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Fundamentals of Earth Sources

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Fundamentals of earthquake source

ICTP Course 2004 Trieste

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Content

What parameter control earthquakes ?

1. Concept of equivalent body forces and earthquake parameter
2. Point source parameter
3. Source time function
4. Source spectra
5. Stress drop

Landers, 28th June 1992, M=7.3

- ≈ 10 Mio USD infra-structural damage, ≈ 4500 damaged buildings
- $34°13'$ N, $116°26'$ W, right-lateral strike-slip
- 85 km rupture length, $\langle \Delta u \rangle = 4\,m$, $\Delta u_{\text{max}} = 6\,m$
- $> 10^5$ located aftershocks in one year

Example for static and dynamic triggering of earthquakes, for fault step-over and verification of dynamic rupture models

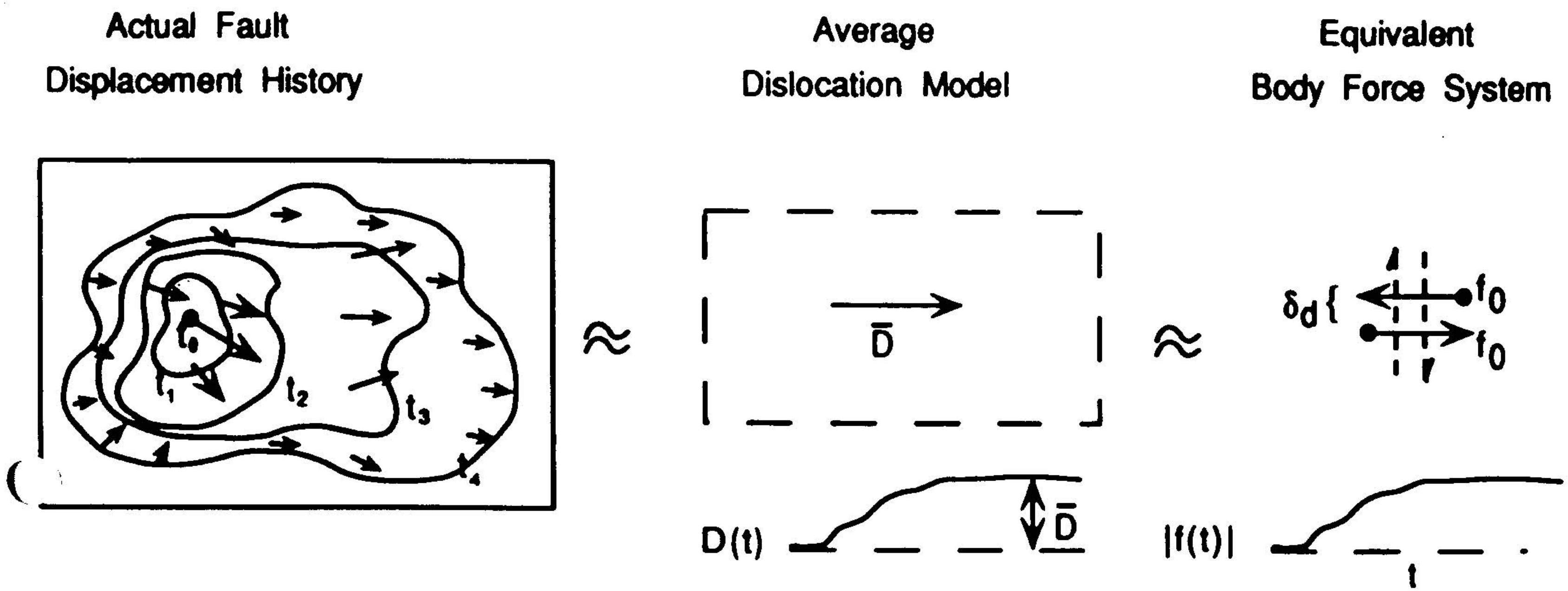
Chi-Chi, Taiwan, 21. Sept. 1999, M=7.6

- 2470 death toll, 11.305 injured persons, 100.000 damaged buildings
- Largest dataset worldwide of near-flield accelerograms from strong earthquake (about 650 3-component and 74 6-component station)
- ”extreme large ground accelerations”, $\dot{u}_{\text{max}} = 3.28\, m/s$, $\ddot{u}_{\text{max}} \geq 1\, g$
- $23, 85^\circ$ N, 120.82° E, oblique reverse faulting
- $100x40\, km$ rupture, $\langle \Delta u \rangle = 8\, m$, $\Delta u_{\text{max}} = 12\, m$
- complex rupture with ”jumping dislocations”, rupturing of asperities and barriers
- $> 10^4$ aftershocks in one year, triggered aftershocks, foreshocks

How does earthquake-rupture work?

1. rupture initiates at the nucleation point
2. a rupture front propagates rapidly over the fault surface
3. high slip-rate occurs at and behind the rupture front
4. rupture may jump between neighbouring faults
5. asperities on the fault may generate inhomogeneous slip distribution
6. abrupt stopping of the rupture front and possible reflection and backward propagation
7. nonuniform radiation of elastic waves depending on fracture mode and fault orientation as well as directivity effects

Earthquakes and body forces

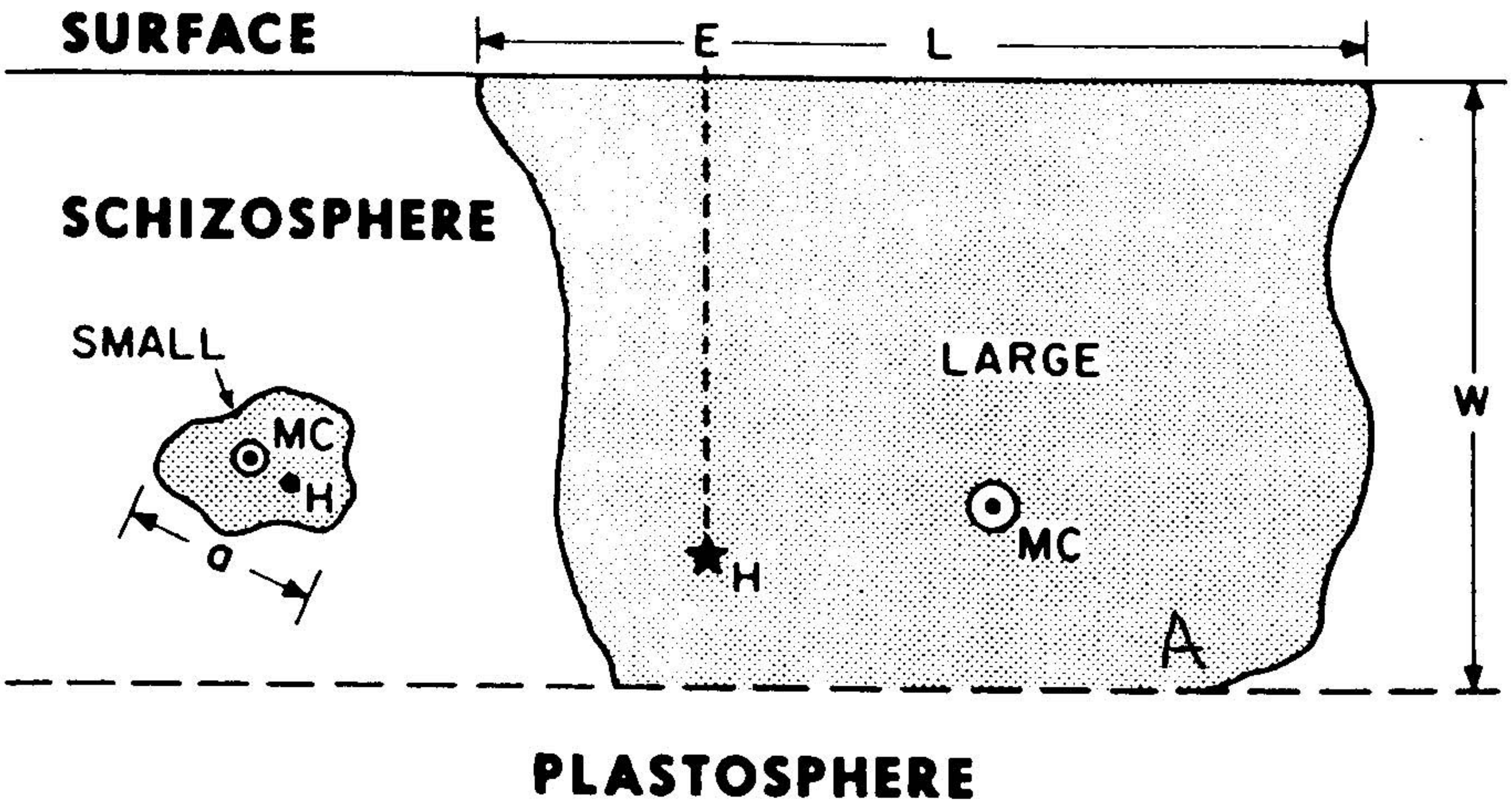


Faulting involves complex cracking and rupturing resulting in a space-time history of slipping motion. The process can be approximated by a dislocation model with dislocation time history $D(t)$. The dislocation model can be idealised by an equivalent force system.

Earthquake parameter

1. time and location of the rupture initiation
2. time and location of the centroid
3. fault plane orientation and dimension
4. average slip vector
5. slip and rupture history

Small and large earthquake location



Hypocenter (H): t^0 and x_k^0 by fitting arrival-times.

