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SMR: 1643/8

WINTER COLLEGE ON OPTICS ON OPTICS AND PHOTONICS
IN NANOSCIENCE AND NANOTECHNOLOGY

(7 - 18 February 2005)

"Biophotonics at the Nanoscale" - II

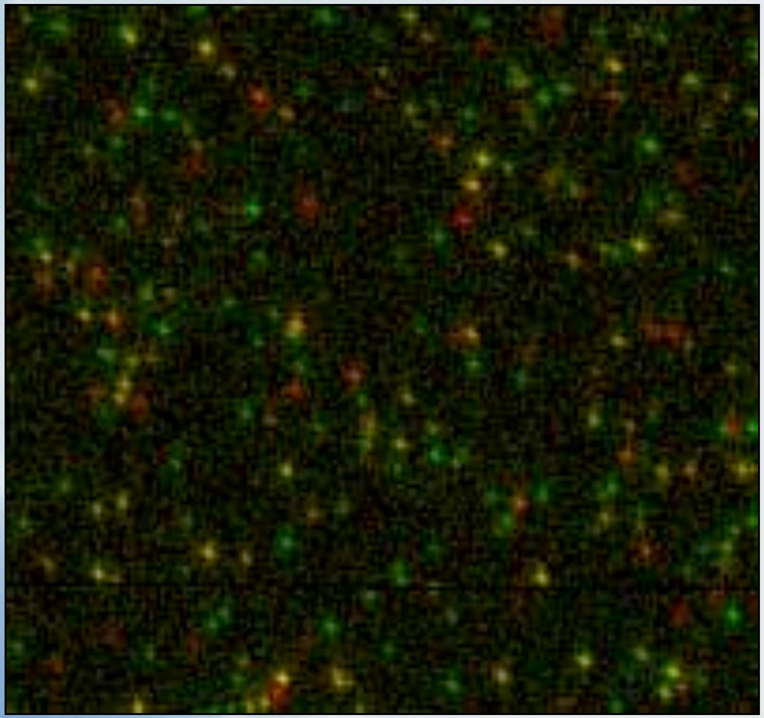
presented by:

L. Novotny
The Institute of Optics
Rochester
U.S.A.

Strada Costiera 11, 34014 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

SP FRET

(single-pair fluorescence resonance energy transfer)

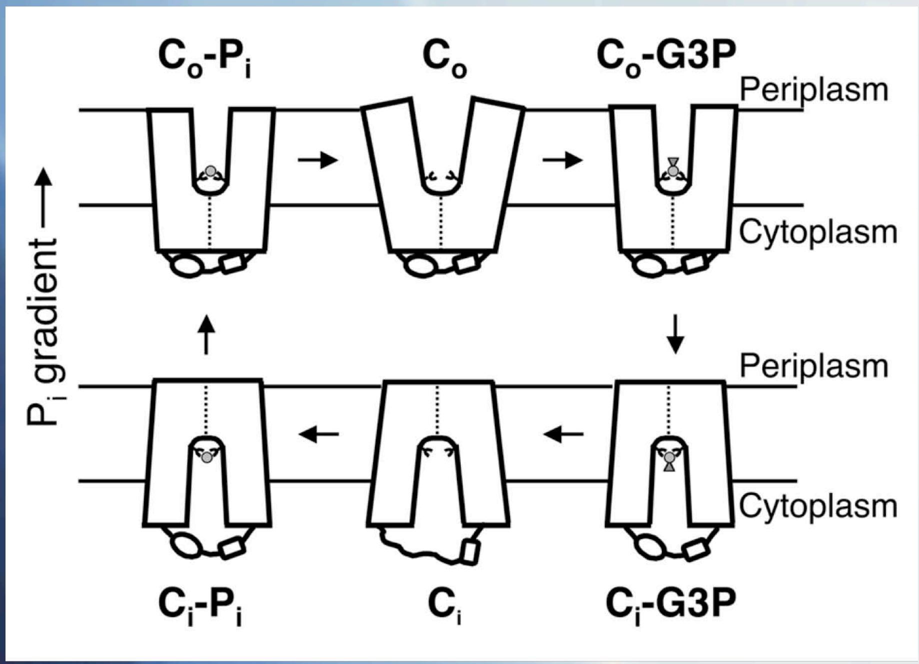


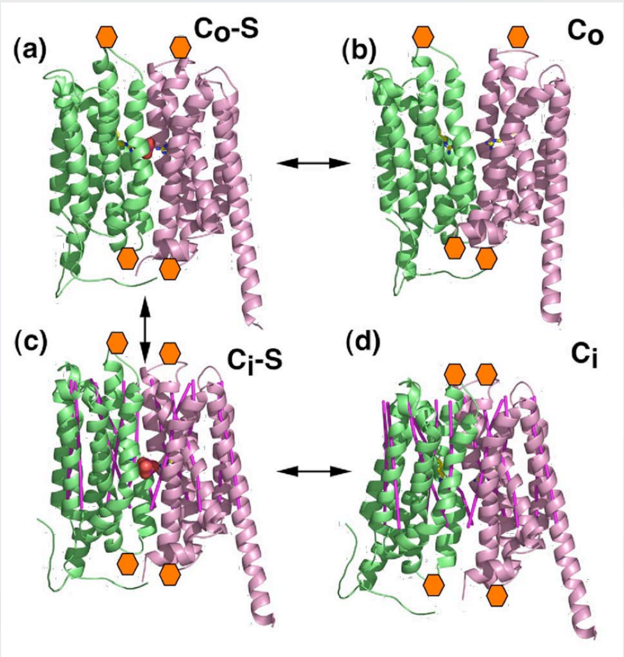
www.nano-optics.org

Lukas Novotny

The Institute of Optics, University of Rochester, Rochester, NY, 14627.

EXAMPLE: ROCKERSWITH MODEL FOR GlpT





Reaction cycle of substrate translocation: Proposed single-binding-site, alternating-access mechanism with a rocker-switch type of movement . Positions of Arg45 and Arg269 are indicated. P_i is represented by a small disk, and the G3P molecule as a small disk and a triangle.

