \frac{\delta m_s}{m_s} \text{ and } \frac{\delta(m_s/\Lambda_{QCD})}{m_s/\Lambda_{QCD}} = (+1.1 \pm 0.3) \times 10^{-3}$$

Oklo natural nuclear reactor

S. Lamoreaux and J. Torgerson, nucl-th/0309048 *PRD(2004)*



$n + ^{149}\text{Sm} \rightarrow ^{150}\text{Sm}$ cross-section.

Two solutions for the change of the resonance position E_r :

1. $\Delta E_r = (-135 \pm 5) \times 10^{-3} \text{ eV}$
2. $\Delta E_r = (-58 \pm 5) \times 10^{-3} \text{ eV}$

V. Flambaum and E. Shuryak, hep-ph/0212403: *PRD (2004)*

$$\Delta E_r = 1.7 \times 10^8 \text{ eV} \frac{\delta(m_s/\Lambda)}{m_s/\Lambda}$$

1. $\frac{\delta(m_s/\Lambda)}{m_s/\Lambda} = (-0.80 \pm 0.03) \times 10^{-9}$
2. $\frac{\delta(m_s/\Lambda)}{m_s/\Lambda} = (-0.34 \pm 0.03) \times 10^{-9}$

Laboratory experiments - atomic clocks

Valuable way to study variation of the fundamental constants due to huge progress in atom cooling and trapping and precise frequency measurements.

Advantages:

1. Very narrow lines (metastable states), sensitivity in $\frac{\Delta\alpha}{\alpha}$ is up to 10^{-18} per year.
2. Larger Z leads to larger q (up to 60000 cm^{-1}).

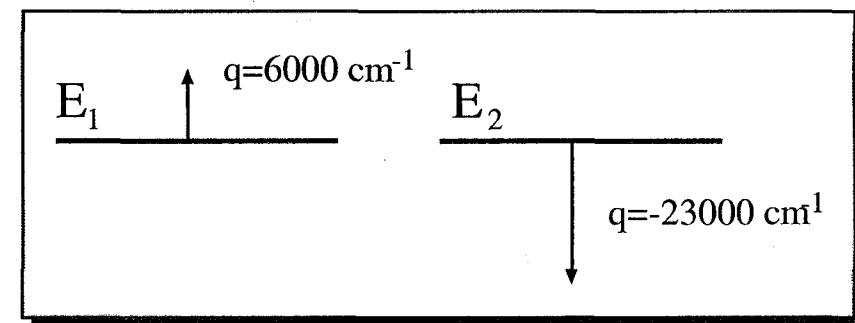
There is a possibility of $\sim 10^8$ times enhancement due to “degenerate” energy levels of different nature!

Example: Dy atom.

$$4f^{10}5d6s, \quad E_1 = 19797.96\ldots \text{ cm}^{-1}$$

$$4f^95d^26s, \quad E_2 = 19797.96\ldots \text{ cm}^{-1}$$

$$\text{Interval } \omega = E_2 - E_1 \sim 10^{-4} \text{ cm}^{-1} \sim 10^{-9} E_1.$$



$$\frac{\Delta\omega}{\omega} = \frac{q_1 - q_2}{\omega} 2 \frac{\Delta\alpha}{\alpha} = 6 \times 10^7 \frac{\Delta\alpha}{\alpha}!$$

Nguyen et AL, physics/0308104:

Sensitivity for Dy $|\dot{\alpha}/\alpha| \sim 10^{-18} \text{ yr}^{-1}$.

Optical atomic clocks

TABLE II: Experimental energies and calculated q coefficients for transitions from the ground state to the state shown.

Atom/Ion	Z	State	Wavelength, Å	q (cm^{-1})	Reference
		Experiment			
Al II	13	$3s3p$	3P_0	2674.30	146
		$3s3p$	3P_1	2669.95	211
		$3s3p$	3P_2	2661.15	343
		$3s3p$	1P_1	1670.79	278
Ca I	20	$4s4p$	3P_0	6597.22	125
		$4s4p$	3P_1	6574.60	180
		$4s4p$	3P_2	6529.15	294
		$4s4p$	1P_1	4227.92	250
Sr I	38	$5s5p$	3P_0	6984.45	443
		$5s5p$	3P_1	6894.48	642
		$5s5p$	3P_2	6712.06	1084
		$5s5p$	1P_1	4608.62	924
Sr II	38	$4d$	$^2D_{3/2}$	6870.07	2828
		$4d$	$^2D_{5/2}$	6740.25	3172
In II	49	$5s5p$	3P_0	2365.46	3787
		$5s5p$	3P_1	2306.86	4860
		$5s5p$	3P_2	2182.12	7767
		$5s5p$	1P_1	1586.45	6467
Ba II	56	$5d$	$^2D_{3/2}$	20644.74	5844
		$5d$	$^2D_{5/2}$	17621.70	5976
Dy I	66	$4f^{10}5d6s$	$^3[10]_{10}$	5051.03	6008
		$4f^95d^26s$	$^9K_{10}$	5051.03	-23708
Yb I	70	$6s6p$	3P_0	5784.21	2714
		$6s6p$	3P_1	5558.02	3527
		$6s6p$	3P_2	5073.47	5883
		$6s6p$	1P_1	3989.11	4951
Yb II	70	$4f^{14}5d$	$^2D_{3/2}$	4355.25	10118
		$4f^{14}5d$	$^2D_{5/2}$	4109.70	10397
		$4f^{13}6S^2$	$^2F_{7/2}$	4668.81	-56737
Yb III	70	$4f^{13}5d$	3P_0	2208.63	-27800
		$6s6p$	3P_0	2656.39	15299
Hg I	80	$6s6p$	3P_1	2537.28	17584
		$6s6p$	3P_2	2270.51	24908
		$6s6p$	1P_1	1849.50	22789
		$5d^96s^2$	$^2D_{5/2}$	2815.79	-56671
Tl II	81	$5d^96s^2$	$^2D_{3/2}$	1978.16	-44003
		$6s6p$	3P_0	2022.20	16267
		$6s6p$	3P_1	1872.90	18845
		$6s6p$	3P_2	1620.09	33268
Ra II	88	$6s6p$	1P_1	1322.75	29418
		$6d$	$^2D_{3/2}$	8275.15	18785
		$6d$	$^2D_{5/2}$	7276.37	17941