Moment centroid (MC): $\Delta\tau = \int\limits_{-\infty}^{\infty} (\tau - \tau^0) f(\tau) d\tau = 0.$ and

$\Delta\xi_k = \int\limits_V (\xi_k - \xi_k^0) g(\xi) dV = 0$ by fitting waveforms.

Earthquake magnitude and moment

Seismic moment: $M_0 = \mu \langle \Delta u \rangle A$

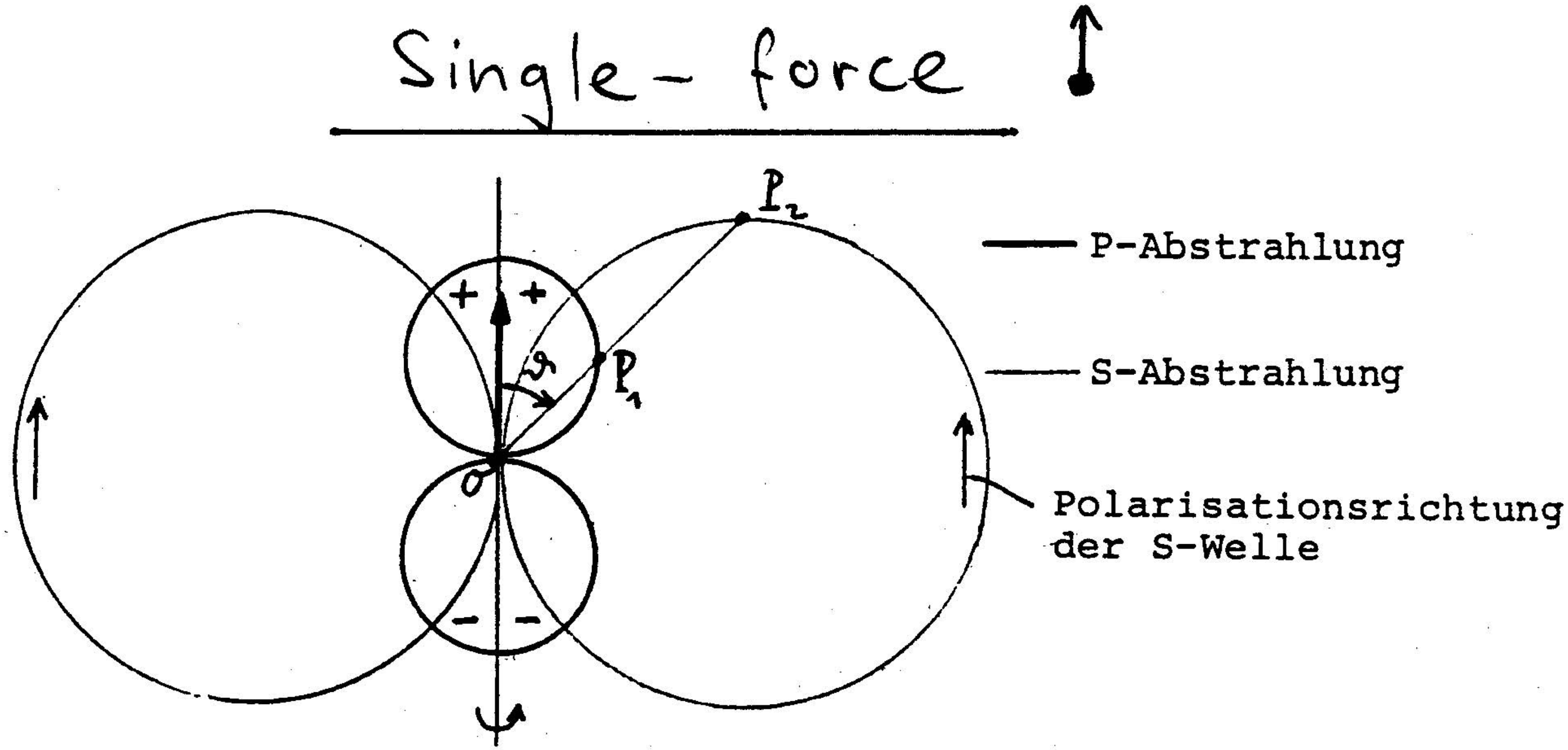
(rectangular planes for large earthquakes, i.e. $A = LW$)

Moment magnitude $M_W = \log M_0 / 1.5 - 10.73$ (in dyne cm).

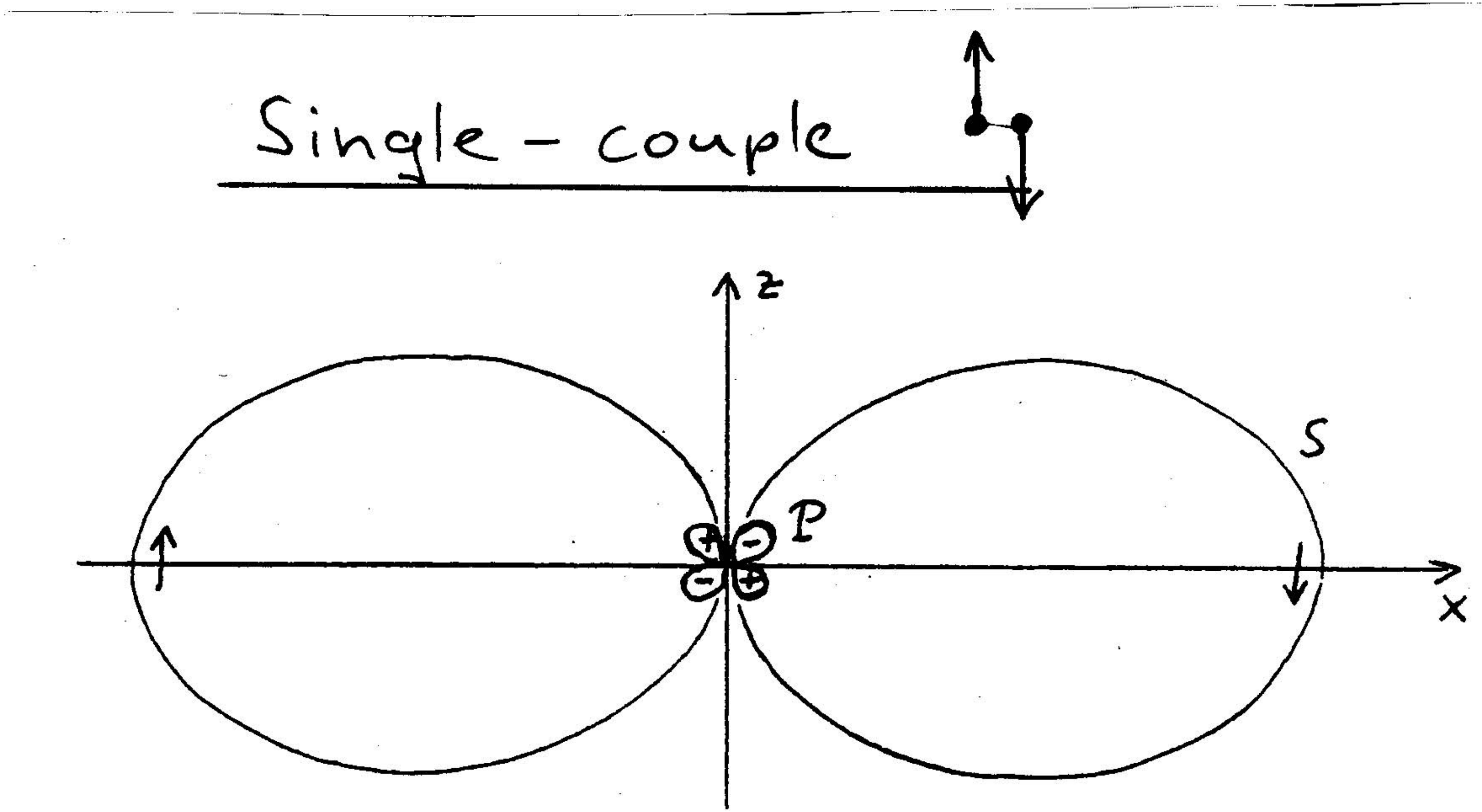
Examples:

event	$A\ (km^2)$	$\langle \Delta u \rangle\ (m)$	M_S	M_W	$M_0\ (dnecm)$
Loma Prieta 1989	40 x 15	1.7	7.1	6.9	$3.0 \cdot 10^{26}$
San Francisco 1906	450 x 10	4	7.8	7.8	$5.4 \cdot 10^{27}$
Alaska 1964	500 x 300	7	8.4	9.1	$5.2 \cdot 10^{29}$
Chile 1960	800 x 200	21	8.3	9.5	$2.4 \cdot 10^{30}$

