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H4.SMR/1519-49

"Seventh Workshop on Non-Linear Dynamics and Earthquake Prediction"

29 September - 11 October 2003

MODELS OF EARTHQUAKES AS A SELF-ORGANIZING PROCESS

An appreciation of the physics of earthquakes

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What have we learned thus far?

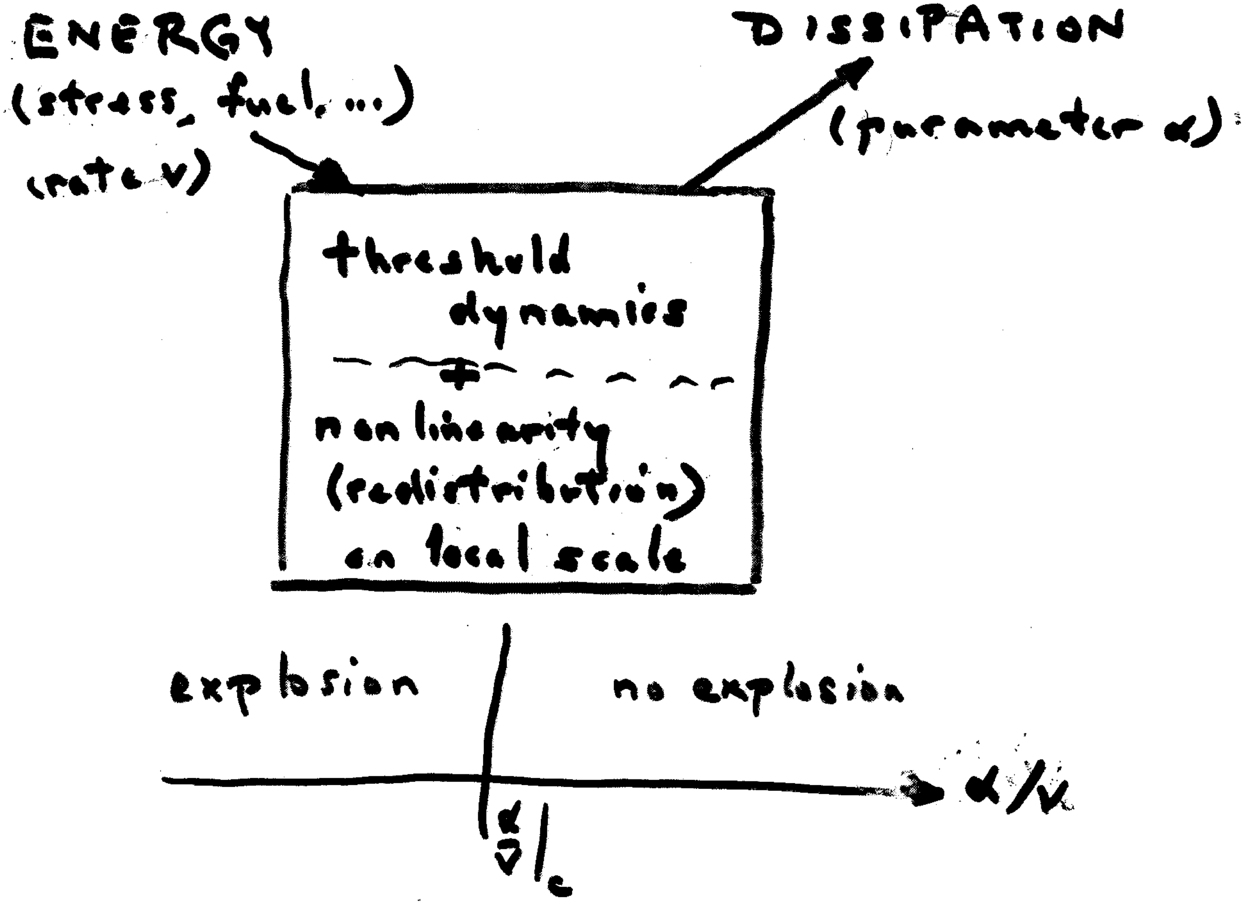
1. Phenomenology: there are many space and time scales.
   ⇒ many physical processes
2. Physics: Rapid rupture ⇒ dynamics*
3. Geometry: many space scales ⇒ heterogeneity*

4. Goal: Self-organization, pattern formation

5. Mathematics: Non-linear dynamics
   ⇒ Stefan problem
   ⇒ Interactions in space & time
6. Phenomenology: Finite Slip Pulse
   No Critical-State event

7. 2-4 km wide transient high compliance, very heterogeneous zone is a precursor (also post [illegible])
8. Subcritical crack growth, strongly influenced by fluids leads to short-term time precursors in heterogeneous zone. (Also aftershocks)

* how important is dynamics
heterogeneity ?


ENERGY
(stress, fuel, ...)
(rate v)
DISSIPATION
(parameter α)
threshold dynamics
+
nonlinearity
(redistribution)
on local scale
explosion
no explosion
α/v
α/v|c

Minimum Ingredients for Physical Models

- 1. Plate tectonics
  — Restores energy dissipated in earthquakes or creep
  — How do we drive the seismogenic zone?

- 2. Ductile-Brittle Fracture Rheology
  — Threshold stress (breaking strength) for fracture
  — Provides for creep
  — Provides time delays between triggering and triggered including aftershocks, foreshocks

  — Together with geometry (see 4) accounts for spatial and temporal localization

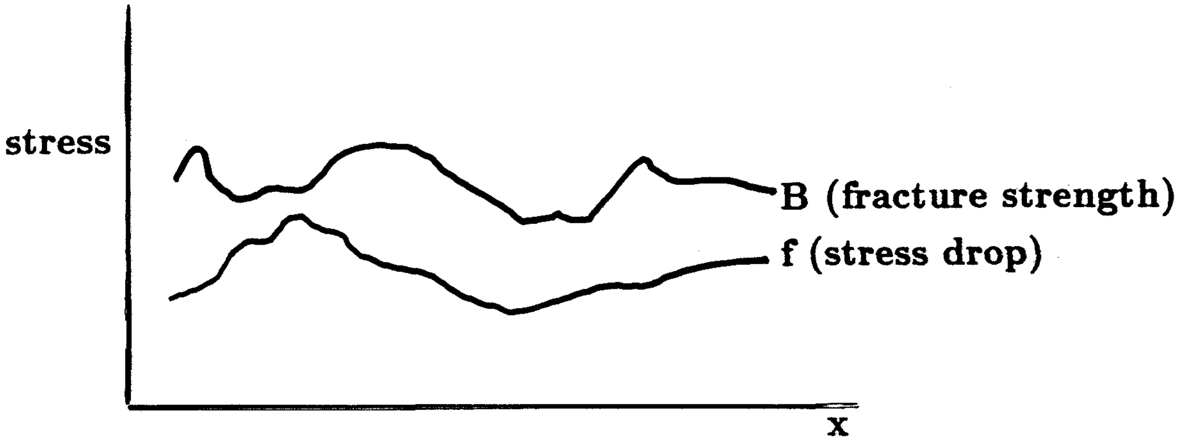
II
4

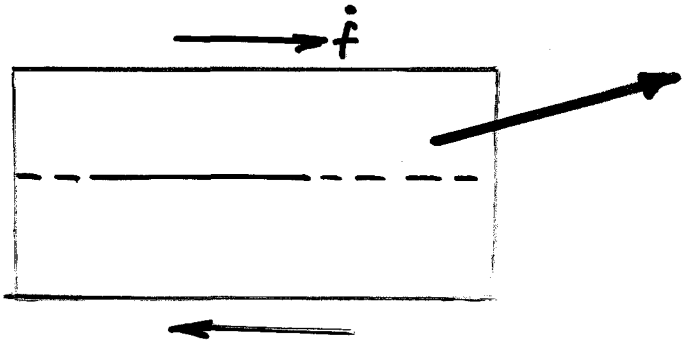
- 3. Stress redistribution after fracture
  — Fluctuations in stress field
  — Scale of fluctuations depends on size of fractures
  — Role of water
  — Nature of sub-seismogenic zone

- 4. Nonuniform geometry
  — Limits sizes of earthquakes
  — Correlated with nonuniformity of fracture strength
  — Provides for localization
  — Irregular shape of major faults
  — Nature of gouge
  — Thickness of seismogenic zone

- 5. Constraint
  — Digitized models should be asymptotic to continuum models in the small scale limit

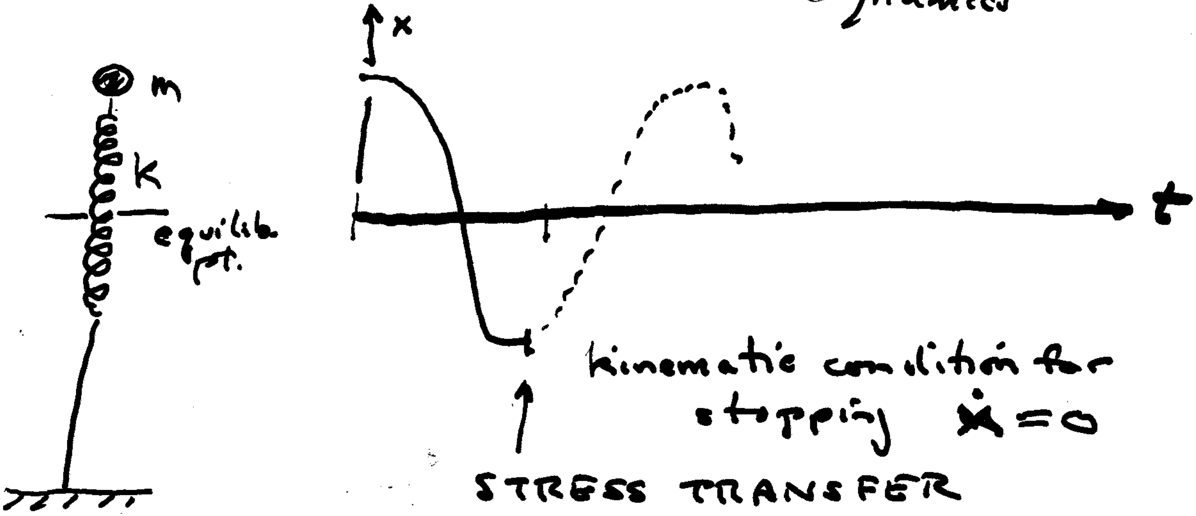
Crack Threshold Dynamics



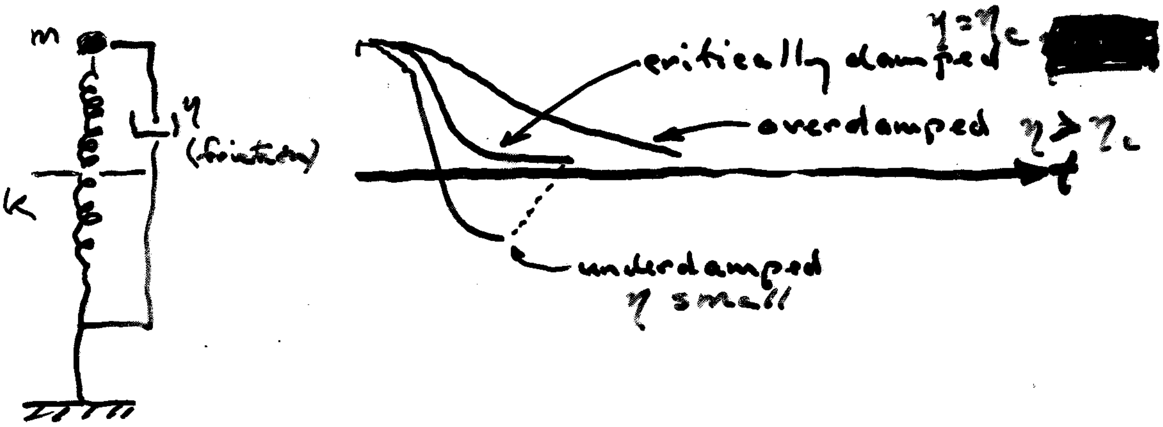


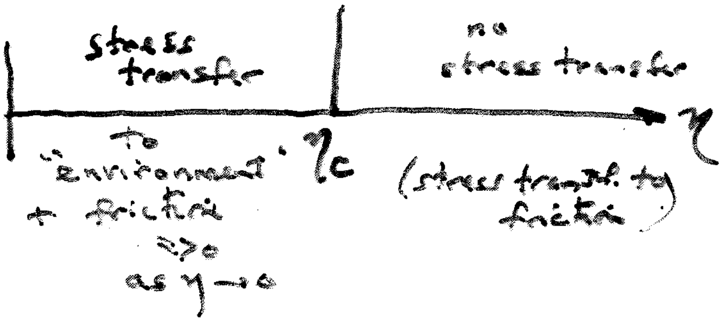
the problem of
continued increase
in <u>stress</u>

importance of Dynamics



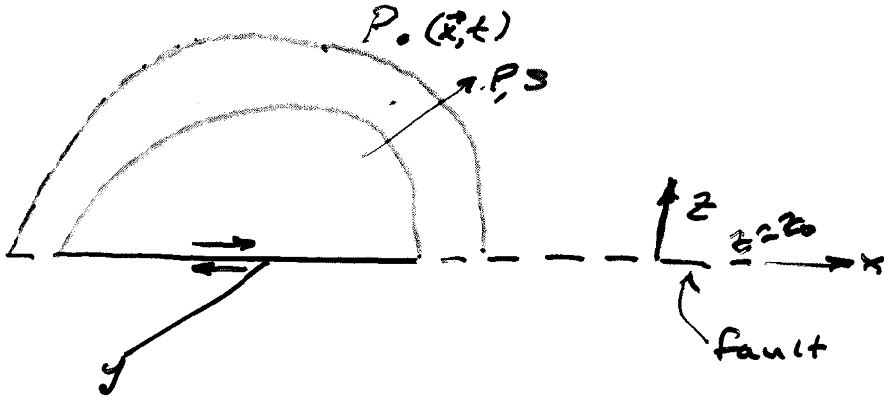
$$m\ddot{x} + \eta\dot{x} + kx = 0$$





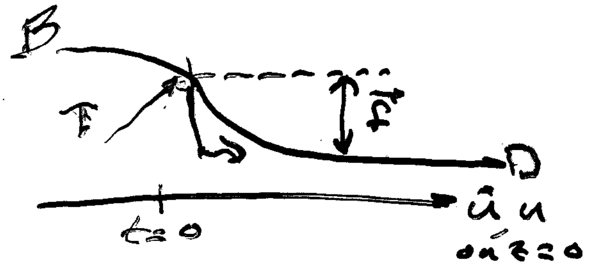
Assume Planar Fault
scale size for
smoothing

Assume stress is
transferred via
elastic processes
in medium adjacent
to fault.



Elasticity at P

$$(\lambda+2\mu)\nabla\nabla\cdot\vec{u}-\mu\nabla\times\nabla\times\vec{u}-\rho\frac{\partial^2\vec{u}}{\partial t^2}=\vec{f}(x,y,0,t)\delta(z-z_0)$$



$\vec{f}=T-D$

prestress

dynamic friction

Solution:

$$\vec{u}(\vec{x},t)=\frac{1}{2}\int\vec{G}(\vec{x}_2',t';\vec{x},t)\,f'\,d\vec{x}_2'\,dt'$$

We are especially interested in $\vec{u}(\vec{x}_2,t)$ $\quad\vec{x}_2\Rightarrow z=$

Green's Functions:

$u(x_2,t)$: $H[c^2(t-t')^2-(x-x')^2]$ 1-D (no energy loss by radiation)

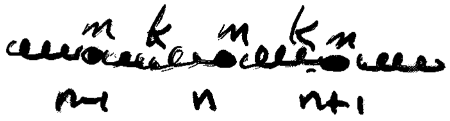
$u(\vec{x}_2,t)$: $\dfrac{1}{\sqrt{c_s^2(t-t')^2-(x-x')^2-(y-y')^2}}$ 2-D antiplane

... 2-D Plane

... 3-D

Finite-Difference (1-D)

$$\left(k\frac{\partial^2 u}{\partial x^2}\rightarrow\right)k\left[u_{n-1}-2u_n+u_{n+1}\right]-m\ddot{u}_n=f_n$$



$v=0$ (fast time)

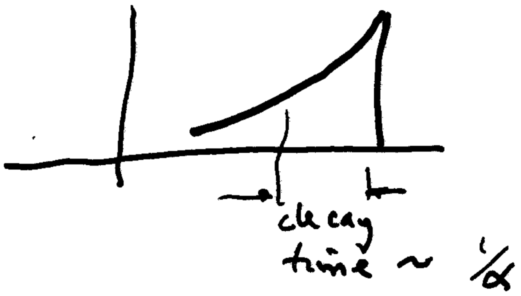
$$+k(2u_n - u_{n-1} - u_{n+1}) + m\ddot{u}_n + \ell(u_n - \cancel{vt}) + \alpha\dot{u}_n = f_n$$

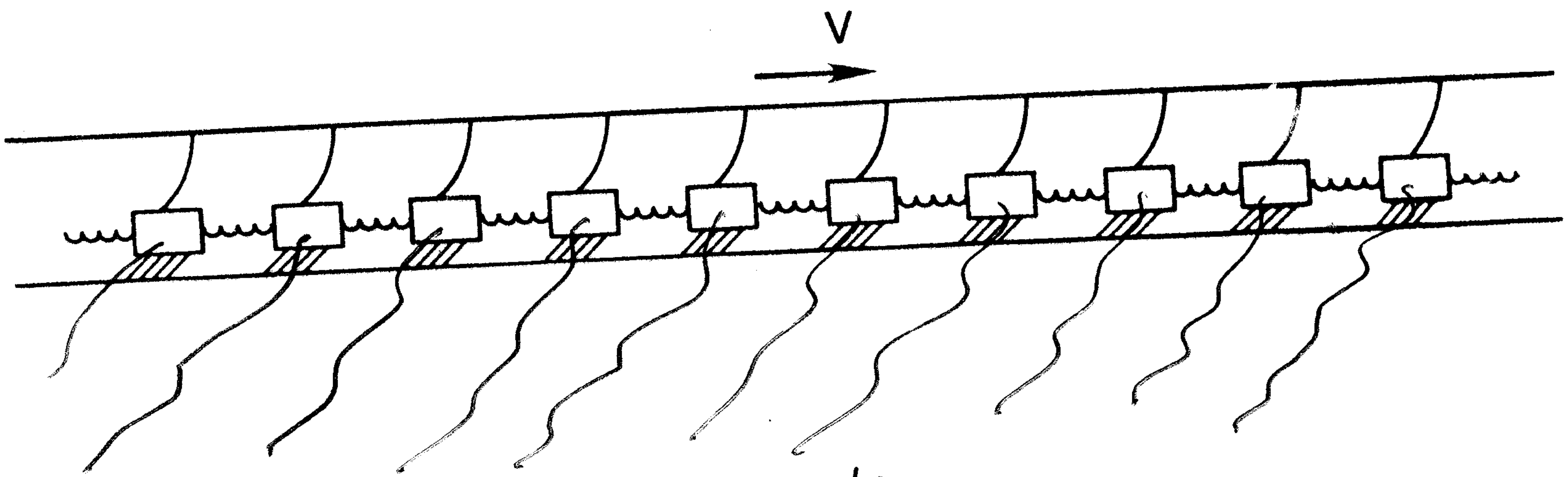
$$\Rightarrow \quad -\frac{\partial^2 u}{\partial x^2} + \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} + \frac{\alpha}{c^2}\dot{u} + \lambda u = f(x)$$

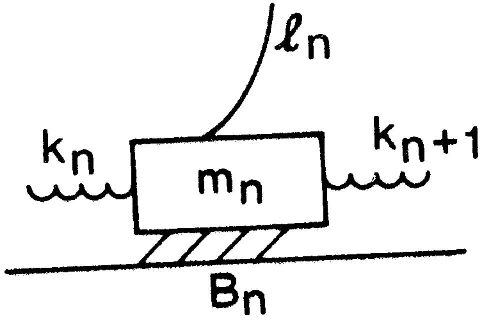
critical damping $\alpha = \frac{2\sqrt{\lambda}}{c}$

mixed Klein-Gordon + Telegraphist's eqn.

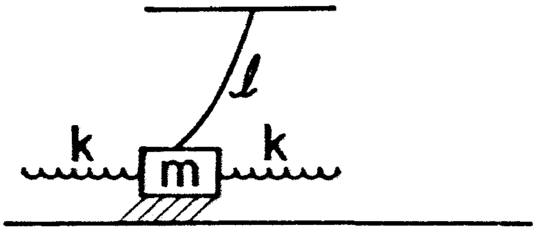
Green's $f^n \Rightarrow e^{-\frac{\alpha t}{2}} I_0\left(\alpha\sqrt{t^2 - x^2/c^2}\right)$

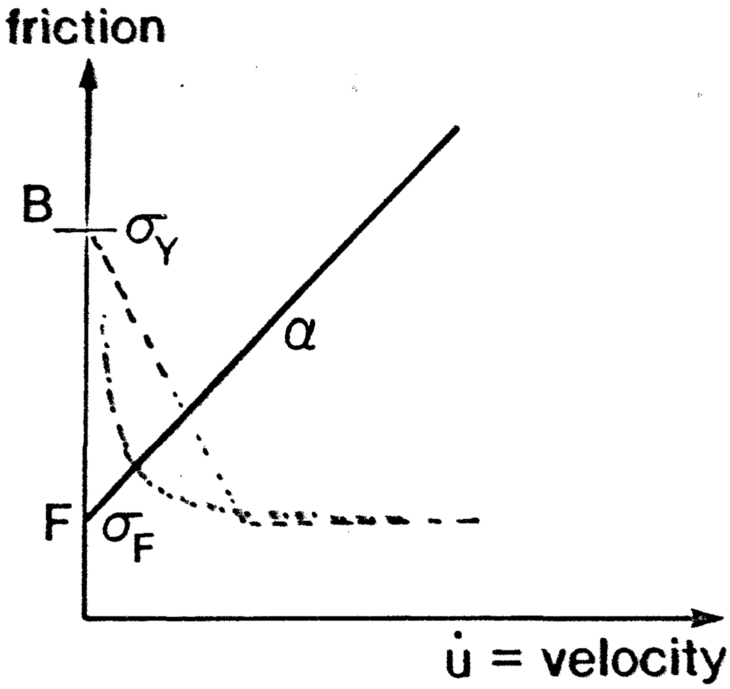






$$m\ddot{u}_n + k(2u_n - u_{n-1} - u_{n+1}) + \ell(u_n - Vt) + \alpha\dot{u}_n = f_n$$
$$\alpha = 2\sqrt{\ell m}$$





Dispersion

I-14

$$m\ddot{u} + k(2u_n - u_{n-1} - u_{n+1}) + \ell u_n = B_n - (F_n + \alpha \dot{u}_n)$$

$$\rho \frac{\partial^2 u}{\partial t^2} - \mu \frac{\partial^2 u}{\partial x^2} + \frac{\lambda u}{d^2} + \alpha \frac{\partial u}{\partial t} = (\sigma_Y - \sigma_F)$$

$$u = u_0 e^{i(kx - \omega t)}$$

Continuum Limit

$$\omega = -\frac{i\alpha}{2\rho} \pm \sqrt{\frac{\mu}{\rho}k^2 + \frac{\lambda}{\rho d^2} - \frac{\alpha^2}{4\rho^2}}$$

$$\text{if} \quad \alpha = \frac{2\sqrt{\lambda\rho}}{d}$$

$$u = u_0 \exp -\left(\frac{\alpha}{2\rho}t\right) \exp\left(i\omega\left(\frac{x}{c} - t\right)\right) \qquad c = \sqrt{\frac{\mu}{\rho}}$$

$\ell / k \Rightarrow$ Measure of dissipation

$U = [u]$

(1) $$\mu \frac{\partial^2 u}{\partial x^2} - \rho \frac{\partial^2 u}{\partial t^2} = -f(x)$$

we consider
$f = \text{const}$

(2) $$\frac{d[u]}{dt} + v \frac{\partial [u]}{\partial x} = 0$$

at edge

v = velocity of edge

at crack tip
$[u] = 0$
$\frac{d[u]}{dt} = 0$

(3) $$\mu \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial t} = -g(x)$$

$c^2 = \mu/\rho$

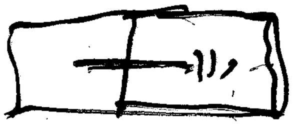
$$\mu \frac{\partial u}{\partial x}\left(1 - \frac{v^2}{c^2}\right) = -g(x)$$

Unilateral fracture: $x > 0$
with const. velocity

$$u = \frac{f}{2\mu} x(vt - x)$$

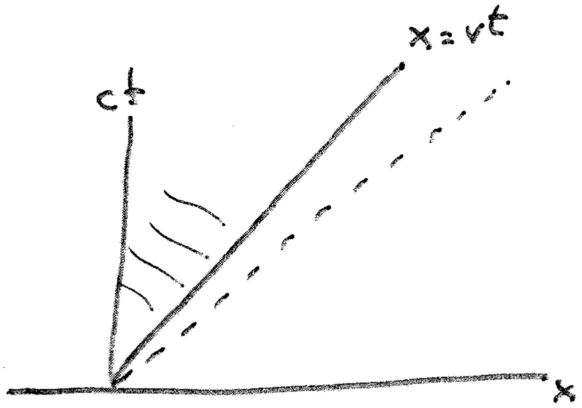
$vt > x$

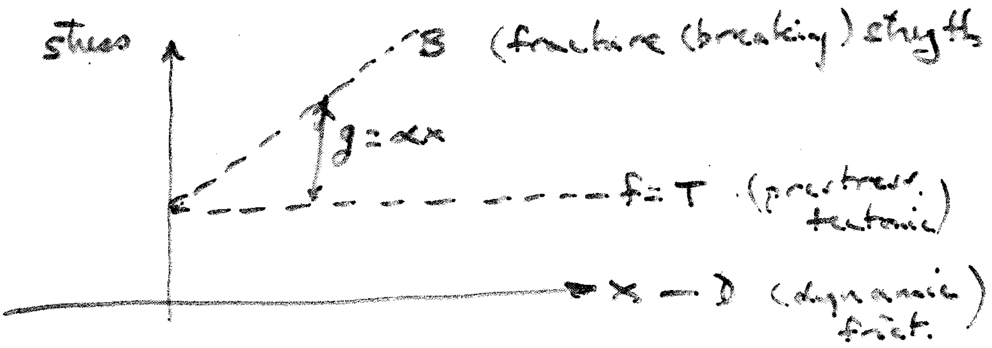
$g = \alpha x$



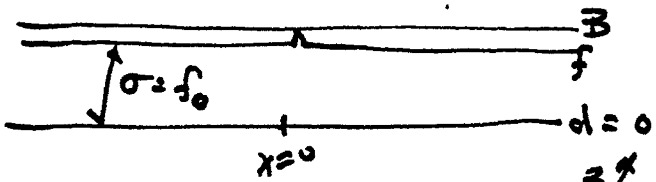
$$\frac{v^2}{c^2} = 1 - \frac{2\alpha}{f}$$

$$\frac{d u(x,t)}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\frac{dx}{dt} \quad ($$





Consider 2-D antiplane crack in case $g=0$



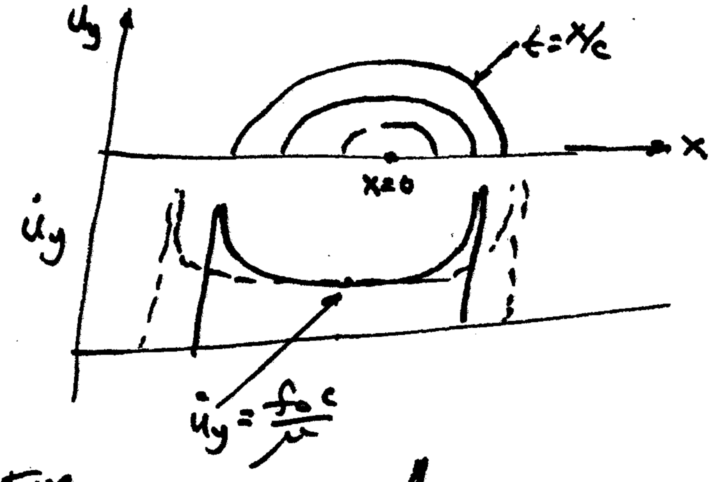
$$\cancel{\mu \frac{\delta^2 u}{\delta z^2}} \mu \frac{\partial^2 u_y}{\partial x^2} - \rho \frac{\partial^2 u_y}{\partial t^2} = -f_0$$

