

**Determination of moment tensor and source depth by joint inversion
of surface wave amplitude spectra and first arrival polarities**

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Computer exercises 2

Moment Tensor & Source Depth Inversion Program Reference Manual

Surface waves processing & seismic source inversion Web site is located at:

<http://www.mitp.ru/soft/index.html>

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4. Calculation and View of results
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MomTens program

Menu and Toolbar







Below we provide first-hand information on the menu and related icons. Further details can be found in the next sections.



File:

- New..... 
- Open..... 
- Save 
- Save as...
- Exit 




Project:


- Records... .. 
- Structural models..... 
- Spectral Discretization.... 
- Parameter Space..... 


Build:

- Polarity..... 
- Moment tensor..... 

View:


- Current Project..... 
- DBase WfDisc..... 
- DBMap..... 

Results.....


 New Project File - Create a new Project File .

 Open Project File.

 Save Project file.


 Selection of records to be used for inversion.

This icon opens a window-based dialog to remove and add records.

 Stations and related structural models. *This icon opens a window-based dialog that relates a station to its structural model.*


 Spectral Range Discretization.

 Specifying a grid in parameter space.


 Selection and Rarefaction of Original Polarities.

 Moment Tensor and Source Depth Calculation.

 View of all project lists and parameters.

 View of DataBase table of selected wfdisc records. It is possible to look at waveforms.

 View of the map of stations for selected wfdisc records.

 Viewing the results.



Input (observed data) and output (results)

I. Input data

1. The program reads filtered records and all the information concerning stations and event from Database generated from SEED files. In the example considered in the section "From SEED to Database" all these data are collected in the "/opt/fmt/example/DB.EX" directory.

1a. Starting a new project (an inversion of a given set of observations) use the icon



or item "New" in the File menu. The following window will appear:

MomTens V App New Project

Define Project Directory: Chile_examp.spr Browse...

Cancel OK

Input File Db Names

Filtered Waveform Disc Db Name:
Flt Browse...

Instrument, Site, Sensor Db Root Names:
97288_Chile_0333 Browse...


Origin Info Event Db Name:
97288_Chile_0333 Browse...

The users have to browse to fill the empty boxes related to the different items displayed in the window. The new project name cannot be inserted in the related box shown in the window above. It has to be browsed and an extension name "spr" is automatically appended.

Note.

If you have filtered any seismograms twice or more times and then saved the results, you will have two or more filtered records in the wfdisc Data base table with the same station and channel names. See **Preparing data for inversion** for removing "doubled" records.

1b. It is possible to start a session using a Project File saved earlier.

Click "Open" button  or "Open" menu item. Using "Browse" button and the directory-file navigator find the Project File you would like to work with.

2. Before starting a new Project you have to create a directory structure containing structural models, first arrival polarity data, and surface wave average phase velocity along the paths. The name of the directories should be as follows: MODELS/ POLAR/ PH_VEL/, and should reside in the directory from where the program MomTens is executed.

2a. MODELS - this directory contains the structural models (ascii files) for the epicentral area and station vicinities.

Every row represents a layer in the model. The format of the ascii file is the following:

```

N
f1 f2 f3 f4
.....
.....
f1 f2 f3 f4
.....
.....
      f2 f3 f4

```

where N is the number of layers in the model (number of rows starting from the 2-nd), f1 is the layer thickness in km, f2 is P wave velocity in km/s, f3 is S wave velocity in km/s, and f4 is density in g/sm³. The last row represents the characteristics of the half-space and does not contain a layer thickness.

The files of the structural models must have the following names:

"name.mod", where extension "mod" is fixed, and "name" is arbitrary for station models.

If for any stations or/and epicentral zone the models are not available they will be calculated by 3SMAC program (see Ricard Y., H-C. Nataf & J-P. Montagner, The 3SMAC model. Confrontation with seismic data, J. Geophys. Res., 1995).

2b. POLAR - this directory contains information concerning the first arrival-polarity data.

It can be given by file "bulletin" - USGS bulletin (monthly <ftp://ghftp.cr.usgs.gov/pub/edr/mchedr/> and weekly <ftp://ghftp.cr.usgs.gov/pub/weekly/>) or/and by ascii file "polarity.sup" with supplementary first arrival-polarity data.