TEXTBOOK (Cambridge Univ. Press)

PRINCIPLES
OF
NANO-OPTICS

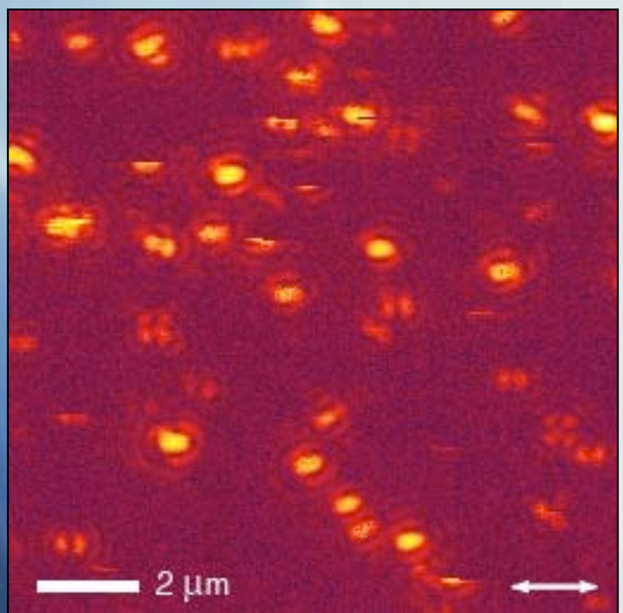
Lukas Novotny

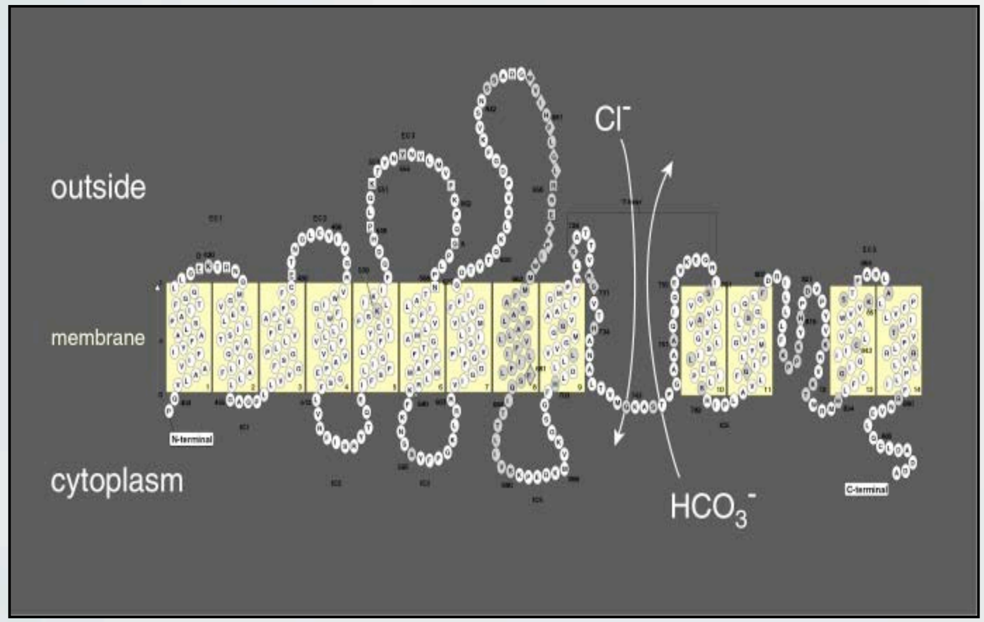
The Institute of Optics
University of Rochester, Rochester, New York

Bert Hecht

Institute of Physics
University of Basel, Basel, Switzerland

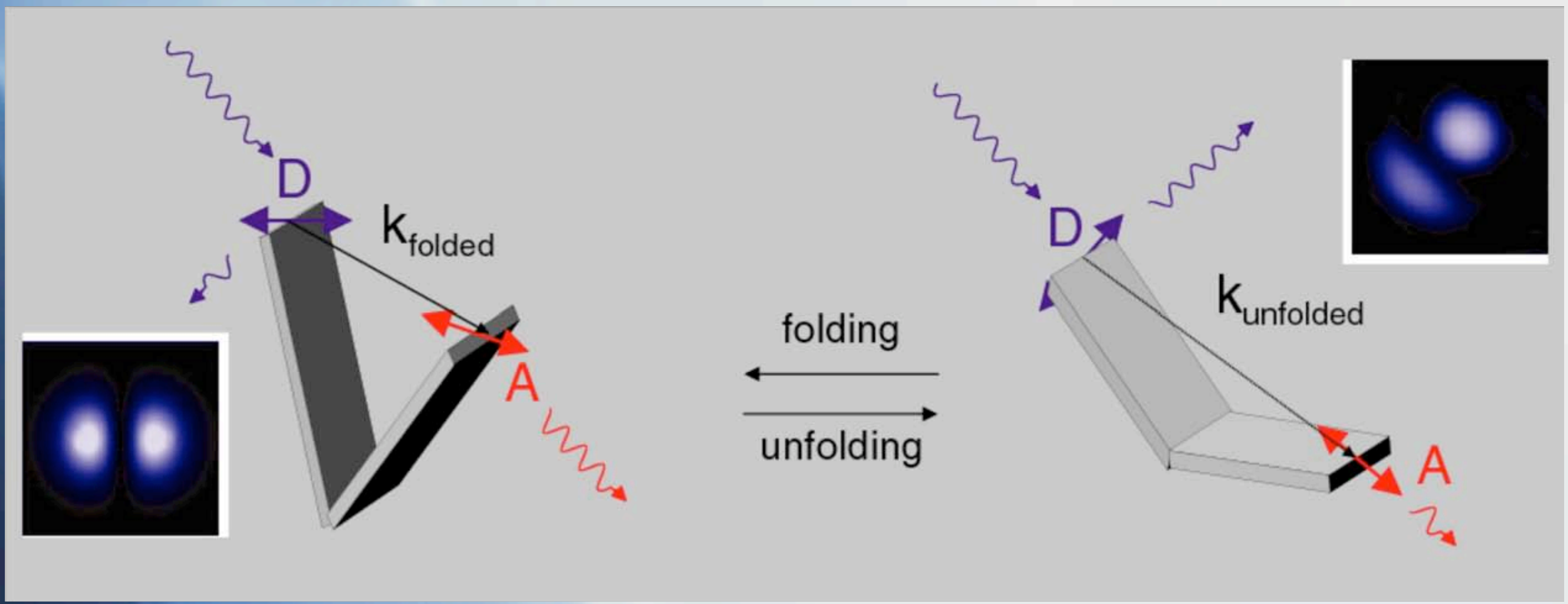
CONFORMATIONAL STATES OF AE1





Collaboration with Prof. P. Knauf (Biochemistry, Rochester)

