Global Navigation Satellite Systems (GNSS) and the International GPS Service

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Introduction to GNSS: GPS / GLONASS / GALILEO

NAVSTAR-GPS: **NAV**igation **S**ystem with **T**ime **A**nd **R**anging –
Global **P**ositioning **S**ystem

GLONASS: **GLO**bal **NA**vigation **S**atellite **S**ystem
(Russian pendant)

GALILEO: future European global navigation satellite system

Characteristics:

- Satellite systems for (real-time) **Positioning** and **Navigation**
- **Global** (everywhere on Earth, up to altitude 5000 km) and **at any time**
- **Unlimited** number of users
- **Weather-independent** (radio signals are passing through the atmosphere)
- **3-dimensional position**, **velocity** and **time** information in **real time**

GPS/GLONASS Segments

The GPS (and the GLONASS) consists of **3 main segments**:

- **Space Segment**: the satellites and the constellation of satellites
- **Control Segment**: the ground stations, infrastructure and software for operation and monitoring of the GPS/GLONASS
- **User Segment**: all GPS/GLONASS receivers worldwide and the corresponding processing software

We should add an important **4th segment**:

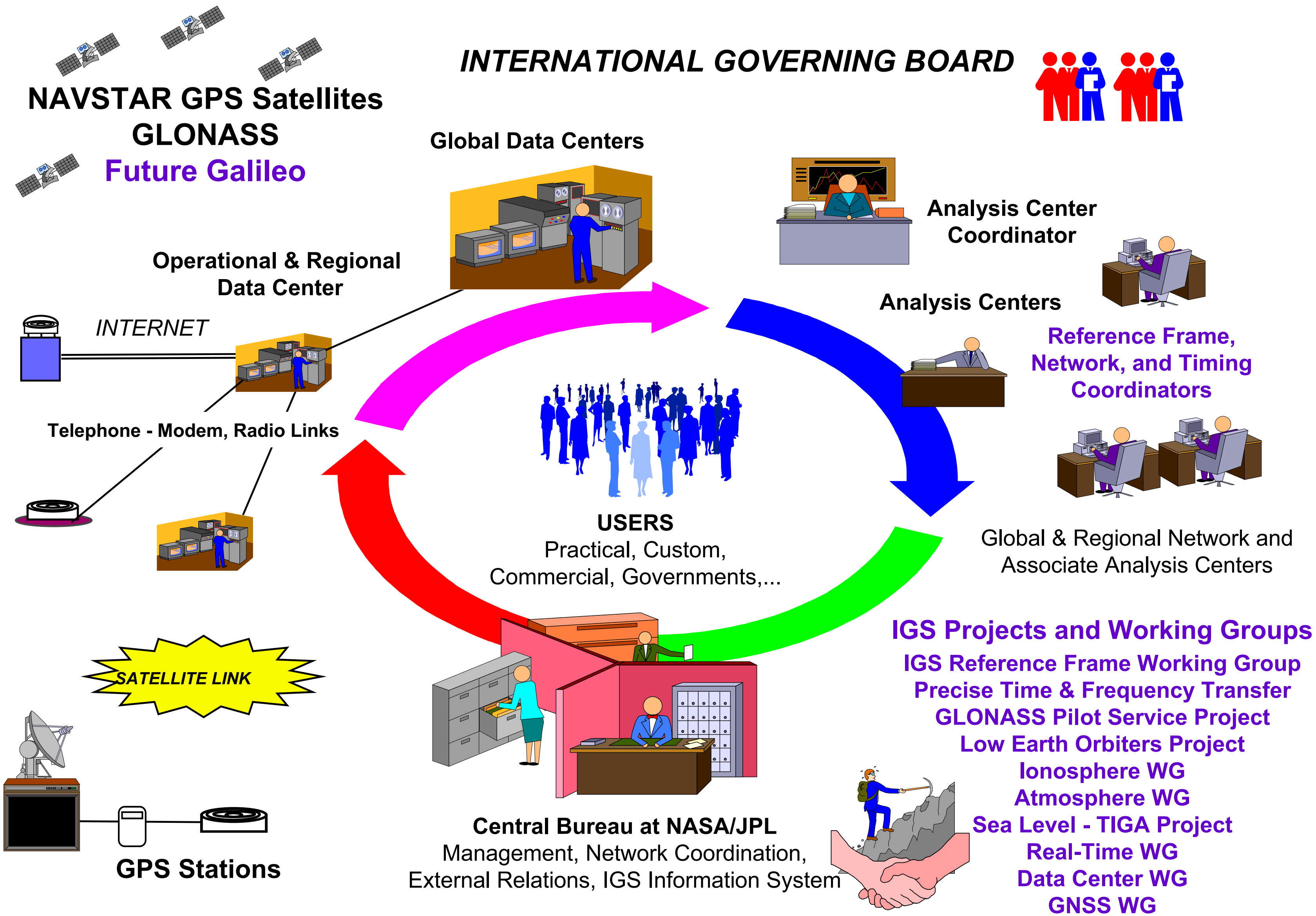
- **Ground Segment**:
  - **Global network** of the International GPS Service (IGS; $\sim$ 300 stations): reference frame (coordinates and velocities), orbits and clocks, Earth rotation, atmospheric products
  - **Regional** and **local** permanent networks (Europe, Japan, US, . . . , SAPOS, . . .): densification of the reference frame, positioning services

International GPS Service (IGS): Mission and Objectives

“. . . committed to providing the highest quality data and products as the standard for global navigation satellite systems (GNSS) in support of Earth science research, multidisciplinary applications, and education. . . . as well as to facilitate other applications benefiting society . . . .”

- Provide the highest quality, reliable GNSS data and products, openly and readily available to all user communities.
- Promote universal acceptance of IGS products and conventions as the world standard.
- Continuously innovate by attracting leading-edge expertise and pursuing challenging projects and ideas.
- Seek and implement new growth opportunities while responding to changing user needs.
- Sustain and nurture the IGS culture of collegiality, openness, inclusiveness, and cooperation.
- Maintain a voluntary organization with effective leadership, governance, and management.

International GPS Service (IGS): Structure

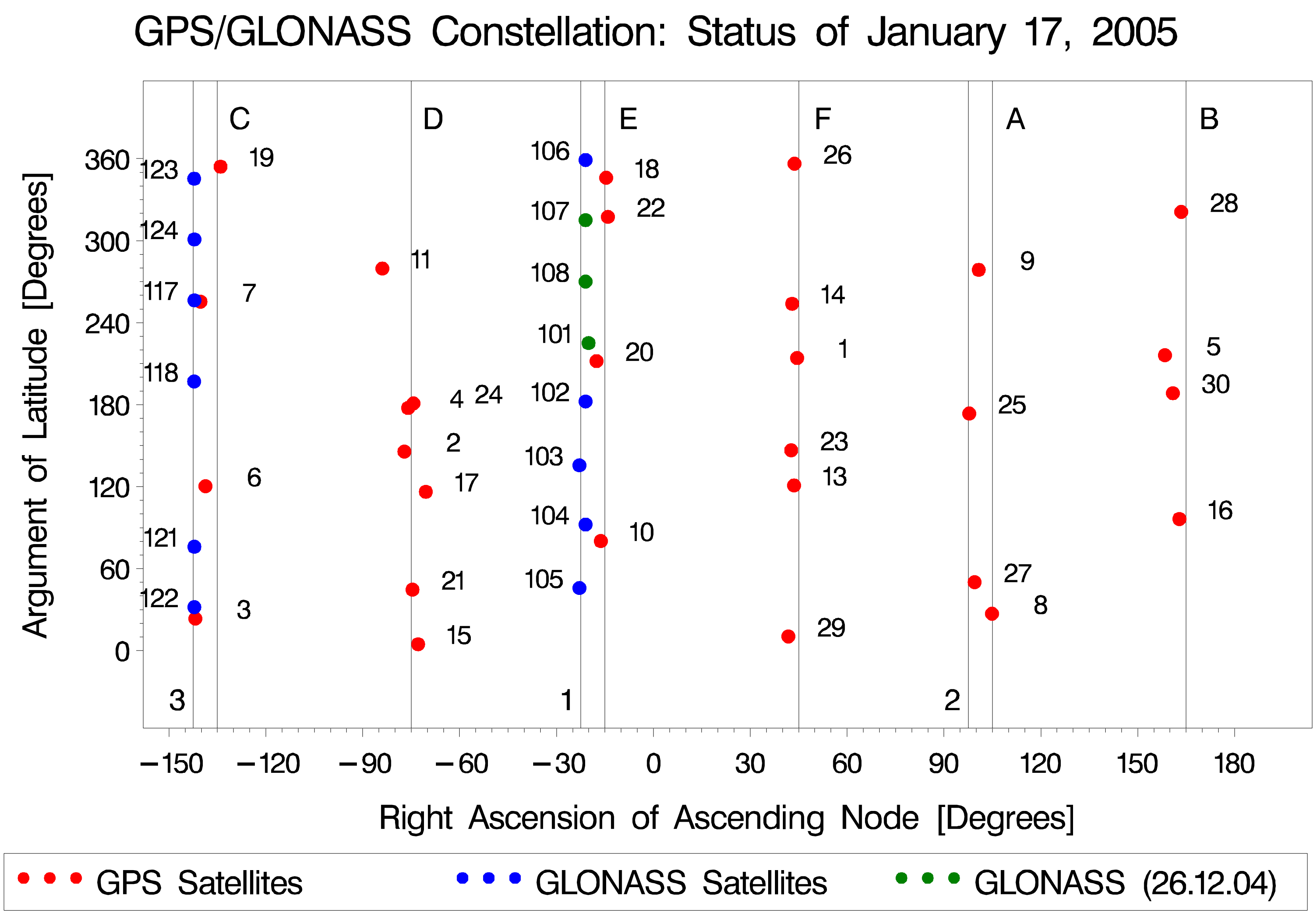


Orbit Characteristics

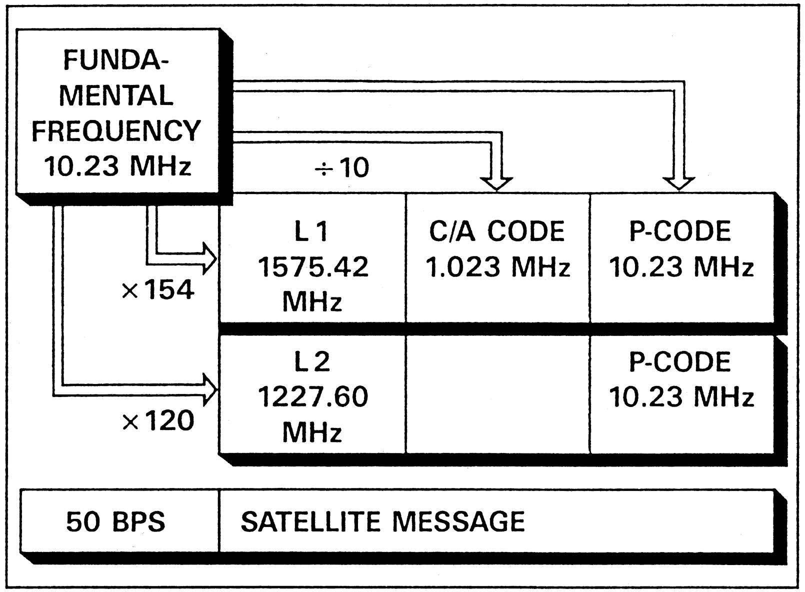
Characteristic	GPS	GLONASS	GALILEO
Semi-Major Axis (Radius)	26'600 km	25'500 km	29'994 km
Orbital Period	11 h 58 min	11 h 16 min	14 h 22 min
Orbit Inclination	55^o	65^o	56^o
Number of Planes	6 (60^o sep.)	3 (120^o sep.)	3 (120^o sep.)
Number of Satellites	29 (nom. 24)	14 (nom. 24)	0 (nom. 30)
PRN Codes	Satellite-dep.	Satellite-indep.	Satellite-dep.
Frequencies	Satellite-indep.	Satellite-dep.	Satellite-indep.
Selective Availability	Yes	No	No
Anti-Spoofing, Encription	Yes	No	Partially

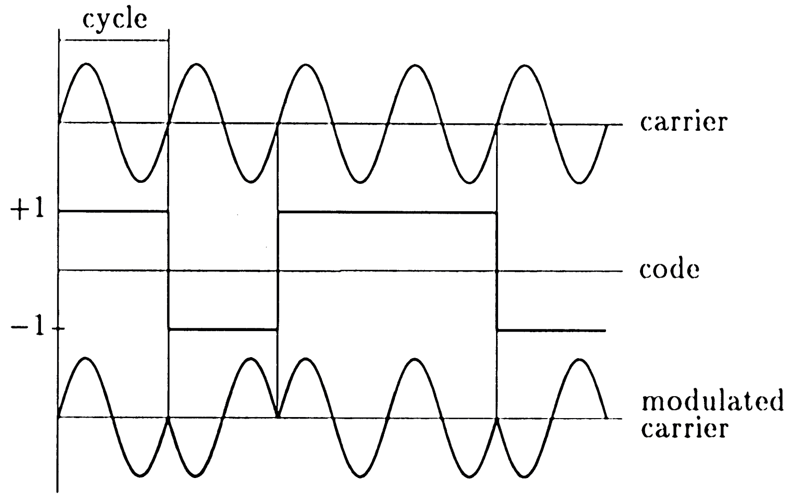
PRN Codes: Pseudo-Random Noise Codes.

GPS/GLONASS Satellite Constellation



GPS Signals





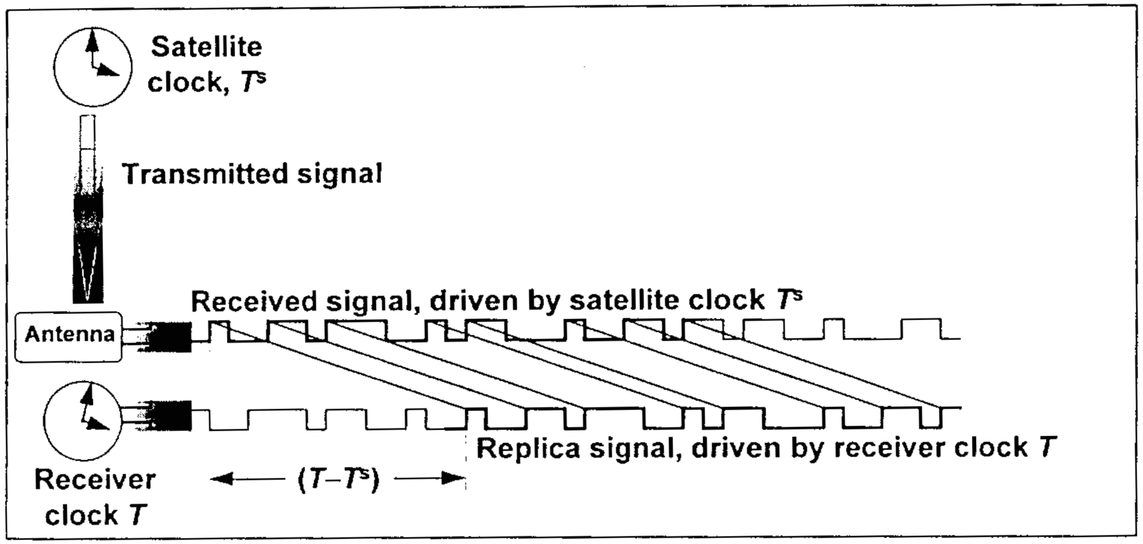
- Signals driven by an **atomic clock**
- Two **carrier signals** (sine waves):
  - L_1: f=1575.43 MHz, λ=19 cm
  - L_2: f=1227.60 MHz, λ=24 cm

- **Bits encoded** on carrier by phase modulation:
  - **C/A-code** (Clear Access / Coarse Acquisition)
  - **P-code** (Protected / Precise)
  - **Satellite/Broadcast/Navigation Message** (Information)

Pseudorange/Code Measurement

- Actual **Pseudorange observation** P_r^s:

$$P_r^s = c\,(T_r - T^s)$$



(Blewitt, 1996)

- c : speed of light (in vacuum)
- No actual **"range"** (distance) because of clock errors

- Clock of receiver r reads T_r when signal is received (T_r in receiver clock time).
- Clock of satellite s reads T^s when signal is emitted (T^s in satellite clock time).
- **Measurement noise**: C/A-code $\sim$ **10 m**; P-code $\sim$ **1 m**

Pseudorange/Code Measurement (2)

$$\begin{aligned} P_r^s &= c\,(T_r - T^s) \\ &= c\,(t_r + \delta t_r - t^s - \delta t^s) \\ &= c\,(t_r - t^s) + c\,\delta t_r - c\,\delta t^s \\ &= \rho_r^s + c\,\delta t_r - c\,\delta t^s \end{aligned}$$

$t_r,\ t^s$	GPS time of reception and emission
$\delta t_r,\ \delta t^s$	Receiver and satellite clock error
ρ_r^s	Range (distance) between receiver and satellite

Simplified model for $\boldsymbol{\rho_r^s}$: atmospheric delay missing, exactly 4 satellites, etc.

$$P_r^{s1} = \sqrt{(x^{s1} - x_r)^2 + (y^{s1} - y_r)^2 + (z^{s1} - z_r)^2} + c\,\delta t_r - c\,\delta t^{s1} \qquad (1)$$

$$P_r^{s2} = \sqrt{(x^{s2} - x_r)^2 + (y^{s2} - y_r)^2 + (z^{s2} - z_r)^2} + c\,\delta t_r - c\,\delta t^{s2} \qquad (2)$$

$$P_r^{s3} = \sqrt{(x^{s3} - x_r)^2 + (y^{s3} - y_r)^2 + (z^{s3} - z_r)^2} + c\,\delta t_r - c\,\delta t^{s3} \qquad (3)$$

$$P_r^{s4} = \sqrt{(x^{s4} - x_r)^2 + (y^{s4} - y_r)^2 + (z^{s4} - z_r)^2} + c\,\delta t_r - c\,\delta t^{s4} \qquad (4)$$

Basic Positioning and Navigation Concept

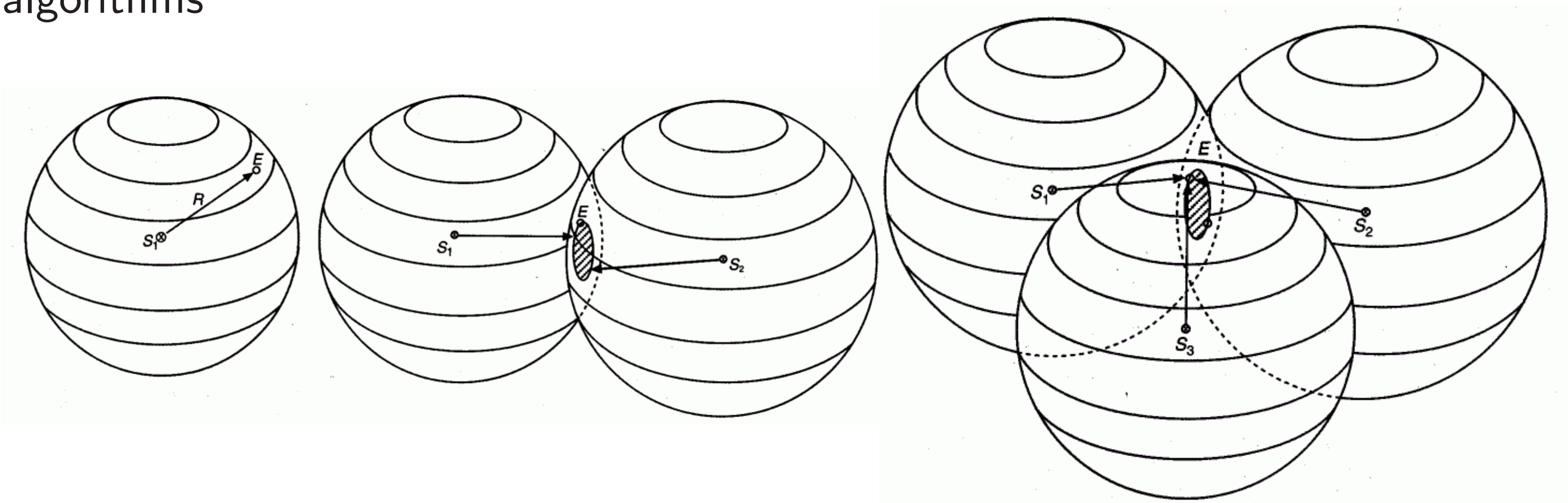
Known from the navigation message:

- satellite position $(x^{s_i},\, y^{s_i},\, z^{s_i})$
- satellite clock error δt^{s_i}

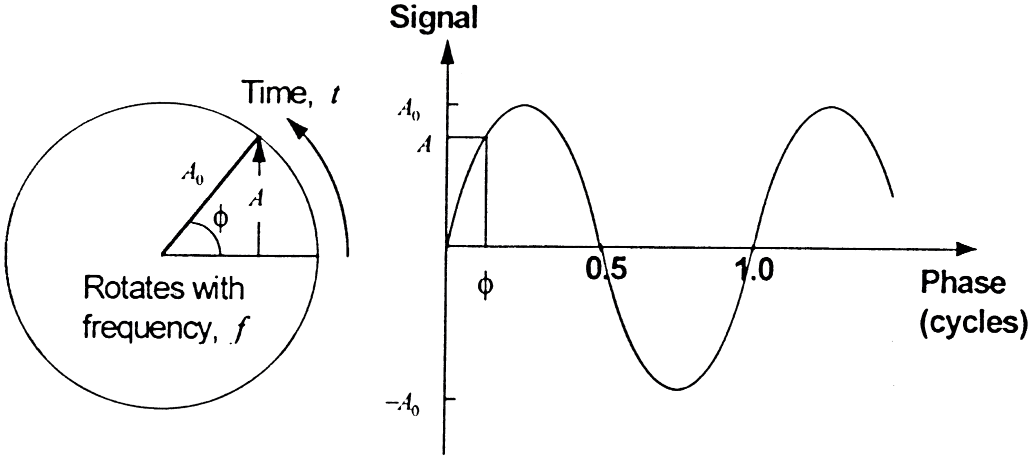
4 unknown parameters:

- receiver position $(x_r,\, y_r,\, z_r)$
- receiver clock error δt_r

More than 4 satellites: best position and clock with **least squares** or **filter** algorithms



Carrier Phase Measurement



Phase Φ (in cycles) increases linearly with time t:

$$\Phi = f \cdot t$$

where f is the frequency.