The values for take-off angles for first arrivals from file "bulletin" will be calculated using IASPEI Travel Time Software (AK135 version). Only those of polarities from "bulletin" which correspond to epicentral distances from 20 to 90 degrees will be used.

If polarity data are not available this subdirectory does not have to be created.

The format of the "polarity.sup" file is the following (there is no any restriction on epicentral distances):

```

f1 f2 f3
.....
.....
f1 f2 f3

```

Every row of this file contains polarity data for one station. f1 is the source-station azimuth , f2 is the take-off angle of the first arrival, f3=-1 for dilatation, and f3=1 for compression.

2c. PH_VEL; this directory contains (for every surface wave record to be used in the inversion) an ascii file with averaged (along path) phase velocity as a function of period. The names of these files have to correspond to the station names. A file containing velocities of Love waves recorded

at the station "XXX" has to be named "XXX.l"; the file with Rayleigh velocities has to be named "XXX.r". If phase velocity data are not available, this subdirectory does not have to be created. In that case PREM velocity model for all traces will be used by default.

The format of "XXX.l" and "XXX.r" files is the following:

```
f1 f2
.....
.....
f1 f2
```

where:

f1 is the period in seconds, and f2 is the corresponding surface wave phase velocity.

II. Output results

If "Chile_examp.spr" is the name of the Project, then the program will create a subdirectory "Chile_examp.SPR" in the "/opt/fmt/example/MOMTENS.EX" directory. The results of the inversion will

be placed in its subdirectory named "RESULTS". The structure of subdirectory "RESULTS" will be organized as follows:

DEPTH.PS/ MAPS.PS/ MECHA.PS/ OBS_CALC.PS/ POLARITY.PS/

All these directories contain output graphics in postscript format. This output is described in the section "View".

