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"Seventh Workshop on Non-Linear Dynamics and Earthquake Prediction"

29 September - 11 October 2003

Boolean Delay Equations (BDEs) as Succinct Models for Climate, Evolution, Earthquakes and other Complex Stuff

Earthquake Applications

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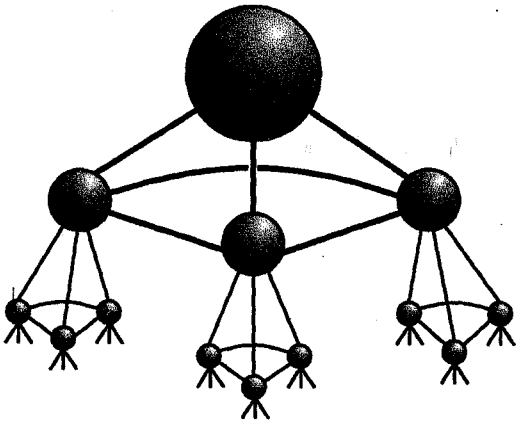
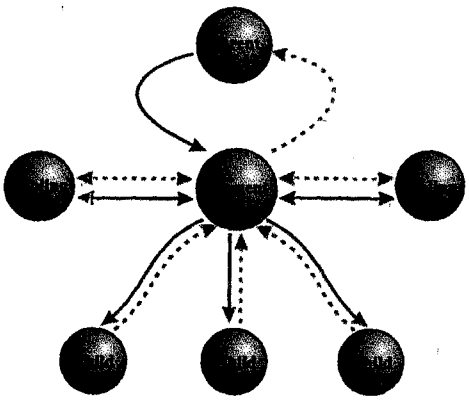
& UCLA, USA

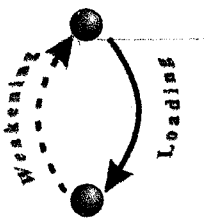
Colliding cascade model

Structure of the model



Interaction between elements

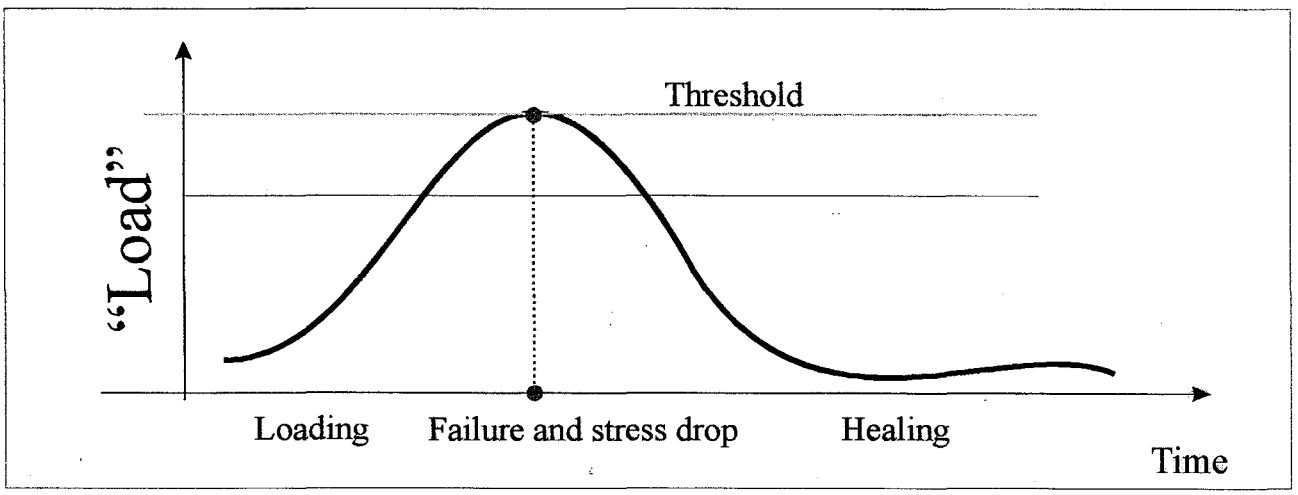




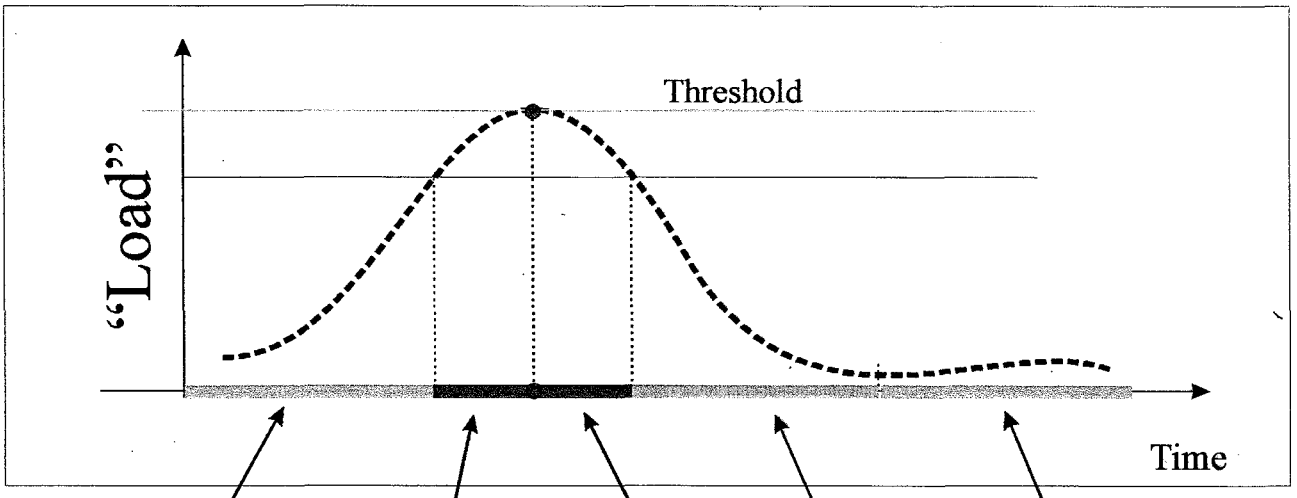
A. Gabrielov, V. Keilis-Borok, W. I. Newman & I. Zaliapin (2000, Phys. Rev. E + Geophys. J. Intl.)

Coarse graining

Physical system



Model



1: Unloaded & intact

2: Loaded & intact

3: Loaded & failed

4: Unloaded & failed

1: Unloaded & intact

Replace elementary interactions of elements in the system by their integral effect: time delays between consecutive switching of element's state.

Δ_L delay between application of load and switching to the loaded state

Δ_H time needed for healing

Model.