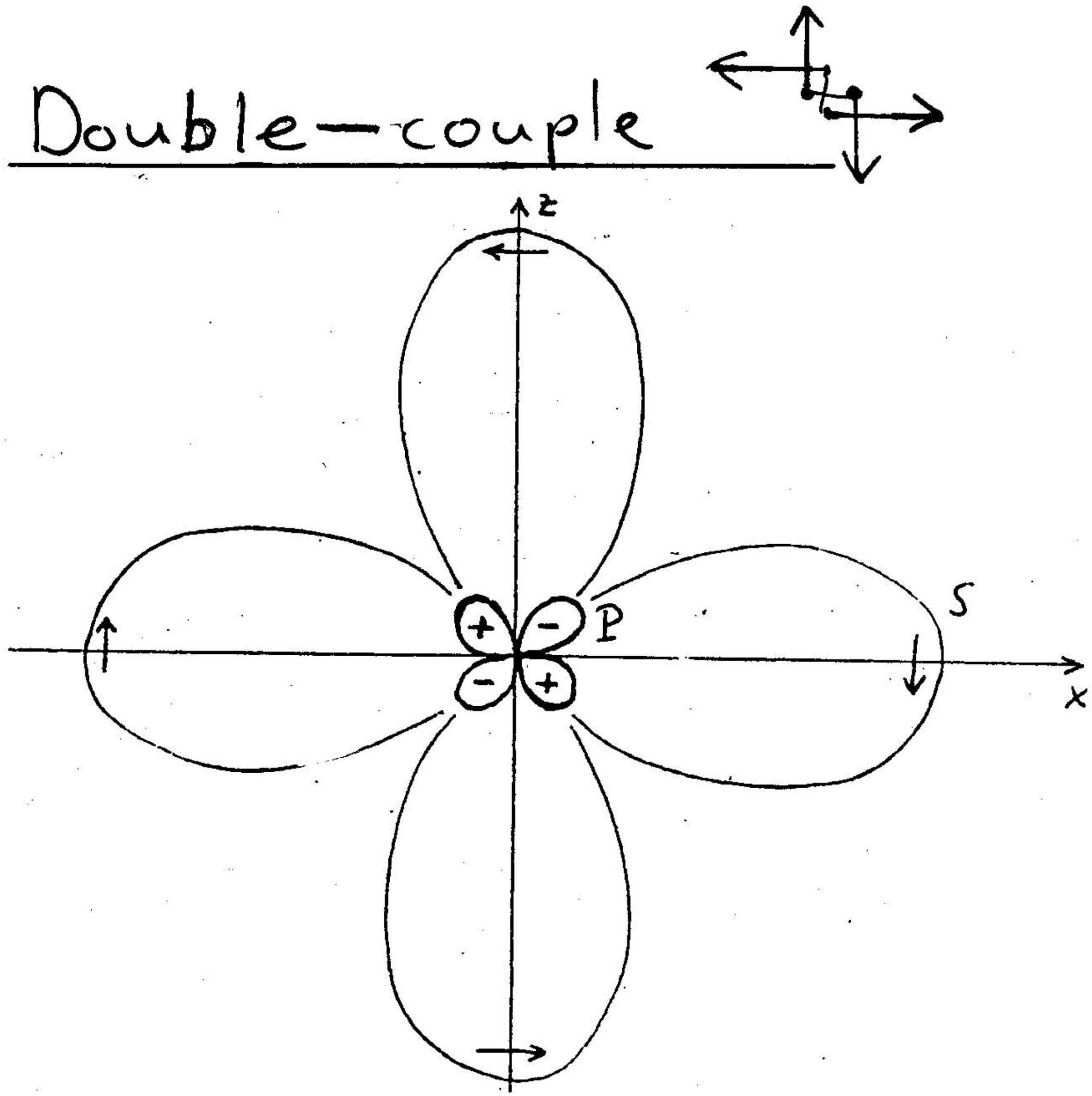
Single force radiation pattern



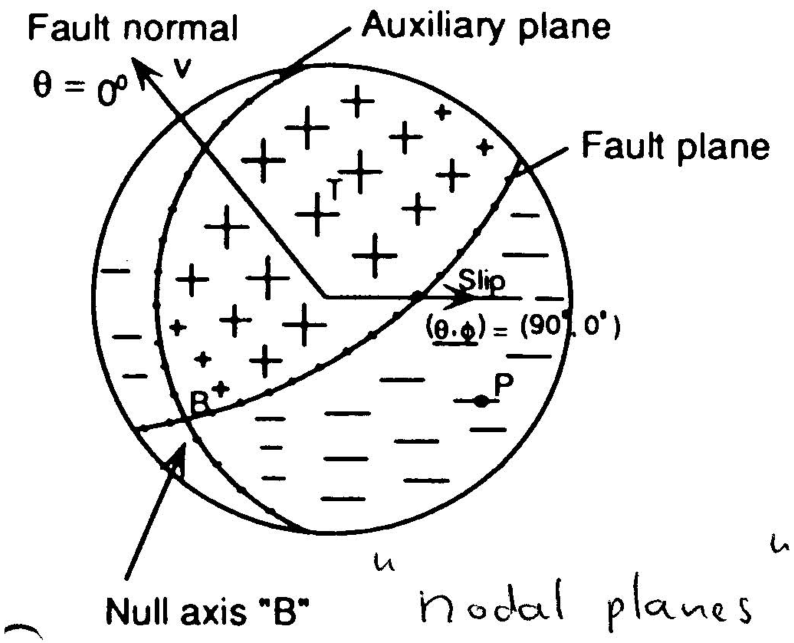
Single couple radiation pattern

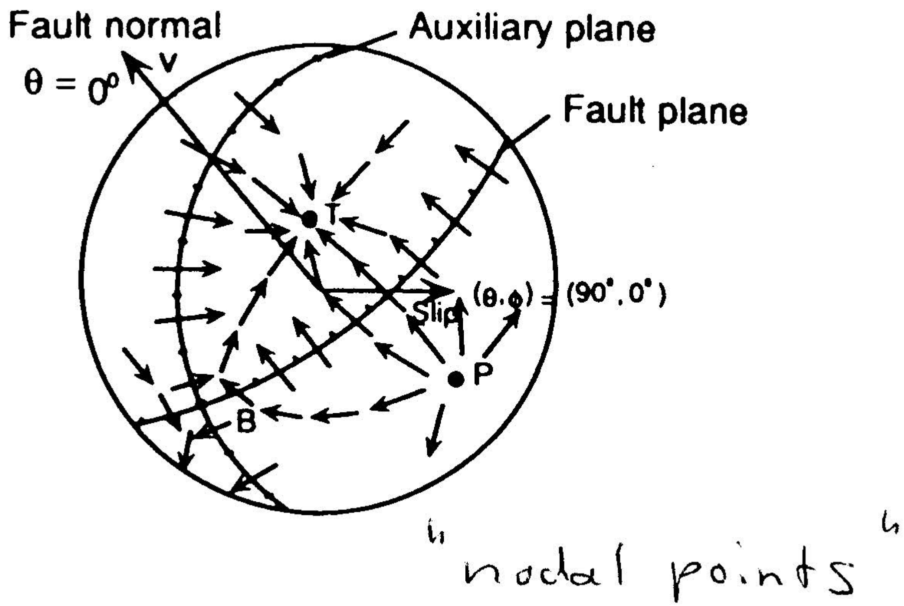


Double couple radiation pattern



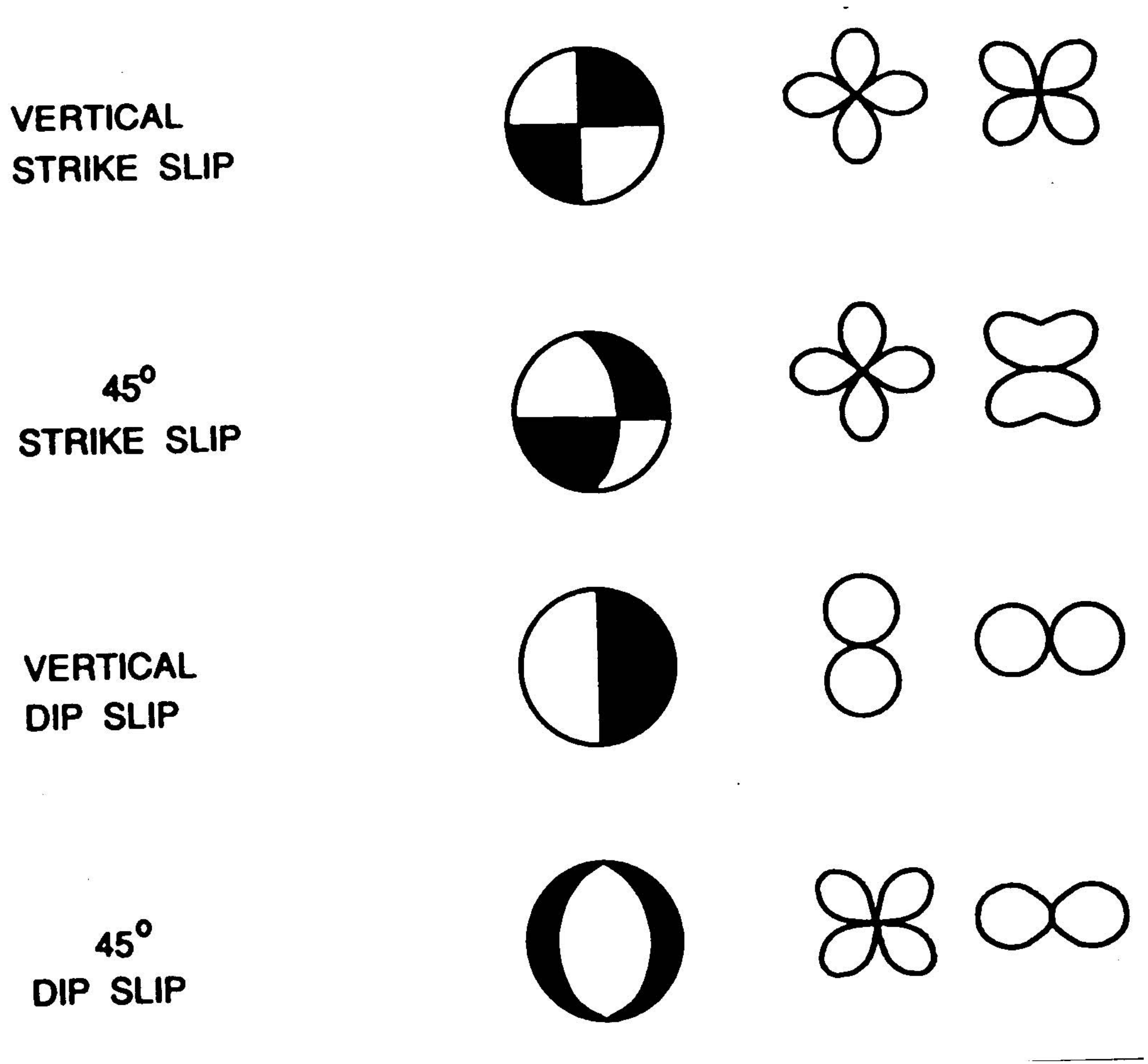
Earthquake radiation pattern





- two orthogonal nodal planes for P
- three nodal points for S
- S-waves are large where P-waves are small
- ambiguity between fault and auxiliary plane