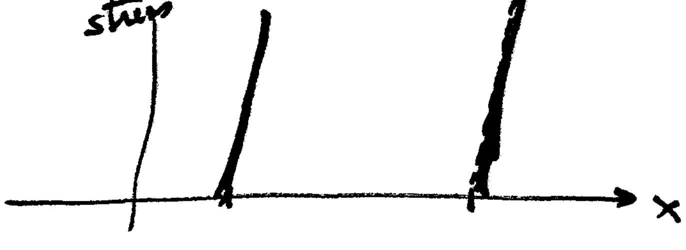
on $y = 0$

solution

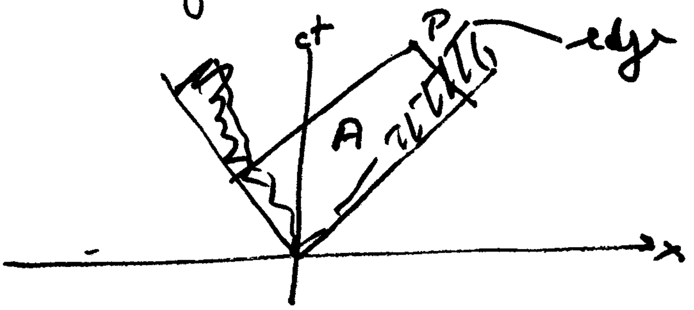
$$u_y = \frac{f_0}{\mu} \sqrt{c^2 t^2 - x^2} \qquad c = \sqrt{\frac{\mu}{\rho}} = V_s .$$

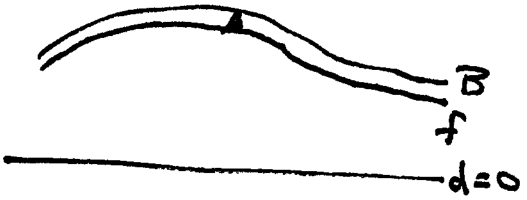
$$\dot{u}_y = \frac{f_0 c}{\mu} \frac{ct}{\sqrt{c^2 t^2 - x^2}} \qquad g = B - f$$





If $g=0$ but f is not const.





$$u_P^{(x,t)} = \int_{A} \frac{f(x_0)\,dx_0\,dt_0}{\sqrt{c^2 t_r^2 - x_r^2}}$$

$t_r = (t - t_0)$
$x_r = (x - x_0)$

these 2-D models include effects of energy loss through radiation

this crack never stops. We need $g(A) \neq 0$

1-D

$$\mu \frac{\partial^2 u}{\partial x^2} - \rho \frac{\partial^2 u}{\partial t^2} = f(x) = \lambda(\overline{vt - u})$$

slow time

Green's f^n is now Heaviside f^n

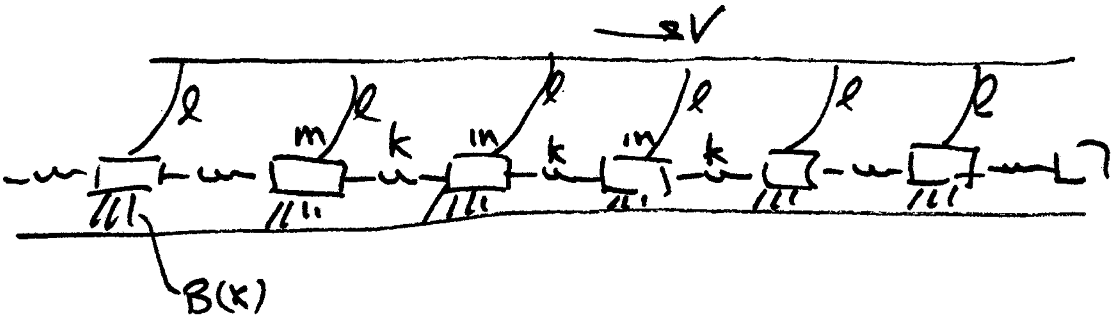
$$u = \int_A f(x)\,dx\,dt\,H(ct - x)$$

But if u is the u in fast time, we solve

$$\mu \frac{\partial^2 u}{\partial x^2} - \rho \frac{\partial^2 u}{\partial t^2} + \lambda u = f(x)$$

which is the Klein-Gordon eqn

No energy loss due to radiation



1-D

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = -\frac{f(x,t)}{\mu}$$

(Wave eq)

no radiation energy loss

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - \alpha\frac{\partial u}{\partial t} = \frac{-f(x,t)}{\mu}$$

(Telegraphist eq.)

add friction to simulate radiation

$$u_{\substack{\text{far}\\\text{field}}} \sim \int_{\substack{\text{crack}\\\text{surface}}} \dot{u}\, d\vec{x}$$

(dispersive)

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - \alpha\frac{\partial u}{\partial t} + \underline{\lambda}\, u = \frac{-f(x,t)}{\mu}$$

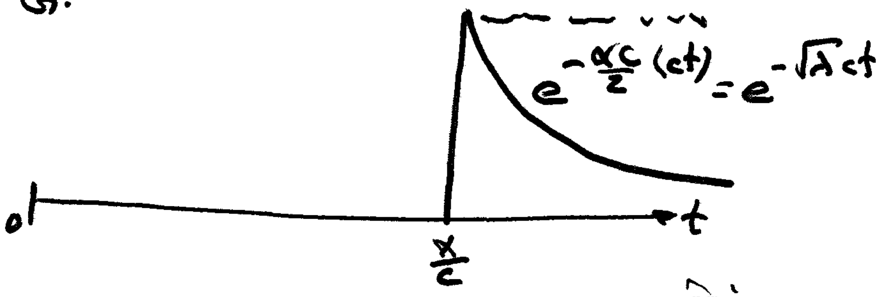
(Telegraphic + Klein Gordon eq)

add stiffness to eliminate dispersion

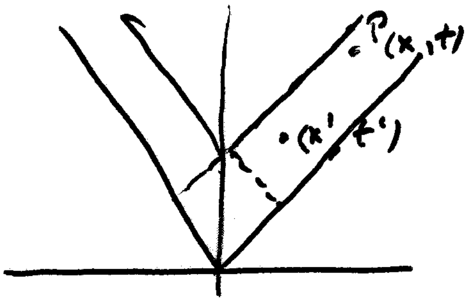
$$\alpha = \frac{2}{c}\sqrt{\lambda}$$

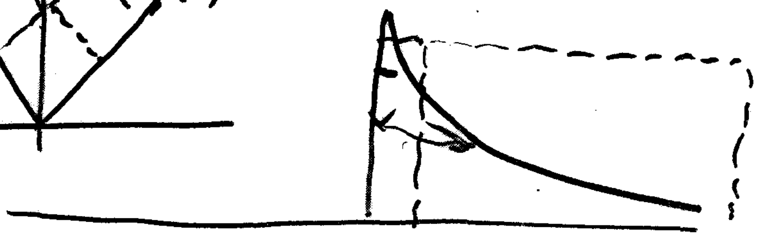
(critical damping)

G:

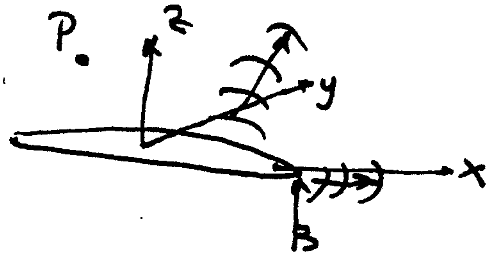


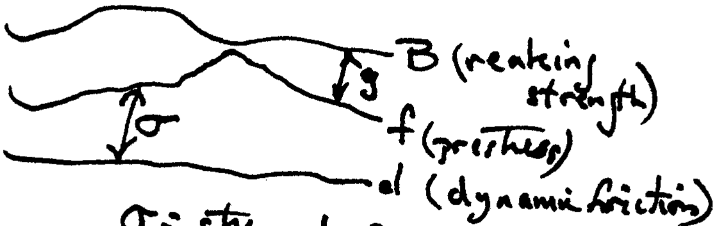
$$e^{-\sqrt{\lambda}ct}\, H(ct-x)$$





Dynamic crack
sends out
elastic waves





$\underline{\sigma: \text{stress drop.}}$

assume homogeneous elastic medium

$$(\lambda+2\mu)\nabla\nabla\cdot\vec{u}_p - \mu\nabla\times\nabla\times\vec{u}_p - \rho\frac{\partial^2\vec{u}_p}{\partial t^2} = -\underset{\substack{\uparrow \\ \text{at time of} \\ \text{fracture}}}{\sigma(x,y)}$$

σ increases with time during slow time.

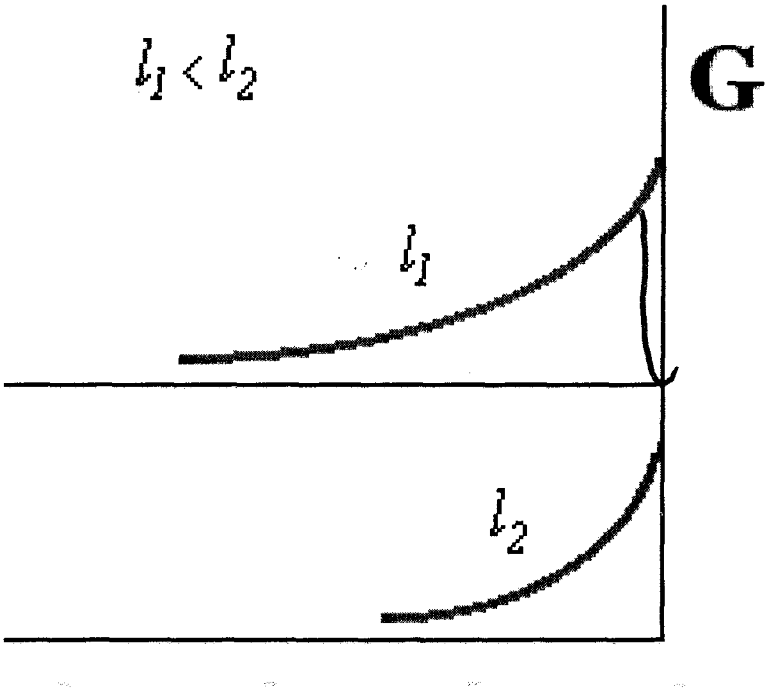
The fracture must supply enough stress to overcome
the fracture strength excess at the crack tip

$$g = B - f$$

if $g = 0$ the crack grows sonically (what does that mean?)

if $g > 0$ The crack is subsonic (")

The Green's function



(scalar) elastic wave equation in 1-D

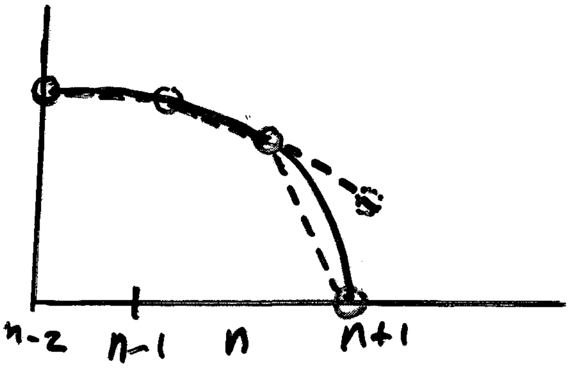
$$\mu \frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2}$$

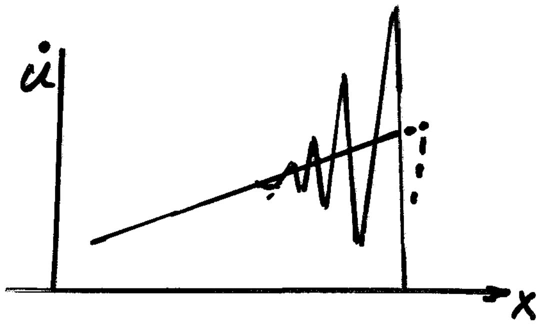
$$\frac{m}{a} \ddot{u}_n$$

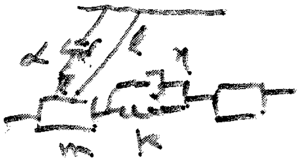
finite difference

$$\mu \frac{\left[\frac{(u_{n+1} - u_n)}{a} - \frac{(u_n - u_{n-1})}{a} \right]}{a}$$

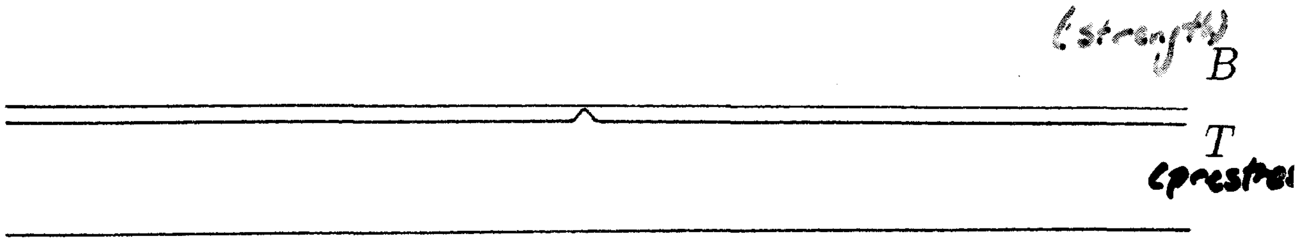
$$= -k \left[2u_n - u_{n-1} - u_{n+1} \right]$$

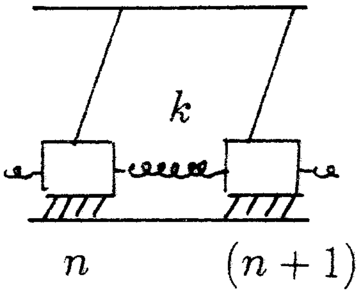


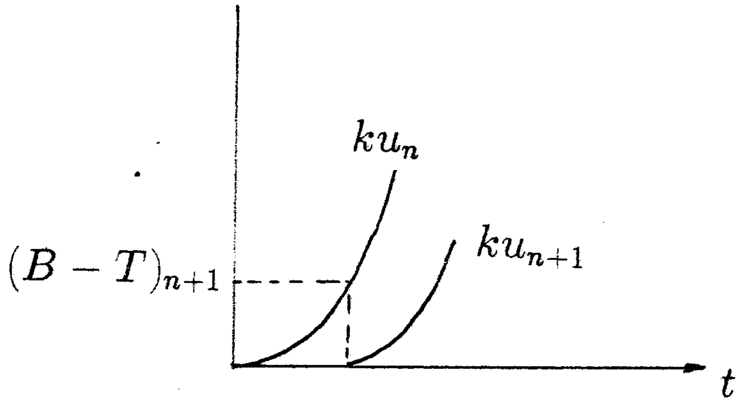




$$-k\left[u_{n+1} - 2u_n + u_{n-1}\right] - \eta\left[\dot{u}_{n+1} - 2\dot{u}_n + \dot{u}_{n-1}\right] = f_n$$
$$+ m\ddot{u}_n \qquad + \ell u_n + \alpha \dot{u}_n$$





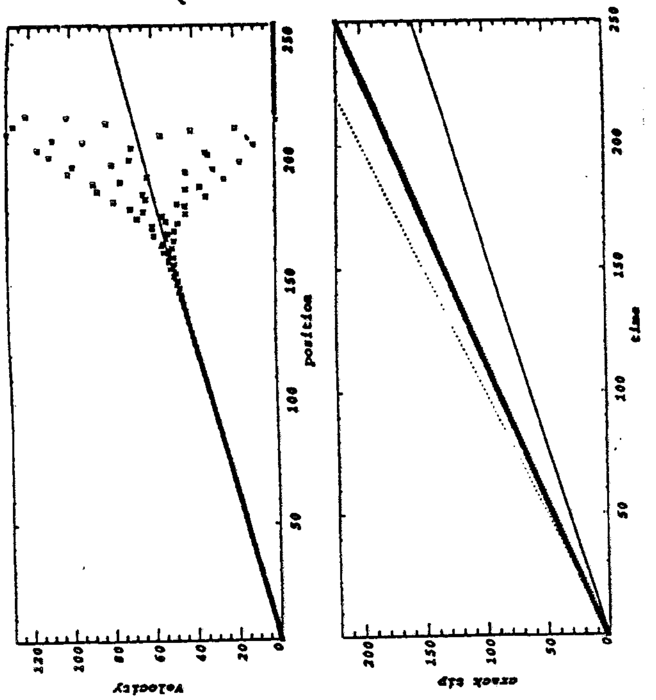


$$k \frac{\partial u}{\partial x}\left(1-\frac{v^2}{c^2}\right) = -q$$

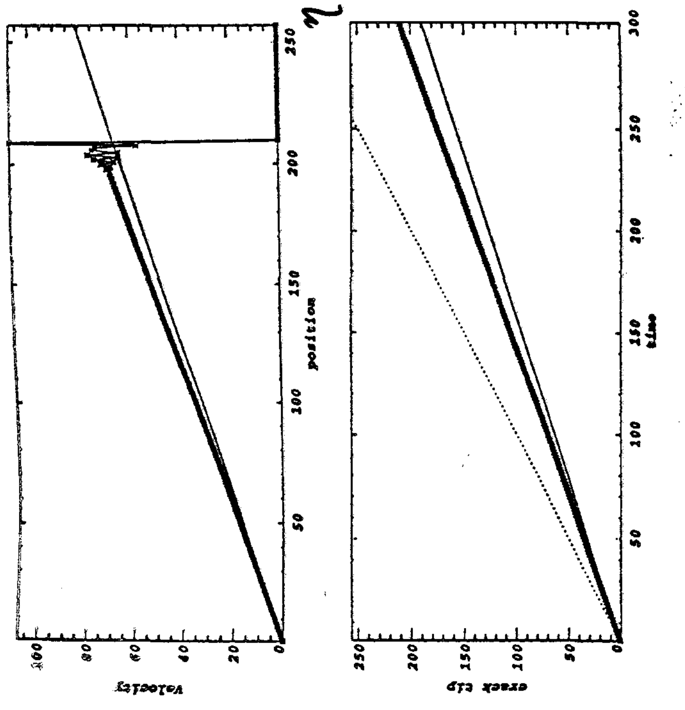
$$k\left(u_n - \cancel{u_{n+1}}\right) = \left(\text{Br. Str.} - T_{(pre)}\right)_{n+1} = 0\ ?$$

$$k u_n \left(1-\frac{a^2}{V_s^2 t^2}\right) = (B-T)_{n+1}$$

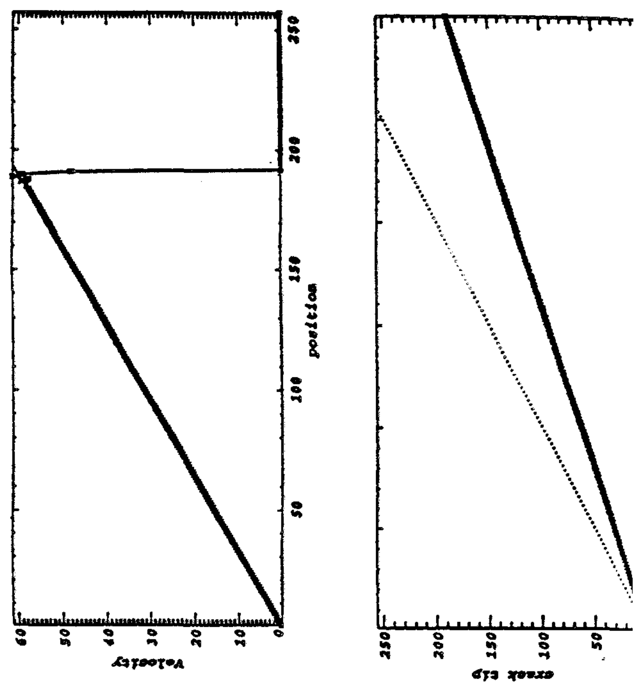
$\eta=0$



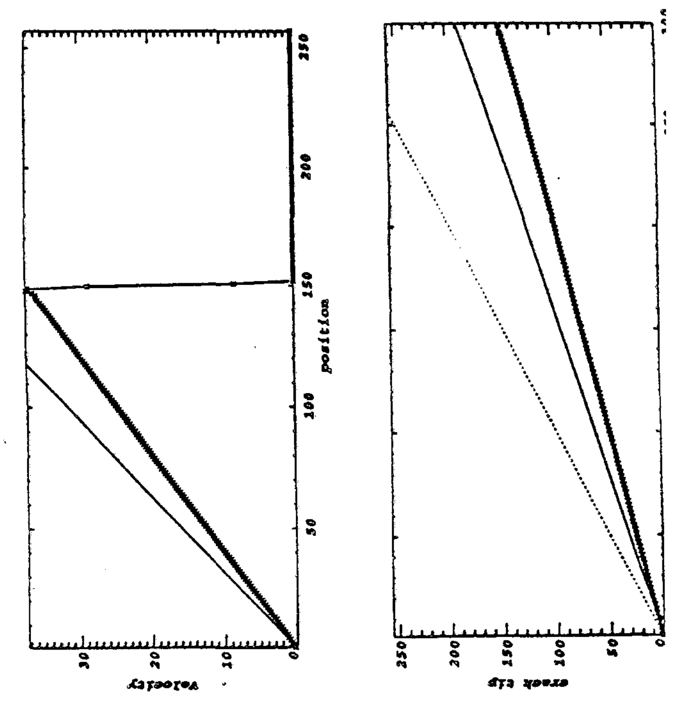
$\eta=.01$

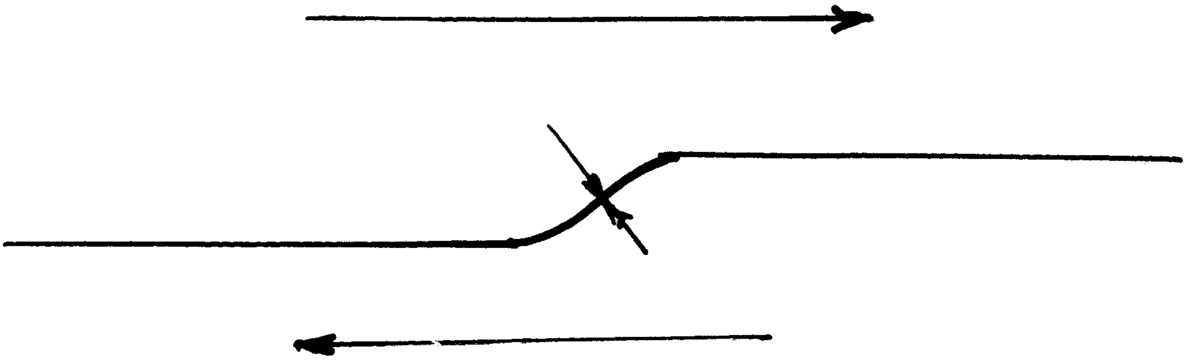


$\eta=.1$

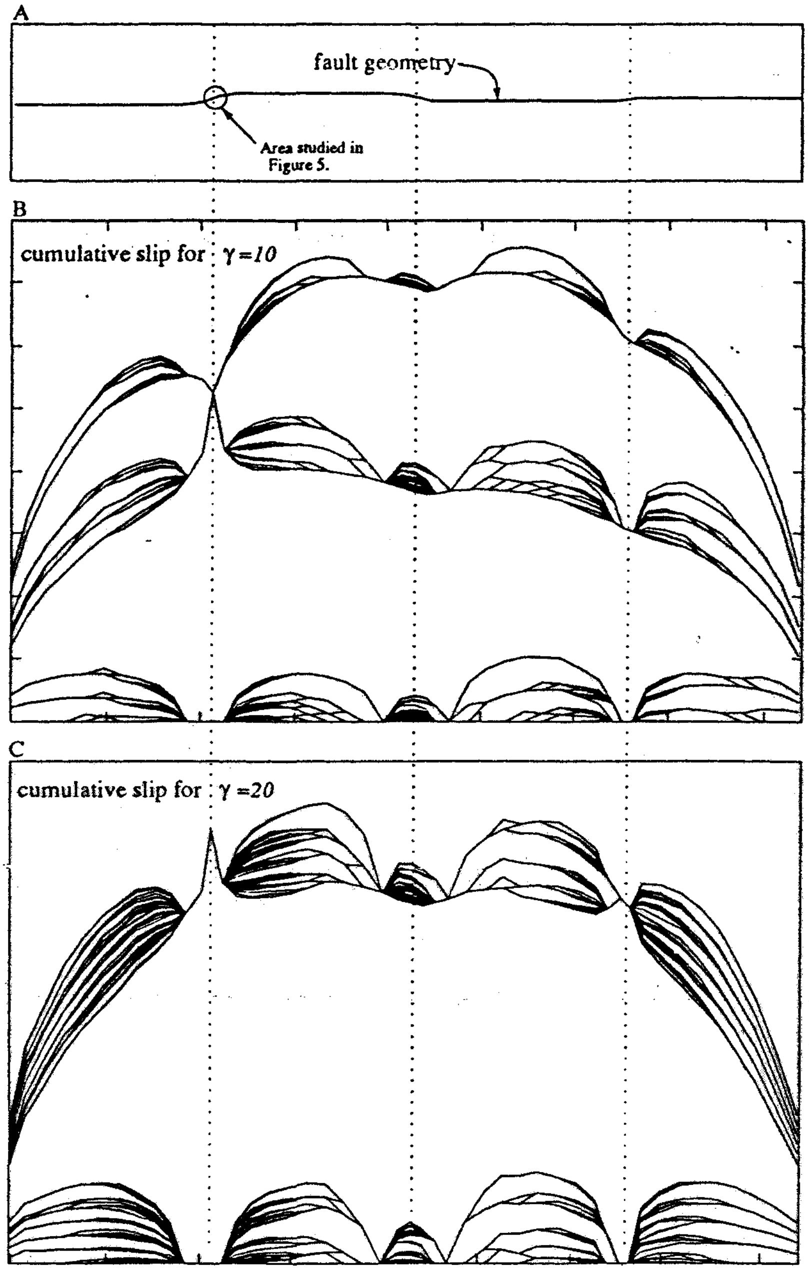


$\eta=1$





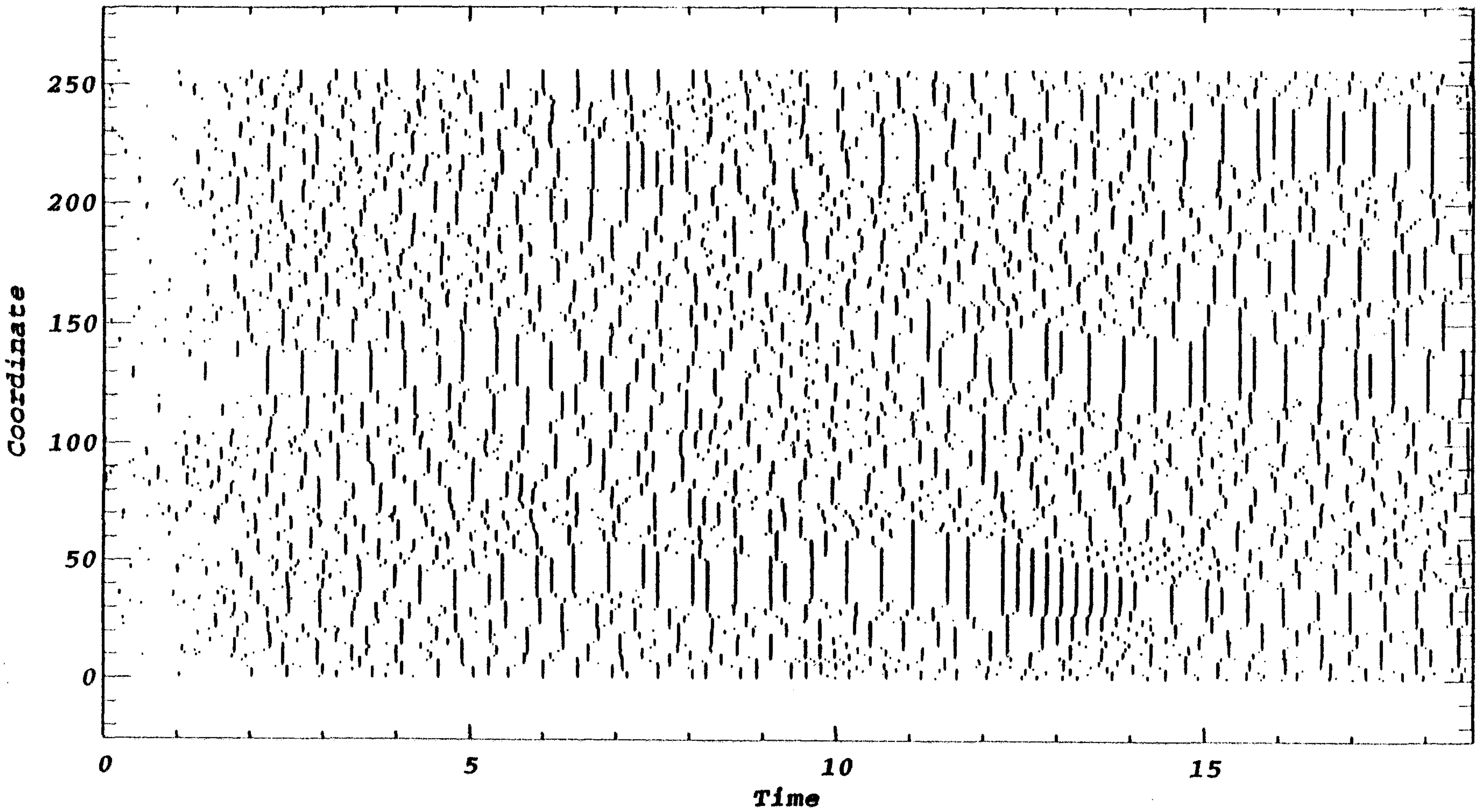
$$\sigma_t = \nu \sigma_n$$



2-D antiplane

Tests of 1-D system

1. Uniform B (homogeneous)
2. Localized fluctuations in B
3. Can we neglect dynamics?
4. Stability of Pattern
5. Fractal (self-affine) B.
6. Two coupled faults
7. Random Power Law B


