



the
abdus salam
international centre for theoretical physics

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**"Seventh Workshop on Non-Linear Dynamics and
Earthquake Prediction"**

29 September - 11 October 2003

Earthquake Dynamics

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RUPTURE MECHANICS

(earthquake as a rupture)

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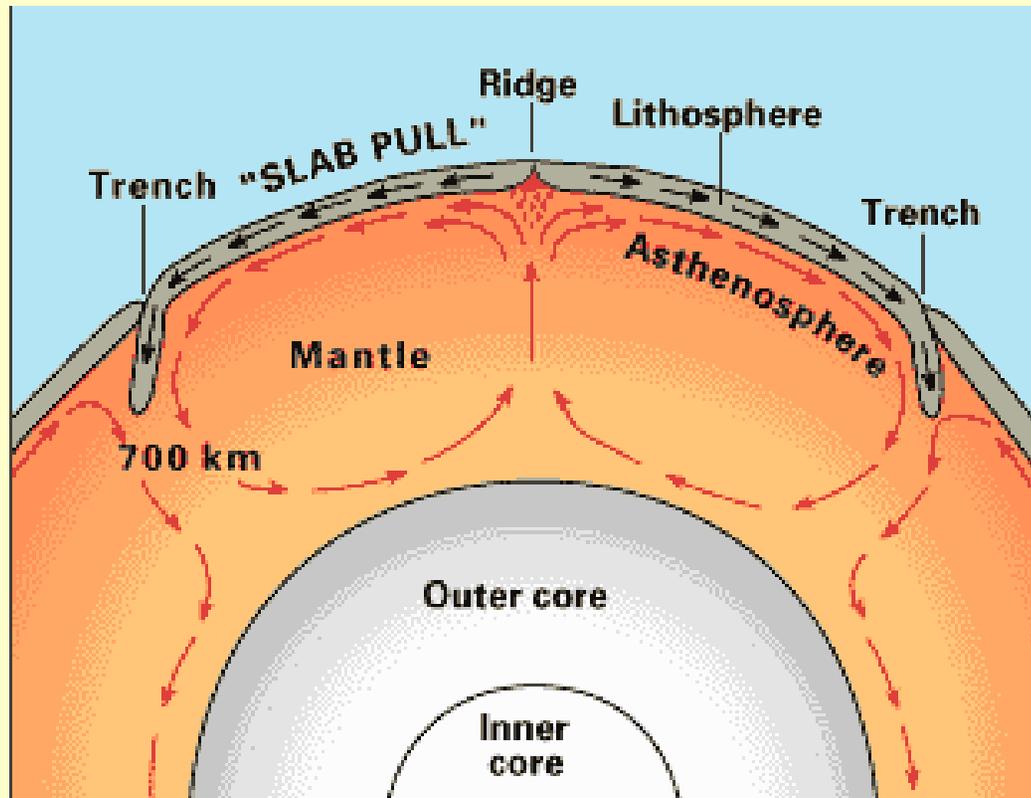
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1

DYNAMICS OF THE MANTLE



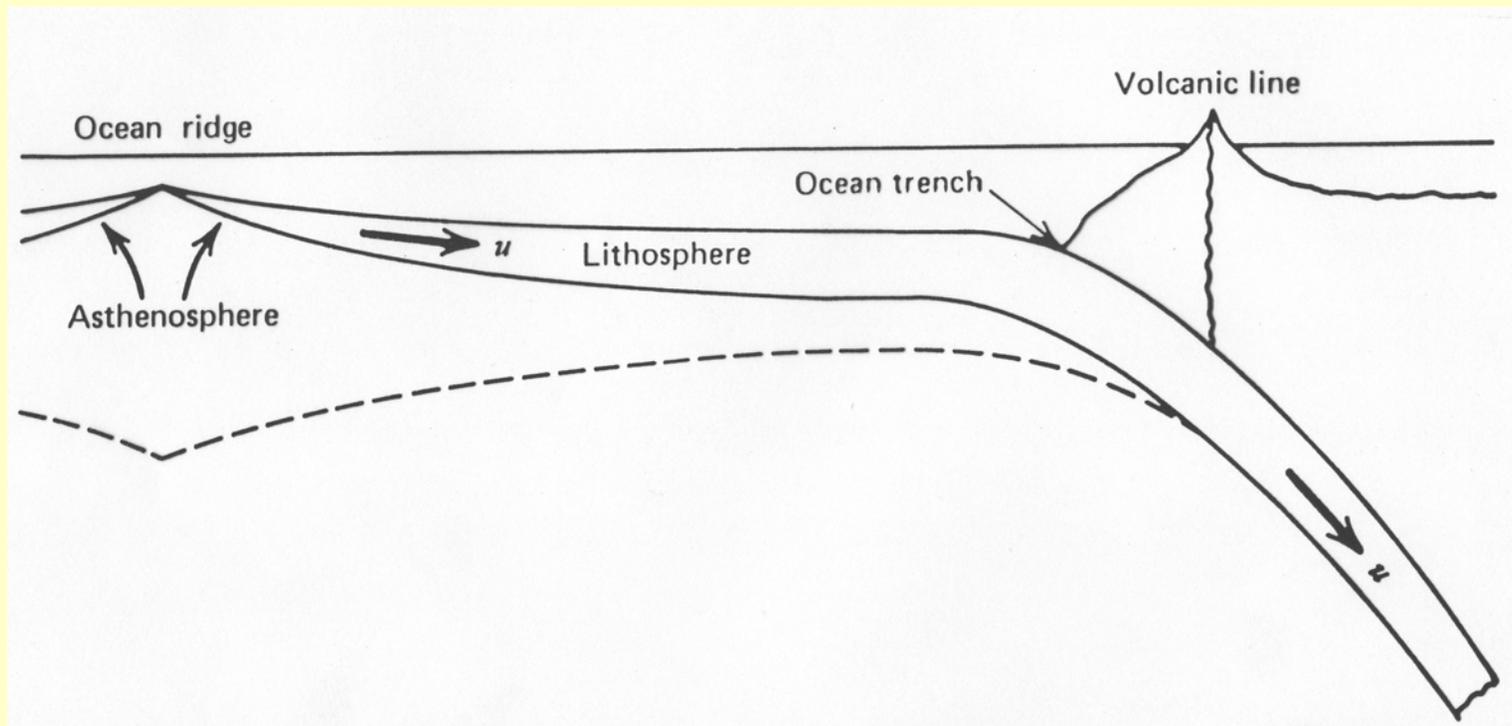
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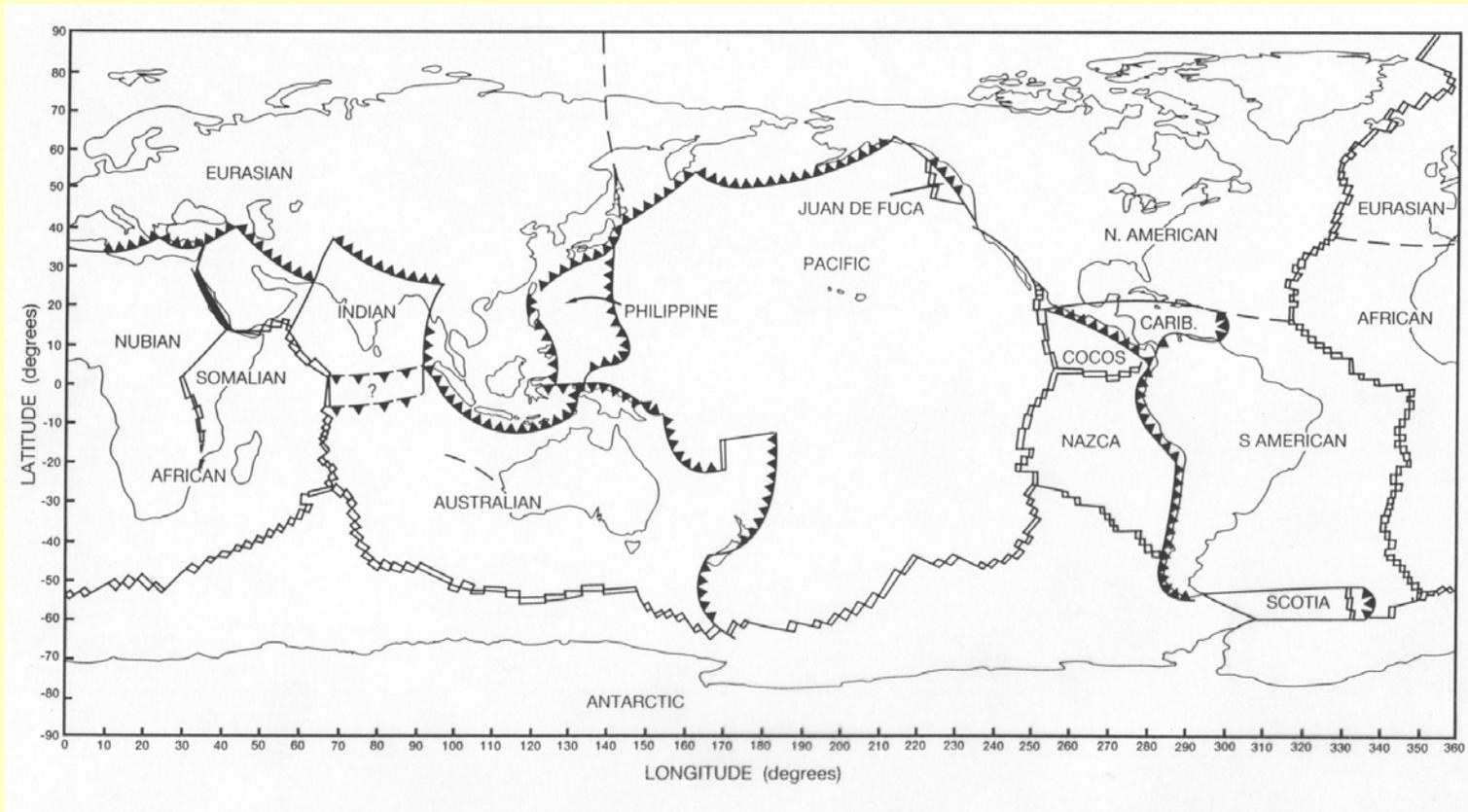
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2