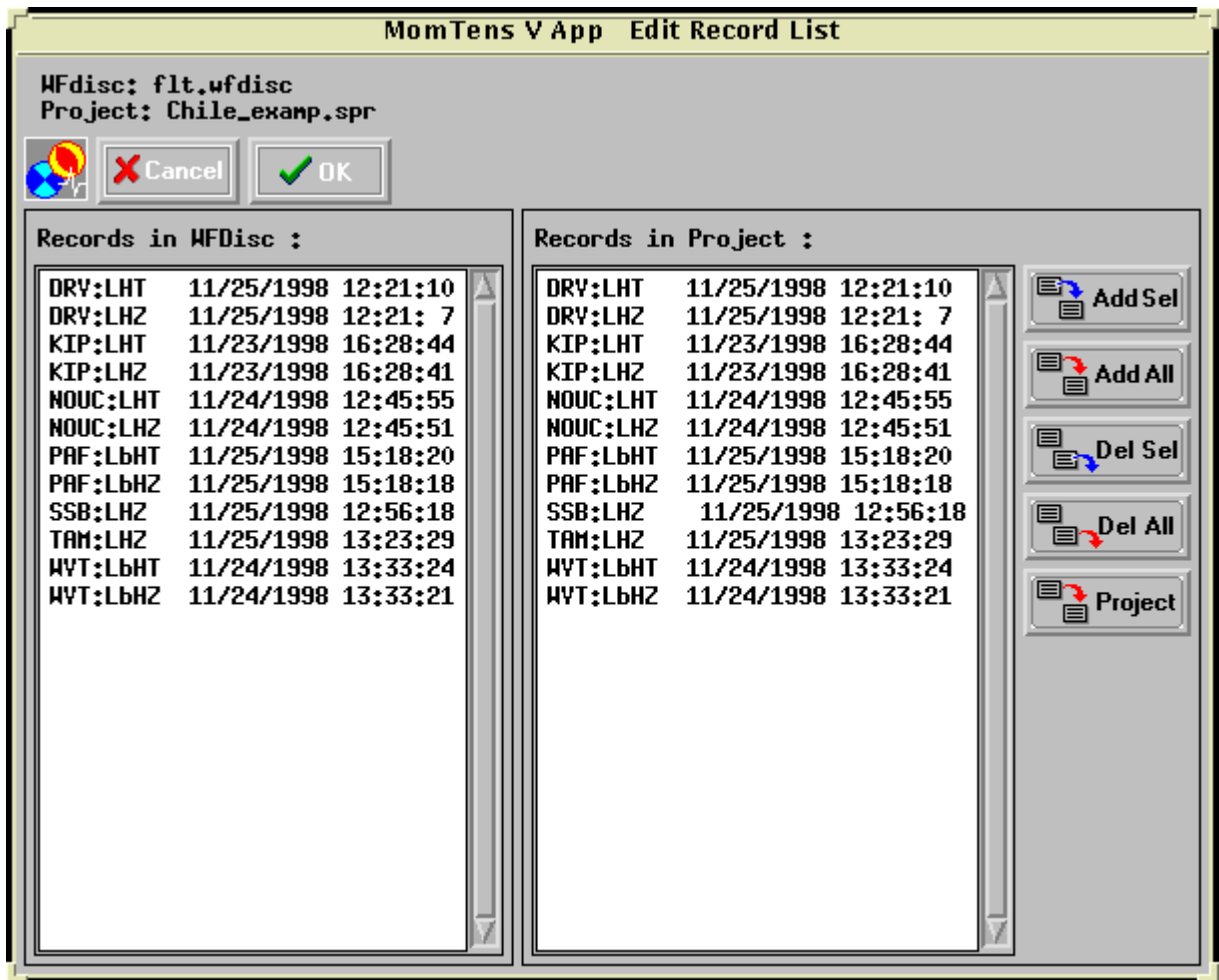
Preparing data for inversion

Before we start the inversion procedure, the following steps have to be followed



1. Selection of records to be used for inversion

The window-based dialog as shown here below selects the records to be used for inversion. The names of available records are listed in the left frame, the names of records selected for inversion – in the right frame. By default the content of these two frames is the same.

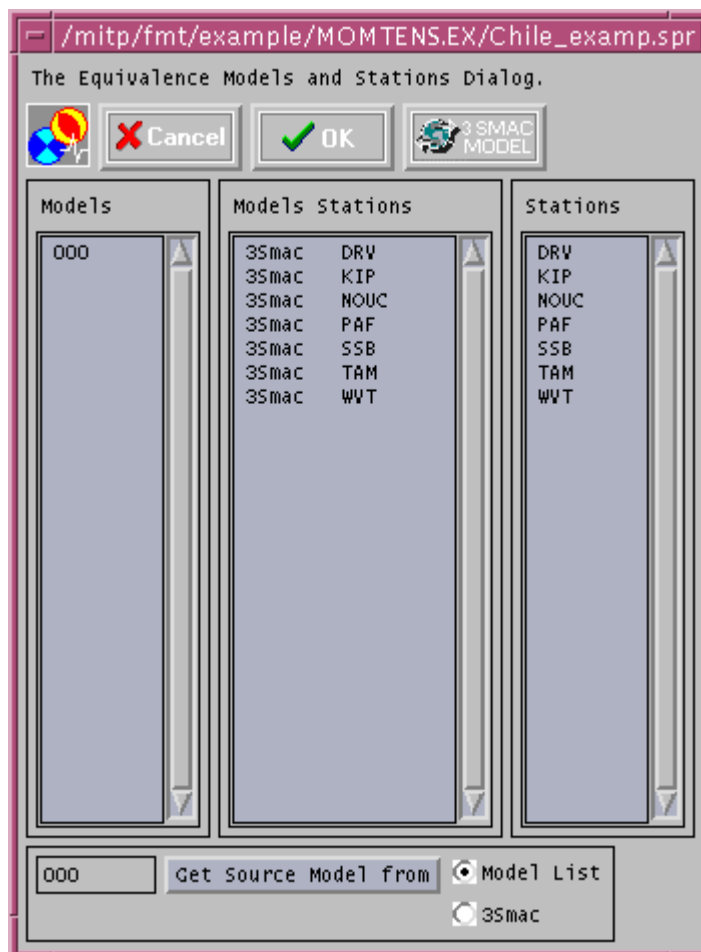


2. Structural models and related stations

The window below provides possibility to calculate the structural model for part of stations by 3SMAC program, and to put in correspondence to the rest of stations given models from directory MODELS. The same can be done for the epicentral zone.