The **satellite** generates with its clock the phase signal Φ^s. At emission time T^s (in satellite time) we have:

$$\Phi^s = f \cdot T^s$$

The same phase signal (e.g. a wave crest) propagates from the satellite to the receiver, but the receiver measures only the fractional part of the phase and doesn't know the **integer number of cycles** N_r^s (phase ambiguity):

$$\Phi_r^s = \Phi^s - N_r^s = f \cdot T^s - N_r^s$$

Carrier Phase Measurement (2)

The **receiver** generates with its clock a **reference phase**. At time of reception T_r of the satellite phase Φ_r^s (in receiver time) we have:

$$\Phi_r = f \cdot T_r$$

The actual **phase measurement** is the difference between receiver reference phase Φ_r and satellite phase Φ_r^s:

$$\psi_r^s = \Phi_r - \Phi_r^s = f \cdot T_r - (f \cdot T^s - N_r^s) = f\,(T_r - T^s) + N_r^s$$

Multiplication with the wavelength $\lambda = c/f$ leads to the **phase observation equation** in meters:

$$\begin{aligned} L_r^s &= \lambda\,\psi_r^s = c\,(T_r - T^s) + \lambda\,N_r^s \\ &= \rho_r^s + c\,\delta t_r - c\,\delta t^s + \lambda\,N_r^s \end{aligned}$$

Difference to the pseudorange observation: **integer ambiguity term** N_r^s.

If the receiver looses the GPS signal (loss of lock), the continuous counting of the arriving wave cycles is interrupted: **jump** of an **integer** number of cycles in the phase (**cycle slip**).

Improved Observation Equation and Parameters

$$
\begin{aligned}
L_r^s \quad = \quad & \rho_r^s + c\,\delta t_r + c\,\delta t_{r,sys} - c\,\delta t^s - c\,\delta t^s_{sys} + \delta\rho_{trp} + \delta\rho_{ion} \\
& + \delta\rho_{rel} + \delta\rho_{mul} + \lambda\, N_r^s + \ldots + \epsilon
\end{aligned}
$$

ρ_r^s	Geometrical distance between satellite and receiver
δt_r	Station clock correction: *receiver clocks* (time and frequency transfer)
$\delta t_{r,sys}$	Delays in receiver and antenna (cables, electronics, . . .)
δt^s	Satellite clock correction: *satellite clocks*
$\delta t_{s,sys}$	Delays in satellite (cables, electronics, . . .)
$\delta\rho_{trp}$	Tropospheric delay: *troposphere parameters* (meteorology, climatology)
$\delta\rho_{ion}$	Ionospheric delay: *ionosphere parameters* (atmosphere physics)
$\delta\rho_{rel}$	Relativistic corrections
$\delta\rho_{mul}$	Multipath, scattering, bending effects
N_r^s	Phase ambiguity: *ambiguity parameters* (ambiguity resolution)
ϵ	Measurement error

Differences between Code and Phase Observation Equation

Phase:

$$\begin{aligned} L_r^s &= \rho_r^s + c\,\delta t_r + c\,\delta t_{r,sys} - c\,\delta t^s - c\,\delta t^s_{sys} + \delta\rho_{trp} + \delta\rho_{ion} \\ &\quad + \delta\rho_{rel} + \delta\rho_{mul} + \lambda\,N_r^s + \ldots + \epsilon \end{aligned}$$

Pseudorange/Code:

$$\begin{aligned} P_r^s &= \rho_r^s + c\,\delta t_r + c\,\delta t_{r,sys} - c\,\delta t^s - c\,\delta t^s_{sys} + \delta\rho_{trp} - \delta\rho_{ion} \\ &\quad + \delta\rho_{rel} + \delta\rho_{mul} + \ldots + \epsilon \end{aligned}$$

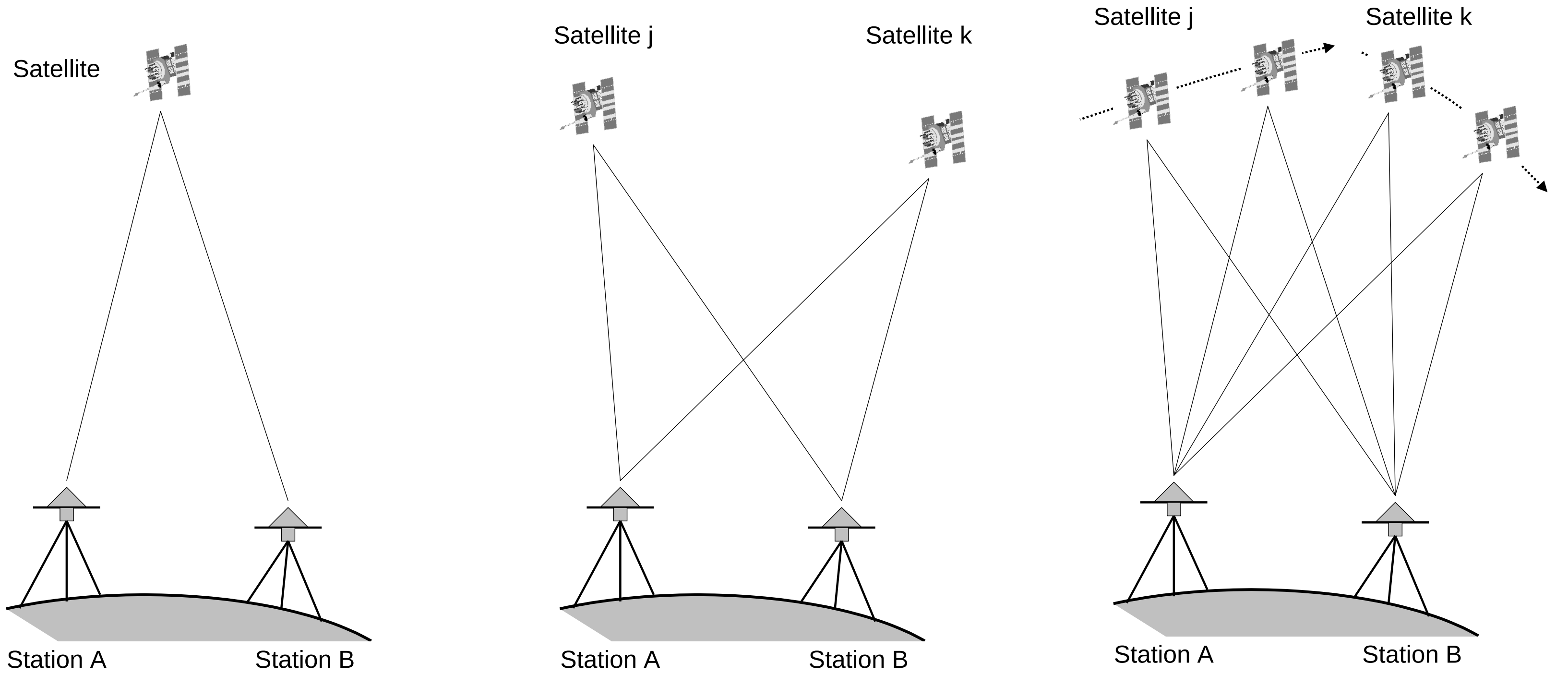
- The ionospheric refraction correction $\delta\rho_{ion}$ has the opposite sign for code measurements.
- There is no **ambiguity term $\lambda\,N_r^s$ for code measurements.**
- **In the case of code measurements the so-called** *differential code biases* **(DCBs) are part of the system delays $\delta t_{r,sys}$ and δt^s_{sys}, i.e., $\mathbf{P}_1 - \mathbf{P}_2$ or $\mathbf{C}_1 - \mathbf{P}_2$ biases for satellites and receivers.**

Geometrical Distance ρ_r^s

$$\rho_r^s \quad = \quad |\boldsymbol{r}_i^s(t^s) - \boldsymbol{r}_{r,i}(t_r)| = |\boldsymbol{r}_i^s(t^r - \tau_r^s) - \boldsymbol{R}(t_r) \cdot \boldsymbol{r}_{r,e}(t_r)|$$

$\boldsymbol{r}_i^s(t^s)$	Satellite position at emission time $t^s = t_r - \tau_r^s$ in **inertial** system (e.g. J2000.0)
$\tau_r^s = \rho_r^s / c$	Light travel time
$\boldsymbol{r}_{r,i}(t_r)$	Receiver position at reception time t_r in **inertial** system
$\boldsymbol{r}_{r,e}(t_r)$	Receiver position at time t_r in **Earth-fixed** system (e.g. ITRF) with $\boldsymbol{r}_{r,i}(t_r) = \boldsymbol{R}(t_r) \cdot \boldsymbol{r}_{r,e}(t_r)$
$\boldsymbol{R}(t_r)$	Transformation from **Earth-fixed** to **inertial** system with $\boldsymbol{R} = \boldsymbol{PNUXY}$, where these rotation matrices contain the *Earth rotation parameters* (IERS, geophysics, . . .): Pole coordinates x_p and y_p in $\boldsymbol{X}$ and $\boldsymbol{Y}$ (polar motion), UT1-UTC in the sidereal rotation matrix $\boldsymbol{U}$, nutation in obliquity $\Delta\epsilon$ and longitude $\Delta\psi$ in $\boldsymbol{N}$

Single, Double, and Triple Differences



- **Single** differences
- Between 2 **sites**
- Elimination: **satellite clock** errors

- **Double** differences
- Between 2 **satellites**
- Elimination: **receiver clock** errors

- **Triple** differences
- Between 2 **epochs**
- Elimination: **ambiguities**

Absolute and Differential/Relative Positioning

- When forming single or double differences we switch from **absolute, geocentric positioning** to **relative positioning** of a receiver relative to a **reference receiver** (baseline).
- In single or double differences there is almost **no information** available on the **absolute position** of the receivers (for short baseline vectors $\Delta \boldsymbol{r}_{r_1,r_2}$):

$$\begin{aligned}\Delta\rho^j_{r_1,r_2} = \rho^j_{r_1} - \rho^j_{r_2} &\approx \rho^j_{r_1} - \{\rho^j_{r_1} + \Delta\boldsymbol{r}_{r_1,r_2}\boldsymbol{\cdot}\,\boldsymbol{e}^j_{r_1} + O(\Delta\boldsymbol{r}^2_{r_1,r_2})\} \\ &\approx -\Delta\boldsymbol{r}_{r_1,r_2}\boldsymbol{\cdot}\,\boldsymbol{e}^j_{r_1} + O(\Delta\boldsymbol{r}^2_{r_1,r_2})\end{aligned}$$

- For **short baselines** various **error sources** are considerably **reduced** when using relative or differential positioning:
  - **Satellite clocks** (selective availability (SA)!)
  - **Orbit** errors
  - **Atmosphere**: tropospheric and ionospheric refraction errors
  - **Antenna phase center** offsets and variations (if same antenna type used)
  - **Motion of the sites**: solid Earth tides, ocean and atmospheric loading

Linear Combinations of GPS Measurements

Very simplified observation equations for the **4 measurement types** in GPS:

$$
\begin{aligned}
L_1 &= \rho' + I_1 + \lambda_1 N_1 \\
L_2 &= \rho' + \frac{f_1^2}{f_2^2} I_1 + \lambda_2 N_2 \\
P_1 &= \rho' - I_1 \\
P_2 &= \rho' - \frac{f_1^2}{f_2^2} I_1
\end{aligned}
$$

with the distance ρ' (incl. clocks, troposphere, relativity) and the ionospheric refraction I_1 for L_1 frequency.

General linear combination (LC) of L_1 and L_2 (or P_1 and P_2):

$$
\begin{aligned}
L_x &= \kappa_{1,x} L_1 + \kappa_{2,x} L_2 \\
P_x &= \kappa_{1,x} P_1 + \kappa_{2,x} P_2
\end{aligned}
$$

Linear Combinations of GPS Measurements (2)

Substituting the right hand sides and setting $\kappa_{1,x} + \kappa_{2,x} = 1$ (conservation of length) we get:

$$L_x = \rho' + (\kappa_{1,x} + \kappa_{2,x}\,\frac{f_1^2}{f_2^2})\,I_1 + (\kappa_{1,x}\lambda_1 N_1 + \kappa_{2,x}\lambda_2 N_2)$$

The ionospheric delay changed by the factor $\alpha_I = \kappa_{1,x} + \kappa_{2,x}\,\frac{f_1^2}{f_2^2}$ compared to L_1.

The ambiguity term $b_x = \kappa_{1,x}\lambda_1 N_1 + \kappa_{2,x}\lambda_2 N_2$, in general, is not integer anymore.

Assuming a similar noise $\sigma(L_1) \approx \sigma(L_2)$ for L_1 and L_2 the noise of the LC L_x is given by:

$$\sigma(L_x) = \sqrt{\kappa_{1,x}^2\,\sigma(L_1)^2 + \kappa_{2,x}^2\,\sigma(L_2)^2} \approx \sqrt{\kappa_{1,x}^2 + \kappa_{2,x}^2}\;\sigma(L_1)$$

Geometry-Free Linear Combination L_I

$$
\begin{aligned}
L_I &= L_1 - L_2 \\
&= (\rho' + I_1 + \lambda_1 N_1) - (\rho' + \frac{f_1^2}{f_2^2} I_1 + \lambda_2 N_2) \\
&= (1 - \frac{f_1^2}{f_2^2})\, I_1 + (\lambda_1 N_1 - \lambda_2 N_2)
\end{aligned}
$$

$$
\begin{aligned}
P_I &= P_1 - P_2 \\
&= -(1 - \frac{f_1^2}{f_2^2})\, I_1
\end{aligned}
$$

All geometry information ρ' (GPS orbits, station coordinates, clocks, tropospheric refraction) is eliminated. Only ionospheric refraction remains (and a non-integer ambiguity term for phase).

Applications: information about the **ionosphere** ; estimation of regional or global **ionosphere maps**; **cycle slip fixing**.

Ionosphere-Free Linear Combination L_c

The coefficients of the ionosphere-free LC are:

$$\kappa_{1,c} = \frac{f_1^2}{f_1^2 - f_2^2}, \quad \kappa_{2,c} = -\frac{f_2^2}{f_1^2 - f_2^2}$$

and we get:

$$\begin{aligned} L_c &= \kappa_{1,c} L_1 + \kappa_{2,c} L_2 \\ &= \kappa_{1,c}(\rho' + I_1 + \lambda_1 N_1) + \kappa_{2,c}(\rho' + \frac{f_1^2}{f_2^2} I_1 + \lambda_2 N_2) \\ &= \rho' + (\kappa_{1,c}\lambda_1 N_1 + \kappa_{2,c}\lambda_2 N_2) \end{aligned}$$

$$\begin{aligned} P_c &= \kappa_{1,c} P_1 + \kappa_{2,c} P_2 \\ &= \rho' \end{aligned}$$

Application: elimination of ionospheric refraction; **most important observable for baselines longer than ca. 10 km**; disadvantage: noise $\sigma(L_c) \approx 3\,\sigma(L_1)$.

Widelane LC L_w and Melbourne-Wübbena LC MW

$$
\begin{aligned}
L_w &= \frac{f_1}{f_1 - f_2} L_1 - \frac{f_2}{f_1 - f_2} L_2 \\
&= \rho' - \frac{f_1}{f_2} I_1 + \frac{c}{f_1 - f_2} (N_1 - N_2)
\end{aligned}
$$

$$
\begin{aligned}
P_w &= \frac{f_1}{f_1 + f_2} P_1 + \frac{f_2}{f_1 + f_2} P_2 \\
&= \rho' - \frac{f_1}{f_2} I_1
\end{aligned}
$$

where

$$\lambda_w = c/f_w = c/(f_1 - f_2) = 86\, cm \quad ; \quad f_w = f_1 - f_2$$

Application: **ambiguity resolution** of **widelane ambiguity** $N_w = N_1 - N_2$; **cycle slip fixing**; advantages: integer ambiguities N_w and long wavelength λ_w.

Melbourne-Wübbena LC (phase and code):

$$MW = L_w - P_w = \lambda_w (N_1 - N_2)$$

Overview of Important LCs of Phase Measurements

LC	λ	$\kappa_{1,x}$	$\kappa_{2,x}$	Geometry		Ionosphere		Noise	
	(m)	(m/m)	(m/m)	(m)	(cycles)	(m)	(cycles)	(m)	(cycles)
L_1	0.190	$+1.000$	0.000	1.000	1.000	$+1.000$	$+1.000$	1.000	1.000
L_2	0.244	0.000	$+1.000$	1.000	0.779	$+1.647$	$+1.283$	1.000	0.779
L_c	0.107	$+2.546$	-1.546	1.000	1.779	0.000	0.000	2.978	5.299
L_I	0.054	$+1.000$	-1.000	0.000	0.000	-0.647	-2.283	1.414	4.991
L_w	0.862	$+4.529$	-3.529	1.000	0.221	-1.283	-0.283	5.742	1.268

Ambiguity Resolution

- **Goal**: resolve phase double difference ambiguities N_{AB}^{jk} to the correct integer numbers (ambiguity fixing).
- Ambiguity resolution only possible on the **double difference level** because of the instrumental biases $\delta t_{r,sys}$ and δt_{sys}^{s}.

Gain in accuracy:

- **Short** observation times (few minutes): **Huge** gain in accuracy, typically an improvement from **1 m** (free) to **1 cm** (fixed).
- **Long** sessions (e.g. 1 day): Gain in accuracy much smaller, typically a **factor of 2** in east–west component of site coordinates.

Resolution strategies:

- Short session (short baselines): **search algorithms** searching for best set of integer ambiguities.
- Long session: resolve **widelane ambiguities** $N_w = N_1 - N_2$ first, then resolve N_1 (**narrow lane**) with the ionosphere-free linear combination.

