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"2nd Workshop on Earthquake Engineering for Nuclear Facilities: Uncertainties in Seismic Hazard"

14 - 25 February 2005

Integration of data - from uncertainties in seismotectonic modelling - to expert elicitation

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IAEA/ICTP Workshop on
Earthquake Engineering for Nuclear Facilities -
Uncertainties in Seismic Hazard Assessment

“Performance-based expert judgement - a structured elicitation approach & case histories”

Trieste, Italy, 14 – 25 February 2005

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INTRODUCTION

In this morning's session, we showed an example illustrating inter-expert variation in a PSHA for a low seismicity area……

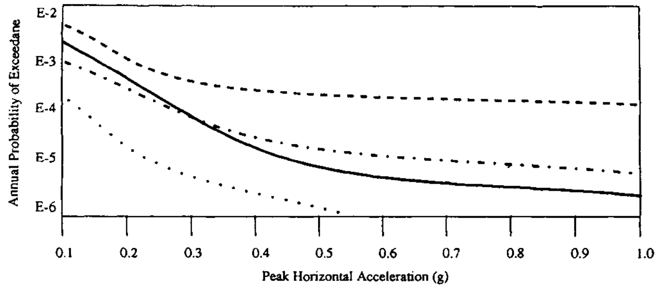


Figure 1: Inter-expert variation in expected seismic hazard curves for a site with sparse regional data

……and a plot of inter-expert variability when providing parameters for a seismic hazard model

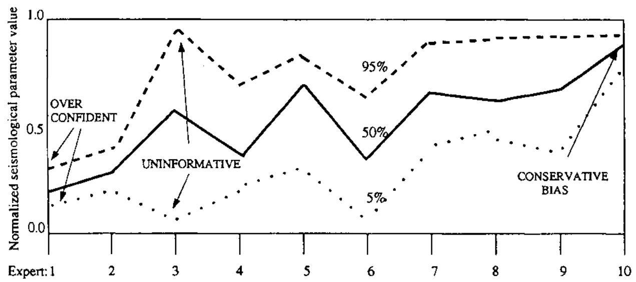


Figure 2. Inter-expert variation in 5%, 50% and 95% estimates of seismological parameters

Recognising that not all subject-matter experts are equal, in this presentation we describe a formal procedure that can be used to provide a performance-based rankings for experts, their judgements and opinions.

We start with a short description of expert judgement elicitation in a volcanic eruption crisis

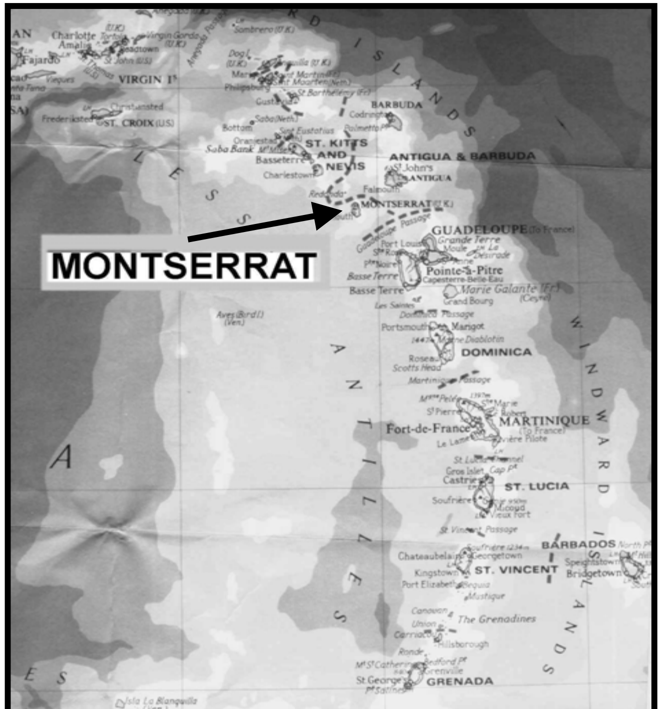
First, I must acknowledge:

Dr. Gordon Woo, Prof. Roger Cooke and Prof. Steve Sparks FRS

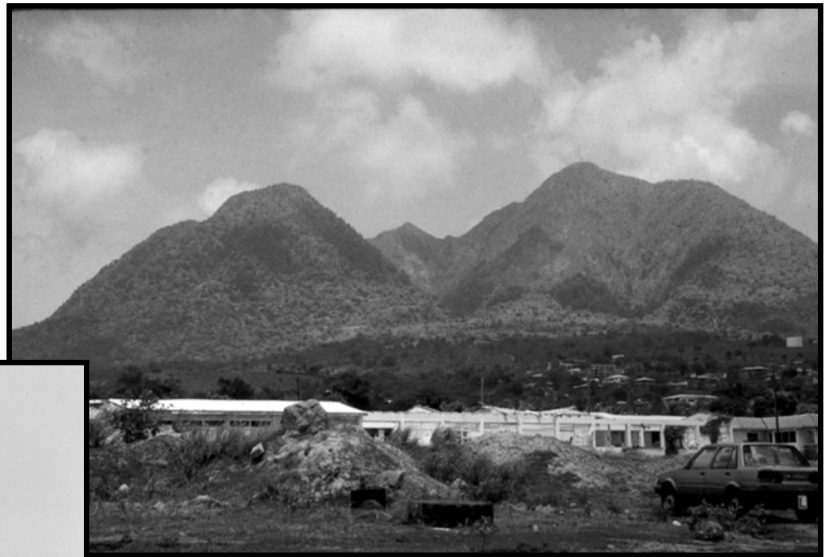
Montserrat Volcano Observatory
British Airways
Kellogg Brown & Root / DEFRA

Institute for Advanced Studies, Bristol University University of BRISTOL

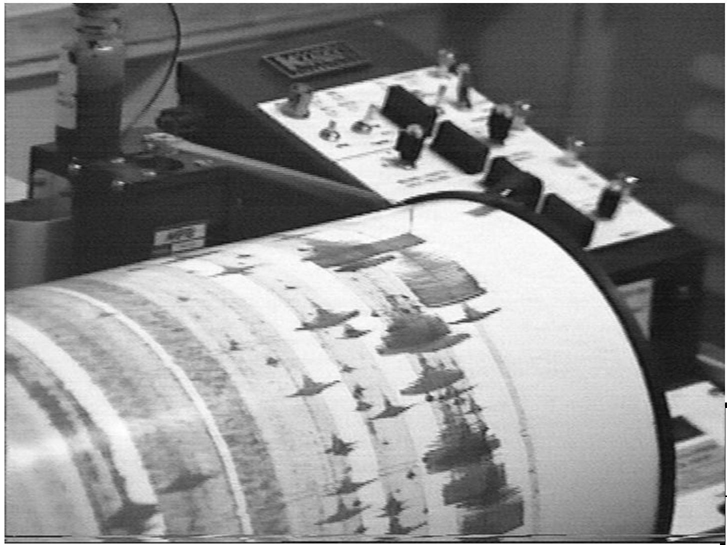
and the Montserrat eruption, 1995…….