Coordinate
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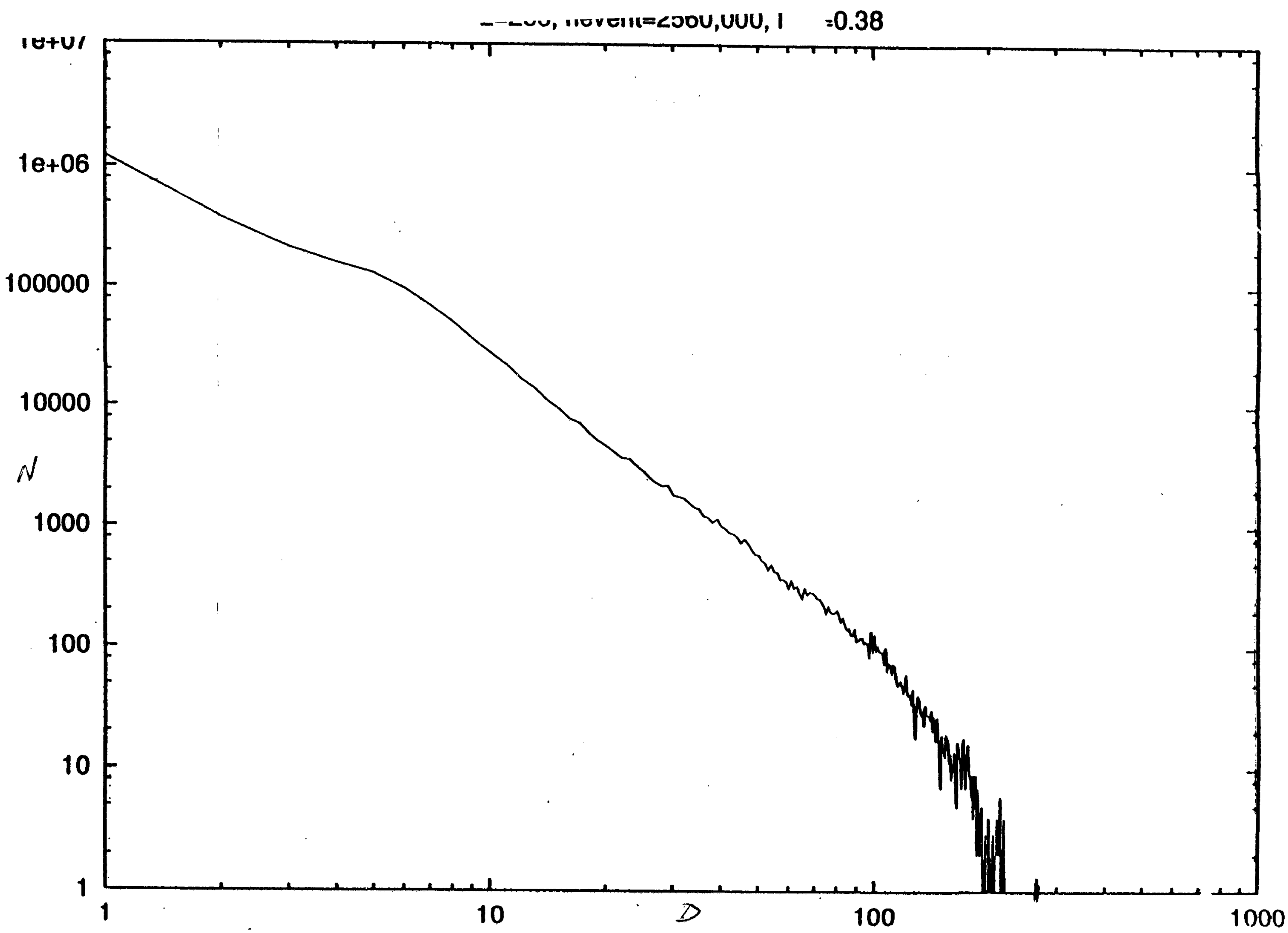
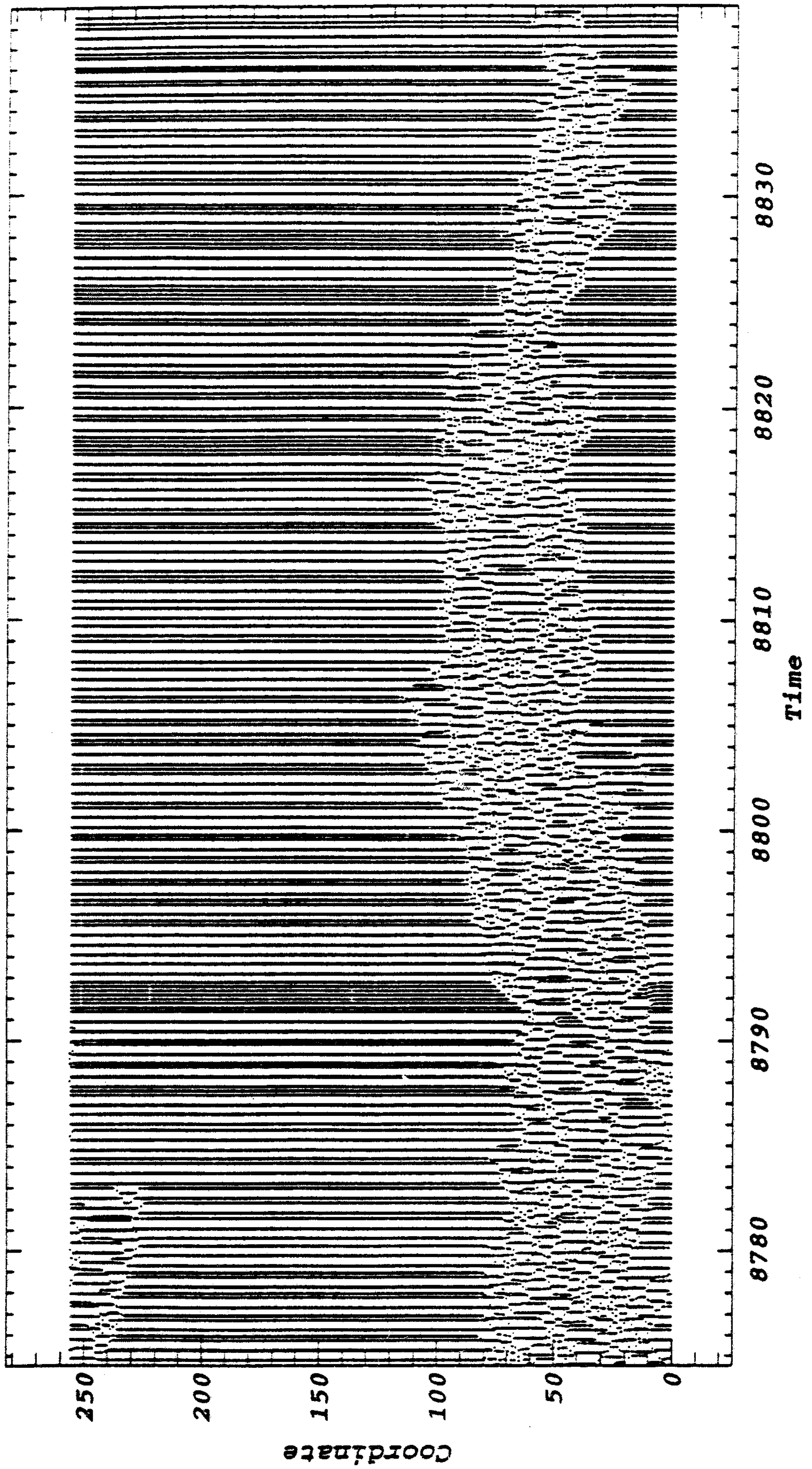
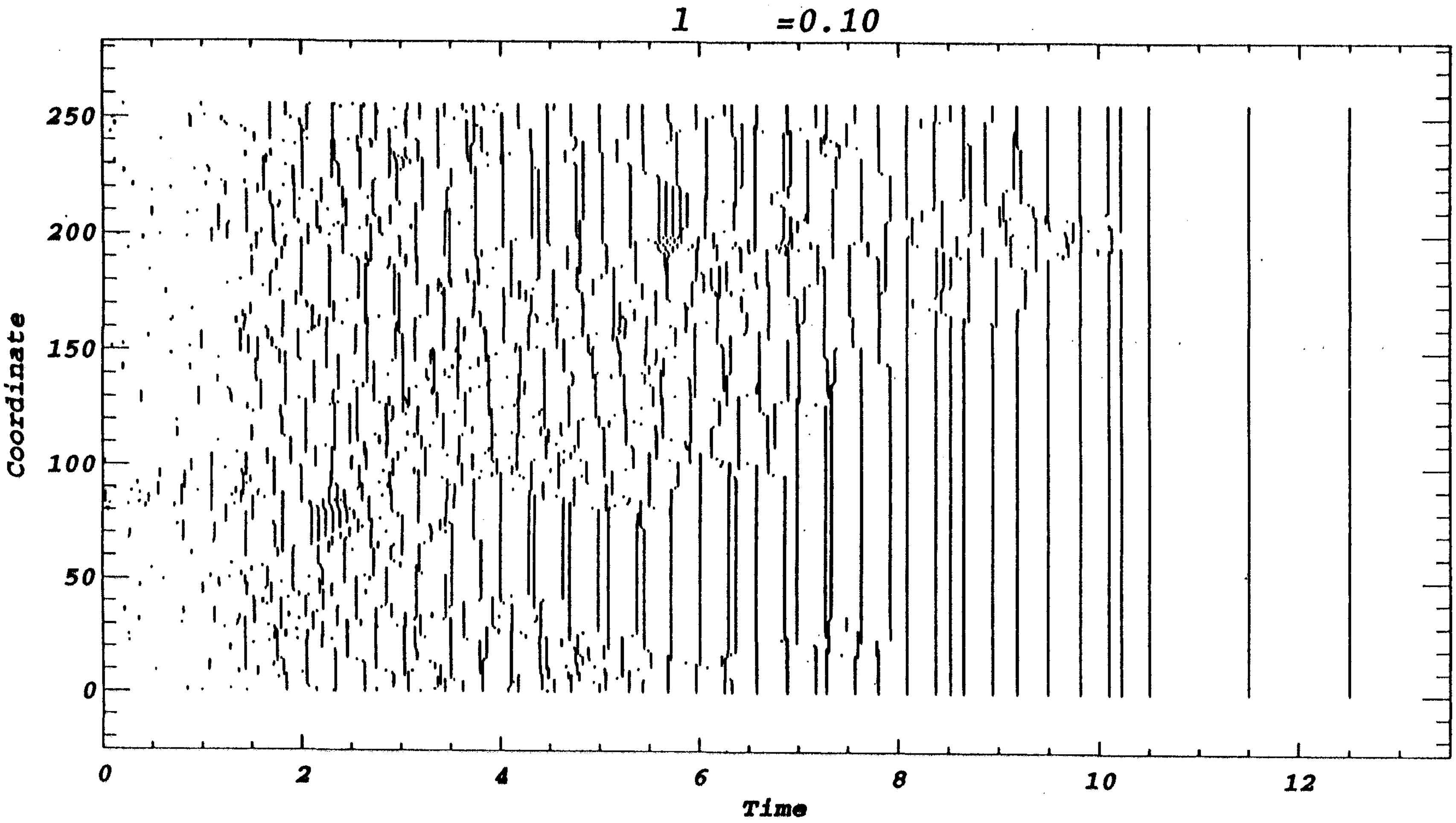
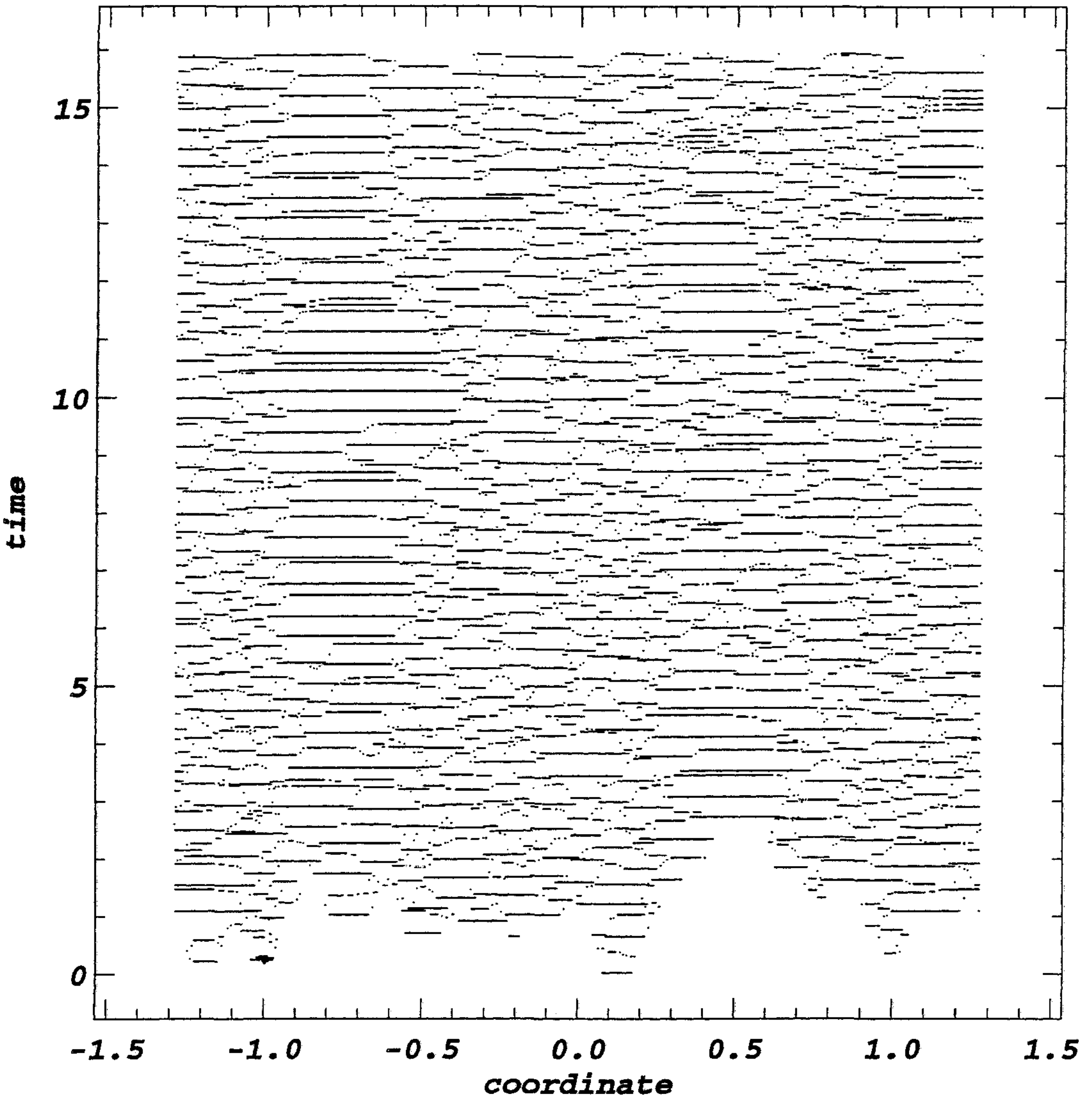
200
150
100
50
0
0
5
10
15
Time


L=256, nevent=2560,000, T =0.38
1e+07
1e+06
100000
10000
1000
100
10
1
N
1
10
100
1000
D

