



The Abdus Salam
International Centre for Theoretical Physics



SMR: 1643/10

*WINTER COLLEGE ON OPTICS ON OPTICS AND PHOTONICS
IN NANOSCIENCE AND NANOTECHNOLOGY*

(7 - 18 February 2005)

*"Optical Response of Semiconductor
Quantum Dots"*

presented by:

G. Cassabois
Laboratoire Pierre Aigrain
Paris
France

These are preliminary lecture notes, intended only for distribution to participants.

Optical response of semiconductor quantum dots

G. Cassabois

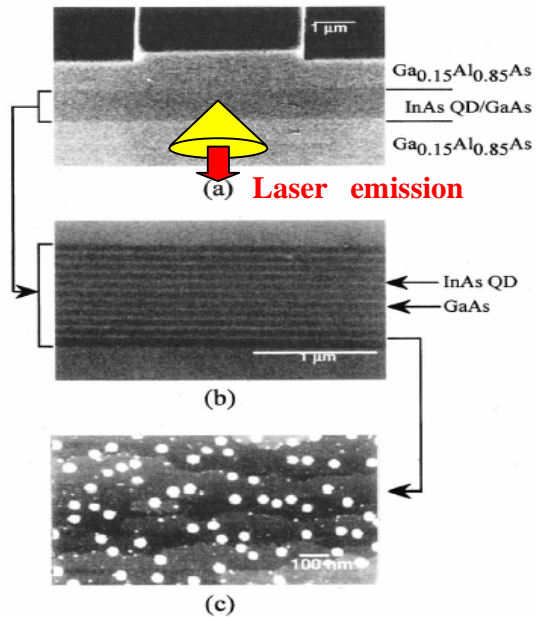
Laboratoire Pierre Aigrain - Ecole Normale Supérieure - Paris

Outline

- fundamental optical properties
- single-photon emission
- spontaneous emission control



Doing optics with quantum dots



Nakamura
J. Appl. Phys. **94**, 1184 (2003)

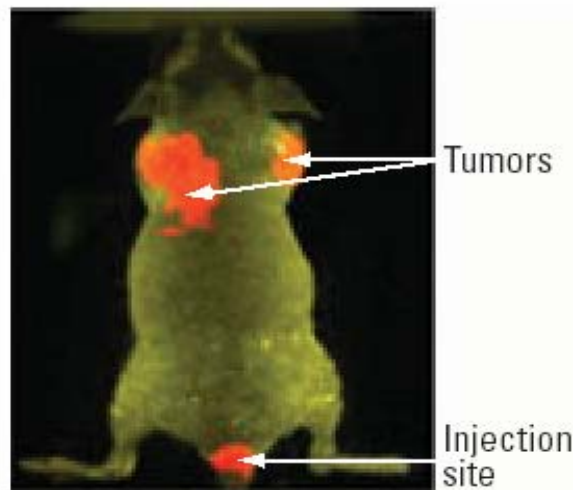
QD laser

Opto-electronics

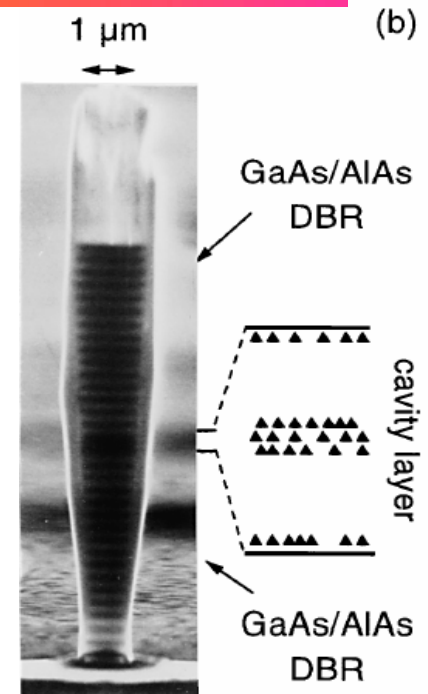
Bio-physics

Biological sensors

In vivo imaging of cancer cells in a living mouse



Gao, Nature Biot. **22**, 969 (2004)



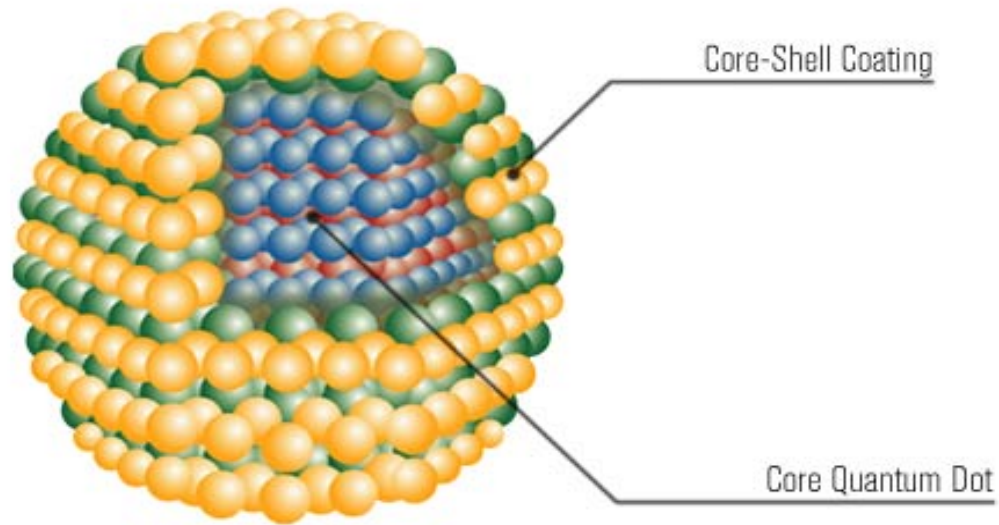
Gérard, PRL **81**, 1110 (1998)

Single photon source

Quantum optics

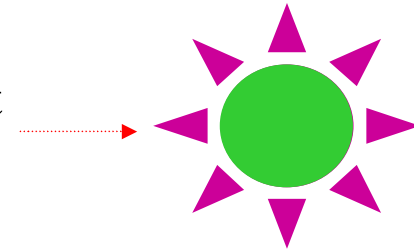


Colloidal QD



- **Chemically synthesized** quantum dots in colloids (charged particles, inverse micelle)

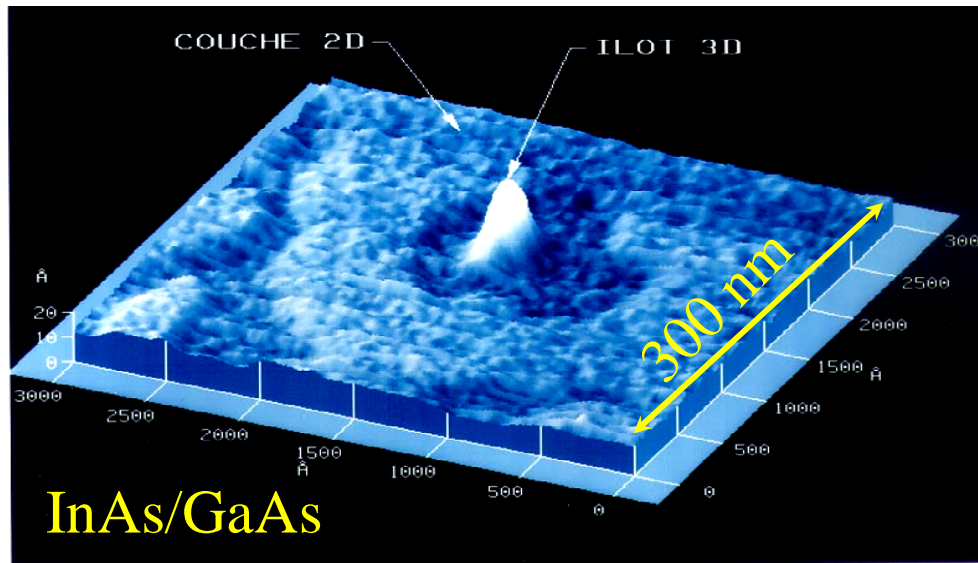
surfactant molecule



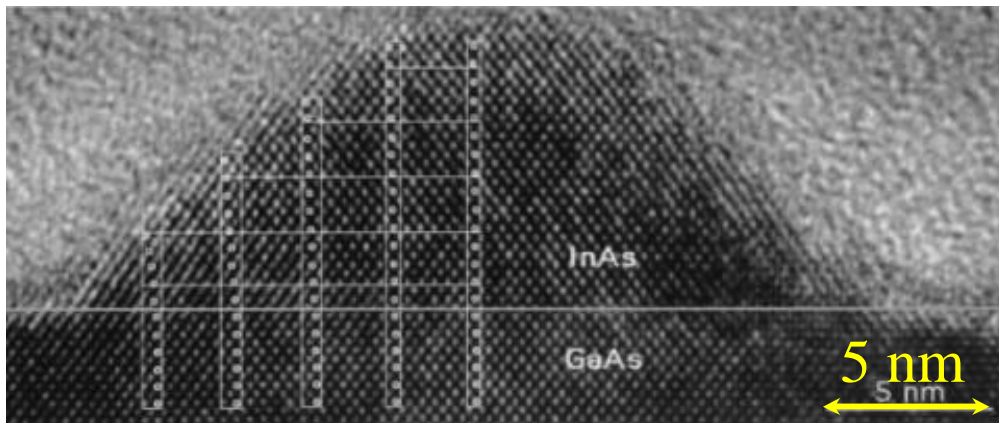
- **Core** material : CdS, CdSe, CdTe
- **Core-Shell** structure
 - surface passivation
 - suppression of non-radiative decay channels
- **Bio-conjugation** of QD with adequate coating



Epitaxially grown QD



J. M. Gérard *et al.*, J. Cryst. Growth **150**, 351 (1995)



- **Molecular Beam Epitaxy**
- **Self-assembled** quantum dots with areal density $< 10^3 \mu\text{m}^{-2}$
- **Stranski-Krastanow** growth mode
 - **strain accumulation** because of lattice mismatch
 - **critical** thickness of wetting layer ~ 1.7 monolayer for InAs
 - **spontaneous** formation of pyramids
- **various** systems : InAs/GaAs, InAs/InP, Ge/Si, GaN/AlN, CdTe/MnTe, CdSe/ZnSe ...



<http://www.wsi.tum.de/E24/research/nanostructures/>

Fundamental optical properties

Outline

- light-matter interaction
- interband and intraband transitions
- single QD spectroscopy
- specific solid-state features



Light-matter interaction

- Single particle Hamiltonian of an electron in a crystal reads

$$H_0 = \frac{\vec{p}^2}{2m} + V(\vec{r})$$

- Interaction of an electron with an electromagnetic field is obtained from **generalized impulsion**

$$\vec{p} = m \vec{v} - e \vec{A} \quad (\text{Lagrangian mechanics})$$

- Single particle Hamiltonian becomes

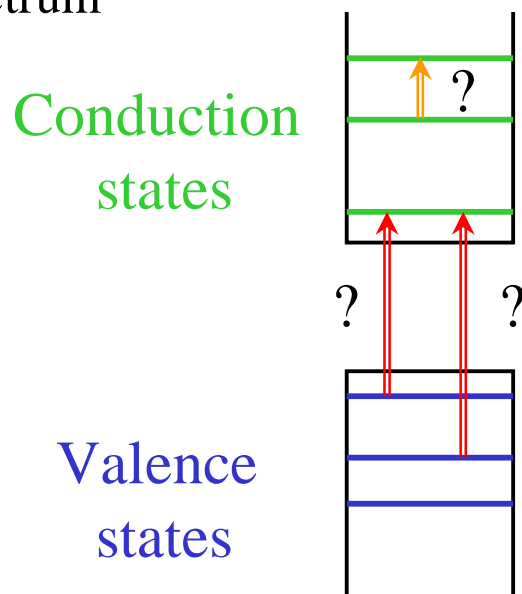
$$\begin{aligned} H &= \frac{(\vec{p} + e \vec{A})^2}{2m} + V(\vec{r}) \\ &= \underbrace{\frac{\vec{p}^2}{2m} + V(\vec{r})}_{H_0} + \underbrace{\frac{e}{2m}(\vec{p} \cdot \vec{A} + \vec{A} \cdot \vec{p})}_{\text{dipole interaction}} + \underbrace{\frac{e \vec{A}^2}{2m}}_{\text{diamagnetic shift}} \end{aligned}$$

the **second term** simplifies because $\vec{p} \cdot \vec{A} - \vec{A} \cdot \vec{p} = \frac{\hbar}{i} \text{div} \vec{A} = 0$
(Coulomb gauge)



Dipole interaction

- **Optical transitions** between electronic states $|i\rangle \longrightarrow |f\rangle$ induced by the dipole interaction term $H_I = \frac{e}{m}(\vec{A} \cdot \vec{p})$
- Their strength depends on the **matrix element** $\langle f|H_I|i\rangle$
- In semiconductor QDs, the 3D confinement leads to a **discrete** electronic spectrum



One expects an absorption spectrum formed by a set of discrete lines

What are the selection rules ?

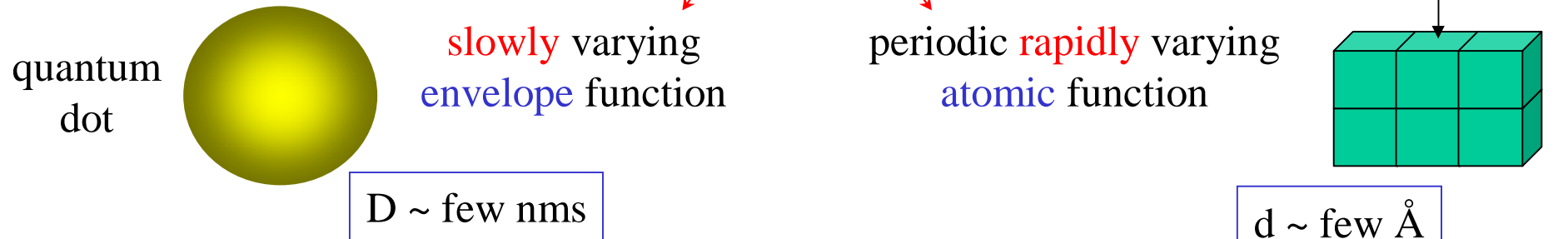
$$\langle f|H_I|i\rangle \neq 0$$



Envelope function approximation

- In the envelope function approximation, electron wave-functions are given by

$$\psi_\alpha(\vec{r}) = \phi_\alpha(\vec{r})u_\alpha(\vec{r})$$



- Matrix element $\langle f|H_I|i\rangle \sim \frac{e}{m} \vec{A} \cdot \langle f|\vec{p}|i\rangle$ (Long wavelength approx.)

$$\vec{p} = \frac{\hbar}{i} \vec{\nabla}$$

2 terms

$$\sim \frac{e}{m} \vec{A} \cdot \int d\vec{r} \phi_f^*(\vec{r}) u_f^*(\vec{r}) \vec{p} (\phi_i(\vec{r}) u_i(\vec{r}))$$

$$\sim \frac{e}{m} \vec{A} \cdot \int d\vec{r} \phi_f^*(\vec{r}) u_f^*(\vec{r}) u_i(\vec{r}) \vec{p} (\phi_i(\vec{r}))$$

$$+ \frac{e}{m} \vec{A} \cdot \int d\vec{r} \phi_f^*(\vec{r}) u_f^*(\vec{r}) \phi_i(\vec{r}) \vec{p} (u_i(\vec{r}))$$



Intraband and interband transitions

$$\langle f|H_I|i\rangle \sim \frac{e}{m} \vec{A} \cdot \int d\vec{r} \phi_f^*(\vec{r}) \vec{p}(\phi_i(\vec{r})) \cdot \frac{1}{\Omega_0} \int_{\Omega_0} d\vec{r} u_f^*(\vec{r}) u_i(\vec{r}) = (1)$$

$$+ \frac{e}{m} \vec{A} \cdot \int d\vec{r} \phi_f^*(\vec{r}) \phi_i(\vec{r}) \cdot \frac{1}{\Omega_0} \int_{\Omega_0} d\vec{r} u_f^*(\vec{r}) \vec{p}(u_i(\vec{r})) = (2)$$

- These two terms can be rewritten as:

$$\langle f|H_I|i\rangle \sim \frac{e}{m} \vec{A} \cdot \langle \phi_f | \vec{p} | \phi_i \rangle \cdot \langle u_f | u_i \rangle = (1)$$

$$+ \frac{e}{m} \vec{A} \cdot \langle \phi_f | \phi_i \rangle \cdot \langle u_f | \vec{p} | u_i \rangle = (2)$$

- The first term (1) accounts for **INTRABAND** transitions

$$\text{if } u_f = u_i = u_c \text{ OR } u_f = u_i = u_v \text{ then } \langle u_f | \vec{p} | u_i \rangle = 0$$

- The second term (2) accounts for **INTERBAND** transitions

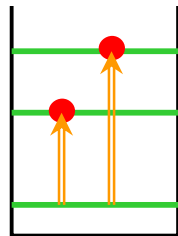
$$\text{if } u_f \neq u_i \text{ then } \langle u_f | u_i \rangle = 0$$



Intraband transitions

$$P_{|i\rangle \rightarrow |f\rangle} = \frac{2\pi}{\hbar} \left| \frac{e}{m} \vec{A} \cdot \langle \phi_f | \vec{p} | \phi_i \rangle \right|^2 \delta(E_f - E_i - \hbar\omega)$$

Conduction states



Polarization of incident electromagnetic field

Energy conservation

Valence states



- Absorption in **energy** range of 10-100 meV
- wavelength** range of 10-100 μm

Mid-Long IR

- Assumption of **initial** state occupied and **final** state empty

Only electrons

or Only holes

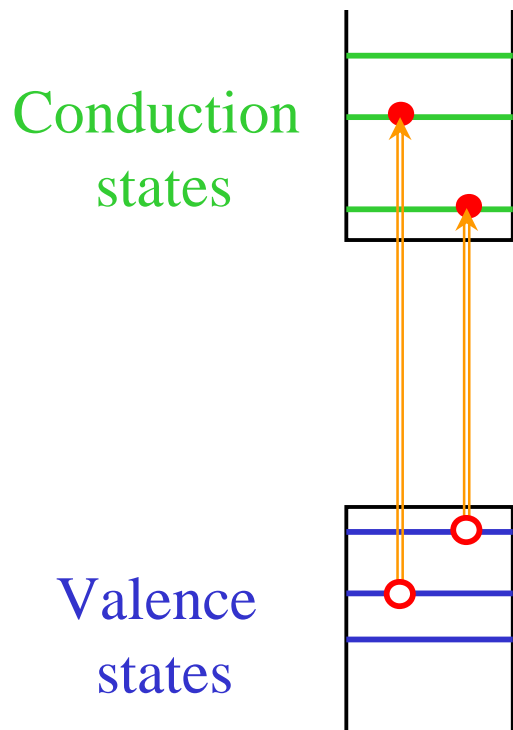


QDs must be doped



Interband transitions

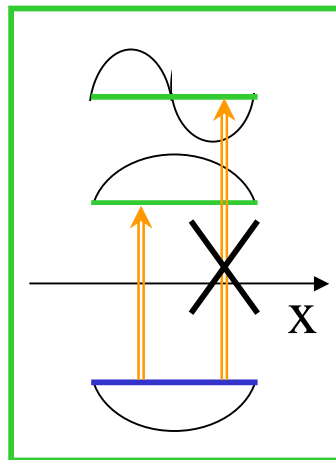
$$P_{|i\rangle \rightarrow |f\rangle} = \frac{2\pi}{\hbar} \left| \frac{e}{m} \cdot \underbrace{\langle \phi_f | \phi_i \rangle}_{\text{Parity selection rule}} \cdot \underbrace{\vec{A} \cdot \langle u_f | \vec{p} | u_i \rangle}_{\text{Polarization selection rule}} \right|^2 \underbrace{\delta(E_f - E_i - \hbar\omega)}_{\text{Energy conservation}}$$



Parity selection rule

Polarization selection rule

Energy conservation



Circularly polarized light propagating along (Oz), couples electronic states

$$\Delta J_z = \pm 1$$

In zinc-blende type semiconductors,

$$|electron\rangle = |J = 1/2, J_z = \pm 1/2, \dots\rangle$$

$$|heavy\ hole\rangle = |J = 3/2, J_z = \pm 3/2, \dots\rangle$$

$$|light\ hole\rangle = |J = 3/2, J_z = \pm 1/2, \dots\rangle$$

• Absorption in

energy range of

1- 2.5 eV

wavelength range of

0.4 - 1 μm

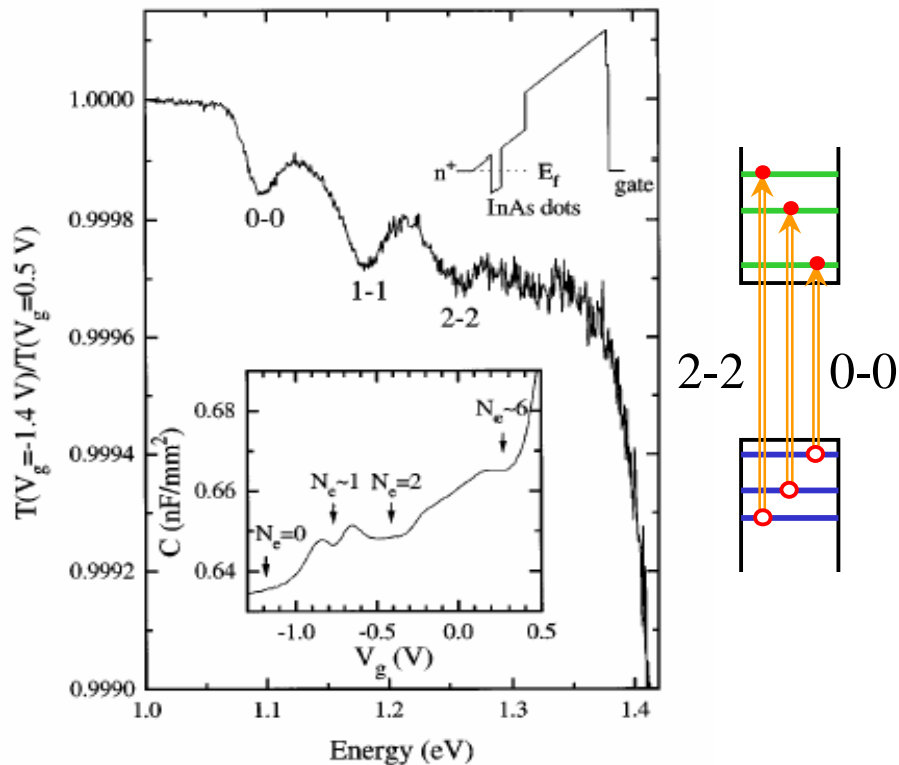
Electron AND hole

Visible -NIR



Interband absorption spectroscopy

Transmission experiment
in an ensemble of InAs QDs



Warburton, PRL **79**, 5282 (1997)

- Observation of **three** interband transitions (0-0, 1-1, and 2-2) in absorption spectrum

absorption spectrum
= set of discrete lines ??

