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# Integrals Over Seismic Sequences

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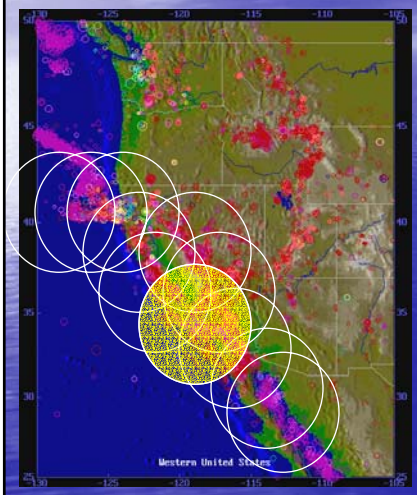
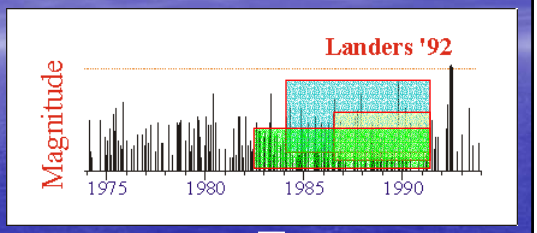
# Outline

- General scheme of analysis
- Functions on a sequence of earthquakes
- Different ways to normalize functions

# General scheme




**F(t)**



# Counts on earthquake sequence

Usually aftershocks are not counted so that large earthquakes do not dominate the count on earthquake sequence.

The number of aftershocks *per se* is one of the parameters of main shocks { i } –

$$\{t_i, m_i, b_i(e)\}$$

## Functions of rate of activity

$N(t \mid m,s)$ - the number of earthquakes with $M \geq m$ in time interval from (t-s) to t, i.e., the number of events of certain size per unit time, *rate of activity*.

A dual function - the time that accommodates the most recent n events with $M \geq m$ ,

$TI(t \mid m,n)$.



## Function K

Increment of activity, *acceleration*, expressed by the difference between the number of earthquakes in the two successive intervals of time (t-s, t) and (t-2s, t-s):

$$K(t \mid m,s) = N(t \mid m,s) - N(t-s \mid m,s).$$

# Function L

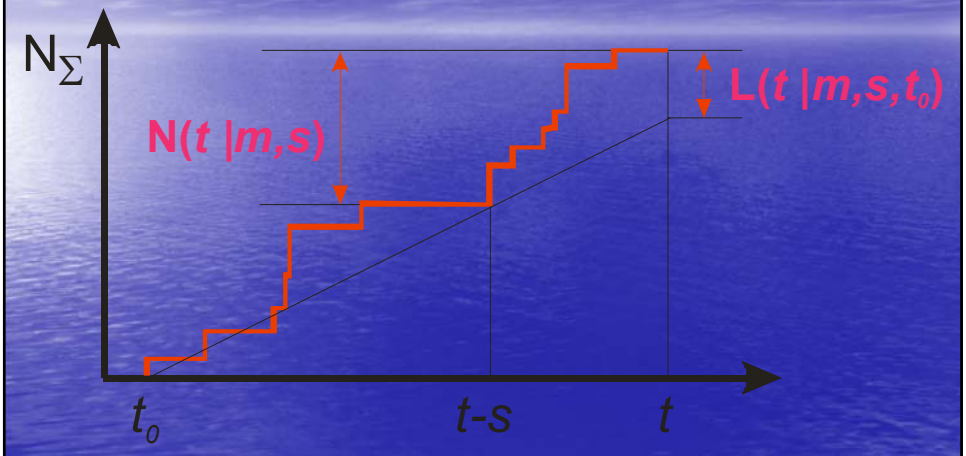
$L(t \mid m, s, t_0)$ - deviation of activity from a longer-term trend over the period from $t_0$ to t.

$$L(t \mid m, s, t_0) = N(t \mid m,s) - N(t \mid m,t-s-t_0) \times s/(t-s-t_0)$$

Usually $t_0$ is the beginning of the catalog.



# Functions of activity



## Function L: Necessary modification

If $t_0$ is far from t, the longer-term rate tends to a constant. Therefore, one should change the definition, if catalogs of extended times are considered, e.g., in model catalogs.

When $t-t_0 \geq 5s$, (i.e., more than 30 years in the applications to real seismic activity) we recommend using trailing $t_0 = t-5s$.