FOERSTER ENERGY TRANSFER



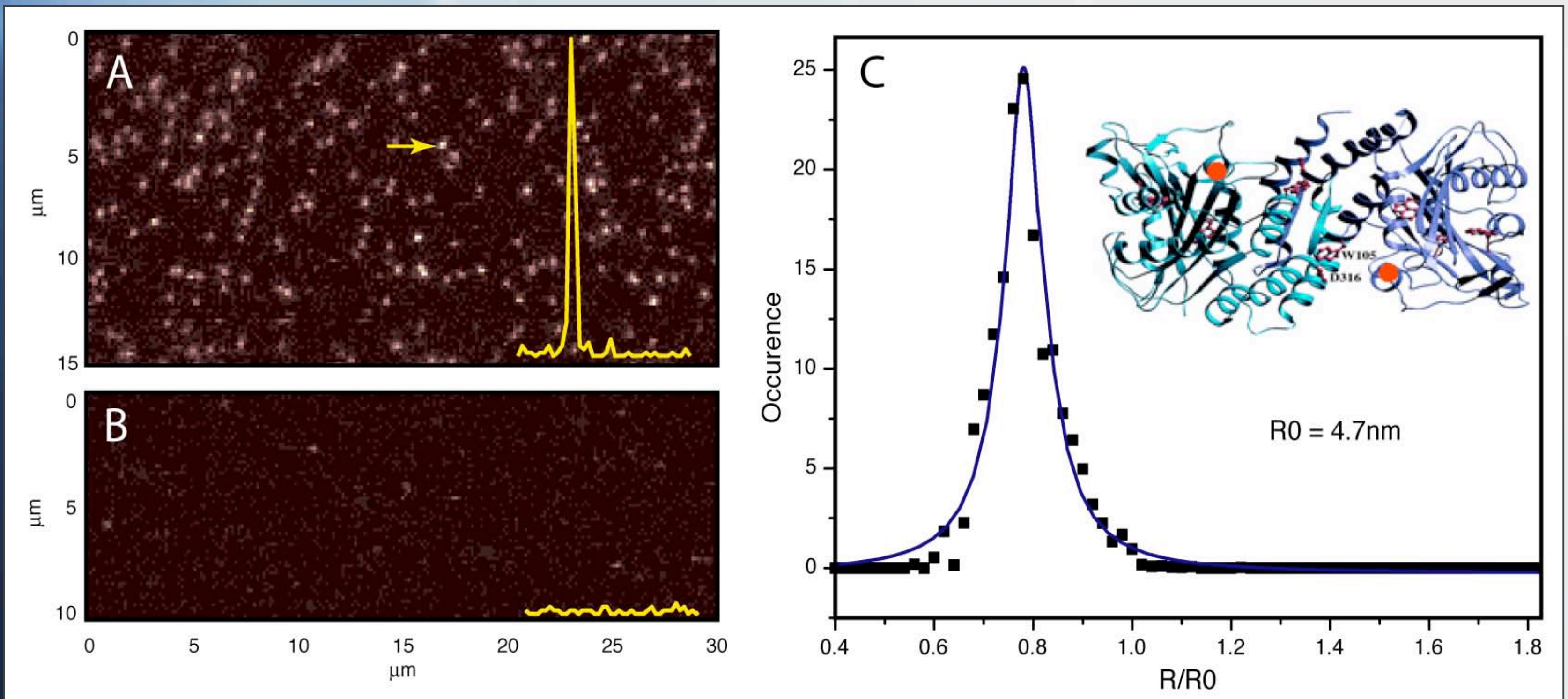
$$R_o^6 = \frac{9000(\ln 10)\kappa^2 \phi_D}{128\pi^2 N_{av}\, n^4}\int_0^\infty F_D(\lambda)\varepsilon_A(\lambda)\lambda^4 d\lambda$$

$$\kappa^2 = [\mathbf{n}_A \cdot \mathbf{n}_D - 3(\mathbf{n}_R \cdot \mathbf{n}_D)(\mathbf{n}_R \cdot \mathbf{n}_A)]^2$$

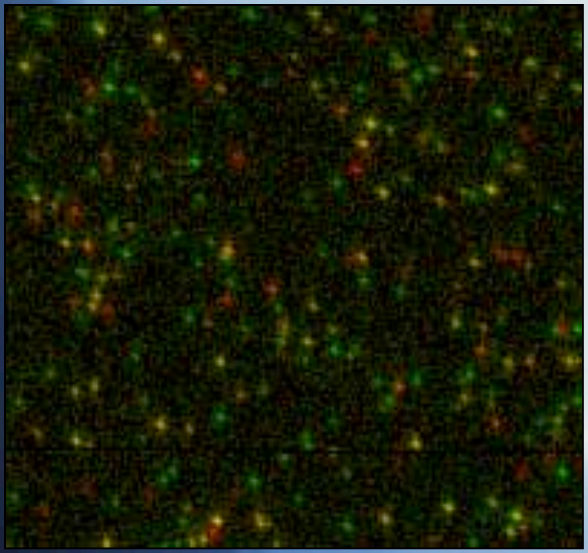
FOERSTER WHO ?

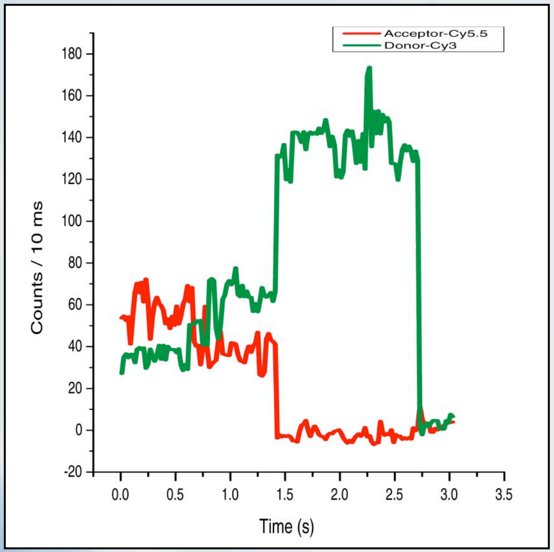
Robert S. Knox (left) and Theodore Foerster (right) preparing for mechanical energy transfer. Springwater, NY, August 1973.