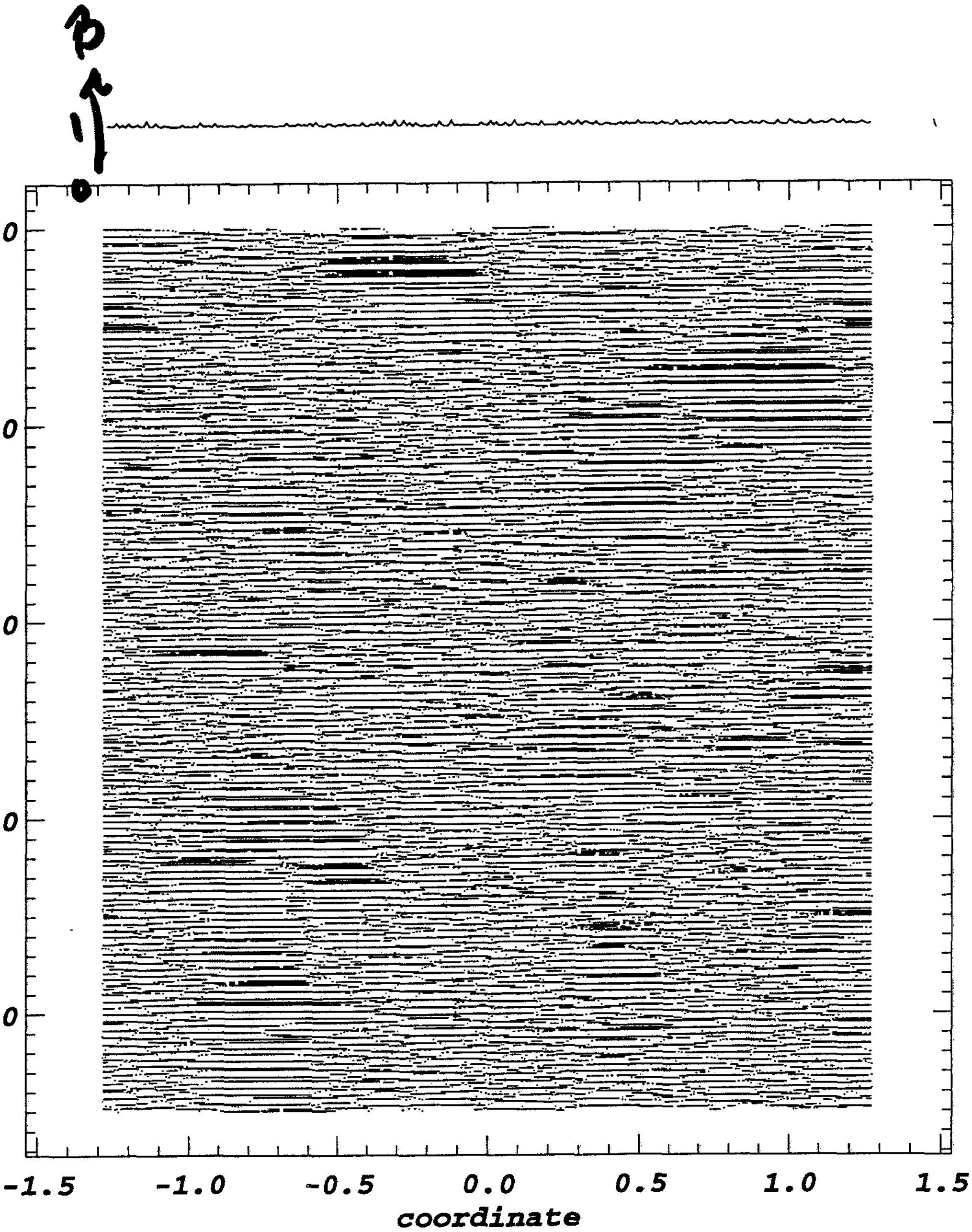

Time
8780
8790
8800
8810
8820
8830
Coordinate
250
200
150
100
50
0


1 =0.10
Coordinate
250
200
150
100
50
0
Time
0
2
4
6
8
10
12

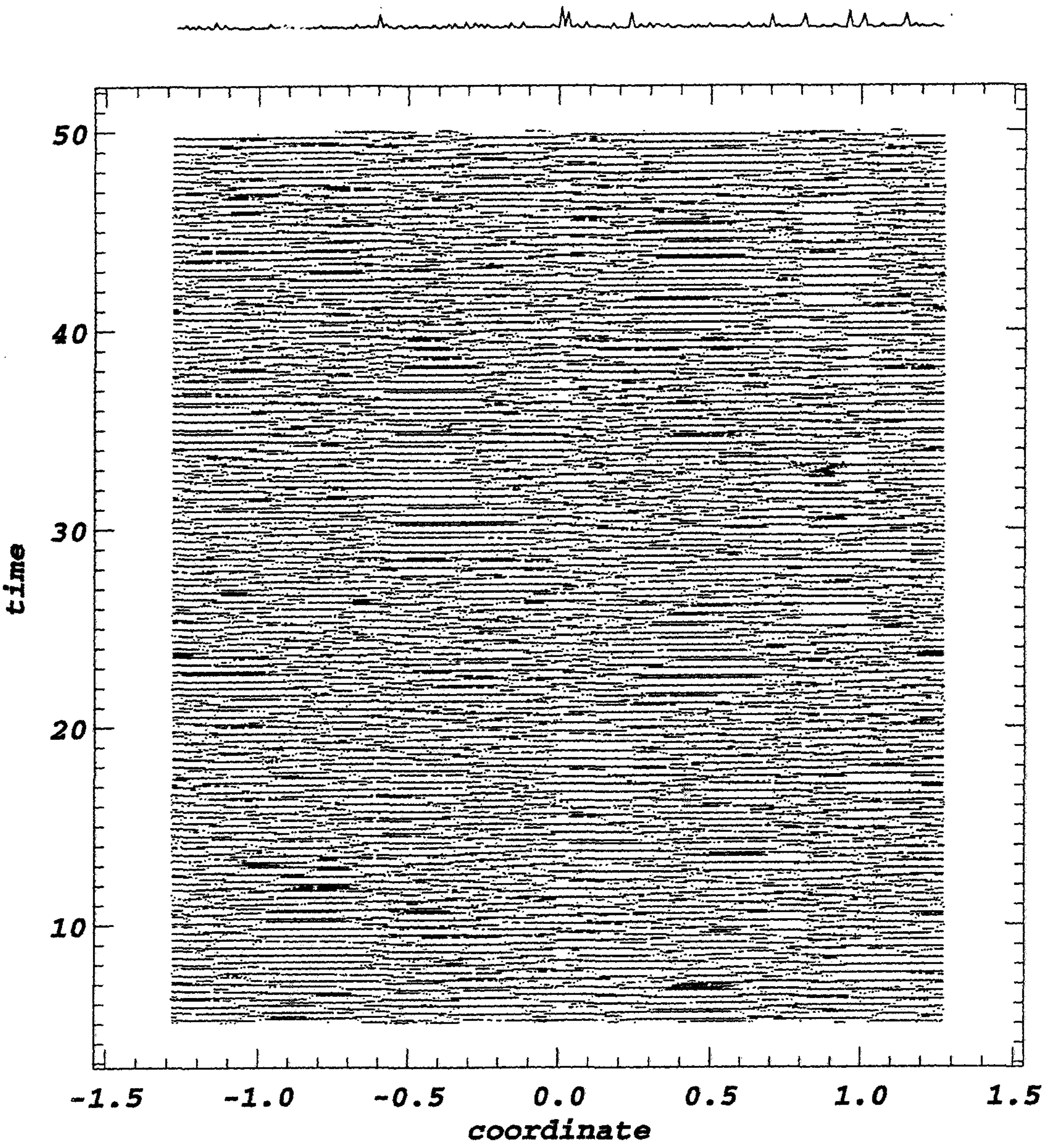


1.08:1.00+0.12

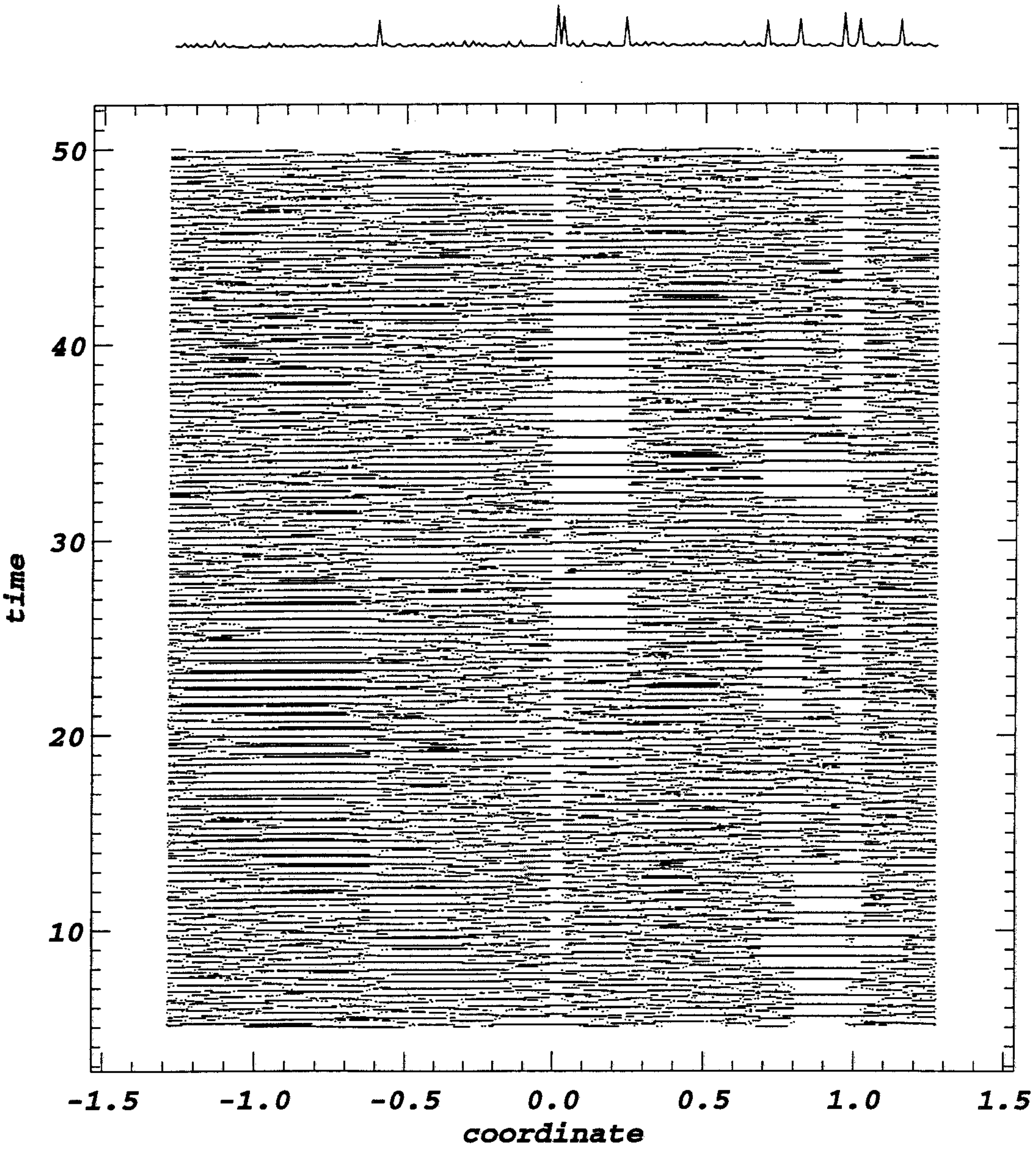
Д-23



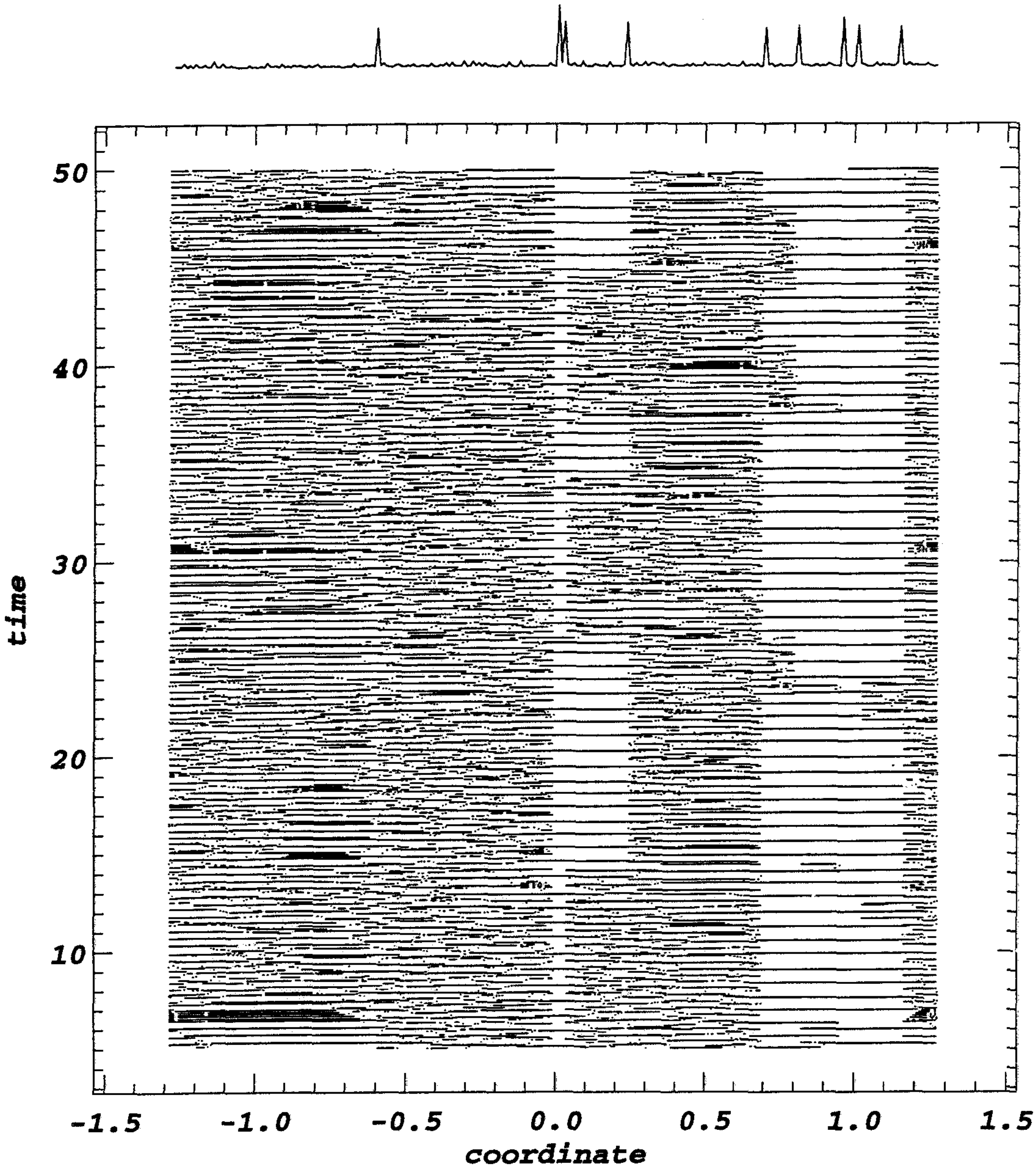
1.08:1.00+0.12



1.4:1.00+0.12

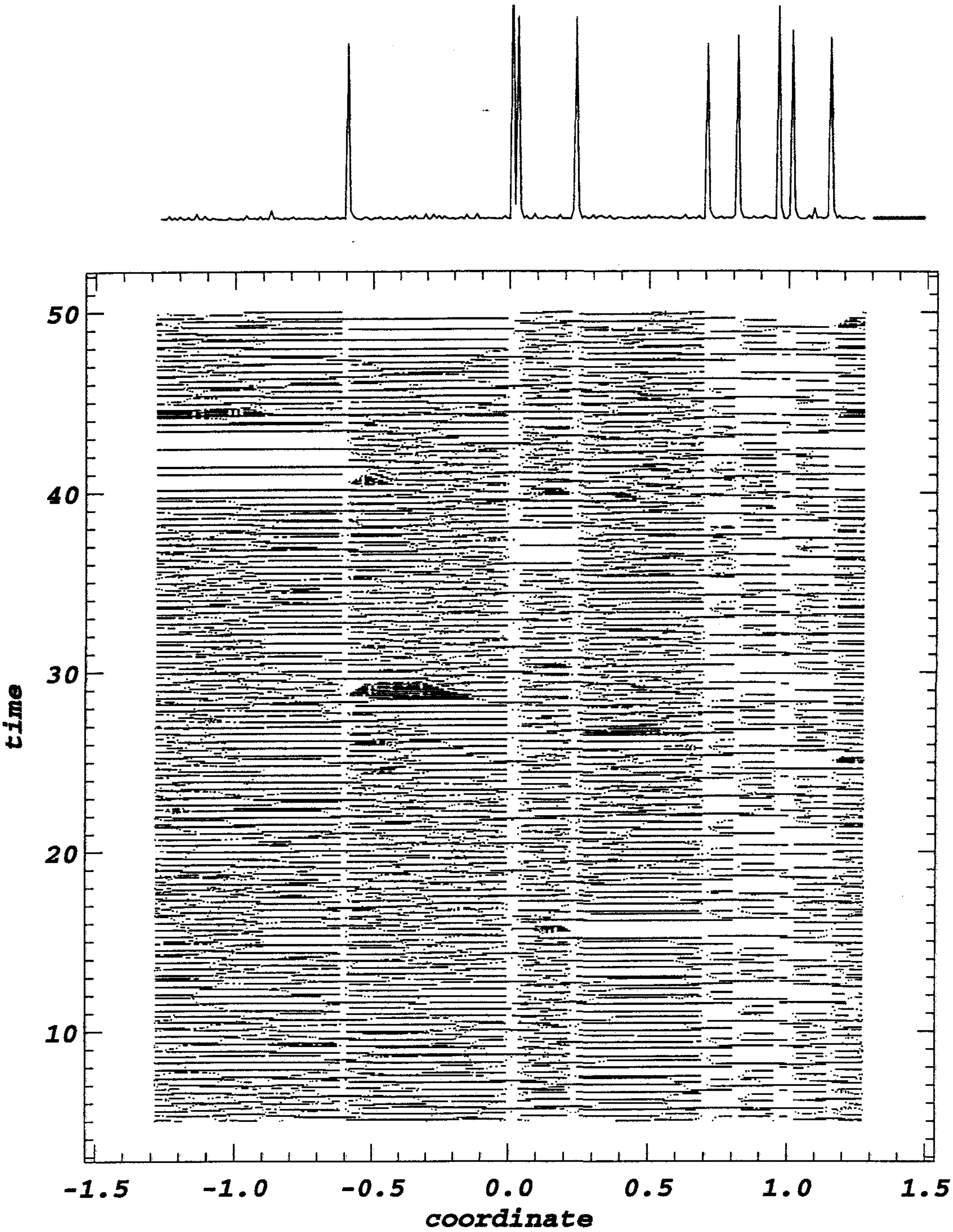


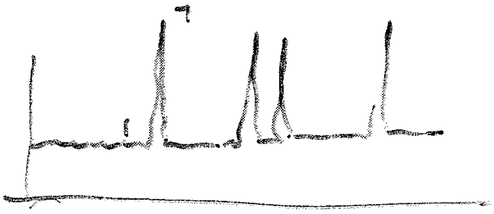
1.8:1.00+0.12



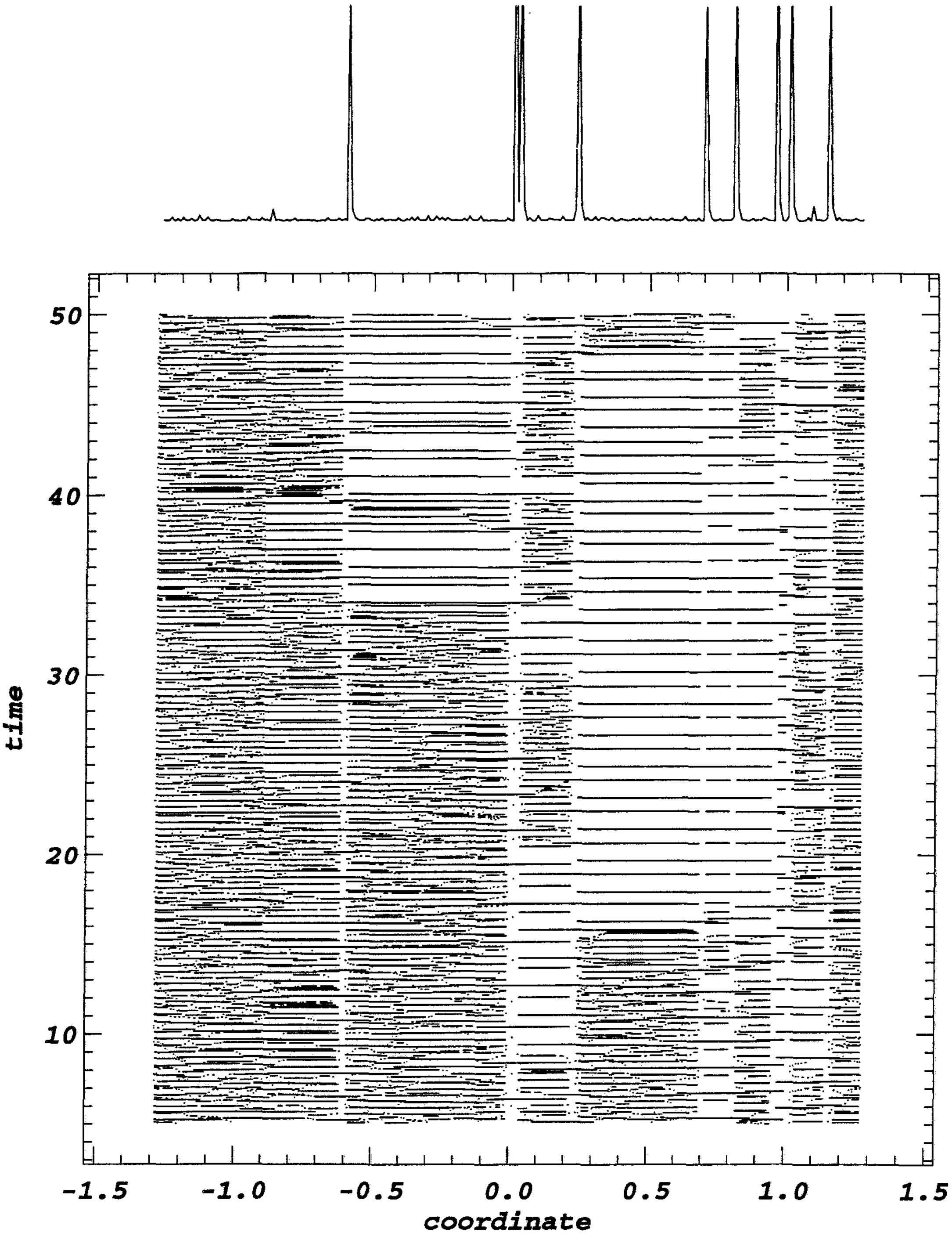
IV
26

2.2:1.00+0.12



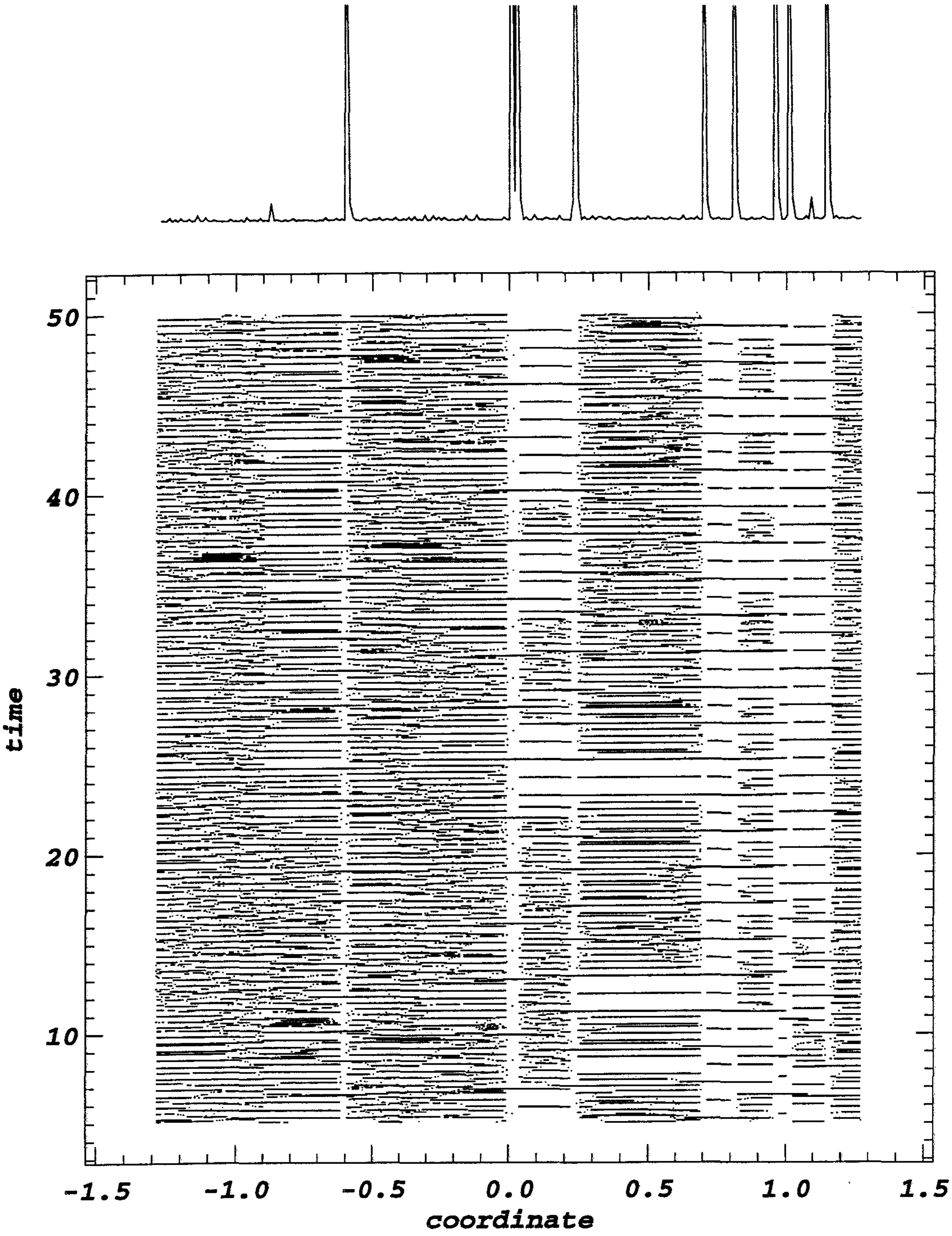


7.0:1.00+0.12

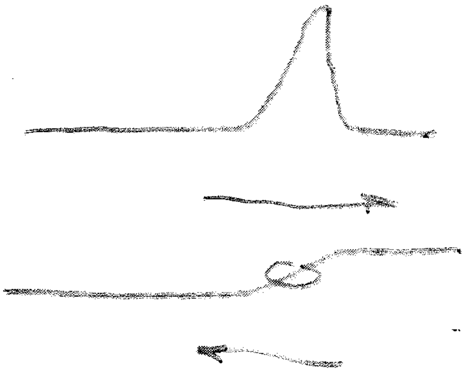


-18

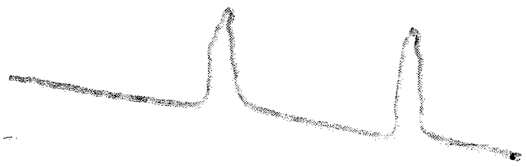
9.0:1.00+0.12

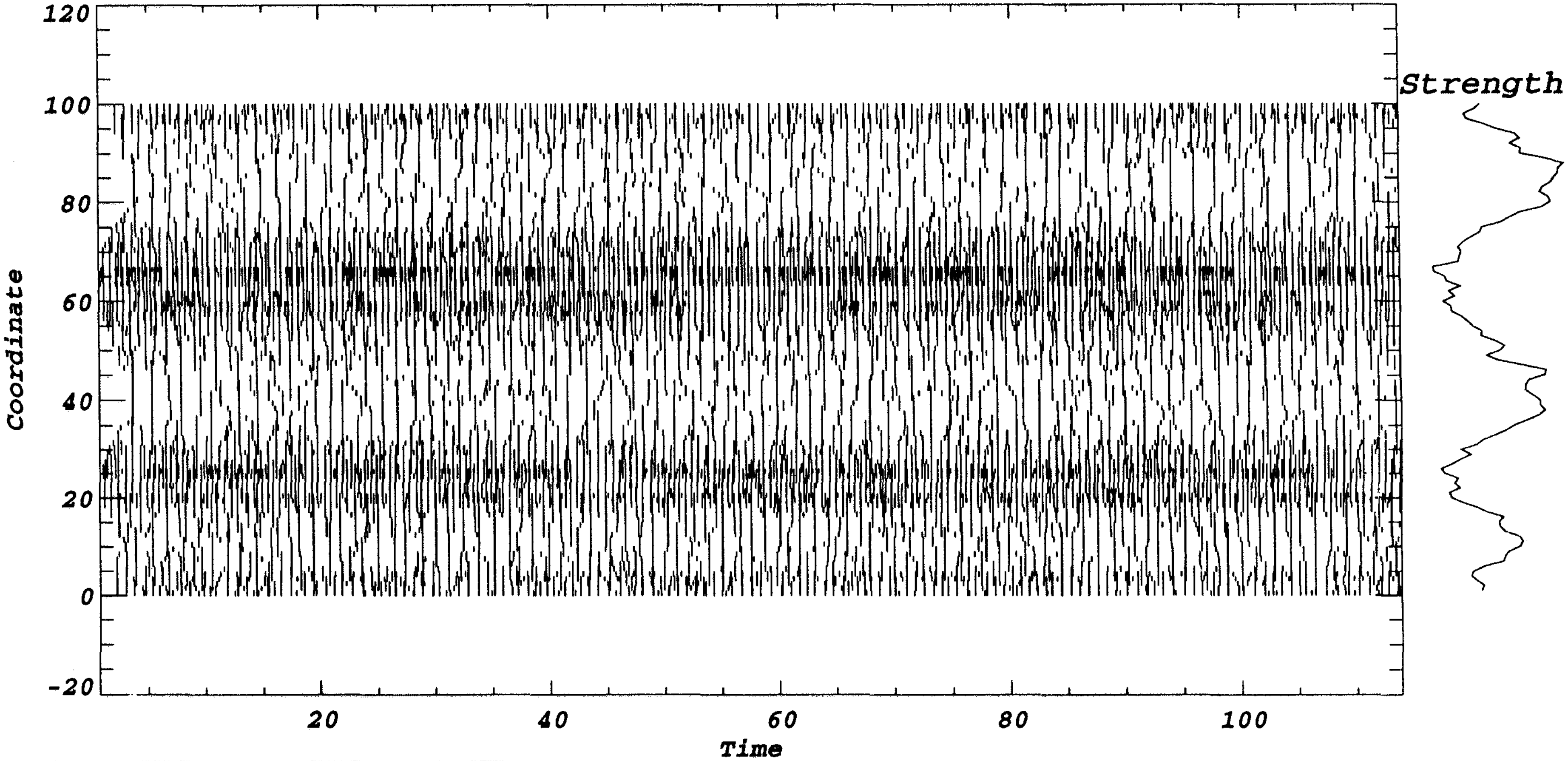


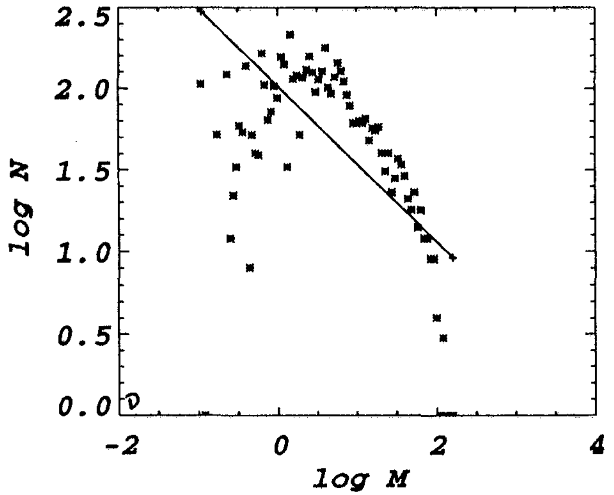
II-29



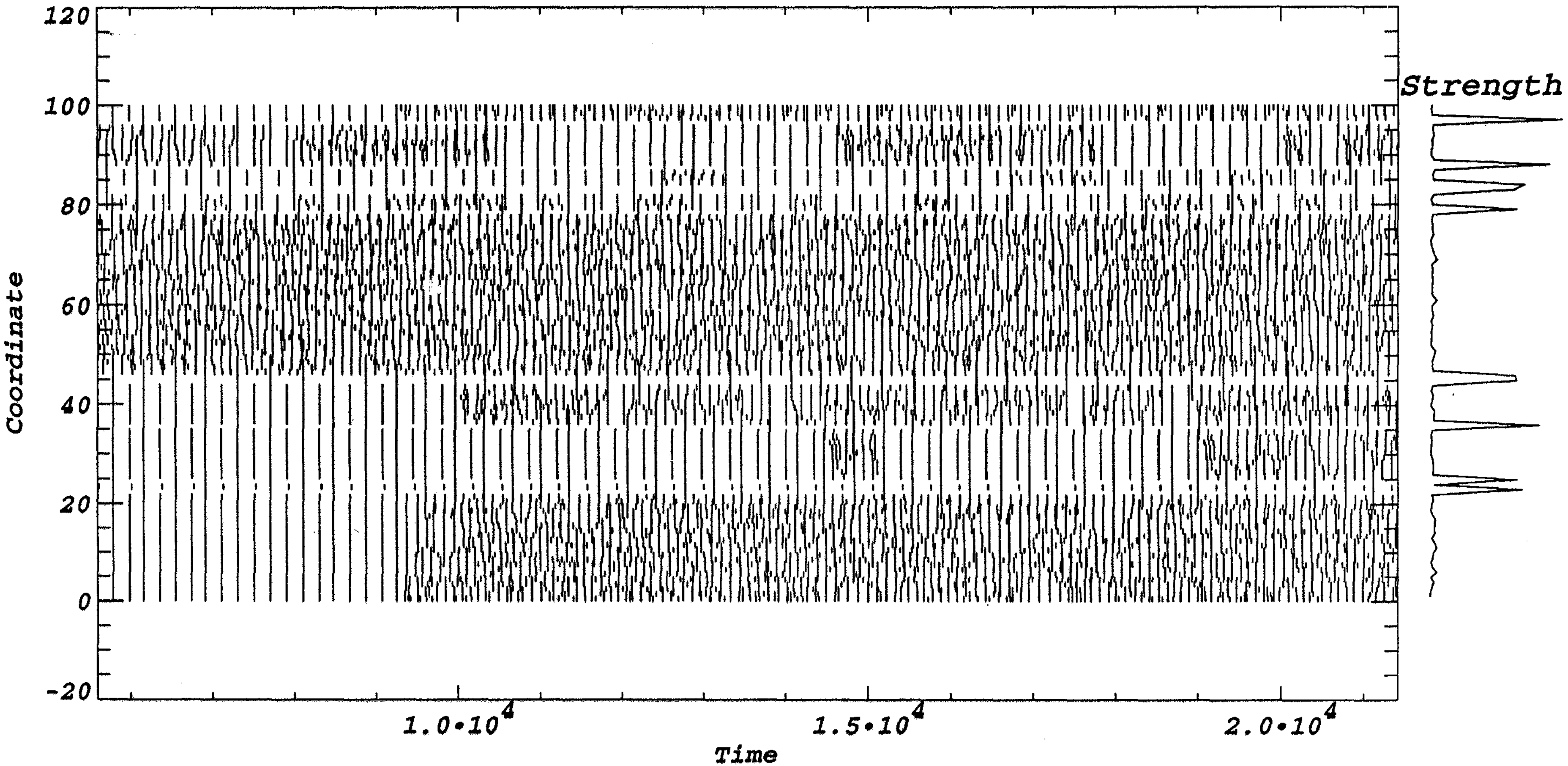
13.0:1.00+0.12

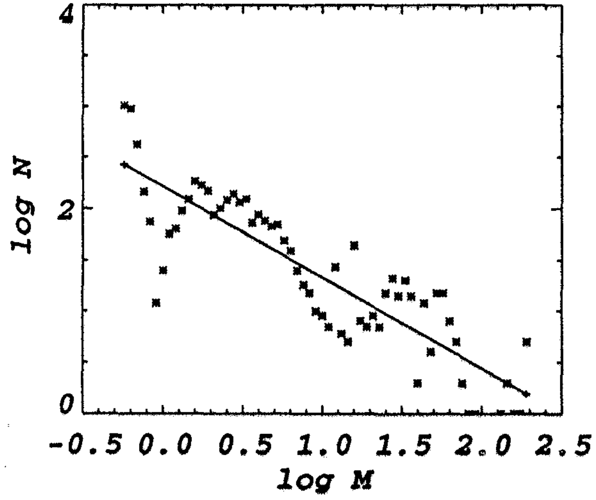






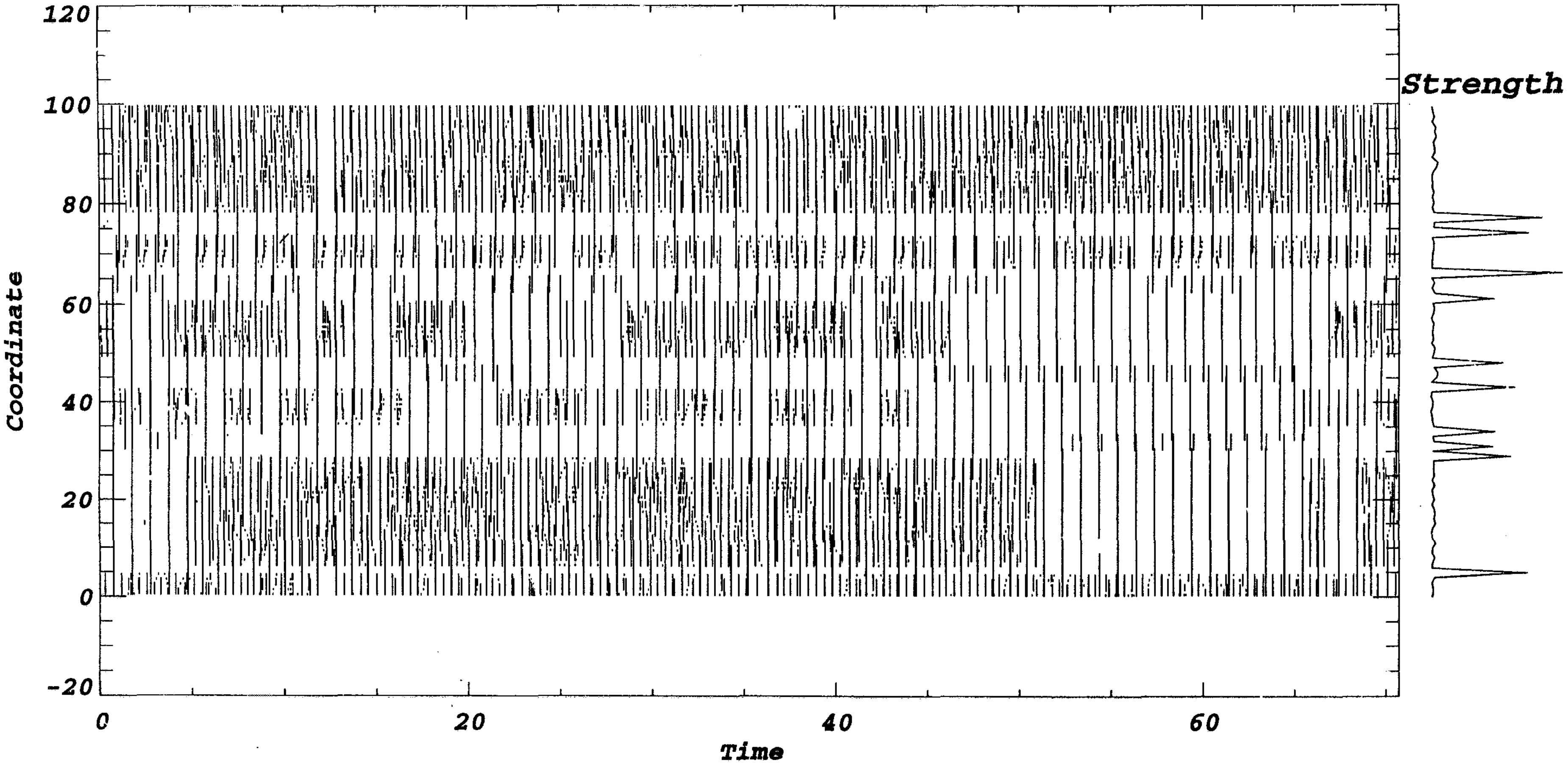
alpha=1.00,   lfast=0.25
Bratio = 9.0,   p=1.30
bmax=4.50 (#88), bmin=0.50 (#67)
bdyn=0.00,   iseed=123457
5000 events,   100 masses
slope = -0.4780 1 s.d. = 0.063535