LITHOSPHERIC PLATES



EARTH'S MAIN PLATES



THE LITHOSPHERE AS A HIERARCHICAL SYSTEM

Boundary zone	Block size (km)
Fault zone	$10^4 - 10^2$
Fault	$10^1 - 10^{-2}$
Crack	$10^{-3} - 10^{-5}$
Microcrack	$10^{-6} - 10^{-7}$
Interface	$10^{-8} - \dots$



A NATURAL DISASTER



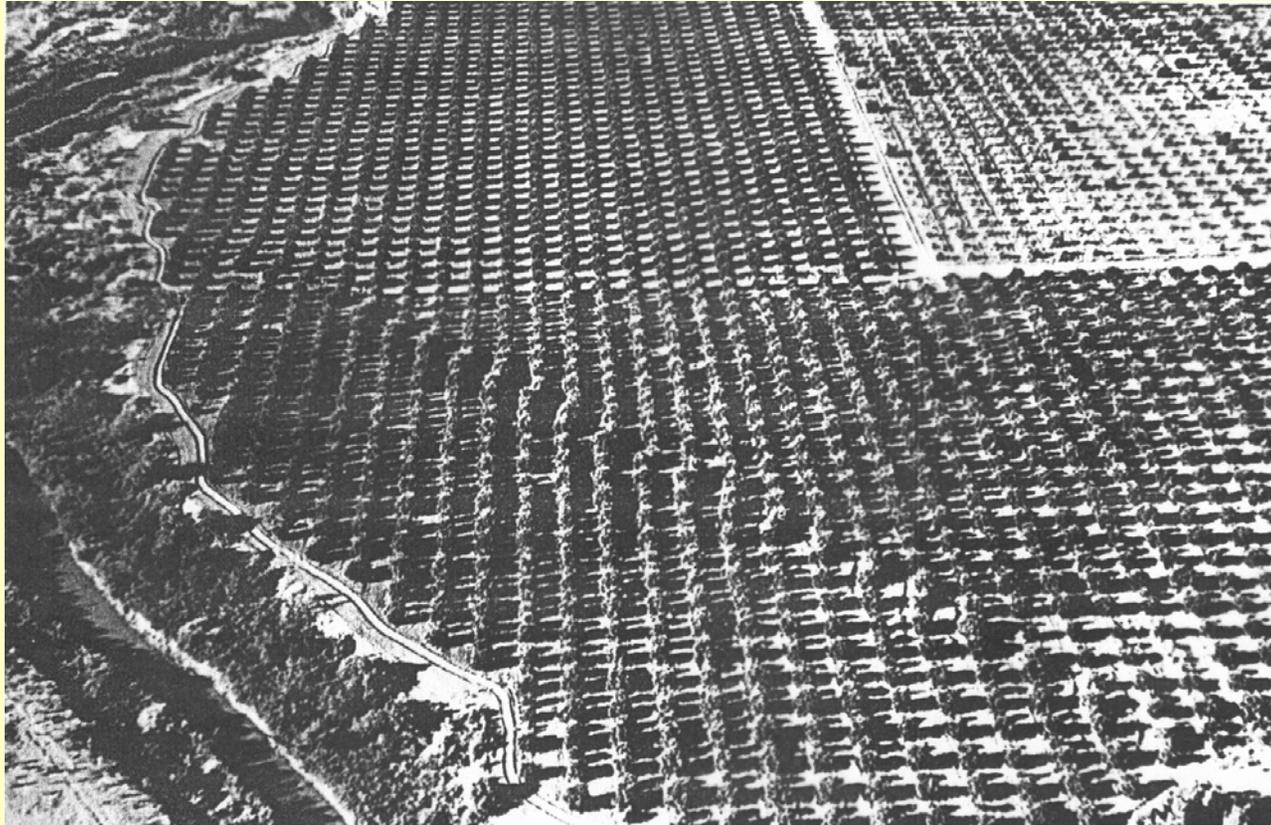
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GROUND DEFORMATION. 1