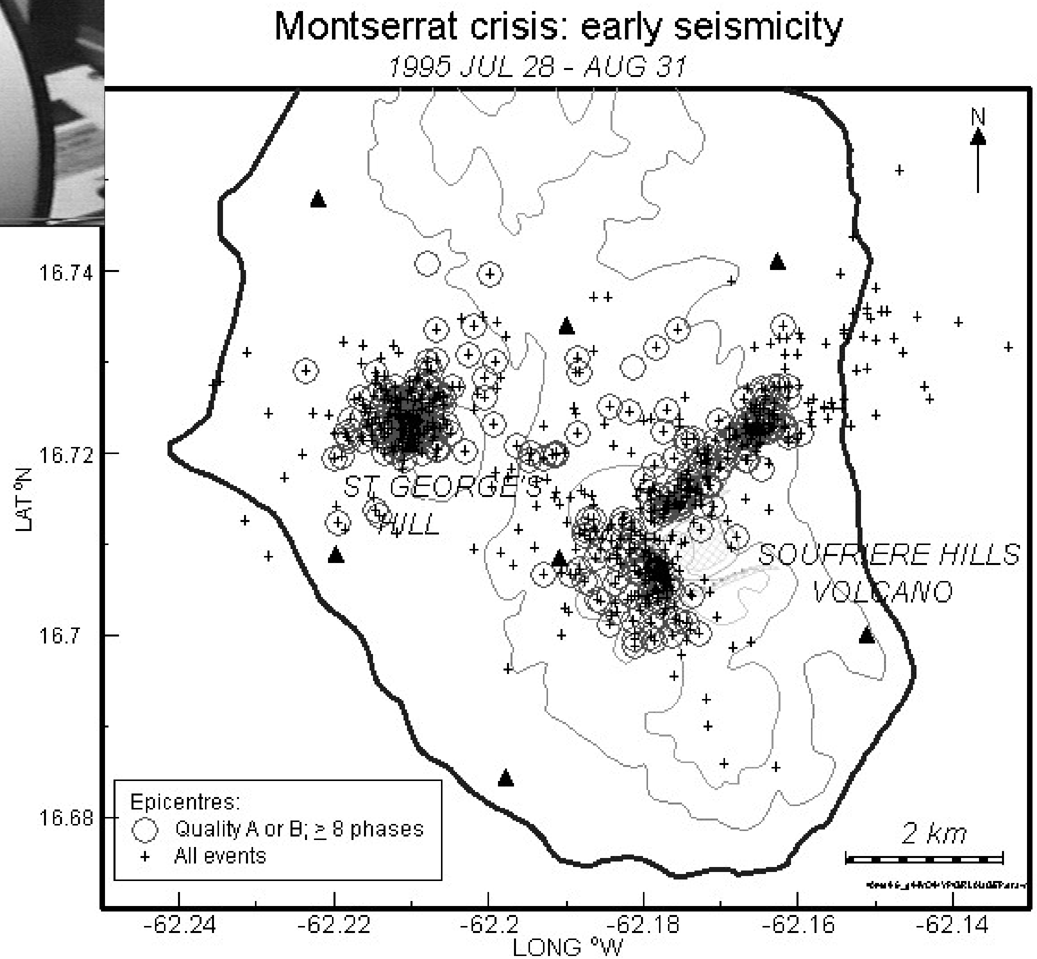


Soufrière Hills, Montserrat, in former times……



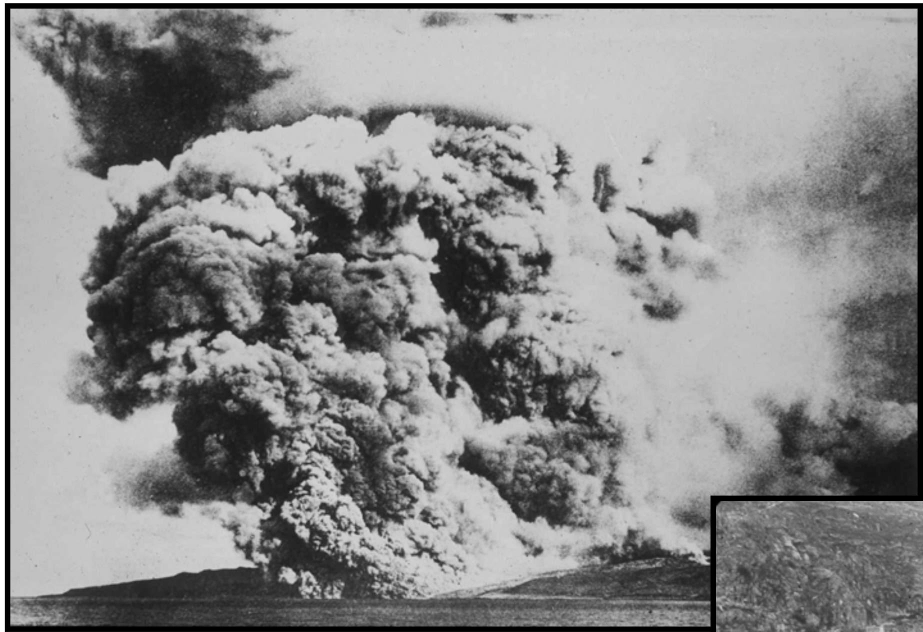
…and in July 1995





Precursory seismic activity………

A regional history of volcanic disasters in the Eastern Caribbean



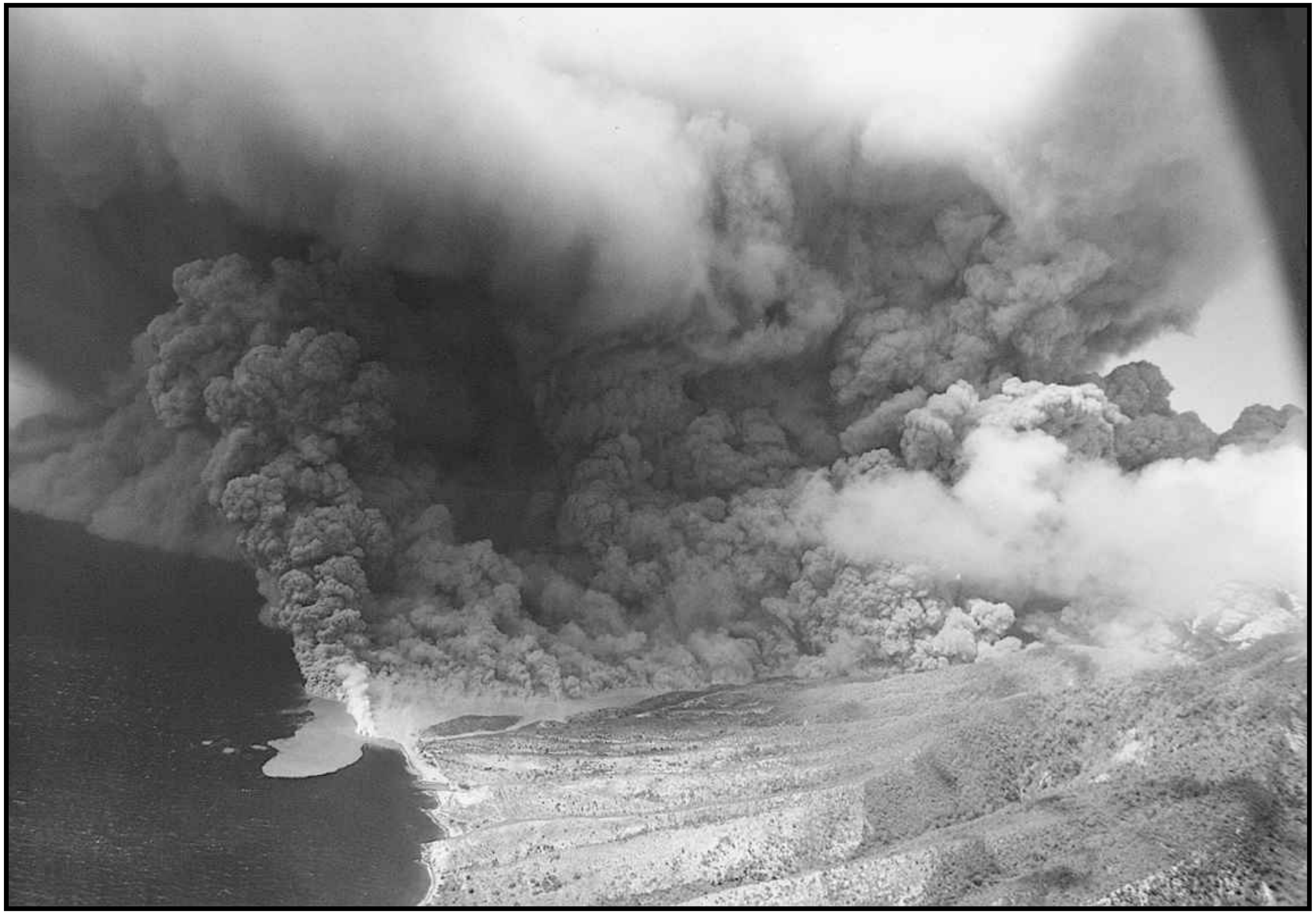
Mt Pelée, Martinique 1902

29,000 people die when political priorities take precedence over public concerns

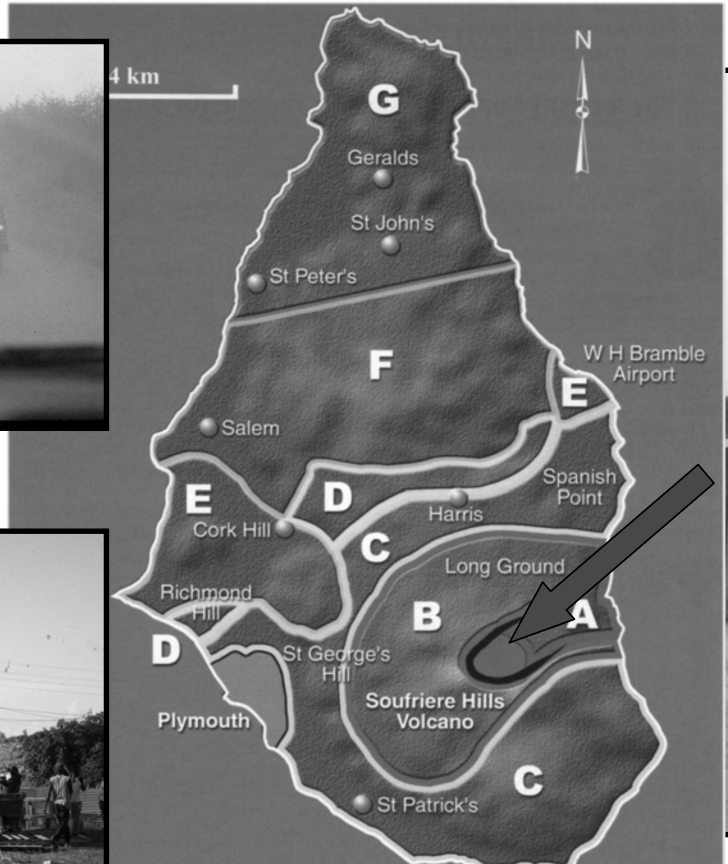
Then, Guadeloupe, 1976……

….a volcanic crisis leads to a major evacuation, but the eruption is stillborn; scientists are embroiled in public controversy, severe criticism and recriminations

In Montserrat, a magmatic eruption is confirmed, and escalates progressively in intensity and danger….



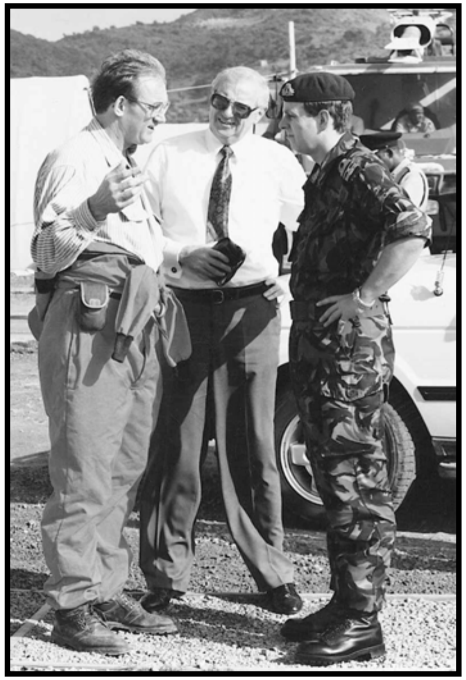
Living with an erupting volcano: hazard zones for crisis micro-management



"…this island is exactly the wrong size for an eruption…"

Prompted by the Guadeloupe 1976 experience….

….in Montserrat, we put in place a formalised procedure for providing scientific advice to the authorities



using a procedure developed originally for the European Space Agency

Alternative approaches to pooling expert opinions:

- simple averaging
- committee
- decision conferencing (Bonano 1990)
- the Delphi method
- equal weights (Coppersmith & Youngs 1990)
- expert self-weighting (TERA 1980)
- group mutual weightings
- mathematical theory of scoring rules $\Rightarrow$

Cooke (1991): "Classical" model for pooling opinions and implementation in the EXCALIBR program