## $\Sigma(t)$

$\Sigma(t \mid m,M',s,\alpha,\beta)$ - the weighted number of earthquakes in time interval from (t-s) to t and magnitude ($m \leq M_i < M'$).

$$\Sigma(t \mid m,M',s,\alpha,\beta) = \Sigma 10^{\beta(M_i-\alpha)}$$

This function estimates different properties depending on the value of $\beta$.

## $\Sigma$ is proportional to

- the total linear size of earthquake sources

  if $\beta=B/3$
- the total area of the earthquake sources

  if $\beta=2B/3$
- the total energy of earthquakes

  if $\beta=B$.

where B comes from the relation between the energy E and magnitude M

$$\log E = A + BM$$



## Normalized counts of $\Sigma(t)$

- Averages –

  $\Sigma(t)/N(t)$
- Concentrations –

  $( \Sigma(t)/N(t) ) / ( N(t) )^{d/3}$

  where d is concentration dimension

# Function G

The ratio of the number of earthquakes from two magnitude ranges –

$$G(t \mid m_1, m_2, s) = 1 - N(t \mid m_2, s) / N(t \mid m_1, s)$$

- The values $m_1$ and $m_2$ may result from inspecting the frequency-magnitude statistics of earthquakes.



# Function G: alternative definition

The ratio of the number of earthquakes from two magnitude ranges –

$$G(t \mid m_1, m_2, s) = \log( N(t \mid m_2, s) / N(t \mid m_1, s) ) / (m_2 - m_1)$$

- In such a definition G is the tangent of slope of the frequency-magnitude graph.

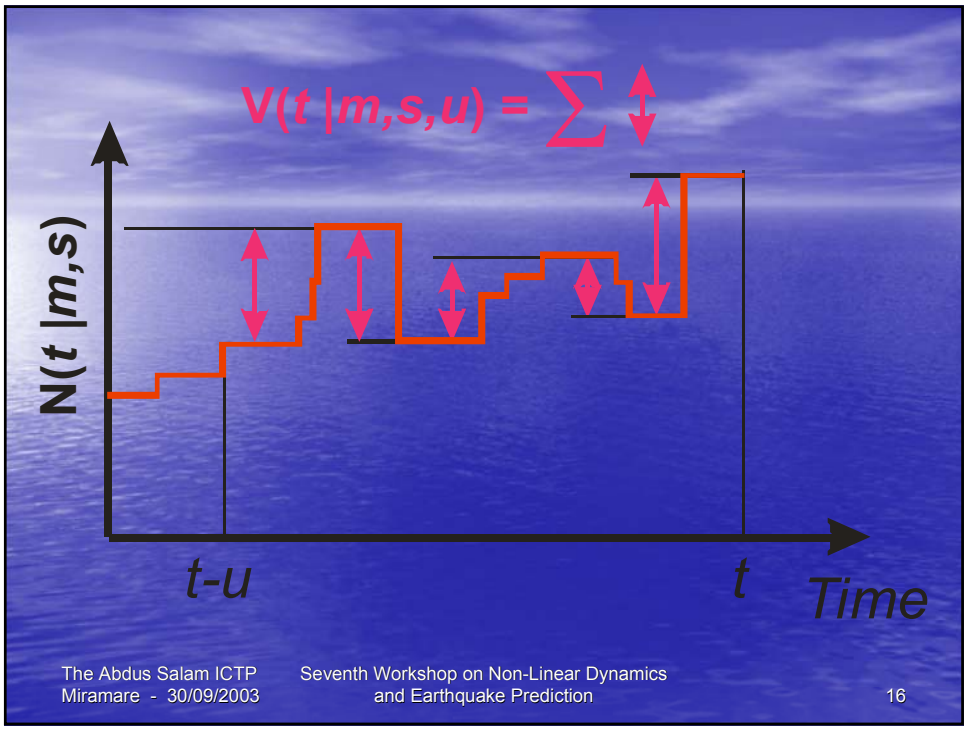
# Variation of seismic activity

$V(t \mid m,s,u) = var\ N(t \mid m,s)$ is variation of the number of earthquakes on the time interval $(t-u,t)$,

$$var\ N(t \mid m,s) = \Sigma \mid N(t_i \mid m,s) - N(t_{i+1} \mid m,s) \mid$$

which is the total of the absolute differences between $N(t \mid m,s)$ at two consecutive times $t_{i+1}$ and $t_i$ from the time interval $(t-u,t)$.





## Average depth

Nadeau et al. (1995) suggested to use the average depth of hypocenters in a given time window claiming that deepening of seismic activity precede large earthquakes.