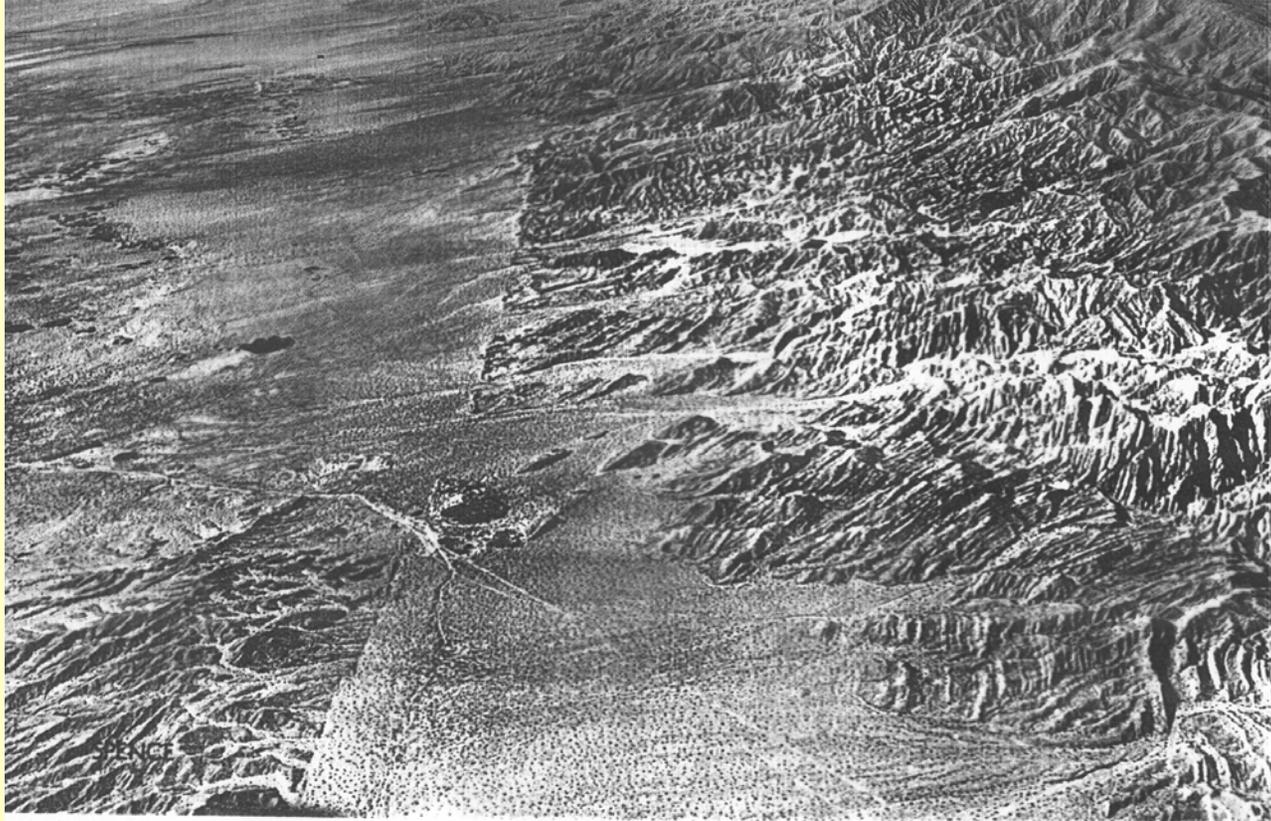
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GROUND DEFORMATION. 2



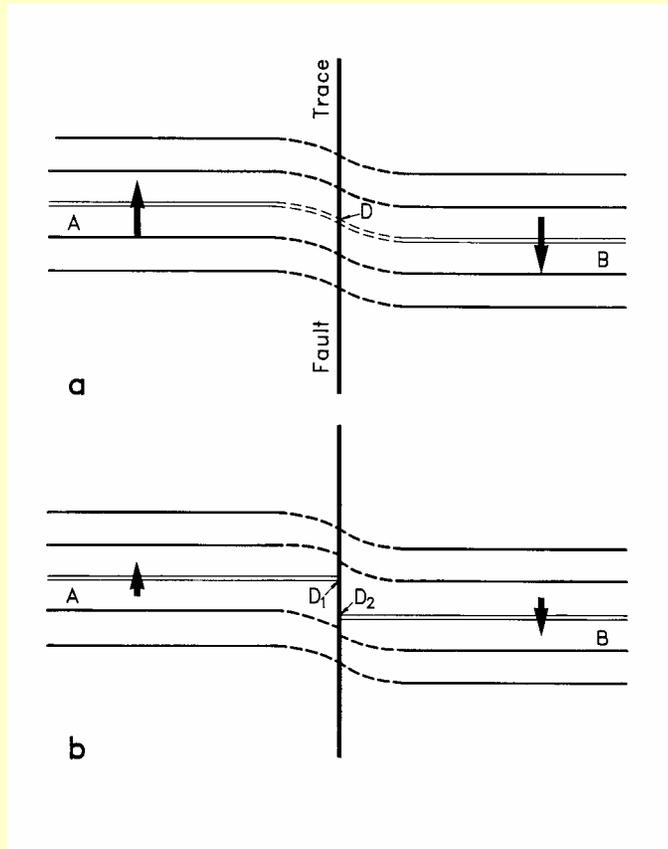
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ELASTIC REBOUND THEORY

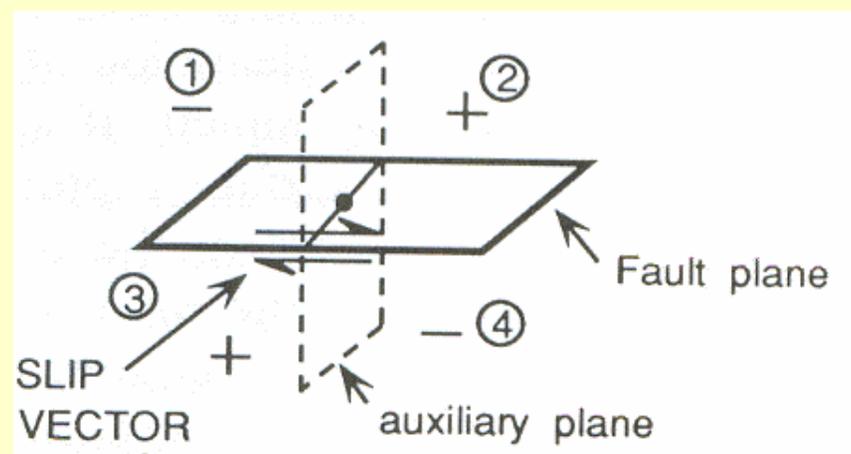
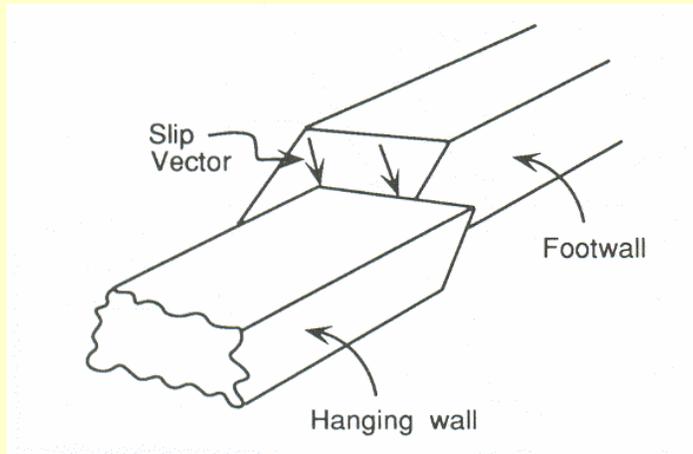
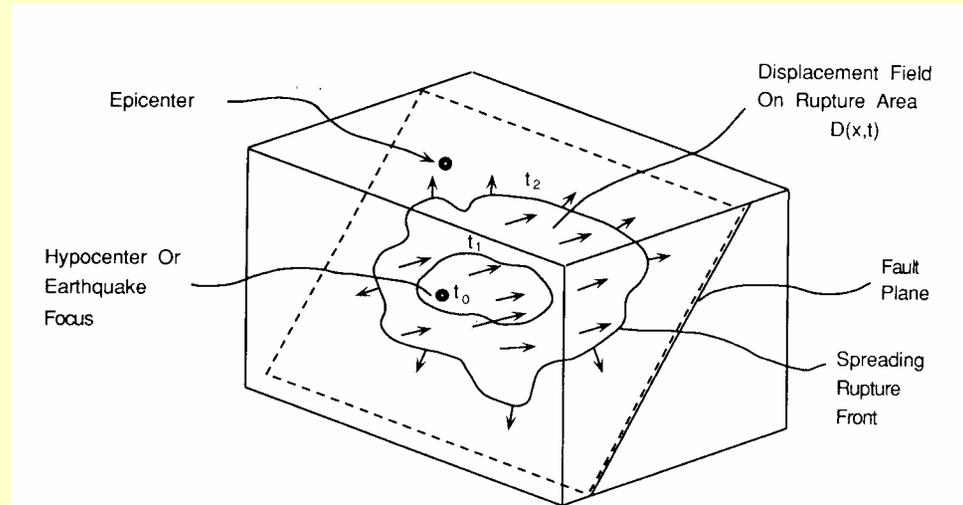