The basis of Cooke's "classical" model

Given a set of known (or knowable) seed items, for each expert

test hypothesis H_0: "This expert is well calibrated", leading to likelihood of acceptance at some defined significance level, and use this likelihood to define his **Calibration score:**

$$C_j = 1 - \chi_R^{\ 2}(2 * M * I(s_j, p) * Power)$$

…where *j* denotes the expert, *R* is no. of quantiles (=degrees of freedom), *M* is the number of seed variables used in calibration, and *I(s,p)* is a measure of information.

Cj **corresponds to the asymptotic probability of seeing a deviation between *s* and *p* at least as great as *I(s,p)*, under the hypothesis.**

The basis of Cooke's "classical" model

– **Entropy score**

estimate individual's information score relative to a uniform or loguniform density function from:

$$I_j(s_j, p) = \frac{1}{n}\sum_{i=1}^{n} s_i \ln\left(\frac{s_i}{p_i}\right)$$

where s_i ***is a sample distribution obtained from the expert on the seed variables, and*** p_i ***is a suitable reference density function, depending on the appropriate scaling for the item.***

The basis of Cooke's "classical" model

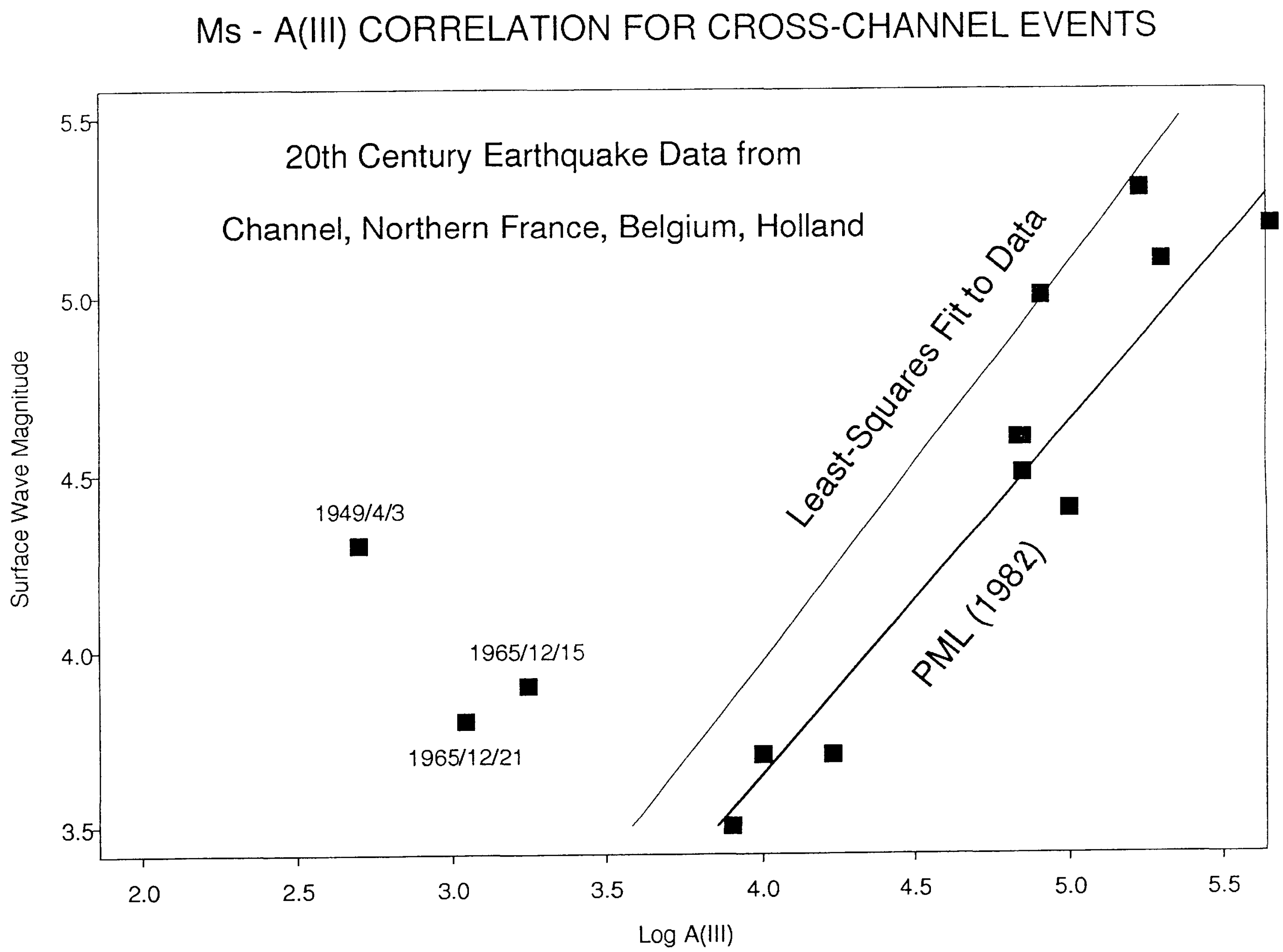
– **Individual's expert weighting**

compute individual's weight from product of his Calibration and Entropy scores (where the latter is now estimated from all variables, seeds and unknowns):

$$W_j = C_j * I_j(s_j, p)$$

and normalise the W_j across all experts to get relative weights.