If there are models to be calculated by 3SMAC program, push at first the button "3 SMAK MODEL". 3SMAC models will be calculated for all stations and for the epicentral zone. If the model for any station must be taken from directory MODELS click on the model name in the left column (files from directory MODELS) and on correspondent station in the right column. If the model for the epicentral zone must be taken from directory MODELS click on correspondent model name in the left column,

push the radio button "Model List", and button "Get Source Model from". In the example below the model from file "000.mod" will be used for the epicentral zone, and for all stations models calculated by 3SMAC program will be used.



3. Spectral Range Discretization

3a. Appropriate Spectral Range for every recorded wave

During preliminary filtering by FTAN program, a spectral domain, defined by maximum and minimum periods, is to be determined for every wave where the signal is of a good quality. These period values go in the text boxes "T min" and "T max" in the uppermost left part of the window. To associate this pair of values to any record, the user has to click on the right record name.

3b. Spectral grid

After the spectral boundaries for every record are fixed, the spectral range for inversion has to be chosen. Minimum and maximum periods have to be typed in "T min" and "T max" text boxes in the central part of the window. A maximum number of spectral samples (≤ 18) has to be typed in the text box N points FFT. The number of samples for the Fast Fourier Transform program calculating the spectra is the last parameter to be defined. Push the arrow in the combo box, choose the number, and push "Get" button. Actual spectral boundaries for every record will appear in the central window. If

you will mark by cursor any record in this window, spectral samples related to this record will appear in the right window.


3c. Minimum period for phase spectra inversion

By default the observed phase spectra will be used for the inversion in the full spectral domain. Model PREM will be used in this case to calculate the surface wave phase velocities. If you will type in the text box T min (in the lowermost left part of the window) any value of the period, then the phase spectra will be used for the inversion only for the periods larger than this value.

If you will click on the check box "PhVel" and the directory PH_VEL containing the files with surface wave average phase velocities is already existing, then these velocities interpolated on the grid defined above will be used for computations.

/data/mostinsk/fmt/example/MOMTENS.EX/Chile_examp.spr

The Spectral Range Discretization.



Spectral Range for every wave

T min(sec):

T max(sec):

Seis	T_min	T_max
DRV:LHT	30.00	180.00
DRV:LHZ	30.00	180.00
KIP:LHT	30.00	180.00
KIP:LHZ	30.00	180.00
NOUC:LHT	30.00	150.00
NOUC:LHZ	30.00	180.00
PAF:LbHT	40.00	160.00
PAF:LbHZ	40.00	180.00
SSB:LHZ	40.00	160.00
TAM:LHZ	45.00	180.00
WVT:LbHT	40.00	180.00
WVT:LbHZ	40.00	130.00

Spectral Range

T min(sec):

T max(sec):

Nw <= 18 : Total:

Seis	T_min	N	T_max	N	W	T
DRV:LHT	81.109	12	178.087	1	0.07747	81.1048
DRV:LHZ	81.109	12	178.087	2	0.07363	85.3346
KIP:LHT	81.109	12	178.087	3	0.06980	90.0170
KIP:LHZ	81.109	12	178.087	4	0.06596	95.2575
NOUC:LHT	81.109	10	146.286	5	0.06213	101.1296
NOUC:LHZ	81.109	12	178.087	6	0.05829	107.7918
PAF:LbHT	81.109	10	146.286	7	0.05446	115.3725
PAF:LbHZ	81.109	12	178.087	8	0.05062	124.1245
SSB:LHZ	81.109	10	146.286	9	0.04679	134.2848
TAM:LHZ	81.109	12	178.087	10	0.04295	146.2907
WVT:LbHT	81.109	12	178.087	11	0.03912	160.6131
WVT:LbHZ	81.109	8	124.121	12	0.03528	178.0948

PhVel T min:



4. Parameter range specification

This window allows you to specify the grid for the source parametric model. The double-couple depth (in km) and three angles (in degrees) specify this model: dip, strike and slip. For each of these parameters you have to specify the starting value, the step, and the number of samples (N Step) using the appropriate text box.

Note:

It will be impossible to close up this window and a warning message appears, in the case where the parameters do not satisfy the following constrains:

$$\text{N Step of depth} \leq 100;$$

$$\text{N Step of dip} \leq 6;$$

$$\text{N Step of strike} \leq 12;$$

$$\text{N Step of slip} \leq 12;$$

$$(\text{start of dip} + \text{step of dip}) * (\text{N Step of dip} - 1) \leq 90.$$

For a complete exploration of the focal sphere, it is enough to assign in the following window half of the full interval for strike and slip angles. The symmetric mechanisms are also investigated during the inversion. An example of parameters specification is shown below.