Geometrical Distance ρ_r^s

$$\rho_r^s \quad = \quad |\boldsymbol{r}_i^s(t^s) - \boldsymbol{r}_{r,i}(t_r)| = |\boldsymbol{r}_i^s(t^r - \tau_r^s) - \boldsymbol{R}(t_r) \cdot \boldsymbol{r}_{r,e}(t_r)|$$

$\boldsymbol{r}_i^s(t^s)$ — Satellite position at emission time $t^s = t_r - \tau_r^s$ in **inertial** system (e.g. J2000.0)

$\tau_r^s = \rho_r^s / c$ — Light travel time

$\boldsymbol{r}_{r,i}(t_r)$ — Receiver position at reception time t_r in **inertial** system

$\boldsymbol{r}_{r,e}(t_r)$ — Receiver position at time t_r in **Earth-fixed** system (e.g. ITRF) with

$$\boldsymbol{r}_{r,i}(t_r) = \boldsymbol{R}(t_r) \cdot \boldsymbol{r}_{r,e}(t_r)$$

$\boldsymbol{R}(t_r)$ — Transformation from **Earth-fixed** to **inertial** system with $\boldsymbol{R} = \boldsymbol{PNUXY}$, where these rotation matrices contain the *Earth rotation parameters* (IERS, geophysics, . . .):

Pole coordinates x_p and y_p in $\boldsymbol{X}$ and $\boldsymbol{Y}$ (polar motion),
UT1-UTC in the sidereal rotation matrix $\boldsymbol{U}$,
nutation in obliquity $\Delta\epsilon$ and longitude $\Delta\psi$ in $\boldsymbol{N}$

Station Position and Modeling of Station Motion

Station position $\boldsymbol{r}_{r,e}(t_r)$ (in Earth-fixed frame) is given by:

$$\boldsymbol{r}_{r,e}(t_r) = \boldsymbol{r}_{r,0} + \boldsymbol{v}_r\,(t_r - t_0) + \delta\boldsymbol{r}_{r,sol} + \delta\boldsymbol{r}_{r,pol} + \delta\boldsymbol{r}_{r,ocn} + \delta\boldsymbol{r}_{r,atm} + \delta\boldsymbol{r}_{r,ant}$$

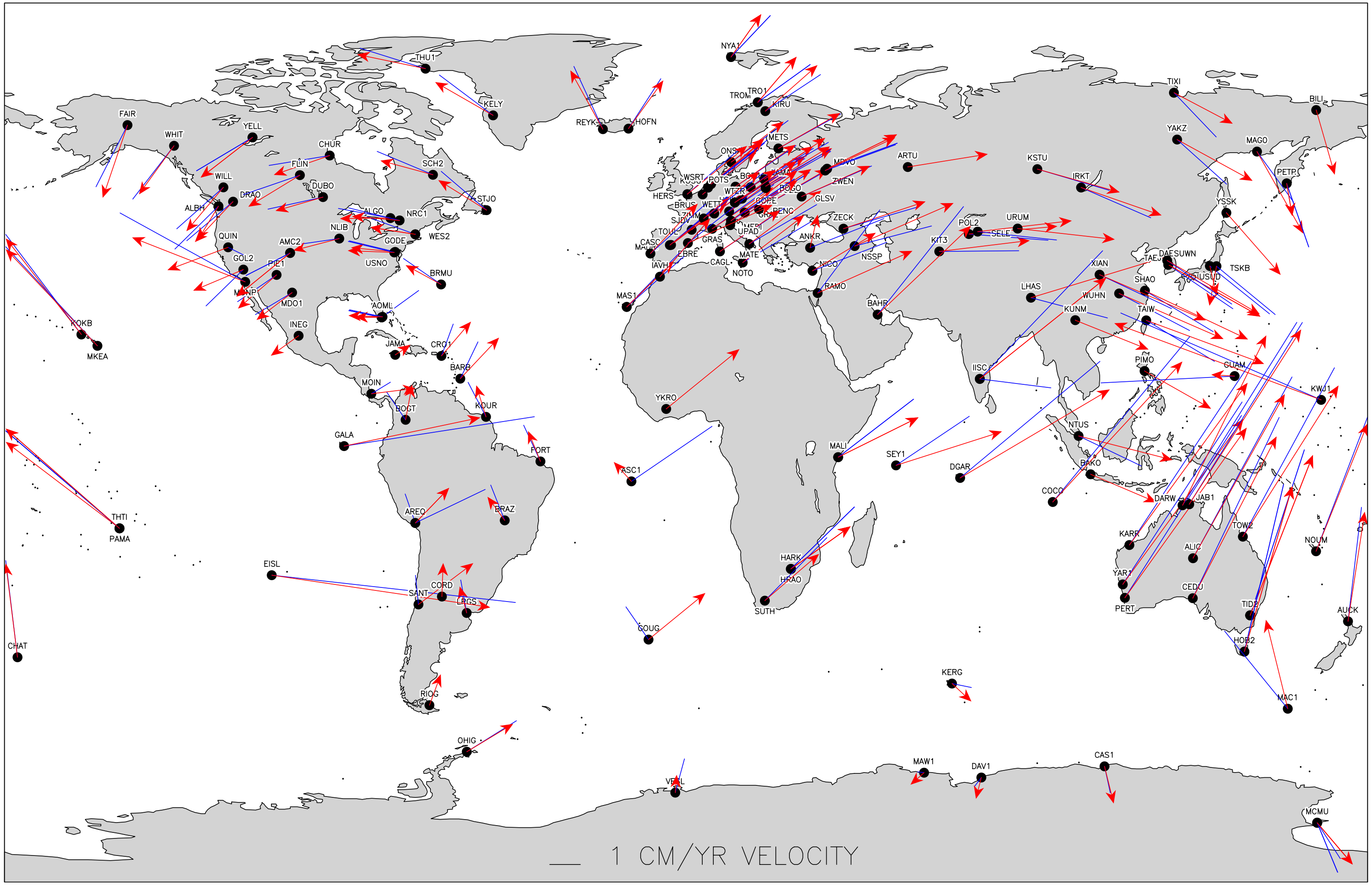
$\delta\boldsymbol{r}_{r,sol}$, $\delta\boldsymbol{r}_{r,pol}$	Solid Earth and pole tides
$\delta\boldsymbol{r}_{r,ocn}$, $\delta\boldsymbol{r}_{r,atm}$	Ocean and atmosphere loading
$\delta\boldsymbol{r}_{r,ant}$	Antenna phase center offset and variation
$\boldsymbol{r}_{r,0}$, $\boldsymbol{v}_r$	*Station coordinates and velocities* (positioning, . . .)

- For a high-accuracy GPS processing (e.g. for **troposphere delay estimation**) all these terms have to be modeled on the **sub-centimeter level**.
- The motion of a plate is modeled as a **rotation** of a rigid plate with the angular velocity $\boldsymbol{\omega}$ according to:

$$\begin{aligned} \boldsymbol{r}_{r,e,pla}(t_r) &= \boldsymbol{r}_{r,e}(t_0) + \boldsymbol{v}_{r,e} \cdot (t_r - t_0) \\ &= \boldsymbol{r}_{r,e}(t_0) + (\boldsymbol{\omega} \times \boldsymbol{r}_{r,e}(t_0)) \cdot (t_r - t_0) \end{aligned}$$

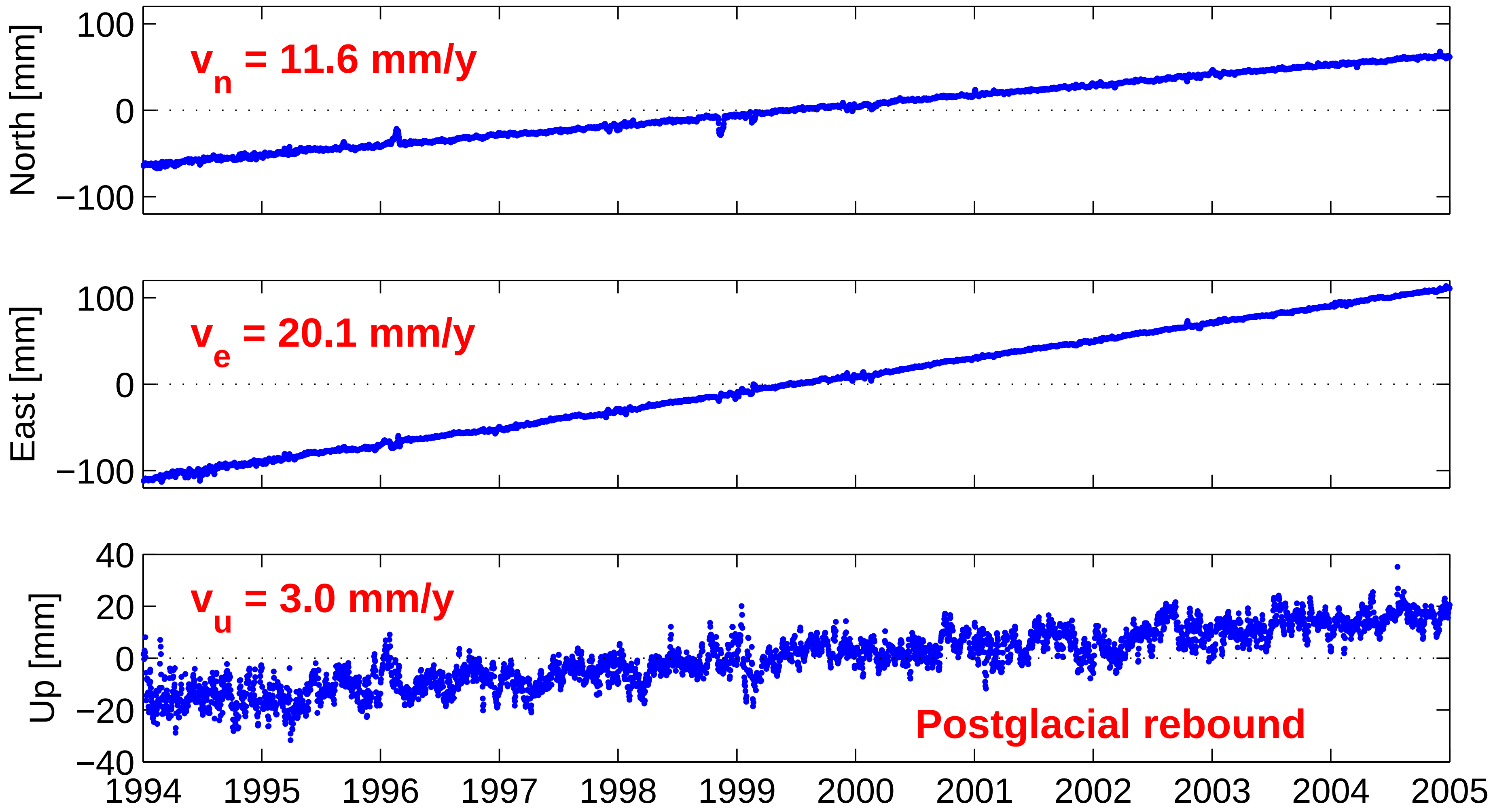
The direction of $\boldsymbol{\omega}$ gives the direction of the pole of rotation.

Station Motion: Global Plate Velocities



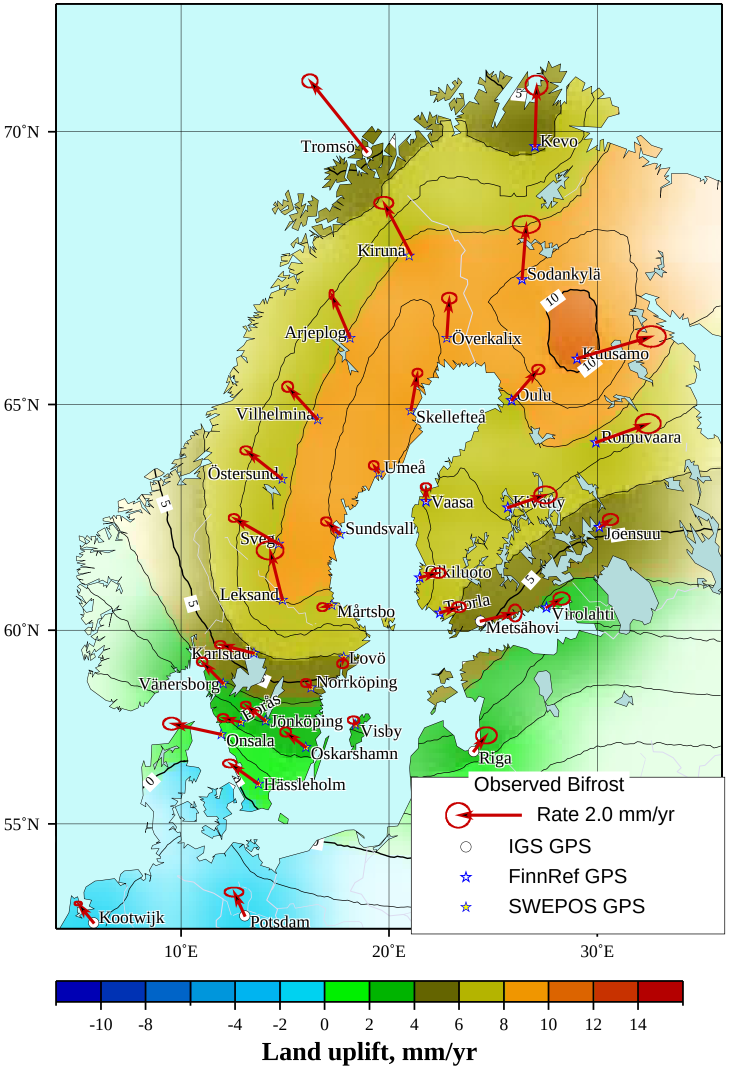
GPS velocities (red) are in good agreement with long-term **geophysical velocities** (blue) derived from paleomagnetic data and hot spots.

Station Motion: Plate Tectonics



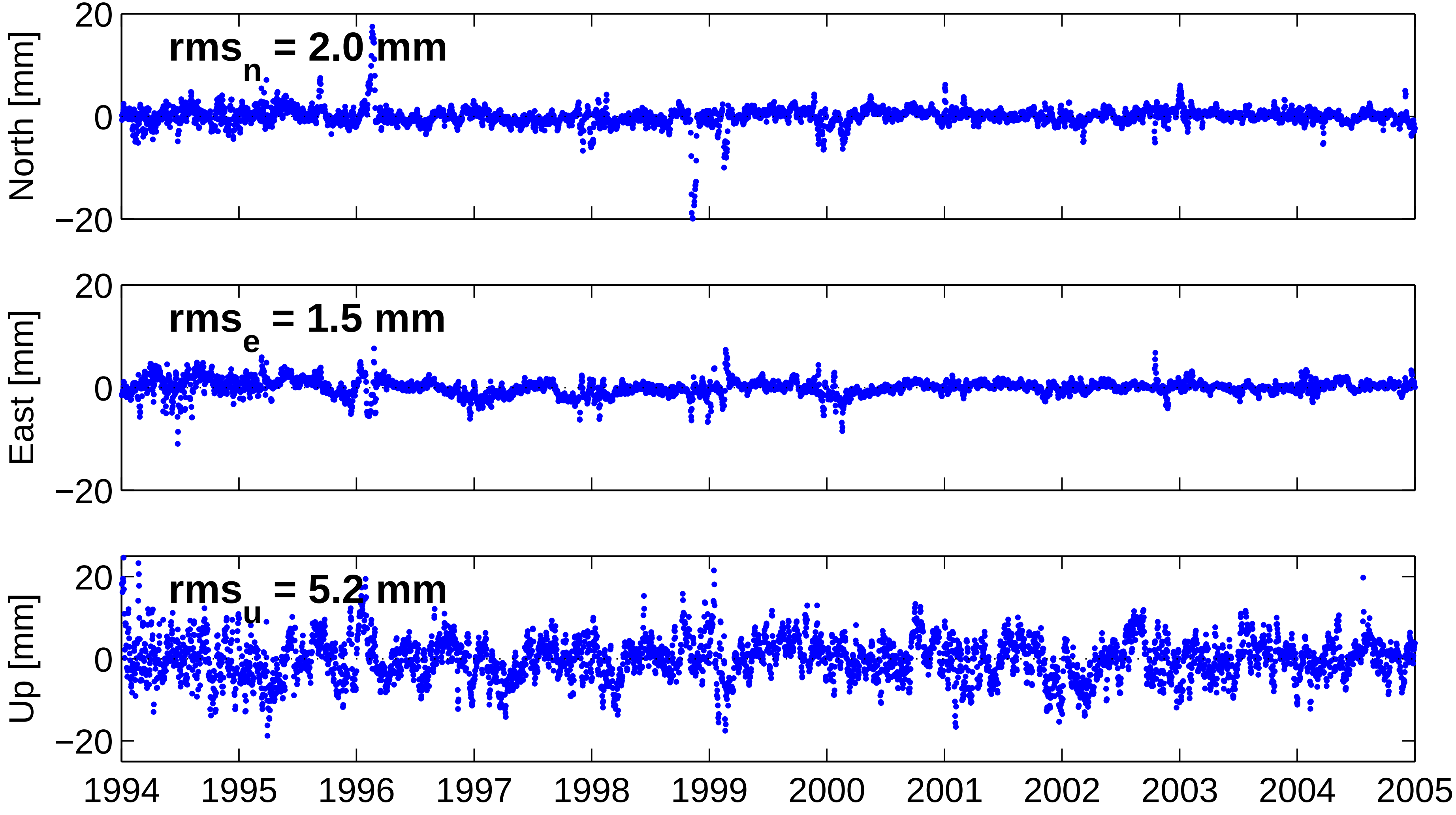
Global motion of **Metsahovi (Finland)** from 10 years of GPS solutions

Station Motion: Postglacial Rebound



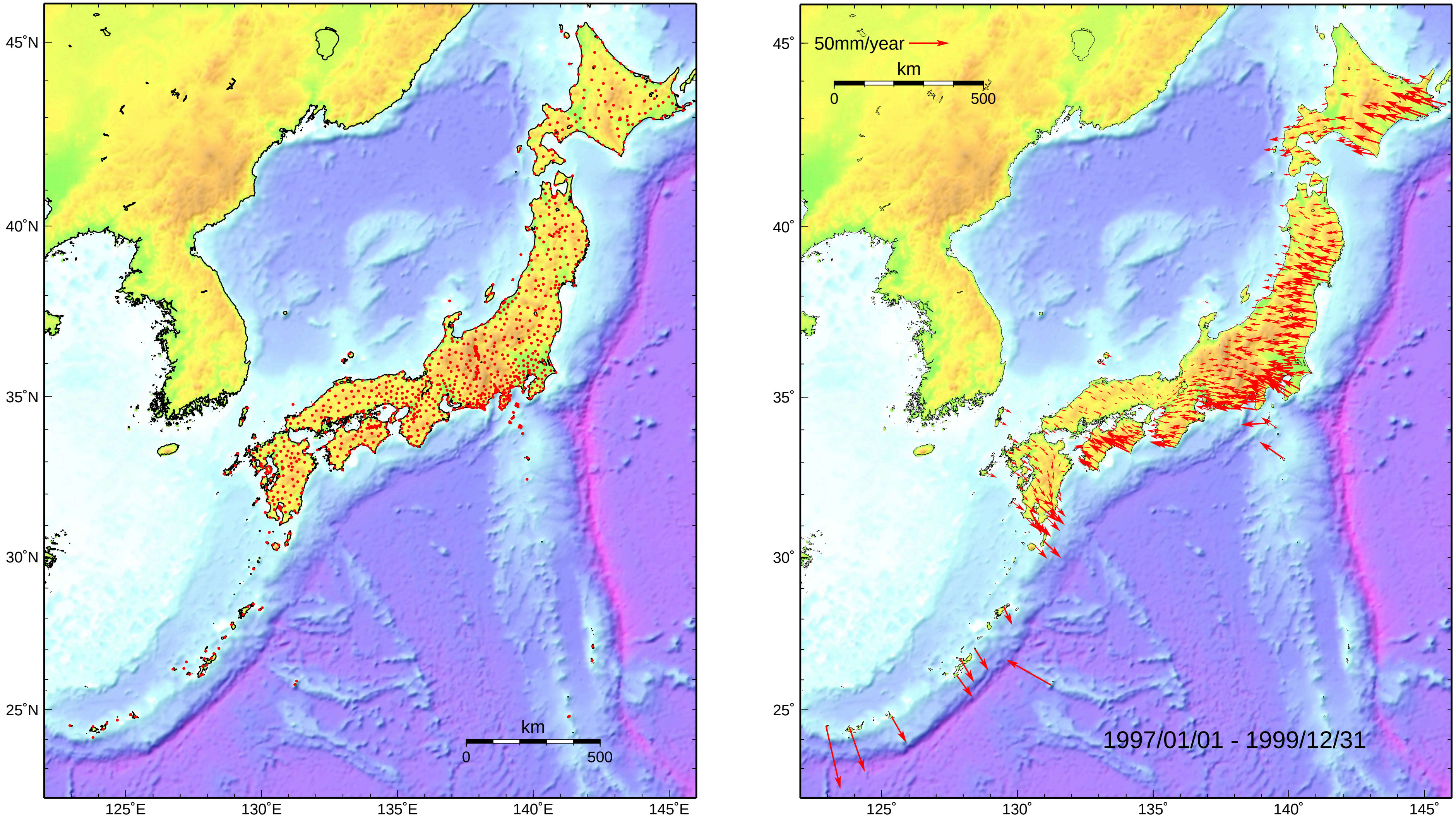
Courtesy: Biforst-Project

Station Motion: Daily Precision



Repeatability for **Metsahovi (Finland)** over 10 years of GPS solutions

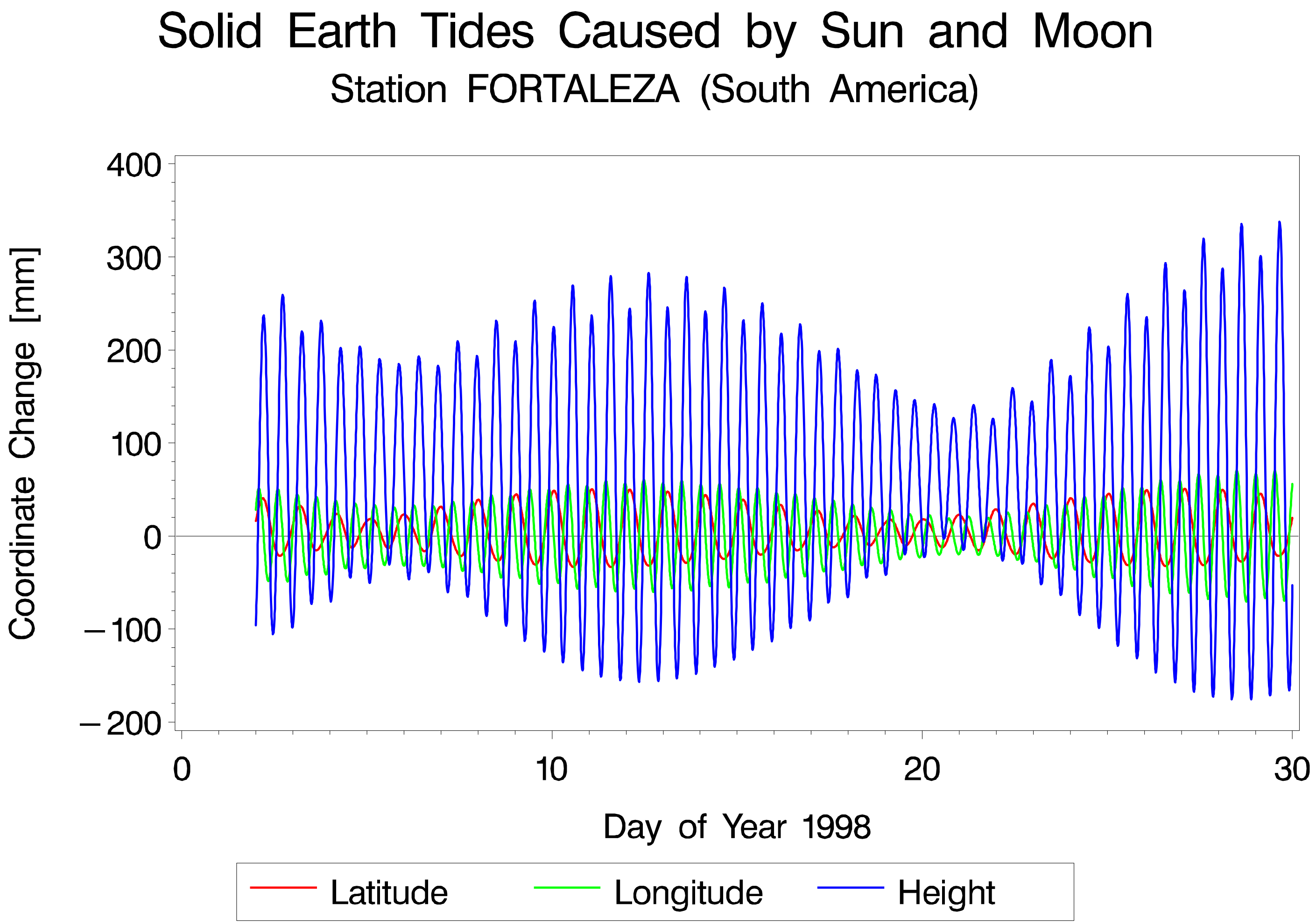
Station Motion: Japanese Permanent Network