- **Large spectral** width of the lines

$$\Delta E \sim 50 \text{ meV}$$

- Coupling to radiation leads to a **natural linewidth** of

$$\Gamma \sim 1 \mu\text{eV}$$

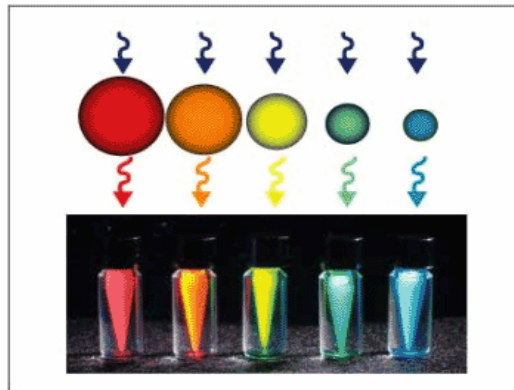


size fluctuations



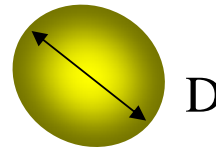
Inhomogeneous broadening

- Optical transition energy

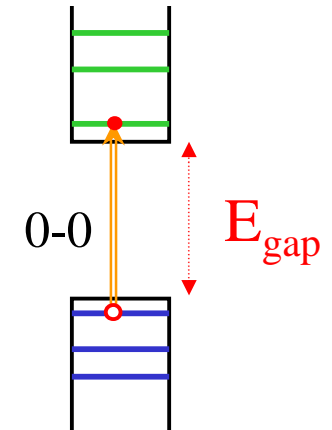


$$E_{n-n} \sim E_{\text{gap}} + E_{\text{confinement}}(n,n)$$

quantum dot



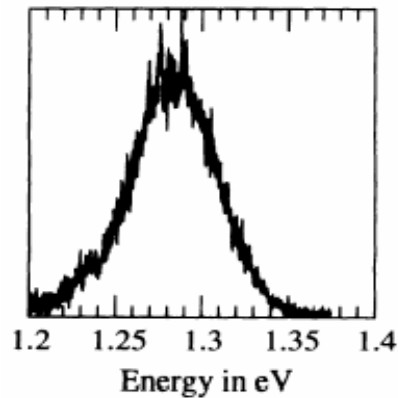
scales like $1/D^2$



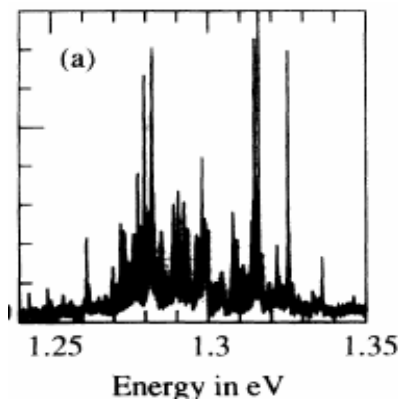
size fluctuations \rightarrow inhomogeneous broadening

- From ensemble measurements to single QD spectroscopy

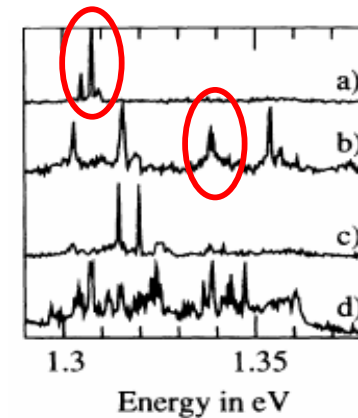
Marzin, PRL 73, 716 (1994)



1000 QDs



10-100 QDs



isolated QDs



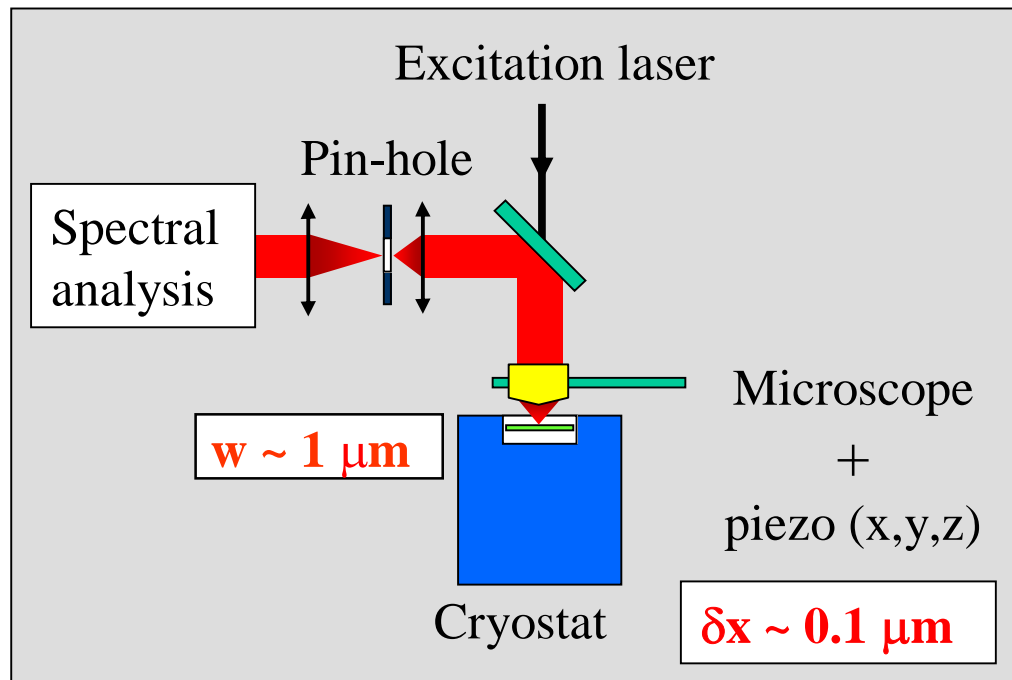
Single QD spectroscopy

- **Near-field** optical spectroscopy (see [previous lectures](#))
- **Far-field** optical spectroscopy

Microscope in confocal geometry

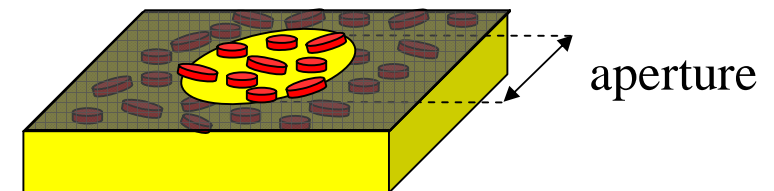
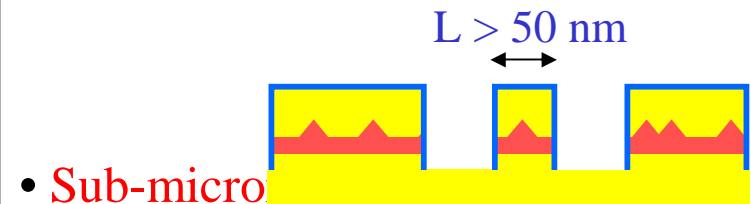


Sample preparation



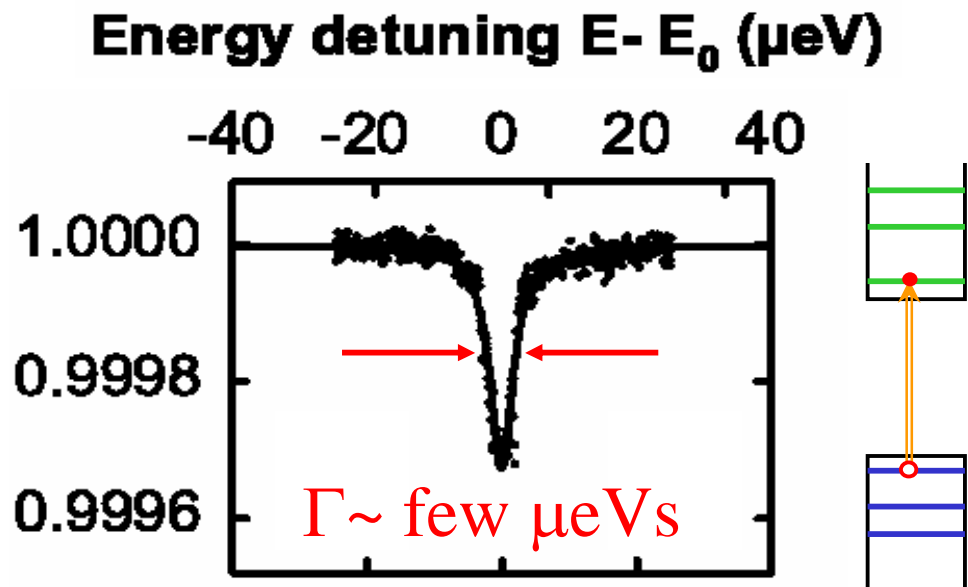
- **Dilute QD arrays**
 - dispersed solutions of colloidal QDs
 - specific epitaxial growth modes

- **Mesa structures**



Absorption of a single QD

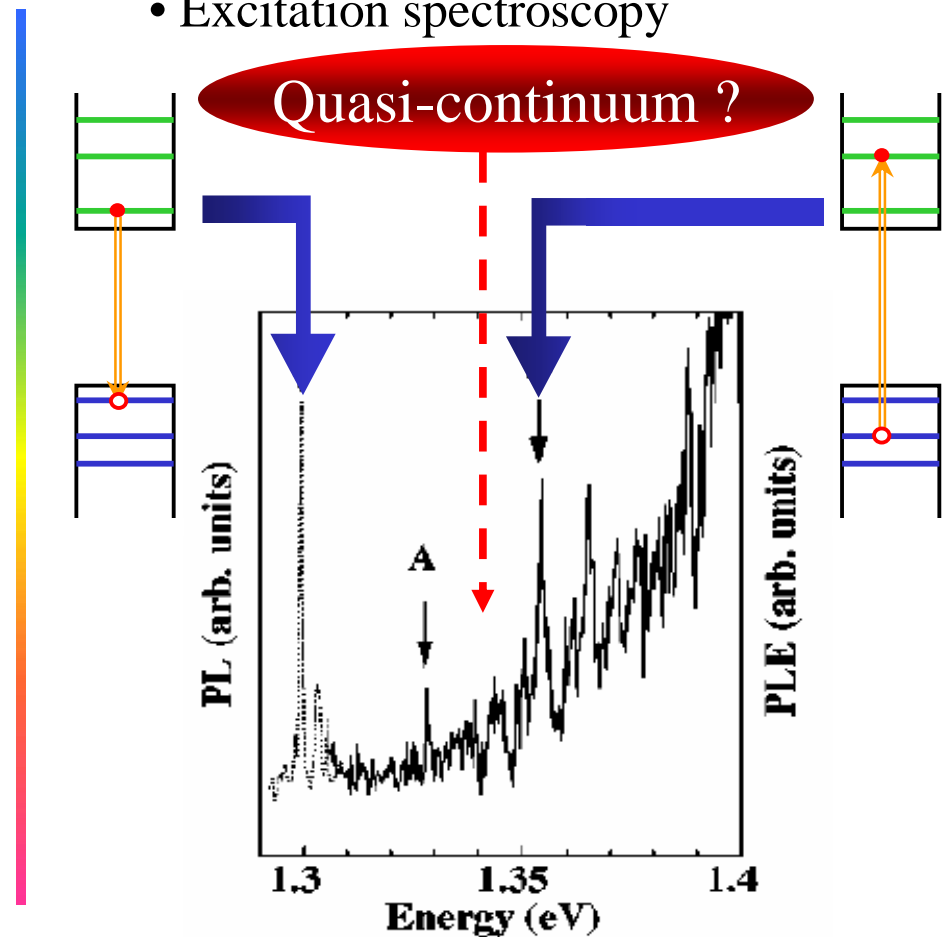
- Absorption spectroscopy



Högele, PRL 93, 217401 (2004)

- **Fundamental** transition = sharp line
- **Homogeneous** linewidth \sim natural linewidth

- Excitation spectroscopy

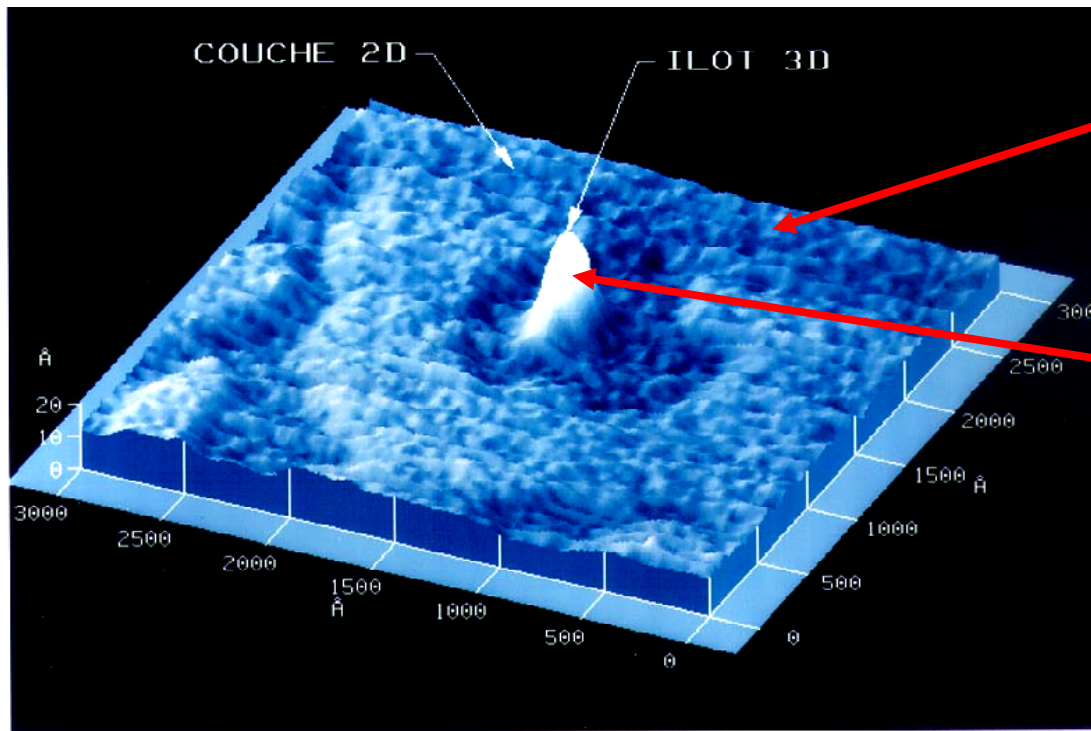


Kammerer, PRB 65, 033313 (2001)



InAs/GaAs self-assembled QD

- Morphology of an epitaxially grown QD



J. M. Gérard *et al.*, *J. Cryst. Growth* **150**, 351 (1995)

InAs **wetting layer**

- thickness ~ 1 nm
(1.7 ML)

InAs **quantum dot**

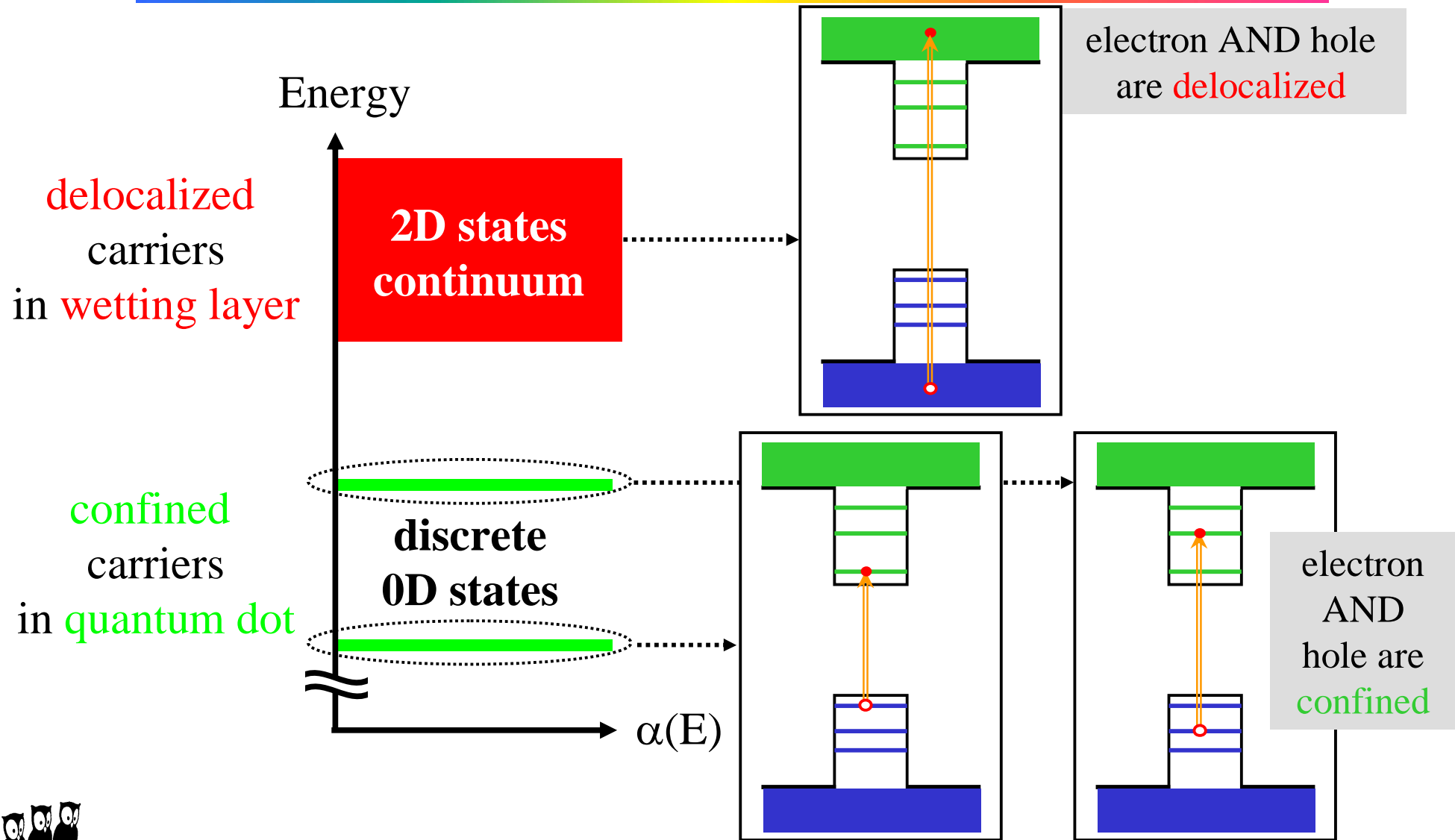
- diameter ~ 20 nm
- height ~ 3 nm



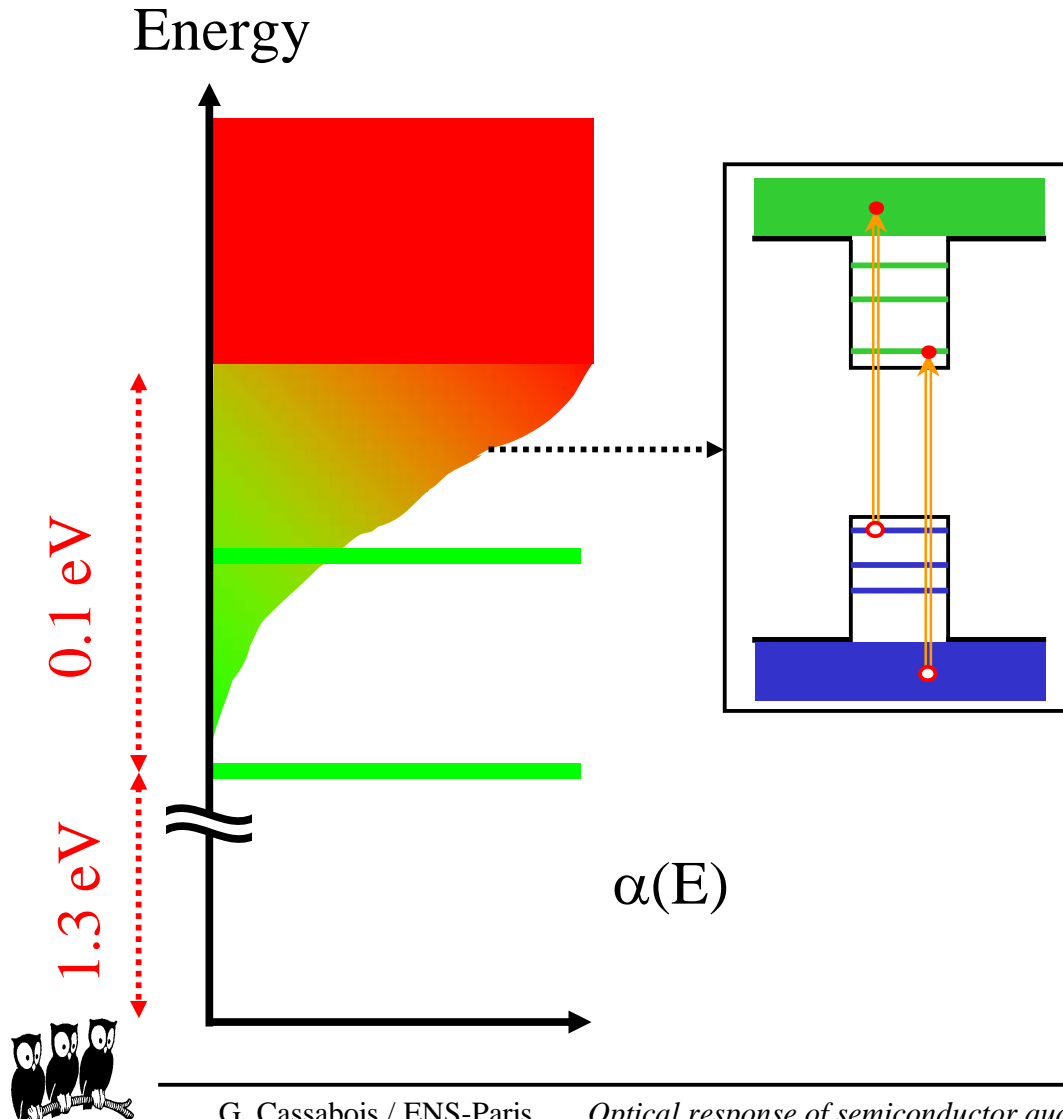
3D confinement of
electrons and holes



2D versus 0D electronic states



2D-0D interband transitions



- Interband transitions involve (electron-hole) pairs

one shall not miss the cases

where { hole confined
electron delocalized

and

{ hole delocalized
electron confined

- **Weak** transition $|\langle \phi_{2D} | \phi_{0D} \rangle| \ll 1$
- But one carrier belongs to a quasi-continuum of states

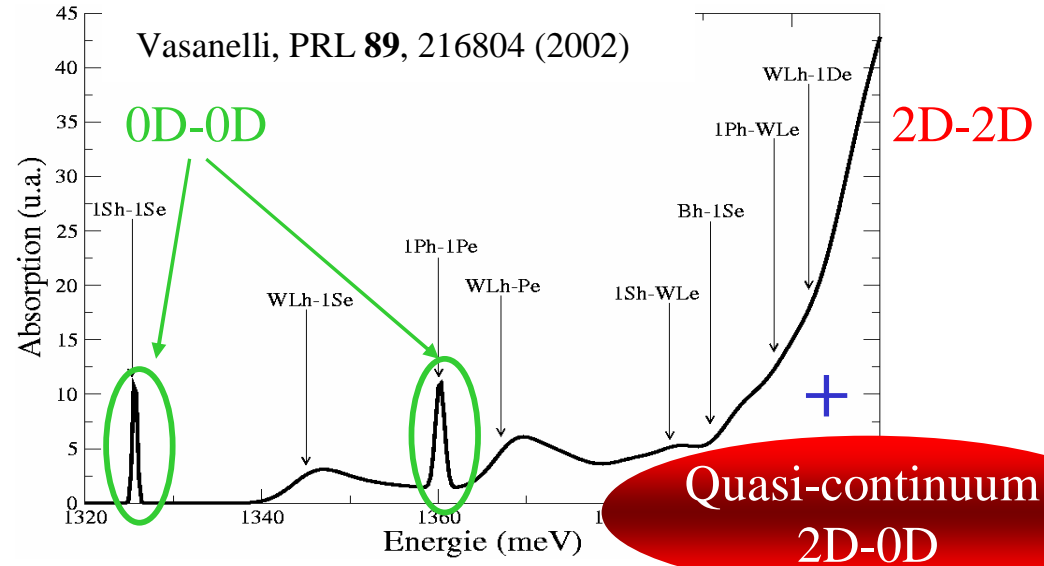
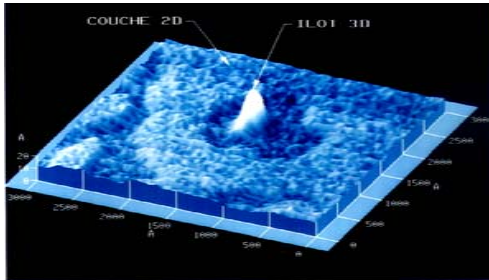
density of **crossed** states $\rho_{2D/0D}(E)$

INTRINSIC EFFECT !

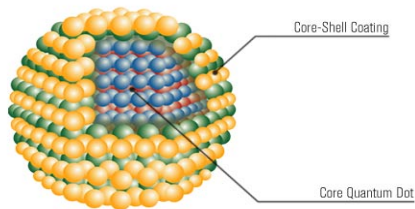


QD = system with discrete lines ?