View of marker lines drawn along a road AB, which crosses a fault trace at the ground surface. (a) In response to the action of tectonic forces, points A and B move in opposite directions, bending the lines across the fault. (b) Rupture occurs at D, and strained rocks on each side of the fault have sprung back to D₁ and D₂.

(Bolt, 1976)



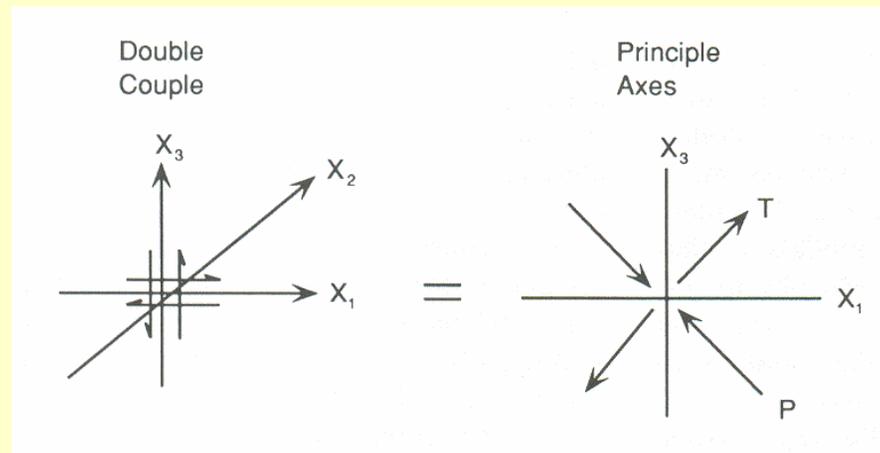
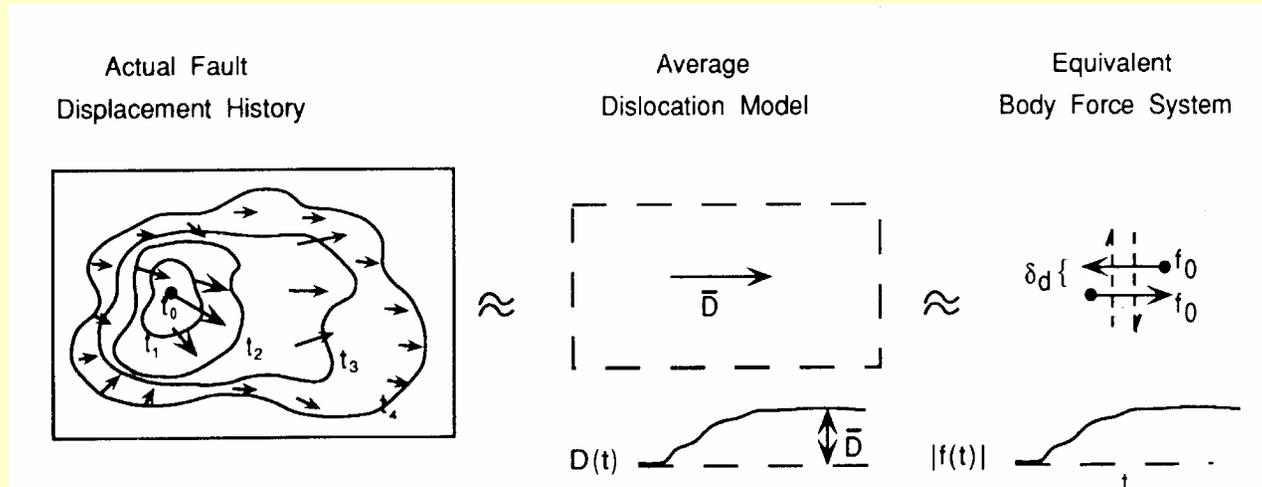
RUPTURE ON A FAULT



(Lay & Wallace, 1995)



BODY FORCE EQUIVALENCE. 1



(Lay & Wallace, 1995)



BODY FORCE EQUIVALENCE. 2

The displacement observed at (\mathbf{x}, t) due to a discontinuity at (ξ, τ) is:

$$u_n(\vec{x}, t) = \iint_{\Sigma} [u_i] v_j c_{ijpq} * \frac{\partial}{\partial \xi_q} G_{np} d\Sigma$$

where $[u(\xi, \tau)]$ is the displacement discontinuity across the surface Σ .