More than 1000 GPS permanent stations

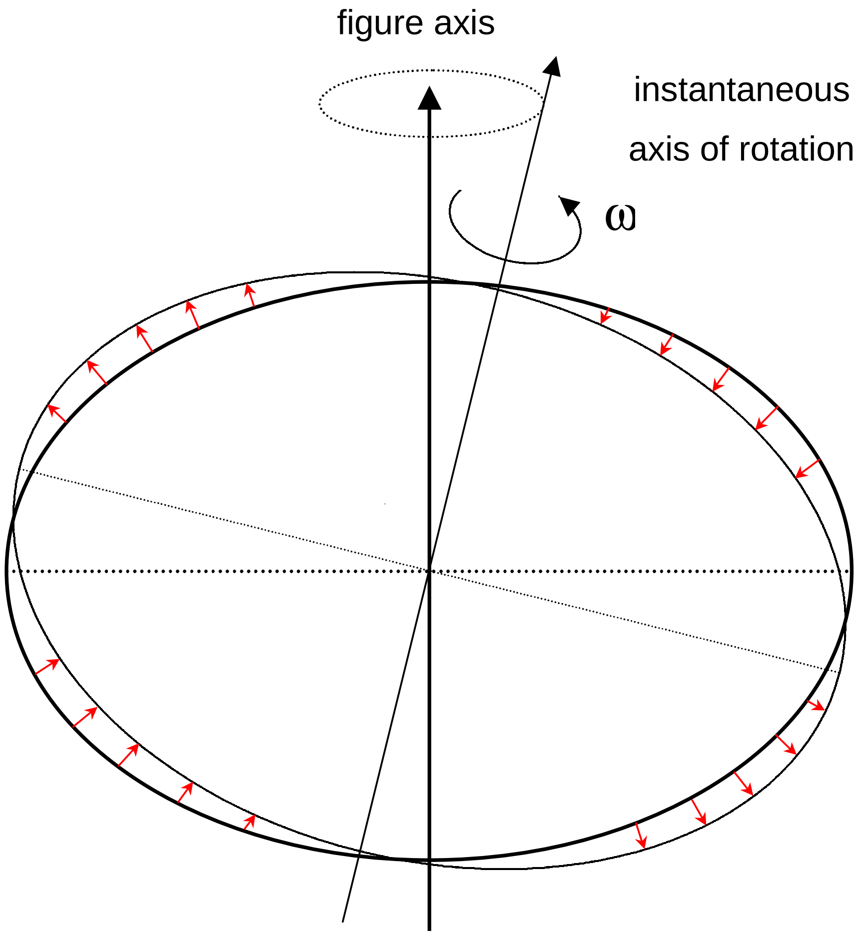
Station Motion: Solid Earth Tides

The non-rigid Earth is deformed by the gravitational forces of Sun and Moon.



Station Motion: Pole Tides

Pole tides: deformation of the Earth by its rotation



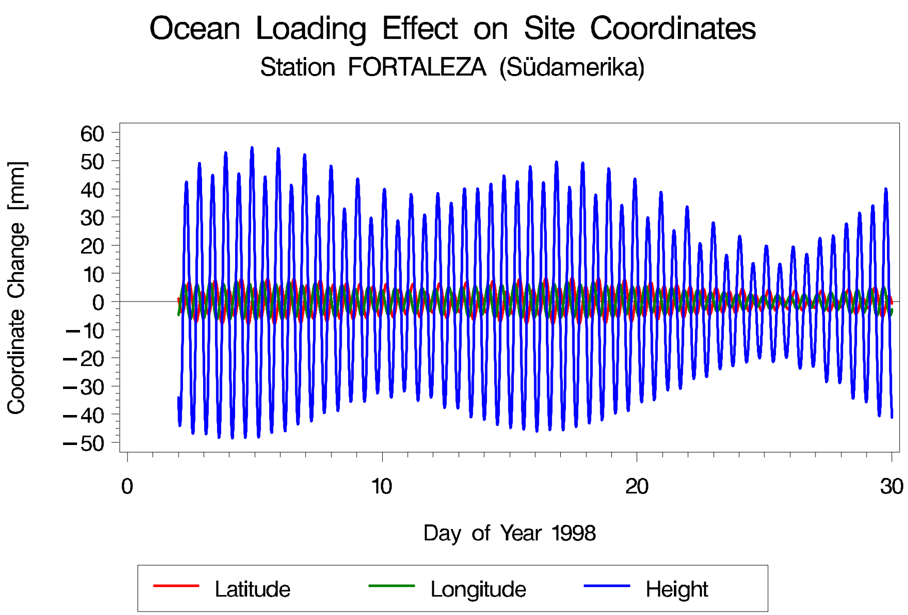
- **Pole tides** are caused by the reaction of the elastic Earth to the **change of the rotation axis**

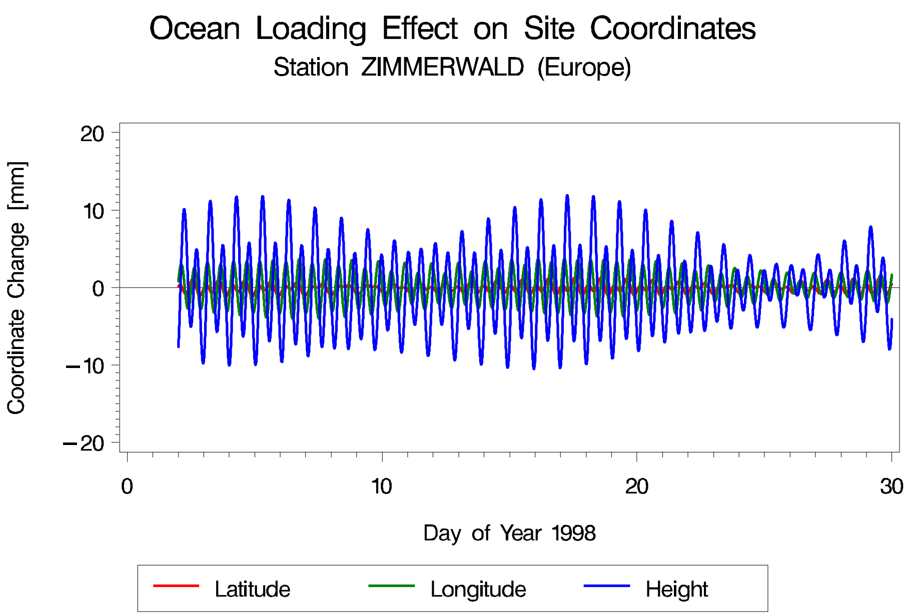
- **Polar motion**: variations of the Earth rotation axis with Chandler ($\approx$ 432 days) and annual periods, "circle" with a diameter of about 15 m.

- Effect on site coordinates up to about **3 cm**.

Station Motion: Ocean Loading

- The station motion due to ocean loading is caused by the **weight of the water** on the continental plate changing with the ocean tides.
- The main effect of up to **several centimeters** for coastal sites is in **station height**.
- Even for Zimmerwald (far away from the sea) the effect in height is still **1 cm**.





Station Motion: Atmospheric Loading and Other Effects

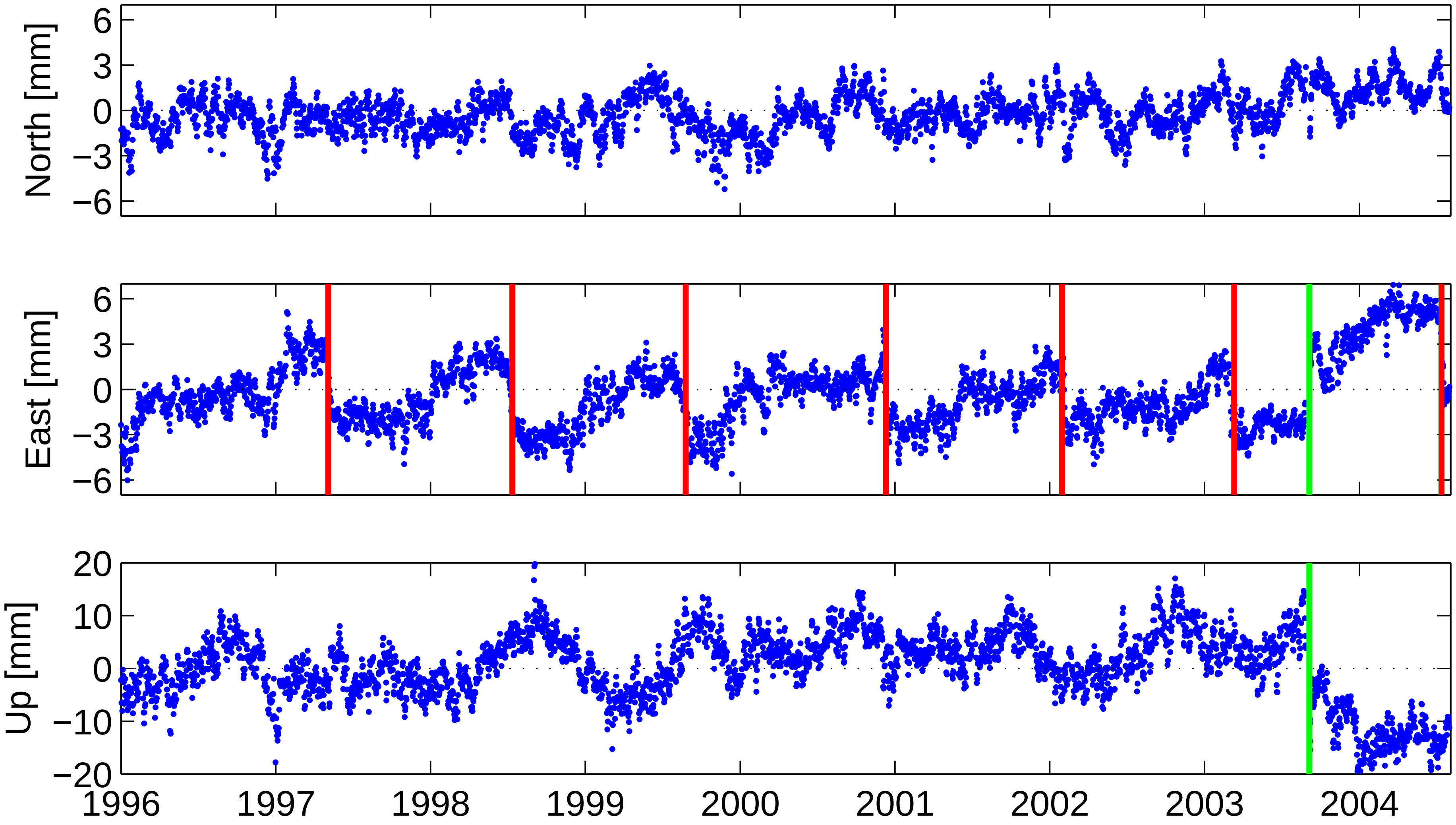
Atmospheric loading:

- Station heights also vary (up to **2 cm**) with the **air pressure**, i.e., the **weight of the atmosphere** on the continental plate.
- The size of the deformation depends on the **local pressure anomaly** (ca. -0.35 mm/mbar) and the **mean pressure anomaly** in a region of 2000 km (ca. -0.55 mm/mbar).

Regional and local effects:

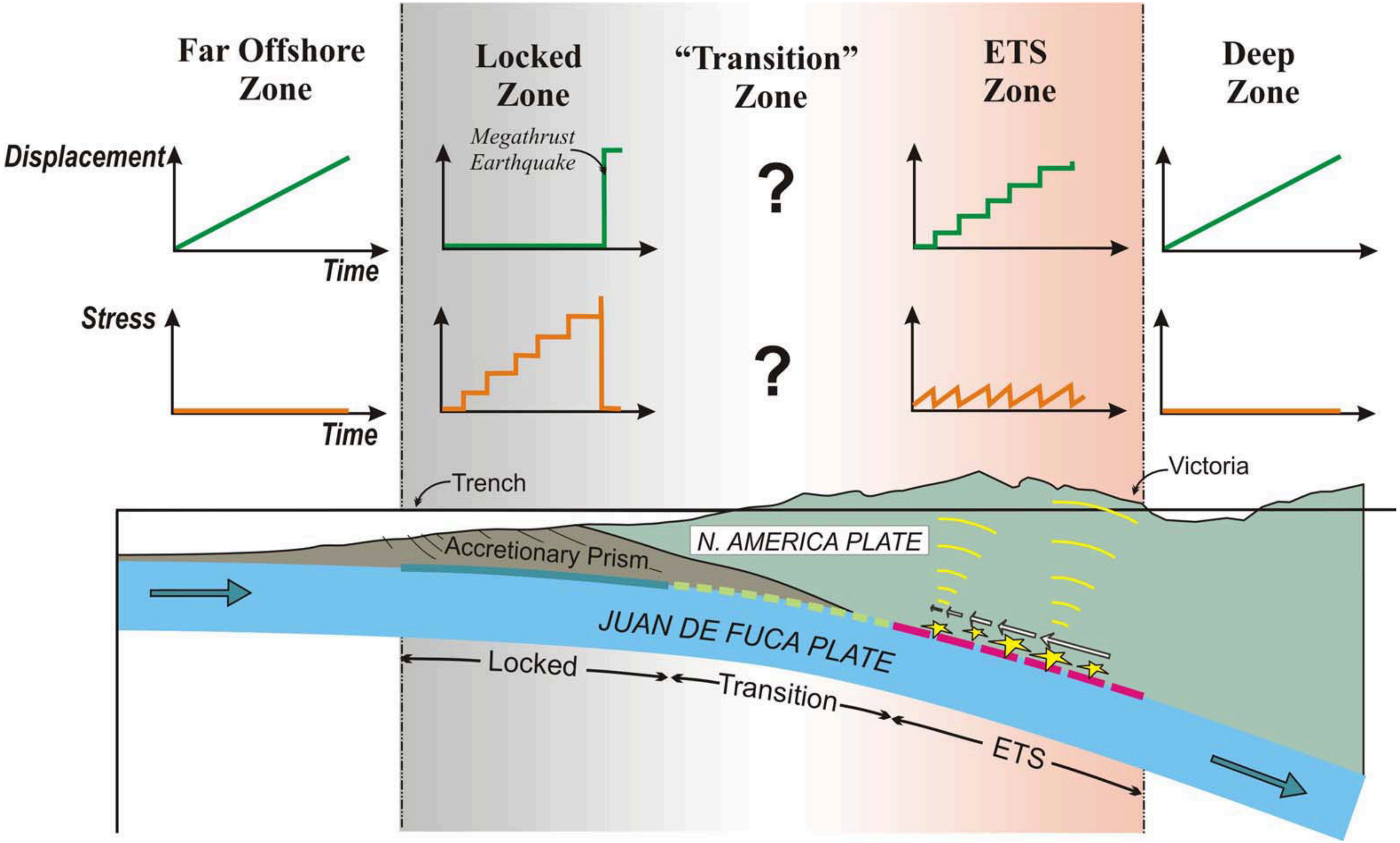
- Difficult **deformations** in the neighborhood of plate boundaries; non-linear motions, **earthquakes** (coseismic and postseismic)
- **Post-glacial rebound**, i.e., the slow uplift of the Earth's crust in Scandinavia, Greenland, or Canada since the melting of the ice caps
- **Instabilities** of the markers due to local changes, e.g., ground water level, mining, monument stability, etc. .

Station Motion: Silent Earthquakes

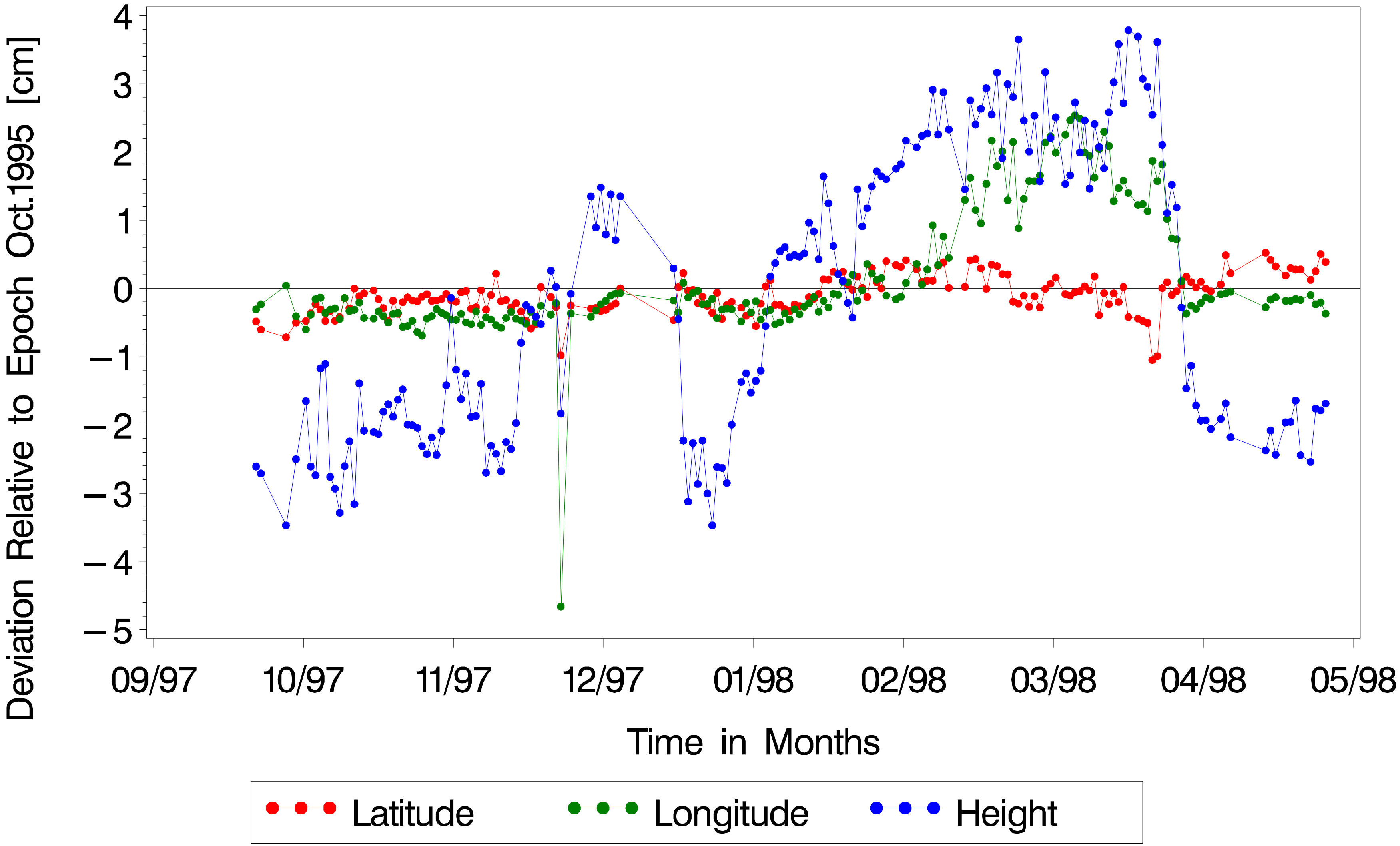


Motion of **Alberthead (western Canada)** from 10 years of GPS solutions

Station Motion: Episodic Tremor and Slip (Silent Earthquakes; courtesy of H. Dragert)



Station Motion: Artifacts



Snow on the antenna at **Sodankyla (Finland)** in winter time

Geometrical Distance ρ_r^s

$$\rho_r^s \quad = \quad |\boldsymbol{r}_i^s(t^s) - \boldsymbol{r}_{r,i}(t_r)| = |\boldsymbol{r}_i^s(t^r - \tau_r^s) - \boldsymbol{R}(t_r) \cdot \boldsymbol{r}_{r,e}(t_r)|$$

$\boldsymbol{r}_i^s(t^s)$ — Satellite position at emission time $t^s = t_r - \tau_r^s$ in **inertial** system (e.g. J2000.0)

$\tau_r^s = \rho_r^s / c$ — Light travel time

$\boldsymbol{r}_{r,i}(t_r)$ — Receiver position at reception time t_r in **inertial** system

$\boldsymbol{r}_{r,e}(t_r)$ — Receiver position at time t_r in **Earth-fixed** system (e.g. ITRF) with

$$\boldsymbol{r}_{r,i}(t_r) = \boldsymbol{R}(t_r) \cdot \boldsymbol{r}_{r,e}(t_r)$$

$\boldsymbol{R}(t_r)$ — Transformation from **Earth-fixed** to **inertial** system with $\boldsymbol{R} = \boldsymbol{PNUXY}$, where these rotation matrices contain the *Earth rotation parameters* (IERS, geophysics, . . .):