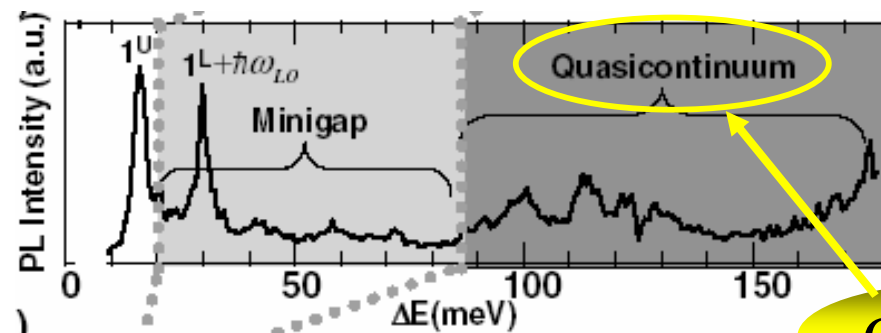
- Epitaxially grown QD



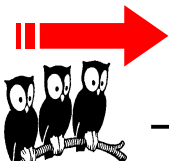
- Colloidal QD



no wetting layer
but an insulating matrix
no 2D-0D quasi-continuum

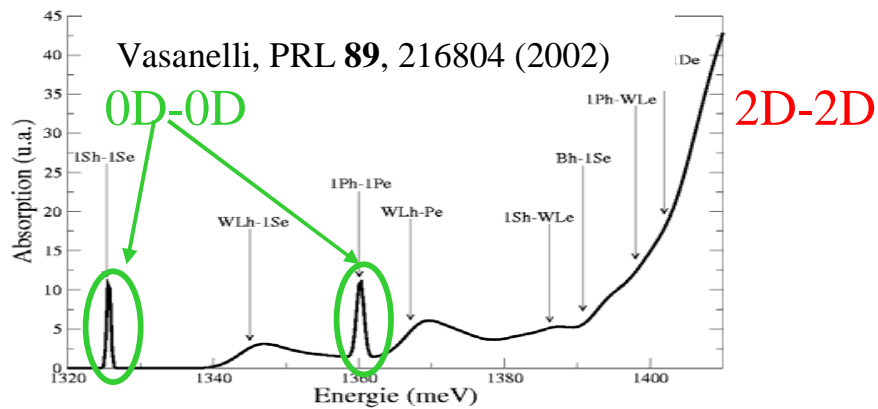
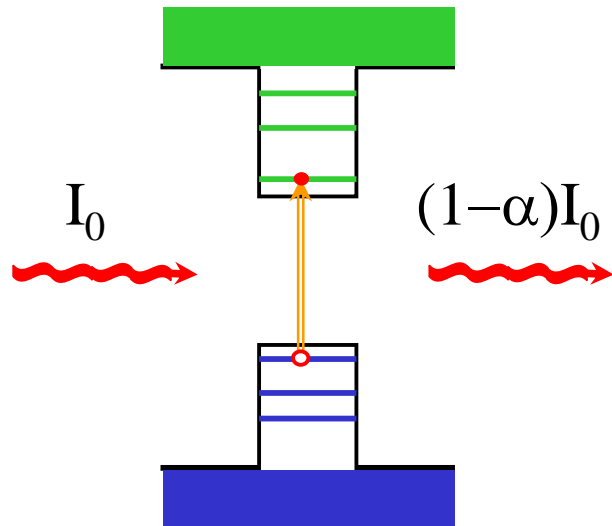


Origin ?

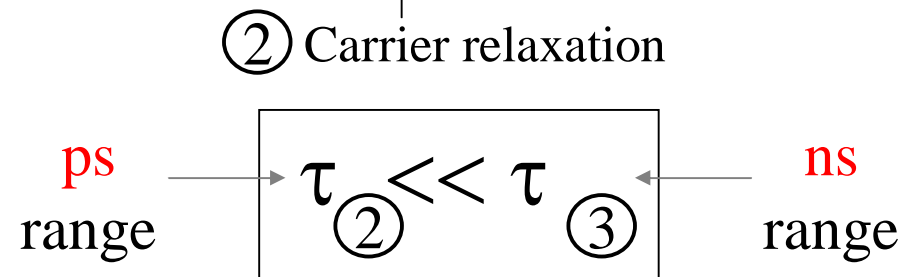
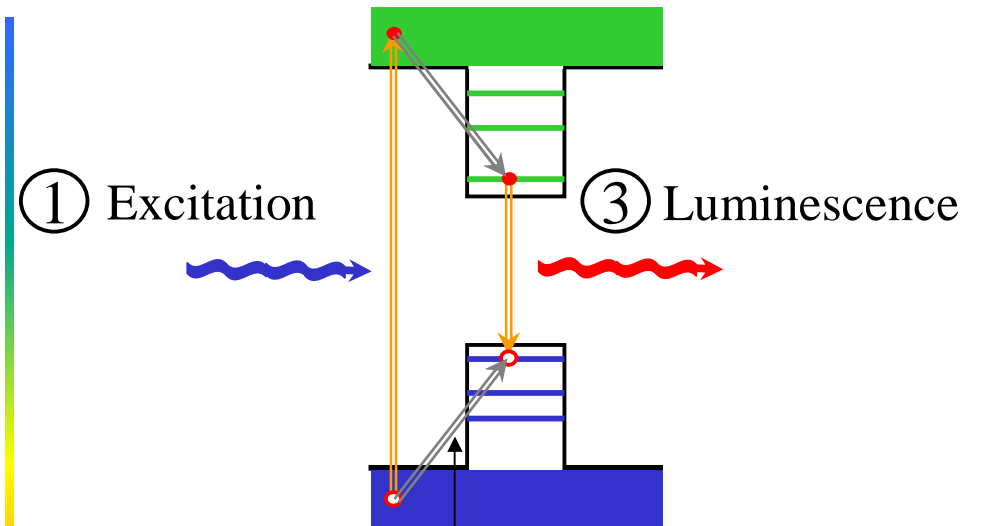


From absorption to emission

• Absorption



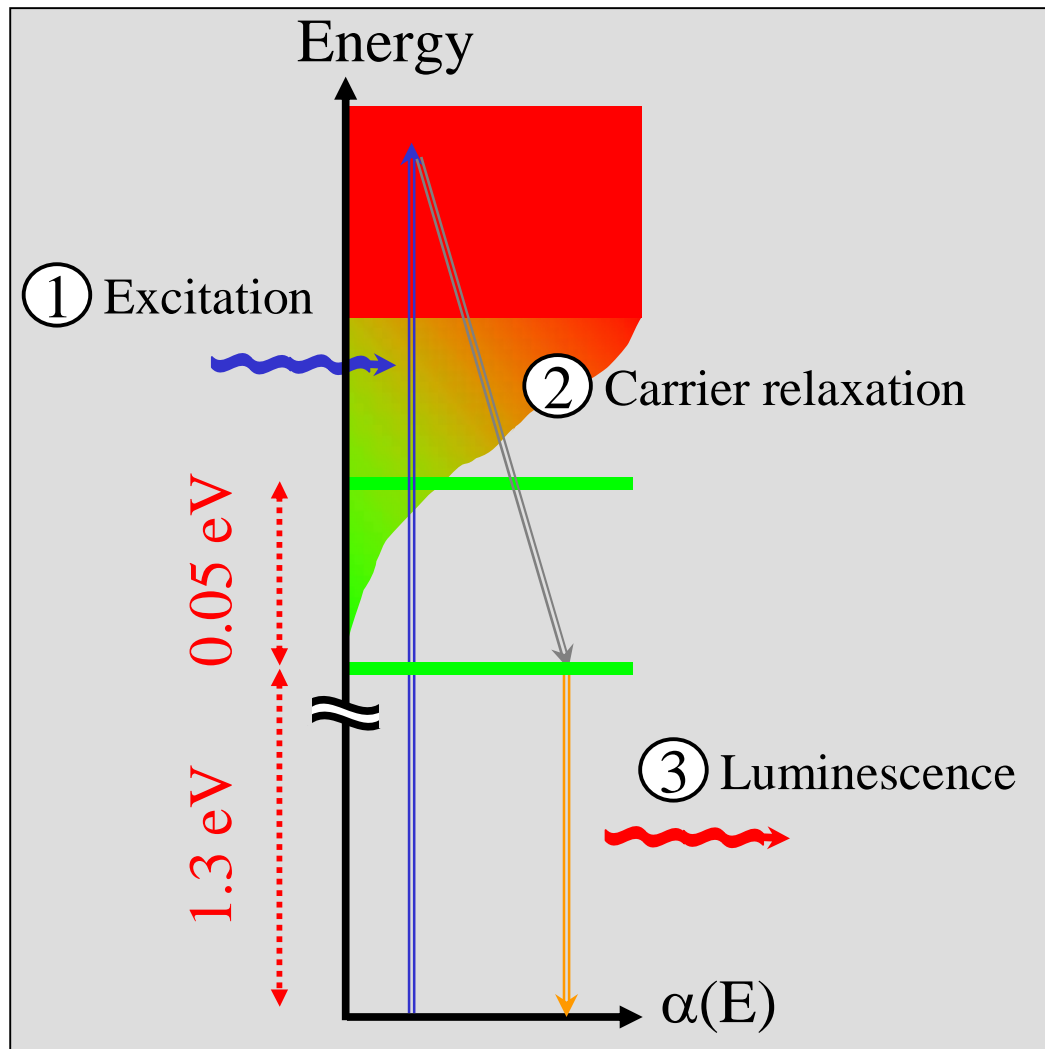
• Emission



Emission comes
only from the lowest transition



Two-level system approximation

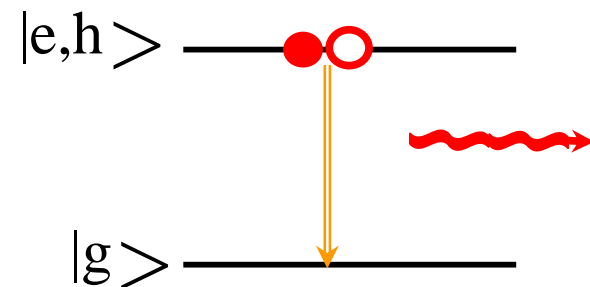


- QD = complex system in solid state physics

BUT

the efficient carrier relaxation shortcuts the **dimensionality transition** in the QD absorption spectrum

- **Two-level system** = first-order approximation for the description of the QD optical response in **emission**



Fundamental transition fine structure

- Fundamental **interband** transition corresponds to (e,h) **pair state** with a **heavy**-like hole

$$\begin{aligned} |electron\rangle &= |J = 1/2, J_z = \pm 1/2, \dots\rangle \\ |heavy\ hole\rangle &= |J = 3/2, J_z = \pm 3/2, \dots\rangle \end{aligned}$$

- Four-fold **degeneracy**

$ e, h\rangle = J = 2, J_z = +1, \dots\rangle$	emits σ_+ light	}	bright states
$= J = 2, J_z = -1, \dots\rangle$	emits σ_- light		
$= J = 2, J_z = +2, \dots\rangle$	no emission	}	dark states
$= J = 2, J_z = -2, \dots\rangle$	no emission		

- **Coulomb** interaction between pair states

$$\langle \psi_{c'}(1), \psi_{v'}(2) | V_c | \psi_c(1), \psi_v(2) \rangle \quad \text{direct interaction term}$$

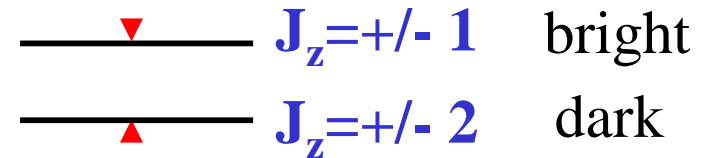
$$\langle \psi_{c'}(1), \psi_{v'}(2) | V_c | \psi_v(1), \psi_c(2) \rangle \quad \text{indirect or exchange interaction term}$$



Dark and bright states

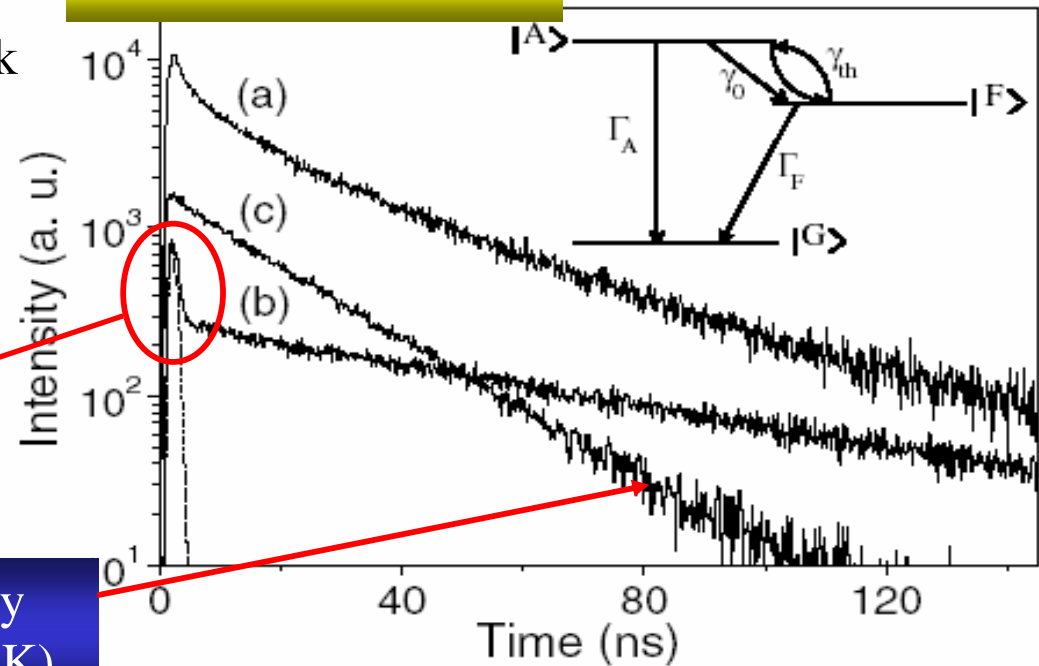
0.5 meV (InAs) – 4 meV (CdSe)

- **Short range** part of exchange interaction **lifts degeneracy** of bright and dark states



- No luminescence from dark states **BUT** interplay between the bright and dark states during **population relaxation dynamics**

Pulsed excitation



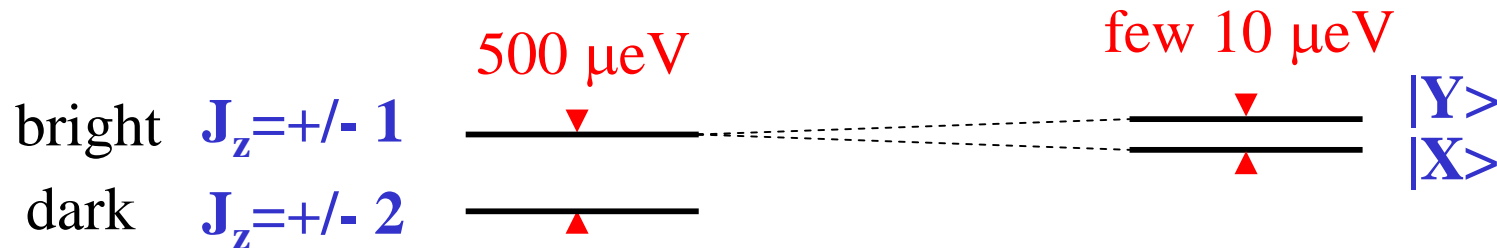
Bi-exponential decay at low temperature (16K)

Mono-exponential decay at high temperature (140K)

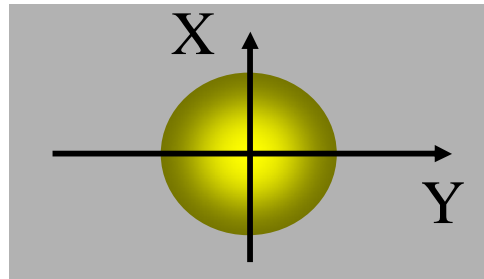
Labeau, PRL **90**, 257404 (2003)



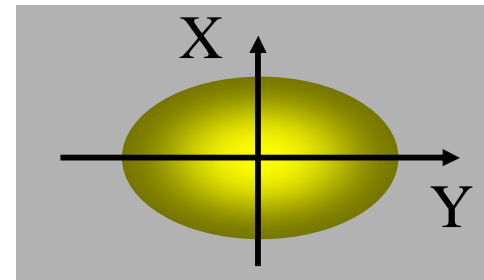
Rotational symmetry breaking



short range
exchange
interaction



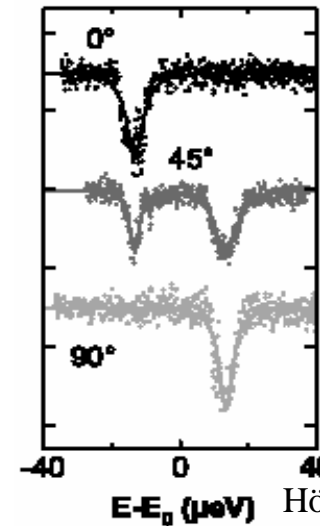
short range
+ long range
exchange
interaction



- In a QD of **broken symmetry** the new bright states emit **linearly** polarized light

$$|X\rangle = \frac{|+1\rangle + |-1\rangle}{\sqrt{2}}$$

$$|Y\rangle = \frac{|+1\rangle - |-1\rangle}{i\sqrt{2}}$$



Analyzer // (Ox)

Analyzer // (Oy)

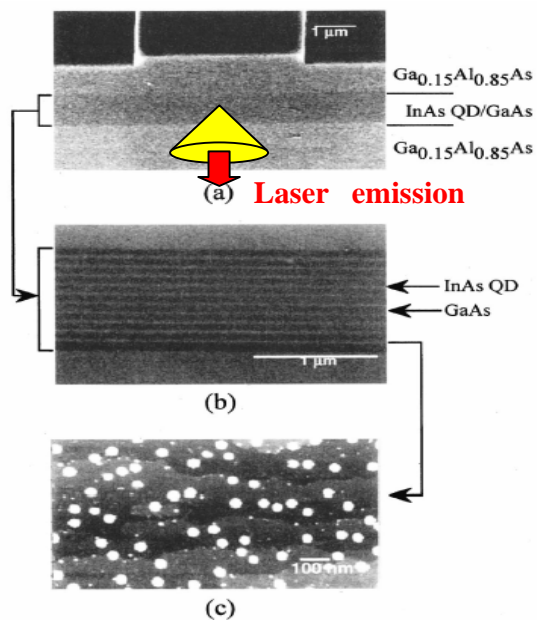
Högele, PRL 93, 217401 (2004)



From fundamental optical properties to optical applications



QD laser



Nakamura
J. Appl. Phys. **94**, 1184 (2003)

QD laser

Opto-electronics

- **Gain** and **threshold current density** of **ideal** QD laser (no inhomogeneous broadening)
Arakawa and Sakaki APL **40**, 939 (1982)
 - higher maximum gain due to narrow gain spectrum (0D density of states)
 - no temperature dependence of threshold current

- size fluctuations in **real** QD laser > inhomogeneous broadening **to be reduced**
- existence of **intrinsic limit** = homogeneous linewidth due to phonon coupling ($\Gamma \sim 10$ meV at 300K)
- no quenching of carrier escape

QW versus QD lasers = ?

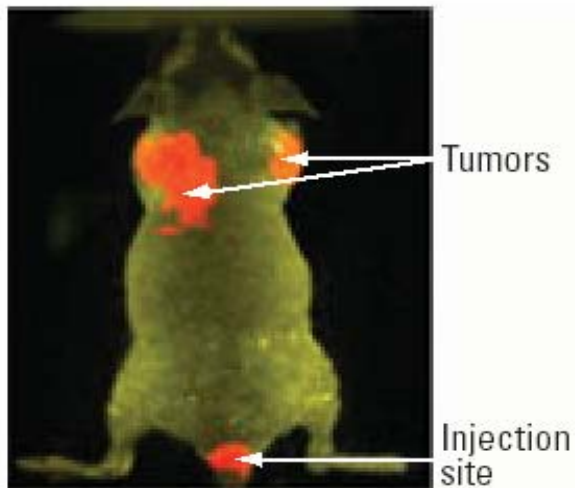


Biological sensors

Bio-physics

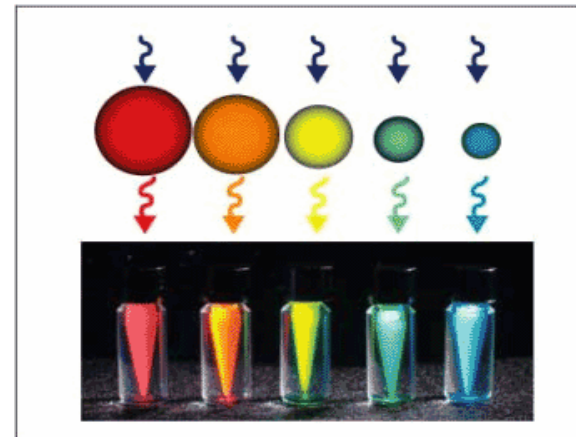
Biological sensors

In vivo imaging of cancer cells in a living mouse



Gao, Nature Biot. **22**, 969 (2004)

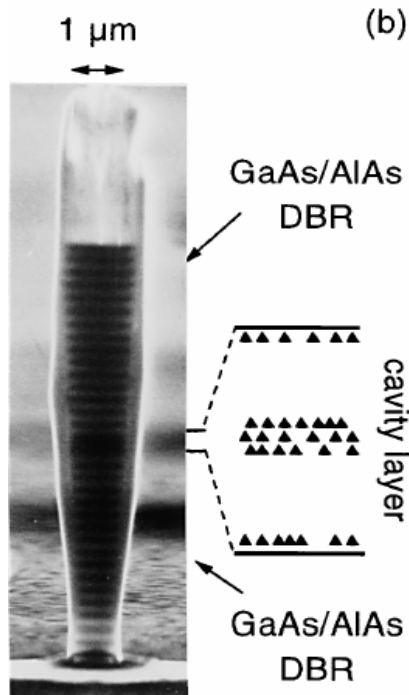
- **colloidal** QDs
- **broadly tunable** in the visible spectrum by size variation



- **high** quantum efficiency
 - **bio-conjugating** colloidal QDs (cell labeling, cell tracking, *in vivo* imaging, DNA detection, multiplexed beads...)
- (see Alivisatos, Nature Biot. **22**, 47 (2004)
and [previous lectures](#))



Single photon source



Gérard, PRL **81**, 1110 (1998)

Single photon source

Quantum optics

- single QD generates **single photon state**
- non-classical source of radiation : **sub-Poissonian** statistics
- **quantum optics** in solid state
- **quantum information** processing