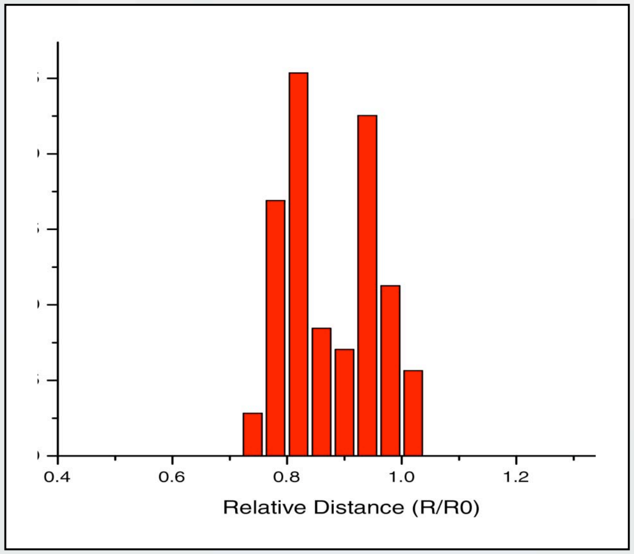
CYTOPLASMIC DOMAIN OF AE1 (CDB3)



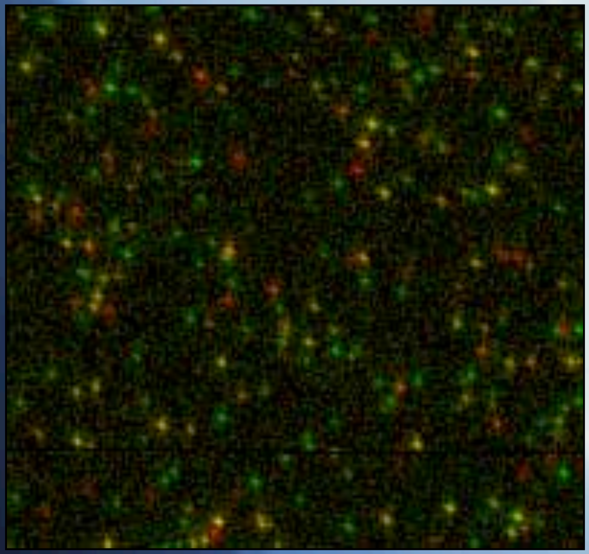
STATIC CONFORMATIONS OF AE1

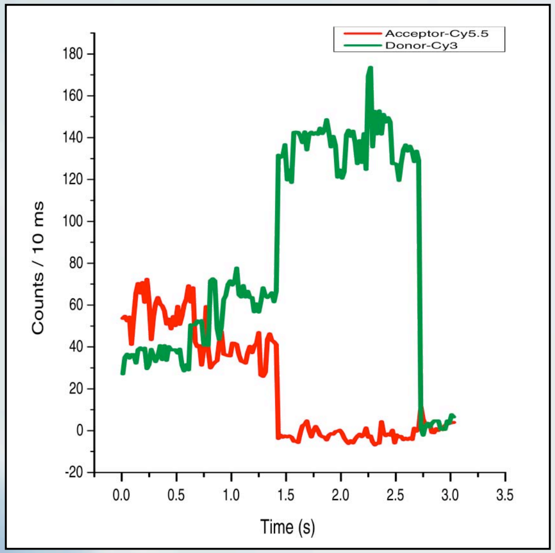


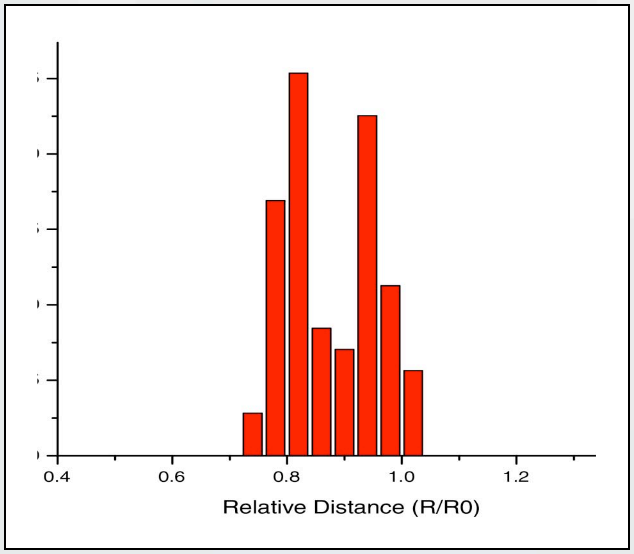




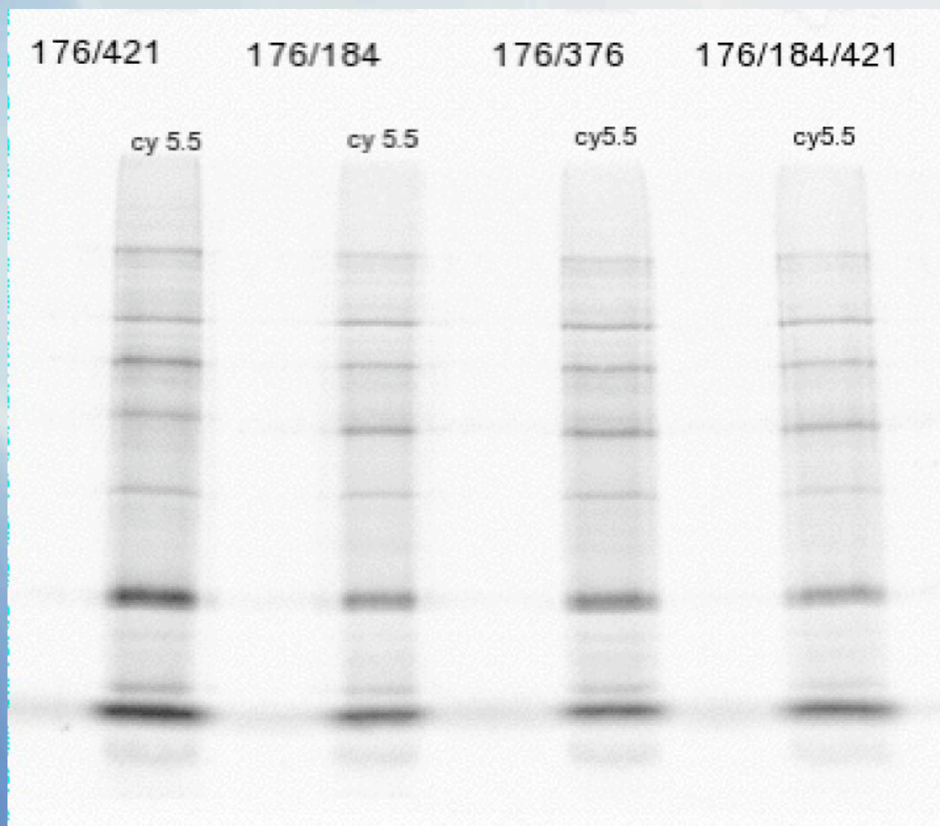
STATIC CONFORMATIONS OF AE1



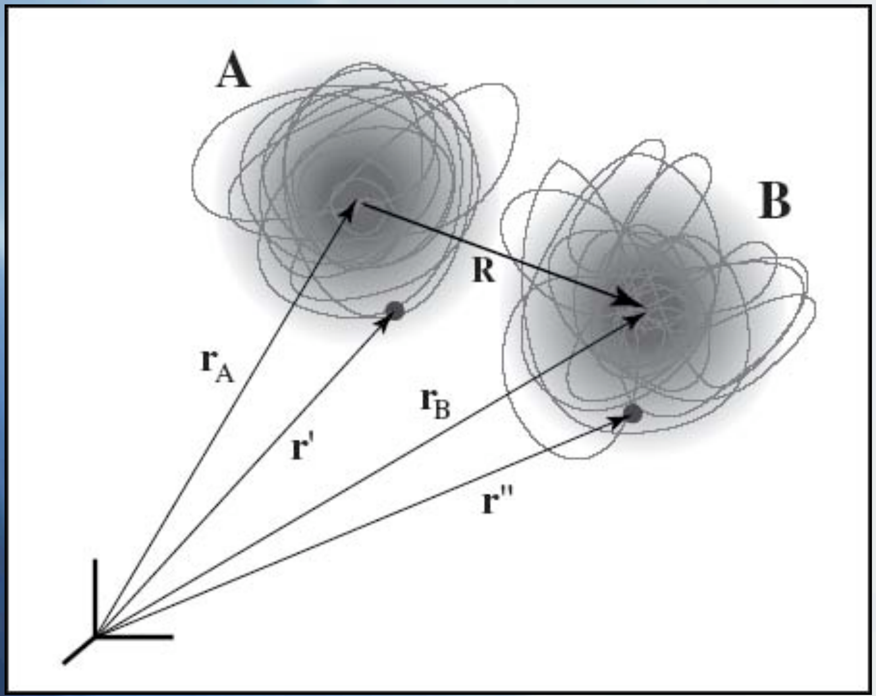




PROBLEM: PURIFICATION



ENERGY TRANSFER THEORY

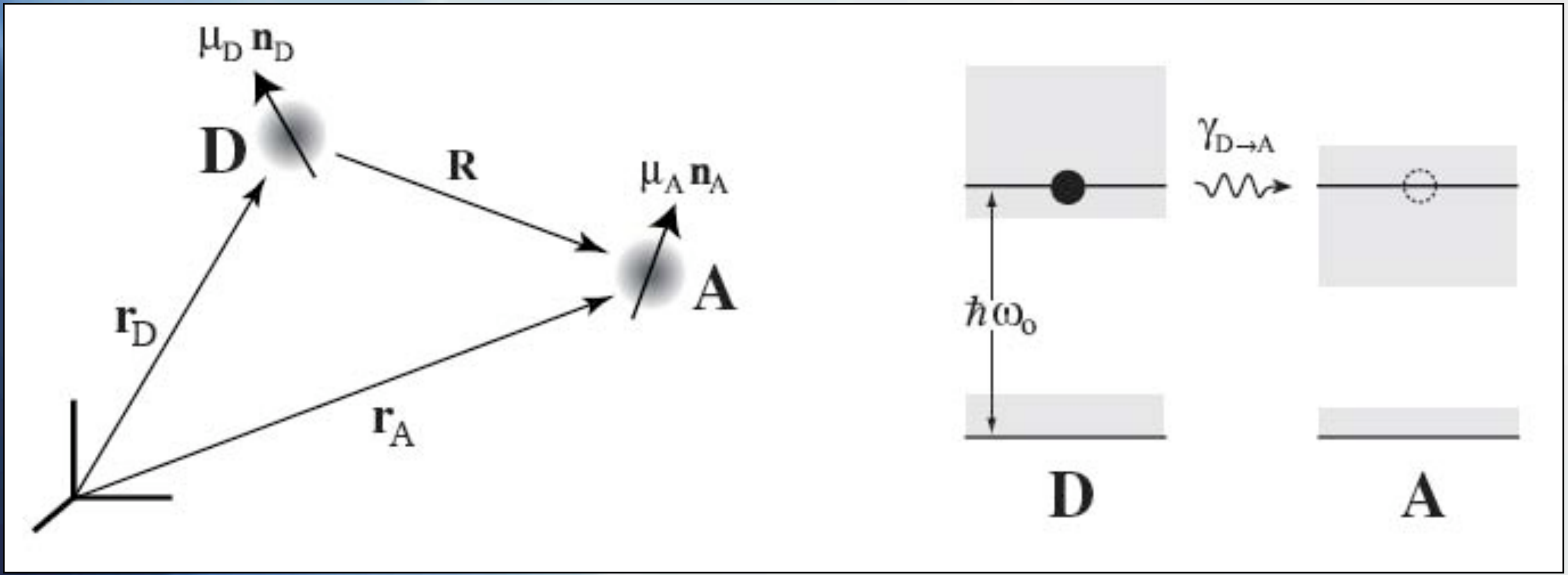


$$q_A = \int \rho_A(\mathbf{r}')\, dV'$$

$$\boldsymbol{\mu}_A = \int \rho_A(\mathbf{r}')\,(\mathbf{r}' - \mathbf{r}_A)\, dV'$$

$$V_{AB}(\mathbf{R}) = \frac{1}{4\pi\varepsilon_o}\left[\frac{q_A q_B}{R} + \frac{q_A\,\boldsymbol{\mu}_B\cdot\mathbf{R}}{R^3} - \frac{q_B\,\boldsymbol{\mu}_A\cdot\mathbf{R}}{R^3} + \frac{R^2\,\boldsymbol{\mu}_A\cdot\boldsymbol{\mu}_B - 3\,(\boldsymbol{\mu}_A\cdot\mathbf{R})\,(\boldsymbol{\mu}_B\cdot\mathbf{R})}{R^5} + \dots\right.$$