Pole coordinates x_p and y_p in $\boldsymbol{X}$ and $\boldsymbol{Y}$ (polar motion),
UT1-UTC in the sidereal rotation matrix $\boldsymbol{U}$,
nutation in obliquity $\Delta\epsilon$ and longitude $\Delta\psi$ in $\boldsymbol{N}$

GPS Satellite Position and Orbit Accuracies

Satellite position $\boldsymbol{r}_i^s(t^s)$ (in inertial frame) is given by:

$$\boldsymbol{r}_i^s(t^s) = \boldsymbol{r}_{i,0}^s(t^s; a, e, i, \Omega, \omega, t_p; p_1, p_2, \ldots, p_d) + \delta\boldsymbol{r}_{ant}^s(t^s)$$

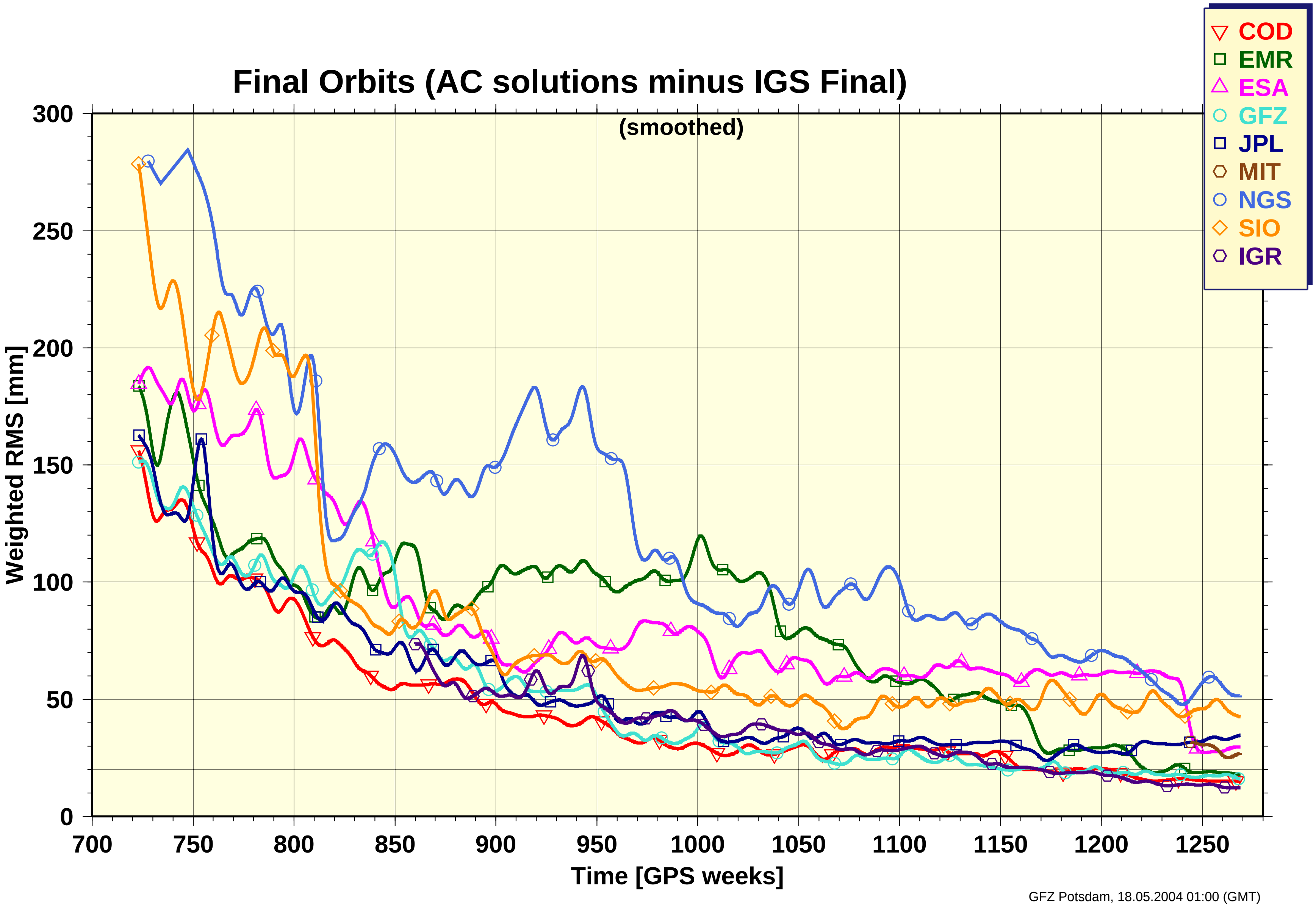
$\boldsymbol{r}_{i,0}^s$, $\delta\boldsymbol{r}_{ant}^s$	Center of mass position and antenna phase center offset
$a, e, i, \Omega, \omega, t_p$	*orbital elements* (orbit determination)
$p_1, p_2, \ldots, p_d$	*gravity field coefficients, air drag and radiation pressure parameters, . . .* (dynamical parameters)

High-precision satellite positions are usually computed by numerical integration and are available from the **International GPS Service**.

IGS GPS/GLONASS Orbit Products Today

IGS/IGEX Product	Availability	Updates	Accuracy
GPS Final Orbits	10 days	weekly	3 cm
GPS Rapid Orbits	17 hours	daily	5 cm
GPS Ultrarapid Orbits	3 hours	twice a day	10 cm
GPS Predicted Orbits	real-time	twice a day	30 cm
GLONASS Final Orbits	6 weeks	weekly	10 cm
GPS Broadcast Orbits	real-time	hourly	300 cm
GLONASS Broadcast Orbits	real-time	30 min	300 cm

Quality of IGS Final Orbits



Influence of Orbit Errors on Station Coordinates

- **Absolute positioning:** orbit errors directly map into station position.
- **Relative positioning: Bauersima (1983)** computes the error Δr in a component of a baseline of length l as a function of an orbit error of size ΔR:

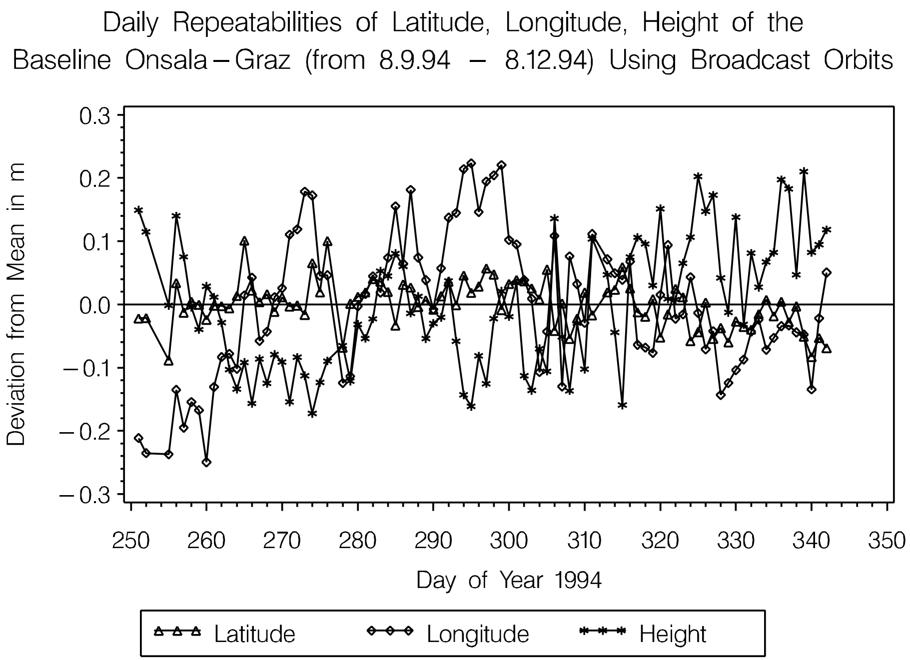
$$\Delta r = \frac{l}{\rho} \cdot \Delta R \approx \frac{l(km)}{25'000(km)} \cdot \Delta R$$

ρ is the mean distance between the survey area and the satellite system.

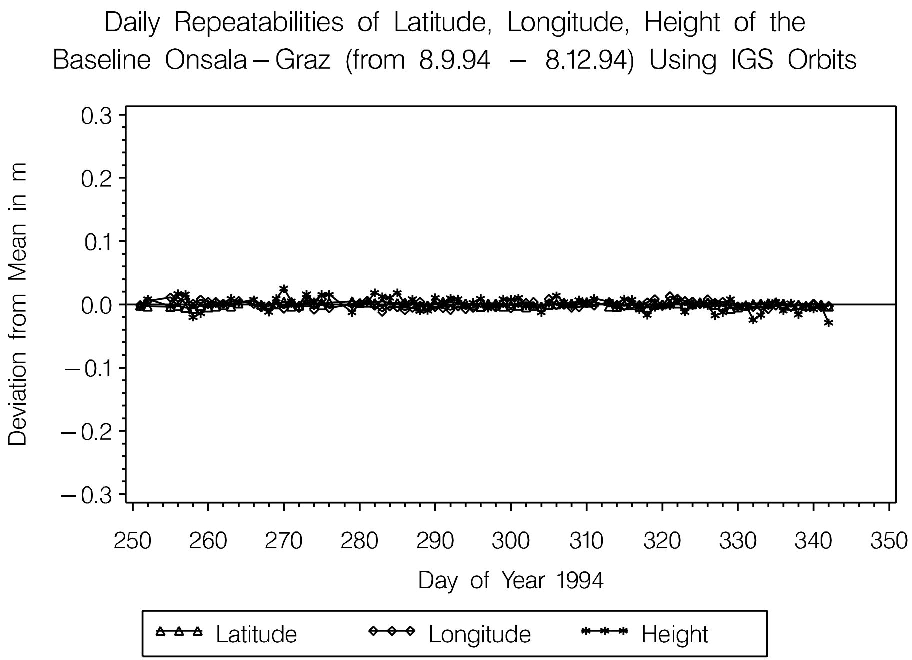
Orbit Error	Baseline Length	Baseline Error	Baseline Error
2.5 m	10 km	.1 ppm	1 mm
2.5 m	100 km	.1 ppm	10 mm
2.5 m	1000 km	.1 ppm	100 mm
.25 m	100 km	.01 ppm	1 mm
.25 m	1000 km	.01 ppm	10 mm
.05 m	100 km	.002 ppm	- mm
.05 m	1000 km	.002 ppm	.5 mm

Influence of Orbit Errors on Station Coordinates

Baseline of about **1000 km** between Onsala (Sweden) and Graz (Austria)



Broadcast Orbits



IGS Precise Orbits

Improved Observation Equation and Parameters

$$L_r^s = \rho_r^s + c\,\delta t_r + c\,\delta t_{r,sys} - c\,\delta t^s - c\,\delta t^s_{sys} + \delta\rho_{trp} + \delta\rho_{ion}$$
$$+\delta\rho_{rel} + \delta\rho_{mul} + \lambda\,N_r^s + \ldots + \epsilon$$

ρ_r^s	Geometrical distance between satellite and receiver
δt_r	Station clock correction: *receiver clocks* (time and frequency transfer)
$\delta t_{r,sys}$	Delays in receiver and antenna (cables, electronics, . . .)
δt^s	Satellite clock correction: *satellite clocks*
$\delta t_{s,sys}$	Delays in satellite (cables, electronics, . . .)
$\delta\rho_{trp}$	Tropospheric delay: *troposphere parameters* (meteorology, climatology)
$\delta\rho_{ion}$	Ionospheric delay: *ionosphere parameters* (atmosphere physics)
$\delta\rho_{rel}$	Relativistic corrections
$\delta\rho_{mul}$	Multipath, scattering, bending effects
N_r^s	Phase ambiguity: *ambiguity parameters* (ambiguity resolution)
ϵ	Measurement error

IGS GPS Satellite Clock Products Today

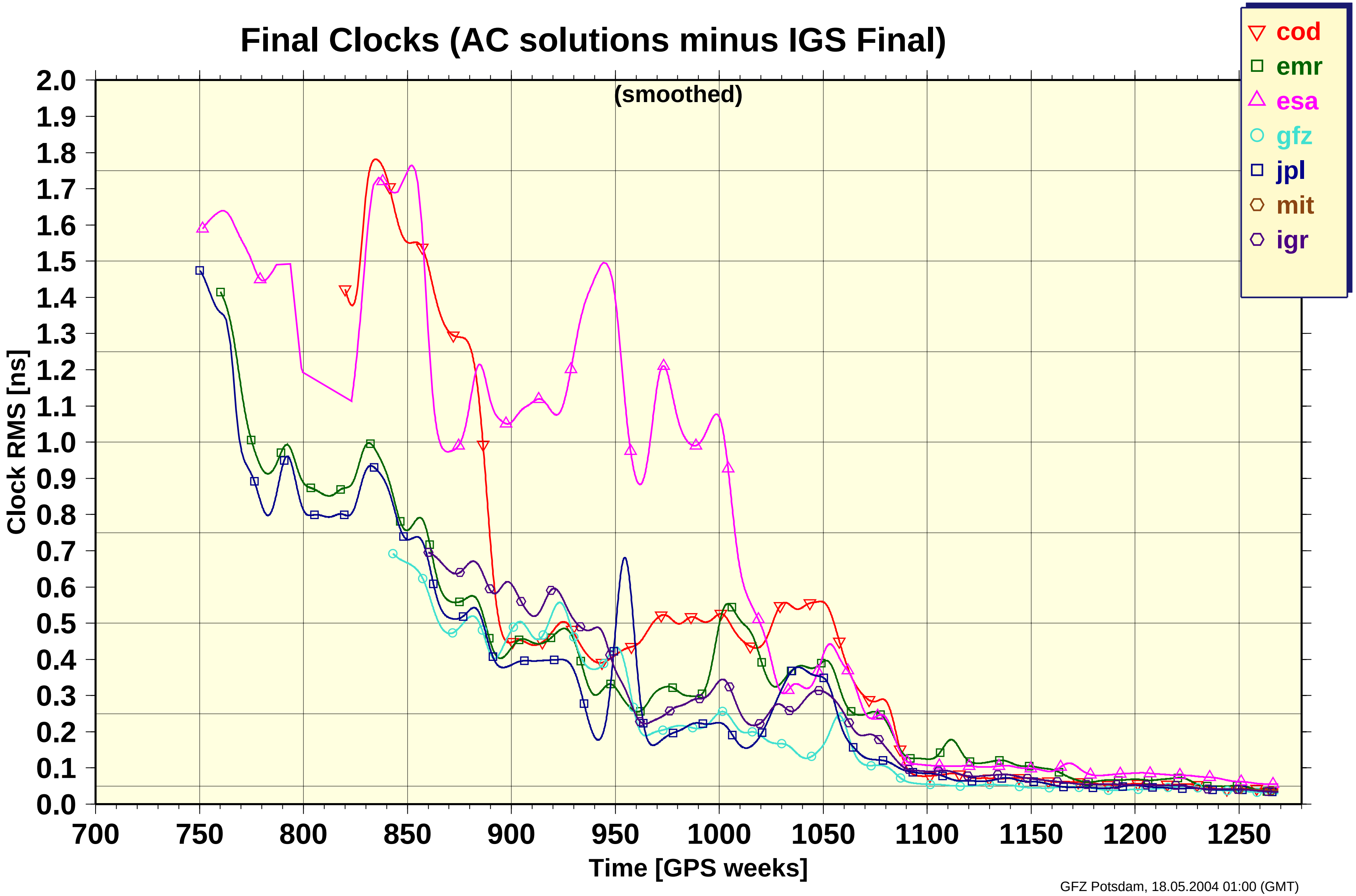
Satellite (and receiver) clock corrections can be estimated using **undifferenced code and phase measurements** of a global network

By convention the IGS clock estimates refer to the **ionosphere-free LC** and thus contain all system delays of this LC ($\delta t_{r,sys}$ and δt^s_{sys}).

The IGS clock products have the following characteristics:

IGS Product	Availability	Updates	Accuracy	
GPS Final Clocks	10 days	weekly	$<$ 3 cm	($<$ 0.1 ns)
GPS Rapid Clocks	17 hours	daily	3 cm	(0.1 ns)
GPS Ultrarapid Clocks	3 hours	twice a day	6 cm	(0.2 ns)
GPS Predicted Clocks	real-time	twice a day	150 cm	(5 ns)
GPS Broadcast Clocks	real-time	hourly	200 cm	(7 ns)

Quality of IGS Final Clocks



Improved Observation Equation and Parameters

$$
\begin{aligned}
L_r^s &= \rho_r^s + c\,\delta t_r + c\,\delta t_{r,sys} - c\,\delta t^s - c\,\delta t^s_{sys} + \delta\rho_{trp} + \delta\rho_{ion} \\
&\quad + \delta\rho_{rel} + \delta\rho_{mul} + \lambda\,N_r^s + \ldots + \epsilon
\end{aligned}
$$

ρ_r^s	Geometrical distance between satellite and receiver
δt_r	Station clock correction: *receiver clocks* (time and frequency transfer)
$\delta t_{r,sys}$	Delays in receiver and antenna (cables, electronics, . . .)
δt^s	Satellite clock correction: *satellite clocks*
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$\delta\rho_{rel}$	Relativistic corrections
$\delta\rho_{mul}$	Multipath, scattering, bending effects
N_r^s	Phase ambiguity: *ambiguity parameters* (ambiguity resolution)
ϵ	Measurement error

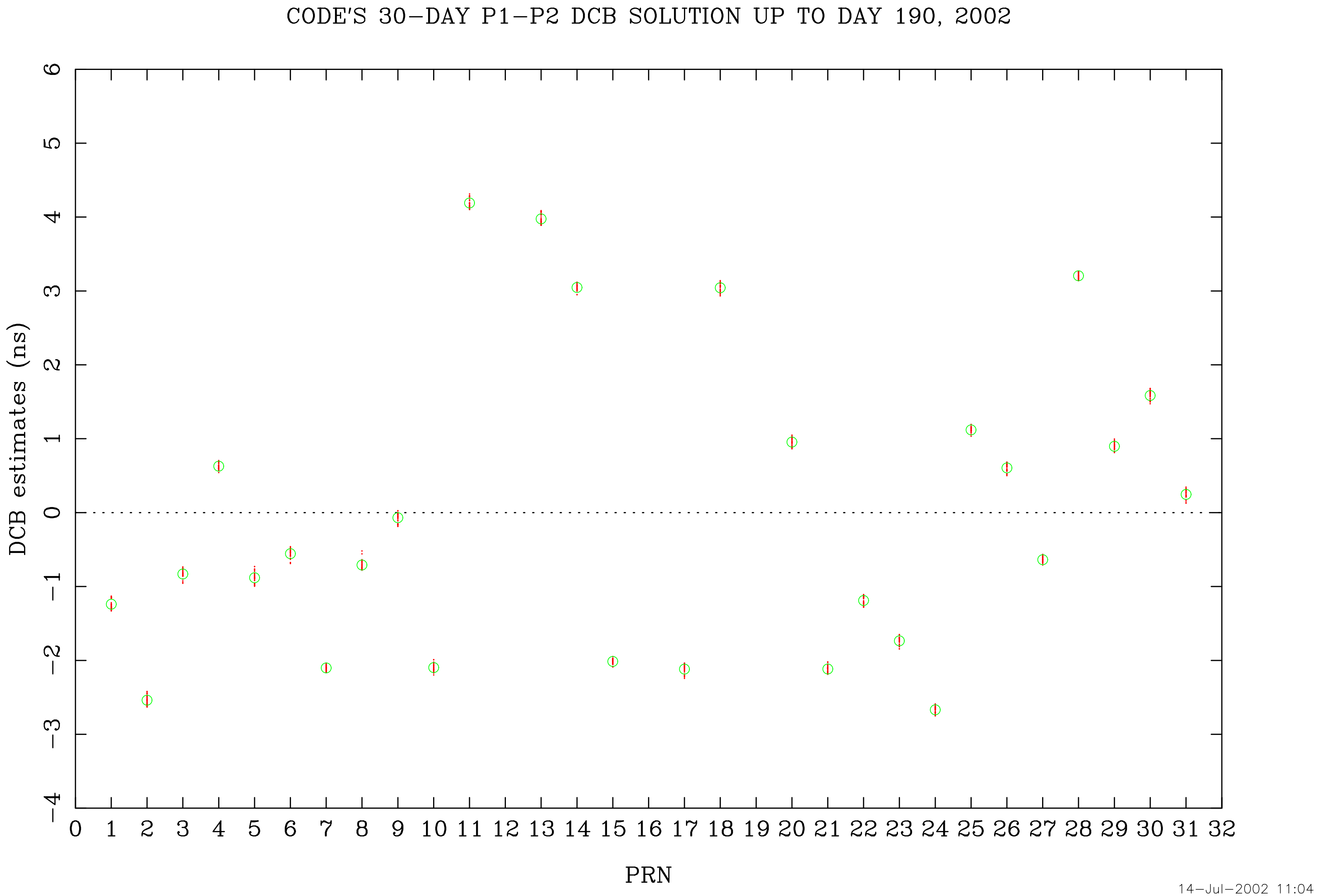
Differential Code Biases: P_1-P_2

- Because the **system delays** in the satellite and receiver, (δt^s_{sys} and $\delta t_{r,sys}$) are different for P_1 and P_2 code measurements, P_1-P_2 corrections have to be applied when processing undifferenced code measurements, so-called **differential code biases (DCBs)**. Differential: they cannot be determined absolutely, but only relatively between different satellites or stations.

- By convention the **ionosphere-free LC** is said to have **no DCBs**. All system delays are **included in the satellite and receiver clocks**, respectively.