The screenshot shows a software window with the following fields:

Parameter	Start	Step	N Point
Depth Range Input (km)	0.0	2.0	51
Dip Range Input	11.0	15.0	6
Strike Range Input	15.0	15.0	12
Slip Range Input	117.0	15.0	12



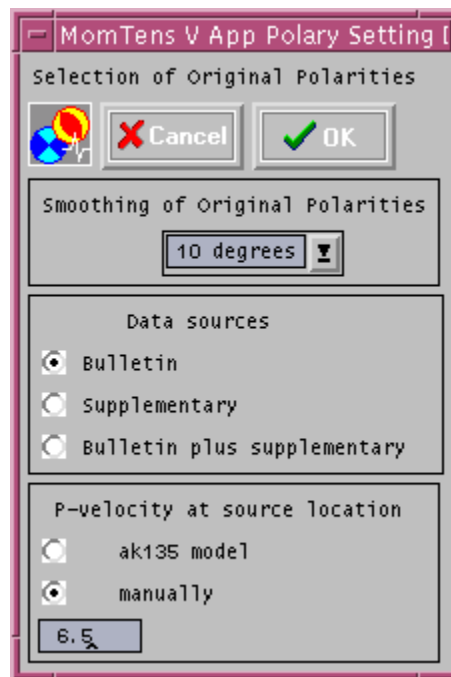
5. Selection and rarefication of polarity data

Upper frame is related to selection of the value of the angle of polarity data smoothing. Push the arrow in the combo box and select the value.

Middle frame shows the list of polarity data sources contained in directory POLAR ("bulletin" or/and "polarity.sup"). Push radio button correspondent to polarity data sources to be used. In example shown below there are both files in directory POLAR, but only bulletin is selected for inversion.

Lower frame provide possibility to set a value of P wave velocity in the source location to be used for take off angle calculation. Push radio button "manually" and type the value in the text box below. The other possibility is to take this value from AK135 model at the source depth (taken from Data Base). To do this push radio button "ak135 model".

After all selections are done push OK. Stereographic projections of focal sphere with original and selected polarities will appear in a GHOSTVIEW window. To go back to the main window, select File and then Quit.





Calculations

Clicking this button when all input data and parameters are ready, you will start the inversion procedure by exploration of the source parameter space.

View of results.



Viewing all project lists and parameters



Viewing the DataBase table of selected wfdisc records.



Viewing the map of stations for selected wfdisc records.



Viewing the results

When this button is clicked the following window will be displayed:

The left panel allows you to choose one of three following items corresponding to the three types of inversion considered in the Appendix.

(a) Amplitude spectra inversion

After you have chosen this item, type in the upper right text box the value defining the numeric difference between the maximum and minimum residual. Only these possible orientations of the double couple principal stress axes which give the residuals between observations and synthetics within this interval will be shown in maps of the quality of different possible axes orientations. Click OK, and the following results will appear in the windows:

- Map of the quality of different possible T and P axes orientation. Every point in the map represents the stereographic projection of the axis orientation on the lower hemisphere. The color shows the quality of this orientation. In the case under consideration the maps for T and P axes are similar because of the symmetry considered in Appendix. That is why only one map is presented.

- Four equivalent best focal mechanisms and seismic moment.
- Residuals as a function of source depth (see [Appendix](#)).
- Observed and synthetic surface wave amplitude spectra calculated for the best values of variable parameters.

(b) Amplitude spectra + P polarity inversion

After you have chosen this item, type in the upper right text box the difference between the maximum and minimum residual which you would like to be shown in the maps. Type in the lower right text box the number of best solutions to be shown in the stereographic projections of nodal planes. Click OK , and the following results will appear in the windows:

- Maps of the quality of different possible T and P axes orientations.
- The best focal mechanism, selected polarities and seismic moment.
- A set of first N (N is the number typed in the lower right text box) best focal mechanisms and seismic moments.
- Residuals as a function of source depth.
- Observed and synthetic surface wave amplitude spectra calculated for the best values of the variable parameters.