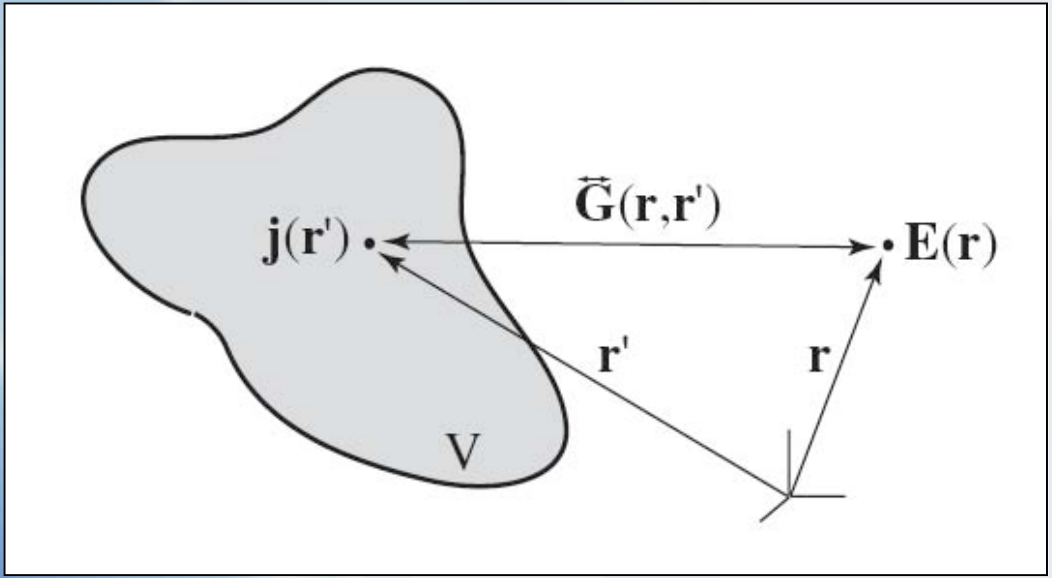
LIMIT: WEAK COUPLING



$$\frac{\gamma_{D\to A}}{\gamma_o} = \frac{P_{D\to A}}{P_o}.$$

$$P_o = \frac{|\mu_D|^2\, n(\omega_o)}{12\pi\,\varepsilon_o\, c^3}\,\omega_o^4$$

GREEN'S FUNCTIONS



$$\mathbf{j}(\mathbf{r}) \;=\; -i\omega\mu\,\delta[\mathbf{r}-\mathbf{r}_o]$$

$$\mathbf{E}(\mathbf{r}) \;=\; \omega^2\mu_o\mu\;\overleftrightarrow{\mathbf{G}}\;(\mathbf{r},\mathbf{r}')\;\boldsymbol{\mu}$$

POWER TRANSFER

$$P_{D\rightarrow A} \;=\; -\frac{1}{2}\int_{V_A} \mathrm{Re}\{\mathbf{j}_A^{*}\cdot\mathbf{E}_D\}\,dV$$

For $\mathbf{j}_A = -i\omega_o \boldsymbol{\mu}_A\, \delta(\mathbf{r}-\mathbf{r}_A)$:

$$P_{D\rightarrow A} \;=\; \frac{\omega_o}{2}\,\mathrm{Im}\{\boldsymbol{\mu}_A^{*}\cdot\mathbf{E}_D(\mathbf{r}_A)\}$$

Induced dipole $\boldsymbol{\mu}_A \;=\; \overleftrightarrow{\alpha}_A\,\mathbf{E}_D(\mathbf{r}_A)$ with $\overleftrightarrow{\alpha}_A = \alpha_A\,\mathbf{n}_A\mathbf{n}_A$:

$$P_{D\rightarrow A} \;=\; \frac{\omega_o}{2}\,\mathrm{Im}\{\alpha_A\}\,\Big|\mathbf{n}_A\cdot\mathbf{E}_D(\mathbf{r}_A)\Big|^2$$

ABSORPTION CROSS-SECTION

$$\sigma(\omega_o) \;=\; \frac{\langle P(\omega_o)\rangle}{I(\omega_o)}$$

$$\sigma(\omega_o) \;=\; \frac{(\omega_o/2)\,\mathrm{Im}\{\alpha(\omega_o)\}\,\langle|\mathbf{n_p}\cdot\mathbf{E}_D|^2\rangle}{(1/2)\,(\varepsilon_o/\mu_o)^{1/2}\,n(\omega_o)\,|\mathbf{E}_D|^2} \;=\; \frac{\omega_o}{3}\sqrt{\frac{\mu_o}{\varepsilon_o}}\,\frac{\mathrm{Im}\{\alpha(\omega_o)\}}{n(\omega_o)}$$

$$P_{D\to A} \;=\; \frac{3}{2}\sqrt{\frac{\varepsilon_o}{\mu_o}}\,n(\omega_o)\,\sigma_A(\omega_o)\,\Big|\mathbf{n}_A\cdot\mathbf{E}_D(\mathbf{r}_A)\Big|^2$$

FIELD OF DONOR EVALUATED AT ACCEPTOR

$$\mathbf{E}_D(\mathbf{r}_A) \;=\; \omega_o^2\,\mu_o\,\overleftrightarrow{\mathbf{G}}(\mathbf{r}_D,\mathbf{r}_A)\,\boldsymbol{\mu}_D$$

Short-hand:

$$T(\omega_o) \;=\; 16\pi^2 k^4 R^6 \left|\mathbf{n}_A\cdot\overleftrightarrow{\mathbf{G}}(\mathbf{r}_D,\mathbf{r}_A)\,\mathbf{n}_D\right|^2$$

where $R = |\mathbf{r}_D - \mathbf{r}_A|$

$k = (\omega_o/c)\,n(\omega_o).$

$$\frac{\gamma_{D\to A}}{\gamma_o} \;=\; \frac{9c^4}{8\pi R^6}\,\frac{\sigma_A(\omega_o)}{n^4(\omega_o)\,\omega_o^4}\,T(\omega_o) \;=\; \frac{9c^4}{8\pi R^6}\int_0^\infty \frac{\delta(\omega-\omega_o)\,\sigma_A(\omega)}{n^4(\omega)\,\omega^4}\,T(\omega)\,d\omega$$

INCLUDE EMISSION SPECTRUM

$$\int_0^\infty \delta(\omega-\omega_o)\, d\omega = 1 \longrightarrow \int_0^\infty f_D(\omega)\, d\omega = 1$$

$$\frac{\gamma_{D\to A}}{\gamma_o} = \frac{9c^4}{8\pi R^6}\int_0^\infty \frac{f_D(\omega)\,\sigma_A(\omega)}{n^4(\omega)\,\omega^4}\, T(\omega)\, d\omega$$