- P_1-P_2 corrections are especially important when estimating ionosphere models or computing L_1 (or L_2) code solutions (e.g. with precise satellite orbits and clocks and an ionosphere model).

Differential Code Biases: P_1-P_2



P_1-P_2 DCBs for all GPS satellites (CODE Analysis Center in Berne)

Differential Code Biases: P_1-C_1

Today there are **three classes of receivers** that provide different types of code (pseudorange) measurements:

- Older **C_1/P_2' or CC receivers** (Turborogue, AOA, older Trimble 4xxx) provide:

C_1	$=$	C/A code at L1 frequency
$L_1(C_1)$	$=$	C/A-based phase at L1 frequency
P_2'	$=$	codeless pseudorange at L2 frequency with $P_2' = C_1 + (P_1\text{-}P_2)$
L_2'	$=$	cross-correlated phase at L2 frequency with $L_2' = L_1(C_1) + (L_2(P_2)\text{-}L_1(P_1))$

- Modern **P_1/P_2 receivers** (Ashtech, older Leica) provide:

C_1	$=$	C/A code at L1 frequency
$L_1(C_1)$	$=$	C/A-based phase at L1 frequency
P_1	$=$	Y_1-codeless pseudorange at L1 frequency
$L_1(P_1)$	$=$	Y_1-codeless-based phase at L1 frequency
P_2	$=$	Y_2-codeless pseudorange at L2 frequency
$L_2(P_2)$	$=$	Y_2-codeless-based phase at L2 frequency

Differential Code Biases: P_1-C_1

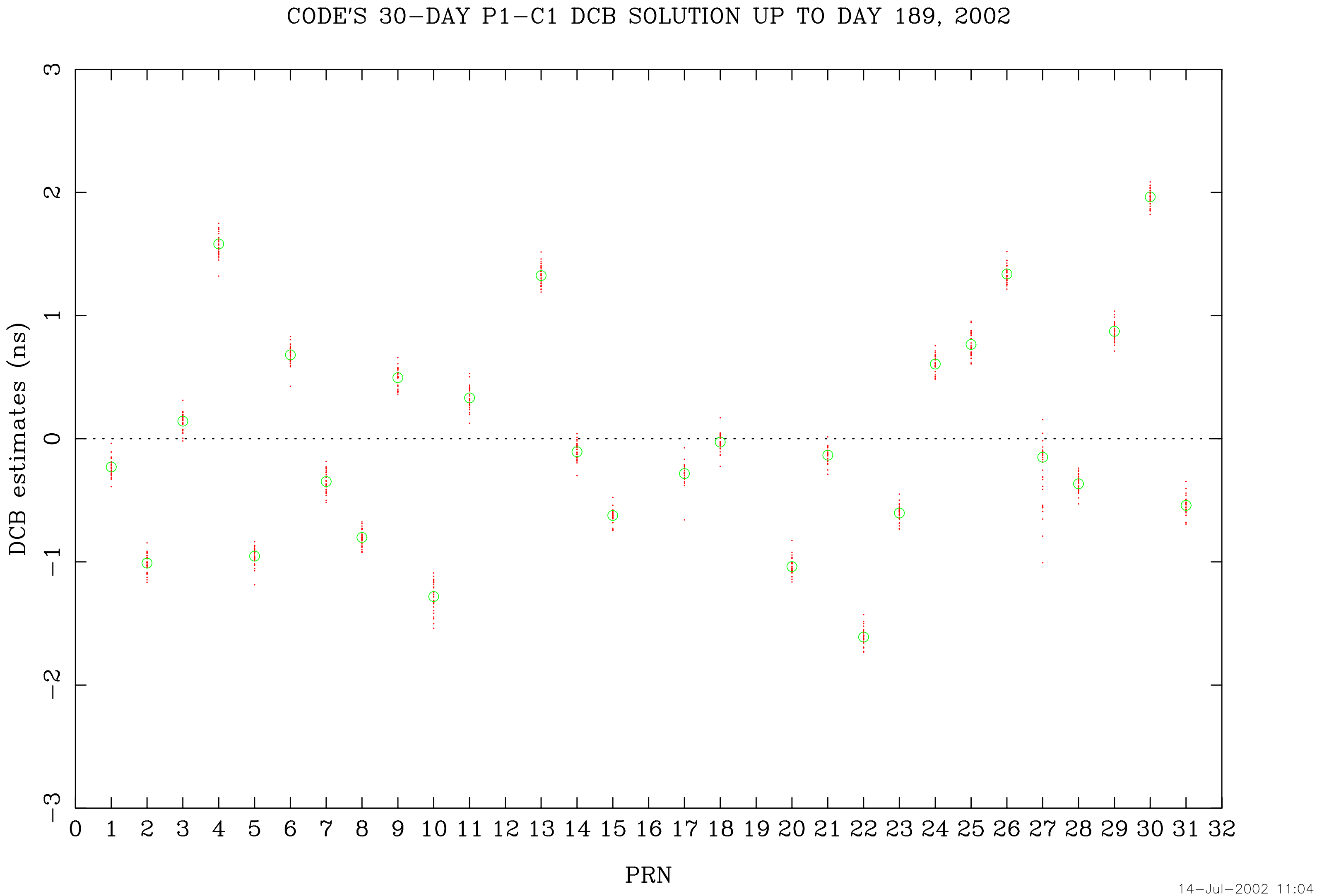
- Very recent **C_1/P_2 receivers** (Leica SR9600, Leica CRS1000, Novatel OEM4, Trimble MS750, Trimble 4700, Trimble 5700) provide:

$$\begin{aligned} C_1 &= \text{C/A code at L1 frequency} \\ L_1(C_1) &= \text{C/A-based phase at L1 frequency} \\ P_2 &= Y_2\text{-codeless pseudorange at L2 frequency} \\ L_2(P_2) &= Y_2\text{-codeless-based phase at L2 frequency} \end{aligned}$$

To obtain **consistent code observations** for all these receiver types, P_1-C_1 (and P_1-P_2) biases have to be applied depending on the LC processed.
This consistency is important for:

- Resolution of **widelane ambiguities** on baselines with receivers from different classes.
- **Satellite clock corrections** to be used for various linear combinations
- Estimation of global, regional or local **ionosphere models** from code measurements of different receiver classes

Differential Code Biases: P_1-C_1



P_1-C_1 DCBs for all GPS satellites (CODE Analysis Center in Berne)

Use of Differential Code Biases

LC	P_1/P_2	C_1/C_2		C_1/P_2	
L_1	$+1.55 \cdot B_{P_1-P_2}$	$+1.55 \cdot B_{P_1-P_2}$	$+B_{P_1-C_1}$	$+1.55 \cdot B_{P_1-P_2}$	$+B_{P_1-C_1}$
L_2	$+2.55 \cdot B_{P_1-P_2}$	$+2.55 \cdot B_{P_1-P_2}$	$+B_{P_1-C_1}$	$+2.55 \cdot B_{P_1-P_2}$	
L_c	0		$+B_{P_1-C_1}$		$+2.55 \cdot B_{P_1-C_1}$
L_I	$-B_{P_1-P_2}$	$-B_{P_1-P_2}$		$-B_{P_1-P_2}$	$+B_{P_1-C_1}$
L_w	$-1.98 \cdot B_{P_1-P_2}$	$-1.98 \cdot B_{P_1-P_2}$	$+B_{P_1-C_1}$	$-1.98 \cdot B_{P_1-P_2}$	$+4.53 \cdot B_{P_1-C_1}$
MW	0		$-B_{P_1-C_1}$		$-0.56 \cdot B_{P_1-C_1}$

L_c, L_I, L_w, and MW denote the ionosphere-free, geometry-free, widelane, and Melbourne-Wübbena linear combination, respectively.

Convention: DCB $= 0$ for L_c (ionosphere-free LC) and DCB $= -B_{P_1-P_2}$ and DCB $= -B_{P_1-C_1}$ for L_I (geometry-free LC).

The factors involved are:

$$\frac{f_1^2}{f_1^2 - f_2^2} = 2.546 \quad , \quad \frac{f_2^2}{f_1^2 - f_2^2} = 1.546 \quad ,$$

$$\frac{f_1 f_2}{f_1^2 - f_2^2} = 1.984 \quad , \quad \frac{f_1}{f_1 - f_2} = 4.529 \quad , \quad \frac{f_1}{f_1 + f_2} = 0.562$$

Improved Observation Equation and Parameters

$$\begin{aligned} L_r^s &= \rho_r^s + c\,\delta t_r + c\,\delta t_{r,sys} - c\,\delta t^s - c\,\delta t^s_{sys} + \delta\rho_{trp} + \delta\rho_{ion} \\ &\quad + \delta\rho_{rel} + \delta\rho_{mul} + \lambda\,N_r^s + \ldots + \epsilon \end{aligned}$$

ρ_r^s	Geometrical distance between satellite and receiver
δt_r	Station clock correction: *receiver clocks* (time and frequency transfer)
$\delta t_{r,sys}$	Delays in receiver and antenna (cables, electronics, . . .)
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$\delta\rho_{ion}$	Ionospheric delay: *ionosphere parameters* (atmosphere physics)
$\delta\rho_{rel}$	Relativistic corrections
$\delta\rho_{mul}$	Multipath, scattering, bending effects
N_r^s	Phase ambiguity: *ambiguity parameters* (ambiguity resolution)
ϵ	Measurement error

Relativistic Corrections (1)

The satellite clocks (as well as the station clocks) are affected by **special** (relative velocity of the satellite) and **general relativistic effects** (gravity field of the Earth).

Special relativity: moving clocks are **slower** than clocks at rest.

General relativity: due to the **weaker gravity field** at the altitude of the GPS satellites the satellite clocks are **faster** by about **40 μs/d** than clocks on the Earth's surface.

Resulting **frequency difference** $\Delta f = f - f_0$ between satellite and receiver:

$$\frac{\Delta f}{f_0} = \frac{f - f_0}{f_0} = \frac{1}{2}\frac{v^2}{c^2} + \frac{\Delta U}{c^2}$$

f_0	Frequency received by the receiver
f	Frequency emitted by the satellite
v	Velocity of the satellite (about 4 km/s)
ΔU	Difference in gravity potential between satellite and receiver

Relativistic Corrections (2)

Assuming a circular orbit and a spherical Earth (with mass M_E) we get approximately:

$$\frac{\Delta f}{f_0} = \frac{1}{2}\frac{v^2}{c^2} + \frac{GM_E}{c^2}\left(\frac{1}{|\boldsymbol{r}^s|} - \frac{1}{|\boldsymbol{r}_r|}\right) \approx -4.464 \cdot 10^{-10}$$

The frequency of the satellite clock is **shifted** at the ground by $\Delta f = 4.464 \cdot 10^{-10} \cdot f_0 = 4.57 \cdot 10^{-3}$ Hz to a value of **10.22999999543 MHz**, so that the receiver will receive the nominal frequency of 10.23 MHz despite the relativistic effects mentioned above.

This frequency shift only corrects for a **constant** clock rate. Because the GPS orbits are not exactly circular, the satellite clock also shows **periodic variations**:

$$\delta\rho_{rel,1} = \frac{2}{c}\sqrt{GM_E\, a}\; e \sin E = \frac{2 \cdot \boldsymbol{r^s} \boldsymbol{\cdot} \dot{\boldsymbol{r}}^s}{c^2}$$

with e the numerical eccentricity, a the semi-major axis and E the eccentric anomaly of the satellite orbit. This distance correction $\delta\rho_{rel,1}$ may amount to more than **10 m**.

Relativistic Corrections (3)

We also have to consider a **small delay** of the GPS signal when it **propagates through the Earth's gravity field** — a signal is slowed down when passing through the gravity field.

The corresponding correction is given by (integration of the gravity field effect from the satellite to the receiver):

$$\delta\rho_{rel,2} = \frac{2\,GM_E}{c^2}\,\ln\left(\frac{r^s + r_r + \rho_r^s}{r^s + r_r - \rho_r^s}\right)$$

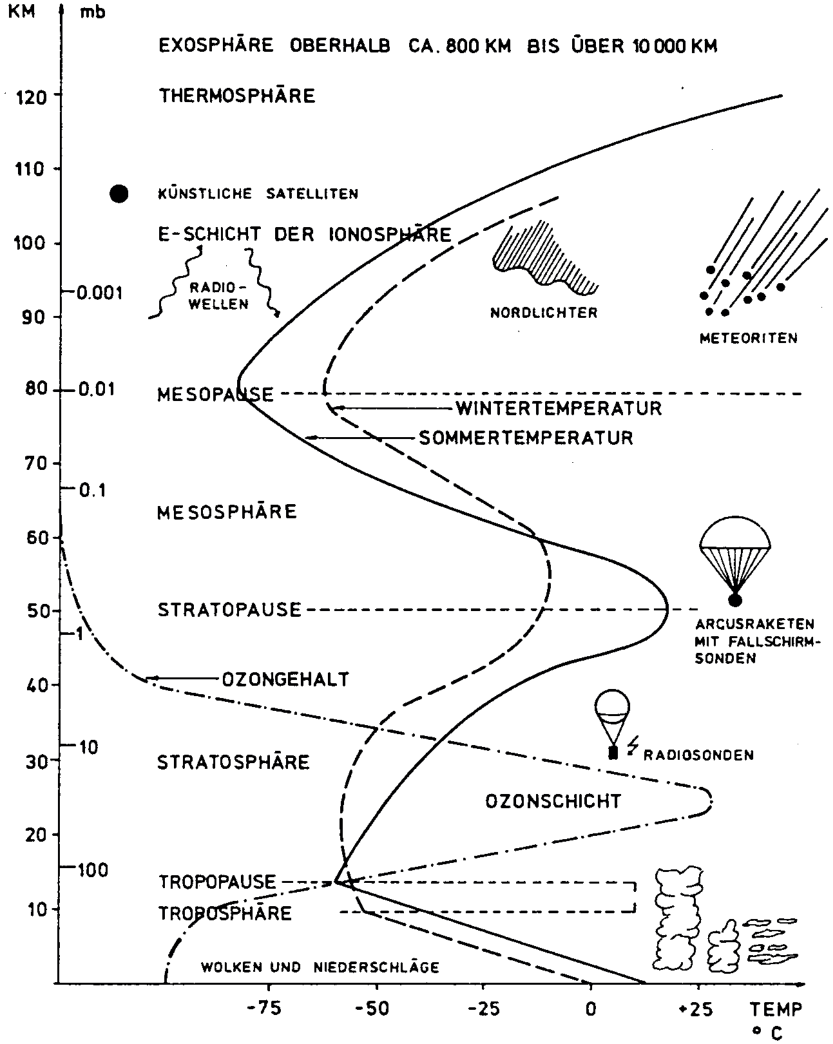
This second term has a maximum size of **18.7 mm** (at low elevations).

The total correction term $\delta\rho_{rel}$ in the observation equation is therefore

$$\delta\rho_{rel} = \delta\rho_{rel,1} + \delta\rho_{rel,2}$$

to be applied independent of whether broadcast or high-precision IGS satellite clock corrections are used.

Atmospheric Refraction



Two regions are important for the propagation of the **microwave** signals:

- **Troposphere** (neutral atmosphere): extending up to a height of **10 km**. The signal delay of microwaves is **non-dispersive**, i.e. frequency-independent.
- **Ionosphere**: **dispersive**, i.e. the two frequencies (L_1 and L_2) are delayed differently. The ionosphere ranges from about **50–1000 km altitude**.

Improved Observation Equation and Parameters

$$
\begin{aligned}
L_r^s \quad = \quad & \rho_r^s + c\,\delta t_r + c\,\delta t_{r,sys} - c\,\delta t^s - c\,\delta t^s_{sys} + \delta\rho_{trp} + \delta\rho_{ion} \\
& + \delta\rho_{rel} + \delta\rho_{mul} + \lambda\,N_r^s + \ldots + \epsilon
\end{aligned}
$$

ρ_r^s	Geometrical distance between satellite and receiver
δt_r	Station clock correction: *receiver clocks* (time and frequency transfer)
$\delta t_{r,sys}$	Delays in receiver and antenna (cables, electronics, . . .)
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$\delta\rho_{rel}$	Relativistic corrections
$\delta\rho_{mul}$	Multipath, scattering, bending effects
N_r^s	Phase ambiguity: *ambiguity parameters* (ambiguity resolution)
ϵ	Measurement error

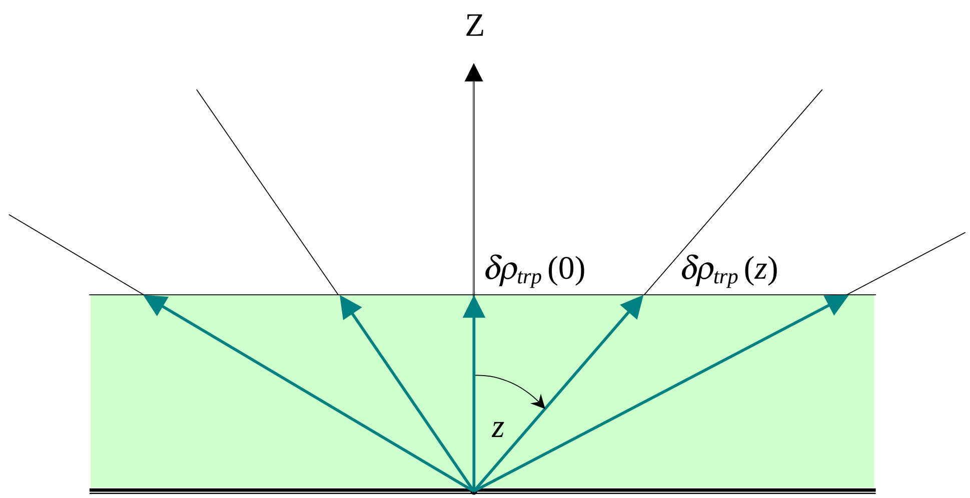
Tropospheric Refraction (signal delay)

- Delay caused by the **refractive index** n (refractivity N) along the path S:

$$\delta\rho_{trp} = \int_S (n-1)\,ds = 10^{-6}\int_S N\,ds$$

- Total delay in zenith direction: $\delta\rho_{trp}(z=0) \approx$ **2.3 m** (8 ns)

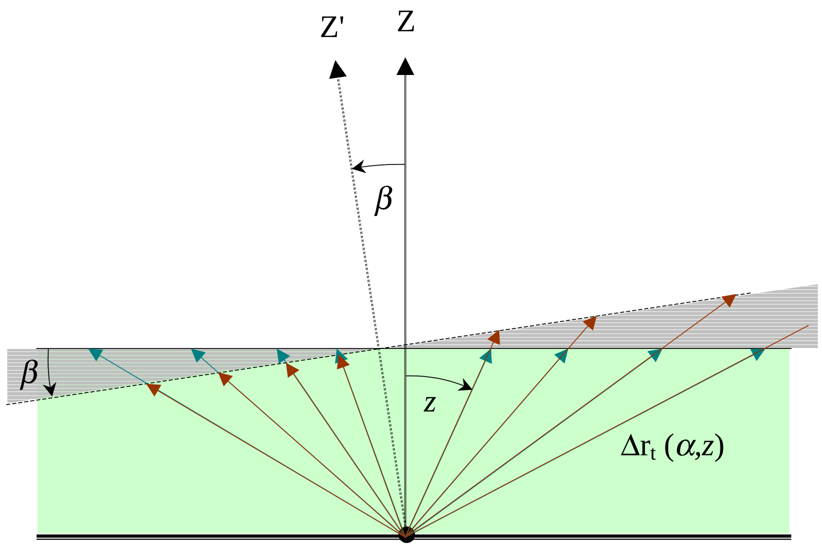
  - **Dry part:** 90% of the delay; mainly depending on pressure; easy to model.
  - **Wet part:** up to **40 cm**; difficult to model (unknown **water vapor** distribution).