It is necessary to note that the depth is one of the most inaccurate parameters of hypocenter determination: the error in depth may be large resulting seismologist attributed values in many cases.



## Clustering of earthquakes

In case of de-clustered catalogs each main shock has its number, $b_i(e,m_{aft})$, of aftershocks of magnitude $M \geq m_{aft}$ in the first e days after the origin time.

$B(t \mid m,M',s,m_{aft},e) = \max b_i(e,m_{aft})$ is the maximum calculated over the main shocks with $m \leq M_i < M'$ and time interval (t-s,t).

## Entropy

Many other measures of clustering were proposed in the literature. For example, *Shannon entropy* –

$$- \Sigma \, q_i \times \ln q_i$$

where $q_i$ is some time dependent probability estimate of occupation of a given cell of the grid, i.e., $n_i/N$.



## Active zone size

Active zone size, AZS, is a measure of the extent to which seismic activity in a given region is diffused. Its premonitory efficiency was first observed in modeling.

AZS is more directly a measure of the broadening of small to moderate size events, which leads to development of a nucleation region associated with a coming large event.

# AZS in seismic regions

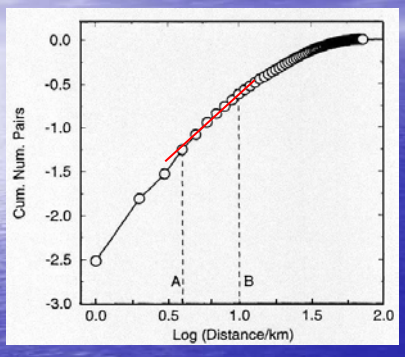
To measure active zone size in the Earth we might use a box counting algorithm, in which the large spatial regions will be subdivided into many smaller regions that contain seismic events of a certain magnitude level.

For example, AZS(t | m,s)= $\Sigma\ \delta(n_i)$,
where $\delta(x) = 0$, if $x = 0$ and $\delta(x) = 1$, if $x > 0$.

In the Earth this measure is somewhat more complex due to the variable complexity of faults networks in different regions.



# Spatial correlation dimension




$C(R) = p(r<R)/P$
$= 2p(r<R)/(N(N-1)$

where $p(r<R)$ is the number of pairs separated by less than $R$ km and $P=N(N-1)/2$ is the total number of pairs for N events.

**The slope of the straight segment over distance range [A,B] is used to estimate the spatial correlation dimension $CD=d_{[A,B]}$.**

## Correlation dimension vs. Clustering

A correlation dimension is inversely related to the degree of spatial clustering, as the slope of the straight segment decreases when the number of event pairs with relatively small inter-event distances increases. Smaller values for CD indicate higher degrees of spatial clustering.



## Long-range interaction of earthquakes

- A. Prozorov (1982) introduced a term-less precursor, which follow shortly a major earthquake but on a large distance from it. He concluded that “long-range aftershocks” mark the location of a future major earthquake.
- The two new phenomena which represent the long-range correlation were found first on a synthetic catalogue.

## Precursor ROC

The name stands for “Range of Correlation” (Shebalin, Zaliapin and Keilis-Borok, 2000) and depicts nearly simultaneous occurrence of two earthquakes at large distances from each other. This is a short-term seismicity pattern of the middle-range spatial uncertainty.



## Pattern *Accord*

Another expression of long-range correlation in seismicity is a premonitory spreading of activity. The seismicity pattern *Accord* (Zaliapin, Keilis-Borok and Axen, 1999) represents this phenomenon. It is defined as a simultaneous rise of seismic activity in a sufficiently large number of fault zones.

## Seismic Reversal

The SR phenomenon (Shebalin and Keilis-Borok, 1999) : Zones of relatively high seismic activity become unusually quiescent while zones of relatively low seismic activity are unusually active. This takes place a few months prior to an approaching strong earthquake within a distance of about 100 km (i.e. about ten times larger than the incipient source) from its future epicenter.



## Normalization of functions

Normalization of earthquake sequences is necessary to ensure adequate uniform application with the same set of adjustable parameters in regions of different seismic activity.

## Magnitude cutoff

We use the normalization by minimal magnitude cutoff $M_{min}$, defined by one of the two conditions:

- $M_{min} = M_0 - C$,

  C being a constant
- $M_{min}$ derived from $N(M_{min}) = A$

  A being a constant rate of activity



## Percentile cutoff

The values of functions are normalized to their empirical probability distribution functions, that is performing a non-linear transformation of different ranges from different seismic environment to [0,1].

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