Atomic clocks based on hyperfine transitions:

- i) variation of α
- ii) variation of nuclear magnetic moments
(Savely Karshenboim).

Calculations:

Ratio of $\boxed{^{199}\text{Hg}^+ / \text{H}}$ hyperfine transition frequencies

$$\frac{\delta [A(\text{Hg}^+) / A(\text{H})]}{[A(\text{Hg}^+) / A(\text{H})]} = 2.3 \frac{\delta \alpha}{\alpha} - 0.02 \frac{\delta [m_q / \Lambda_{\text{QCD}}]}{m_q / \Lambda_{\text{QCD}}}$$

sensitive to $\delta \alpha$ only!

$\boxed{^{133}\text{Cs} / ^{87}\text{Rb}}$ hyperfine transitions

$$\frac{\delta [A(\text{Cs}) / A(\text{Rb})]}{[A(\text{Cs}) / A(\text{Rb})]} = 0.49 \frac{\delta \alpha}{\alpha} + 0.17 \frac{\delta [m_q / \Lambda_{\text{QCD}}]}{m_q / \Lambda_{\text{QCD}}}$$

sensitive to variation of quark mass and strong interactions.

$\boxed{^{133}\text{Cs} / \text{H}}$ hyperfine transitions

$$\frac{\delta [A(\text{Cs}) / A(\text{H})]}{[A(\text{Cs}) / A(\text{H})]} = 0.8 \frac{\delta \alpha}{\alpha} + 0.2 \frac{\delta [m_q / \Lambda_{\text{QCD}}]}{m_q / \Lambda_{\text{QCD}}}$$

$\boxed{^{133}\text{Cs (hyperfine)} / \text{Hg} (\lambda = 282\text{nm})}$ optical

$$\frac{\delta [A(\text{Cs}) / E(\text{Hg})]}{[A(\text{Cs}) / E(\text{Hg})]} = 6 \frac{\delta \alpha}{\alpha} + 0.1 \frac{\delta [m_q / \Lambda_{\text{QCD}}]}{m_q / \Lambda_{\text{QCD}}} + \frac{\delta [m_e / \Lambda_{\text{QED}}]}{m_e / \Lambda_{\text{QED}}}$$

Measurements

$$\frac{1}{\alpha} \frac{d\alpha}{dt} (10^{-7}/\text{year})$$

Marion et.al. 2003	$\frac{Rb(\text{hfs})}{Cs(\text{hfs})}$	$(0.05 \pm 1.3) \frac{\text{m}_q}{\Lambda}$
Bize et.al. 2003	$\frac{Hg^+(\text{opt})}{Cs(\text{hfs})}$	$(-0.03 \pm 1.2) \frac{\text{m}_q}{\Lambda}$
Fisher et.al. 2004	$\frac{H(\text{opt})}{Cs(\text{hfs})}$	$(-1.1 \pm 2.3) \frac{\text{m}_q}{\Lambda}$
Peik et.al 2004	$\frac{Yb^+(\text{opt})}{Cs(\text{hfs})} + \uparrow$	-0.2 ± 2.0

Fisher et.al 2004 combination $\frac{Rb}{Cs}, Hg^+(\text{opt})/Cs$

$$\frac{d \ln(N_{Re}/Cs)}{dt} = (-0.7 \pm 1.7) \cdot 10^{-15} / \text{year}$$

$$\rightarrow \frac{d \ln(m_q/\Lambda_{QCD})}{dt} = (-4 \pm 10) \cdot 10^{-15} / \text{year}$$

$$\frac{d \ln \alpha}{dt} = (-0.9 \pm 2.9) \cdot 10^{-15} / \text{year}$$

Conclusions

MM method provided increase of sensitivity ~ 100 times.
Larger effect, larger statistics, all observed lines can be used -
we can study variation from first quasars to present time.
Anchors, positive and negative shifters - control of systematics.

Keck data (3 independent samples!) - varying α ,
VLT data - no variation.

Undiscovered systematic effect? Spatial variation?

21 cm hydrogen/mm molecules - no variation at $z \approx 0.68$.

BBN/CMB data may be interpreted as variation of
 m_q/Λ_{QCD} (4σ if there is no other explanation).

Oklo data- strong interaction dominates in nuclei, interpretation in terms of variation of m_q/Λ_{QCD} .

Atomic clocks are sensitive to variation of α and m/Λ_{QCD} .
Transition between close levels with different q - a billion times enhancement.