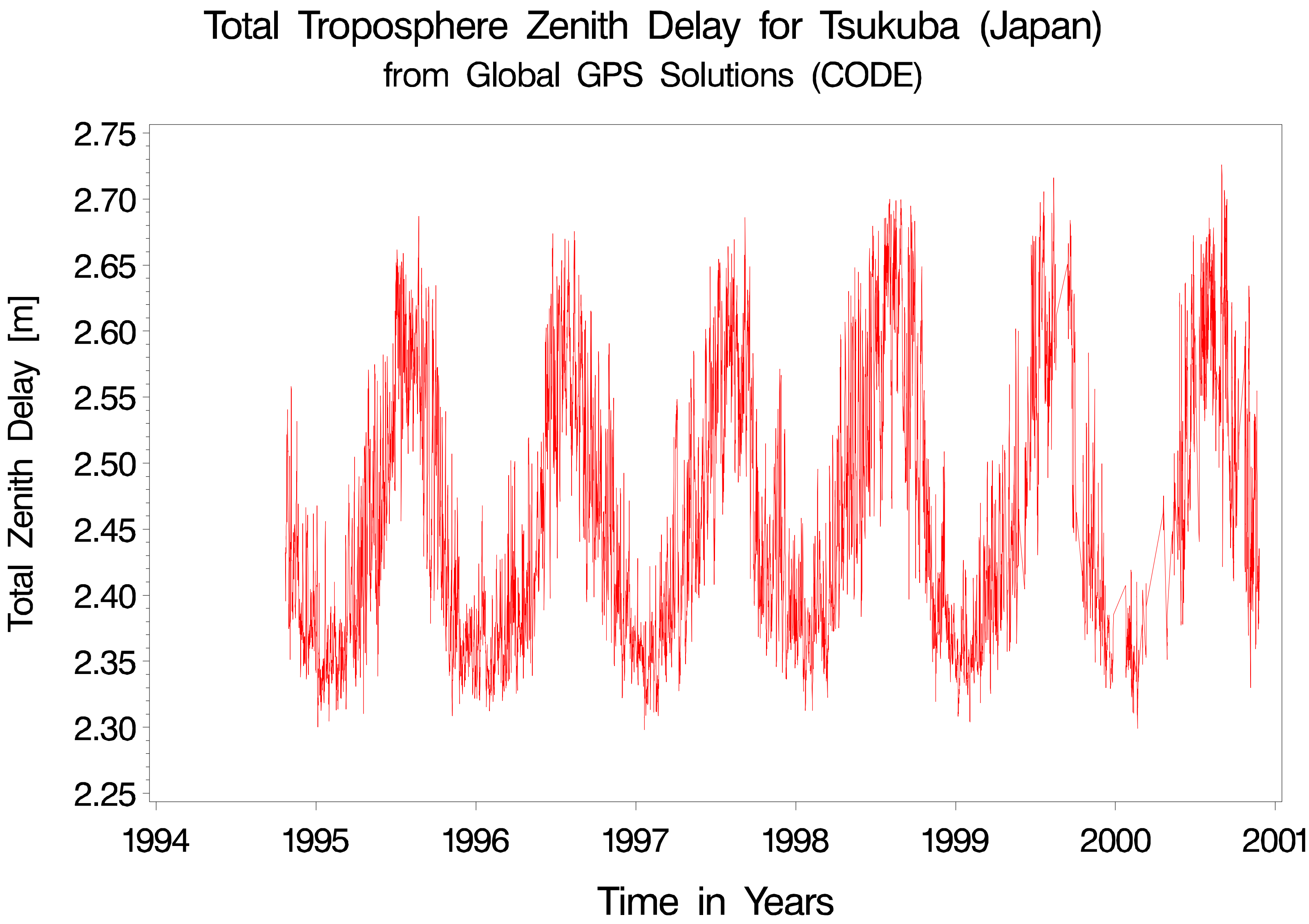
$$\delta\rho_{trp}(z) = \frac{1}{\cos(z)}\cdot\delta\rho_{trp}(0)$$

Modeling and Estimation of Troposphere

- Models compute the tropospheric delay $\delta\rho_{trp}$ from pressure P, temperature T, and relative humidity H.
- Meteorological data (P, T, H): **measurements at the station** or **standard atmosphere**.
- **Problems:** Standard atmosphere not representative; met data not good enough (error in temperature of **1 C^o** may cause height error of **8 cm**!).
- Therefore: **estimation of zenith troposphere delays** from the GPS/GLONASS data.
- **Long sessions** needed to separate troposphere delay and height.
- **Tilting of the zenith** can be used to represent troposphere gradient parameters.



Total Delay from GPS at Tsukuba (Japan)



Improved Observation Equation and Parameters

$$L_r^s = \rho_r^s + c\,\delta t_r + c\,\delta t_{r,sys} - c\,\delta t^s - c\,\delta t^s_{sys} + \delta\rho_{trp} + \delta\rho_{ion}$$
$$+\delta\rho_{rel} + \delta\rho_{mul} + \lambda\,N_r^s + \ldots + \epsilon$$

ρ_r^s	Geometrical distance between satellite and receiver
δt_r	Station clock correction: *receiver clocks* (time and frequency transfer)
$\delta t_{r,sys}$	Delays in receiver and antenna (cables, electronics, . . .)
δt^s	Satellite clock correction: *satellite clocks*
$\delta t_{s,sys}$	Delays in satellite (cables, electronics, . . .)
$\delta\rho_{trp}$	Tropospheric delay: *troposphere parameters* (meteorology, climatology)
$\delta\rho_{ion}$	Ionospheric delay: *ionosphere parameters* (atmosphere physics)
$\delta\rho_{rel}$	Relativistic corrections
$\delta\rho_{mul}$	Multipath, scattering, bending effects
N_r^s	Phase ambiguity: *ambiguity parameters* (ambiguity resolution)
ϵ	Measurement error

Ionospheric Refraction

- The ionospheric refraction $\delta\rho_{ion}$ is caused by the interaction of the microwave signal with the **free electrons**:

$$\delta\rho_{ion} = \int_0^\infty (n_e - 1)\, ds = \pm \frac{a}{f^2} \int_0^\infty N_e\, ds = \pm \frac{a}{f^2} E$$

f	Frequency
n_e	Refractive index $n_e = 1 \pm \frac{a}{f^2} N_e$
N_e	Electron density (electrons/m^3)
E	Number of free electrons in a column of 1 m^2 cross-section along the signal path (**TEC: total electron content** in TECU: 10^{16} free electrons/m^2.
a	Constant: $a = 40.3 \cdot 10^{16}\, m\, s^{-2}\, TECU^{-1}$

- **Positive sign** (delay) for pseudorange/code, **negative sign** (advance) for phase measurements (group and phase velocity).
- The number of free electrons (generated by ionization) depends on the **solar activity** (geographic latitude, time of day, 11-year solar cycle).

Impact of Ionospheric Refraction

- The **total delay** in zenith direction varies between **1–20 m** (3–70 ns):

$$\delta\rho_{ion}\ \ [\mathrm{m}]\ \approx\ \frac{1}{6}\,E\ [\mathrm{TECU}] \qquad \text{for } L_1$$

  Low elevation: delays of up to **100 m** possible.

- Absolute positioning: primary effect in **height** (several meters).

- Main effect on relative positioning (baselines): **scaling** of the network (neglect of ionosphere: distances are **too short**).

- Estimate of the **scale factor** (in ppm) for a baseline of length l:

$$\frac{dl}{l} \approx -0.10 \cdot E \quad [ppm] \quad (E\ in\ TECU)$$

- **Example**: Network of 4 by 4 km in Switzerland in 1989: $E \approx 50$ TECU, $dl/l \approx$ **-5 ppm**, and $dl \approx$ **-2 cm** for the baseline error. Network of 20 km: error of already **-10 cm**.

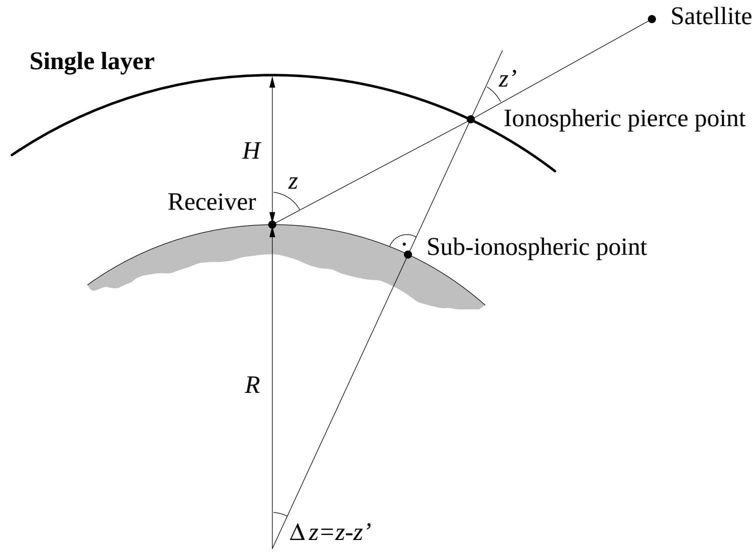
Modeling and Estimation of the Ionosphere

- Elimination by forming the **ionosphere-free** linear combination of L_1 and L_2:

$$L_c = \frac{f_1^2}{f_1^2 - f_2^2} L_1 - \frac{f_2^2}{f_1^2 - f_2^2} L_2$$

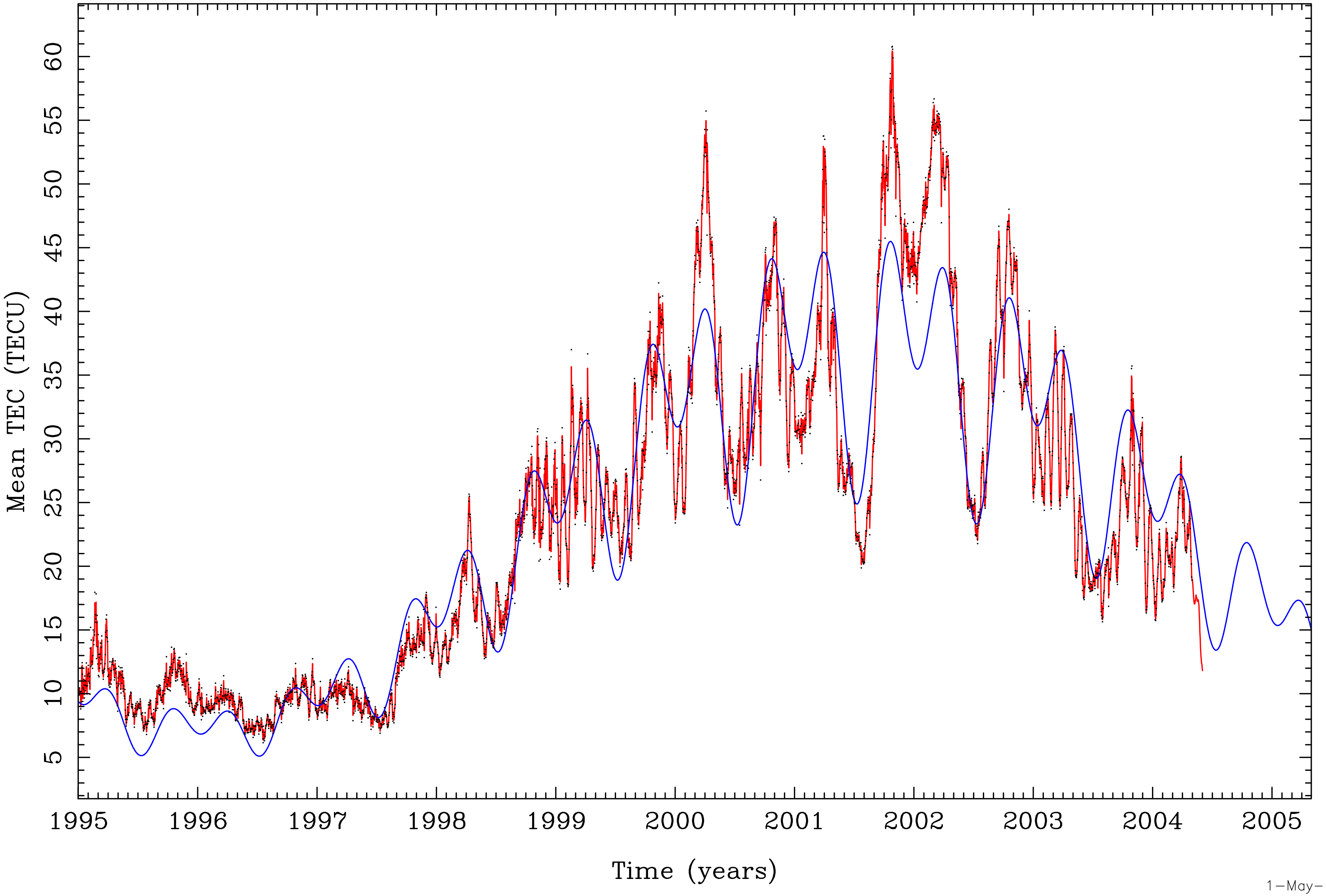
But:

  - Noise $\sigma(L_c) \approx 3\,\sigma(L_1)$.
  - Non-integer ambiguities.



- **Short baselines** ($<$ 10 km): use L_1 and L_2; **longer baselines**: use L_c.
- Simple ionosphere model available in the **broadcast message**.
- Daily regional and global **ionosphere models** (maps) computed by the **IGS**.
- Assumption: all free electrons located in an **infinitesimal layer** at a height of about **450 km** height (**single layer**).
- Benefit: elimination of **scale factor**, improved **ambiguity resolution**.

Development of the Ionosphere derived from GPS



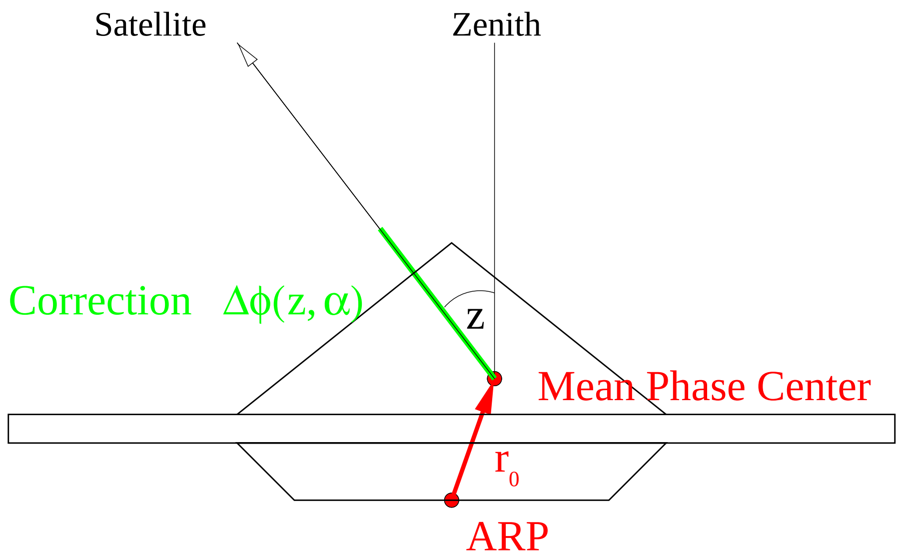
Diversity of GPS Receiver Antennas

Ashtech 700936 mit Radom **Dorne Margolin T (JPL)**

Leica SR399 **Trimble 22020 (Compact L1/L2)**

Antenna Phase Center Offsets and Variations

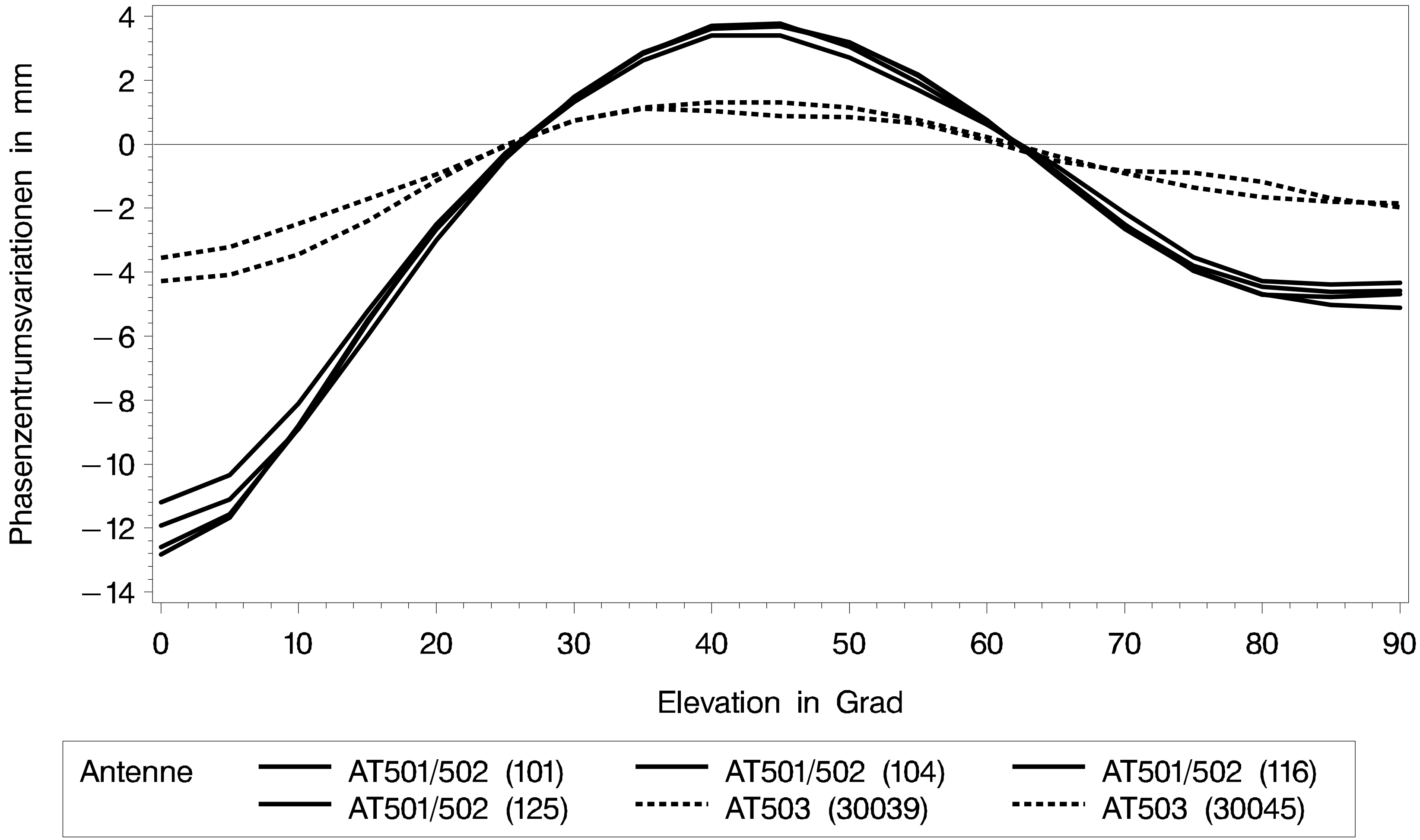
- The antenna phase center is not a fixed point; it varies with the direction of the incoming signal (phase center variations).
- Combining different antenna types: errors of up to **10 cm** in **height**.
- Two corrections to be applied:
  - **mean phase center offsets** r_0
  - **elevation- and azimuth-dependent variations** $\Delta\phi(z, \alpha)$ of the phase center.



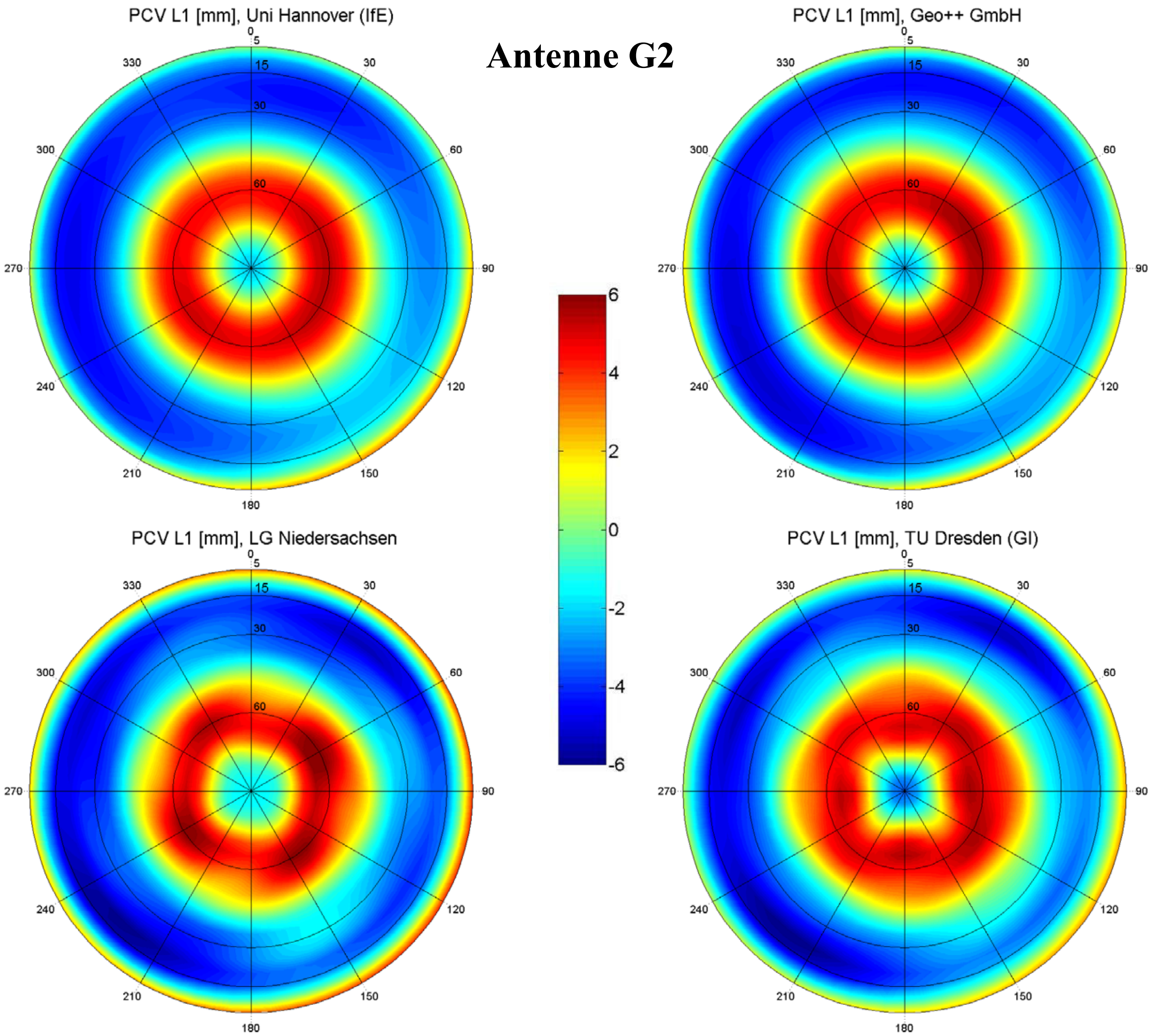
- Antenna offsets and variations available at the **IGS**: `ftp://igscb.jpl.nasa.gov/igscb/station/general/igs_01.pcv`