Quantum optics in solid state with a non-classical source of radiation

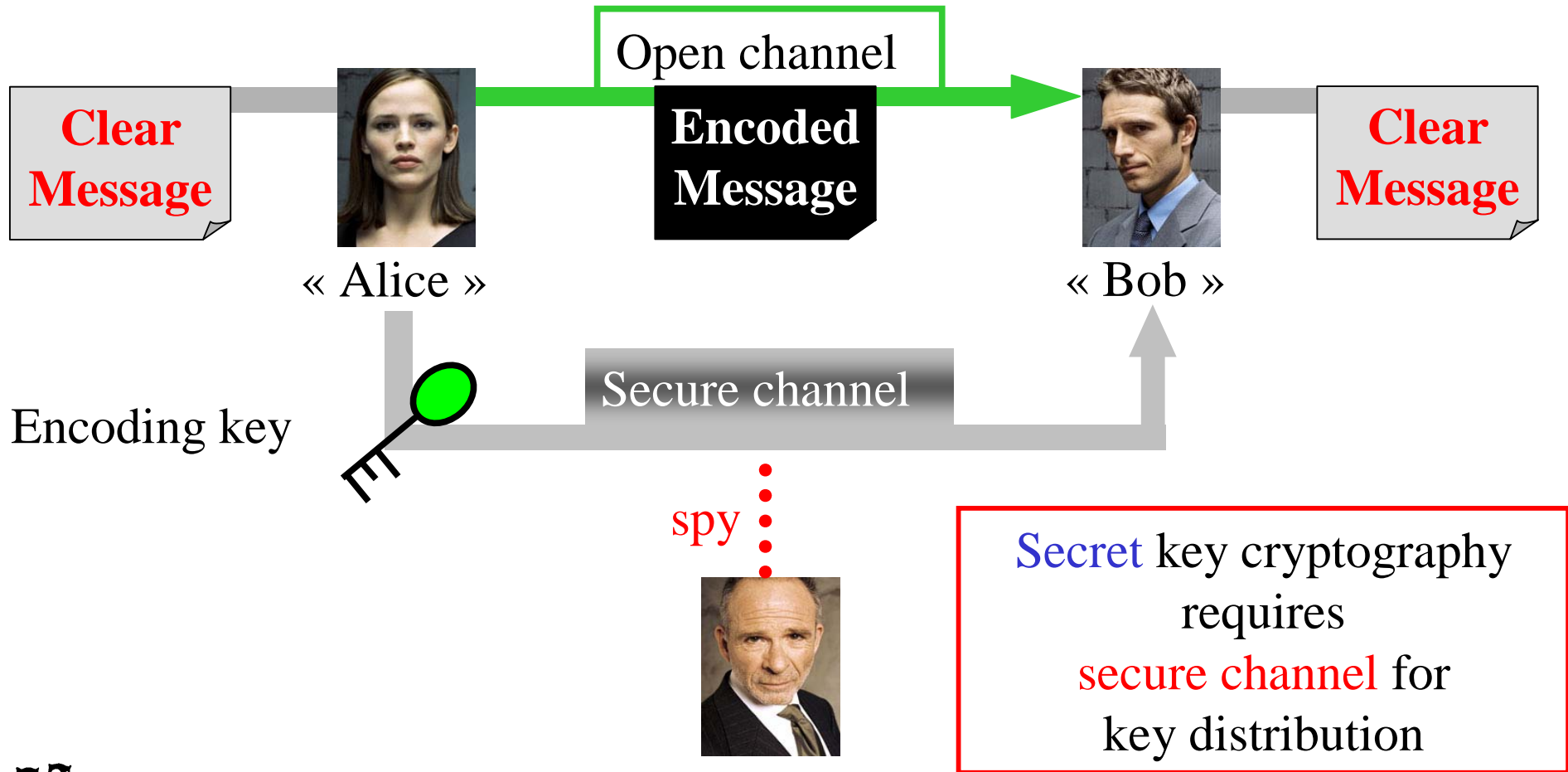
Outline

- quantum cryptography
- single-photon source
- carrier quantum cascade



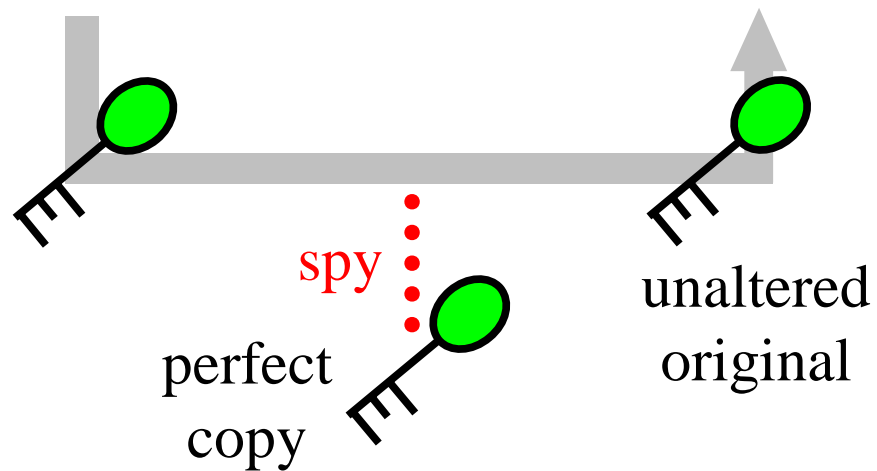
Secret key cryptography

- Goal : transmission of **secret** message



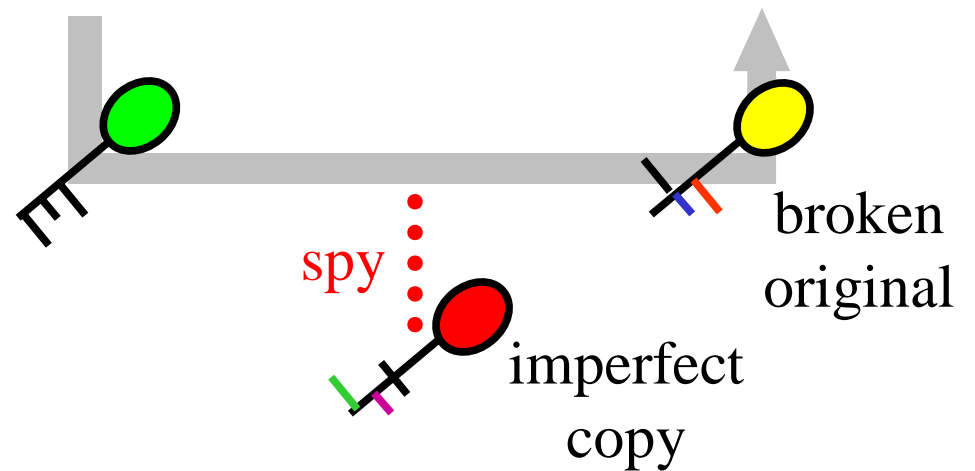
Classical versus quantum information

- **Classical** information



in principle every classical channel can be monitored passively

- **Quantum** information

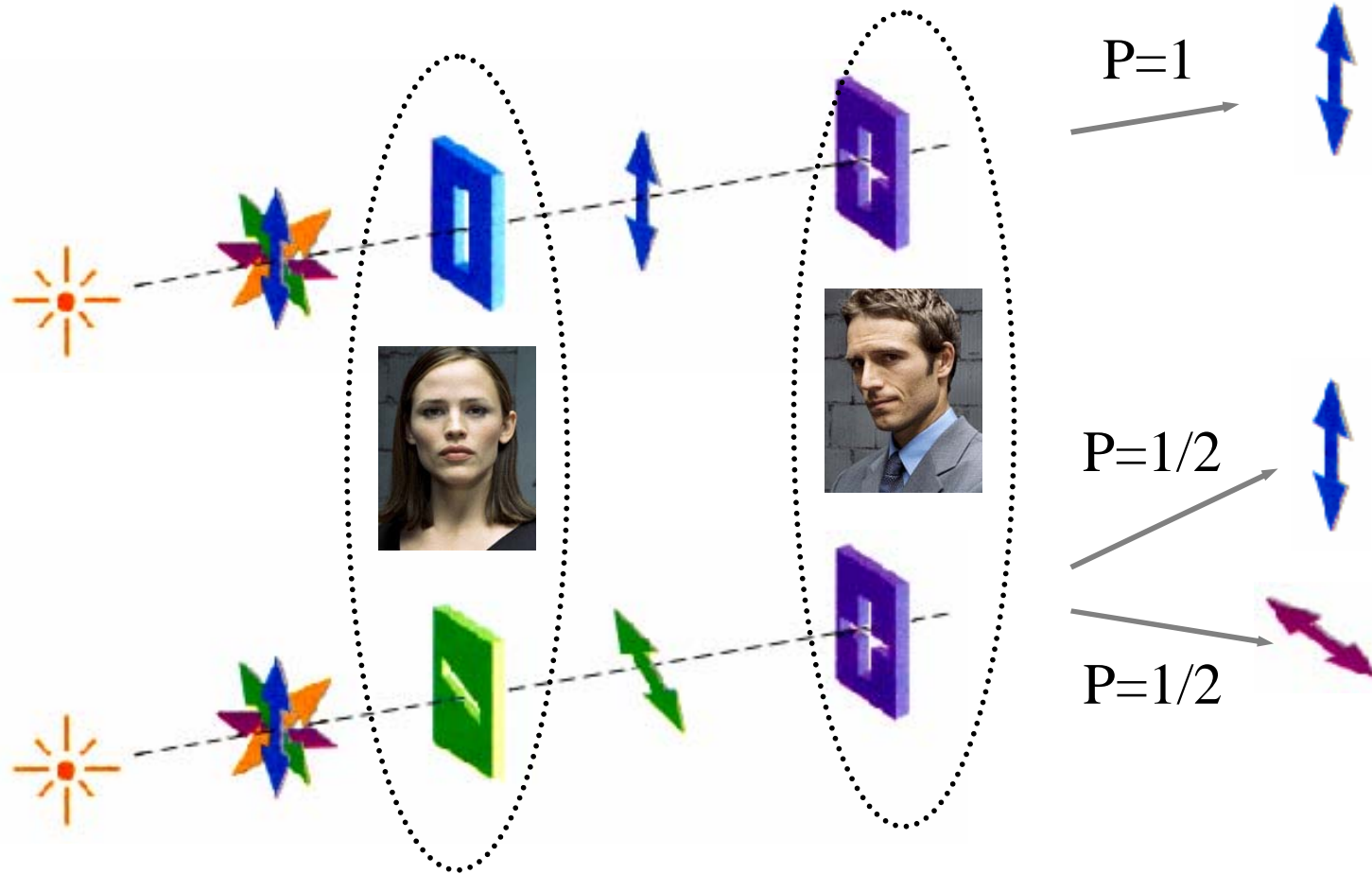


- quantum cryptography distributes the key by transmitting quantum photon states in an **open channel**
- this **open channel** is **intrinsically secure** because a spy destroys the encoding key

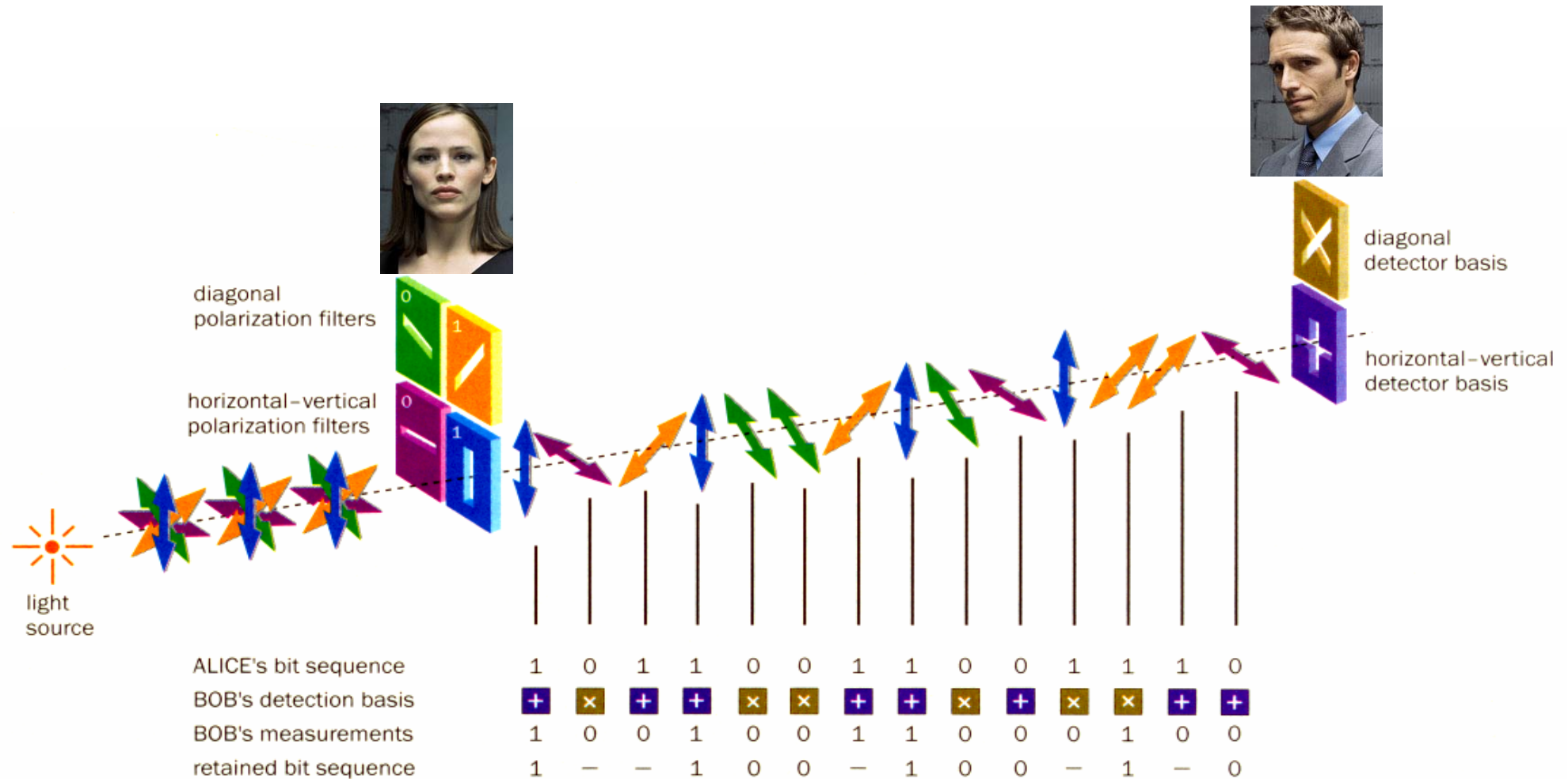


Polarization state of a single photon

Polarizer orientation Measurements

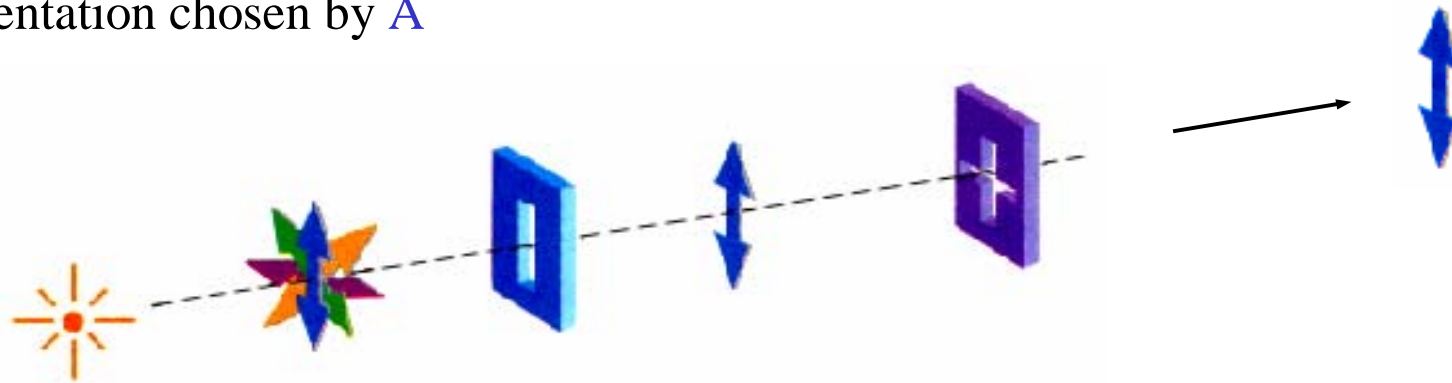


Quantum key distribution



Key distribution analysis

- B sends to A by open channel his set of detection basis and half of his measurements
- A selects the cases where the detection basis corresponds to the polarizer orientation chosen by A



in that cases B must have found the same result as A
if not, this means there is a spy which has generated a single photon with an incorrect polarization orientation !

- among the retained bit sequence, A chooses a subset of bits to compose the encoding key and sends the location of these bits in the total message to A by open channel

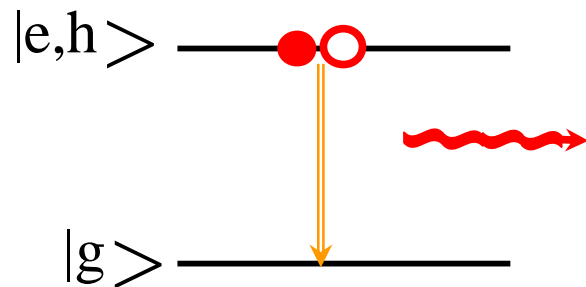


Single QD as single-photon source



Solid-state single-photon source

- single QD

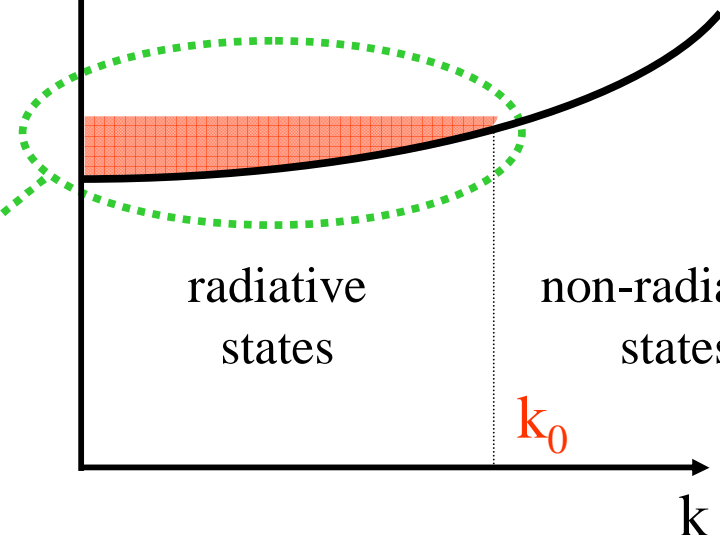


- radiative recombination of **single (e,h) pair** generates single photon
- this picture breaks down for a **QDs ensemble**

- quantum **wire** (1D), quantum **well** (2D)

continuum of electronic states

$E_{|e,h\rangle}$

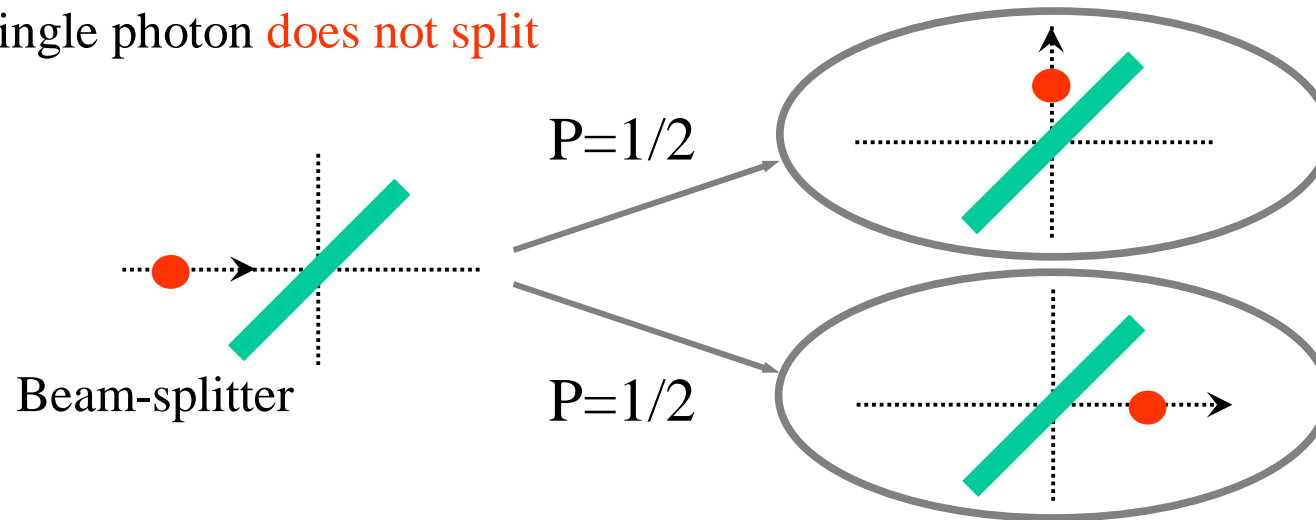


translational quantum number

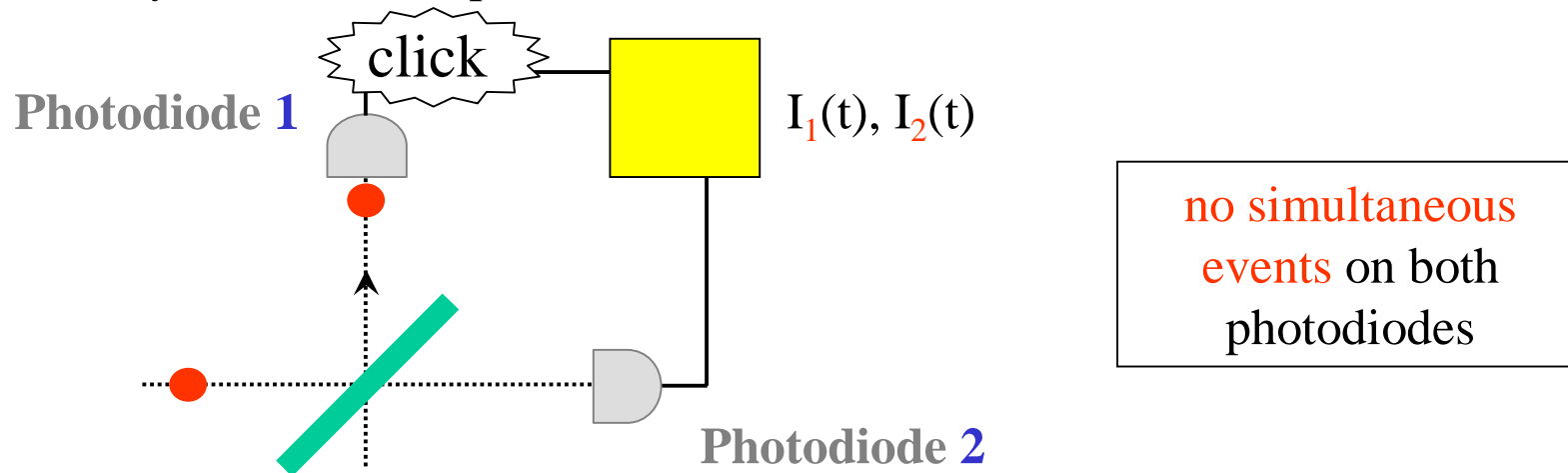


Coincidence detection

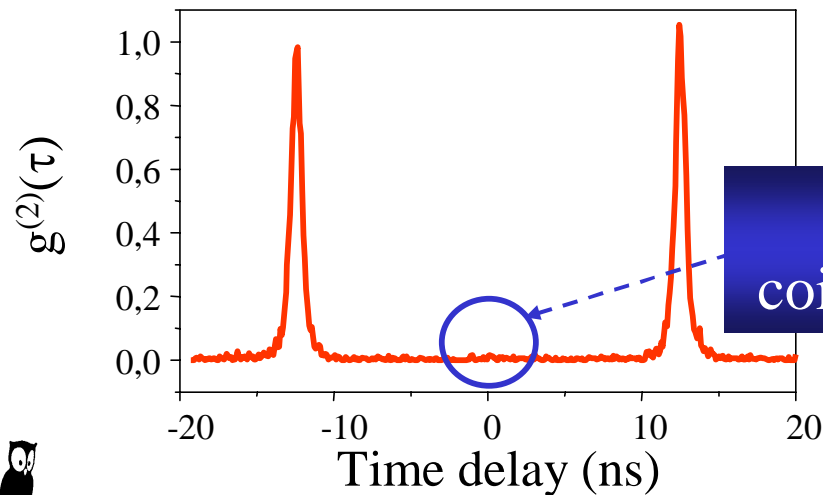
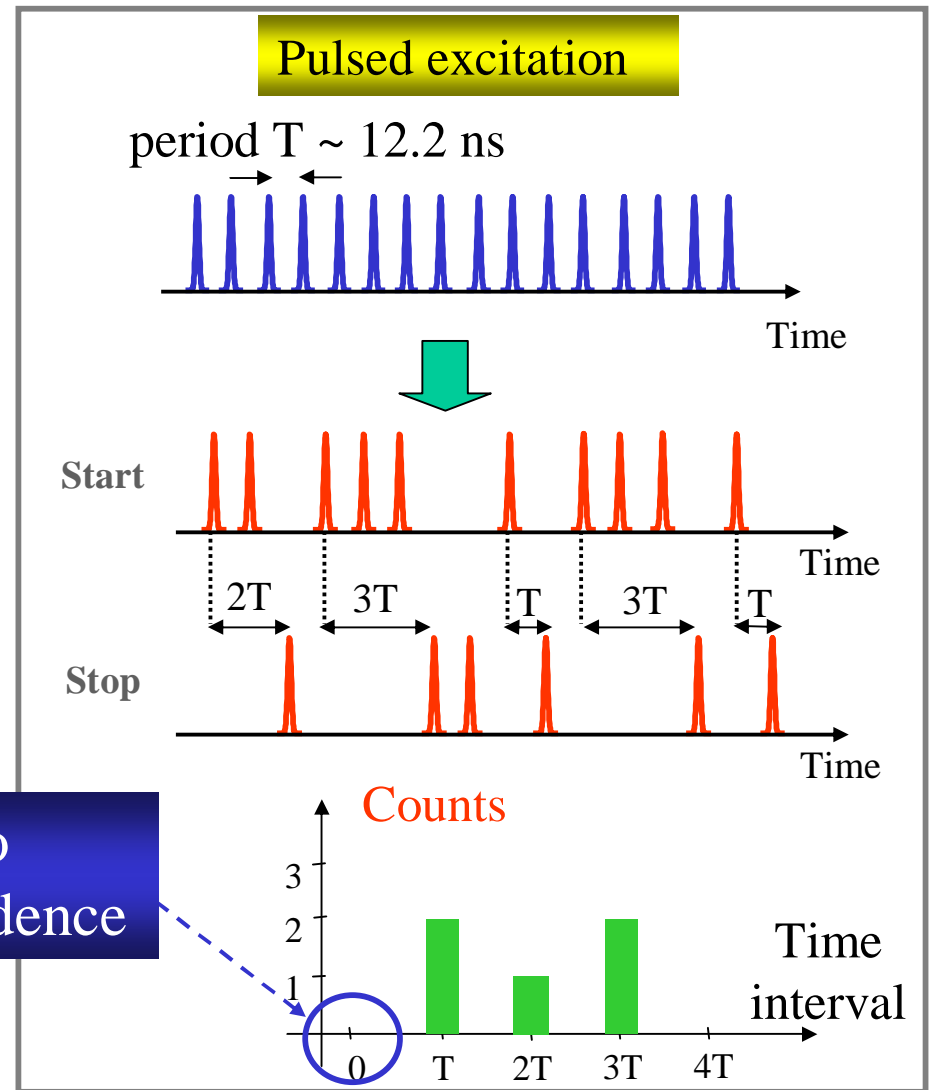
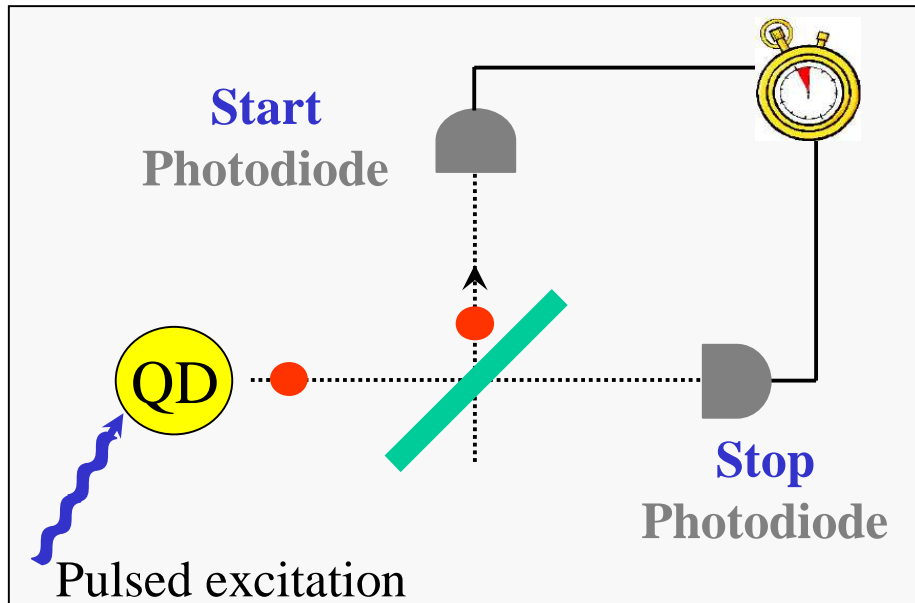
- single photon **does not split**