Surface wave radiation pattern



Far-field body-wave representation

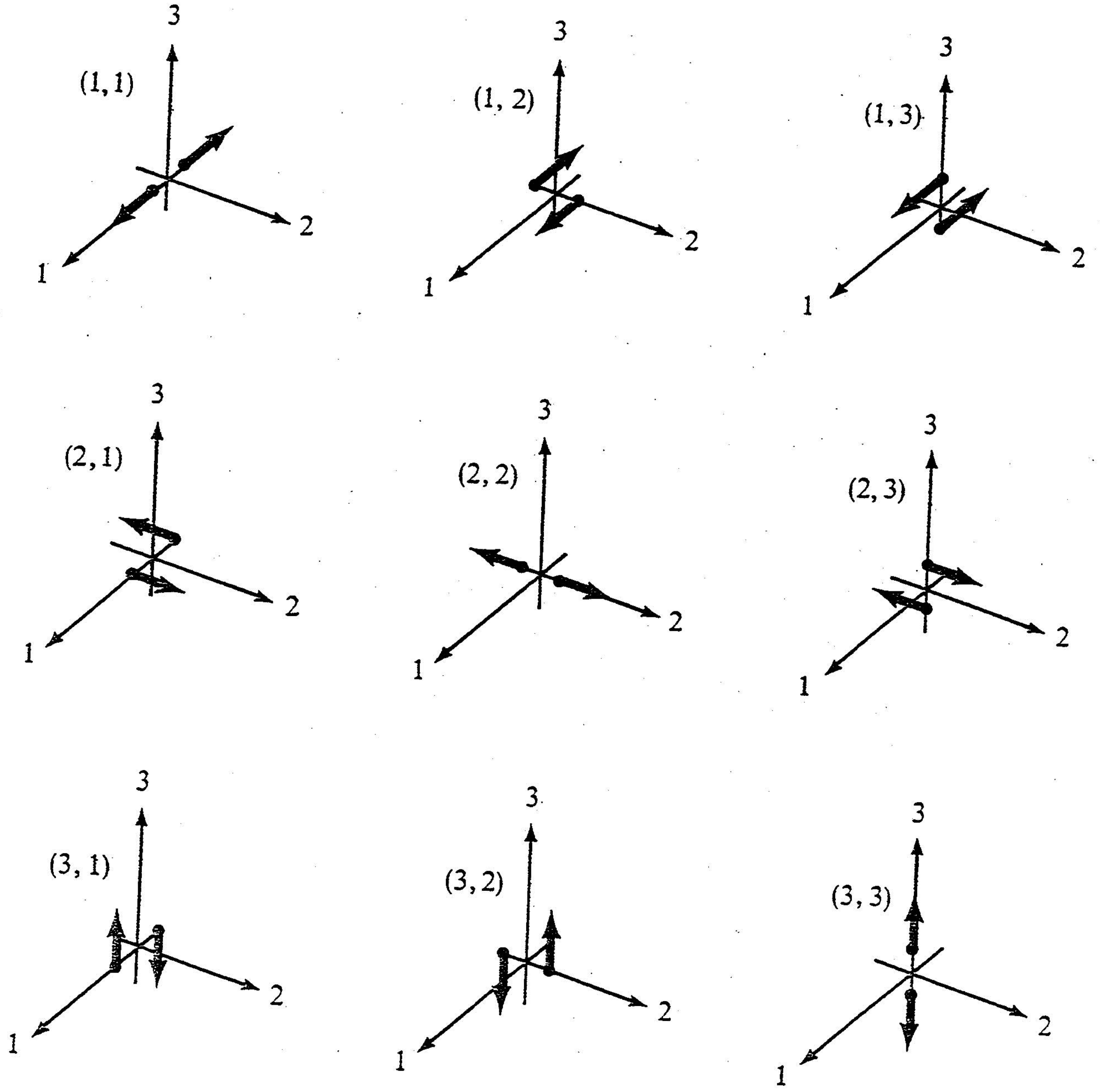
$$u_n(\mathbf{x},t) \approx M_{pq}\,G_{np}(\mathbf{x},t)\,\frac{\gamma_q}{c}$$

with

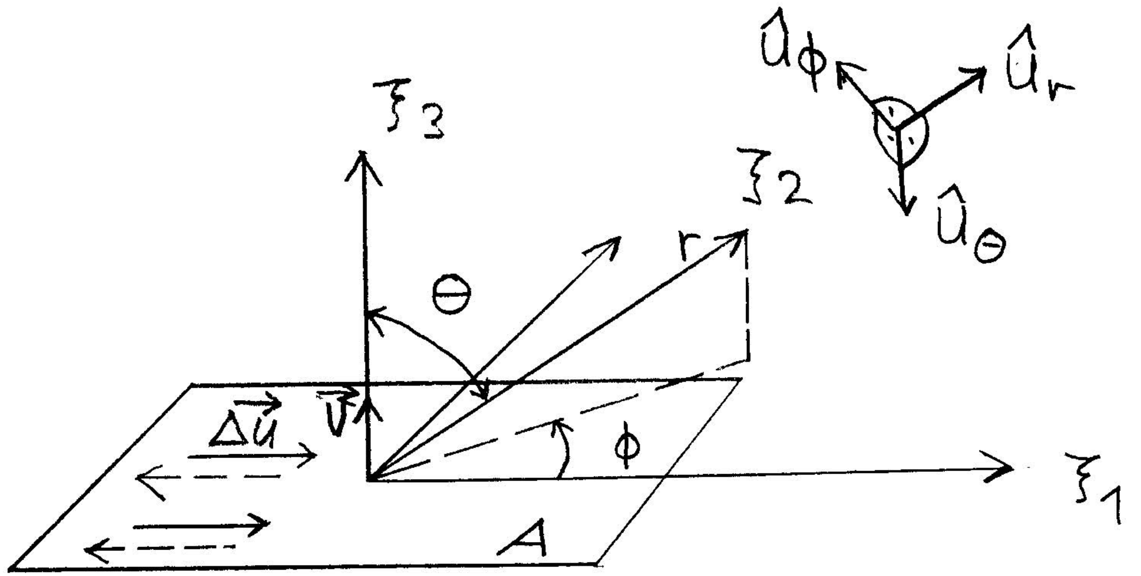
$\mathbf{u}$	$=$	ground displacement
$\mathbf{M}$	$=$	moment tensor
$\mathbf{G}$	$=$	Green tensor
γ	$=$	direction cosine x_q/r
$\mathbf{x}$	:	spatial vector measured from source origin
t	:	time measured from origin time
c	:	wave velocity

(spatial-temporal point source and body-waves assumed!)

Generalised force dipoles



Slip on horizontal plane



$$\begin{pmatrix} 0 & 0 & M_0 \\ 0 & 0 & 0 \\ M_0 & 0 & 0 \end{pmatrix}$$

$$M_{pq} = M_0(\nu_p \Delta\hat{u}_q + \nu_q \Delta\hat{u}_p)$$

Homogeneous full space Green function

$$4\pi\rho G_{np} = \gamma_n\gamma_p \frac{\delta(t - r/\alpha)}{\alpha^2 r} + (-\gamma_n\gamma_p + \delta_{np})\frac{\delta(t - r/\beta)}{\beta^2 r}$$

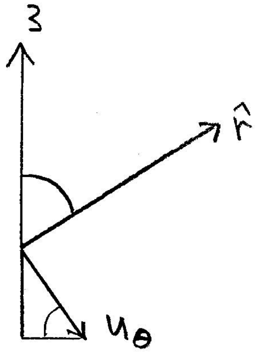
This leads for P-waves to

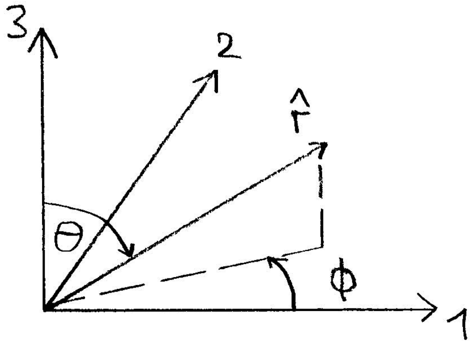
$$\begin{aligned} 4\pi\rho u_n^{(P)} &= 4\pi\rho M_0 \left(G_{n1}\frac{\gamma_3}{\alpha} + G_{n3}\frac{\gamma_1}{\alpha}\right) \\ &= 2\gamma_n\gamma_1\gamma_3 \frac{M_0\delta(t - r/\alpha)}{\alpha^3 r} \end{aligned}$$

and for S-waves to

$$4\pi\rho u_n^{(S)} = (-2\gamma_n\gamma_1\gamma_3 + \delta_{n1}\gamma_3 + \delta_{n3}\gamma_1)\frac{M_0\delta(t - r/\beta)}{\beta^3 r}$$