Elevation-Dependent Phase Center Variations (PCV)

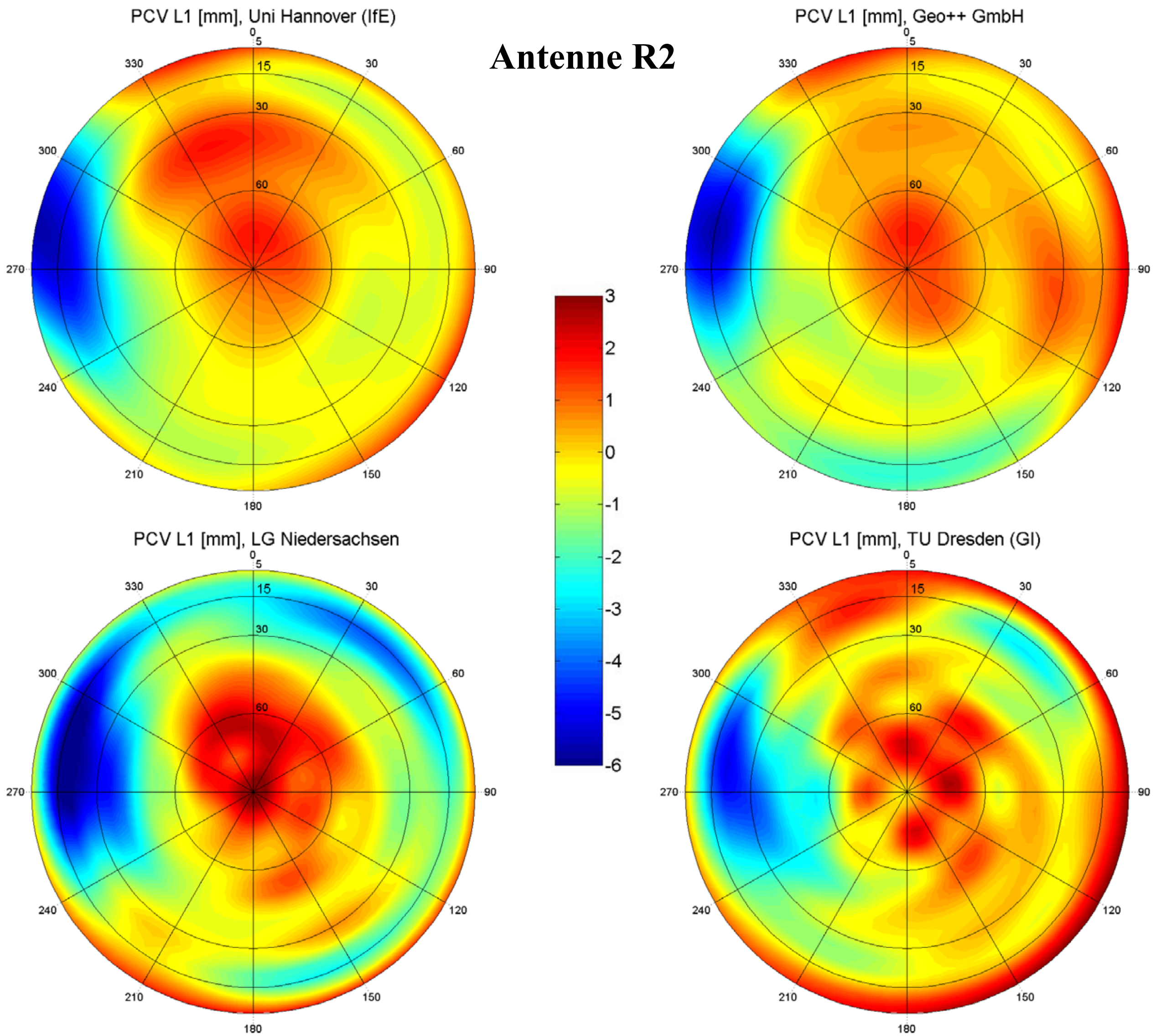
Example: Two different Leica antenna types



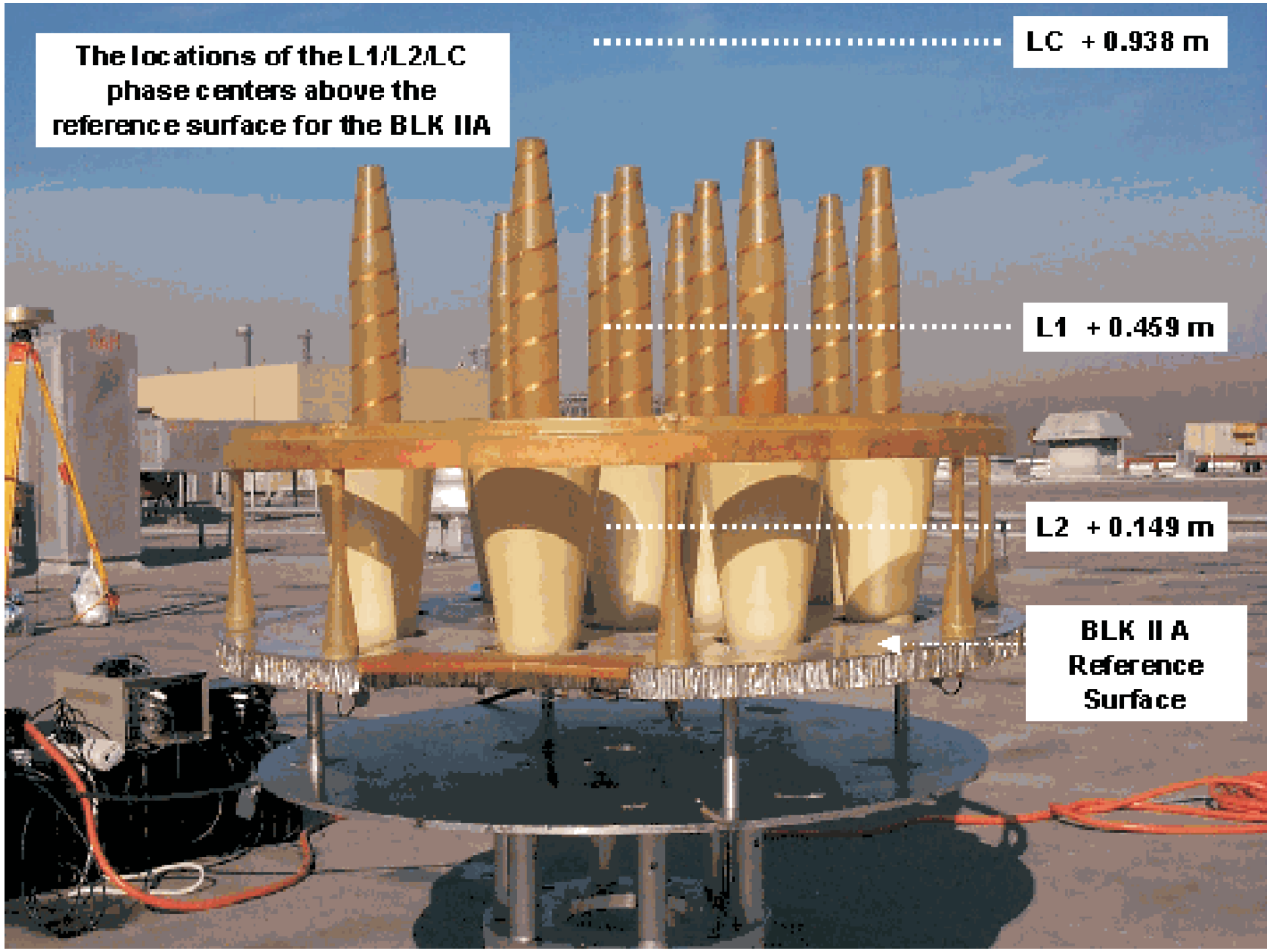
Elevation- and Azimuth-Dependent PCV (Geodetic Antenna)



Elevation- and Azimuth-Dependent PCV (Rover Antenna)

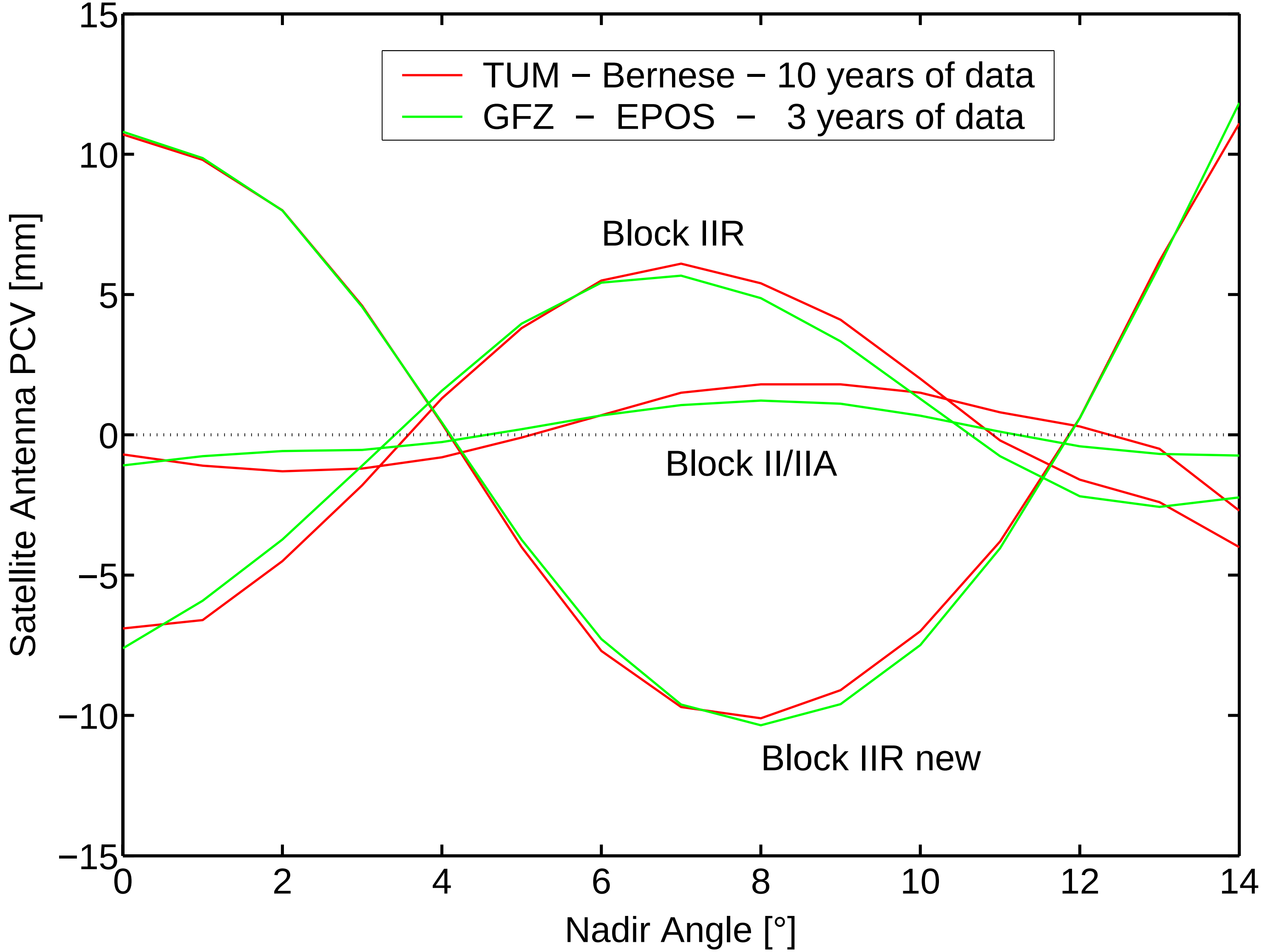


GPS Satellite Antenna Offsets

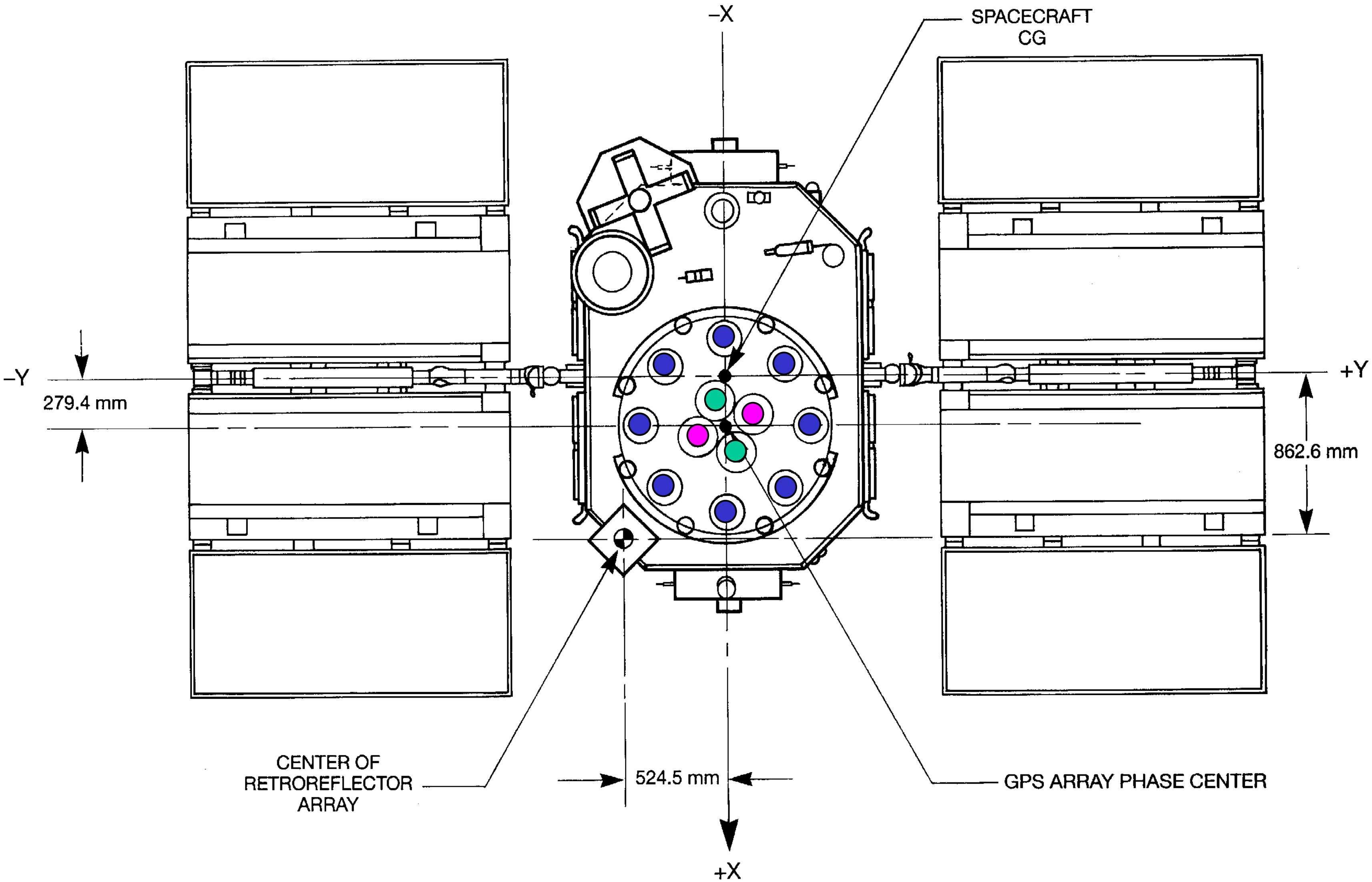


(Gerry Mader, NGS)

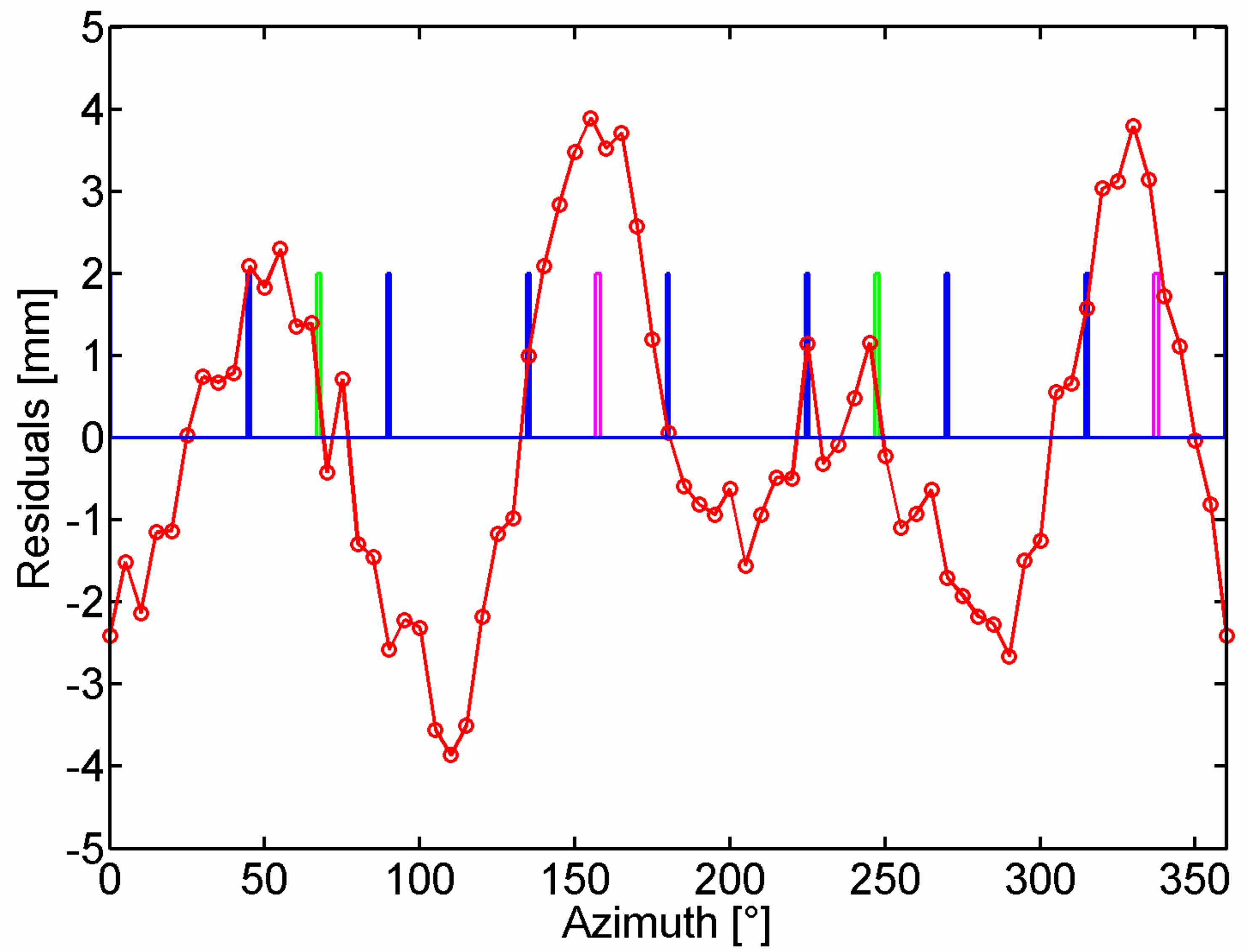
Elevation-Dependent PCV: GPS Satellites



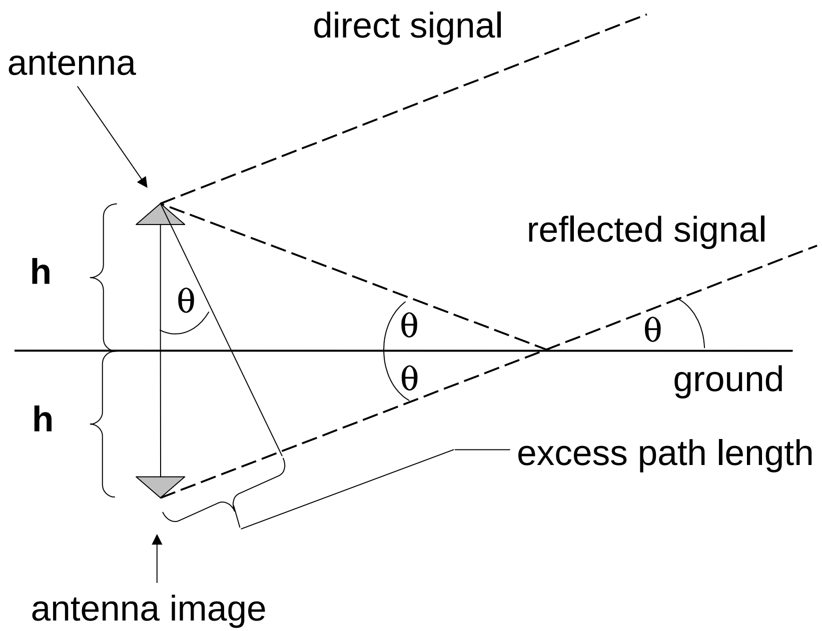
GPS Block II/IIA Antenna Design



Azimuth-Dependent PCV: Block II/IIA Satellites



Multipath Effects



- The antenna receives (apart form the direct signal) **reflections** of the signal from **objects in the vicinity**.
- **Superposition** of direct and indirect signal.
- Systematic deviations in code up to **50 m**, in phase up to **5 cm**.
- Variations with periods of **5-30 min**.
- Most critical at **low elevation**, with **short** observations times.
- **Package of measures**: Antenna design (choke ring, ground plane, . . .); selection of site (free horizon, . . .); longer observation sessions (averaging of effects).