The (symmetric) moment *density tensor* \mathbf{m} is defined:

$$m_{pq} \equiv [u_i] v_j c_{ijpq}$$

So that

$$u_n(\vec{x}, t) = \iint_{\Sigma} m_{pq} * G_{np, q}$$



SHEAR PLANAR FAULT

$$m_{pq} = \mu(v_p[u_q] + v_q[u_p]) = \mu(u[a_{pq}] + u[a_{qp}])$$

$u = |u_q| = |u_p|$, $t \rightarrow \infty$. **u: final dislocation**

$a_{ij} \in [A]$, **[A] : slip spatial orientation**

$$\mathbf{m}_{pq} = \mu \mathbf{u} [A]$$



POINT SOURCE REPRESENTATION

$$u_n(\vec{x}, t) = M_{pq} * G_{np, q} \quad M_{pq} = \iint_{\Sigma} m_{pq} d\Sigma$$

$$M_{pq} = \mu u S [A] = M_0 [A] : \quad \text{seismic source}$$

$$M_0 = \mu u S : \quad \text{scalar seismic moment}$$

(strength of the source)

$$P = u S : \quad \text{seismic potency}$$

$$M_W = (\log M_0 - 9.1)/1.5 \quad \text{moment magnitude}$$



SEISMIC CATALOG

seismic source + earthquake coordinates

Seismic source:

- strength M_0 or M_w
- spatial orientation of slip [A]
(usually not accounted for)

Earthquake coordinates:

- hypocentral location
- origin time



MECHANICS OF EARTHQUAKES. 1

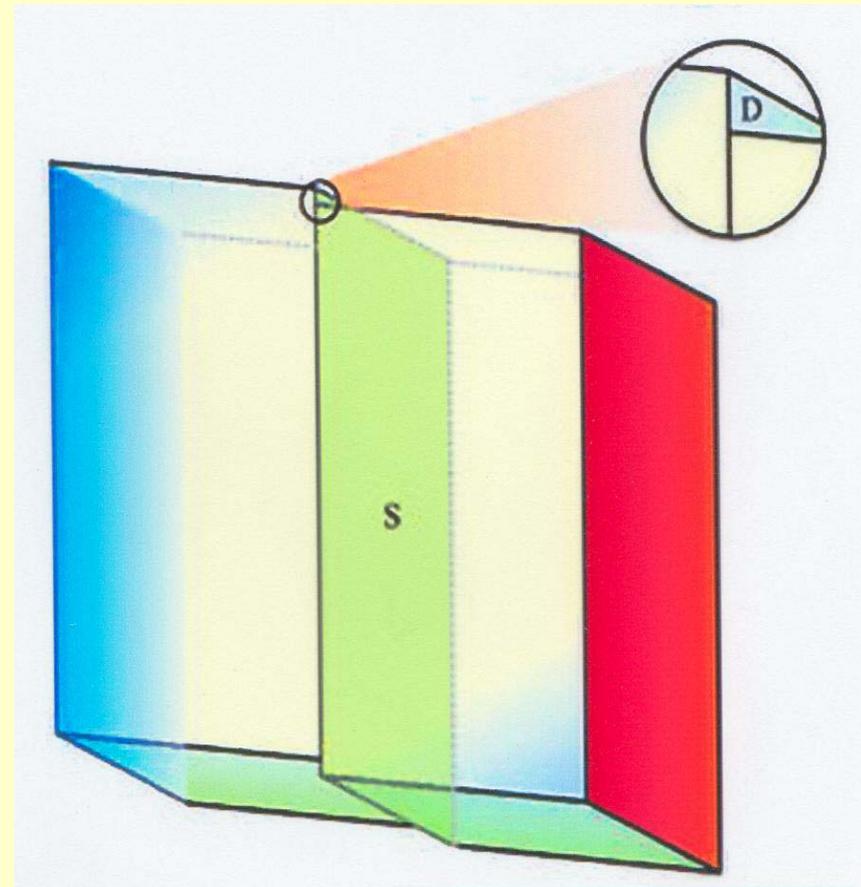
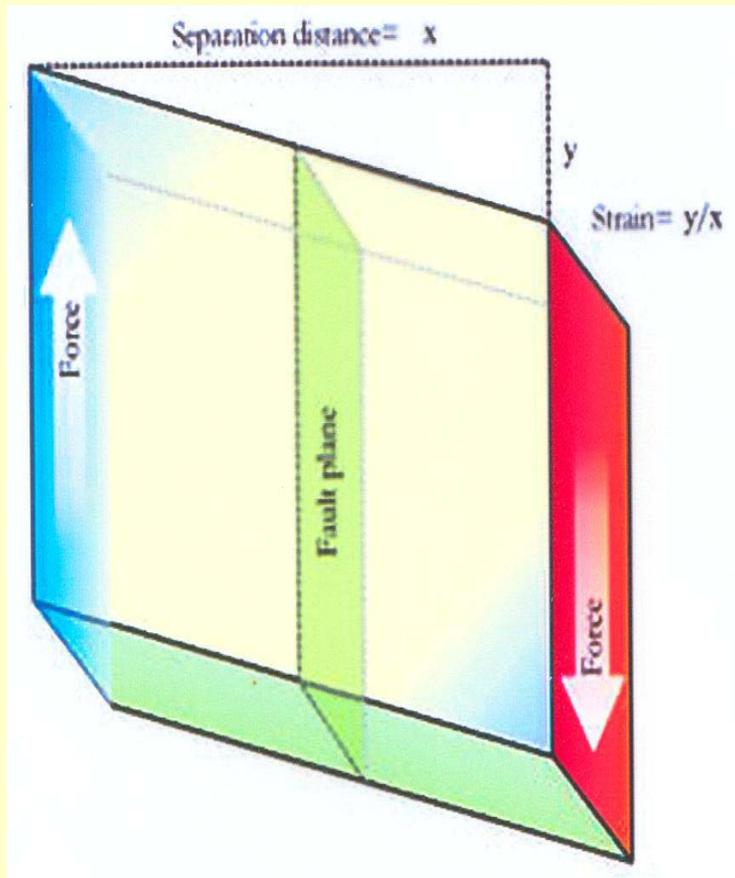
Earthquake: a sudden fracture in the earth's crust followed by ground shaking.

- dynamics of fracture
- wave propagation
- earthquake prediction

Earthquake: a long term process triggered by a short term process.



MECHANICS OF EARTHQUAKES. 2



(Kanamori, 2001)



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LONG TERM PROCESSES

Loading of tectonic stress (shear forces) due to the relative motion of tectonic plates.

- relative motion: 2 – 7 cm / year
- strain accumulation rate (plate boundaries): $\sim 3 \times 10^{-7}$ / year
- stress accumulation rate: $\sim 10^{-2}$ M Pa / year
- repeat times of major earthquakes: $\sim 100 - 1000$ years



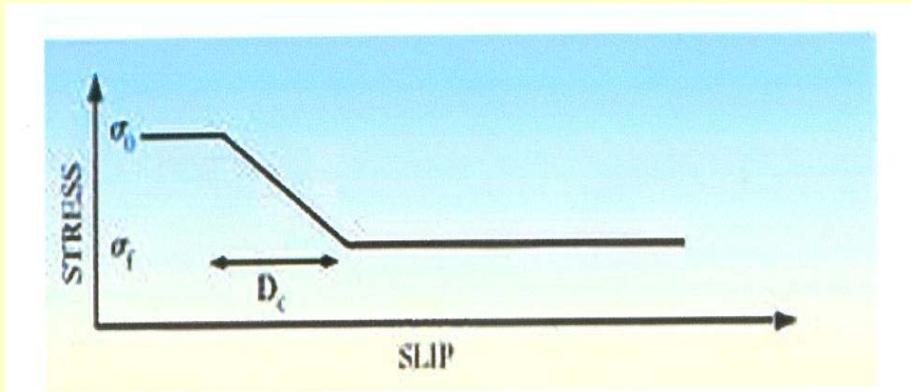
SHORT-TERM PROCESSES

Earthquake fault motion: a frictional sliding on a fault plane

- The friction changes as a function of slip (relative displacement of the two sides of the fault plane), velocity or history of contact
- Frictional stress controls seismic motion
- In general fault motion occurs in a stop-and-go fashion *stick-slip* motion
- Slip displacement: up to 10 m
- Particle velocity: up to 3m/s