(i) The model runs in discrete time $n = 0, 1, \ldots$. At each epoch a given element may be either *intact* or *failed*, and either *loaded* or *unloaded*. The state of an element e at a moment n is defined by two Boolean functions:

$$s_e(n) = \begin{cases} 0, \text{ if element is intact,} \\ 1, \text{ if element is failed.} \end{cases}$$

$$l_e(n) = \begin{cases} 0, \text{ if element is unloaded,} \\ 1, \text{ if element is loaded.} \end{cases}$$

Thus, an element may be in one of the four states $(s_e,\ l_e)$

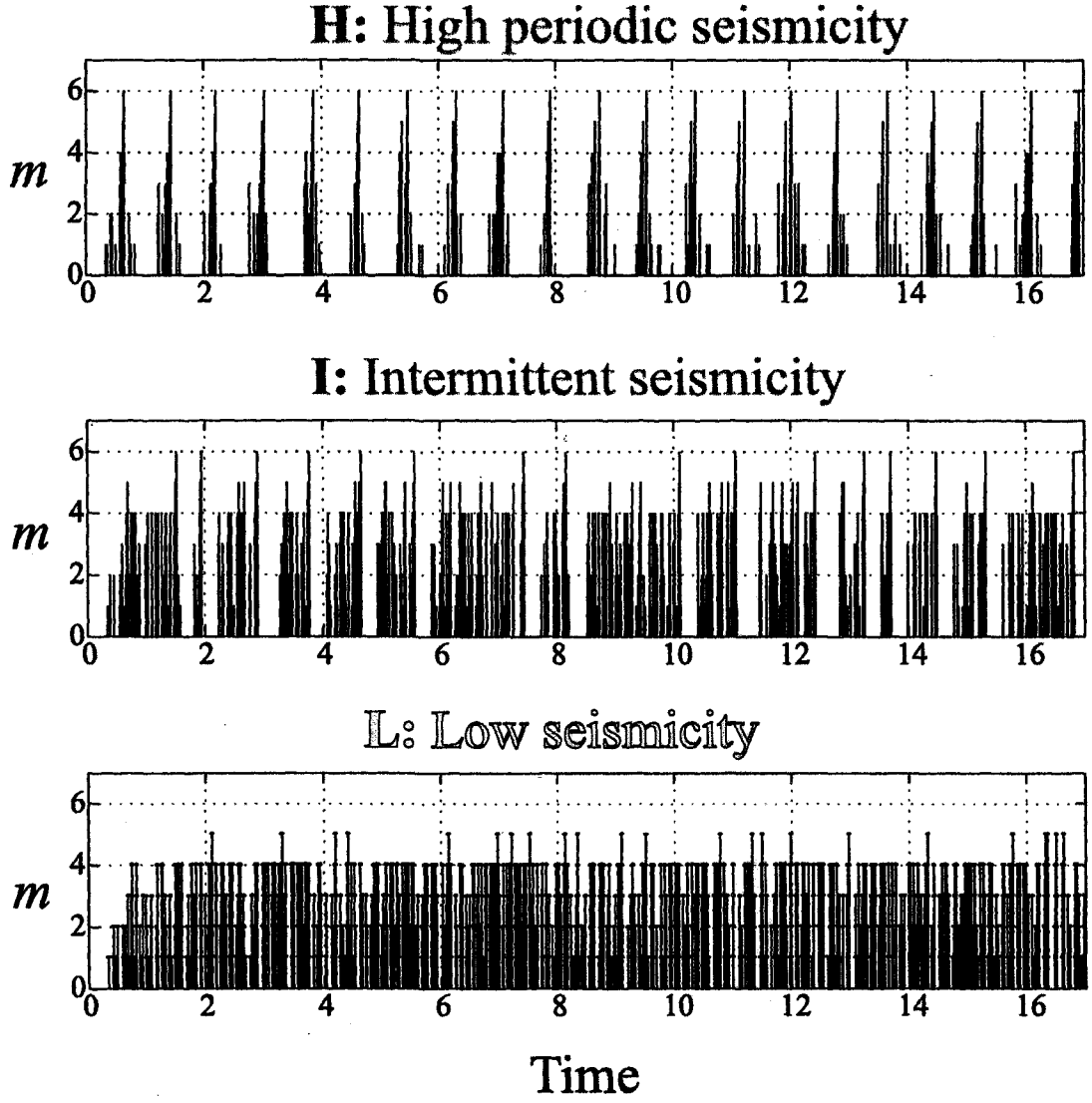
s_e	l_e	Element's state
0	0	*intact and unloaded*
0	1	*intact and loaded*
1	0	*failed and unloaded*
1	1	*failed and loaded*

(ii) An element of the system may switch from one state to another under an impact from its nearest neighbors. The dynamics of the system is controlled by the time delays between the given impact and switching to another state. The time delays are the following:

- Δ_L - between an element being impacted by the load and switching to the loaded state;
- Δ_F - between the increase in weakness and switching to the failed state;
- Δ_D - between failure and switching to the unloaded state;
- Δ_H - between the moment when healing conditions are established and switching to the intact (healed) state.

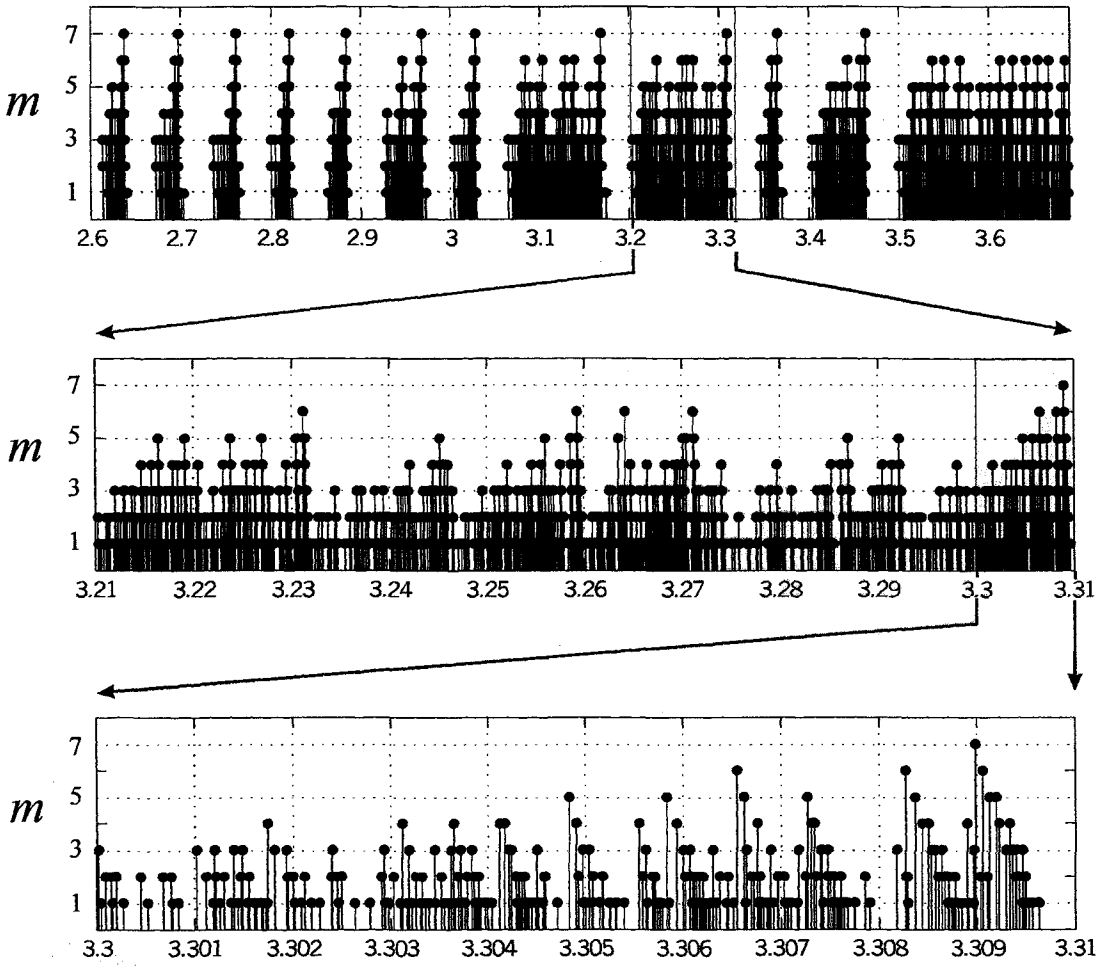
(iii) At the start, $n = 0$, all elements are in the state $(0, 0)$, intact and unloaded.