- intensity **correlation** experiments



Start-stop correlation experiments



No coincidence

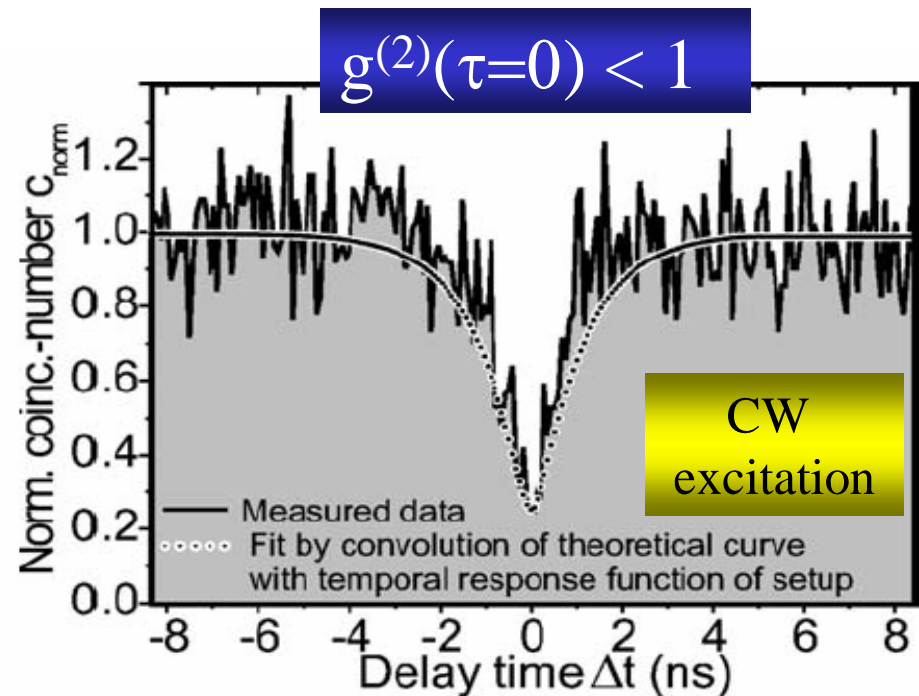
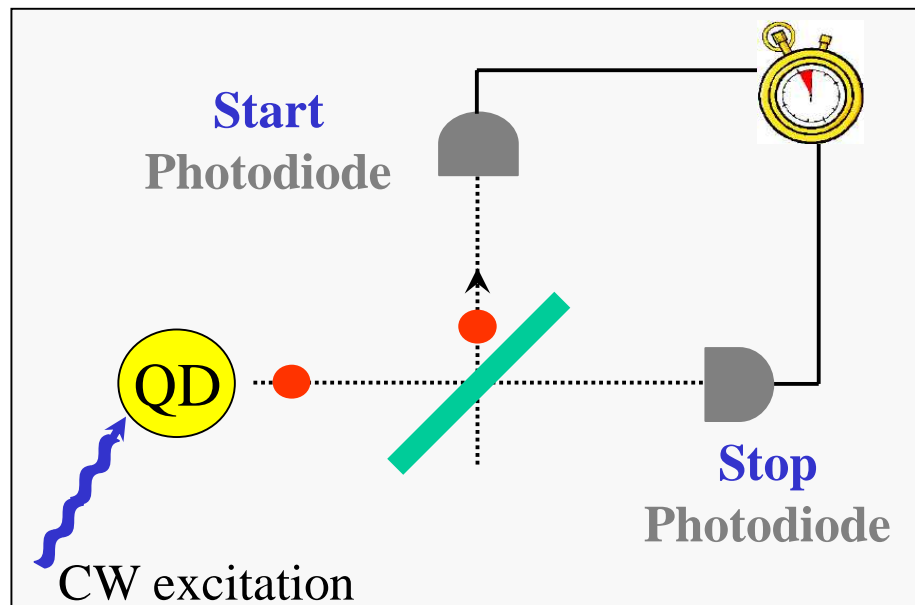


Photon anti-bunching

- normalized intensity correlation function $g^{(2)}(\tau)$

$$g^{(2)}(\tau) = \frac{\langle I_1(t)I_2(t + \tau) \rangle}{\langle I_1(t) \rangle \langle I_2(t) \rangle}$$

- start-stop correlation experiments under **continuous-wave excitation**

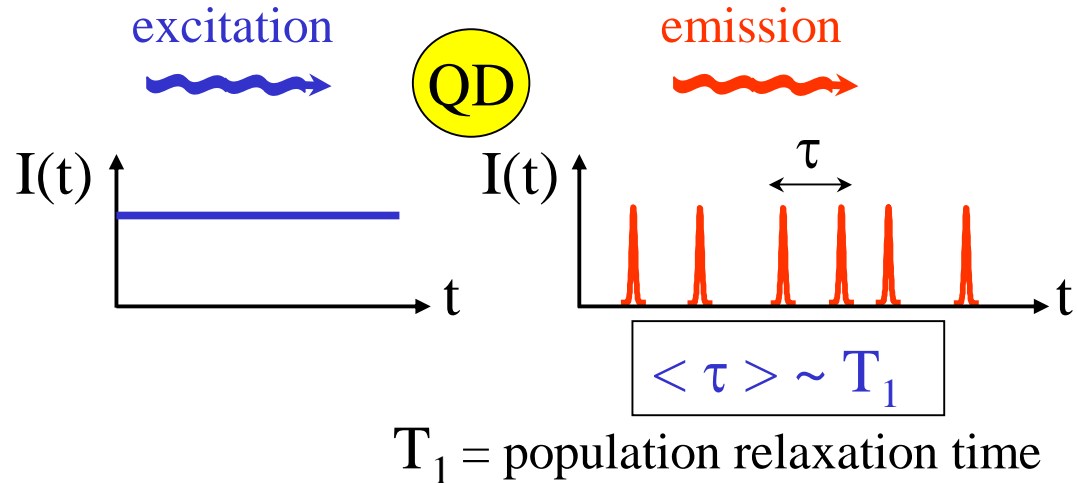
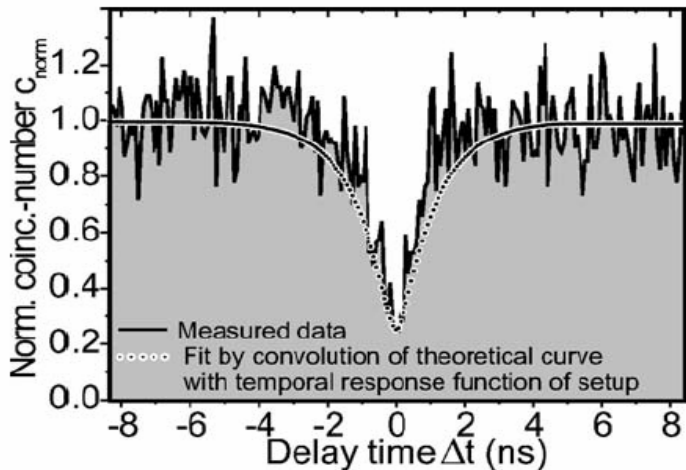


Baier, APL **84**, 648 (2004)

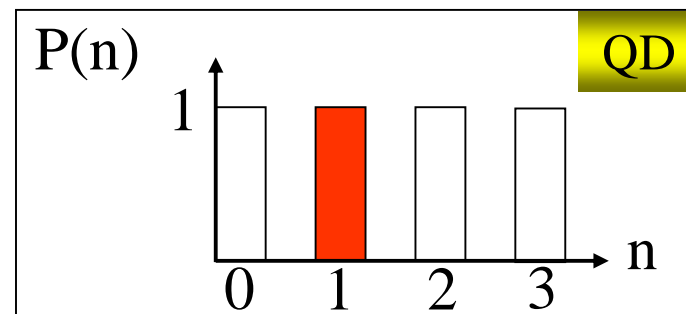
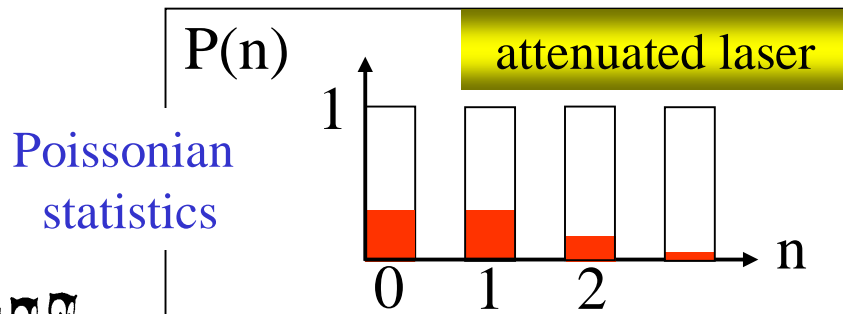


Sub-poissonian statistics

- $g^{(2)}(\tau=0) < 1$ reveals non-classical character of light source
- **sub-poissonian** photon statistics



- **attenuated laser** versus single QD when $\langle n \rangle = 1$



Photon coalescence

- **One photon** impinges on beam-splitter

$$|1_1\rangle |0_2\rangle \longrightarrow r |1_3\rangle |0_4\rangle + t |0_3\rangle |1_4\rangle$$

$$|0_1\rangle |1_2\rangle \longrightarrow t |1_3\rangle |0_4\rangle - r |0_3\rangle |1_4\rangle$$

- **Two photons** impinge on beam-splitter

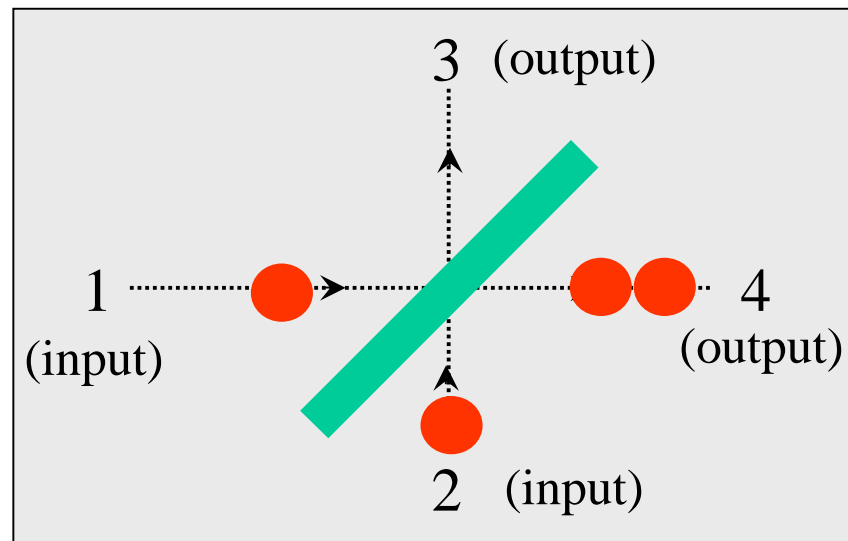
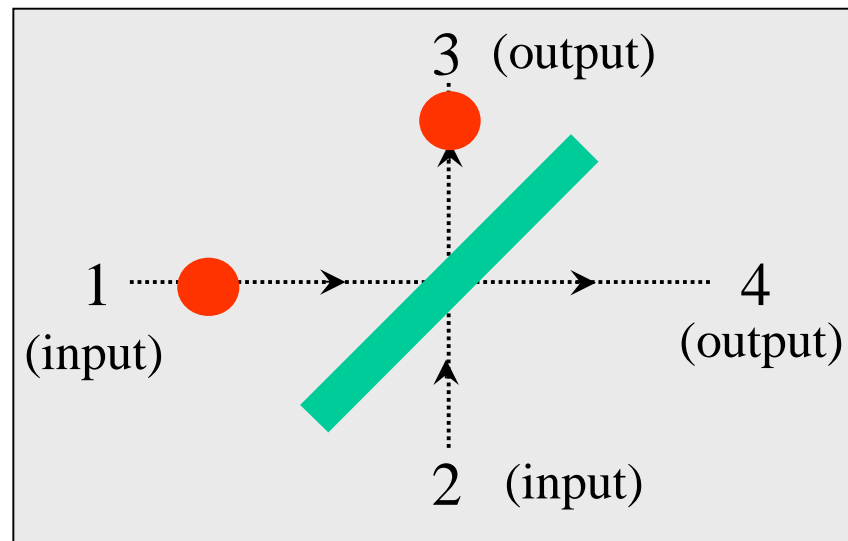
$$|1_1\rangle |1_2\rangle$$



$$t^2 |1_3\rangle |1_4\rangle - r^2 |1_3\rangle |1_4\rangle$$

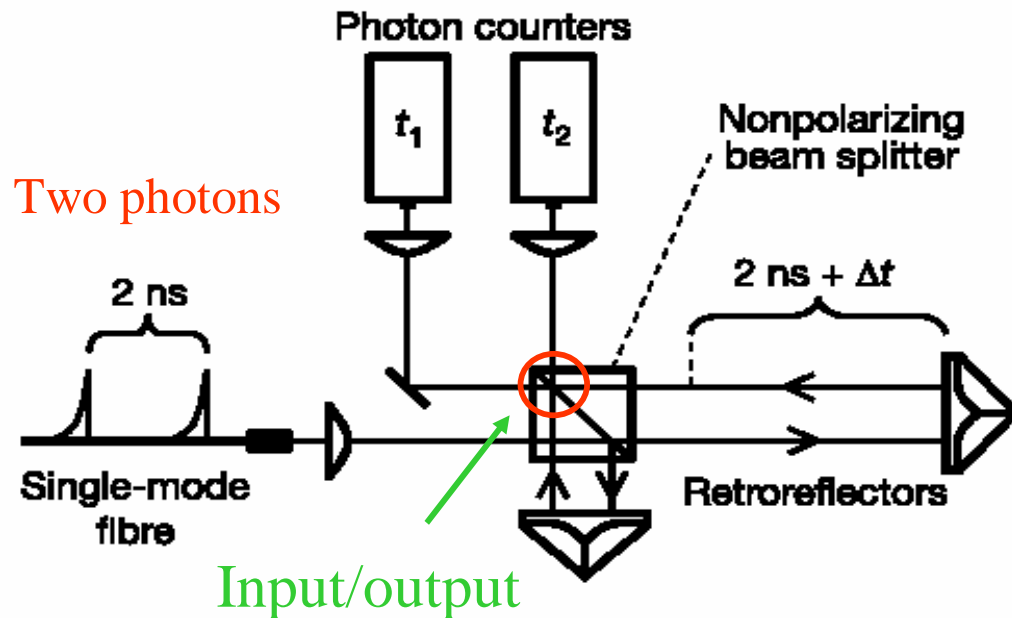
$$+ rt |2_3\rangle |0_4\rangle - rt |0_3\rangle |2_4\rangle$$

if $r = t$, only **two-photon states**



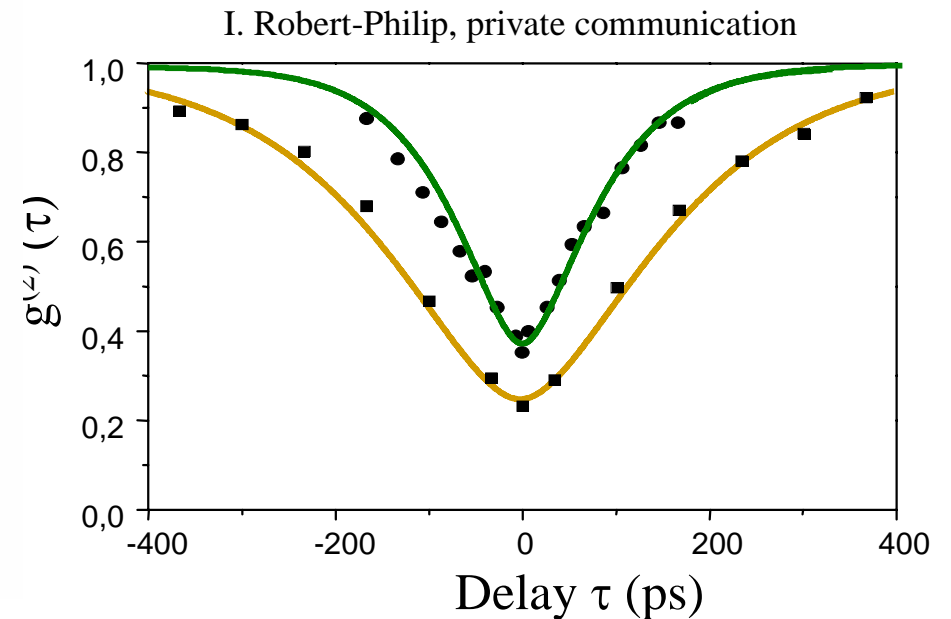
Indistinguishable photons

- Coalescence requires generation of **two indistinguishable photons**



Santori, Nature **419**, 594 (2002)

Sequential emission from the same QD



No coincidence :
coalescence of the two photons



Single-mode single-photon emission

AND ALSO

- **Quantum teleportation** Fattal, PRL **92**, 037904 (2004)
- **Bell's inequality violation** Fattal, PRL **92**, 037903 (2004)

Demonstration of **single-mode single-photon emission**
(polarization, space, spectrum) from a single QD

see also Moreau, APL **79**, 2865 (2001)

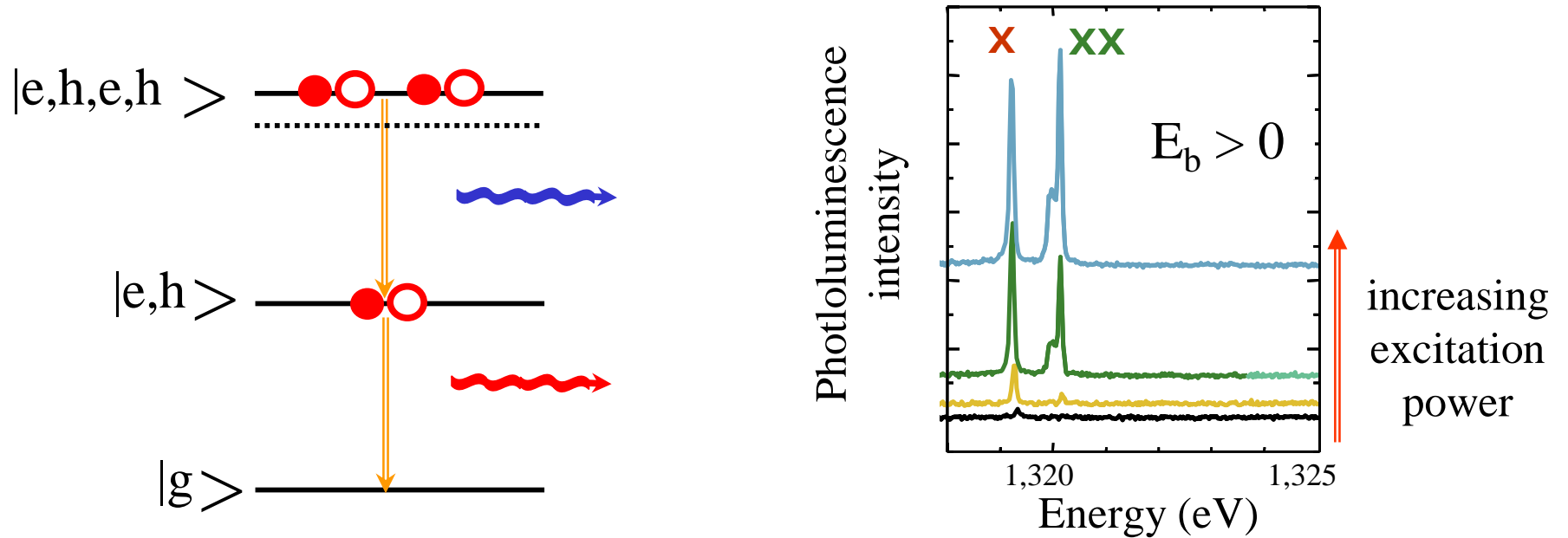


Single QD as two-photon source



Exciton and biexciton

- two (e,h) pairs are photogenerated in a single QD

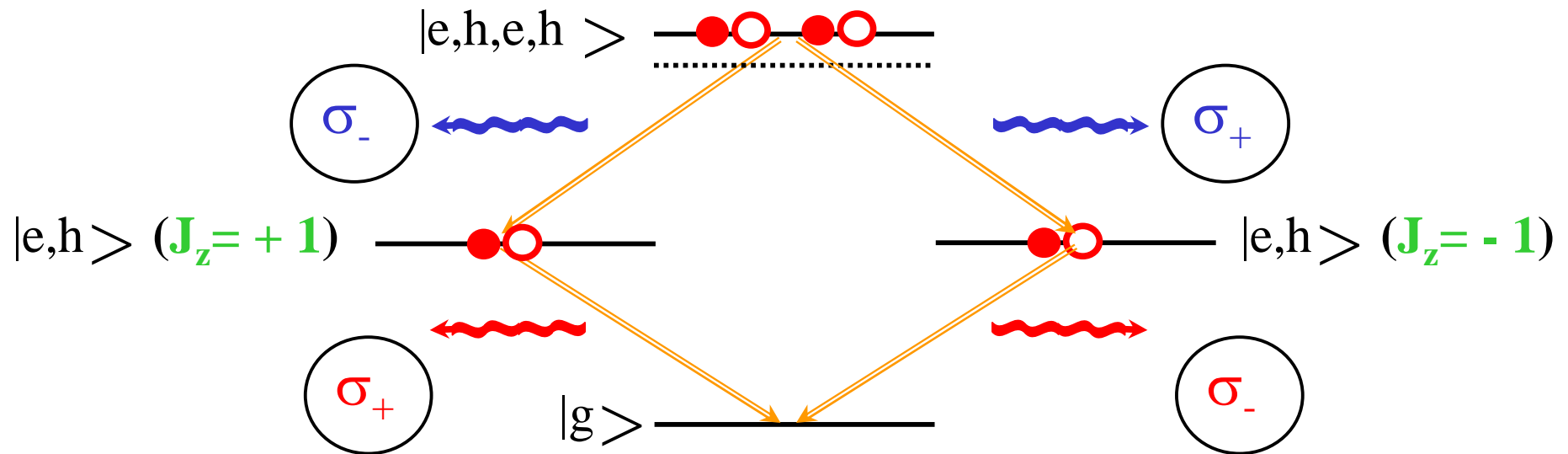


- Coulomb correlated state with 1 (e,h) pair = exciton (X)
 - Coulomb correlated state with 2 (e,h) pairs = biexciton (XX)
- $$E_{XX} = 2E_X + E_b$$
- binding energy