MECHANICS OF RUPTURE



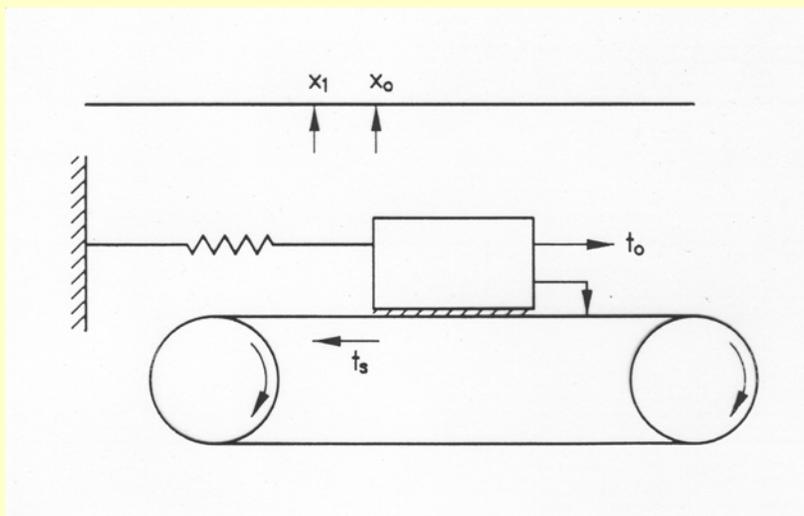
σ_0 : static frictional stress

σ_f : kinematic stress

- For a small slip D_c , the dynamic stress drop $\Delta\sigma_d = \sigma_0 - \sigma_f$ initially drives the sliding
- The sliding stops when the shear stress drops below a final frictional stress σ_1 (in general, $\sigma_1 \neq \sigma_f$)
- Static stress drop: $\Delta\sigma = \sigma_0 - \sigma_1$



ENERGY BUDGET – SPRING SYSTEM



Consider a simple mass-spring-slider model. The mass rides on a conveyor belt and is restrained by the spring

- t_0 : driving force of the conveyor on the block
- t_s : kinetic friction force
- an arm is attached to the block that scratches the conveyor

When friction drops to f_f the spring begins to recoil under a driving force $f_0 - f_f$, scratching the conveyor as it goes.

Eventually the spring stops at x_0 with a displacement $d = x_0 - x_1$, when the force is f_1 . Several mechanisms can stop the motion, so that f_1 is not necessarily equal to f_f .



ENERGY BALANCE

- Frictional energy loss: $E_f = d f_f$
- Total potential energy change: $\Delta W = \frac{1}{2}(f_0 + f_1) d$
- Radiated energy: E_R
- Fracture energy: E_G
- Conservation of energy: $E_R = \Delta W - E_f - E_G$



ANALOGY SPRING SYSTEM – EARTHQUAKE MODEL

Replace: $f_0 \rightarrow S \sigma_0$ S: fault plane
 $f_1 \rightarrow S \sigma_1$ d \rightarrow D
 $f_f \rightarrow S \sigma_f$

Thus: $\Delta W = D S (\sigma_0 + \sigma_1) / 2$
 $E_f = D S \sigma_f$
 $E_R = \Delta W - E_f$

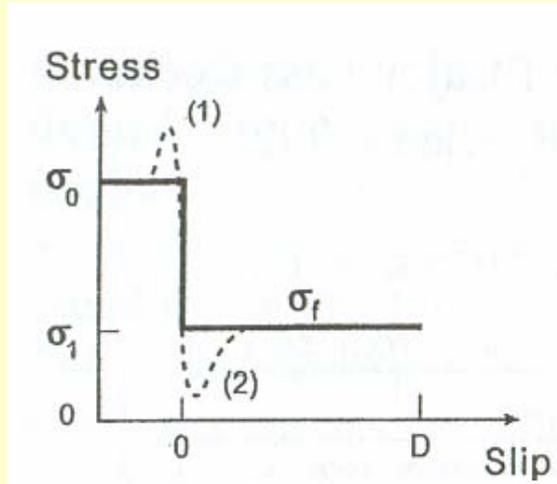
- ΔW cannot be determined. A lower bound can be estimated as

$$\Delta W_0 = D S \Delta \sigma_s / 2, \quad \sigma_s = (\sigma_0 - \sigma_1) / 2$$

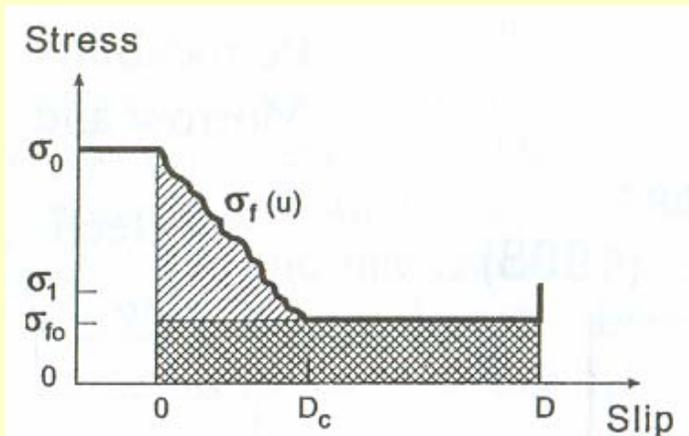
- The radiated energy E_R (elastic waves) is about a 10% of the total released energy. What about the remaining 90%?



MODEL OF ENERGY RELEASE



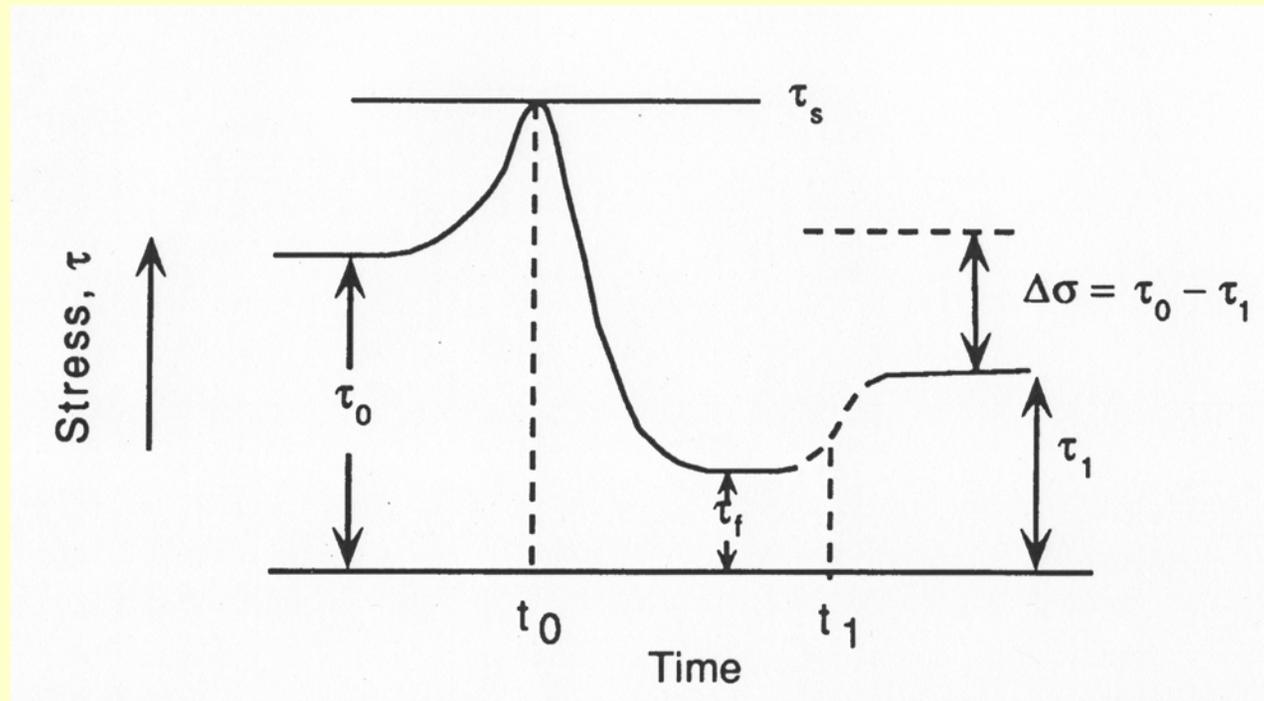
Simple model of stress release: $\Delta\sigma = \sigma_0 - \sigma_1 = \sigma_0 - \sigma_f$ (solid line). However, the rupture process may be more complex: the stress may increase at the beginning of the slip (dashed curve 1) or the friction may drop dramatically at the beginning, recovering later on its normal value (dashed curve 2).