BDE model of colliding cascades: three seismic regimes



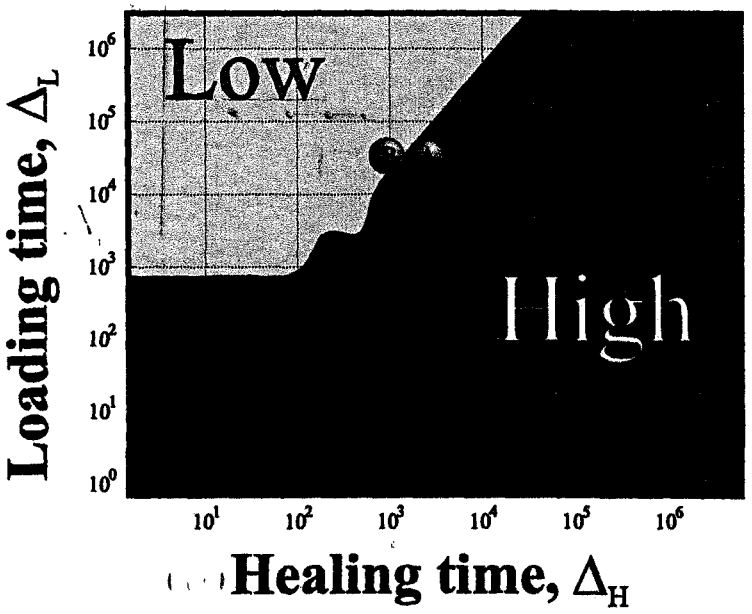
I. Zaliapin, V. Keilis-Borok & M. Ghil (2003a, b, JSP)

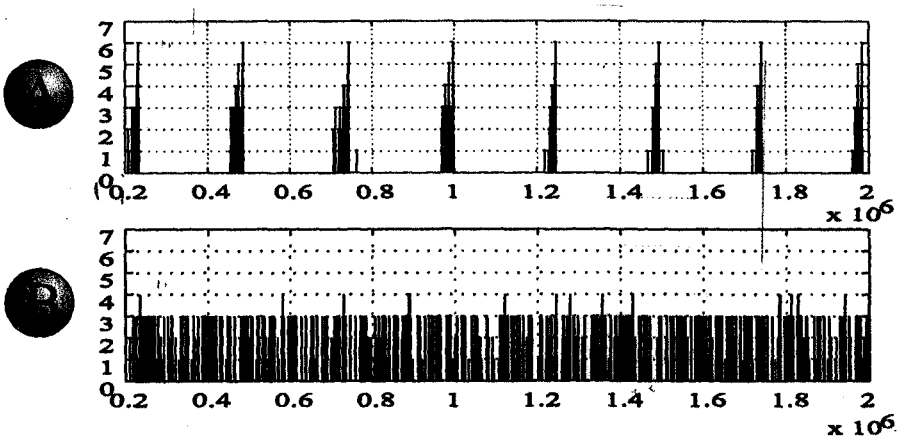
Synthetic sequence: zoom.



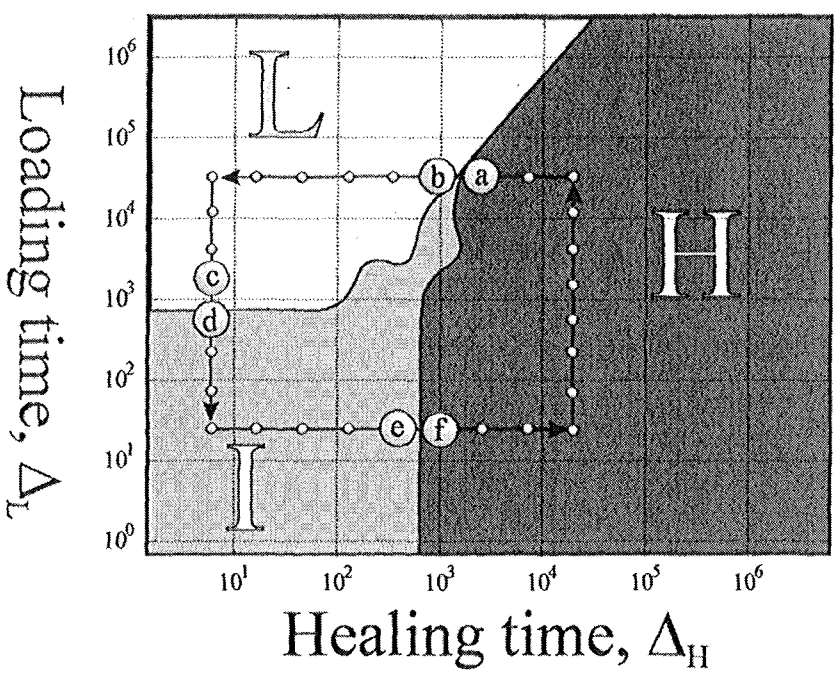
Time in 10^6 time units.

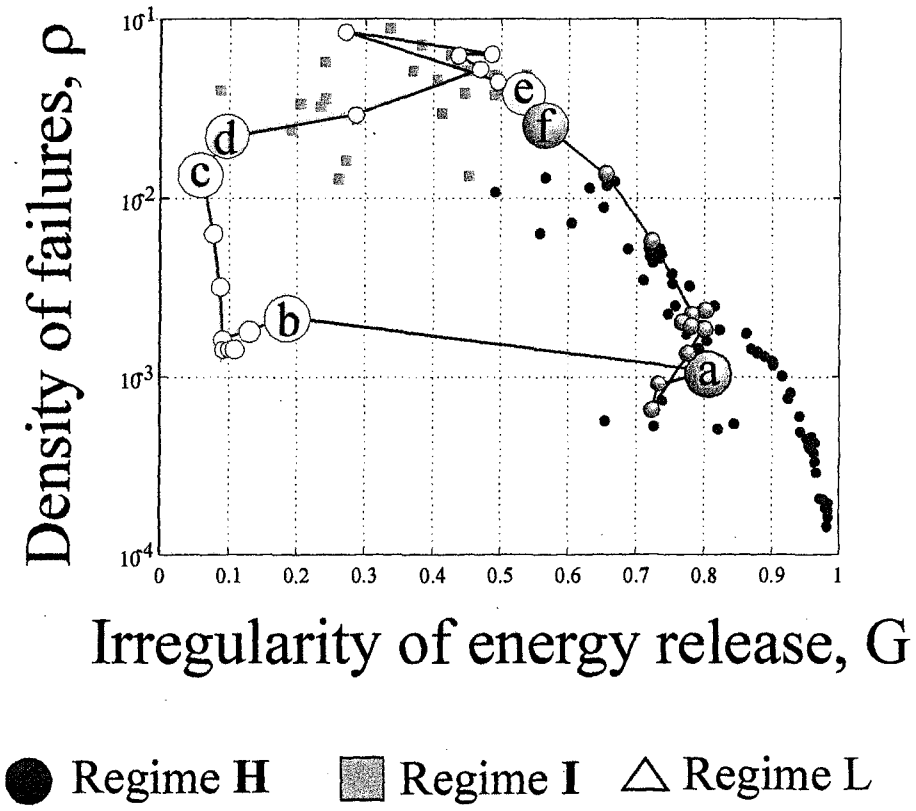
Regime diagram: instability near the triple point