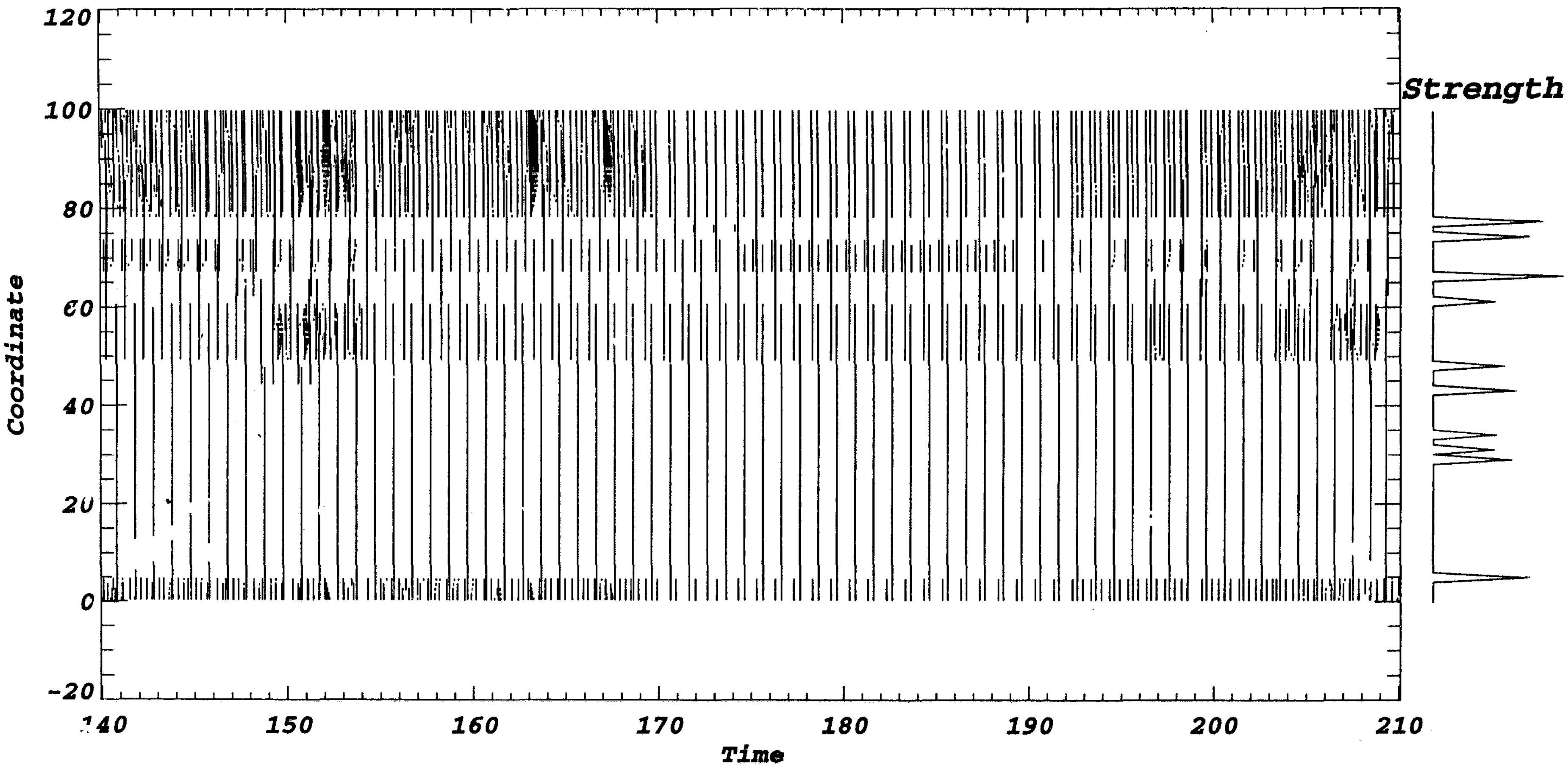


alpha=1.41,   lfast=0.50
Bratio = 5.0,   p=1.20
bmax=5.00 (#97), bmin=1.00 (# 5)
bdyn=0.00,   iseed=712345
6000 events,   100 masses
slope = -0.8869   1 s.d. = 0.067799

[redacted]



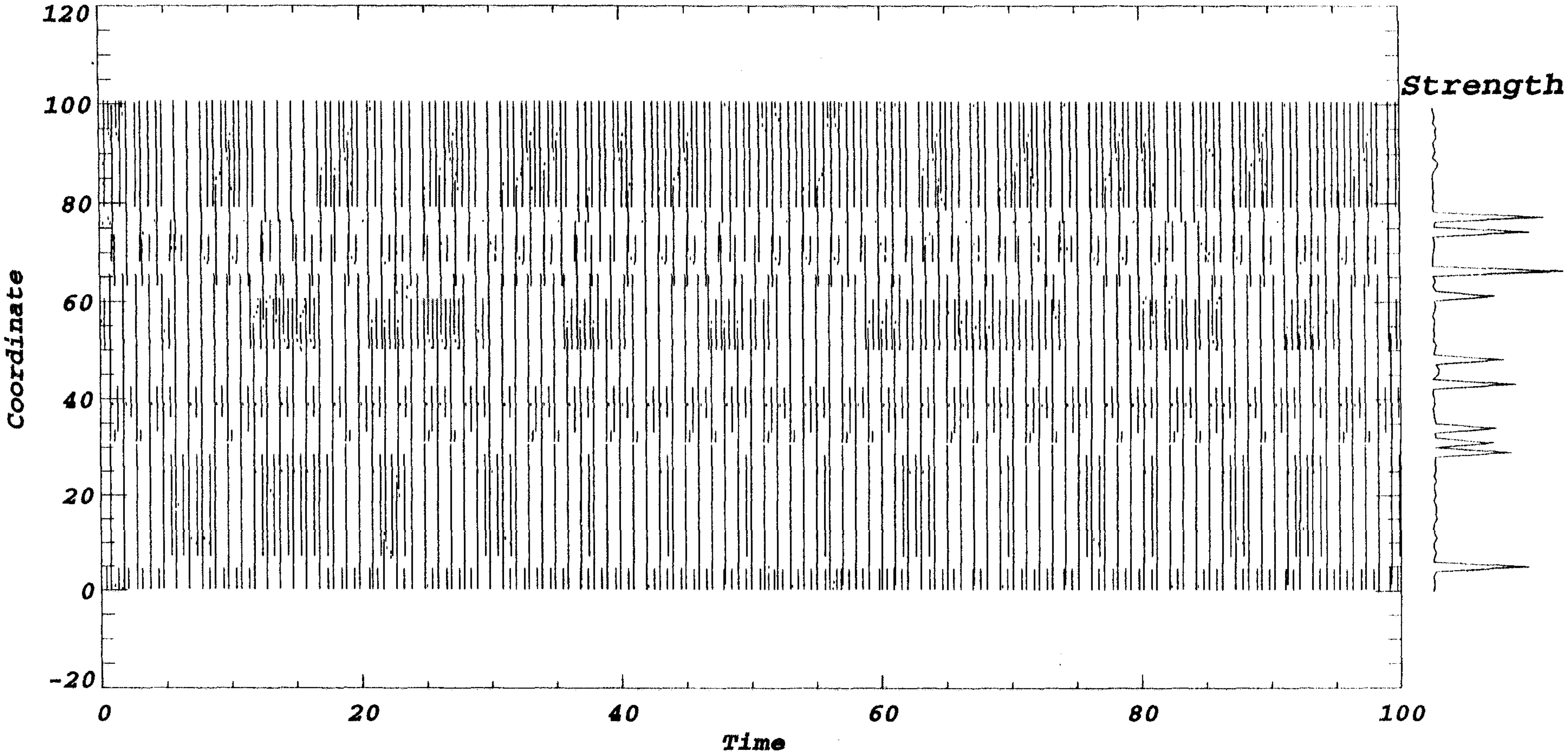
$\alpha = 0.75$


Strength
Coordinate
120
100
80
60
40
20
0
-20
Time
140
150
160
170
180
190
200
210

$\alpha=0.6$

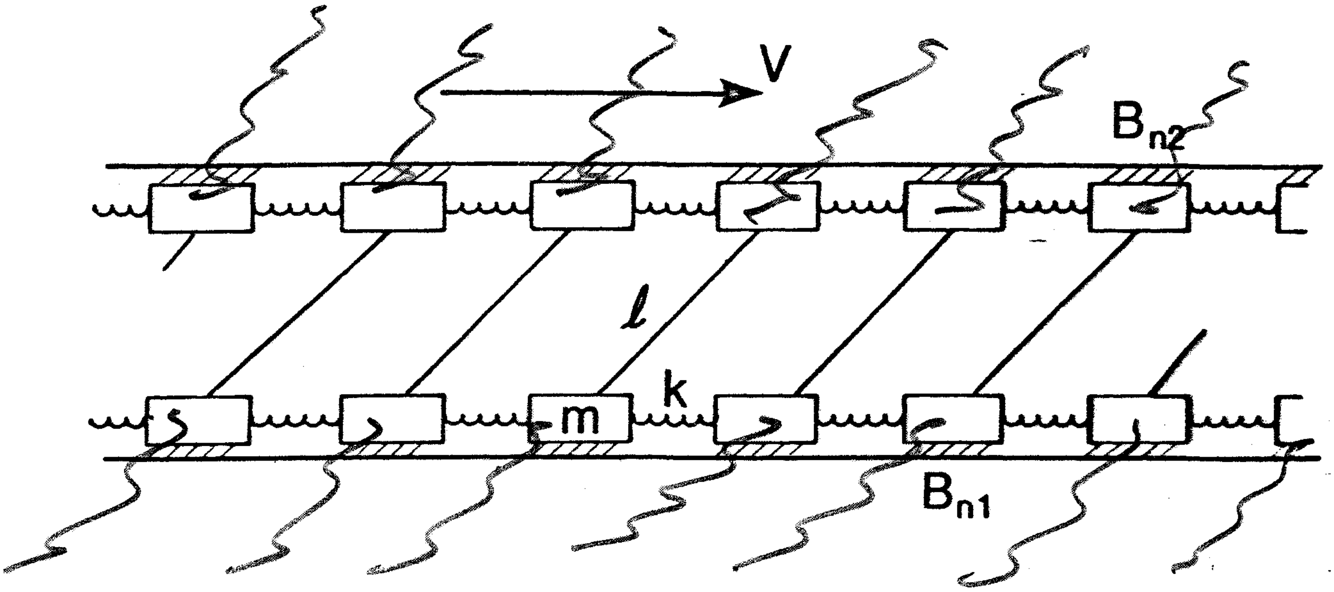
15-I

g.S.

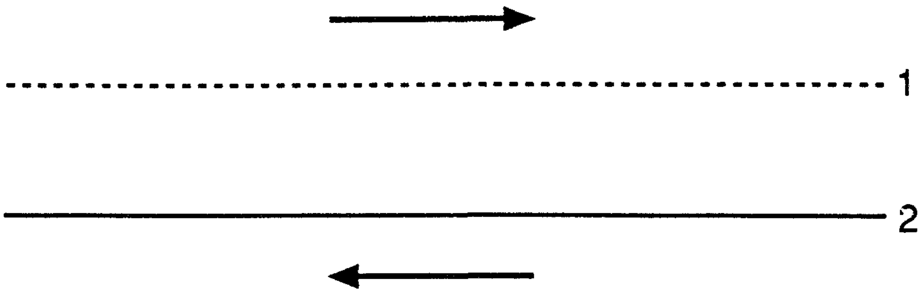

Strength
120
100
80
60
40
20
0
-20
Coordinate
0
20
40
60
80
100
Time

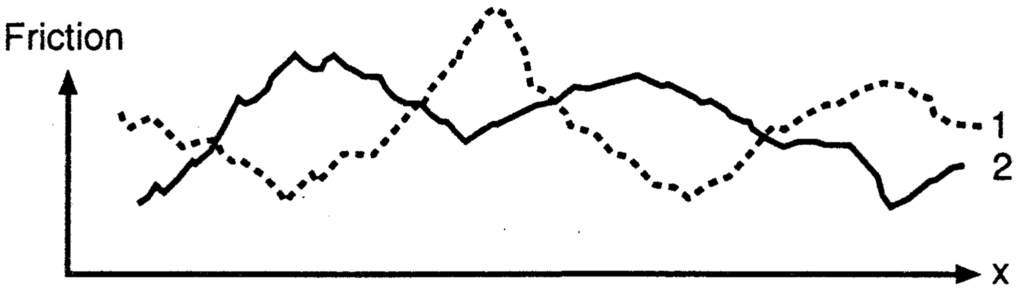
g.S.

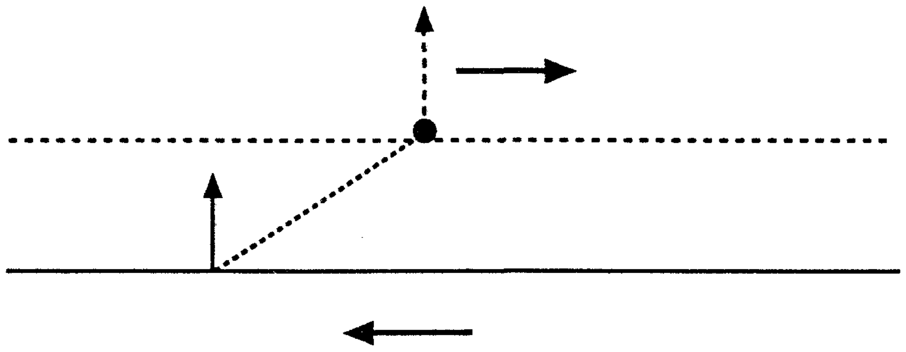
IF ONE FAULT WAS FUN, HOW ABOUT TWO?

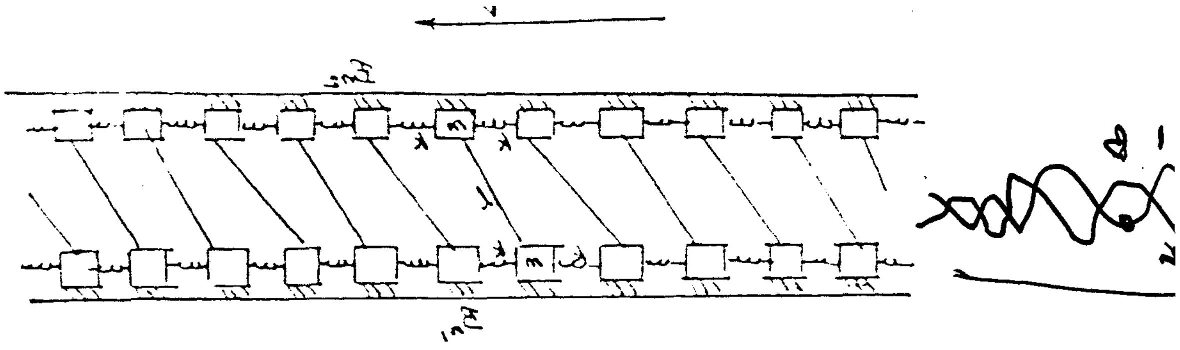

V
B_{n2}
l
k
m
B_{n1}

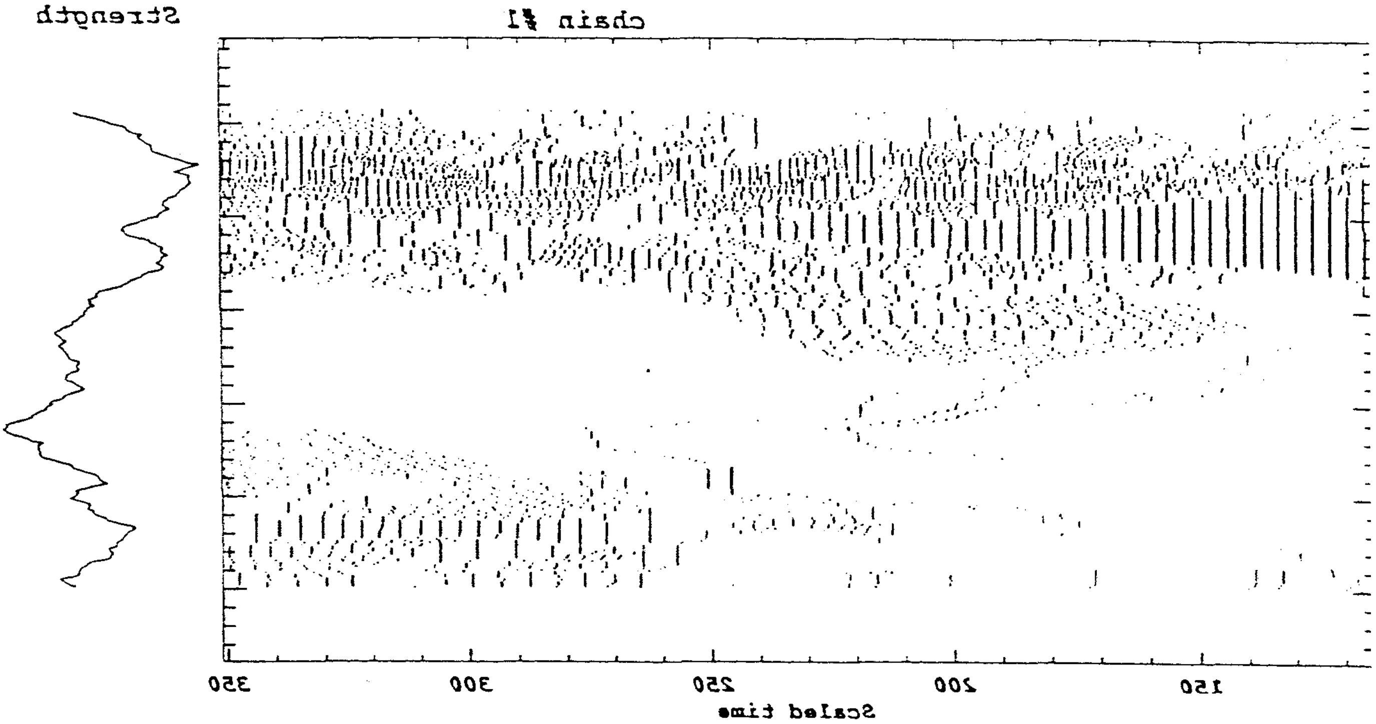
II
3

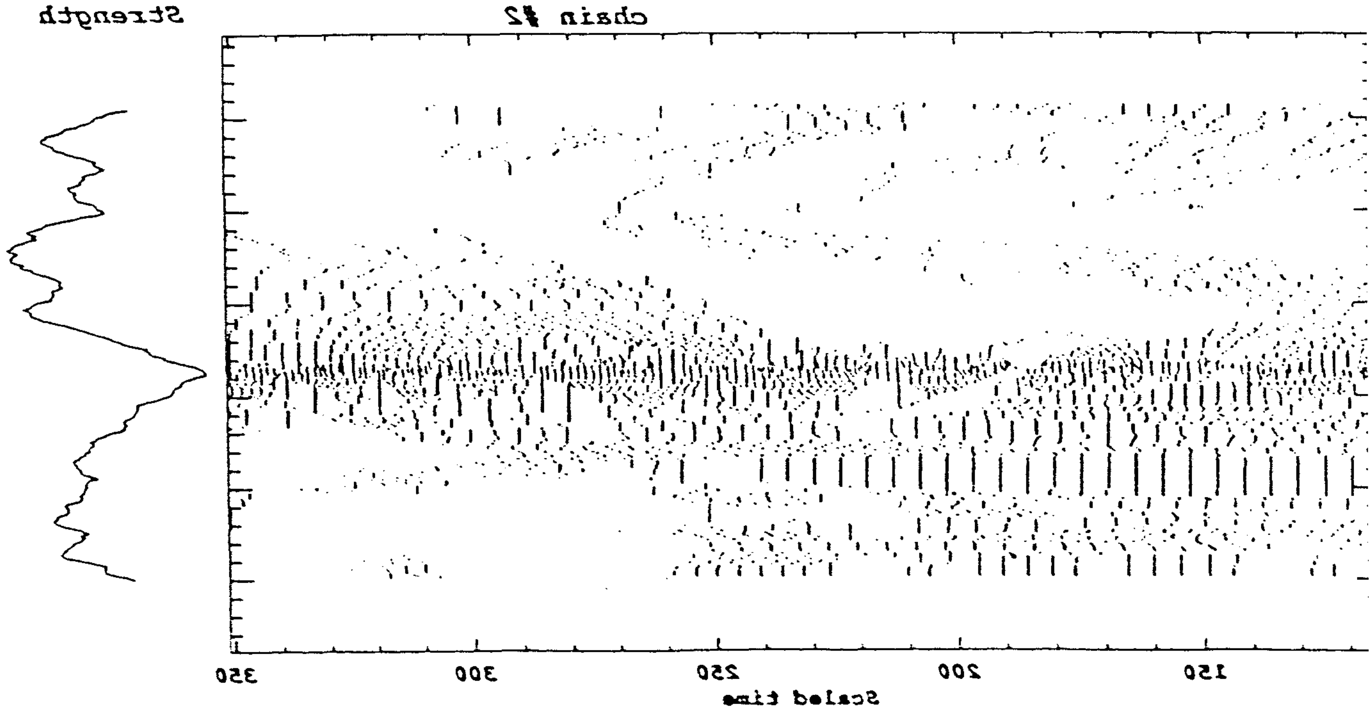


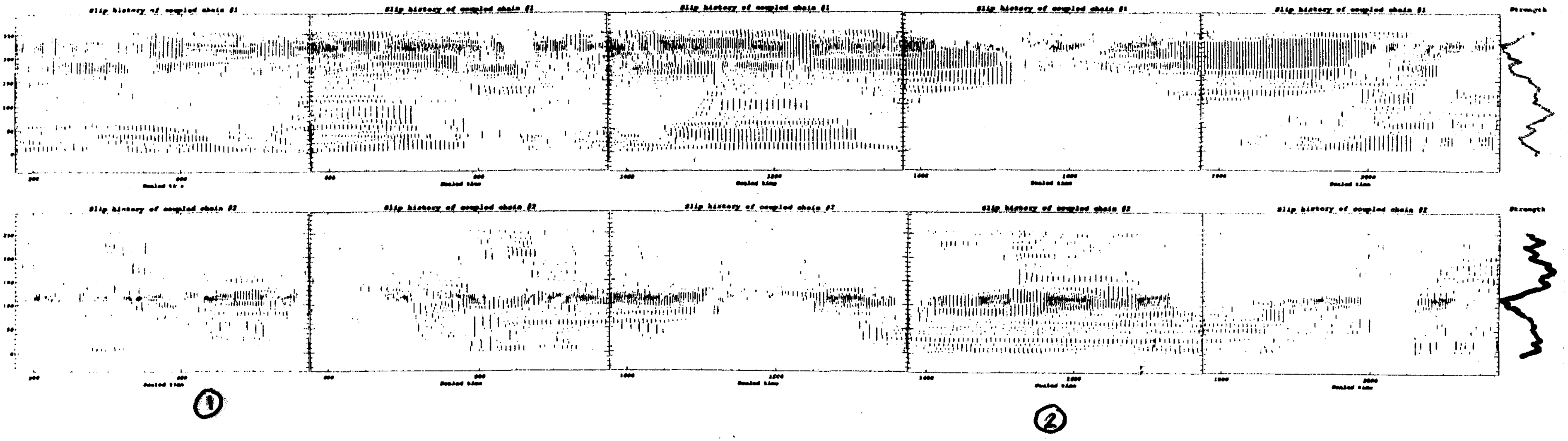


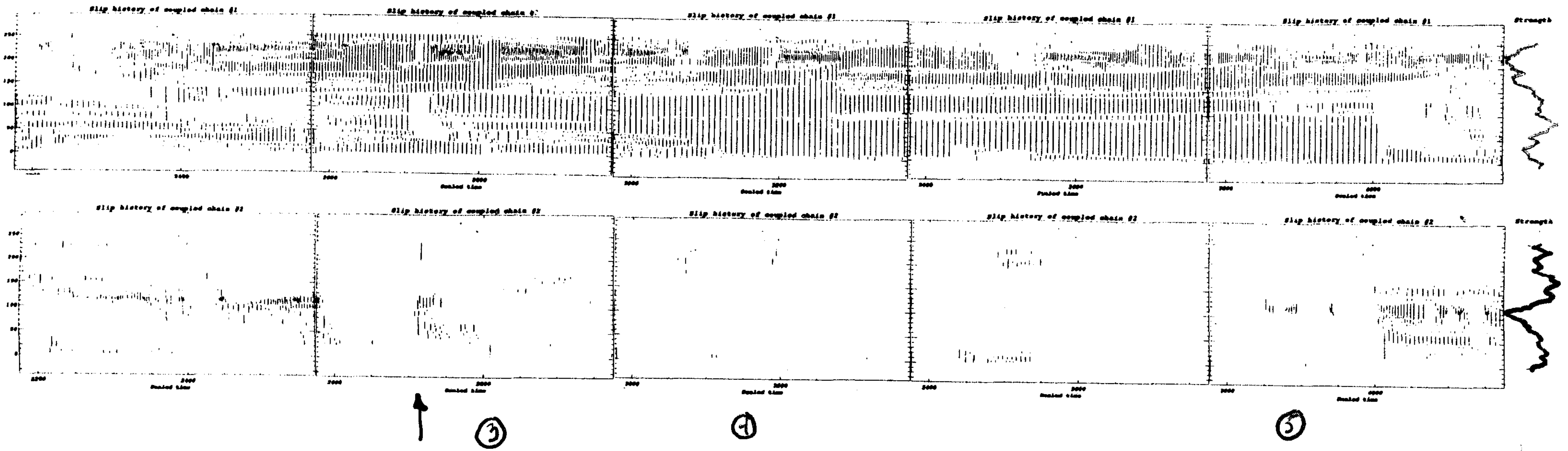



v
k
m
k
l
k
m


Strength
chain #1
350
300
250
200
150
Scaled time

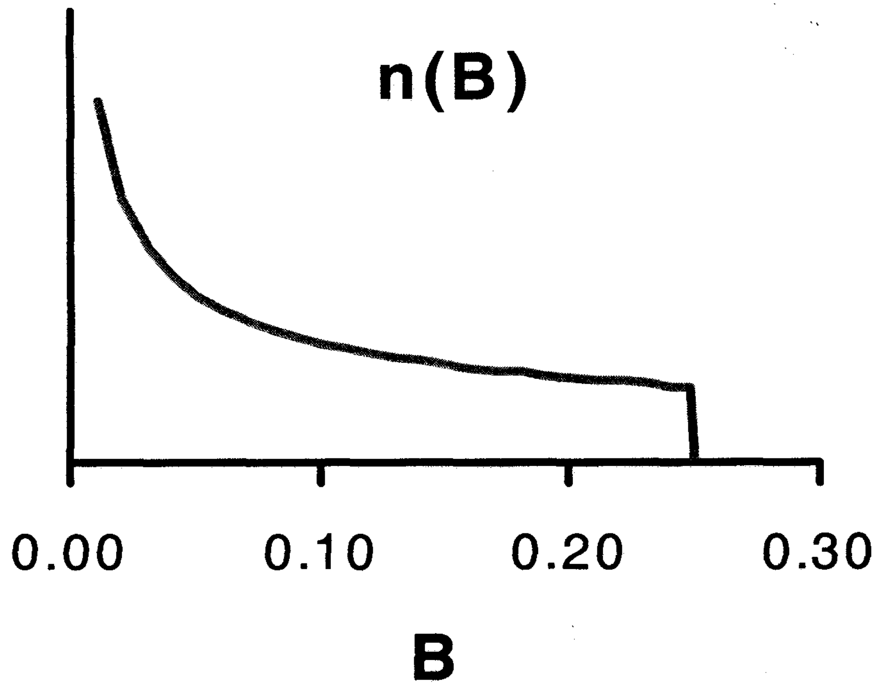

Strength
chain #2
350
300
250
200
150
Scaled time


Slip history of coupled chain #1
Slip history of coupled chain #1
Slip history of coupled chain #1
Slip history of coupled chain #1
Slip history of coupled chain #1
Strength
Scaled time
Slip history of coupled chain #2
Slip history of coupled chain #2
Slip history of coupled chain #2
Slip history of coupled chain #2
Slip history of coupled chain #2
Strength
Scaled time
①
②

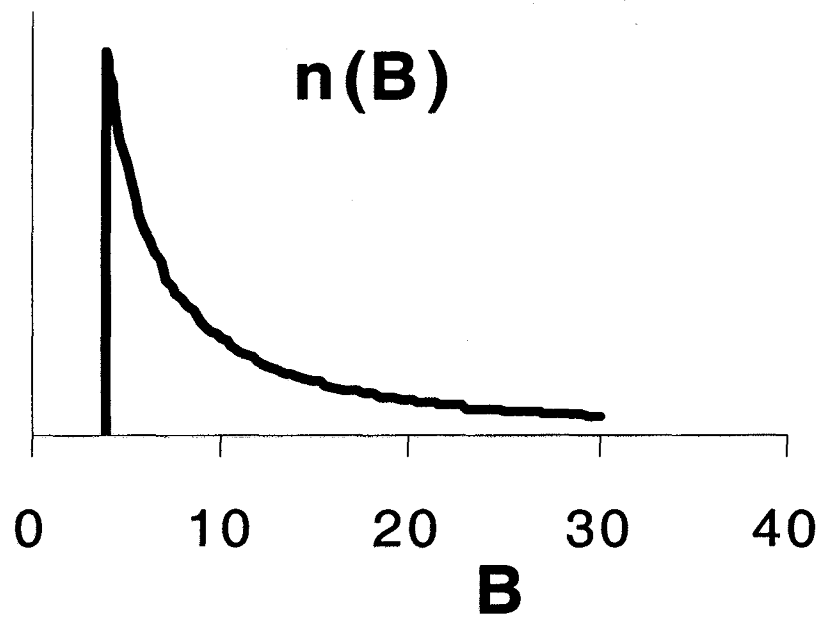

Slip history of coupled chain #1
Slip history of coupled chain #1
Slip history of coupled chain #1
Slip history of coupled chain #1
Slip history of coupled chain #1
Strength
Scaled time
Slip history of coupled chain #2
Slip history of coupled chain #2
Slip history of coupled chain #2
Slip history of coupled chain #2
Slip history of coupled chain #2
Strength
Scaled time
③
④
⑤