Macroseismic area - instrumental magnitude correlation across borders



The outliers….

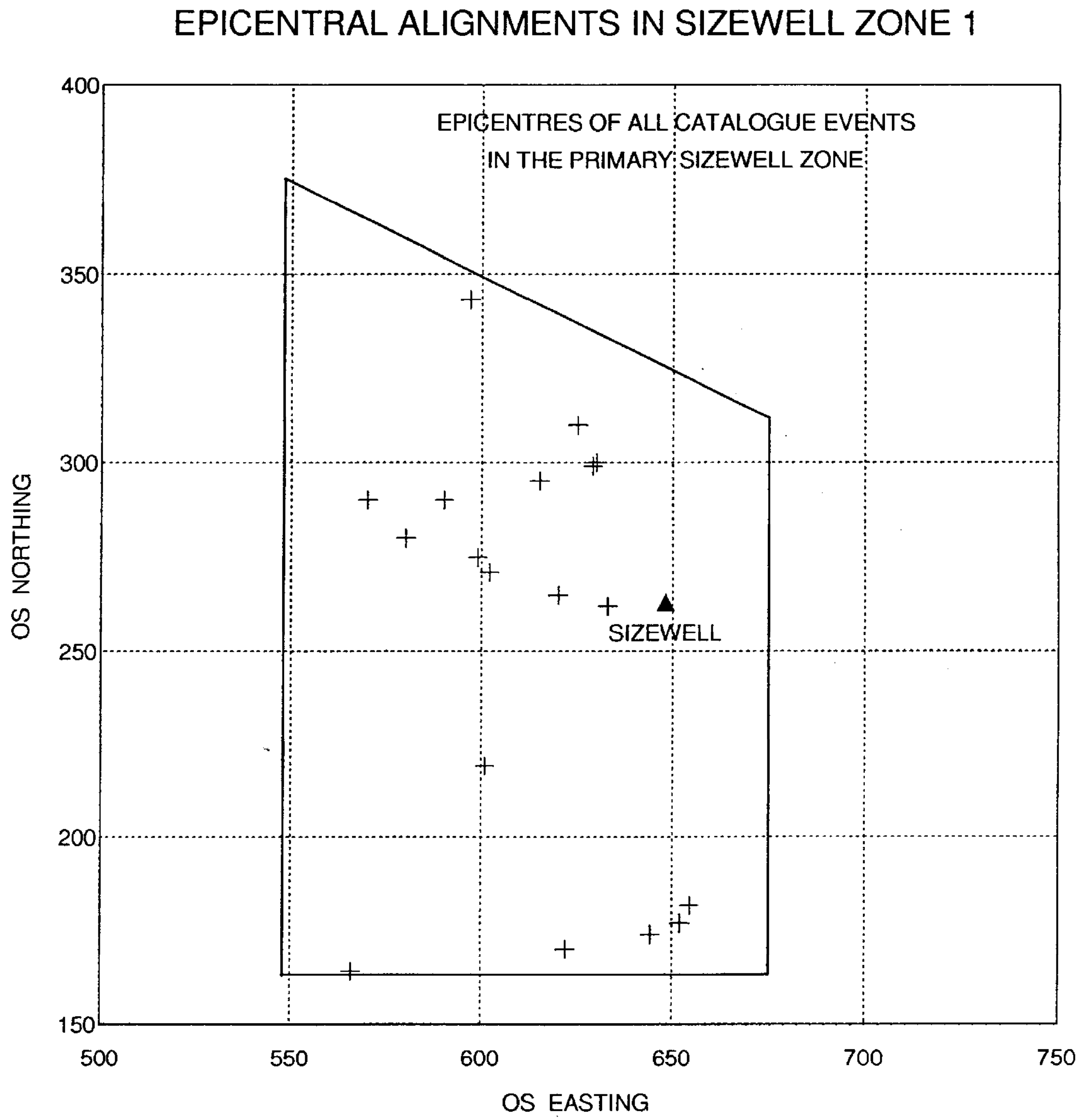
Date	Lat	Lon	I_0	Ms	A(IV) sq. km	A(III) sq. km	Location
1925/02/01	49.16	-5.22	6	5.1	50000	200000	Eastern Approaches
1926/07/30	49.17	-1.62	6	5.1	70000	200000	Channel Islands
1927/02/17	49.17	-1.62	6	4.6	30000	70000	Channel Islands
1927/11/19	48.78	-0.50	6	4.6	18500	68000	Calvados-Orne
1928/01/14	50.52	6.22	5	3.7	5000	10000	Liege
1928/12/13	50.92	6.53	6	3.5	4000	8000	Duitsland
1932/11/20	51.71	5.61	6	4.5	18500	70000	North Brabant
1938/06/11	50.78	3.58	7	5.0	31000	80500	Belgium
1949/04/03	**50.45**	**4.00**	**7**	**4.3**	**300**	**500**	**Mons-Havre**
1951/03/14	50.64	6.73	7	5.3		170000	Euskirchen
1960/06/25	51.20	5.70	5	3.7	8000	17000	Stramproy
1965/12/15	**50.48**	**4.09**	**7**	**3.8**	**750**	**1100**	**Hainaut**
1965/12/21	**50.65**	**5.58**	**7**	**3.9**	**750**	**1750**	**Hainaut**
1983/11/08	50.65	5.54	7	4.4	40000	100000	Liege
1992/04/13	51.18	6.00	7	5.2	145000	450000	Roermond

Solution: return to original accounts, re-interpret and re-plot isoseismals to a uniform standard for the whole dataset - *don't rely totally on some other expert's judgements !*

Collinearity of epicentres??

….and potential association with mapped geological faults.

First question: are the apparent lineations of epicentres meaningful, or simply stochastic chance variations?



Collinearity of epicentres??

Stochastic geometry: definition of collinearity of points in a plane…….

Three points are collinear within tolerance ε if the largest angle in the triangle formed by the points is $>(\pi - \varepsilon)$ radians - this is called a 'triad' [Broadbent, 1980]

a 'quadrad' is formed by 4 points collinear, within tolerance ε

and

a 'pentad' is formed by 5 points collinear, within tolerance ε

(Broadbent was interested in the apparent alignment of standing stones or megaliths - see: Broadbent S. 1980 Simulating the Ley hunter. Proc. J. Roy. Stat. Soc., 143, 109-140)

Collinearity of epicentres??

For *n* epicentres in a zone, uniformly randomly distributed, the expected number of triads is:

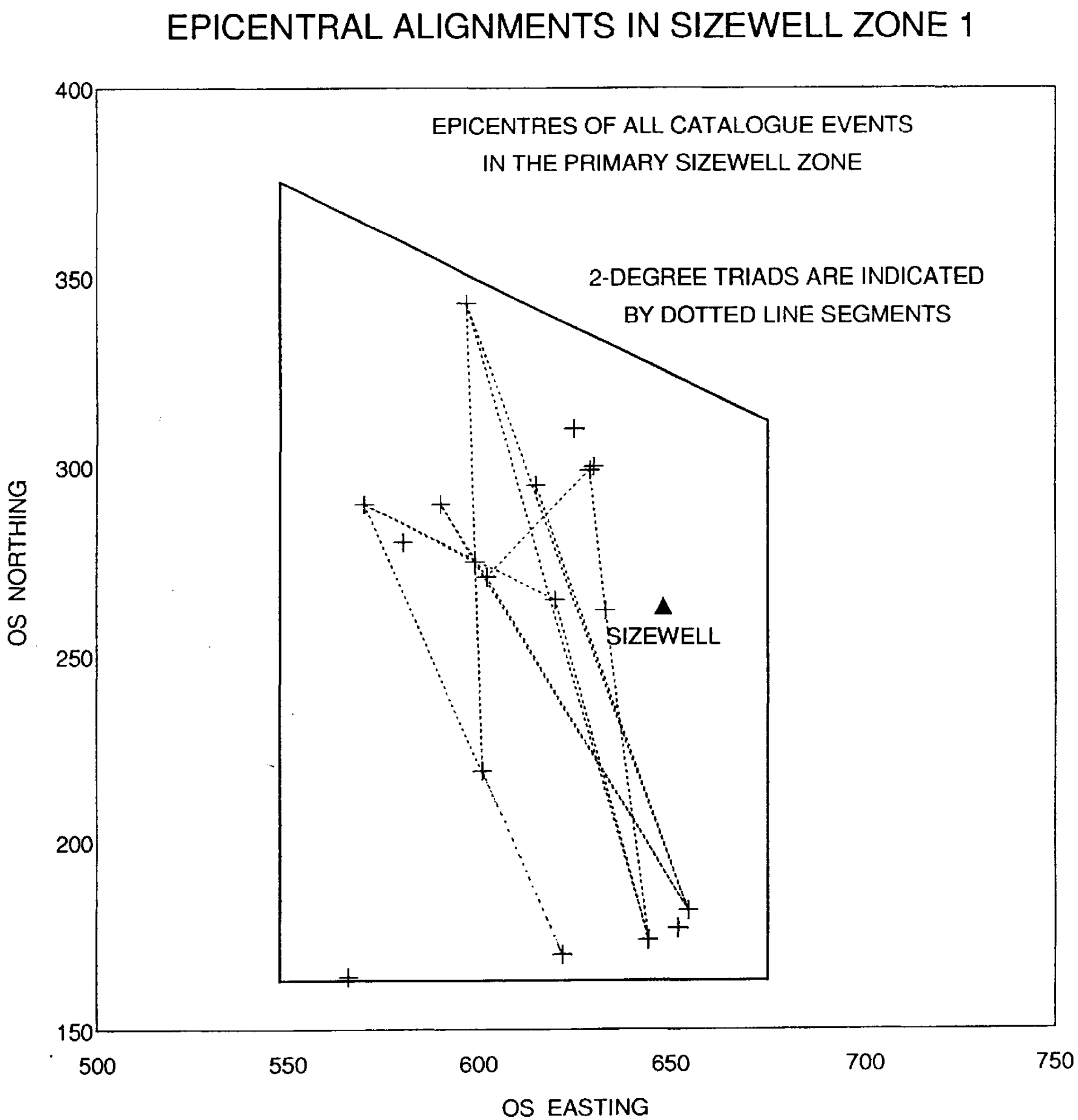
$$\binom{n}{3}\lambda\varepsilon$$

where λ is the first collinearity constant for the area, given by $2k^2/A$ where A is zone area and k^2 is the radius of gyration about an axis through the centroid perp. to the plane containing the points.

The s.d. of the number of triads is approx.:

$$\sqrt{\binom{n}{3}\lambda\varepsilon(1-\lambda\varepsilon)}$$

Collinearity of epicentres??



9 triads found,

for a tolerance angle of 2°

Collinearity of epicentres??

In the Sizewell case, 9 triads were found

With 18 epicentres randomly distributed, Monte Carlo resampling of location uncertainties produces the expected no. of triads (using the Broadbent formula):

$$10.5 \pm 3.2$$

for a tolerance angle of 2°

That is, the epicentres are consistent with randomness of location, and the apparent collinearities do not require a hypothesis of underlying structural control.

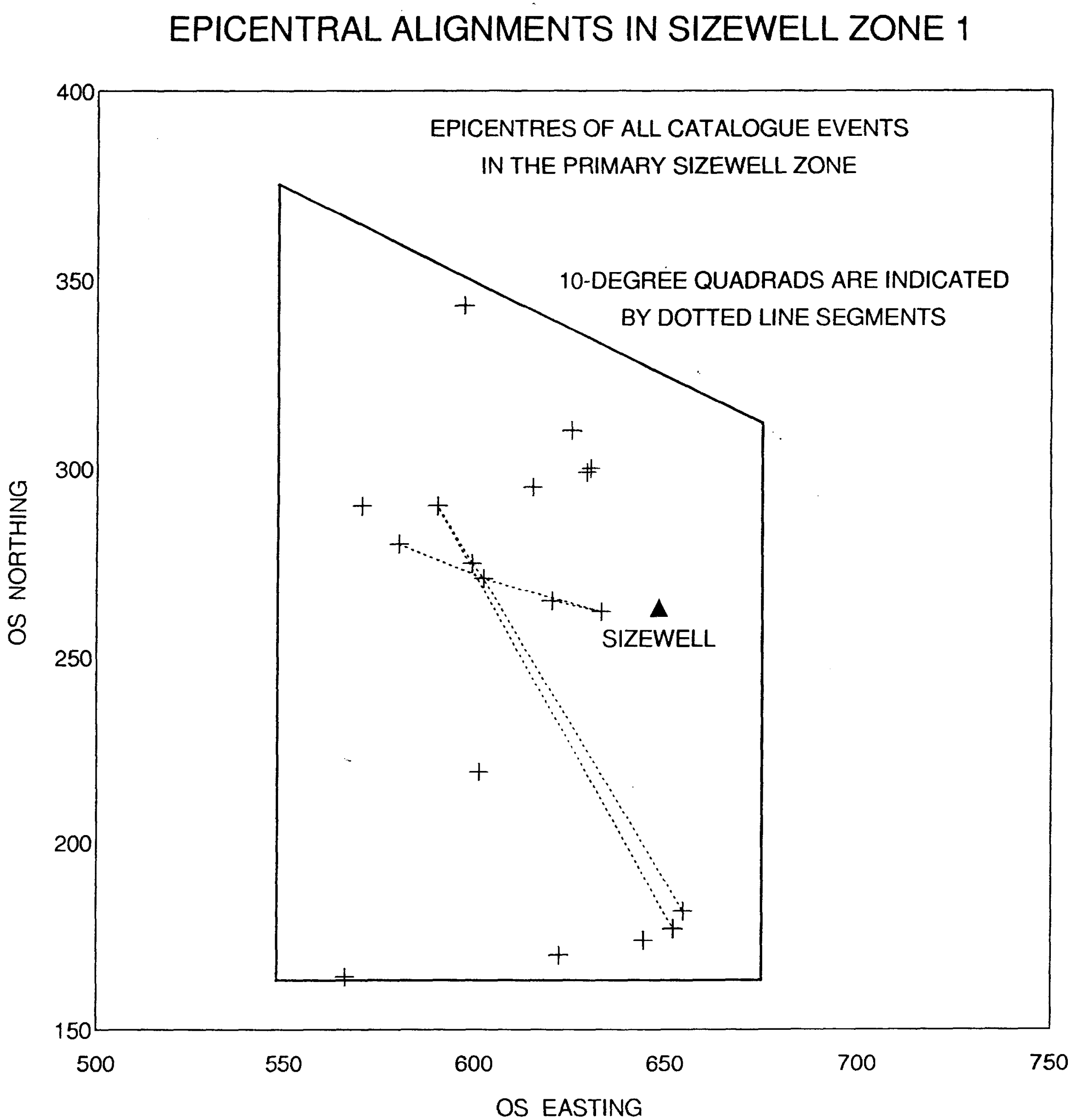
The same conclusion is drawn for a quadrad analysis……

Collinearity of epicentres??

3 quadrads found,

for a tolerance angle of 10°

expected no. ~ 10



Associating earthquake lineaments with mapped faults

A full and formal probabilistic interpretation of the possible association of linear epicentre patterns with mapped geological faults has to take into account the probability of such lineations arising by chance in a sample of data……..

….questions of hypocentral locational accuracy (we have already seen that even the best instrumental solutions seldom reach +/-1km precision), and some network geometries can generate lineations……

….correspondence of focal mechanisms with fault movement style……

……..as well as issues over the completeness, spatial reliability, and very existence of mapped faults (especially at depth) - let alone their active seismogenic status under the current seismotectonic regime.

All this calls for a very rigorous approach to the appraisal of different strands of evidence, and the avoidance of the 'prosecutor's fallacy'....the transposed conditional :

$$\Pr[\text{evidence}|\text{hypothesis}] \neq \Pr[\text{hypothesis}|\text{evidence}]$$

(Good I. 1995 The defender's fallacy. Roy. Statist. Soc. News, 4-5)

Extreme value probabilities

For Britain, highest magnitude recorded in last 900 years is 5.5Ms.

Extreme value (Gumbel) analysis indicates Mmax ~ 6Ms.

Suppose there is a population of larger faults, dormant in recent history, which might produce a magnitude >6Ms….

Indexing the two populations as 1 and 2, the distribution *f* of annual max magnitudes from both populations is:

$$f = pf_2 + (1-p)f_1$$

with the mixing probability *p* small.

Over the next *N* years (eg 50 yrs), the probability of observing an event from f_2, given no event was observed in the previous *kN* years is:

$$\Pr\left[obs_N | \overline{obs_{kN}}\right] = (1-p)^{kN}\left(1-(1-p)^N\right)$$

Extreme value probabilities, cont…

In a worst case choice of *p*, this reduces to:

$$\Pr[\ ] \rightarrow \left(\frac{1}{(1+1/k)}\right)^{k}\left(\frac{1}{1+k}\right)$$

and taking *k* = 900/50 (*i.e.* 900 yrs for British eq. history, and 50 yrs for an NPP lifetime),

Pr ~ 0.02

If experts are assigning weights to values of Mmax, exceeding the catalogue extreme (*e.g.* 6.5Ms), their weights should be approximately consistent with the probability calculated above.