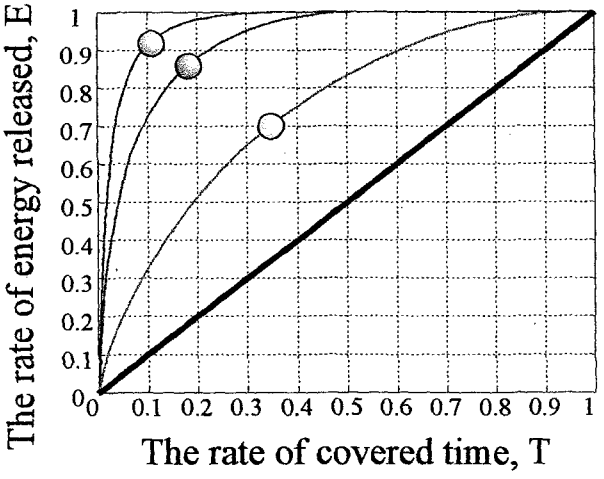
Regime diagram: switching between regimes

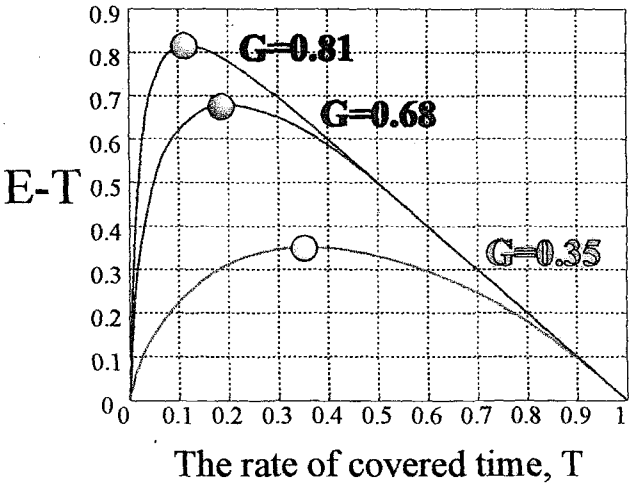




● Regime **H** ▩ Regime **I** △ Regime L

Definition of a measure of clustering, G = max{E - T}

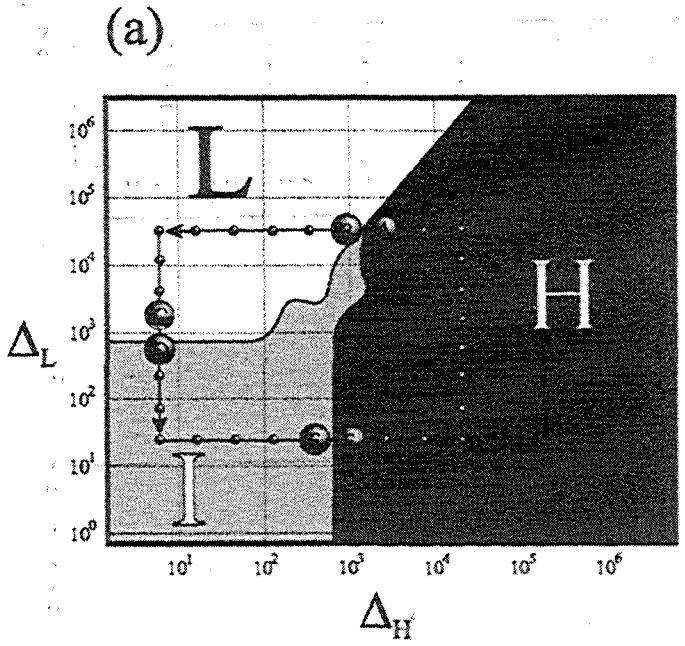