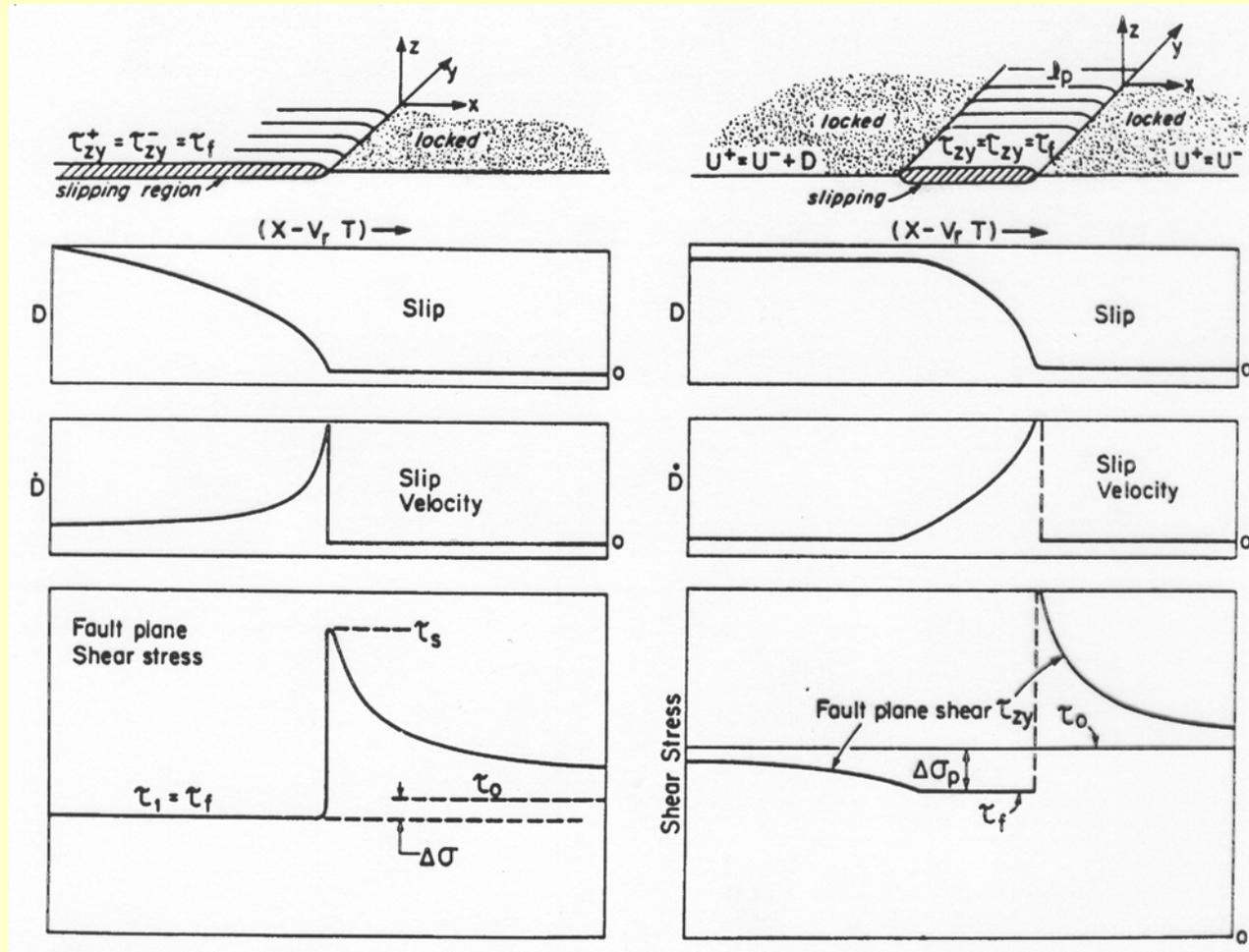
As a variation of curve (2), a gradual decrease of σ_f may occur as a function of slip (or slip velocity): **slip-weakening process (velocity weakening)**.