Note for very short catalogues (e.g. instrumental), the formula gives:

for *k* = 0.5 Pr = 0.4

for *k* = 0.1 Pr = 0.7

The tail behaviour of such distributions can be examined with recent advances in statistical modelling. A distribution which is said to be 'heavy-tailed', can be conveniently represented by a suitable peak-over-threshold model, such as the generalised Pareto Distribution (GPD)

The Generalised Pareto distribution is a two- or three-parameter statistical distribution, one of a family of distributions that describe tail behaviour for extreme values.

One functional form looks like this:

$$S(t) := 1 \cdot \left[1 + -\frac{\kappa}{\alpha} \cdot \left(t + T_{trsh} + \mu \right) \right]^{\left(\frac{1}{\kappa} \right)}$$

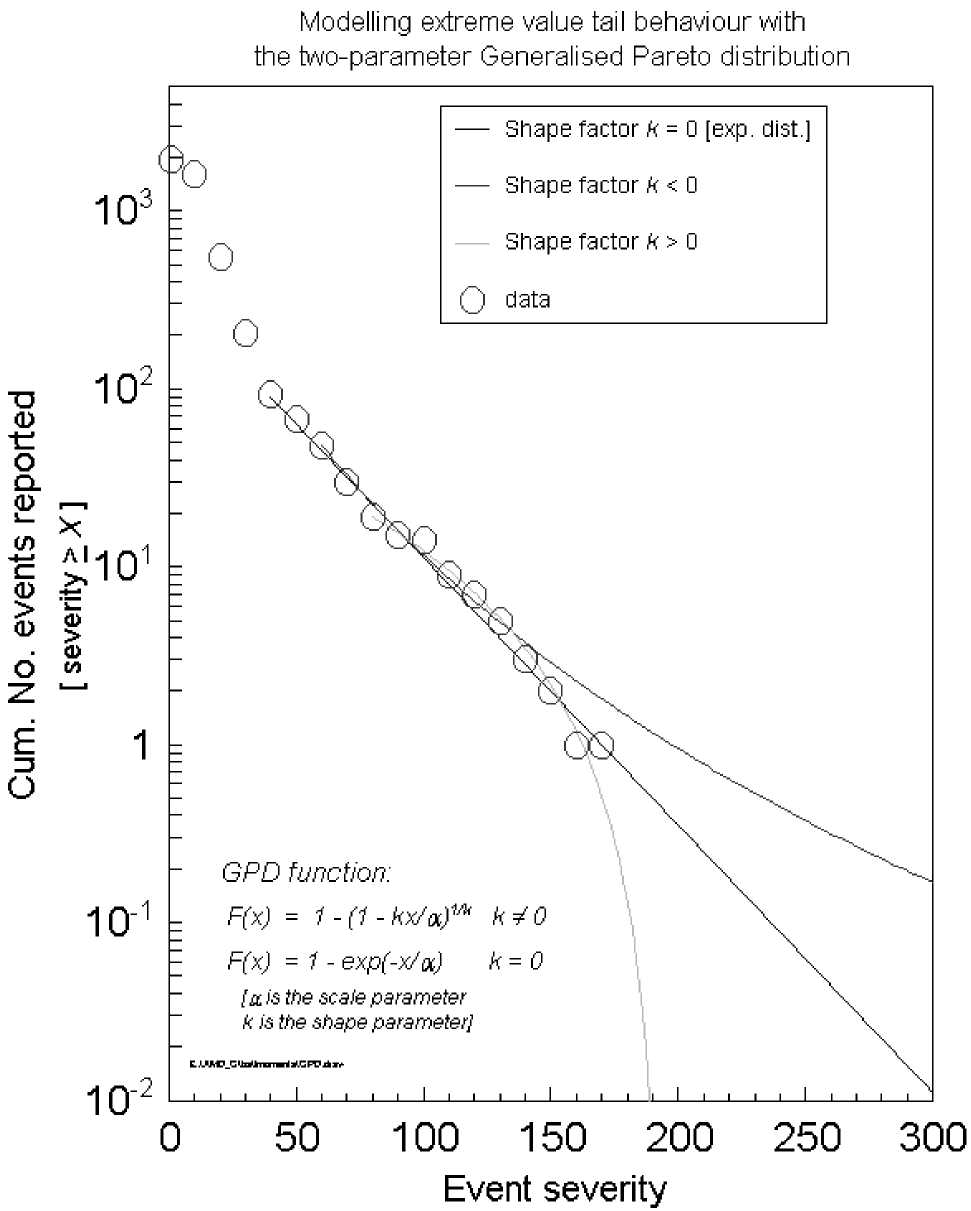
Having selected T_{trsh} on empirical or other grounds, the three parameters α [scale], κ [shape], and μ [location] have to be evaluated for the given dataset using a suitable procedure, such as maximum likelihood estimation.

Illustration of different modes of tail behaviour that can be modelled with the GPD

(Example from British Airways 'deep landing' dataset)

This type of analysis can be applied to extreme magnitudes in an earthquake catalogue, when the data are sufficiently numerous……

….and can provide guidance on weighting alternative Mmax values in logic tree models, for example

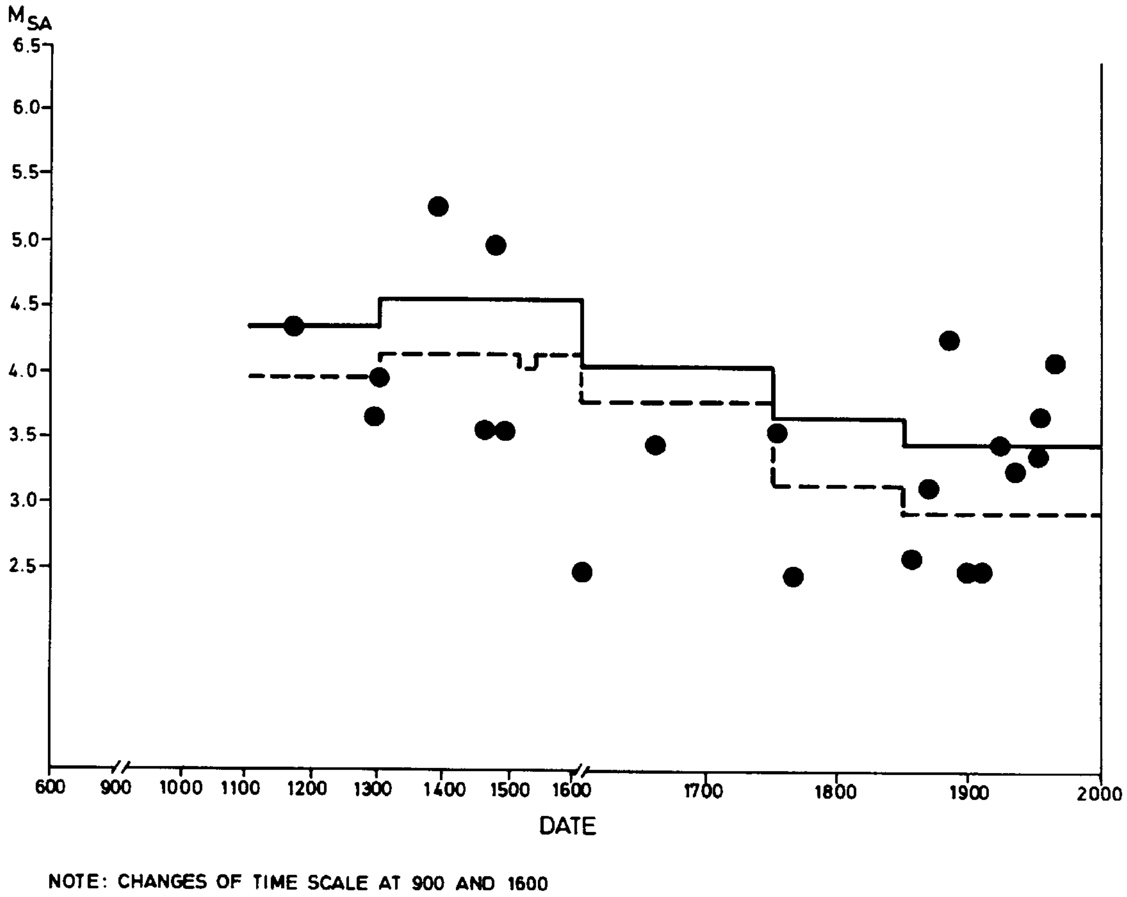


Estimating activity rate and b-value distributions for logic tree models from data

Once the spatio-temporal patterns of seismicity have been checked for statistical uniformity, and partitioned into areas where the Poissonian assumption holds, the next step is to derive suitable distributions of activity rate and b-value, which accord with the data available - *recognising that it is only a sample of a long-term process*.