Regime H

Regime I

Regime M



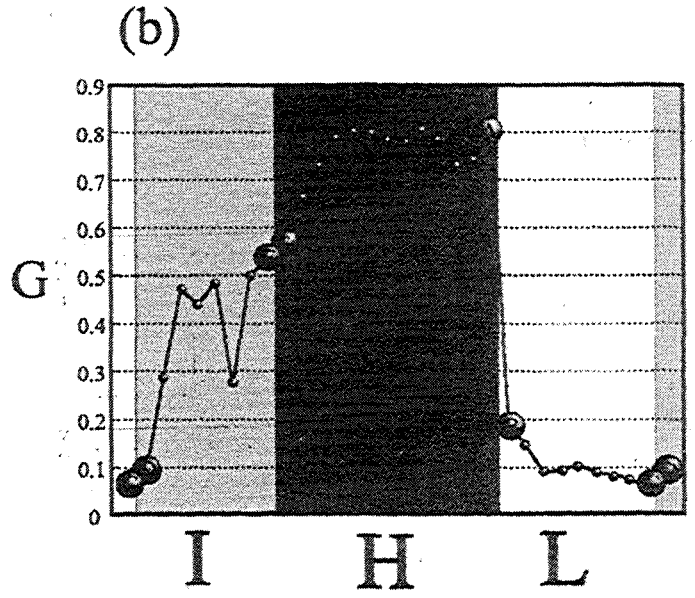


Fig. 8

CCM-BDB 2001. Part I.
I. Zaliapin, V. Kelliis-Borok, M. Ghil

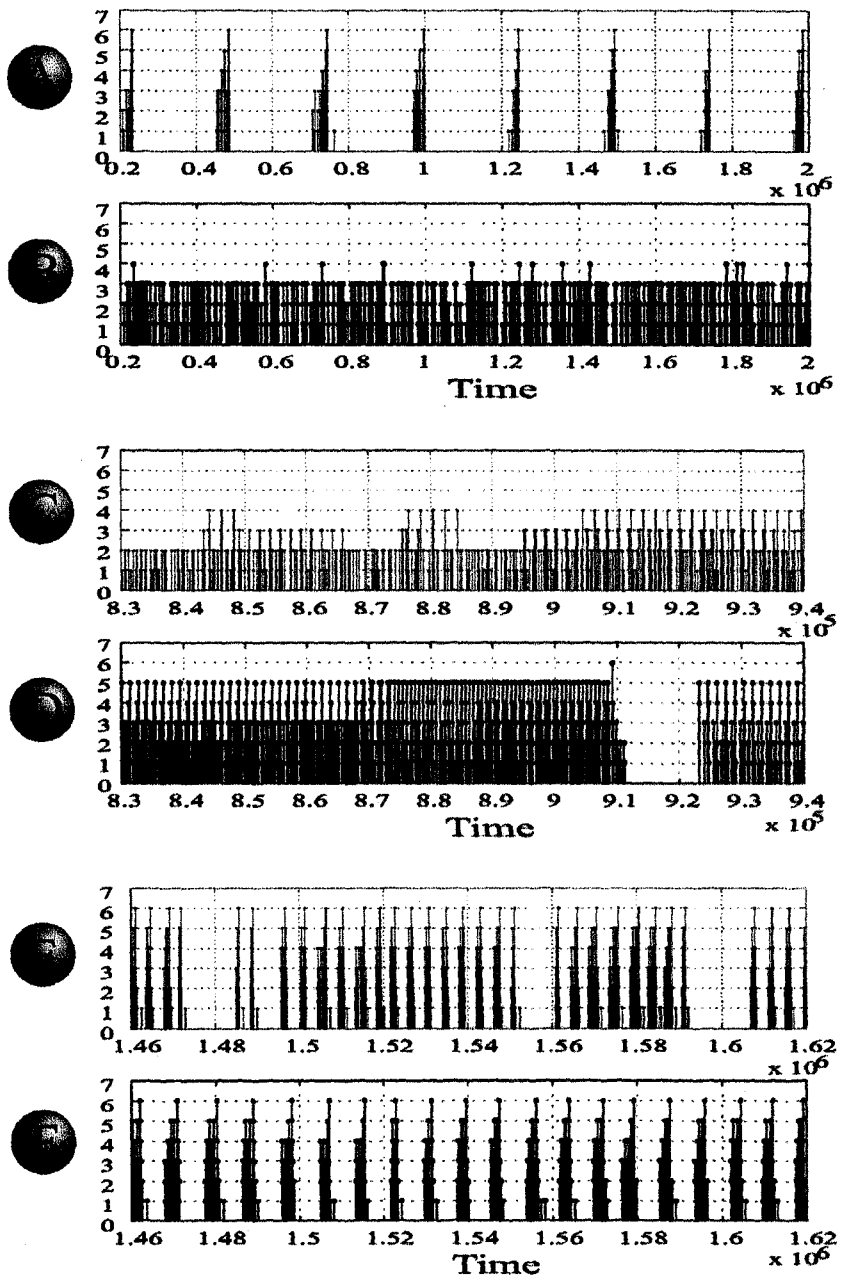
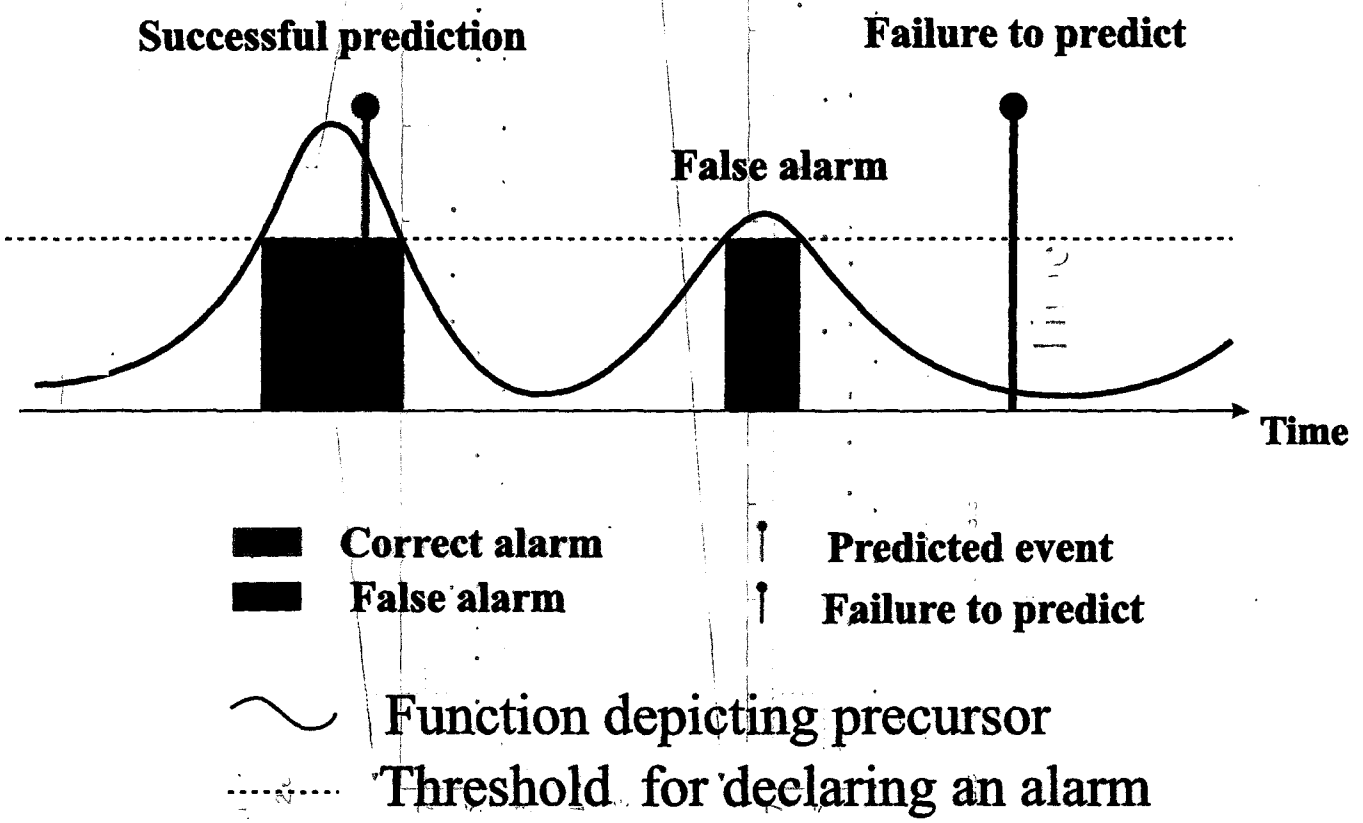