STRESS AT A POINT ON A FAULT SURFACE



IDEALIZED MODELS OF RUPTURE

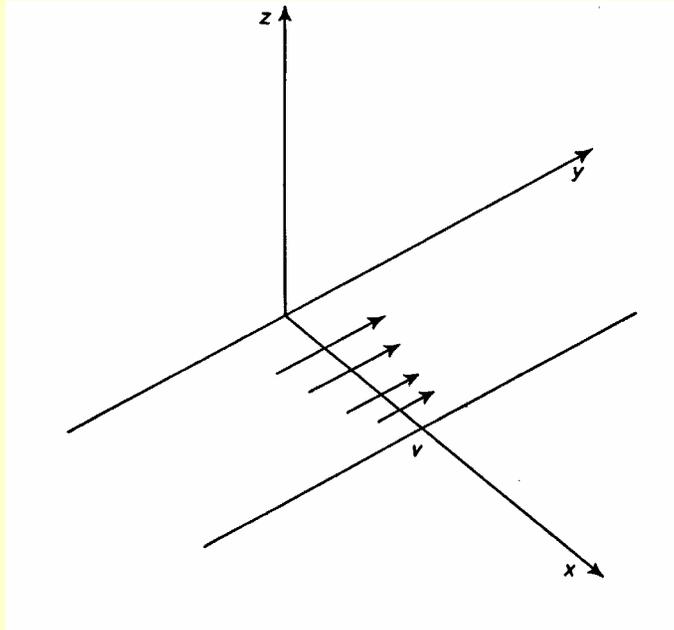


Crack model

Self-healing slip pulse



ANTIPLANE FAULT GEOMETRY

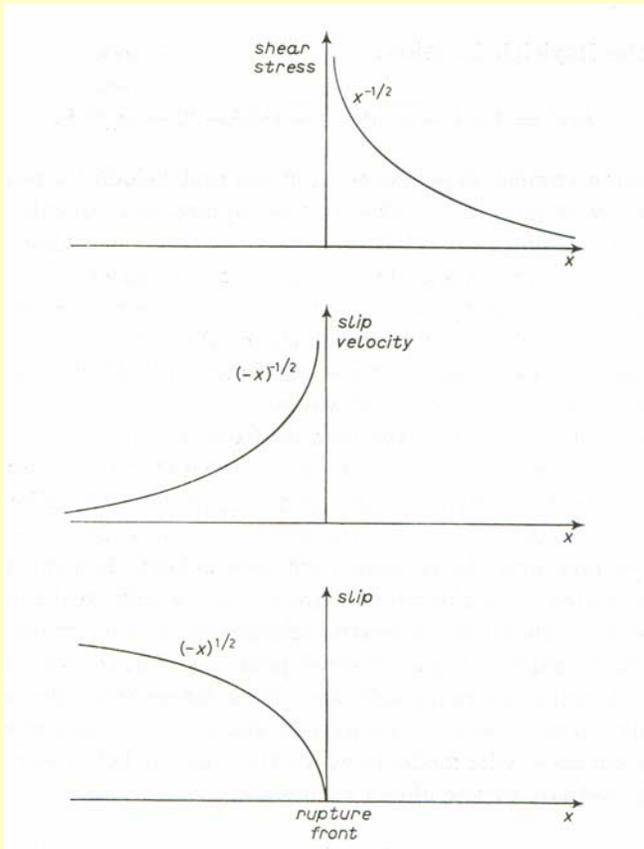


Three modes of fracture may be defined:

- **Antiplane, mode III or SH**, where stress drops in the direction y , *i.e.*, σ_{zy} is relaxed. Slip is in the direction y .
- **Plane, mode II**, where the component σ_{zx} of stress is relaxed. Slip is in the direction x . Both P and SV waves are excited.
- **Opening or mode I cracks**, where the crack opens in the direction z due to the relaxation of the σ_{zz} stress on the fault.



RUPTURE FRONT



Shear stress

$$\sigma(x) = K [x - l(t)]^{\frac{1}{2}}, x > l(t)$$

Slip velocity

$$\Delta \dot{u}(x) = V [l(t) - x]^{\frac{1}{2}}, x < l(t)$$

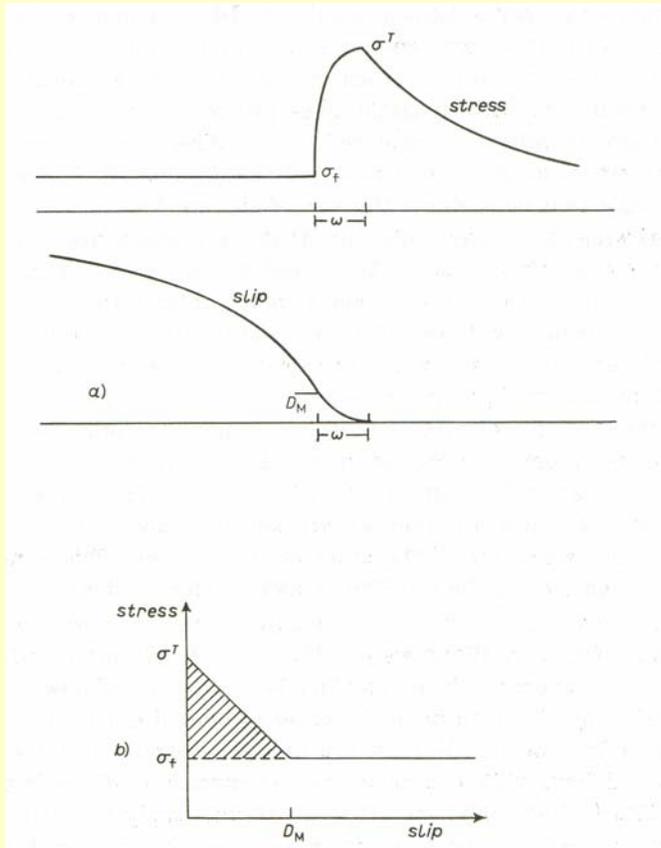
Slip

$$\Delta u \propto [l(t) - x]^{\frac{1}{2}}, x < l(t)$$

K, V: dynamic stress intensity and velocity intensity factors



COHESIVE END ZONE



Linear elastic fracture mechanics: coarse-grained approximation to fracture mechanics.

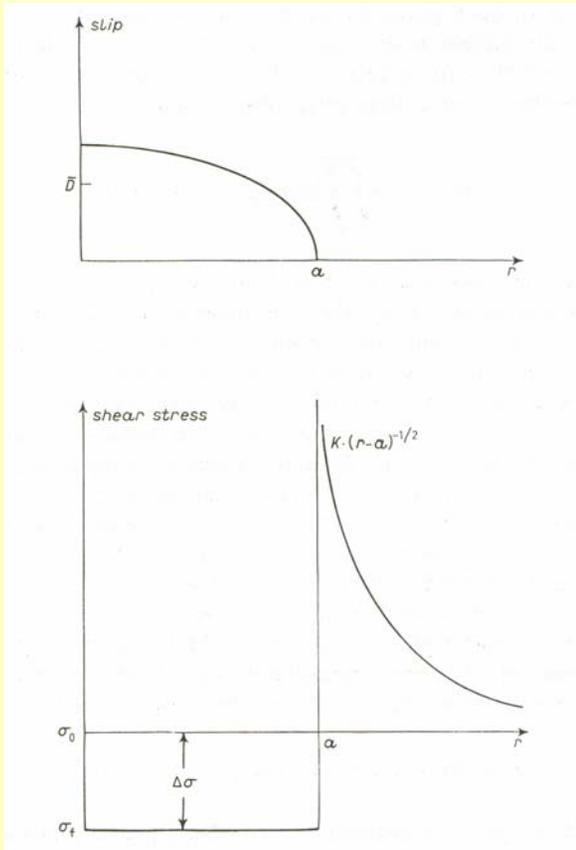
Physics of the process: needs the incorporation of the nonlinear processes near the crack tip.

A solution: consider a cohesive model of the breakdown zone, as for example a slip-weakening model to relate stress and slip.

As slip increases, stress decreases from σ^T to σ_f . Once the slip reaches the maximum shearing slip D_M , the stress remains fixed and equal to the frictional stress σ_f .



THE STATIC CIRCULAR CRACK MODEL



$$\Delta\sigma = \sigma_{xz}^0 - \sigma_{xz}^f$$

$$\Delta u_x(r) = D(r) = \frac{24}{7\pi} \frac{\Delta\sigma}{\mu} (a^2 - r^2)^{\frac{1}{2}}$$

$$\bar{D} = \frac{16}{7\pi} \frac{\Delta\sigma}{\mu} a$$

$$M_0 = \frac{16}{7} \Delta\sigma a^3$$

$$\Delta\varepsilon = \frac{\Delta\sigma}{\mu}$$



OCCURRENCE AND PREDICTION OF EARTHQUAKES

- I. Rupture Mechanics (earthquake as a rupture)
- II. Earthquake Occurrence (the seismic catalog)
- III. Synthesis: Earthquake Prediction (an emergent structure)