Magnitude detection thresholds and reporting thresholds vary with time, as does the length of an earthquake catalogue, and for any given area the record can be divided into N time intervals, of different durations D_1, $D_2 \ldots D_N$, with magnitude thresholds M_1, $M_2 \ldots M_N$.

Estimating activity rate and b-value distributions for logic tree models from data, cont…..



Typical set of historical completeness thresholds and durations,…based on historiographic analysis

Estimating activity rate and b-value distributions for logic tree models from data, cont…..

Let *Mmax* be the zone maximum magnitude, M_0 the engineering threshold magnitude, and *a* the zone activity rate (no. events / year / unit area exceeding $M_{0,}$ usually 4Ms).

As usual, let the relative frequency of events in the range M_i to M_{i+1} be defined as:

$$f_i = \frac{\left(10^{-bM_i} - 10^{-bM_{i+1}}\right)}{\left(10^{-bM_0} - 10^{-bM_{MAX}}\right)}$$

..then if n_i events are observed in the magnitude range M_i to M_{i+1} during the cumulative time period $D_1 + D_2 + \ldots + D_{i,}$ the probability of this occurring is is proportional to the product:

$$a^{(n_1+n_2+..n_N)} \exp\left(-a*\left(D_1\left(f_1+f_2+\ldots f_N\right)+D_2\left(f_2+f_3+..f_N\right)+\ldots+D_N f_N\right)\right)$$

Estimating activity rate and b-value distributions for logic tree models from data, cont…..

Writing $F_i = f_i + f_{i+1} + \ldots f_N = \dfrac{\left(10^{-bM_i} - 10^{-bM_{MAX}}\right)}{\left(10^{-bM_0} - 10^{-bM_{MAX}}\right)}$

…the product can be expressed simply as: $a^K \exp\left(-a * D_{SUM}\right)$

…where $K = n_1 + n_2 + \ldots + n_N$ **, and** $D_{SUM} = D_1F_1 + D_2F_2 + \ldots + D_NF_N$

The product above has the form of the standard Gamma distribution:

$$f(x) = x^{\alpha-1} \exp\left(\frac{-x}{\beta}\right) * \frac{1}{\beta^{\alpha}\Gamma(\alpha)}$$

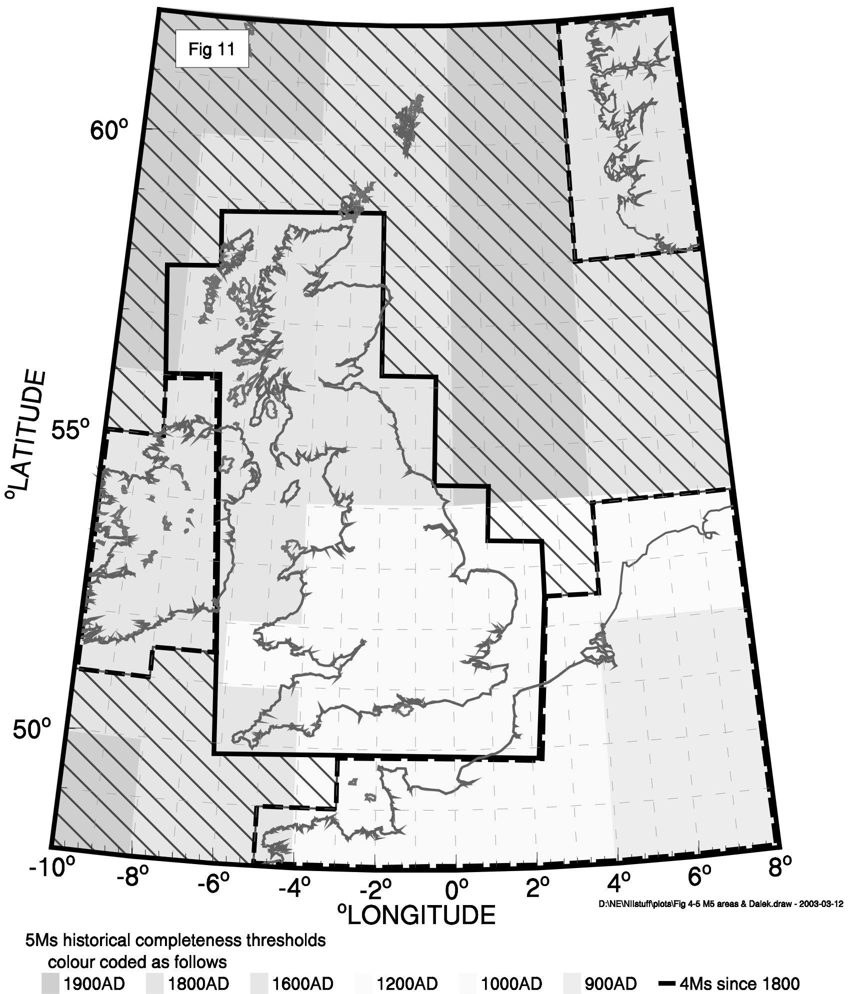
..for $x > 0$**.**

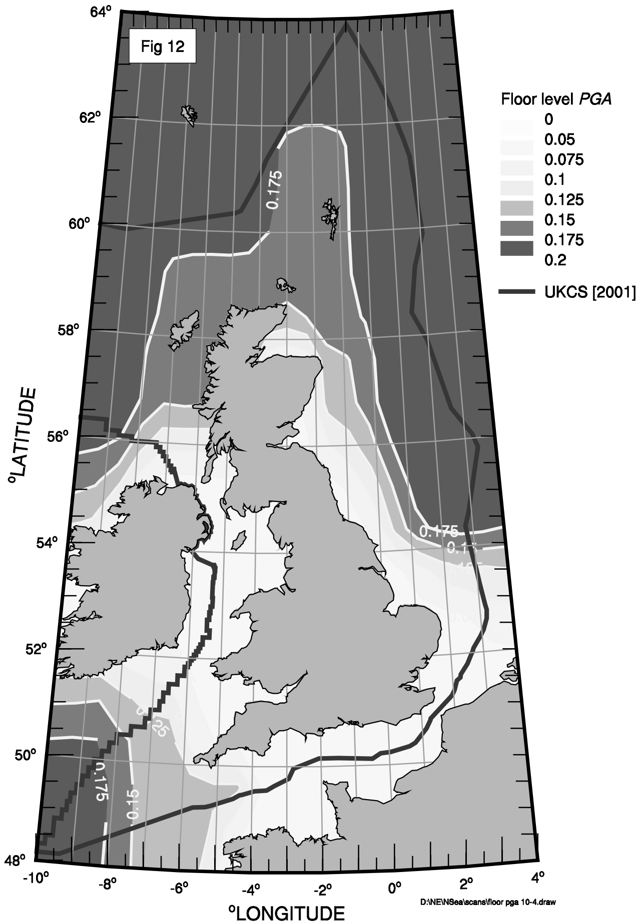
Thus, the equivalent Gamma distribution can be used to define the statistical variability of the activity rate parameter, given the data sample available, and this can be easily implemented in a spreadsheet:

	Mthres	Mo	Mmax	b			
1st	4.6	4	6.5	-1.28			
2nd	4.8	4	6.5	-1.28			
3rd	4.4	4	6.5	-1.28			
4th	4.0	4	6.5	-1.28			
5th	3.6	4	6.5	-1.28			
6th	2.7	4	6.5	-1.28			
F1=	0.170085		No. yrs	eq. M4 sample	Ni (+1)		
			150	25.51268965	5		
		reduced tir	25.51269				
F2=	0.094052		No. yrs	eq. M4 sample			
			100	9.405210166			
		reduced tir	9.40521				
F3=	0.307173		No. yrs	eq. M4 sample			
			100	30.71725369			
		reduced tir	30.71725				
F4=	1		No. yrs	eq. M4 sample			
			50	50			
		reduced tir	50				
F5=	3.252294		No. yrs	eq. M4 sample			
			0	0			
		reduced tir	0			0.042903	wt ave
F6=	46.16025		No. yrs	eq. M4 sample			
			0	0		**Prob**	**invGamma**
		reduced tir	0			0.05	0.0170
						0.15	0.0241
		M4 cmp	0	115.6351535		0.25	0.0291
						0.35	0.0337
						0.45	0.0381
F = (10^bMt - 10^bMmax)/(10^bMo - 10^bMmax)						0.55	0.0428
						0.65	0.0480
						0.75	0.0543
						0.85	0.0628
						0.95	0.0792