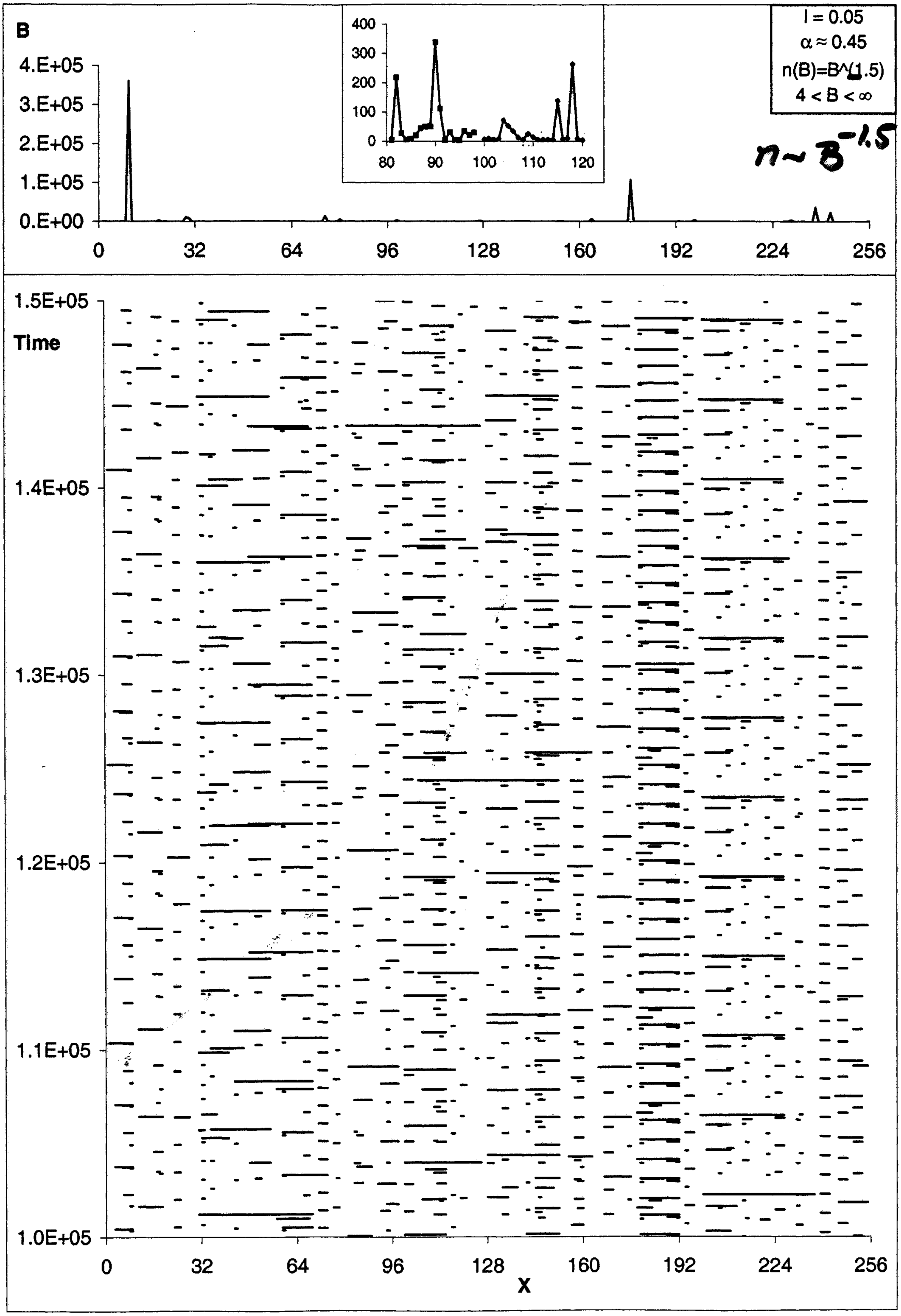
Distributions of the Breaking Strength

$$n(B) \sim B^{-0.5}$$

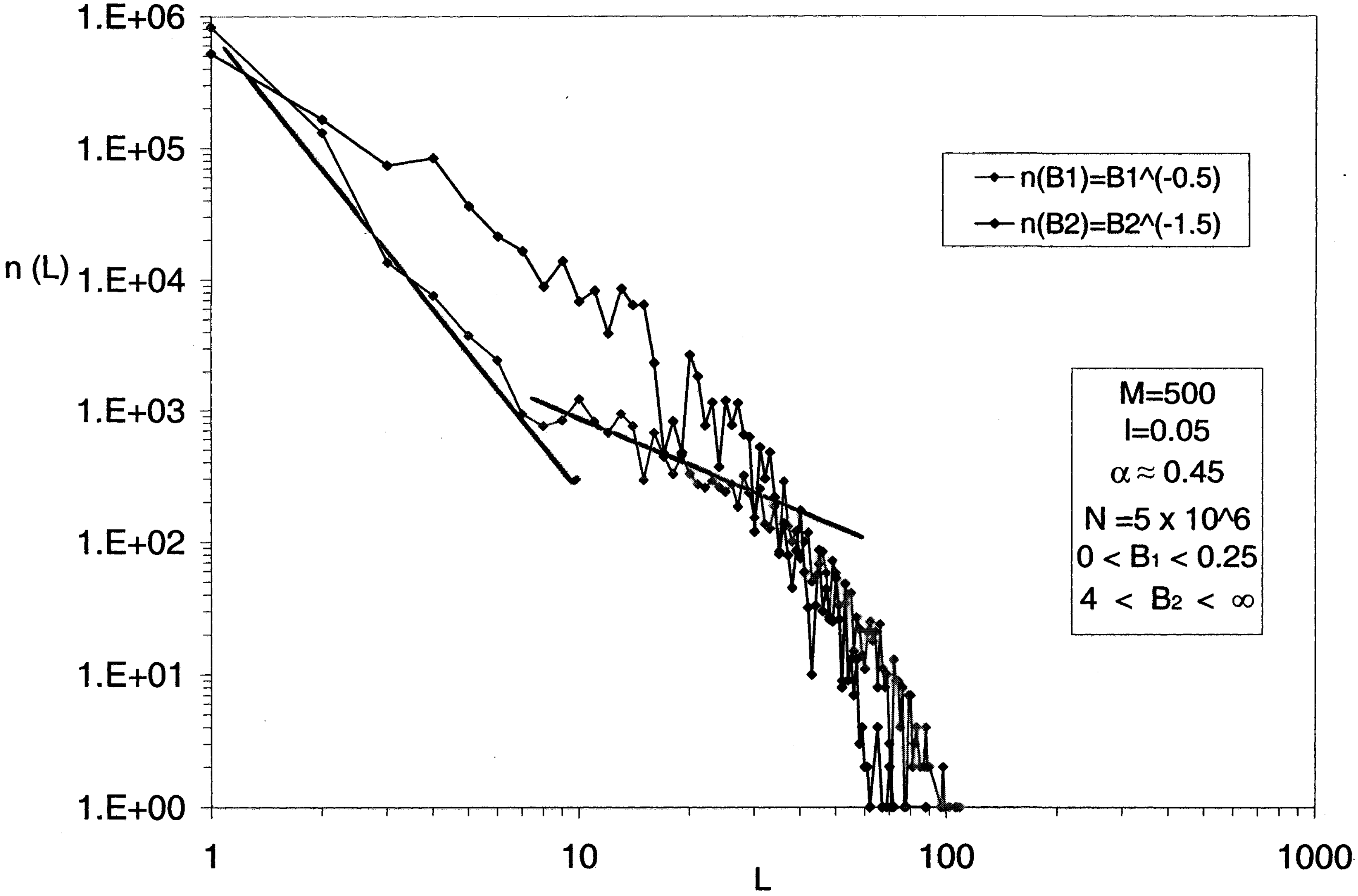


$$n(B) \sim B^{-1.5}$$




B
4.E+05
3.E+05
2.E+05
1.E+05
0.E+00
0
32
64
96
128
160
192
224
256
400
300
200
100
0
80
90
100
110
120
l = 0.05
α ≈ 0.45
n(B)=B^(1.5)
4 < B < ∞
1.5E+05
Time
1.4E+05
1.3E+05
1.2E+05
1.1E+05
1.0E+05
0
32
64
96
128
160
192
224
256
X

$n \sim B^{-1.5}$



n (L)
1.E+06
1.E+05
1.E+04
1.E+03
1.E+02
1.E+01
1.E+00
1
10
100
1000
L
n(B1)=B1^(-0.5)
n(B2)=B2^(-1.5)
M=500
l=0.05
α ≈ 0.45
N =5 x 10^6
0 < B1 < 0.25
4 < B2 < ∞

The question of self-healing pulses

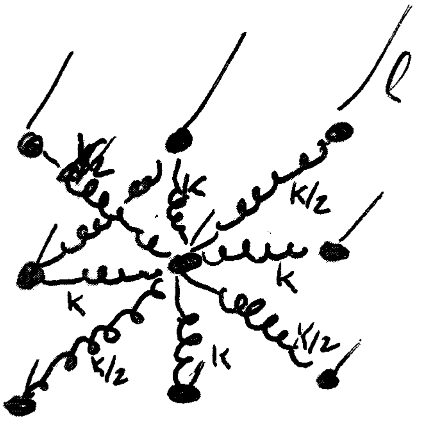
2-D models: In-plane slip

$$\rho\ddot{\vec{u}} = (\lambda+2\mu)\nabla\nabla\cdot\vec{U} - \mu\nabla\times\nabla\times\vec{U} - \ell(\vec{u}-V_0 t\hat{e}_x)$$
$$- \alpha\dot{\vec{u}} + \vec{T} - f_{frict}\frac{\dot{\vec{u}}}{|\dot{\vec{u}}|}$$

$$\alpha = 2\sqrt{\ell\rho}$$

$$\rho\ddot{u} = (\lambda+2\mu)\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial^2 u}{\partial y^2} + (\lambda+\mu)\frac{\partial^2 v}{\partial x\partial y} - \ell(u-V_0 t)$$
$$- \alpha\frac{\partial u}{\partial t} + T_x - f_f\frac{\dot{u}}{\sqrt{\dot{u}^2+\dot{v}^2}}$$

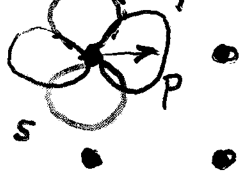
$$\rho\ddot{v} = (\lambda+2\mu)\frac{\partial^2 v}{\partial y^2} + \mu\frac{\partial^2 v}{\partial x^2} + (\lambda+\mu)\frac{\partial^2 u}{\partial x\partial y} - \ell v$$
$$- \alpha\frac{\partial v}{\partial t} + T_y - f_f\frac{\dot{v}}{\sqrt{\dot{u}^2+\dot{v}^2}}$$



1820
Cauchy
Poisson
P. ratio = 1/4
$\lambda=\mu$

4
15
32

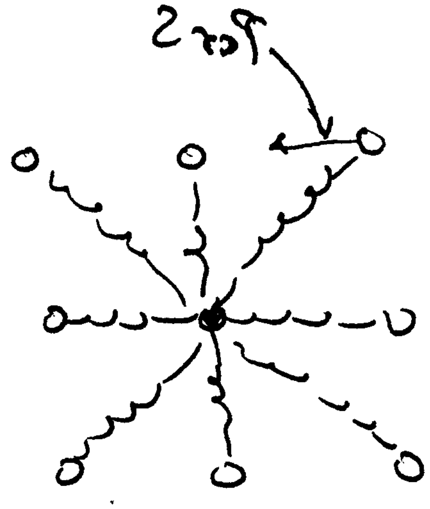
causality:

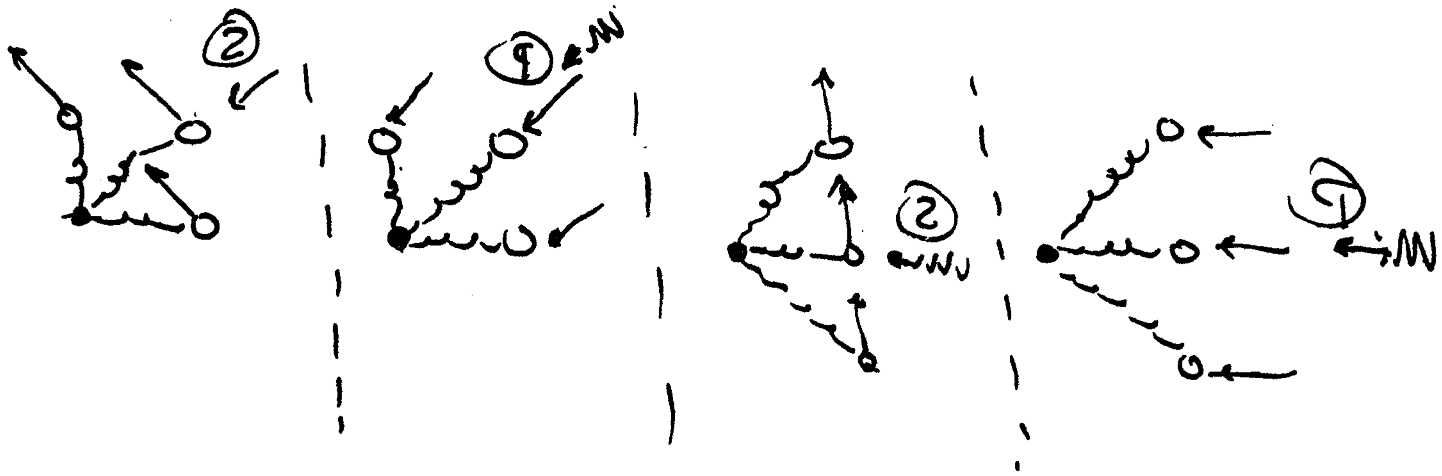


$$\nu_n\left(1-\frac{\omega^2}{v^2t^2}\right)=-g$$

in 1-D

but what is v? in the 2-D case

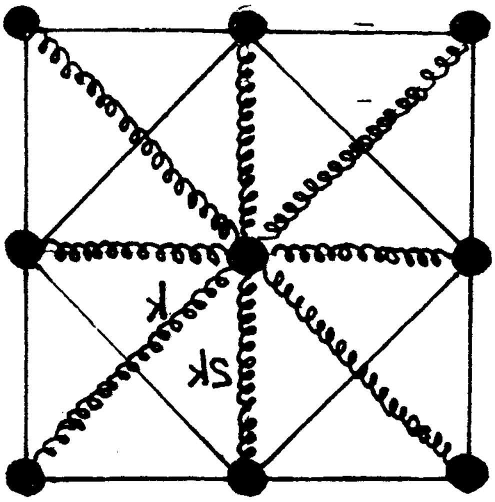


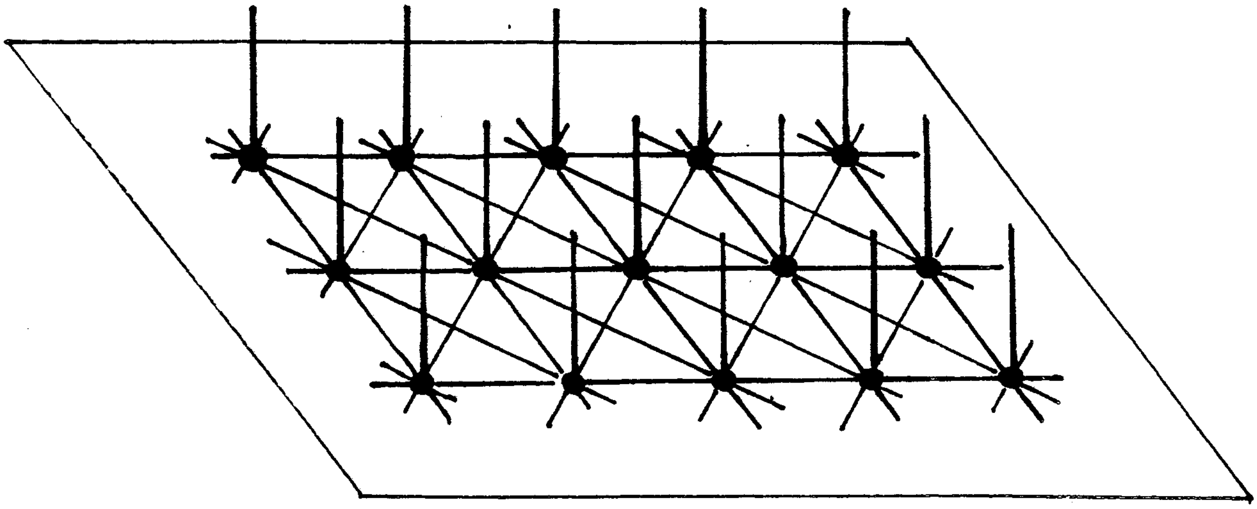


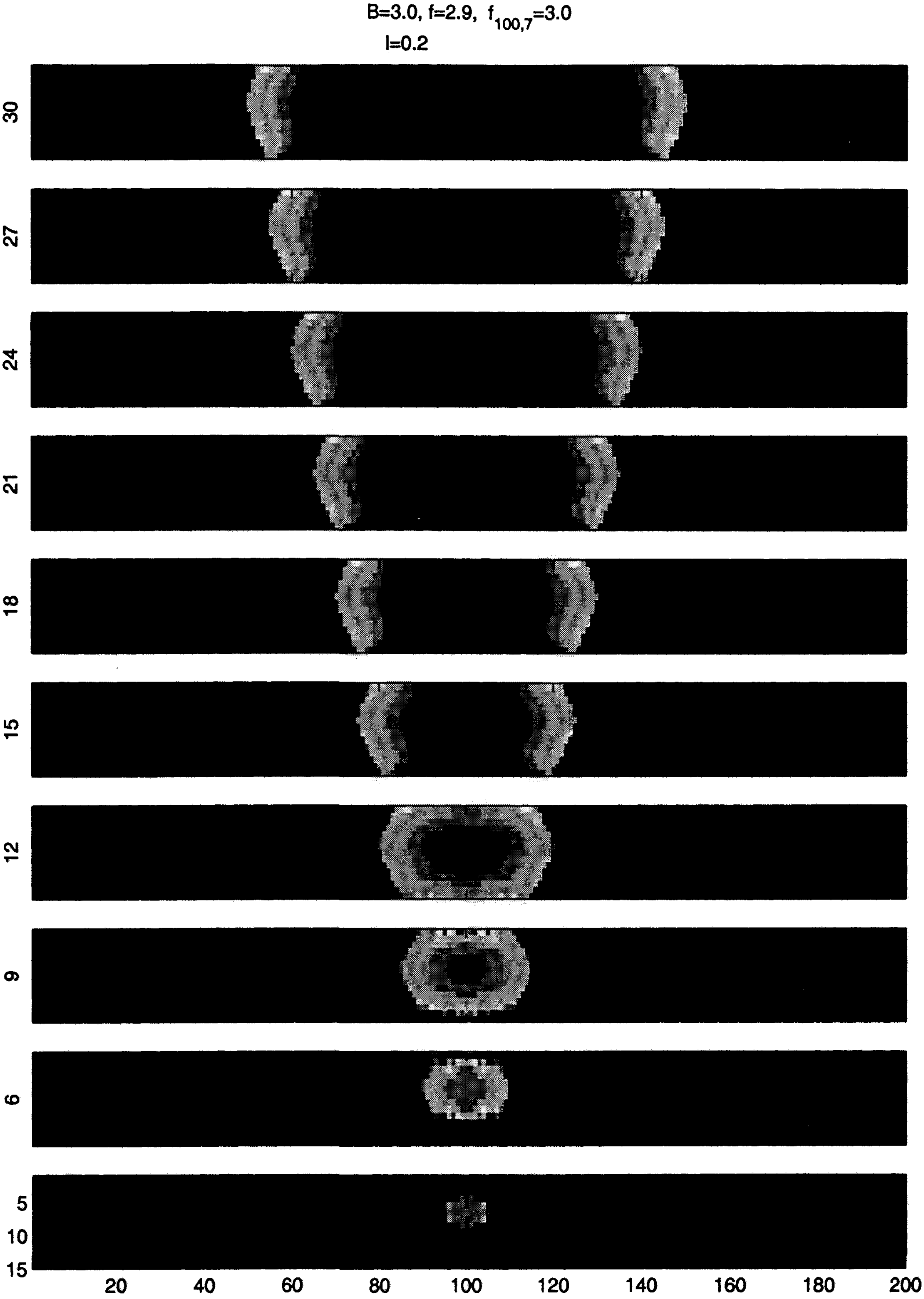
8 P-mode, 8 S-mode

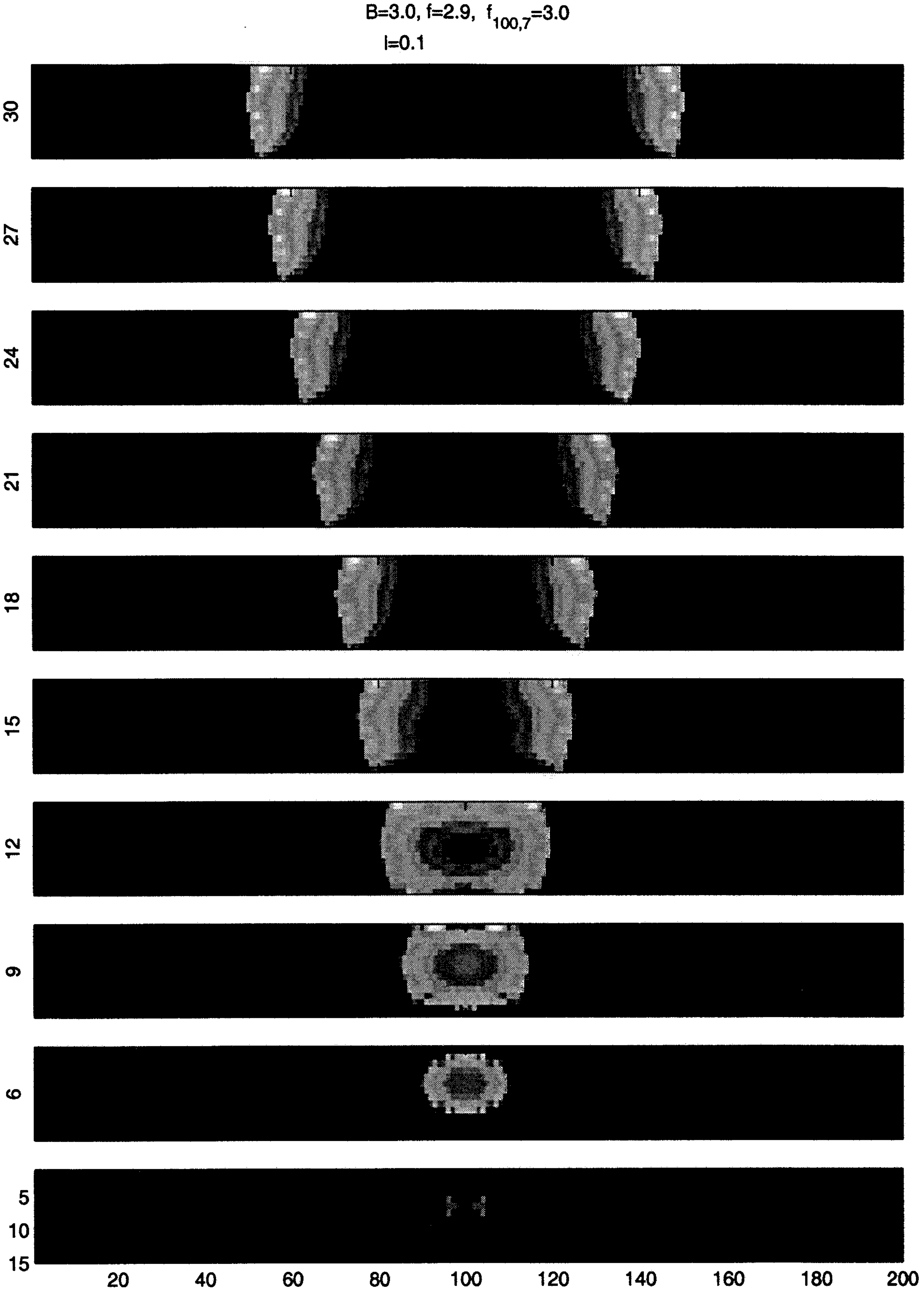
Mode j

$$\nu_j n\left(1-\frac{\omega_j^2}{v_j^2t^2}\right)=g_n$$

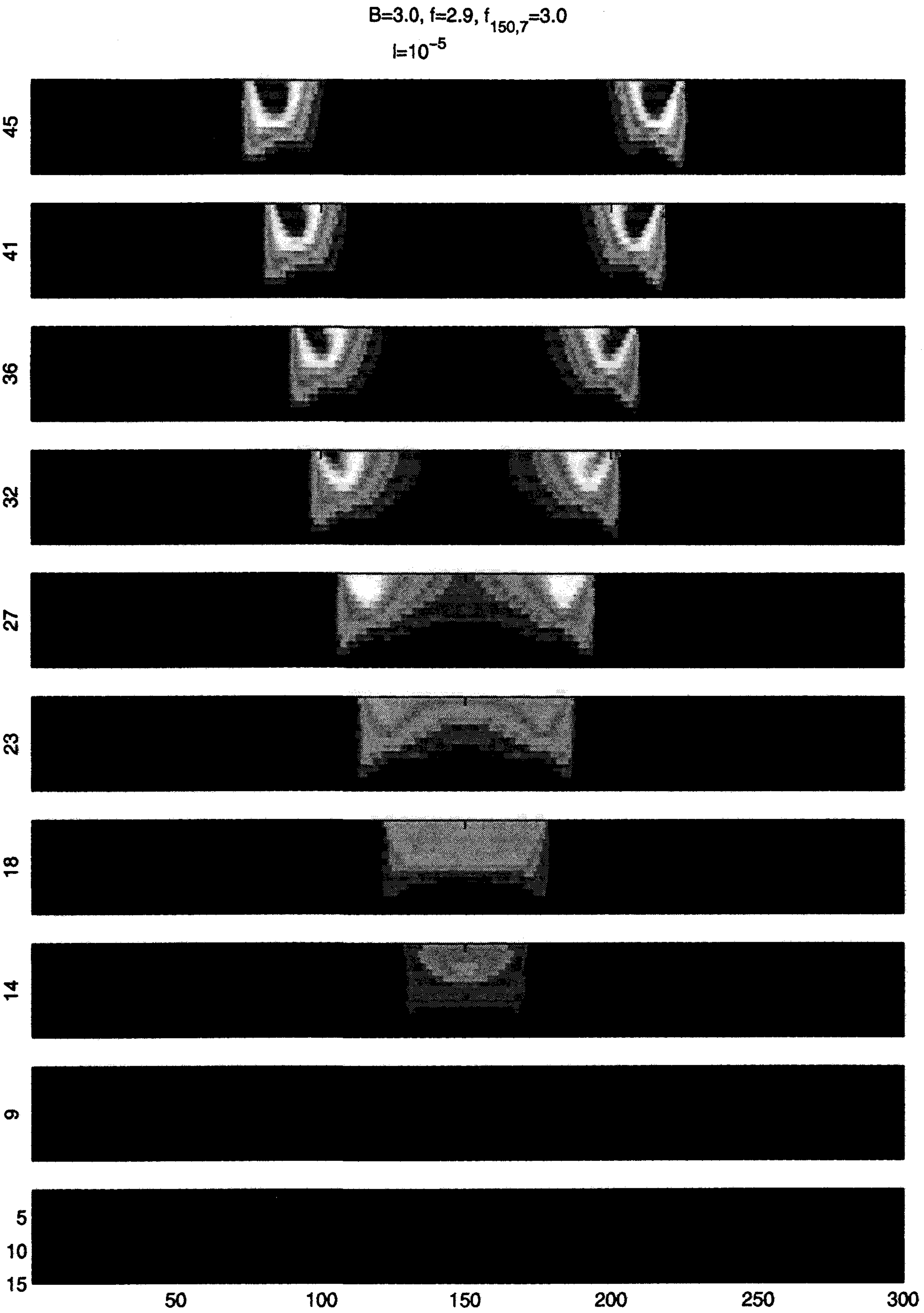

k
2k

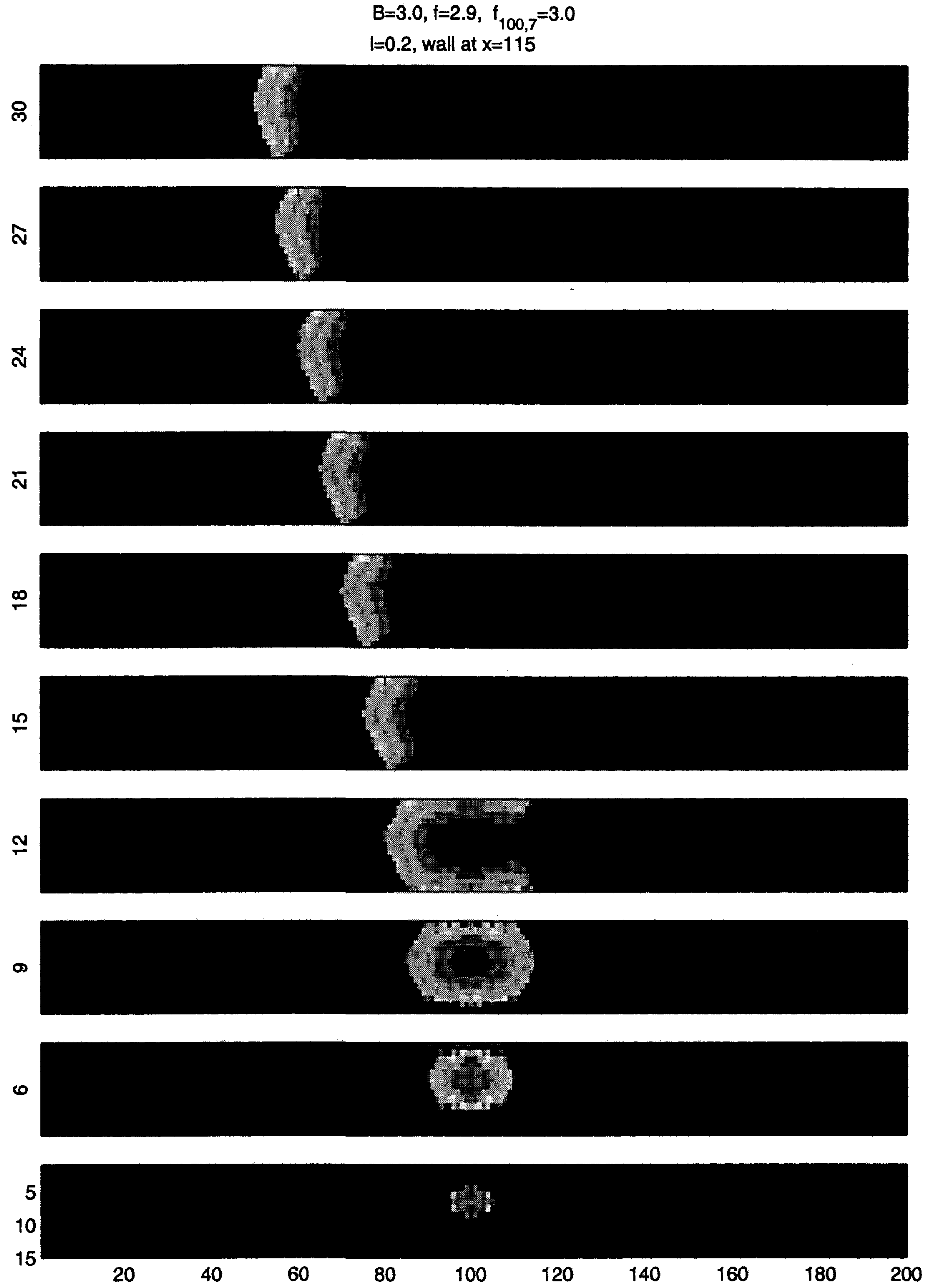


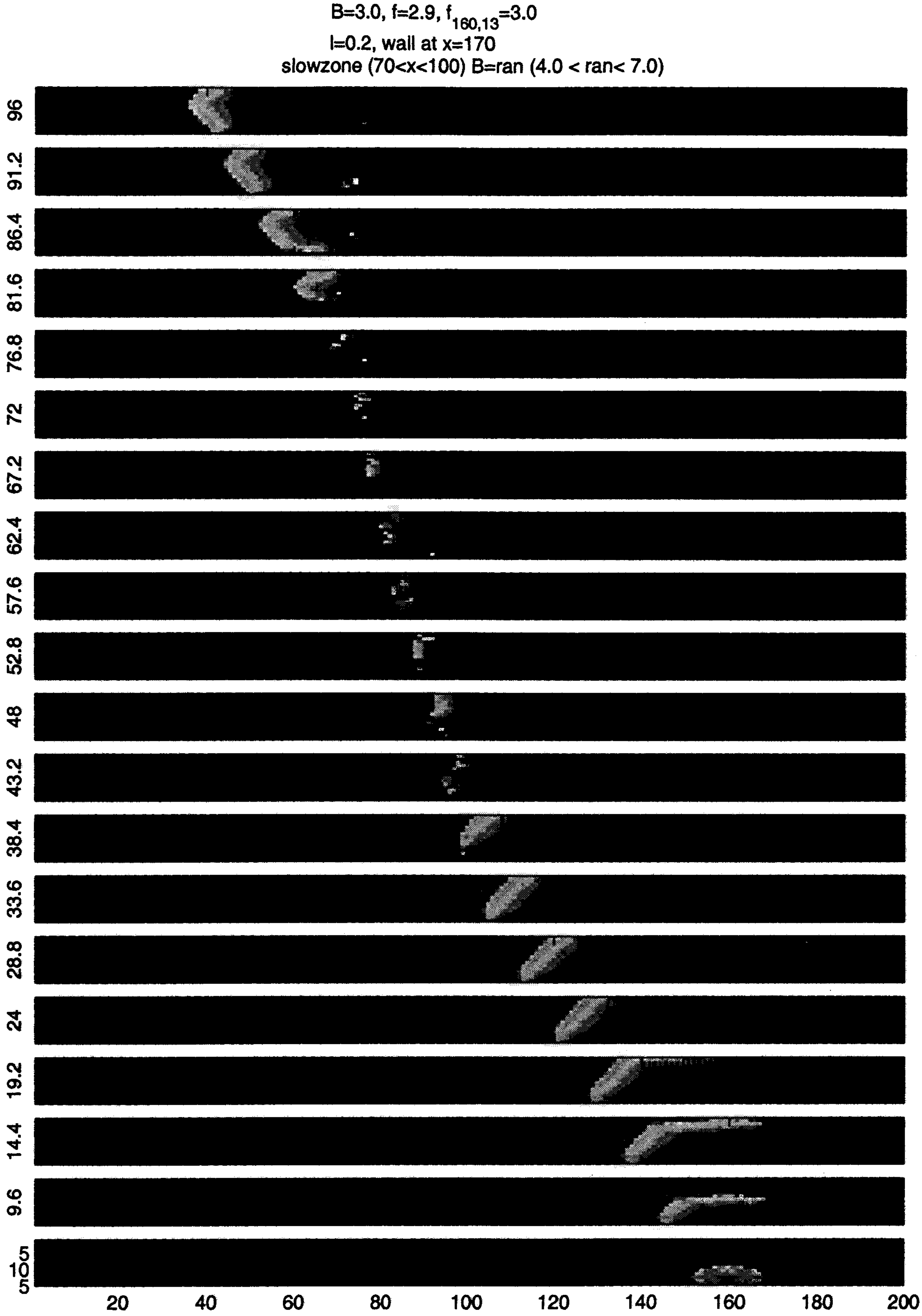

B=3.0, f=2.9, f100,7=3.0
I=0.2
30
27
24
21
18
15
12
9
6
5
10
15
20
40
60
80
100
120
140
160
180
200

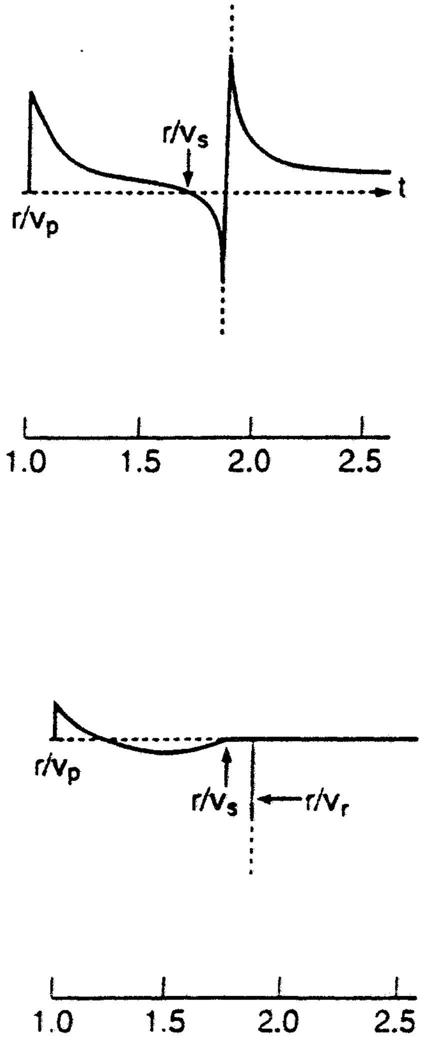


B=3.0, f=2.9, $f_{100,7}$=3.0
I=0.1
30
27
24
21
18
15
12
9
6
5
10
15
20
40
60
80
100
120
140
160
180
200

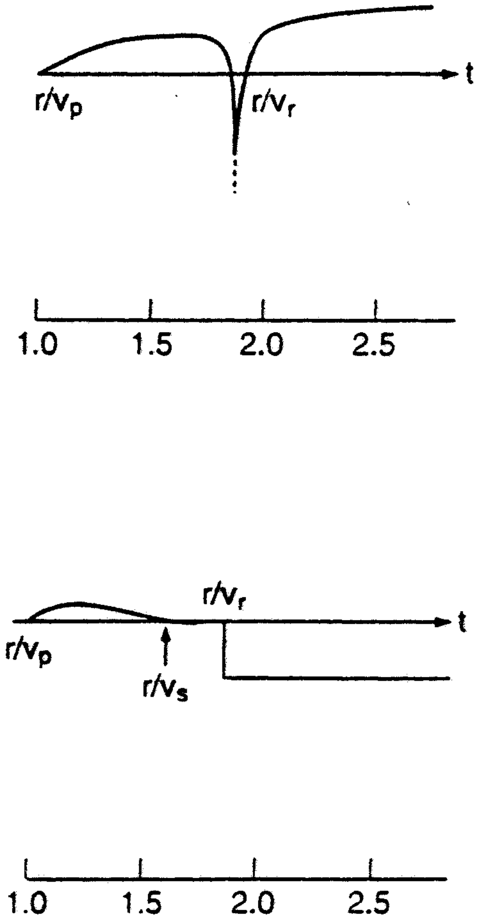

B=3.0, f=2.9, $f_{150,7}$=3.0
I=10^{-5}
45
41
36
32
27
23
18
14
9
5
10
15
50
100
150
200
250
300


B=3.0, f=2.9, $f_{100,7}$=3.0
I=0.2, wall at x=115
30
27
24
21
18
15
12
9
6
5
10
15
20
40
60
80
100
120
140
160
180
200


B=3.0, f=2.9, $f_{160,13}$=3.0
I=0.2, wall at x=170
slowzone (70<x<100) B=ran (4.0 < ran< 7.0)
96
91.2
86.4
81.6
76.8
72
67.2
62.4
57.6
52.8
48
43.2
38.4
33.6
28.8
24
19.2
14.4
9.6
5
10
5
20
40
60
80
100
120
140
160
180
200


r/v_s
t
r/v_p
1.0
1.5
2.0
2.5
r/v_p
r/v_s
r/v_r
1.0
1.5
2.0
2.5

a)


t
r/v_p
r/v_r
1.0
1.5
2.0
2.5
r/v_r
t
r/v_p
r/v_s
1.0
1.5
2.0
2.5

b)

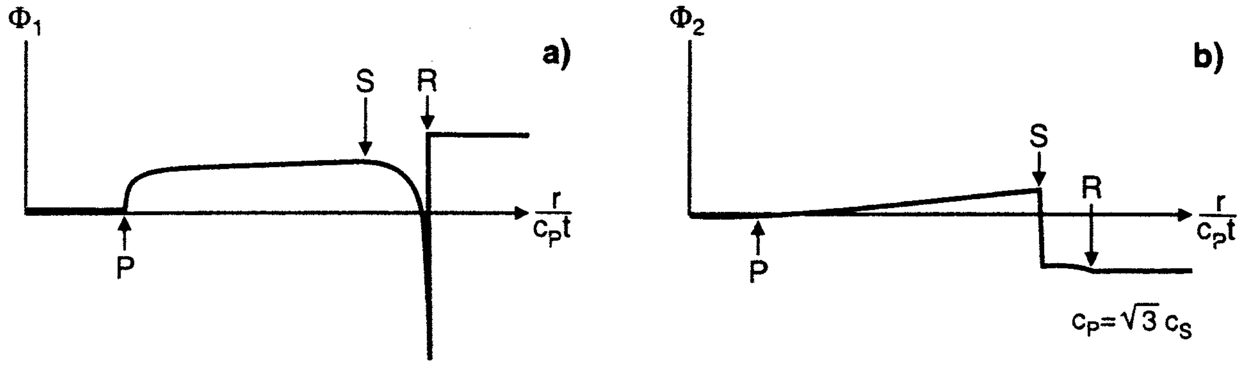


Figure 3. Half-space Green's function components for a linear crack.
a) Radial slip along the prolongation of the crack. b) Transverse slip on the normal to the crack. Both functions to be multiplied by 1/r.

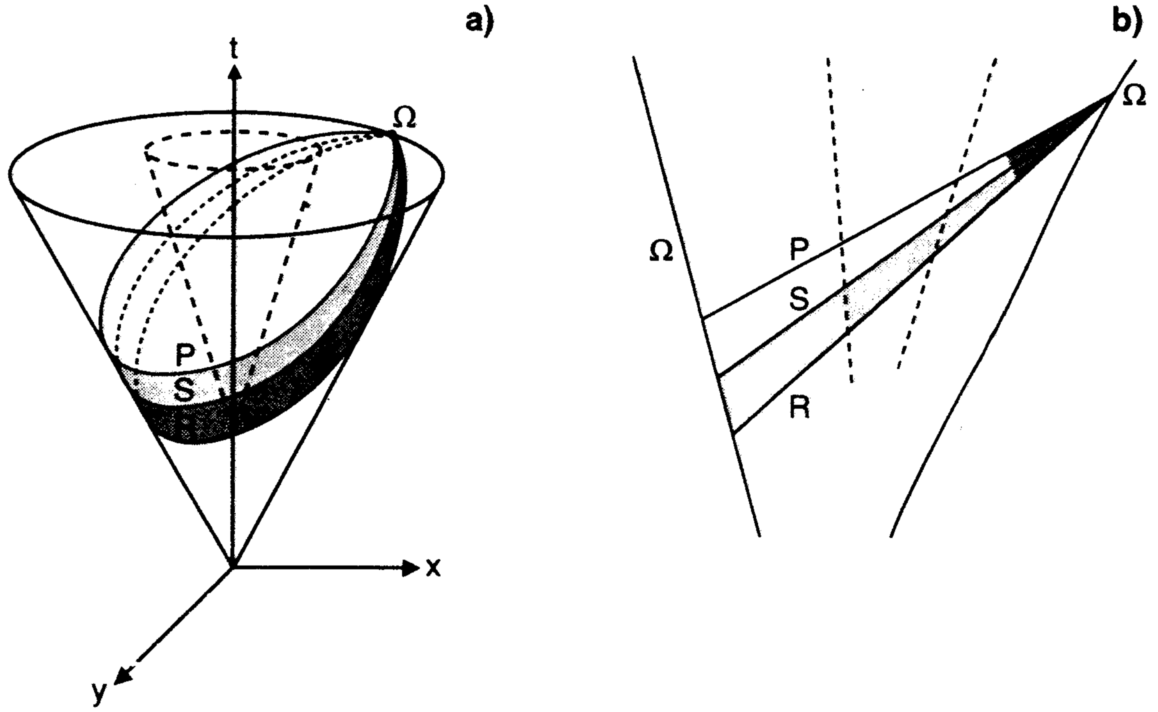


Figure 4. a) Intersection of surface Ω with three backward cones with P- S- and R-wave slopes. b) Plane cross-section of a) showing region of integration (shaded): integrate over zone between P and R cones close to edge and between S and R cones at greater distance.

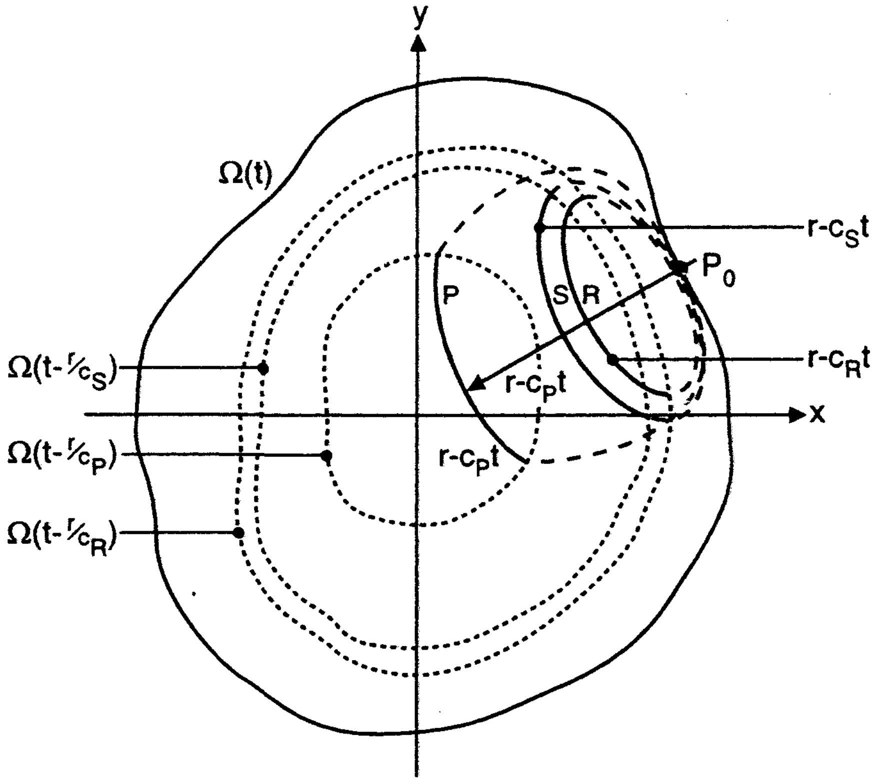
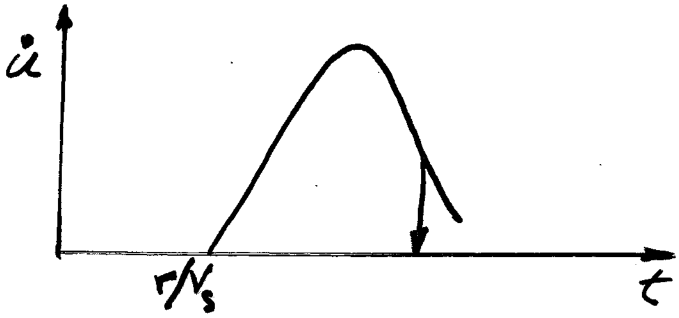
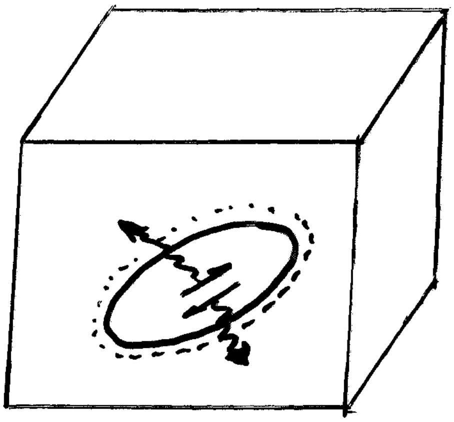


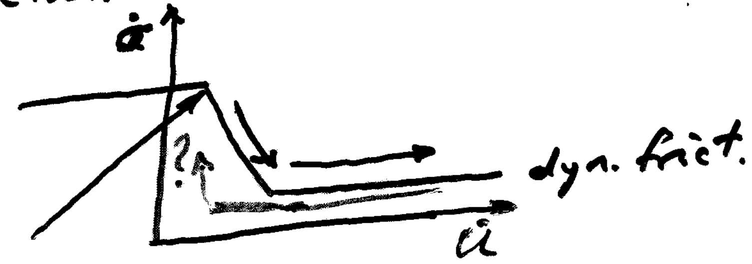
Figure 5. Growth of a 2-D crack with sub-R wave speed at its edge. All points outside a circular arc P of radius c_Pt centered at P_0 transmit no slip motions to P_0. No points outside arcs S, R of radius c_St, c_Rt transmit slip motions earlier than the arrival of the S-R wave. The three cones intersect the edge at times indicated by (dots) curves. The path of these intersections is given by the larger dashes. Integration takes place in the region interior to the closed curve P.

SELF-HEALING PULSES

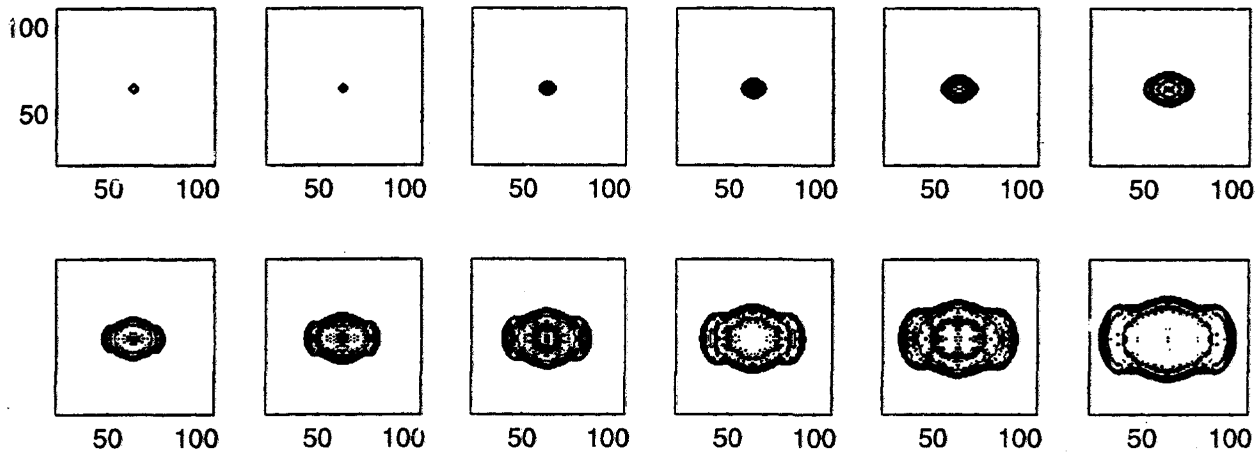




Internal friction



Insufficient Nucleation



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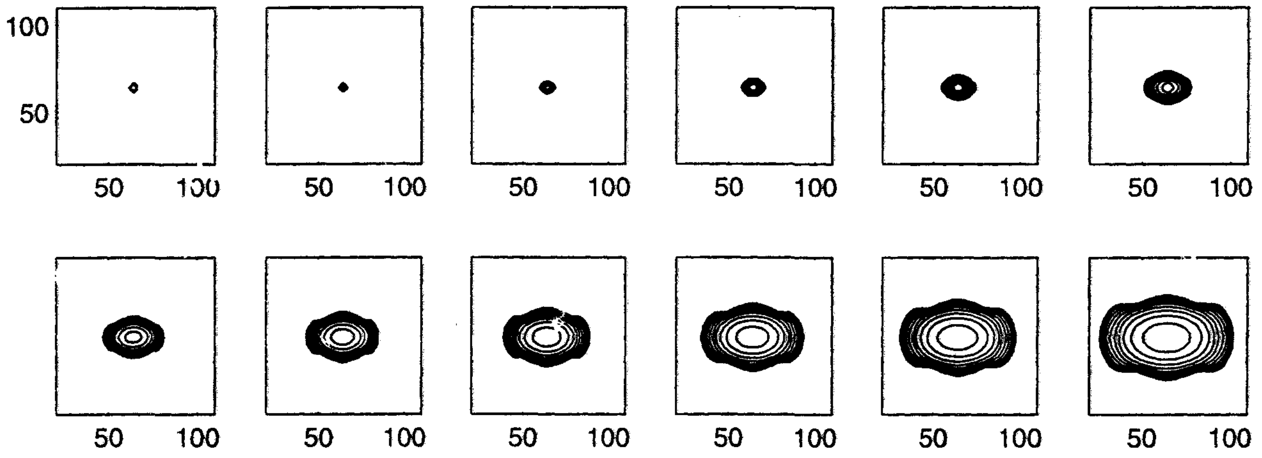

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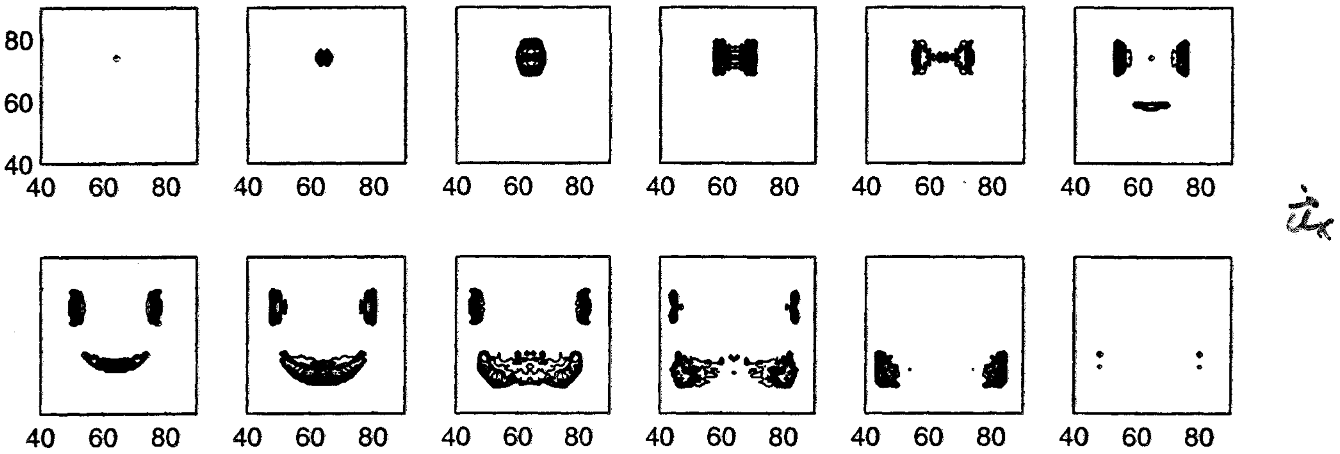
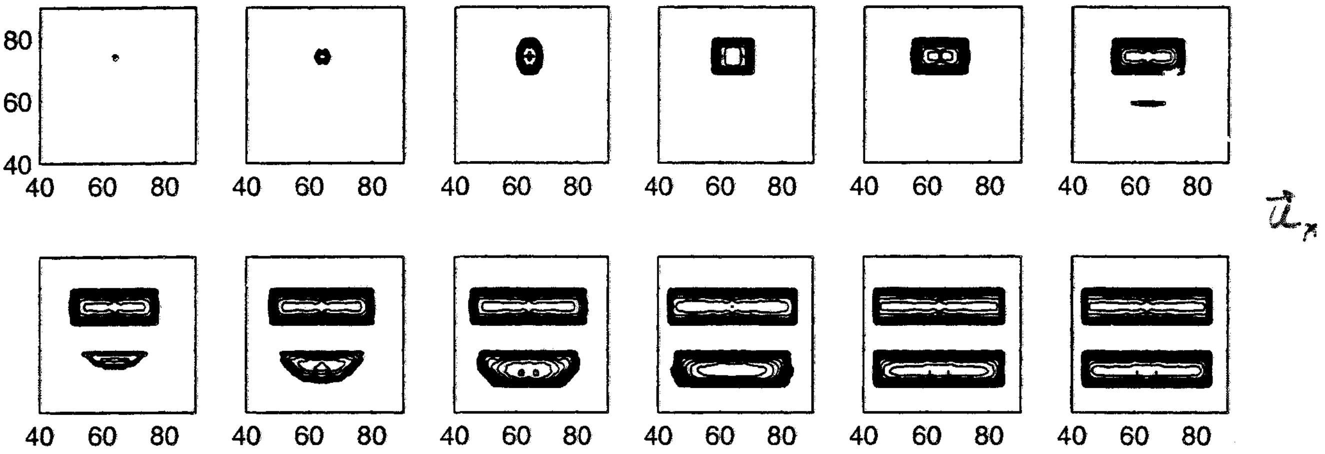
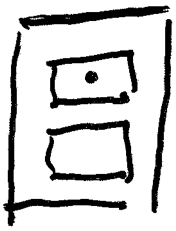
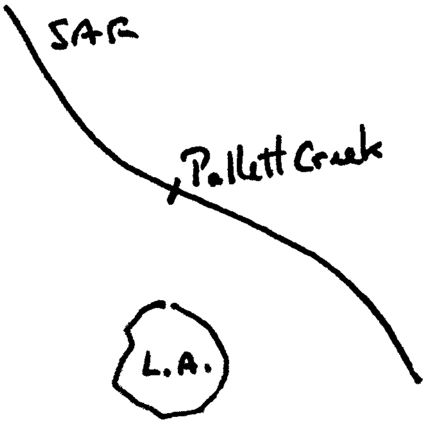
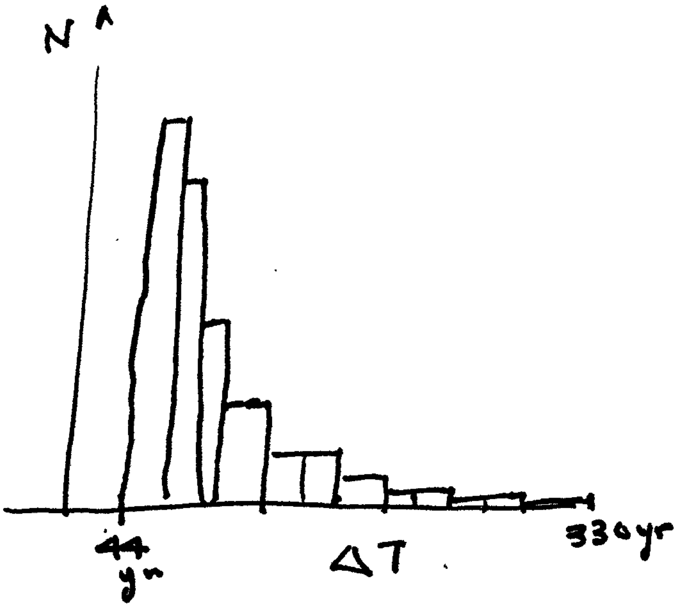