Evaluate T(ω) :

$$\begin{aligned} T(\omega) = {} & (1-k^2R^2+k^4R^4)\,(\mathbf{n}_A\cdot\mathbf{n}_D)^2 + \\ & (9+3k^2R^2+k^4R^4)\,(\mathbf{n}_R\cdot\mathbf{n}_D)^2(\mathbf{n}_R\cdot\mathbf{n}_A)^2 + \\ & (-6+2k^2R^2-2k^4R^4)\,(\mathbf{n}_A\cdot\mathbf{n}_D)(\mathbf{n}_R\cdot\mathbf{n}_D)(\mathbf{n}_R\cdot\mathbf{n}_A) \end{aligned}$$

$$\langle T(\omega)\rangle = \frac{2}{3} + \frac{2}{9}k^2R^2 + \frac{2}{9}k^4R^4$$

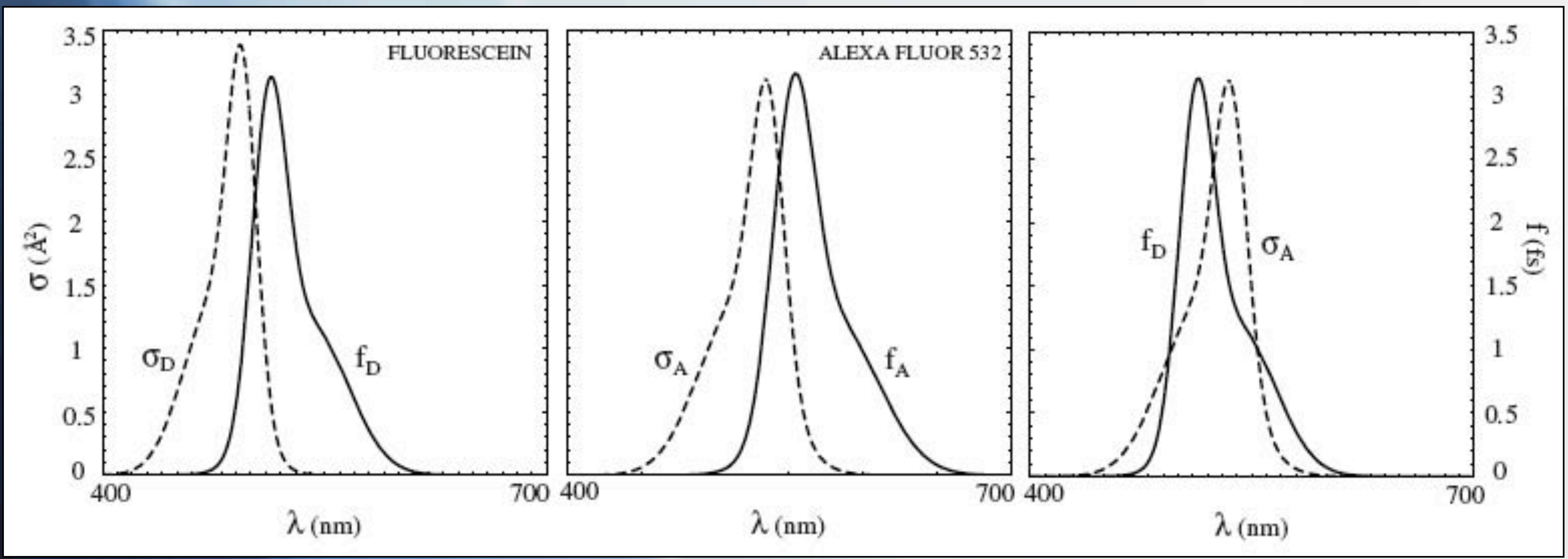
ONLY NEAR-FIELD TERMS

$$\frac{\gamma_{D\to A}}{\gamma_o} = \left[\frac{R_o}{R}\right]^6 , \qquad R_o^6 = \frac{9\,c^4\,\kappa^2}{8\pi}\int_0^\infty \frac{f_D(\omega)\;\sigma_A(\omega)}{n^4(\omega)\,\omega^4}\,d\omega$$

$$\kappa^2 = [\mathbf{n}_A\!\cdot\!\mathbf{n}_D - 3\,(\mathbf{n}_R\!\cdot\!\mathbf{n}_D)(\mathbf{n}_R\!\cdot\!\mathbf{n}_A)]^2$$

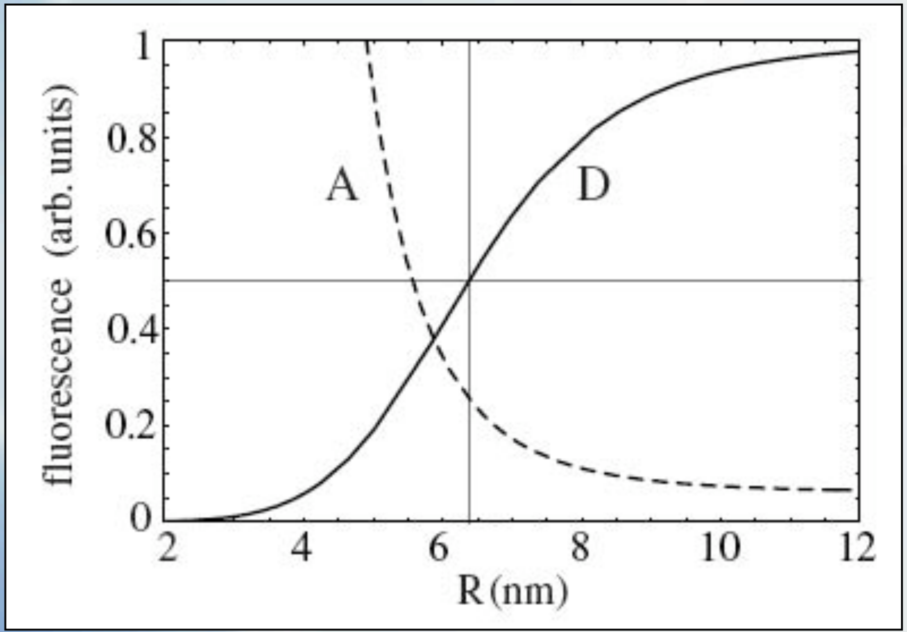
EXAMPLE:
Alexa Fluor 532 (A) and Fluorescein (D)