Bell states generation

- Fundamental transition **degeneracy**



- Two parallel paths  possible generation of **entangled photon states**

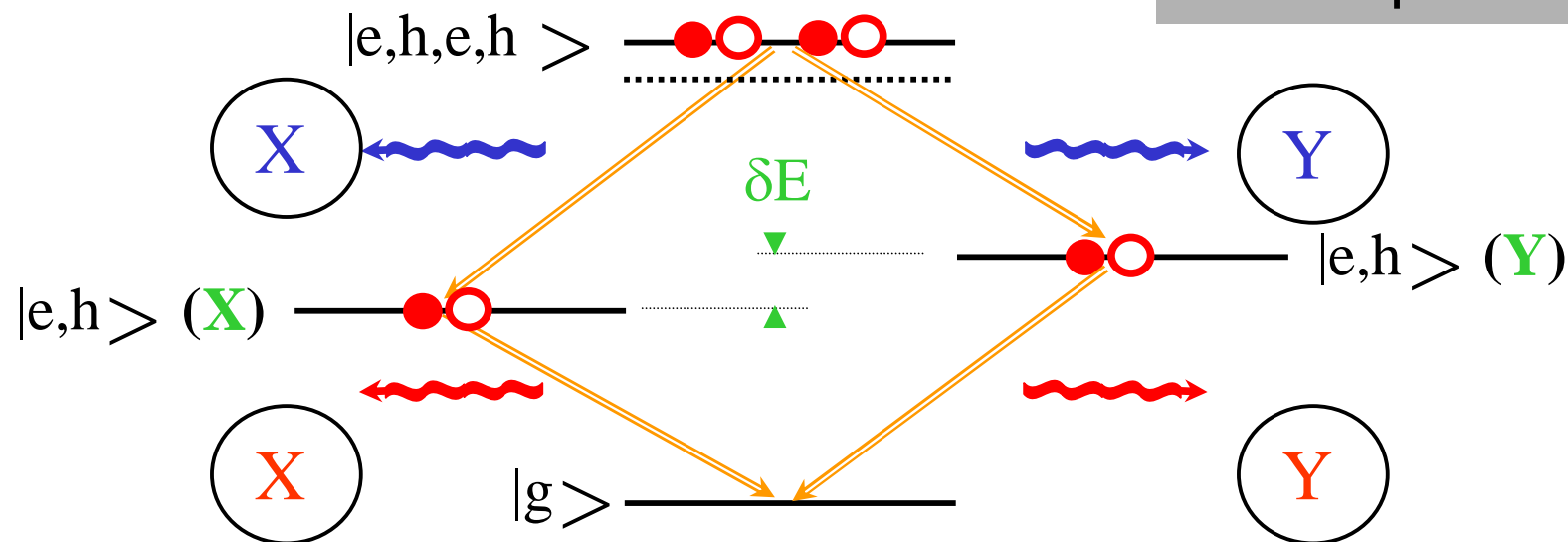
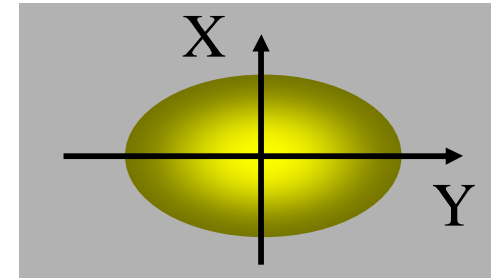
$$|\psi\rangle = \frac{|\sigma_-\rangle_{xx}|\sigma_+\rangle_x + |\sigma_+\rangle_{xx}|\sigma_-\rangle_x}{\sqrt{2}} \neq |\phi\rangle_{xx}|\phi\rangle_x$$

Benson, PRL **84**, 2513 (2000)



Rotational symmetry breaking

- **Degeneracy lifted** in asymmetric QDs



- Observation of **polarization correlations** but not entanglement

$$\text{if } \delta E \gg \Gamma_X, \Gamma_Y$$

Santori, PRB **66**, 045308 (2002)



Towards efficient single photon sources

All these experiments are **performed at low temperature ($T \sim 5-10\text{K}$)**
to avoid phonon-assisted dephasing processes

All these experiments use **single QDs embedded in micropillar** to
control spontaneous emission



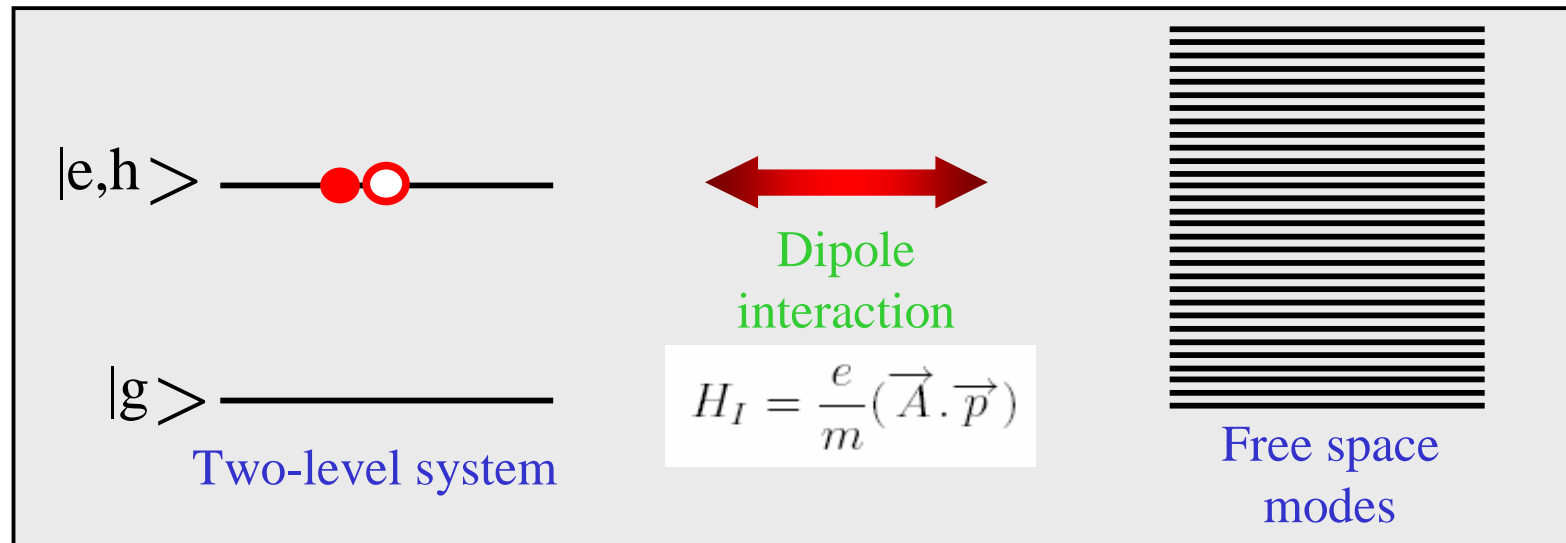
Spontaneous emission control

Outline

- Fermi's golden rule
- Purcell effect
- from weak to strong coupling



Fermi's golden rule



- Time domain analysis

$$|i\rangle = |e, h\rangle |0_\omega\rangle$$

$$|f\rangle = |g\rangle |1_\omega\rangle$$

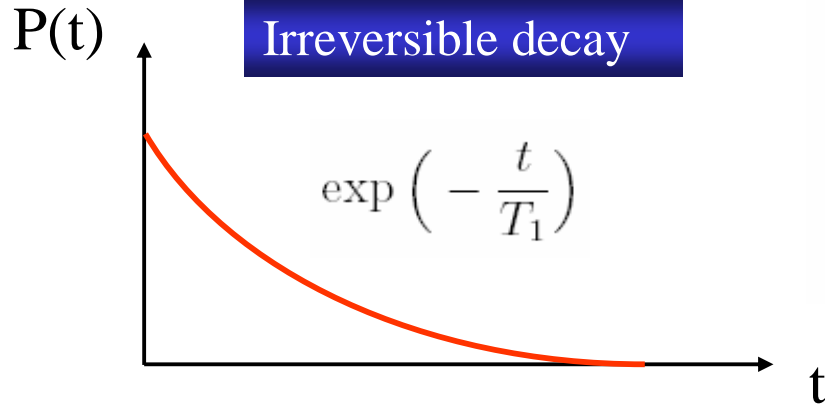
$$P_{|i\rangle}(t) = \exp\left(-\frac{t}{T_1}\right)$$

where

$$\frac{1}{T_1} = \frac{2\pi}{\hbar} \sum_{\{|1_\omega\rangle\}} |\langle f | \widetilde{H}_I | i \rangle|^2 \delta(E_{|e,h\rangle} - \hbar\omega)$$



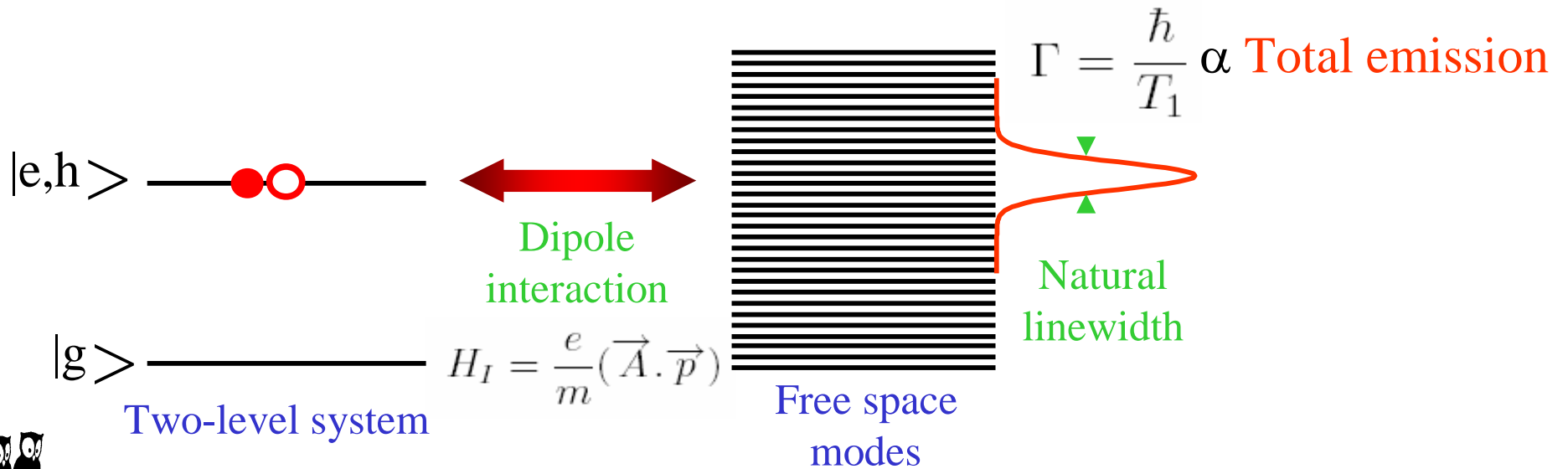
Natural linewidth



$$\frac{1}{T_1} \sim \frac{2\pi}{\hbar} |\langle f | \widetilde{H}_I | i \rangle|^2 \sum_{\{|1_\omega\rangle\}} \delta(E_{|e,h\rangle} - \hbar\omega)$$

$$\sim \frac{2\pi}{\hbar} |\langle f | \widetilde{H}_I | i \rangle|^2 \underbrace{\rho_{e.m.}(E_{|e,h\rangle})}_{\text{free-space density of electromagnetic modes}}$$

- Spectral domain analysis

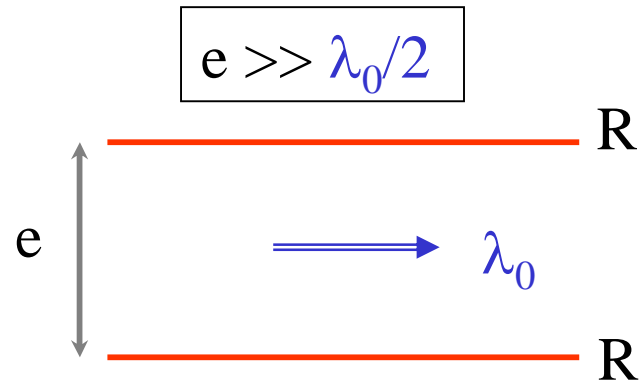


From free-space to cavities



Fabry-Pérot cavity

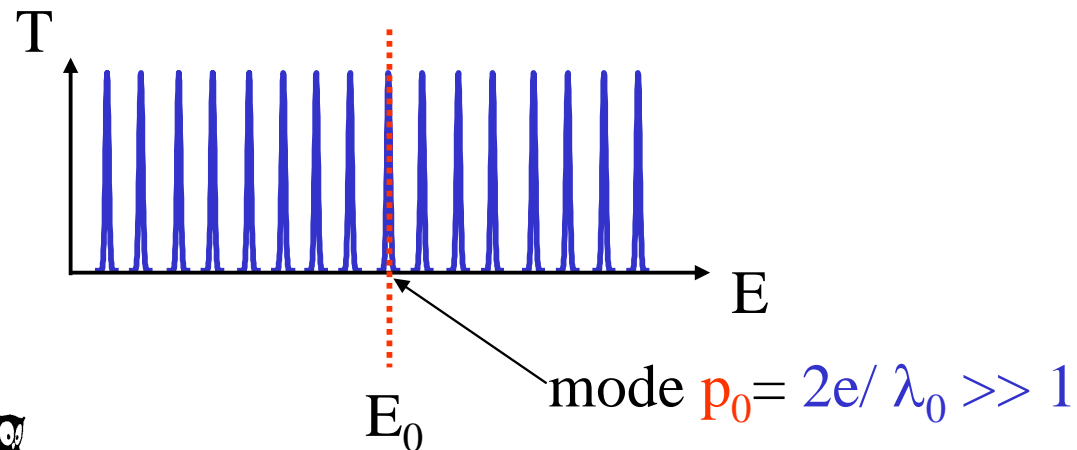
- Fabry-Pérot cavity with **metallic mirrors**



$$\nu_p = p \frac{c}{2e \cos(\theta_p)}$$

$$E_p = E_0 \frac{\lambda_0/2}{e} \frac{p}{\cos(\theta_p)} \quad \text{with} \quad E_0 = \frac{hc}{\lambda_0}$$

- Fabry-Pérot transmission at **normal incidence**



- **Free spectral range**

$$\frac{E_0}{p_0}$$

- for **large** cavities

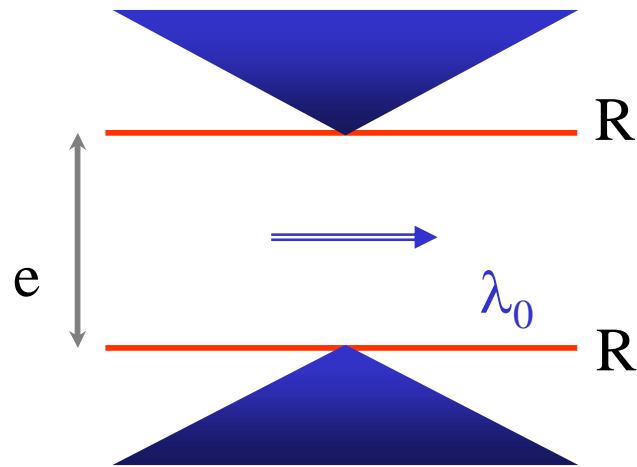
$$\frac{E_0}{p_0} \ll E_0$$



Angular redistribution in planar cavities

A. Kastler, Appl. Optics 1,17 (1962)

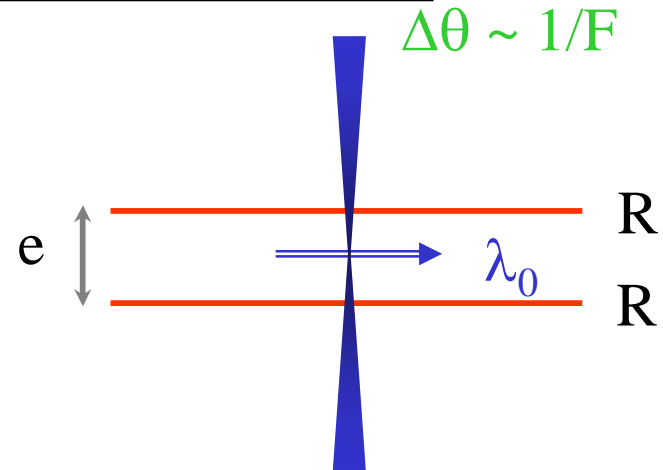
$$e \gg \lambda_0/2, p_0 \gg 1$$



$$E_p = E_0 \frac{p/p_0}{\cos(\theta_p)}$$

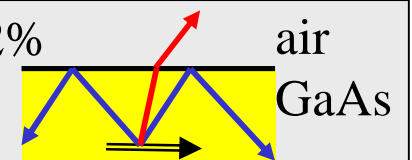
resonant interaction of emitter with a large number of modes at different emission angles

$$e = \lambda_0/2, p_0 = 1$$



- resonant interaction of emitter with a single mode
- angular redistribution of the dipole emission
- enhanced extraction efficiency $\eta \sim 25\%$

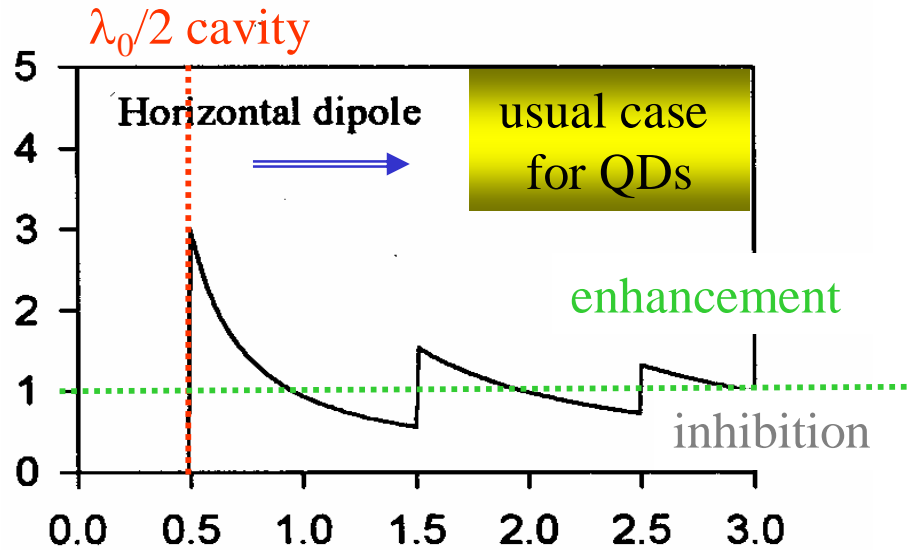
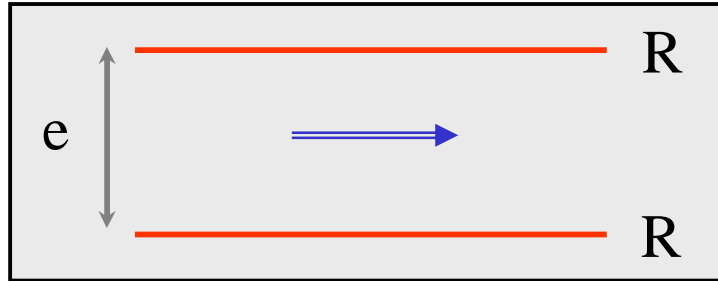
Total internal reflection $\eta \sim 2\%$ for $i > 13^\circ$



Enhancement and inhibition

Brorson, IEEE JQE 26, 1492 (1990)

Horizontal dipole



Normalized cavity thickness e/λ_0

$$\frac{1}{T_1} \sim \frac{2\pi}{\hbar} |\langle f | \widetilde{H}_I | i \rangle|^2 \rho_{e.m.}(E_{|e,h\rangle})$$

- free space $E = \hbar ck$

$$\rho_{3D}(E) = \frac{\Omega}{\pi^2} \frac{E^2}{(\hbar c)^3}$$

- $\lambda_0/2$ cavity

$$E = \hbar c \sqrt{\left(\frac{\pi}{e}\right)^2 + k_{\parallel}^2} \quad [E_0, 2E_0]$$

$$\rho_{2D}(E) = \frac{2\Sigma}{\pi} \frac{E}{(\hbar c)^2}$$

for $\lambda_0/2$ cavity $\frac{\rho_{2D}(E_0)/(e\Sigma)}{\rho_{3D}(E_0)/\Omega} = 2$??