P-radiation in spherical coordinates





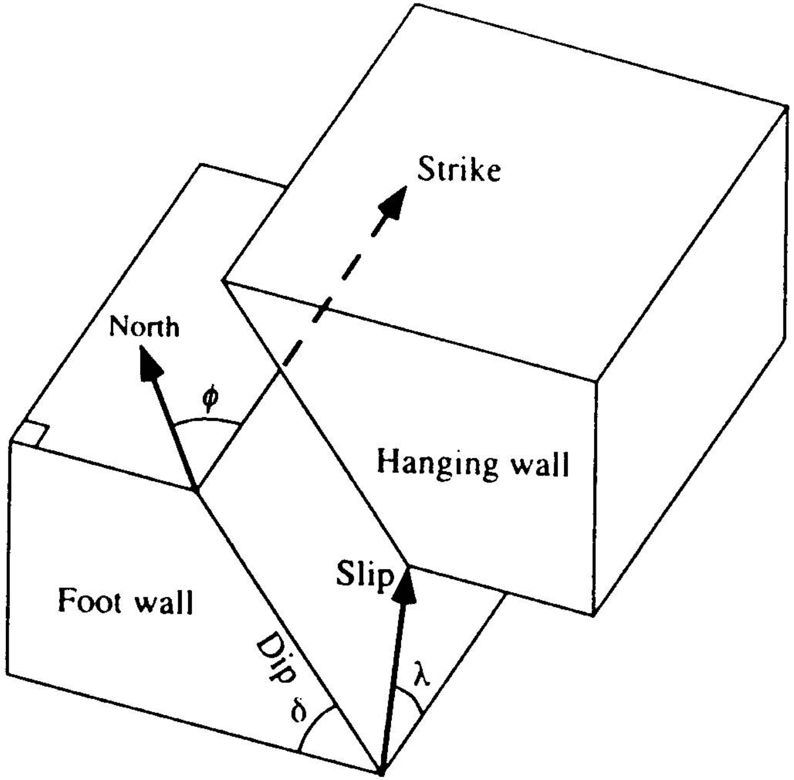
$$x_1 = r \sin\Theta cos\Phi = r\gamma_1$$
$$x_2 = r \sin\Theta \sin\Phi = r\gamma_2$$
$$x_3 = r \cos\Theta = r\gamma_3$$

or

$$\hat{r}_n = \gamma_n$$

$$u_n \sim 2\gamma_n\gamma_1\gamma_3 = \hat{r}_n 2 \sin\Theta \cos\Theta cos\Phi = \hat{r}_n \sin 2\Theta \cos\Phi$$

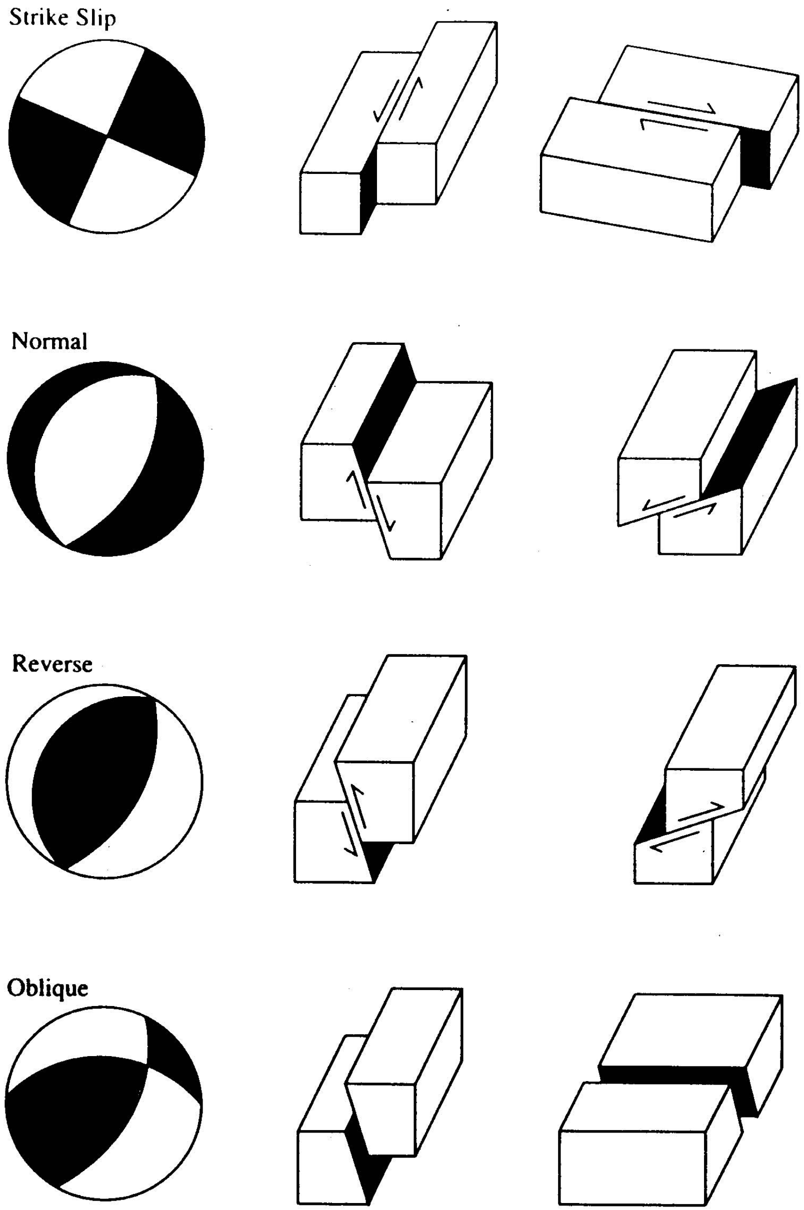
Fault plane parameter



Strike Φ ($0°$ - $360°$)
Dip δ ($0°$ - $90°$)
Rake λ ($-180°$ - $180°$)

reverse faulting: upward movement of hanging wall ($\lambda > 0°$)
normal faulting: downward movement of h.w. ($\lambda < 0°$)
strike slip: right lateral and left lateral
oblique faulting:
thrust and overthrust: ($\delta < 45°$)

Basic fault types



Effects of extended fault and rupture

1. Far-field body-wave representation of spatial point source:

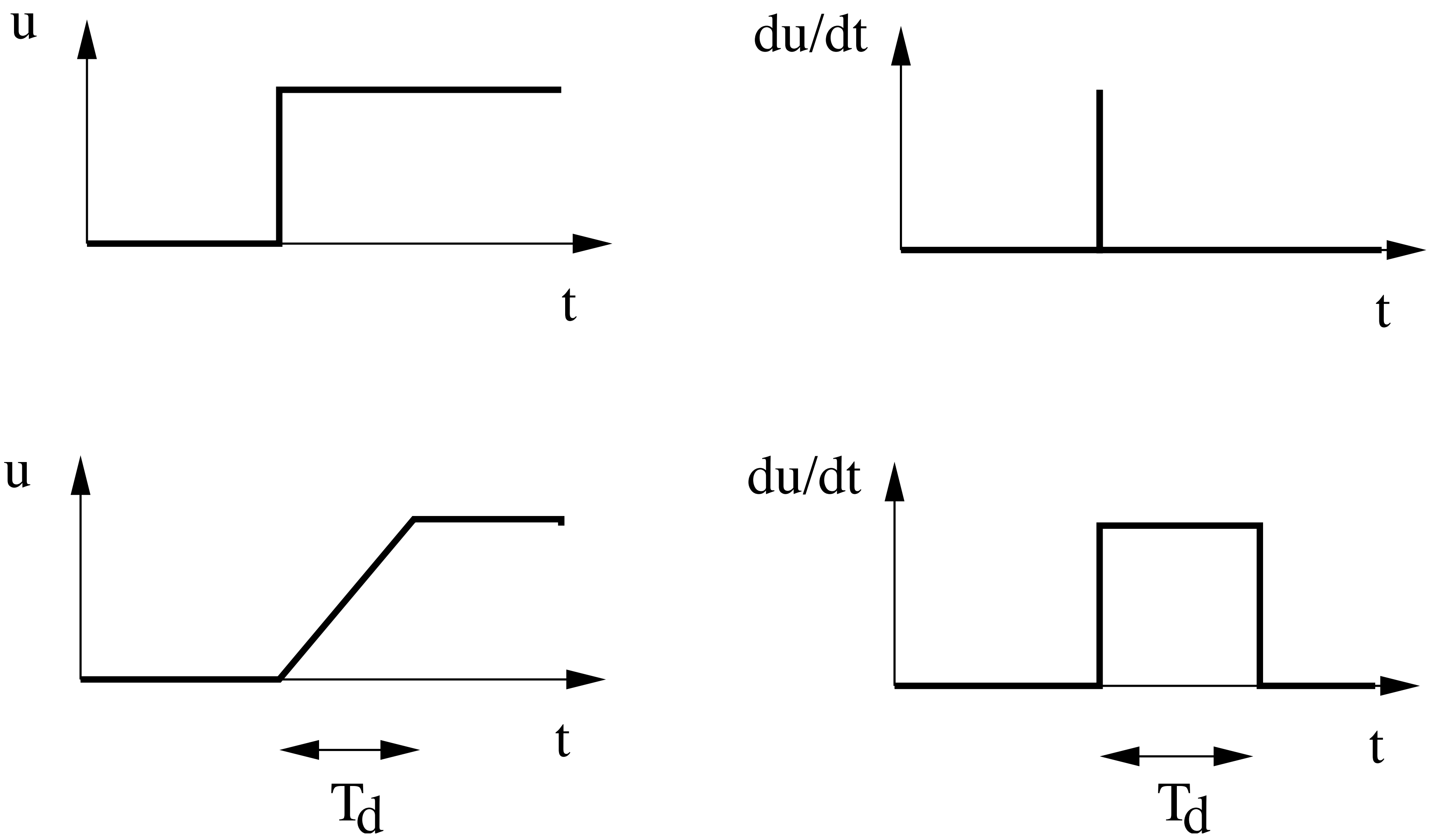
$$u_n(\mathbf{x}, t) \;=\; M_{pq}^0 \, S(t) \star G_{np}(\mathbf{x}, \xi, t) \, s_q$$