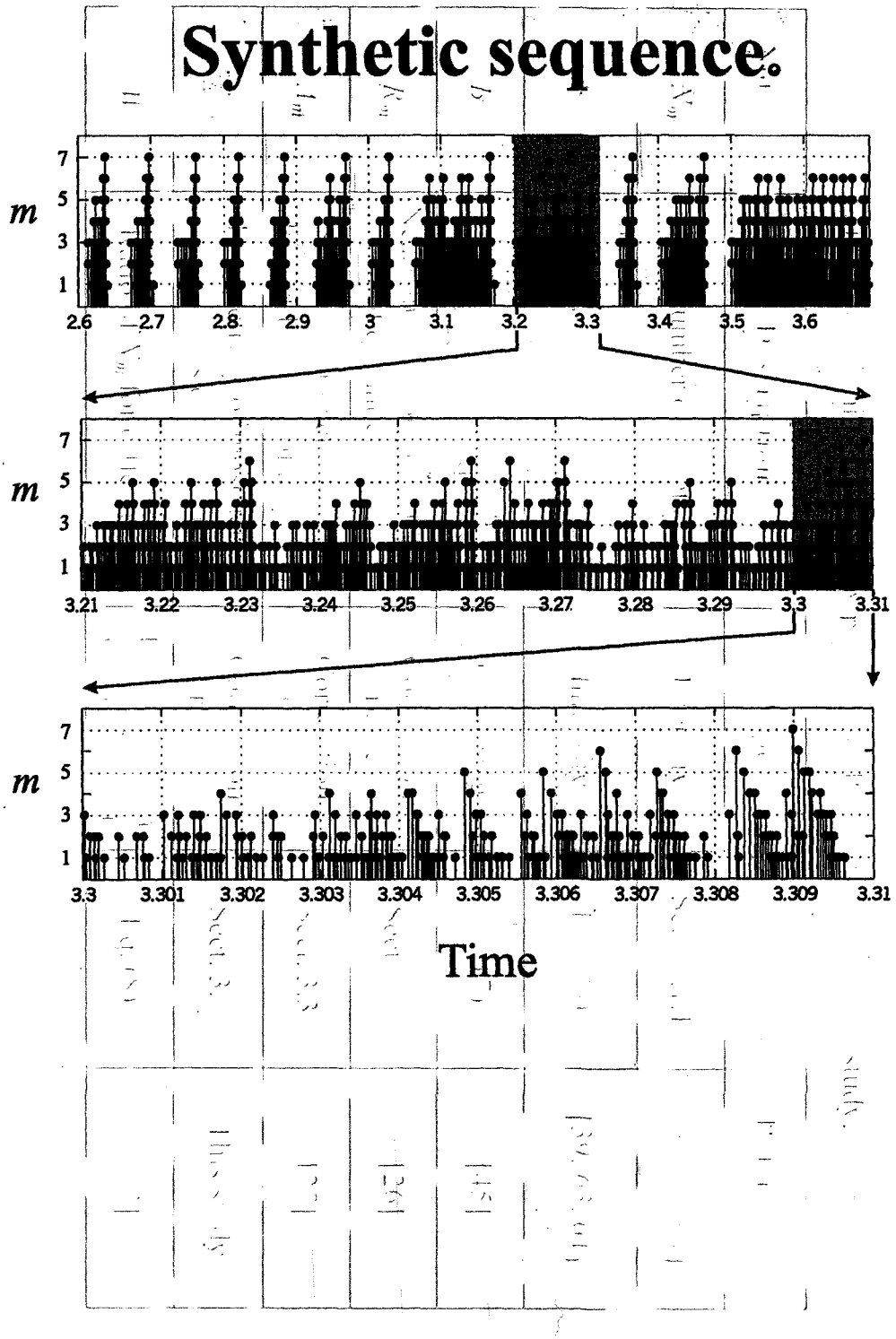
Fig. 9

CCM-BDE 2001. Part I.
I. Zaliapin, V. Keilis-Borok, M. Ghil

Possible outcomes of prediction



V. I. Keilis-Borok et al. (MITP)

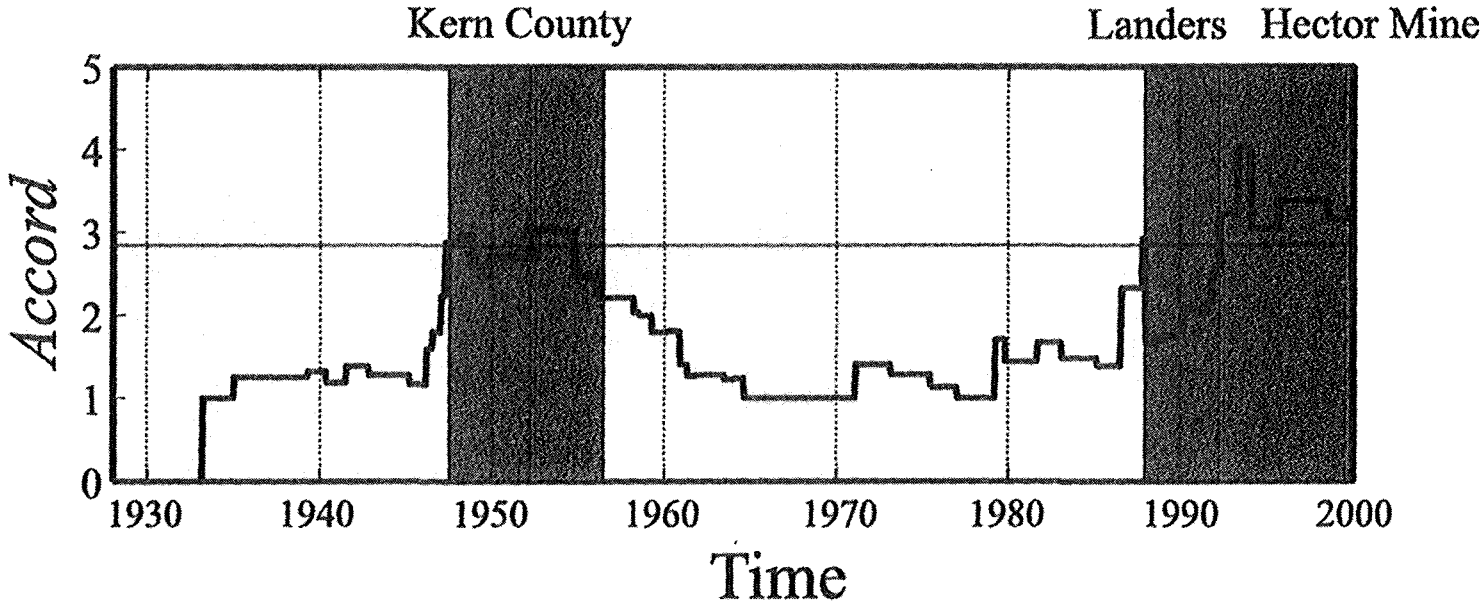


Zaliapin, Keilis-Borok & Ghil (2026, J. Stat. Phys.)

Table 1. Premonitory seismicity patterns (PSPs) considered in this study.

Notation	Description	Type	Definition in the text	References
N_m	Number of events	Intensity	Sect. 3.1.1	[40, 63, 61]
S_m	Weighted number of events; coarsely estimates the area of faultbreaks	Intensity	Eq. (3a)	[39, 63, 61]
B_m	Weighted number of immediate aftershocks	Clustering	Eq. (7)	[45]
R_m	Near-simultaneous occurrence of distant events	Correlation range	Sect. 3.3	[26]
A_m	Simultaneous activation of distinct branches of a system	Correlation range	Sect. 3.3	[27]
π_m	Total activity of most active branches of a system	Correlation range	Sect. 3.3	This study
W	Ratio of N_m for different m	Transformation of GR relation	Eq. (8)	[63]

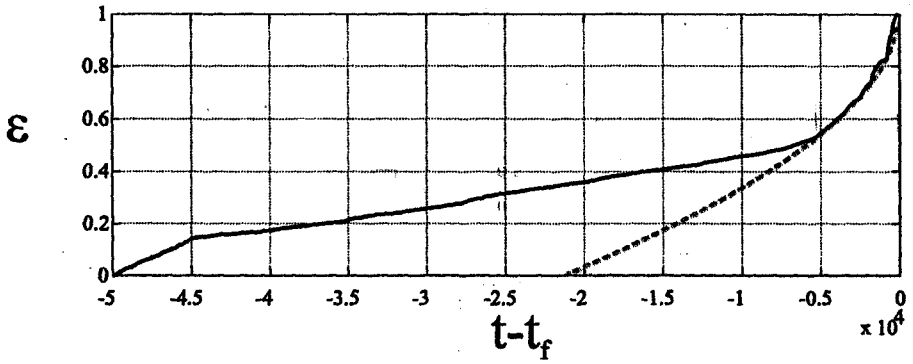
Precursor Accord in S. California: Analysis of M7.5+ earthquakes.