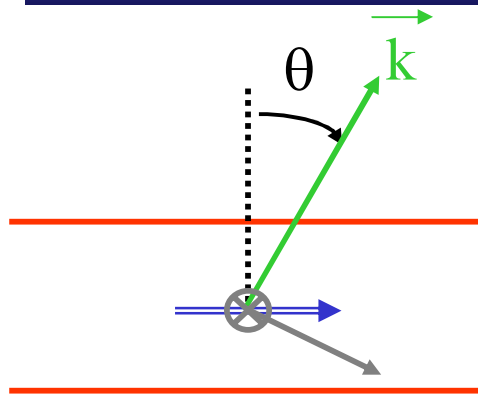
Dipole-field orientation

$$\frac{1}{T_1} = \frac{2\pi}{\hbar} \sum_{\{\lvert 1_\omega \rangle\}} \underbrace{|\langle f | \widetilde{H}_I | i \rangle|^2}_{\substack{\uparrow \\ \vec{A} \cdot \langle u_c | \vec{p} | u_v \rangle = \vec{A} \cdot \vec{P}_{cv} = AP_{cv} \cos \alpha}} \delta(E_{|e,h\rangle} - \hbar\omega)$$

mode-dependent
term in matrix element

$$\vec{A} \cdot \langle u_c | \vec{p} | u_v \rangle = \vec{A} \cdot \vec{P}_{cv} = AP_{cv} \cos \alpha$$

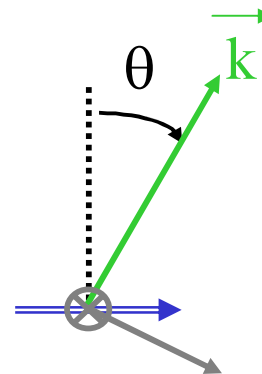
Fabry-Pérot cavity



given E \Rightarrow single angle θ

two modes $\begin{cases} \alpha = \pi/2 \\ \alpha = \theta \end{cases}$ 1/2

Free space



given E $\Rightarrow \theta : [0, \pi]$

two modes $\begin{cases} \alpha = \pi/2 \\ \alpha = \theta \end{cases}$ 1/2

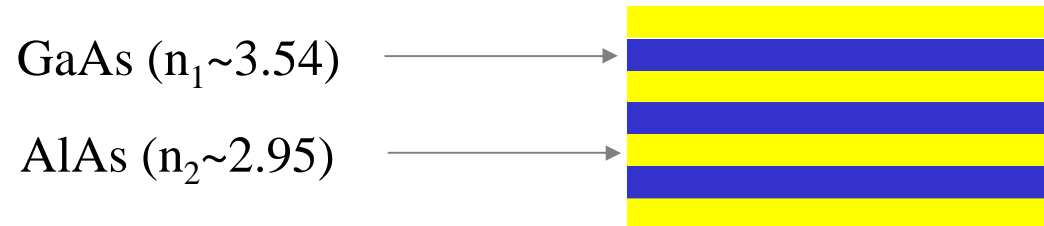
$$\int_0^\pi \cos^2 \theta \sin \theta d\theta d\varphi = \frac{2}{3} d\varphi$$

for $\lambda_0/2$ cavity $\frac{\rho_{2D}(E_0)/(e\Sigma) * 1/2}{\rho_{3D}(E_0)/\Omega * 1/3} = 3$



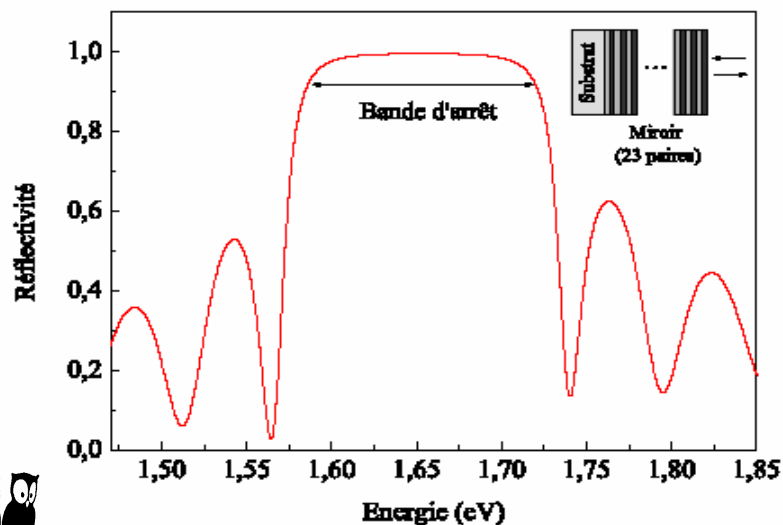
Semiconductor microcavities

- metallic mirrors = **lossy** at optical frequencies
- epitaxially grown **Distributed Bragg Reflectors**



stacked pairs of GaAs/AlAs with $n_1 e_1 = n_2 e_2 = \lambda_0/4$

- **reflectivity spectrum**



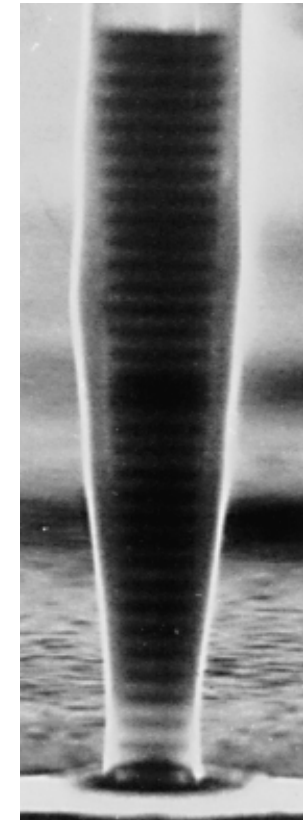
- **High** reflectivity

$$R_{\max} \sim 99.5\%$$

with **23** pairs

- **Stop-band** width ΔE

$$\Delta E/E_0 \sim (n_1 - n_2) / \langle n \rangle$$

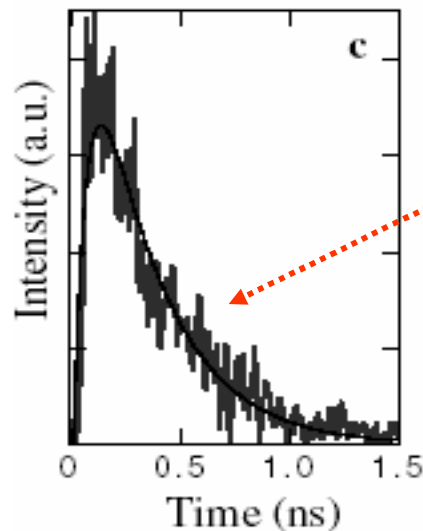


Gérard, PRL **81**, 1110 (1998)



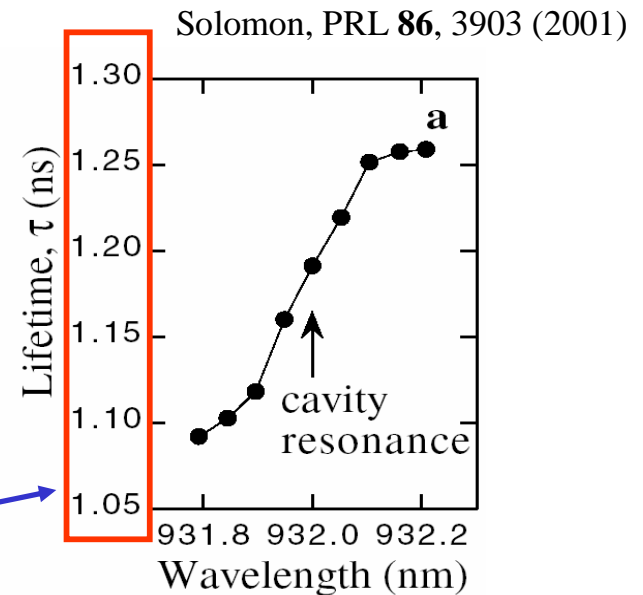
QDs in planar semiconductor microcavities

- time-resolved photoluminescence experiments



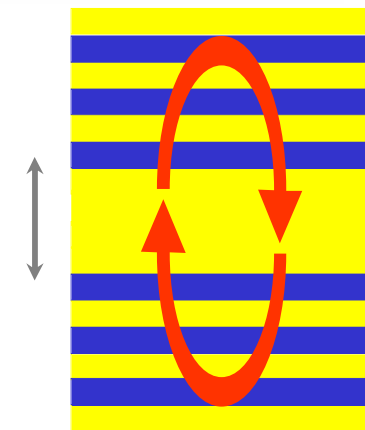
$$\exp\left(-\frac{t}{T_1}\right)$$

+/- 10%

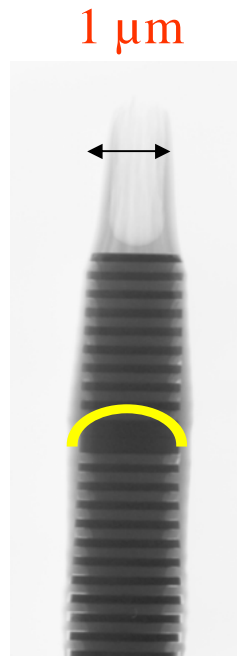


- very weak effect on QD spontaneous emission dynamics !

- in DBR based-cavity, the effective cavity length ne is around $4\lambda_0$

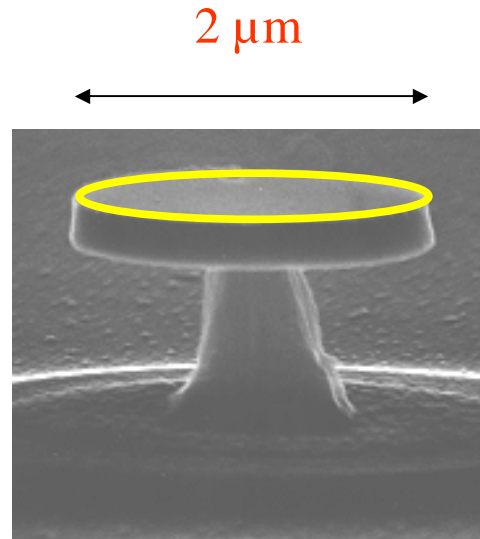


Some solid-state 0D microcavities



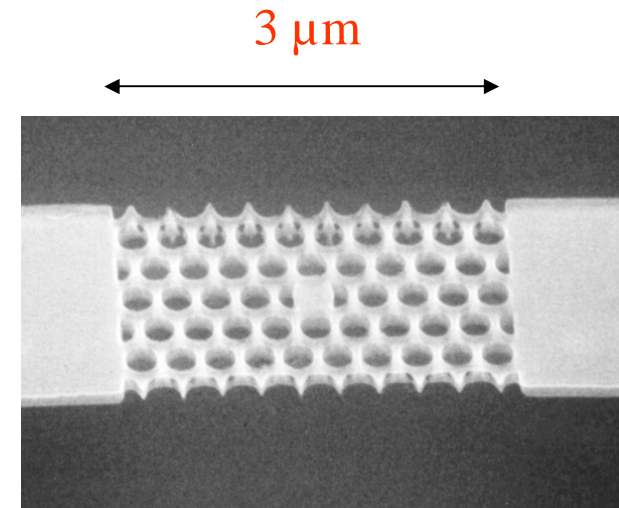
Micropillars

lateral etching
of planar
semiconductor
DBRs

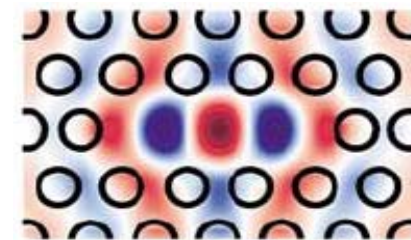


Microdisks

whispering
gallery modes



Photonic crystals defects



Akahane, Nature **425**, 944 (2003)

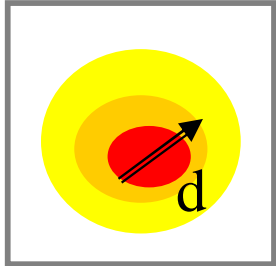


Two important figures of merit


- **spatial** confinement of electromagnetic field

$V_{eff} :$ $\mathcal{E}_{max} = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 n^2 V_{eff}}}$

effective mode volume



Actual cavity



Equivalent cavity
volume V_{eff}

- **temporal** confinement of electromagnetic field

$Q :$ $Q = \frac{E_0}{\Gamma_{cav}} \longrightarrow \tau_{photon} = \frac{\hbar}{\Gamma_{cav}}$

mode quality factor

around 1 eV, $Q = 1000$ \longleftrightarrow $\tau_{photon} = 0.6$ ps



High-Q small-mode volume 0D cavities

1 μm

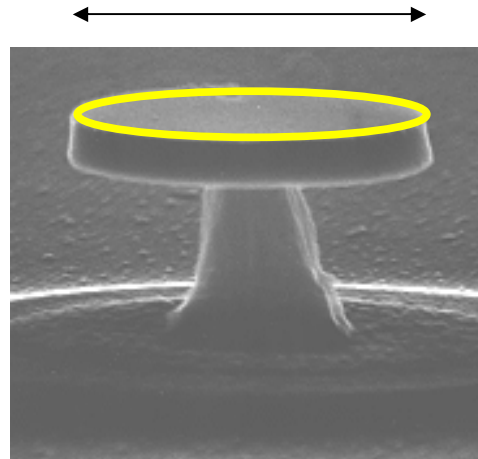


Micropillars

$$V=5 (\lambda/n)^3$$
$$Q \sim 7000$$

Reithmaier, Nature **432**, 197 (2004)

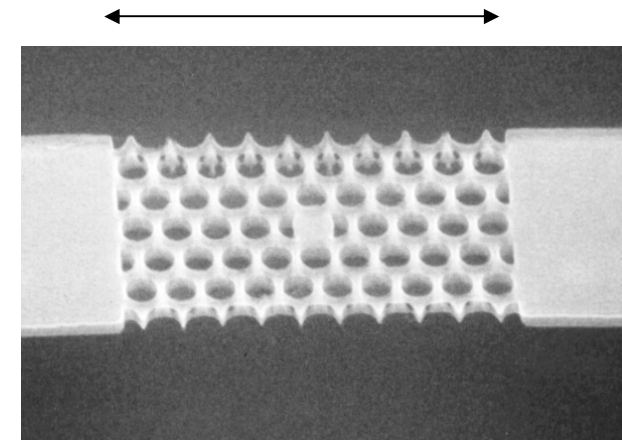
2 μm



Microdisks

$$V=6 (\lambda/n)^3$$
$$Q \sim 12000$$

3 μm



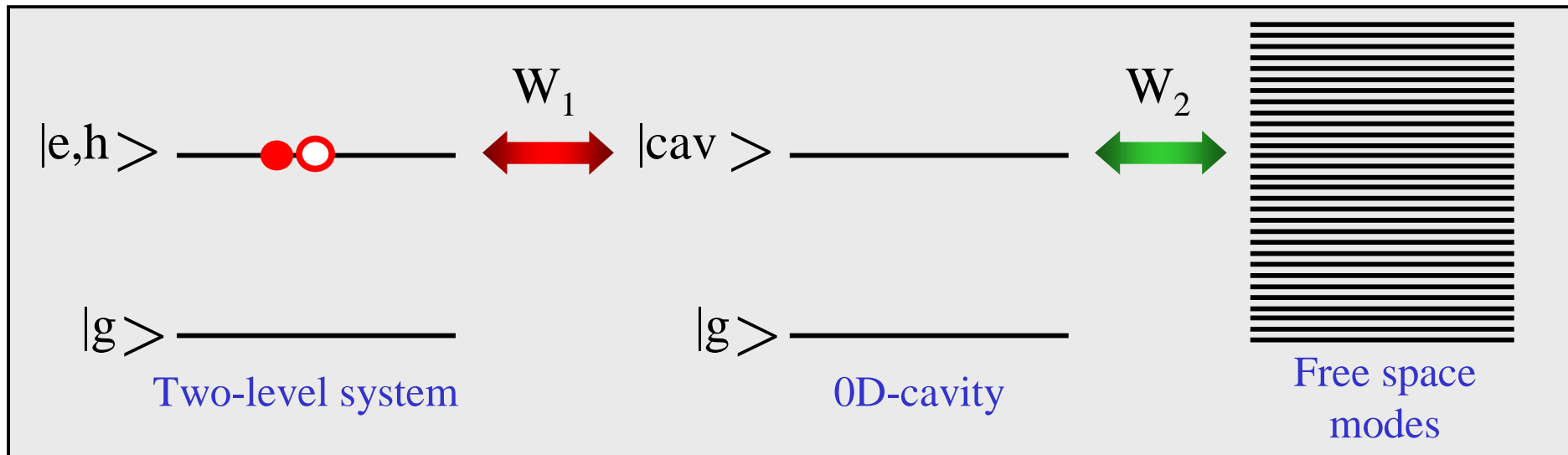
Photonic crystals defects

$$V=1.2(\lambda/n)^3$$
$$Q \sim 20000$$

Yoshie, Nature **432**, 200 (2004)



Quasi-mode approximation



- two cases : $W_1 > W_2$ **strong coupling** regime
 $W_1 < W_2$ **weak coupling** regime

- in the **weak coupling** regime, QD is coupled to the electromagnetic field which mode density is given by :

$$\rho(E) = \frac{2Q}{\pi E_{cav}} \frac{E_{cav}^2}{4Q^2(E - E_{cav})^2 + E_{cav}^2}$$

damped 0D-cavity mode
= quasi-mode

Fano, Phys. Rev. **124**, 1866 (1961)



Purcell factor

$$\frac{1}{T_1} = \frac{2\pi}{\hbar} \sum_{\{|1_\omega\rangle\}} |\langle f | \widetilde{H}_I | i \rangle|^2 \delta(E_{|e,h\rangle} - \hbar\omega)$$

with $\vec{A}(\vec{r}) = i\sqrt{\frac{\hbar}{2\varepsilon_0\omega V_{eff}}} (a\vec{f}^*(\vec{r}) + a^+\vec{f}(\vec{r}))$ where $\begin{cases} |\vec{f}(\vec{r})|_{max} = 1 \\ \int d^3\vec{r} |\vec{f}(\vec{r})|^2 = V_{eff} \end{cases}$

free space

$$\left(\frac{1}{T_1}\right)_{free\ space} = \frac{\Gamma}{\hbar} = \frac{1}{3} \frac{P_{cv}^2}{\varepsilon_0} \frac{E_0}{\pi\hbar^2 c^3}$$

0D cavity

- dipole at the field maximum
- dipole parallel to field
- $E_0 = E_{cav}$

$$\left(\frac{1}{T_1}\right)_{cav} = \frac{\Gamma_p}{\hbar} = 2 \frac{P_{cv}^2}{\varepsilon_0} \frac{\hbar Q}{E_0^2 V_{eff}}$$

Purcell factor

Purcell, Phys. Rev. **69**, 681 (1946)

$$F_p = \frac{\Gamma_p}{\Gamma}$$

and we find

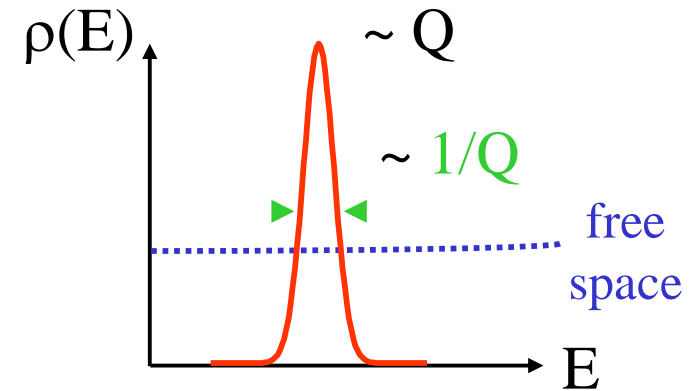
$$F_p = \frac{3}{4\pi^2} \frac{\lambda_0^3 Q}{V_{eff}}$$



Purcell effect in 0D-cavity

- **QD** = dipole in a dielectric 0D-cavity

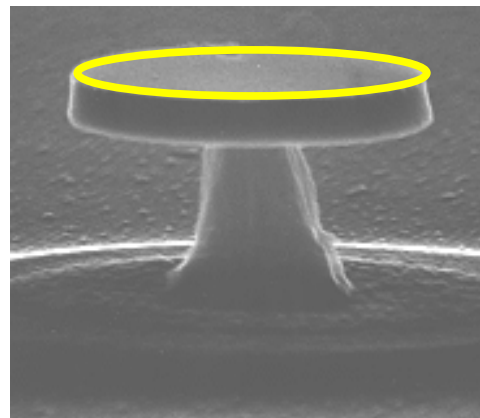
$$F_p = \frac{3}{4\pi^2} \frac{(\lambda_0/n)^3 Q}{V_{eff}}$$



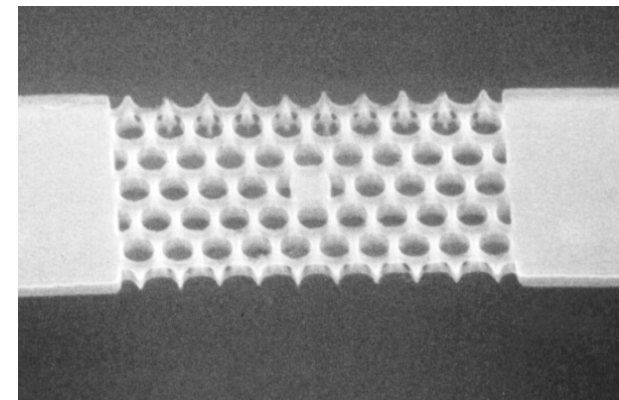
- **Calculated** Purcell factors



$V=5 (\lambda_0/n)^3$
 $Q \sim 7000$
 $F_p \sim 100$



$V=6 (\lambda_0/n)^3$
 $Q \sim 12000$
 $F_p \sim 150$



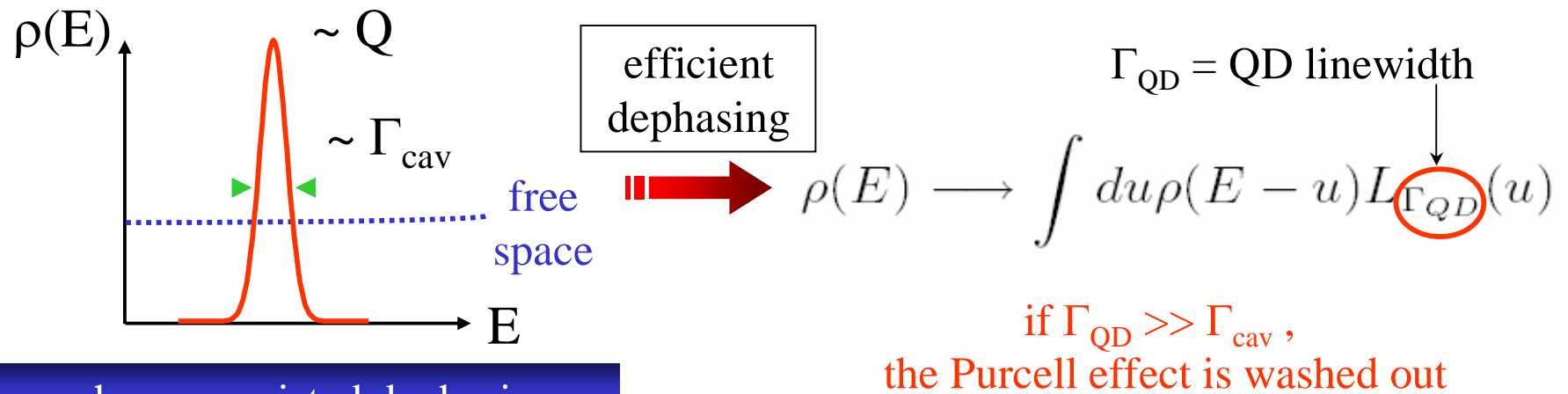
$V=1.2(\lambda_0/n)^3$
 $Q \sim 20000$
 $F_p \sim 800$



Purcell effect in a non-ideal case

$$F_p = F_p^0 \underbrace{\frac{E_{cav}^2}{4Q^2(E_0 - E_{cav})^2 + E_{cav}^2}}_{\text{Spectral detuning}} \underbrace{\left| \vec{f}(\vec{r}_0) \right|}_{\text{Spatial detuning}} \underbrace{\cos^2 \alpha}_{\text{Dipole misorientation}}$$

- **broadening** of the QD emission line

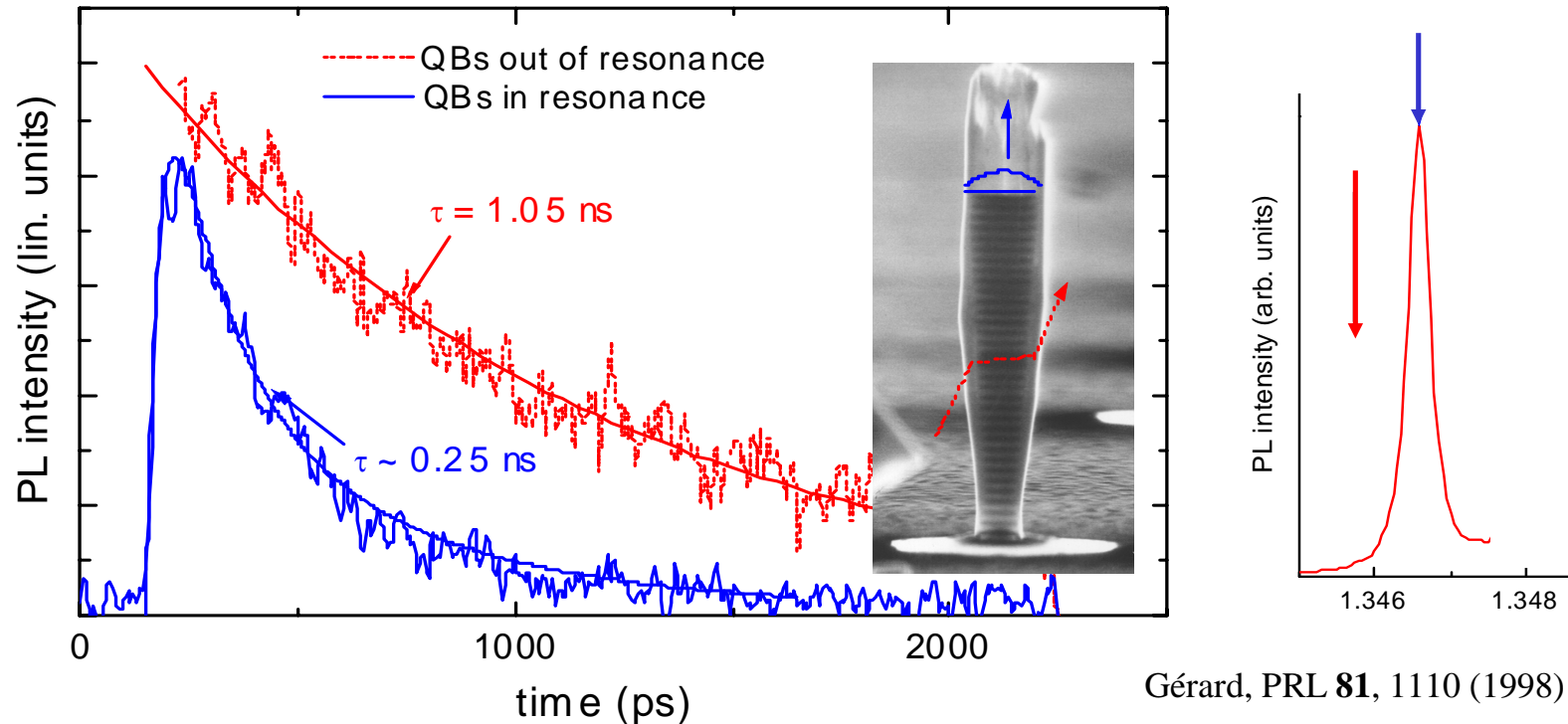


no phonon-assisted dephasing



Purcell effect in InAs QDs

- time-resolved experiments in QD arrays : **spatial** + **spectral** averaging

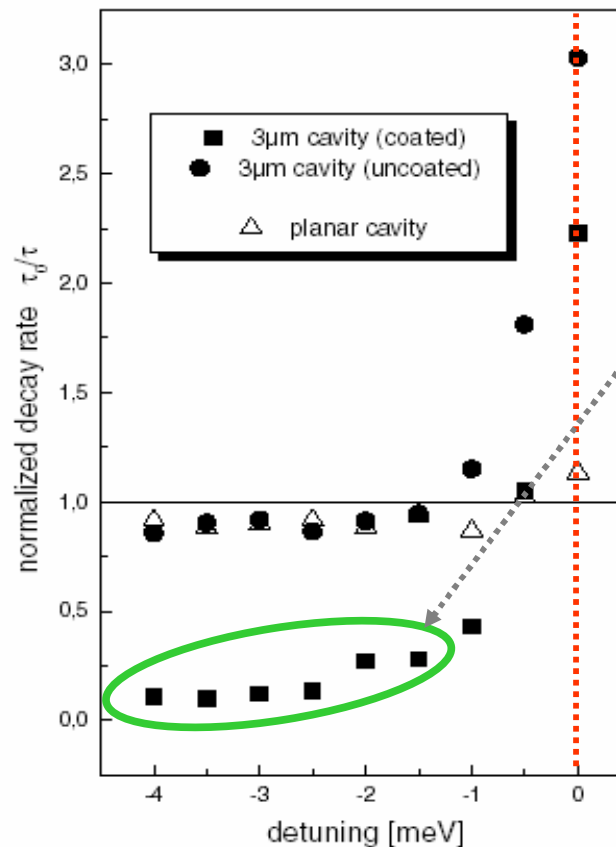
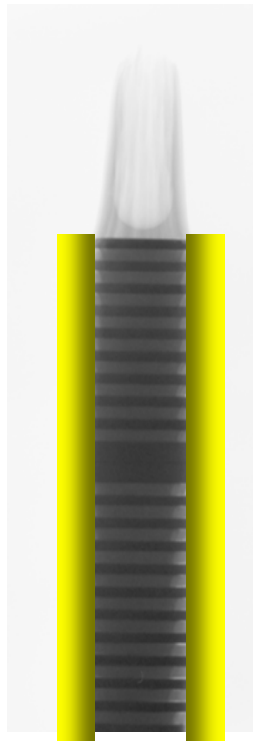