$M_{pq}^0 S(t) = \dot{M}_{pq}(t)$ is the moment rate function. The source time function $S(t)$ is the time derivative of the point source slip function.

2. finite fault (m_{pq} is the moment tensor density):

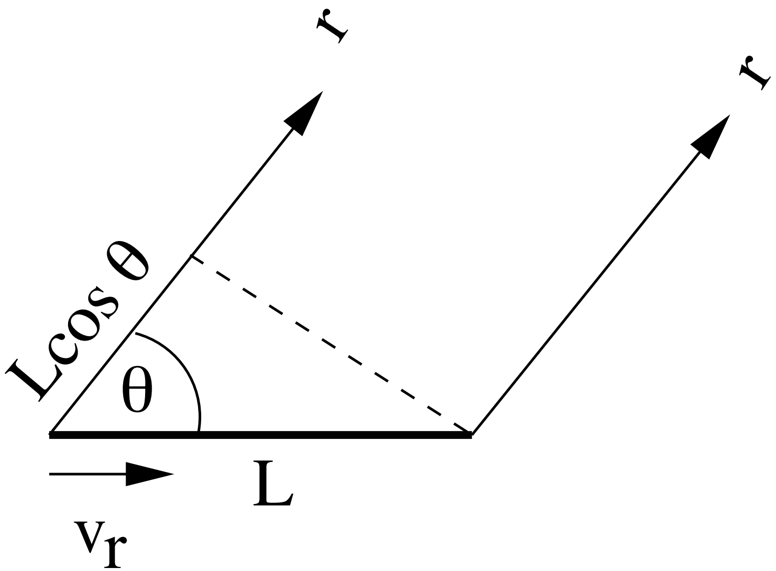
$$u_n(\mathbf{x}, t) \;=\; \int_{\xi_1, \xi_2} \dot{m}_{pq}(\xi, t) \star G_{np}(\mathbf{x}, \xi, t) \, s_q \, d\xi_1 d\xi_2$$

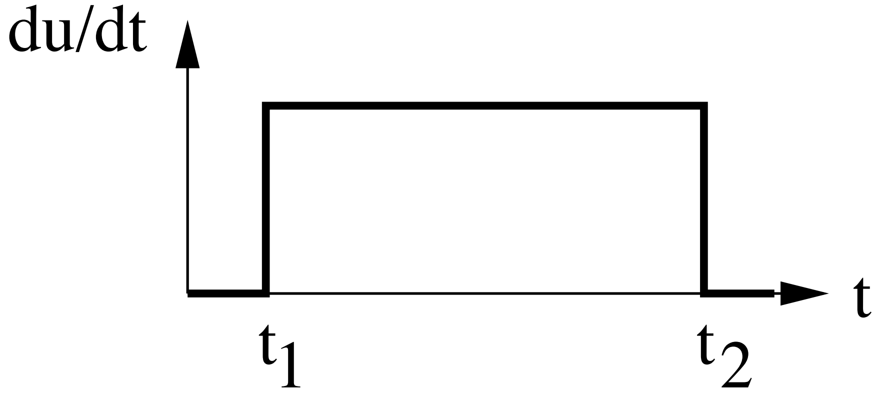
point source moment rate functions



rupture time ($T_r = 0$) , rise time (T_d)

effect of finite rupture

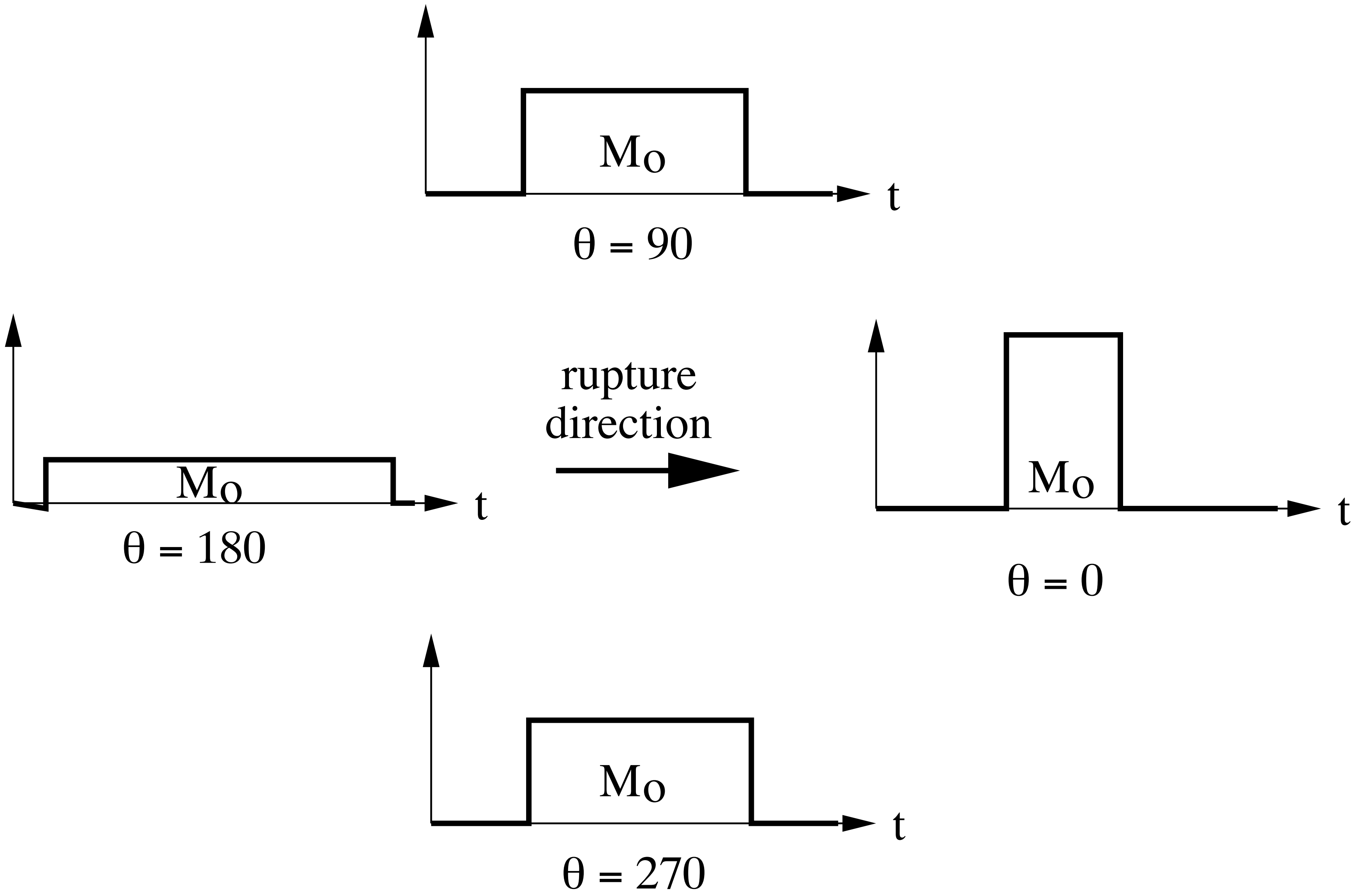




rupture time

$$
\begin{aligned}
T_r &= t_2 - t_1 \\
&= \frac{L}{v_r} + \left(\frac{r}{\beta} - \frac{L\cos\Theta}{\beta}\right) - \frac{r}{\beta} \\
&= L\left(\frac{1}{v_r} - \frac{\cos\Theta}{\beta}\right)
\end{aligned}
$$