Fitting : $$\sum_{n=1}^{N} A_n e^{-(\lambda-\lambda_n)^2/\Delta\lambda_n^2}$$



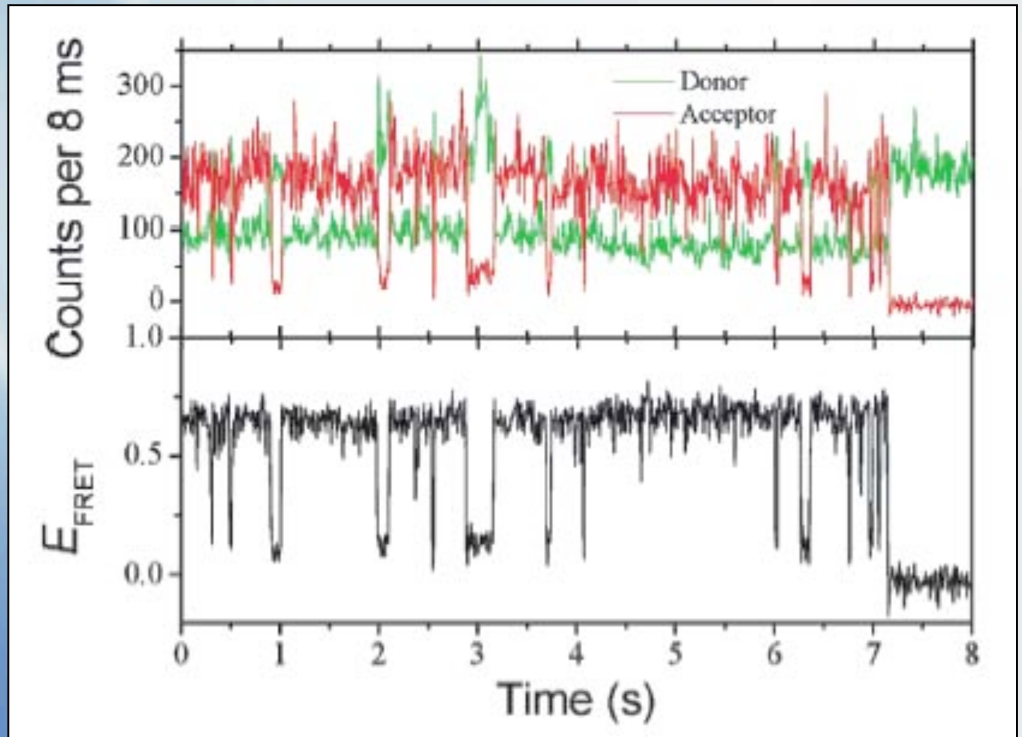
Water: R_o = 6.3nm ; Air: R_o = 7.6nm

DONOR / ACCEPTOR FLUORESCENCE



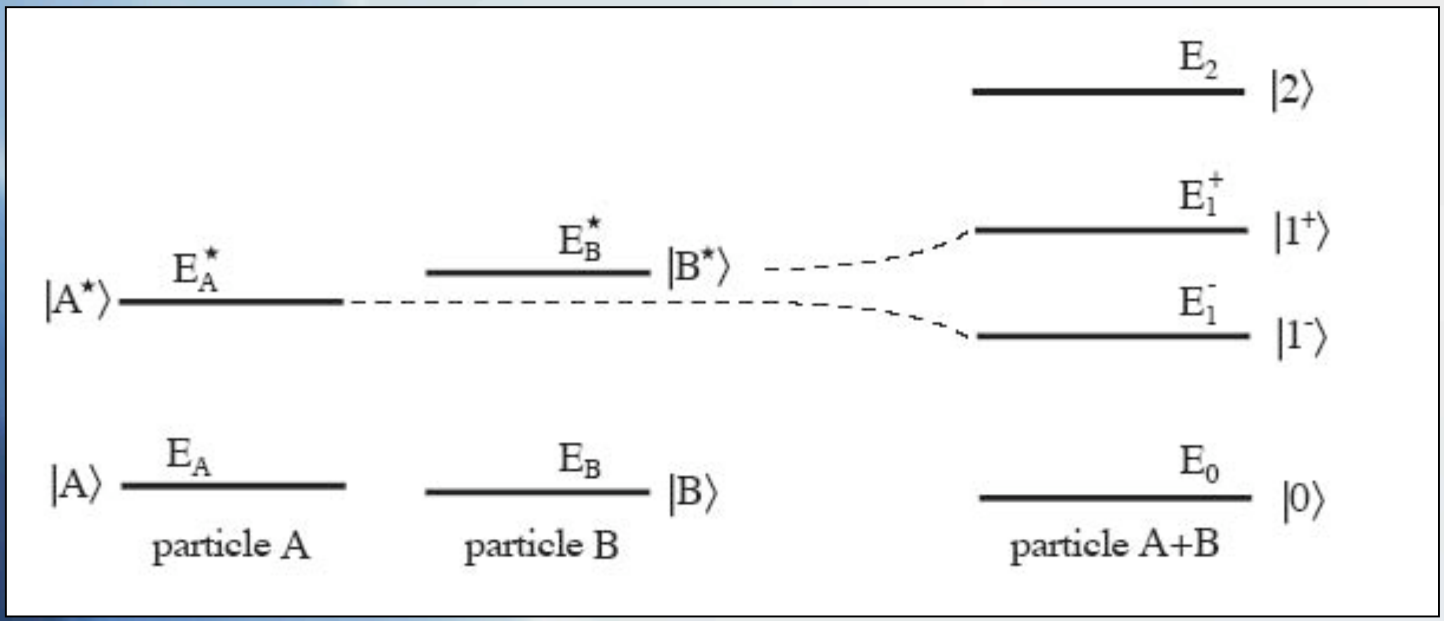
$$E = \frac{P_o}{P_o + P_{D\to A}} = \frac{1}{1 + (\gamma_o/\gamma_{D\to A})} = \frac{1}{1 + (R/R_o)^6}$$

EXAMPLE: DNA HOLIDAY JUNCTION



Taekjip Ha (Urbana-Champaign)

STRONG COUPLING REGIME



$$
\begin{aligned}
\left|1^{+}\right\rangle &= \cos\alpha\,|A^{\star}B\rangle + \sin\alpha\,|AB^{\star}\rangle \\
\left|1^{-}\right\rangle &= \sin\alpha\,|A^{\star}B\rangle - \cos\alpha\,|AB^{\star}\rangle
\end{aligned}
$$

SUMMARY

FRET IS A POWERFUL TECHNIQUE TO MEASURE NANOSCALE DISTANCES

EFFECT CAN BE UNDERSTOOD BASED ON ANTENNA THEORY

- Gives correct Result for Near-field, Intermediate Field, and Farfield
- Strong Coupling Regime requires QM