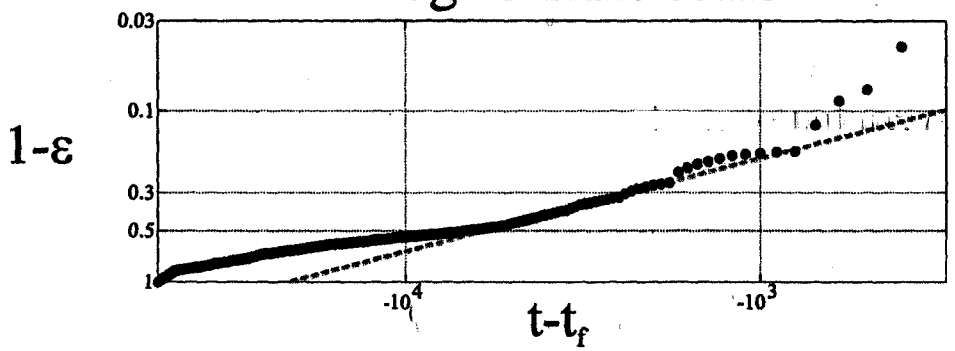
Zaliapin, Keilis-Borok & Axen (2002, JGR)

Acceleration of Benioff strain release

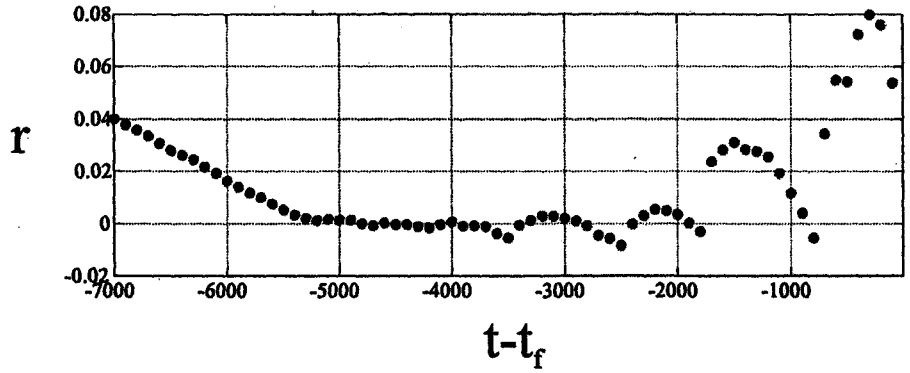
Linear scale



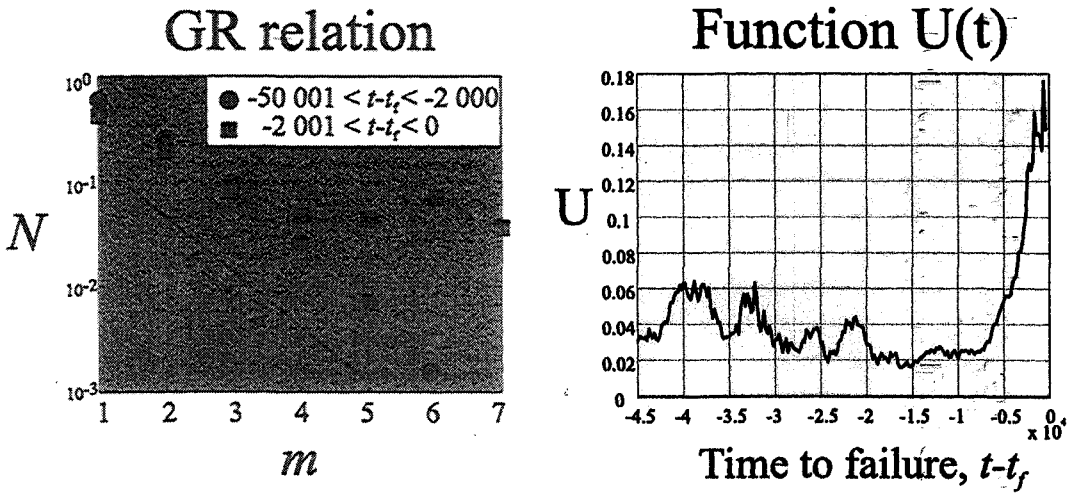
Logarithmic scale



Residuals in linear scale



Premonitory change of GR relation.



$$U(t) = \frac{\#\{\text{EQs with } 4 \leqslant M \leqslant 6\}}{\#\{\text{EQs with } 1 \leqslant M \leqslant 3\}}$$