directivity effect



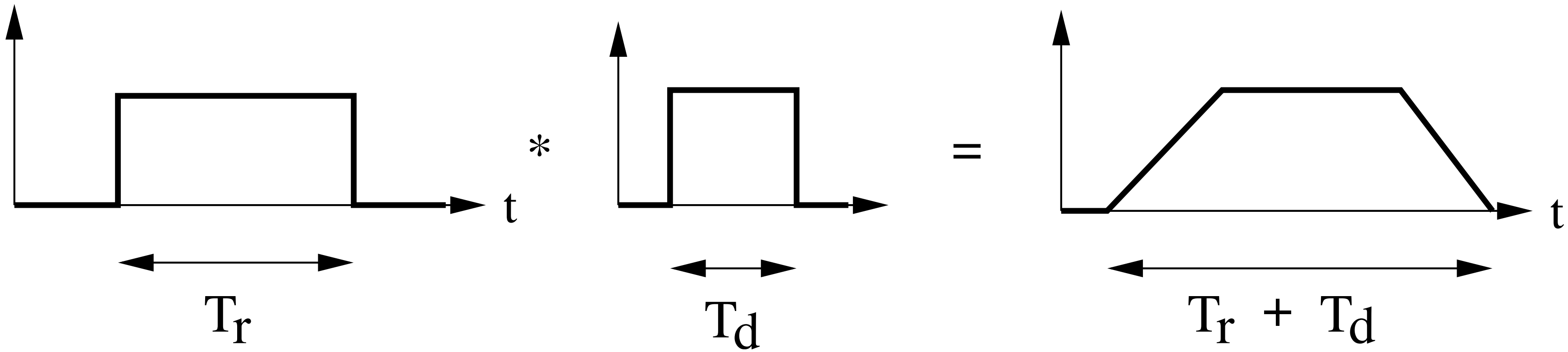
temporal point source approx.

the rupture time is roughly $T_r \approx \frac{L}{\beta}$

leading to the condition $\frac{T}{T_r} = \frac{\lambda/\beta}{L/\beta} = \frac{\lambda}{L} \gg 1$

Note that the temporal point source approximation may be fulfilled for long period surface waves but not for body waves.

rupture and rise time



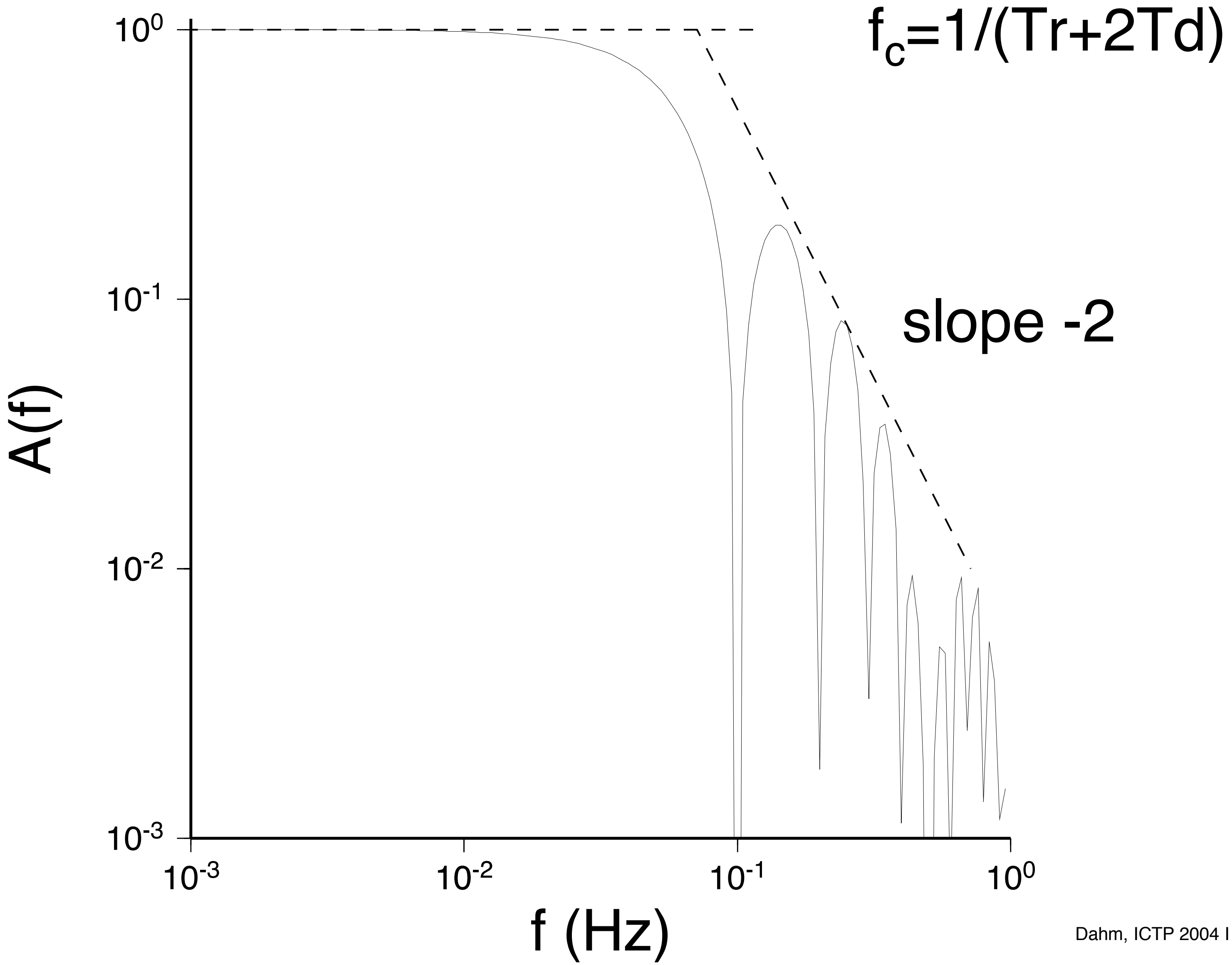
Deconvolution of rupture duration ”boxcar” with rise time ”boxcar” gives a trapezoidal source time.

Frequency domain:

$$A(f) \;\sim\; M_0 \left|\frac{\sin\,\pi f T_r}{\pi f T_r}\right| \left|\frac{\sin\,\pi f T_d}{\pi f T_d}\right| \;\sim\; f^{-2} \text{ for } f > f_c$$

amp.-spectra of trapezoidal function

T_r=10. T_d=2.



average stress drop

assuming an average coseismic strain change of

$$e_{xx} = \partial u_x / \partial x \approx \langle \Delta u \rangle / L \, ,$$

the average stress drop over the fault is:

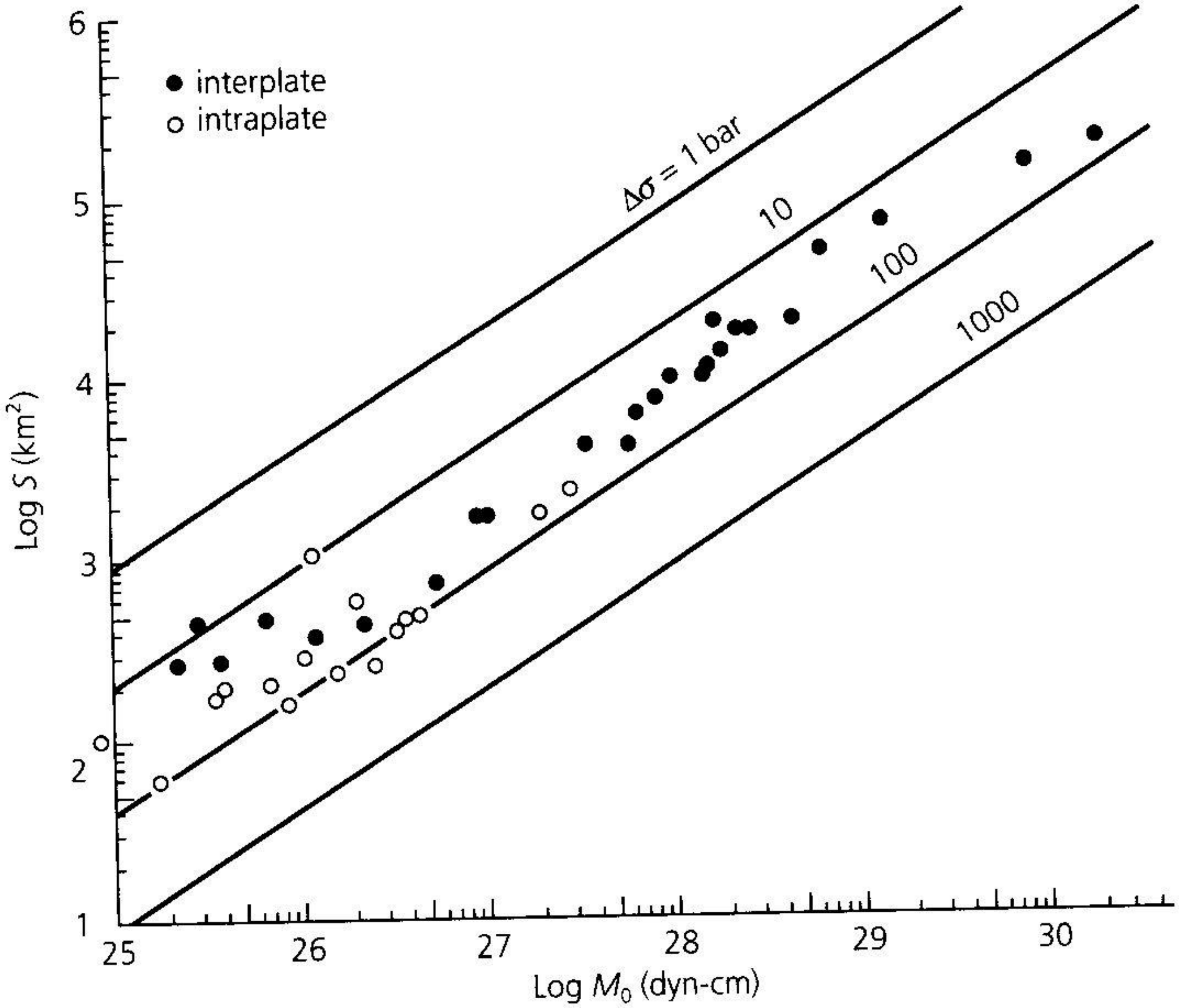
$$\Delta\sigma \approx \frac{\mu \langle \Delta u \rangle}{L} = \frac{c M_0}{L^3} \, ,$$

where c depends on the fault shape and rupture dimension.