- PL decay **faster** only for **on-resonance** QDs : (x5) enhancement of spontaneous emission rate (much smaller than F_p)
- for **off-resonance** QDs, **free-space** like recombination dynamics : **leaky modes**



Spontaneous emission inhibition

- suppression of leaky modes in **gold-coated** micropillars



- **strong inhibition** (/10) of spontaneous emission rate
- **efficient reduction of the density** of non-resonant modes by the metallic coating

Bayer, PRL **86**, 3168 (2001)

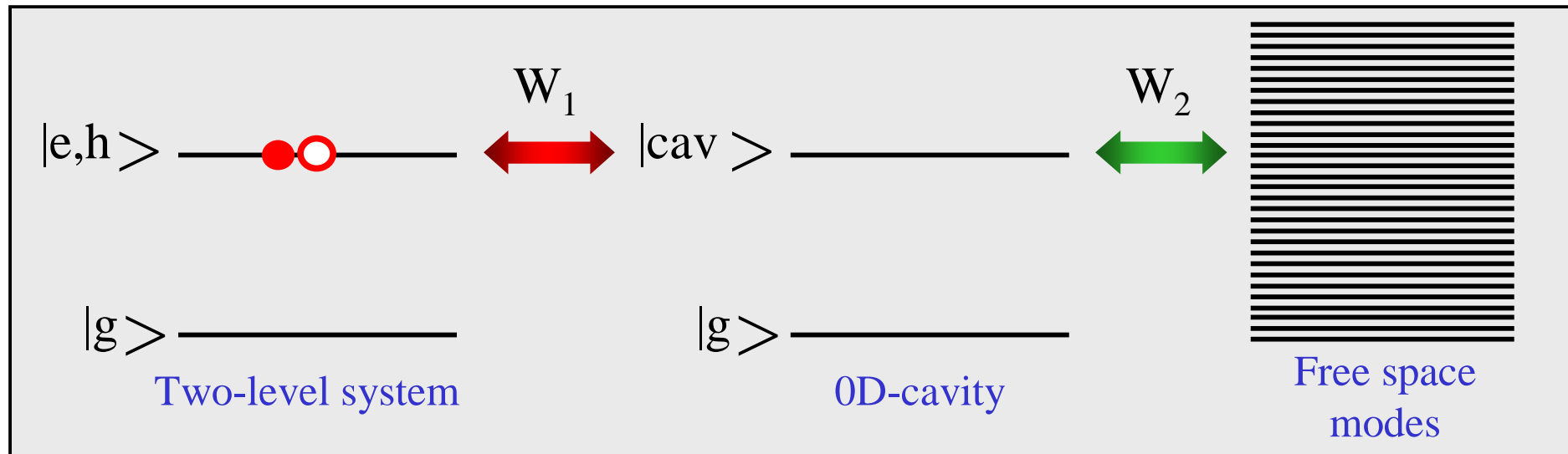
off-resonance QDs on-resonance QDs



Strong coupling regime



From weak to strong coupling



- two cases : $W_1 > W_2$ **strong coupling** regime
 $W_1 < W_2$ **weak coupling** regime

- in the **strong coupling** regime, the coupling to free space modes is a **perturbation to (QD-cavity) sub-system**

existence of mixed electron-photon states = microcavity polaritons



Microcavity polaritons

- **two states** of microcavity polariton

$$|-\rangle = \alpha|e, h\rangle + \beta|cav\rangle$$

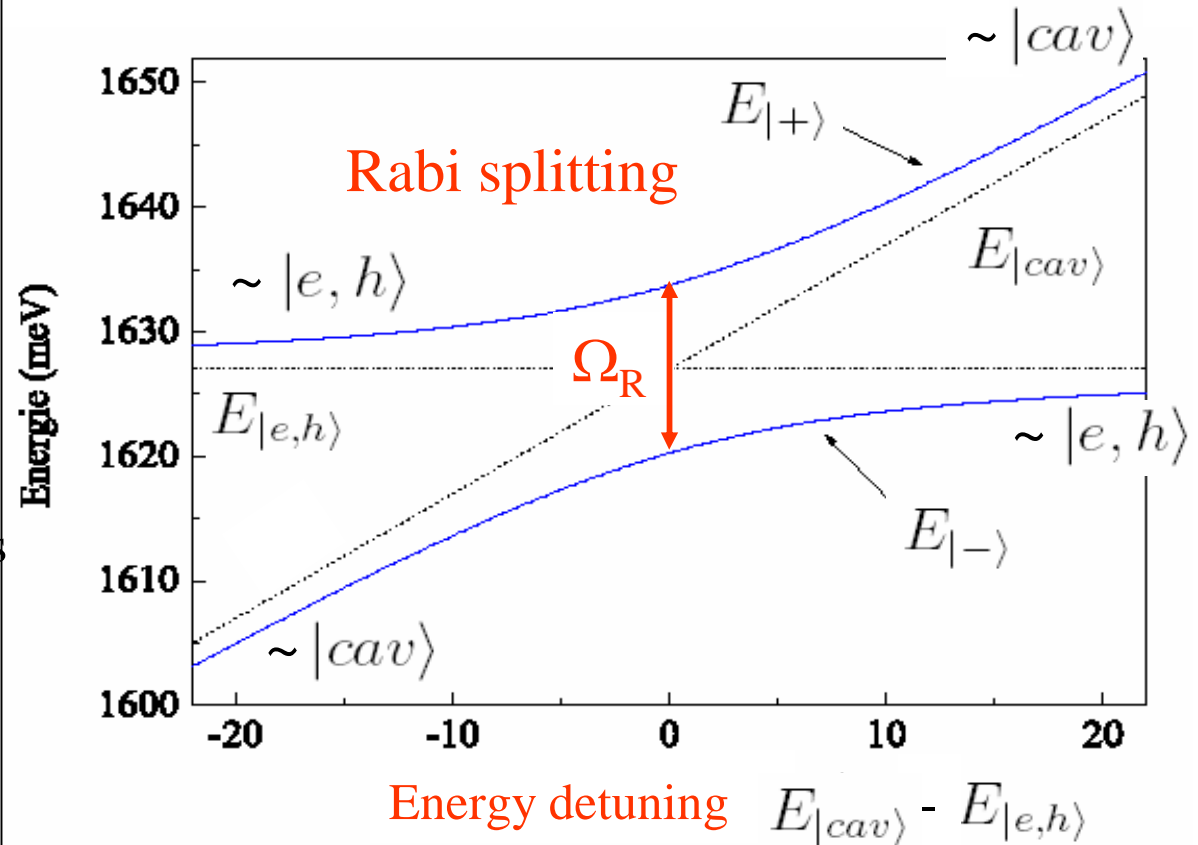
$$|+\rangle = \beta|e, h\rangle - \alpha|cav\rangle$$

- **anti-crossing** at zero detuning

$$E_{|+\rangle} - E_{|-\rangle} = \Omega_R$$

- at zero-detuning, the polaritons are **half electron** } **half photon**

$$|\alpha|^2 = |\beta|^2 = \frac{1}{2}$$



Weak to strong coupling crossover

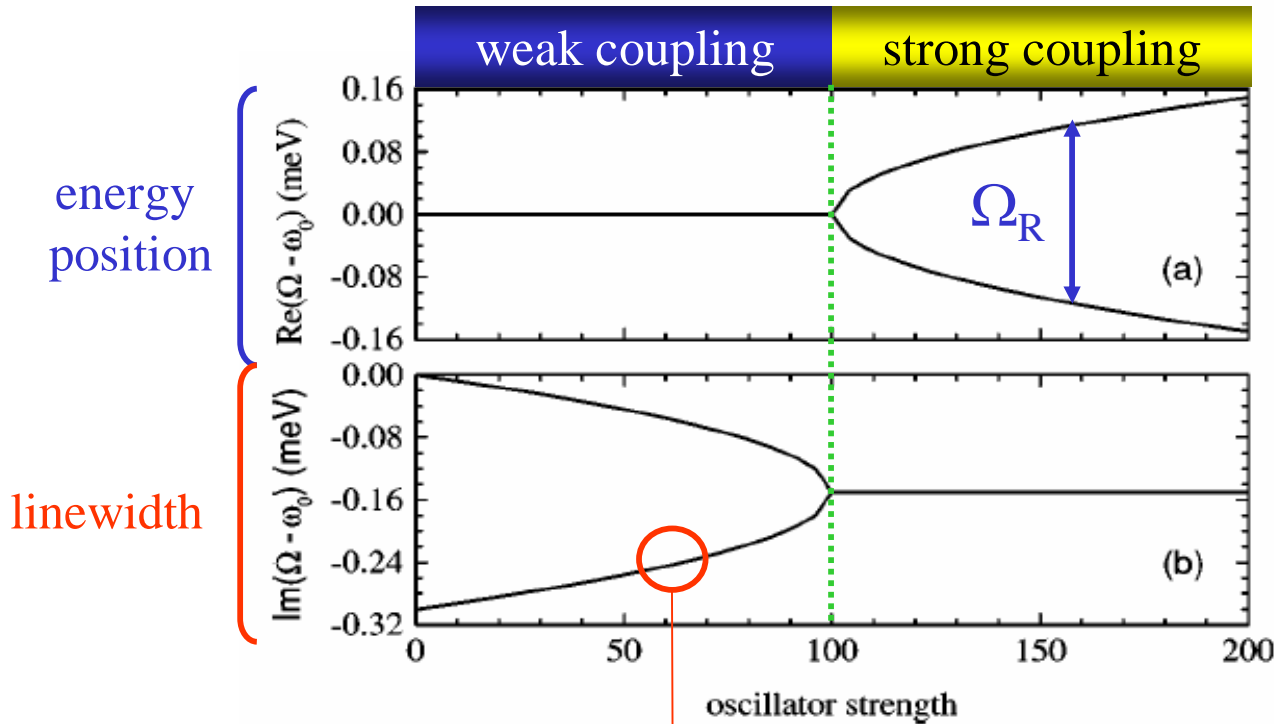
Andreani, PRB **60**, 13276 (1999)

$$\Omega_{\pm} = \omega_0 - \frac{i}{4}(\gamma_a + \gamma_{c,\mu}) \pm \sqrt{g^2 - \left(\frac{\gamma_a - \gamma_{c,\mu}}{4}\right)^2}$$

$$\hbar g \propto |\langle \vec{p} \cdot \vec{A} \rangle| \propto \frac{1}{\sqrt{V_{eff}}}$$

$$\hbar \gamma_a = \Gamma_{QD}$$

$$\hbar \gamma_{c,\mu} = \Gamma_{cav} \propto \frac{1}{Q}$$



weak coupling

$$g \ll |\gamma_a - \gamma_{c,\mu}|$$

$$g \ll \gamma_{c,\mu}$$

high V_{eff}
small Q

strong coupling

$$g \gg |\gamma_a - \gamma_{c,\mu}|$$

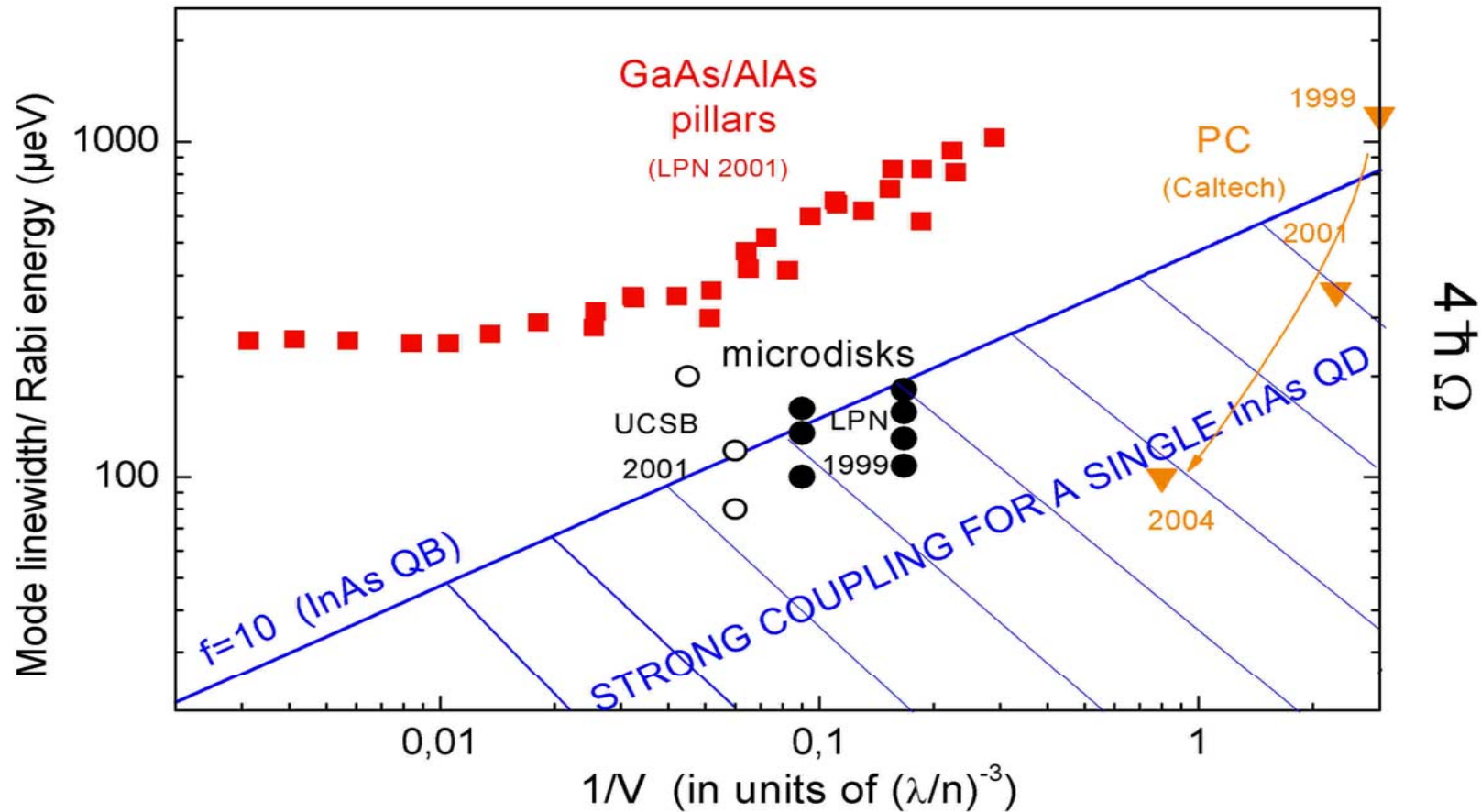
$$g \gg \gamma_{c,\mu}$$

small V_{eff}
high Q

$\Gamma_p = F_p \Gamma$, modified spontaneous emission rate (Purcell effect)



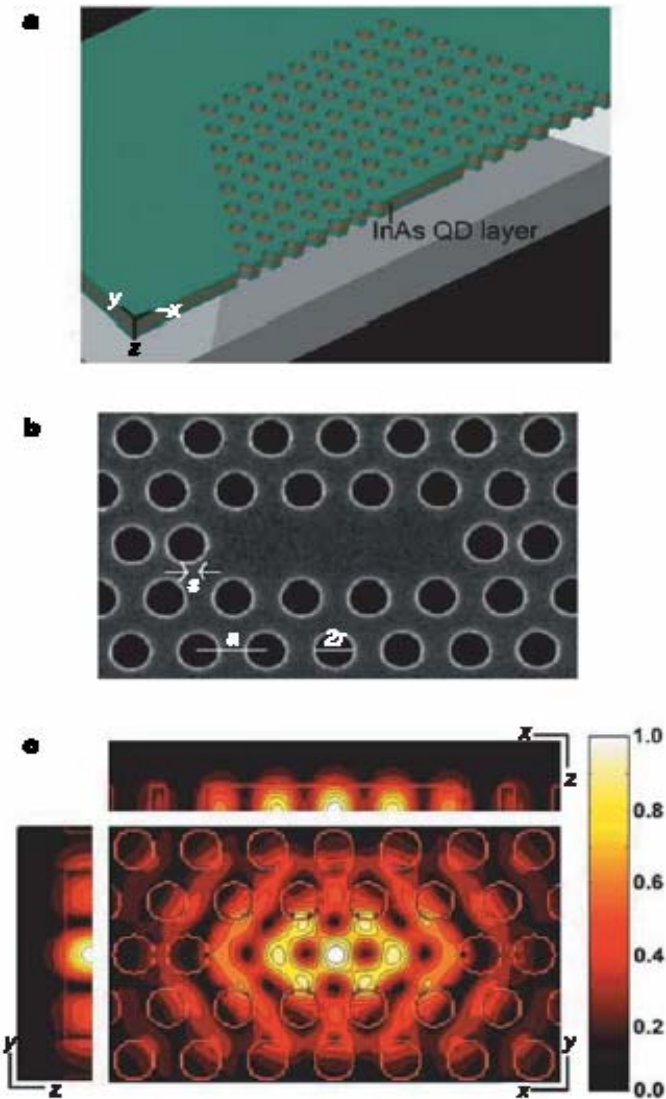
Towards strong coupling for a single QD



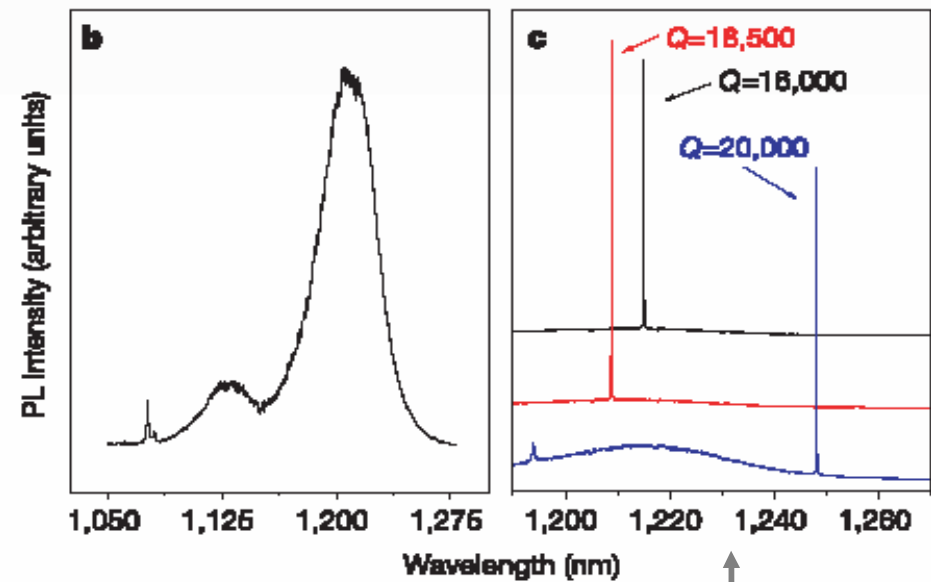
For a single InAs QD, strong coupling already observable with microdisks and PC cavities



Photonic crystals high-Q microcavities



Yoshie, Nature **432**, 200 (2004)

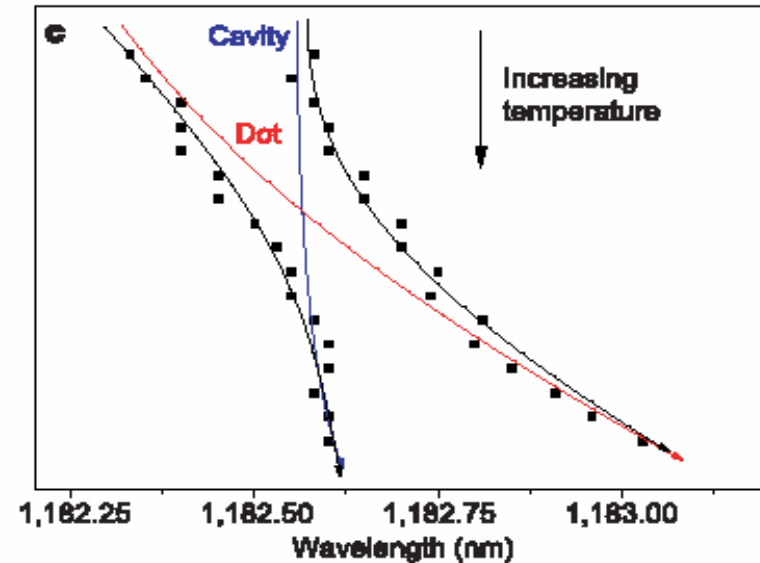
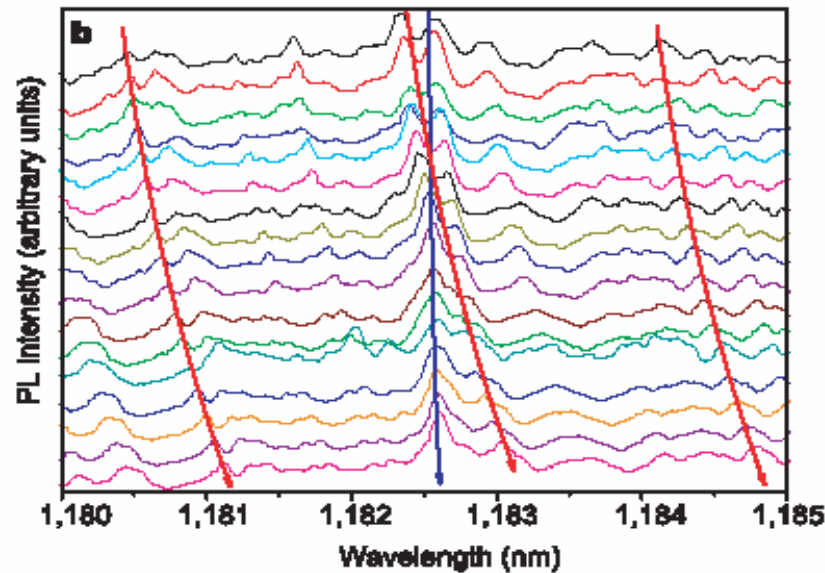


three different
cavities of high Q



Strong coupling for a single InAs QD

Yoshie, Nature **432**, 200 (2004)



- evidence for a single InAs QD in the strong coupling regime : $\Omega_R \sim 170 \mu\text{eV}$
- strong coupling also obtained (**simultaneously !!**) for
 - a single InGaAs QD in **micropillar** Reithmaier, Nature **432**, 197 (2004)
 - a single GaAs QD in **microdisk** Peter (LPN-France), submitted to PRL