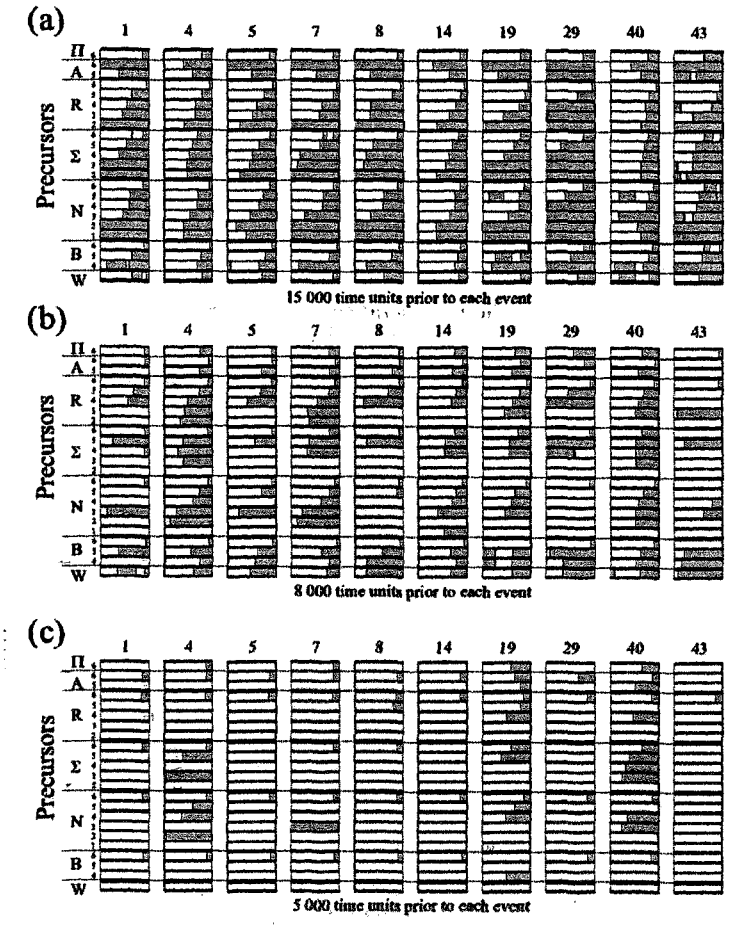
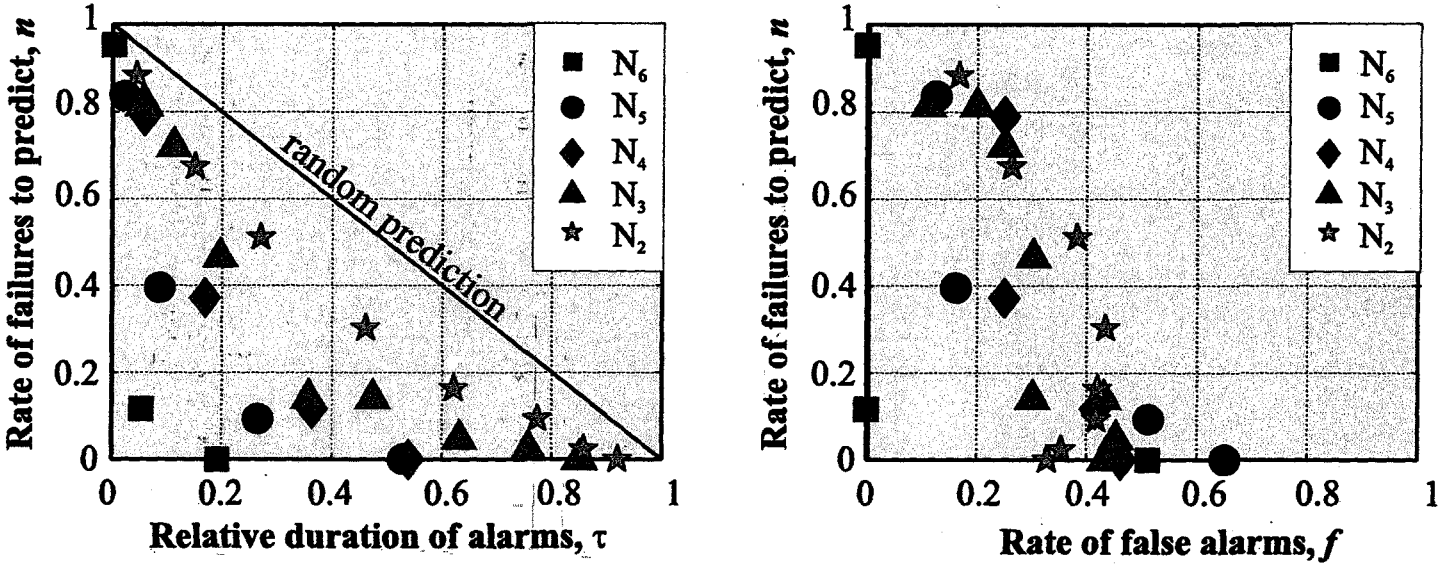


Figure 6.

CCM-BDE. PartII. 2001

I. Zaliapin, V. Keilis-Borok, M. Ghil

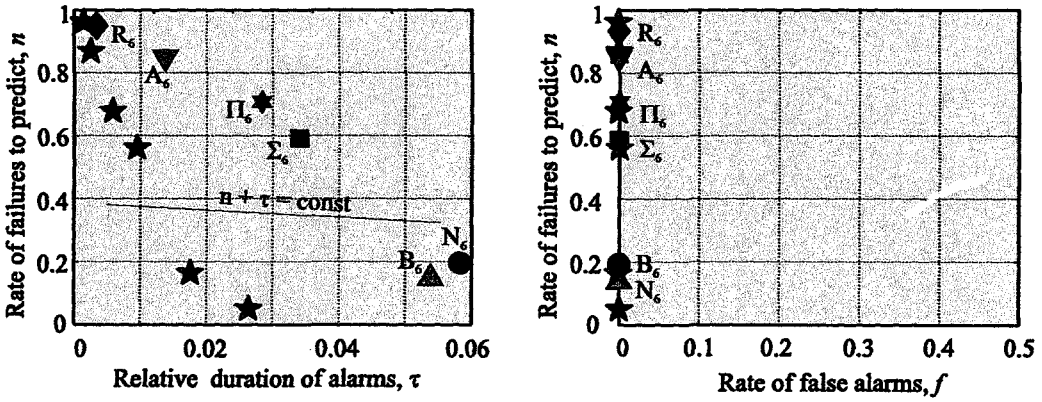
Boolean delay equation model of colliding cascades. Error diagram* for precursor N (Number of earthquakes)



* G. M. Molchan et al. (1990, PEPI)

Minimax prediction strategy

Individual PSPs are tuned to eliminate false alarms at the cost of having more failures to predict. Collectively, errors of both kinds are drastically reduced.



★ Alarm is declared when N out of 6 individual PSPs produce an alarm; N=1,...,6 from bottom to top.

BDE Bibliography

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1. Dee, D., and M. Ghil, 1984: Boolean difference equations, I: Formulation and dynamic behavior, *SIAM J. Appl. Math.*, **44**, 111–126.
2. Ghil, M., and A. Mullhaupt, 1985: Boolean delay equations, II: Periodic and aperiodic solutions, *J. Stat. Phys.*, **41**, 125–174.

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1. Ghil, M., A. Mullhaupt, and P. Pestiaux, 1987: Deep water formation and Quaternary glaciations, *Climate Dyn.*, **2**, 1–10.
2. Mysak, L. A., D. K. Manak, and R. F. Marsden, 1990: Sea-ice anomalies observed in the Greenland and Labrador Seas during 1901–1984 and their relation to an interdecadal Arctic climate cycle. *Climate Dyn.*, **5**, 111–113.
3. Wright, D.G., T.F. Stocker, and L.A. Mysak, 1990: A note on Quaternary climate modelling using Boolean delay equations, *Climate Dyn.*, **4**, 263–267.
4. Darby M.S., and L.A. Mysak, 1993: A Boolean delay equation model of an interdecadal Arctic climate cycle. *Climate Dyn.*, **8**, 241–246.
5. Saunders, A., and M. Ghil, 2001: A Boolean delay equation model of ENSO variability, *Physica D*, **160**, 54–78.

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1. Zaliapin, I., V. Keilis-Borok, M. Ghil, M., 2002a: A Boolean delay model of colliding cascades. I: Multiple seismic regimes. *J. Stat. Phys.*, **111**, 815–837.
2. Zaliapin, I., V. Keilis-Borok, M. Ghil, M., 2002b: A Boolean delay model of colliding cascades. II: Prediction of critical transitions. *J. Stat. Phys.*, **111**, 839–861.

Concluding remarks

1. BDEs have rich behavior:
   periodic, quasi-periodic, aperiodic, increasing complexity.

2. They are relatively easy to study.

3. Are natural in a digital world.

4. Two types of applications
   - strictly discrete (genes, computers)
   - saturated, threshold behavior (nonlinear oscillations, climate dynamics, population biology; earthquake dynamics).

5. Can provide insight on a very qualitative level (~symbolic dynamics).

6. Generalizations possible (space dependence – partial BDEs; stochastic delays).

Conclusions

1. *Hmmm, this is interesting!*
2. *But what does it all mean?*
3. *Needs more work!!*

http://www.atmos.ucla.edu/tcd/

Ghil/BDEs-KB's80thB'day