e.g. for a circular fault with radius R: $\Delta\sigma \approx \frac{7}{16} \frac{M_0}{R^3}$

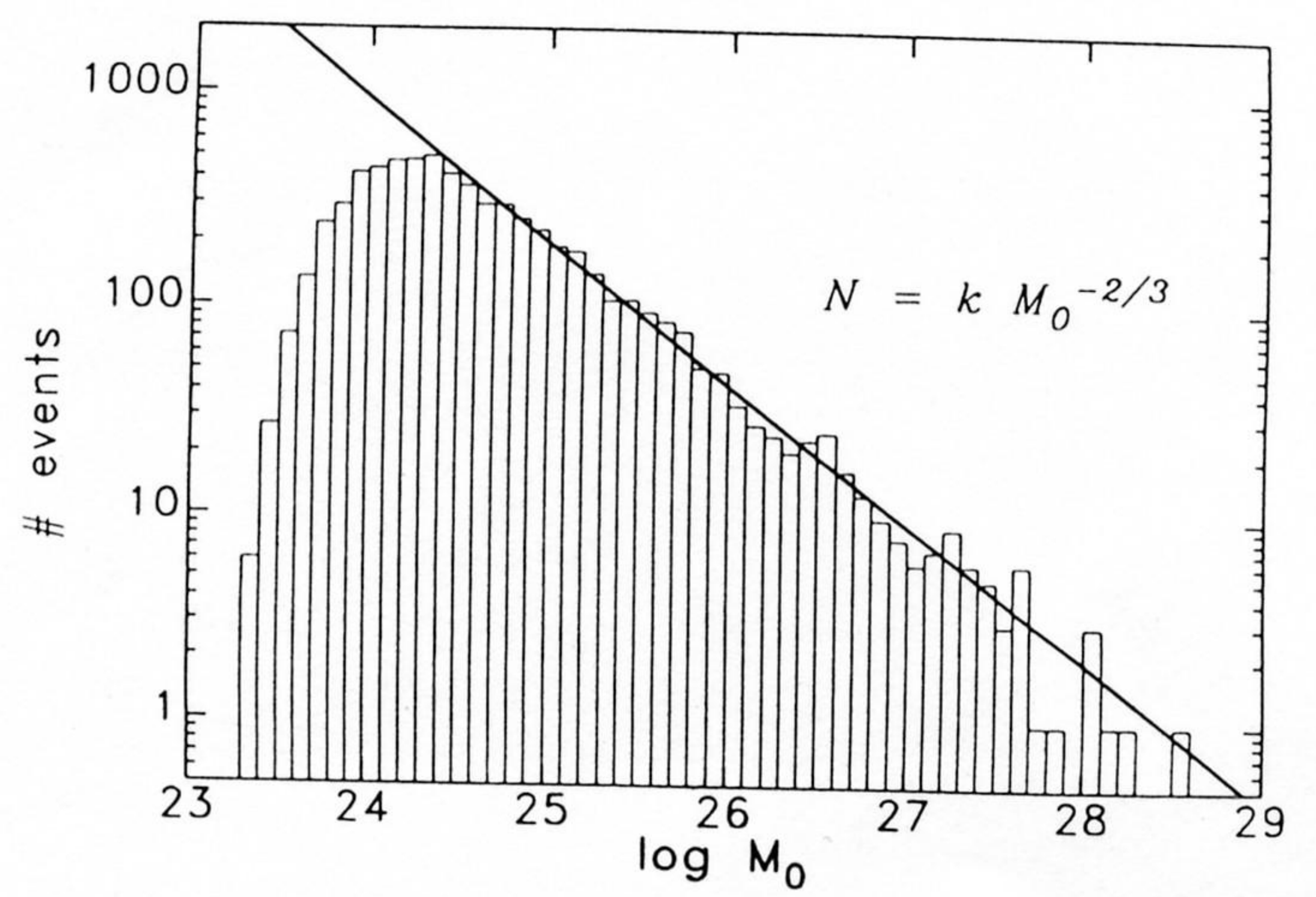
Typically L or R is estimated from aftershocks or from f_c.

is stress drop constant ?



earthquake statistics I

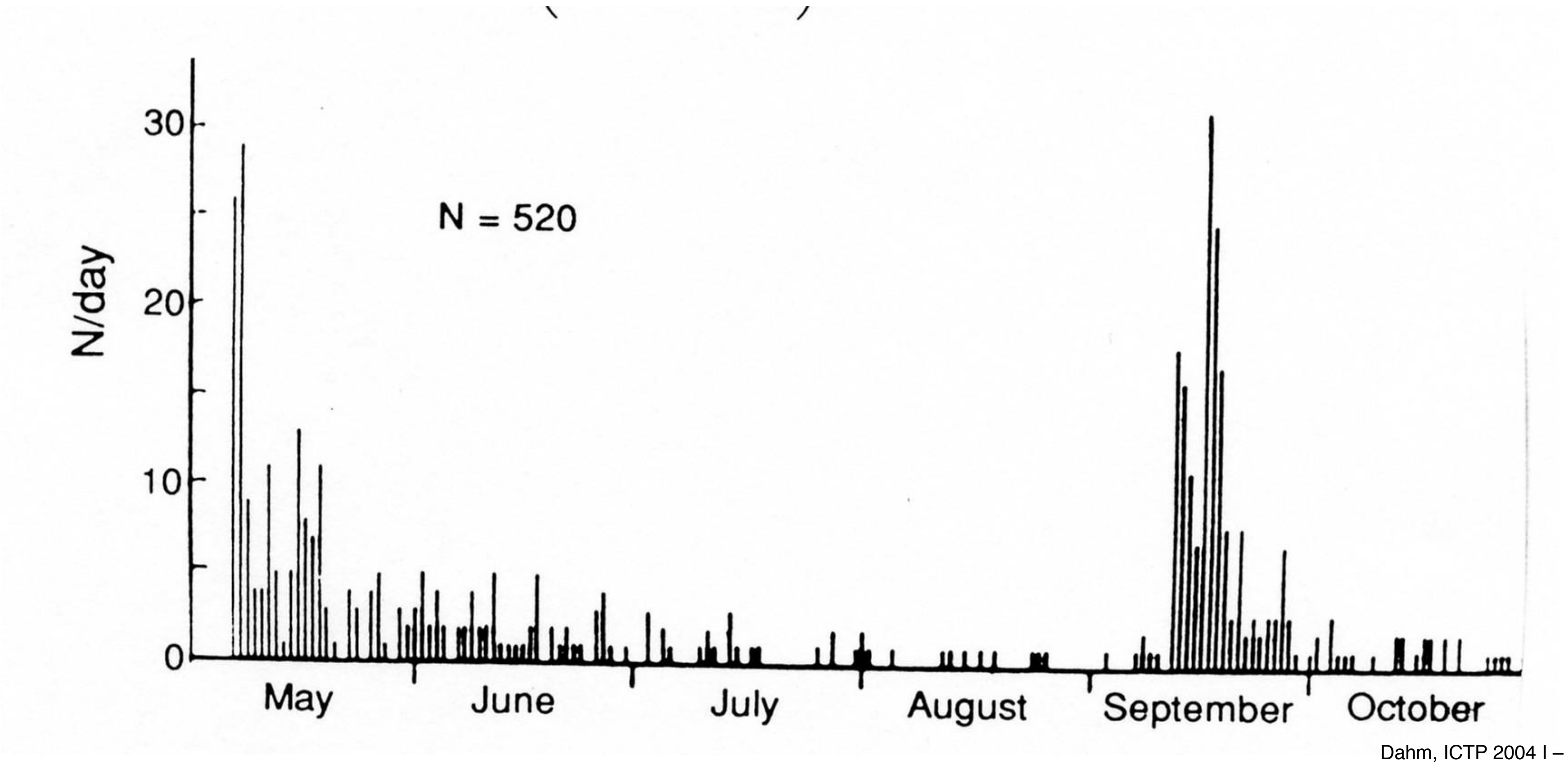
mag.-freq. relation: $N(M_0) = A\, M_0^{-2b/3} \quad 0.6 \leq b \leq 1.5$



earthquake statistics II

frequency of aftershocks (Omoris law):

$$n(t) = \frac{C}{(K+t)^P} \qquad 1 \leq P \leq 1.4$$



Summary

1. Point source parameter are sufficient to explain seismograms below the corner frequency of the event
2. Rupture and extended fault can only be studied at higher frequencies
3. Moment tensor (equivalent force-couples) is a general description covering most point and extended source problems

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