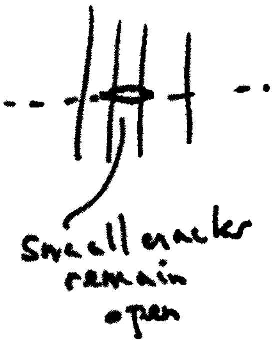


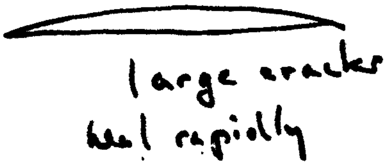


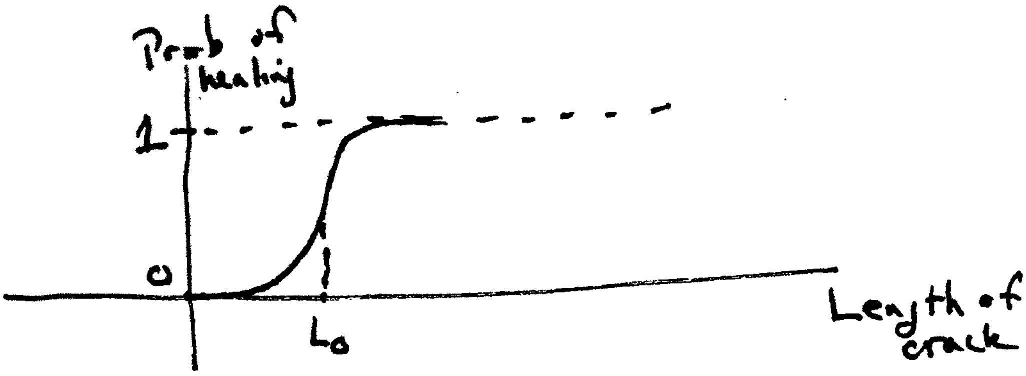
linear slip weakening





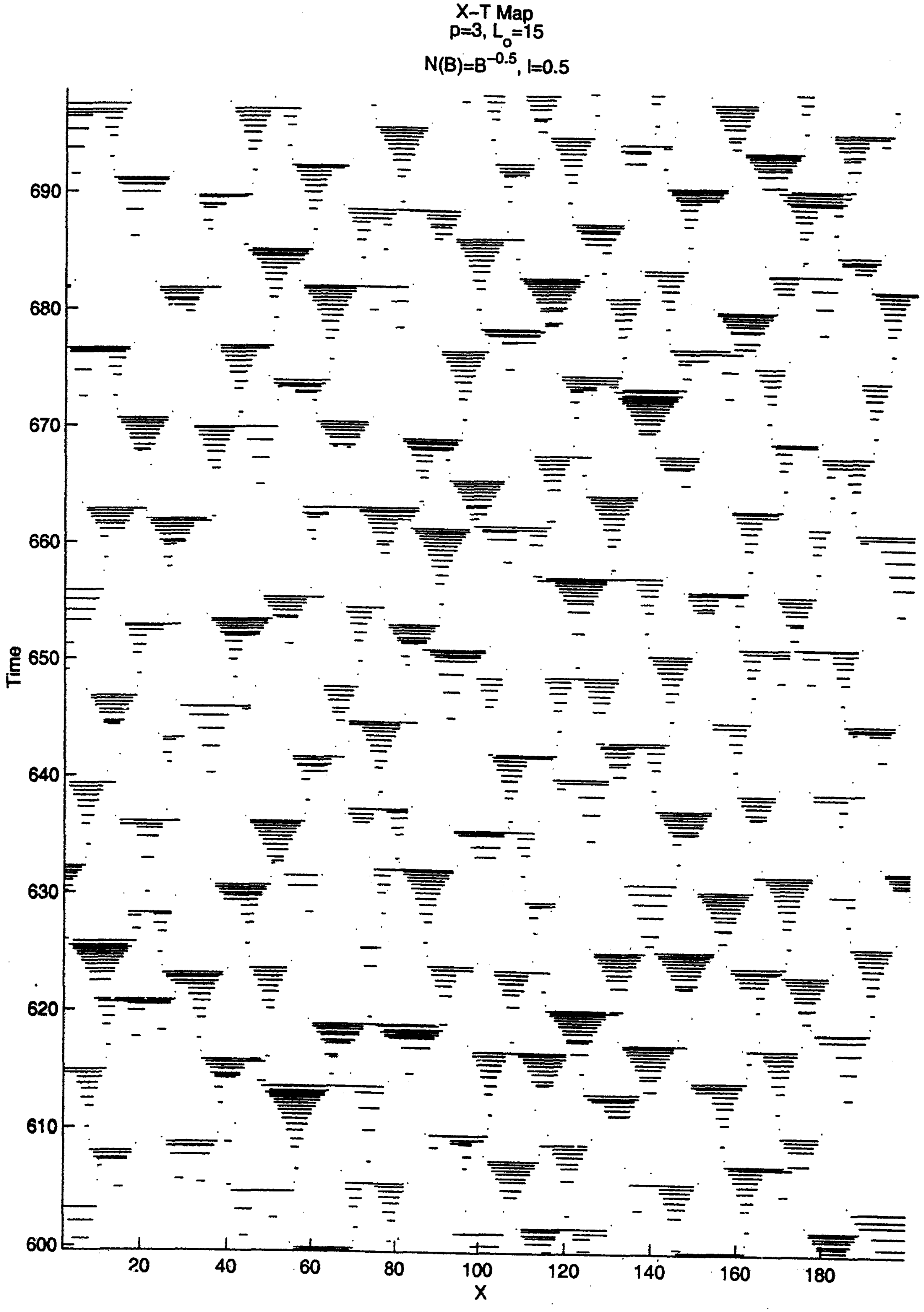




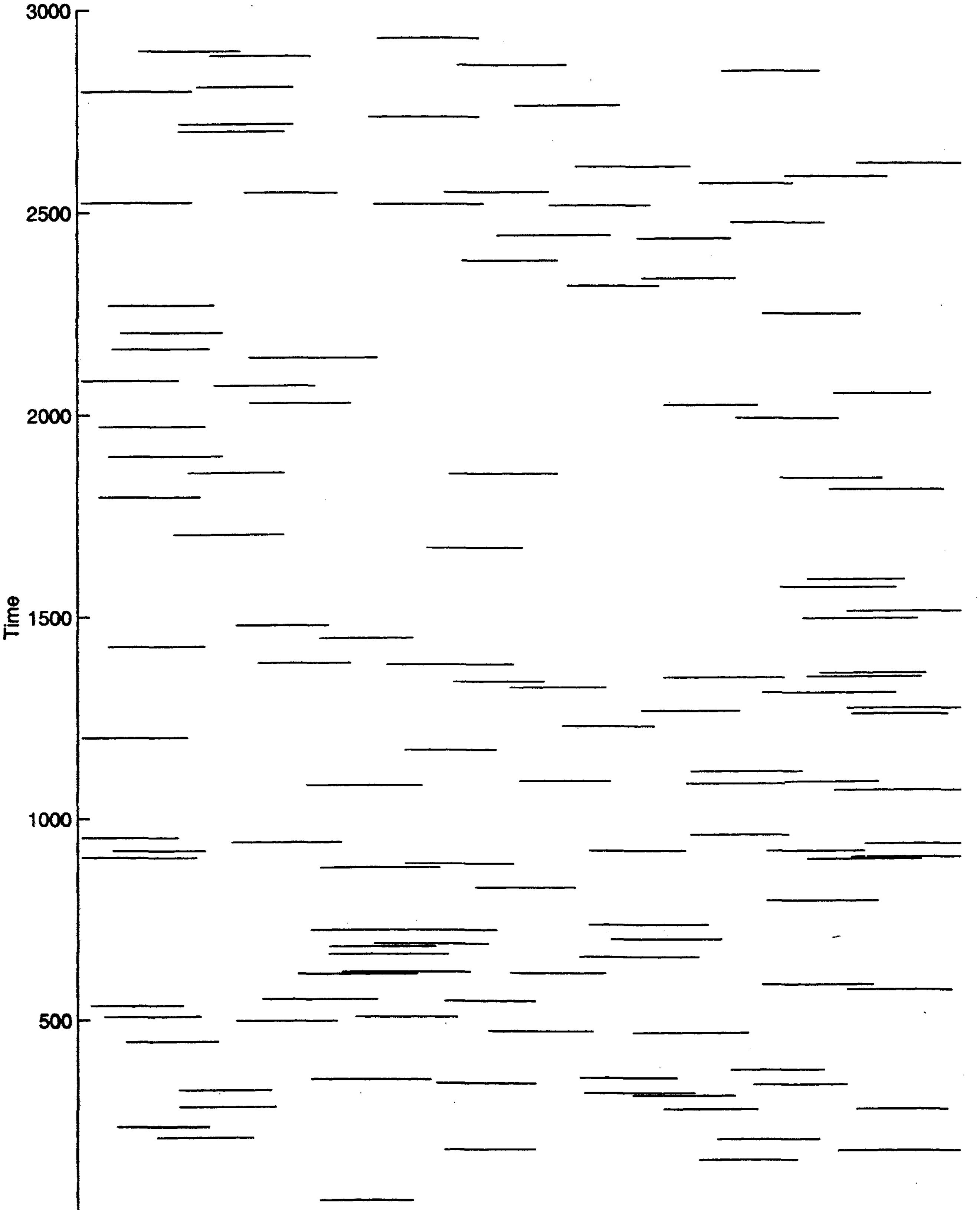


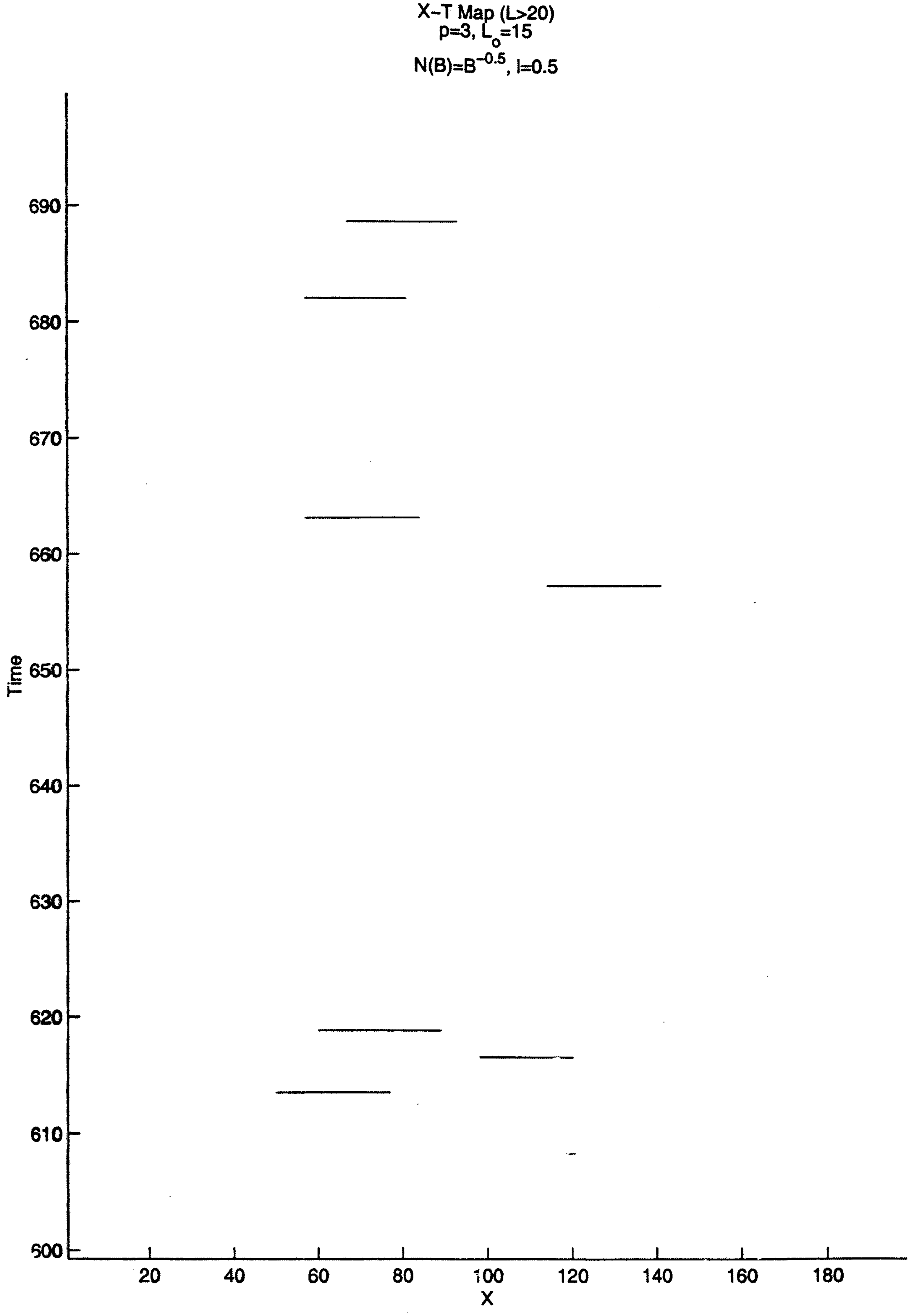
$$\text{Prob. healing} = \frac{2}{\pi}\tan^{-1}\frac{\beta L}{L_0}$$

$$\text{or } \tanh\frac{\beta L}{L_0}$$

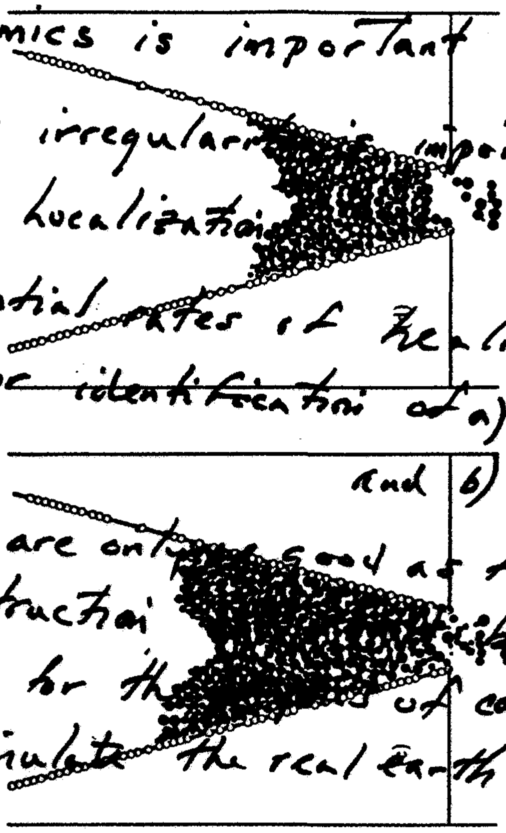

X-T Map
p=3, L_o=15
N(B)=$B^{-0.5}$, l=0.5
Time
690
680
670
660
650
640
630
620
610
600
20
40
60
80
100
120
140
160
180
X

X-T Map (L>20)
$p=3$, $L_o=15$
$N(B)=B^{-0.5}$, $l=0.5$



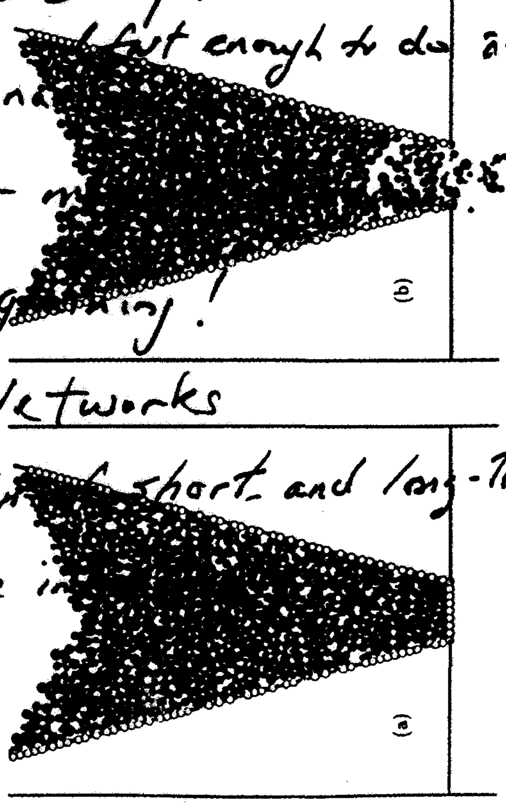

X-T Map (L>20)
p=3, L_o=15
N(B)=B^-0.5, l=0.5
Time
690
680
670
660
650
640
630
620
610
600
20
40
60
80
100
120
140
160
180
X

CONCLUSIONS

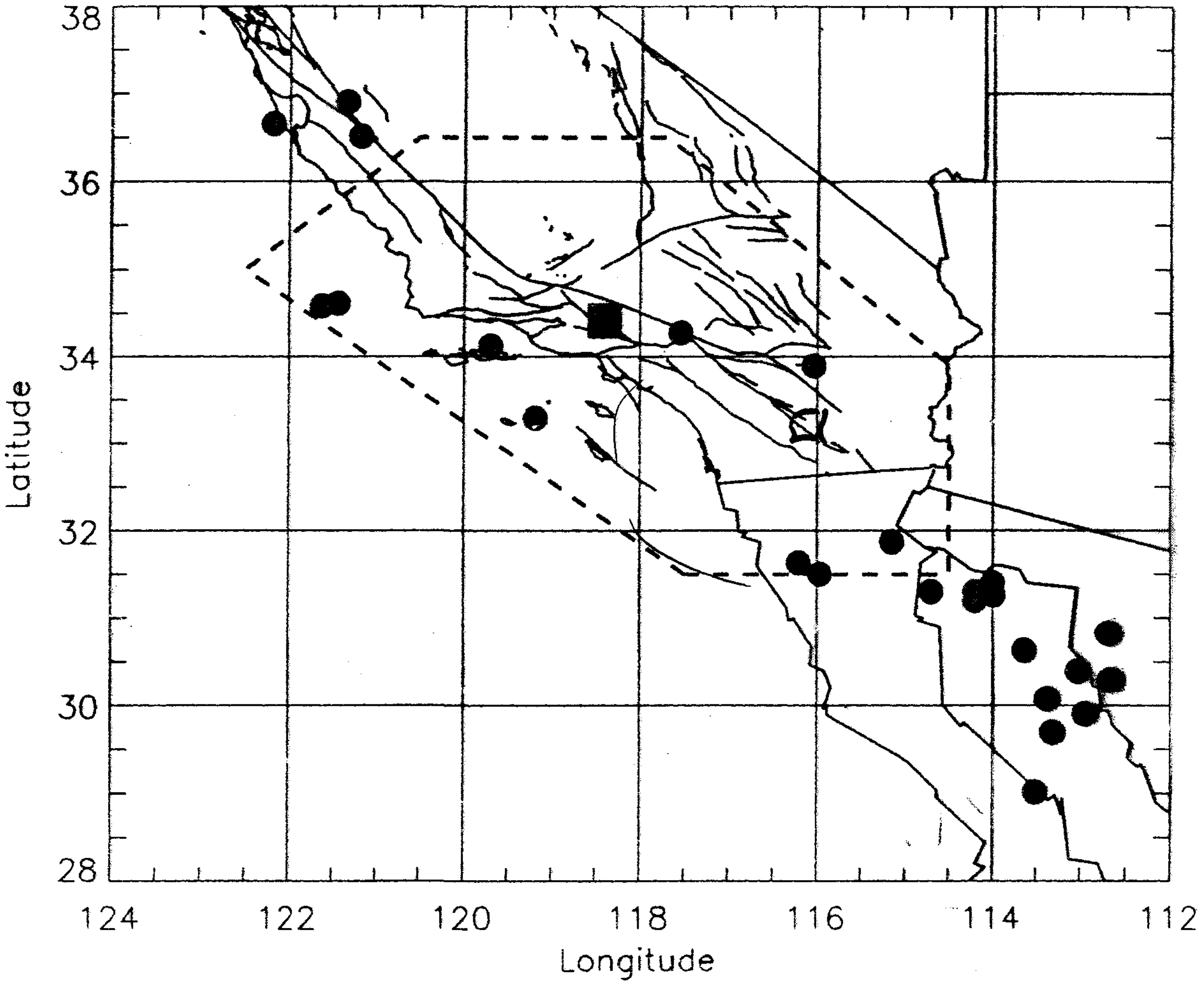


1. Dynamics is important
2. Fault irregularity is important and lead to localization
3. Differential rates of healing hold promise for identification of a) fluctuations in pattern and b) precursors.
4. Models are only as good as the assumptions in their construction [illegible] used as learning tools for the [illegible] of earthquakes. Their use to simulate the real earth is very premature.

Major problems for the future



1. Fast 3-D systems (2-D cracks in 3-D media) (fast enough to do extensive seismicity ⇒ statistical studies)
2. Non-plana [illegible]
3. Granular m [illegible]
4. Coarse g[illegible]ing!
5. Fault Networks
6. Interaction of short and long-term organization
7. Long-range in [illegible]



Seismicity $6.3 \geq M \geq 4.7$ in Southern California between Borrego Mountain and San Fernando earthquakes (4-9-68) to (2-9-71)

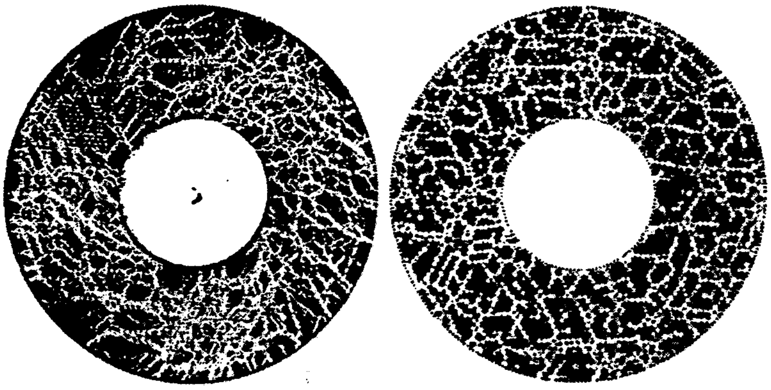
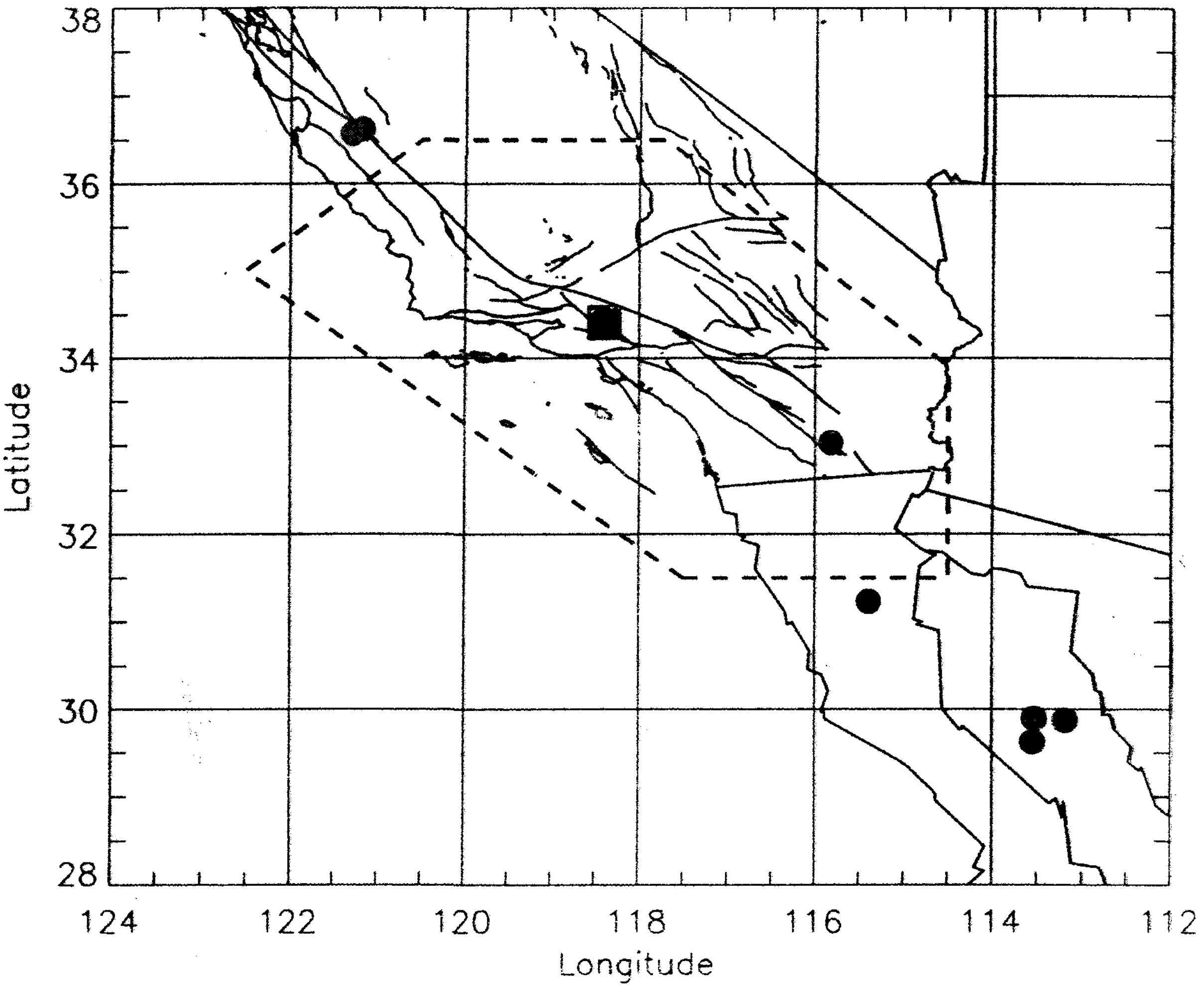


Figure 4. Pictures of the force-chains. (left) is an experiment showing the transparent light intensity. (right) is a simulation showing the potential energy stored in all contacts of one particle.



Seismicity $6.3 \geq M \geq 4.7$ in Southern California after San Fernando earthquake
(2-9-71) to (2-8-73)

PHYSICS OF THE EARTHQUAKE SOURCE: SUMMARY

- Inhomogeneity of geometry is an important determinant of seismicity
- Relaxation of stress at barriers must be understood
- Models of seismicity appear to be unstable with regard to many important ingredients
  - We have to get the geometry and the physics correct
  - What is the level of sensitivity that is needed so that the systems stabilize?
- It is doubtful that seismic time series are stationary, because of space-time interactions
  - We must consider the entire network of faults and not merely one linear structure
  - It is doubtful that the strain rates on individual faults are constants over time
- Dynamics and Dissipation are important ingredients in modeling seismicity
- Interaction on multiple time scales
- Fluid migration in faults plays an important part in the evolution of seismicity
- It is premature to construct the definitive model of seismicity of Southern California. Models that claim to give the "solution" to problems of seismicity are too simplistic, unstable, and unreliable.