(c) Amplitude + phase spectra inversion

After you have chosen this item, follow the instructions given in the previous paragraph (b).

Appendix

Description of technique

Instant point source can be described by the moment tensor - a symmetric 3x3 matrix \mathbf{M} . Seismic moment M_0 is defined by equation $M_0 = \sqrt{\frac{1}{2} \text{tr}(\mathbf{M}^T \mathbf{M})}$, where \mathbf{M}^T is transposed moment tensor \mathbf{M} , and $\text{tr}(\mathbf{M}^T \mathbf{M}) = \sum_{i,j=1}^3 M_{ij}^2$. Moment tensor of any event can be presented in the form $\mathbf{M} = M_0 \mathbf{m}$, where matrix \mathbf{m} is normalised by condition $\text{tr}(\mathbf{m}^T \mathbf{m}) = 2$.

We are considering a double couple instant point source (a pure tangential dislocation) at a depth h . Such a source can be given by 5 parameters: double couple depth, its focal mechanism which is characterising by three angles: strike, dip and slip or by two unit vectors (direction of principal tension \mathbf{T} and direction of principal compression \mathbf{P}) and seismic moment M_0 . Four of these parameters we determine by a systematic exploration of the four dimensional parametric space, and the 5-th parameter M_0 - solving the problem of minimisation of the misfit between observed and calculated surface wave amplitude spectra for every current combination of all other parameters.

Under assumptions mentioned above the relation between the spectrum of the displacements $u_i(\mathbf{x}, \omega)$ in any surface wave and the total moment tensor \mathbf{M} can be expressed by following formula

$$u_i(\mathbf{x}, \omega) = \frac{1}{i\omega} [M_{jl} \frac{\partial}{\partial y_l} G_{ij}(\mathbf{x}, \mathbf{y}, \omega)] \quad (1)$$

$i, j = 1, 2, 3$ and the summation convention for repeated subscripts is used. $G_{ij}(\mathbf{x}, \mathbf{y}, \omega)$ in equation (1) is the spectrum of Green function for the chosen model of medium and wave type (see Levshin, 1985; Bukchin, 1990), \mathbf{y} - source location. We assume that the paths from the earthquake source to seismic stations are relatively simple and are well approximated by weak laterally inhomogeneous model (Woodhouse, 1974; Babich *et al.*, 1976). The surface wave Green function in this approximation is determined by the near source and near receiver velocity structure, by the mean phase velocity of wave, and by geometrical spreading. The amplitude spectrum $|u_i(\mathbf{x}, \omega)|$ defined by formula (1) does not depend on the average phase velocity of the wave. In such a model the errors in source location do not affect the amplitude spectrum (Bukchin, 1990). The average phase velocities of surface waves are usually not well known. For this reason as a rule we use only amplitude spectra of surface waves for determining source parameters under consideration. We use observed surface wave phase spectra only for very long periods.

Surface wave amplitude spectra inversion

If all characteristics of the medium are known the representation (1) gives us a system of equations for parameters defined above. Let us consider now a grid in the space of these 4 parameters. Let the models of the media be given. Using formula (1) we can calculate the amplitude spectra of surface waves at the points of observation for every possible combination of values of the varying parameters. Comparison of calculated and observed amplitude spectra give us a residual $\varepsilon^{(i)}$ for every point of observation, every wave and every frequency ω . Let $u^{(i)}(\mathbf{x}, \omega)$ be any observed value of the spectrum,

$i = 1, \dots, N$; $\varepsilon_{\text{amp}}^{(i)}$ - corresponding residual of $|u^{(i)}(\mathbf{x}, \omega)|$. We define the normalised amplitude residual by formula

$$\varepsilon_{\text{amp}}(h, \varphi, \mathbf{T}, \mathbf{P}) = \left[\left(\sum_{i=1}^N \varepsilon_{\text{amp}}^{(i)2} \right) / \left(\sum_{i=1}^N |u^{(i)}(\mathbf{x}, \omega)|^2 \right) \right]^{1/2} . \quad (2)$$

The optimal values of the parameters that minimize ε_{amp} we consider as estimates of these parameters. We search them by a systematic exploration of the five dimensional parameter space. To characterize the degree of resolution of every of these source characteristics we calculate partial residual functions. Fixing the value of one of varying parameters we put in correspondence to it a minimal value of the residual ε_{amp} on the set of all possible values of the other parameters. In this way we define one residual function on scalar argument and two residual functions on vector argument corresponding to the scalar and two vector varying parameters: $\varepsilon_h(h)$, $\varepsilon_{\mathbf{T}}(\mathbf{T})$ and $\varepsilon_{\mathbf{P}}(\mathbf{P})$. The value of the parameter for which the corresponding function of the residual attains its minimum we define as estimate of this parameter. At the same time these functions characterize the degree of resolution of the corresponding parameters. From geometrical point of view these functions describe the lower boundaries of projections of the 4-D surface of functional ε on the coordinate planes.

It is well known that the focal mechanism can not be uniquely determined from surface wave amplitude spectra. There are four different focal mechanisms which will radiate this same surface wave amplitude spectra. These four equivalent solutions represent two pairs of mechanisms symmetric with respect to the vertical axis, and within the pair differ from each other by the opposite direction of slip.

To get a unique solution for the focal mechanism we have to use in the inversion additional observations. For these purpose we use very long period phase spectra of surface waves or polarities of P wave first arrivals.

Joint inversion of surface wave amplitude and phase spectra

Using formula (1) we can calculate for chosen frequency range the phase spectra of surface waves at the points of observation for every possible combination of values of the varying parameters. Comparison of calculated and observed phase spectra give us a residual $\varepsilon_{\text{ph}}^{(i)}$ for every point of observation, every wave and every frequency ω . We define the normalised amplitude residual by formula

$$\varepsilon_{\text{ph}}(h, \varphi, \mathbf{T}, \mathbf{P}) = \frac{1}{\pi} \left[\left(\sum_{i=1}^N \varepsilon_{\text{ph}}^{(i)2} \right) / N \right]^{1/2} . \quad (3)$$

We determine the joint residual ε by formula

$$\varepsilon = 1 - (1 - \varepsilon_{\text{ph}})(1 - \varepsilon_{\text{amp}}) . \quad (4)$$

Joint inversion of surface wave amplitude spectra and P wave polarities

Calculating radiation pattern of P waves for every current combination of parameters we compare it with observed polarities. The misfit obtained from this comparison we use to calculate a joint residual of surface wave amplitude spectra and polarities of P wave first arrivals. Let ε_{amp} be the residual of

surface wave amplitude spectra, ε_p - the residual of P wave first arrival polarities (the number of wrong polarities divided by the full number of observed polarities), then we determine the joint residual ε by formula

$$\varepsilon = 1 - (1 - \varepsilon_p)(1 - \varepsilon_{amp}) . \quad (5)$$

Before inversion we apply to observed polarities a smoothing procedure (see Lasserre *et al.*, 2001) which we will describe here briefly.

Let us consider a group of observed polarities (+1 for compression and -1 for dilatation) radiated in directions deviating from any medium one by a small angle. This group is presented in the inversion procedure by one polarity prescribing to this medium direction. If the number of one of two types of polarities from this group is significantly larger than the number of opposite polarities, then we prescribe this polarity to this medium direction. If no one of two polarity types can be considered as preferable, then all these polarities will not be used in the inversion. To make a decision for any group of n observed polarities we calculate the sum $m = n_+ - n_-$, where n_+ is the number of compressions and $n_- = n - n_+$ is the number of dilatations. We consider one of polarity types as preferable if $|m|$ is larger than its standard deviation in the case when +1 and -1 appear randomly with this same probability 0.5. In this case n_+ is a random value distributed following the binomial law. For its average we have $M(n_+) = 0.5n$, and for dispersion $D(n_+) = 0.25n$. Random value m is a linear function of n_+ such that $m = 2n_+ - n$. So following equations are valid for the average, for the dispersion, and for the standard deviation σ of value m

$$M(m) = 2M(n_+) - n = n - n = 0, \quad D(m) = 4D(n_+) = n, \quad \text{and} \quad \sigma(m) = \sqrt{n} .$$

As a result, if the inequality $|m| \geq \sqrt{n}$ is valid then we prescribe +1 to the medium direction if $m > 0$, and -1 if $m < 0$.

References

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