10-pt weighted Gamma distribution for activity rate for 4 events above relevant thresholds in 400 years

Illustration of effect on hazard levels of Gamma distribution activity rate estimation for no recorded seismicity above completeness threshold(s):



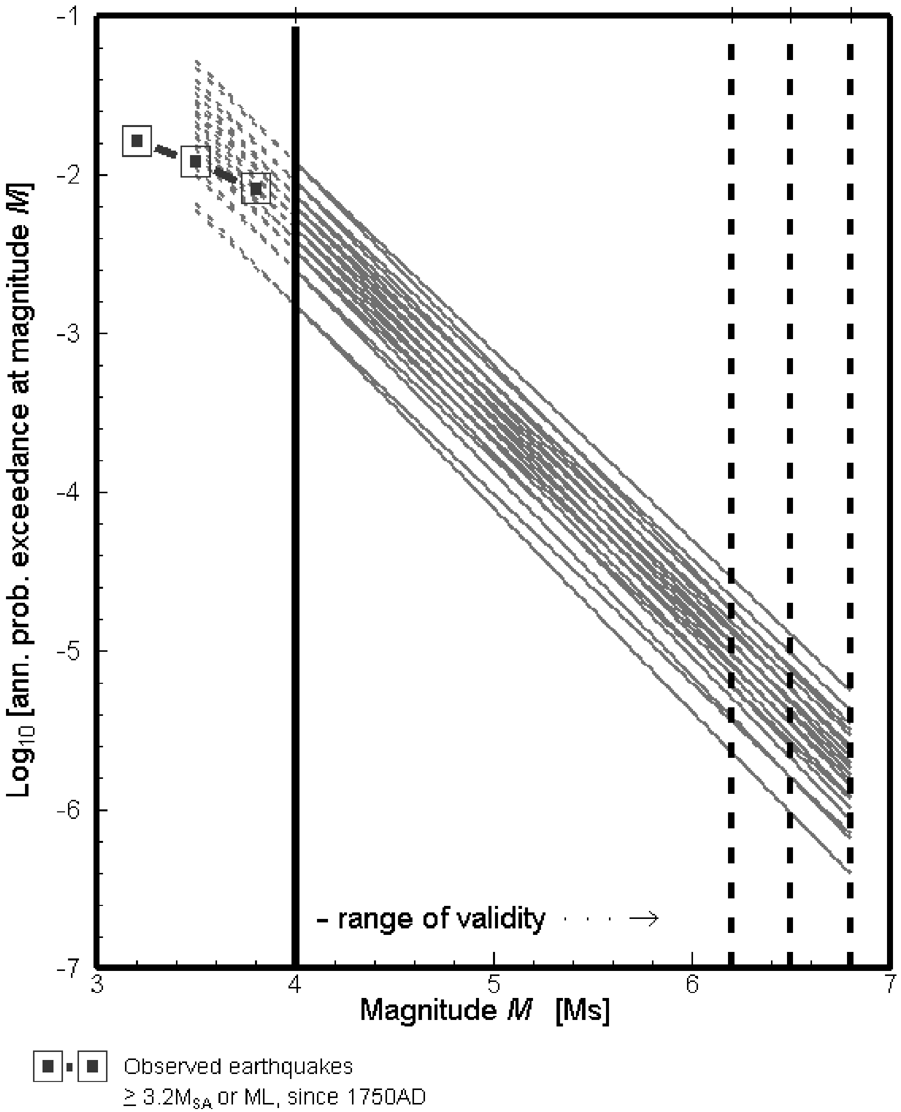


Updating alternative b-value weights for logic tree input….

	BAYESIAN UPDATING OF B-VALUE WEIGHTS, GIVEN SPECIFIC DATASET							
first b-val	1.19					alt b-val	1.28	
prior wt	0.5					prior wt	0.5	
			likelihood	0.02951252		likelihood	0.02184228	
		Mo						
beta1:	obs. Data	Mmax				beta2:		
2.7401	**4.9**	4		0.232928657		2.9473	0.20782595	
	4.1	6.5		2.08556112			2.19634099	
	4.0			2.742981774			2.94916972	
	4.8	denom for col E		0.306353553	denom for col H		0.2790614	
	4.4	0.998941		0.916690904		0.999369	0.90719316	
	4.6			0.529935388			0.50315265	
	4.2			1.585706922			1.63568536	
	4.7			0.402923801			0.37471387	
	4.9			0.232928657			0.20782595	
					Posterior wt for b2=		**0.425**	
					Posterior wt for b1=		**0.575**	

Typical logic-tree combinations of different active rates and b-values, representing uncertainty in the true Gutenberg-Richter relationship…..

…and comparison with actual experience in a low seismicity area !!



Site-specific model: zonal magnitude recurrence relations for 10-point activity rates at M = 4.0Ms, with b-values of 1.19 or 1.28 and three values for M_{MAX}, extrapolated down to 3.5Ms

Using expert judgement for PSHA…..

Well-known plot from Reiter (1990), after Bernreuter et al. (1989)...........

......influence of Expert 5

Challenge is how to best use expert judgements, when they have to be used.........

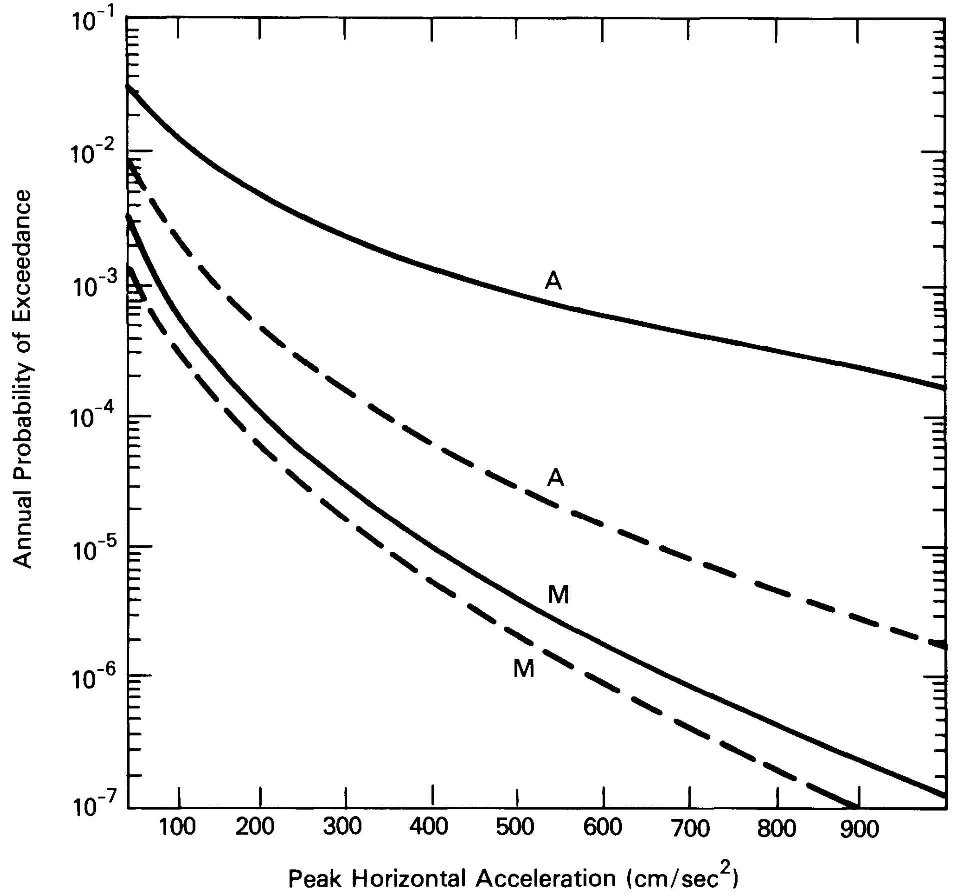


FIGURE 11.13 Seismic hazard at the Browns Ferry Nuclear Power Plant site in Alabama integrating all models and uncertainties. A is (arithmetic) mean and M is median. Results are shown with (solid line) and without (dashed line) the input of ground motion expert 5 (after Bernreuter and others 1989).

Example of inter-expert variation in PSHA, for a low seismicity area

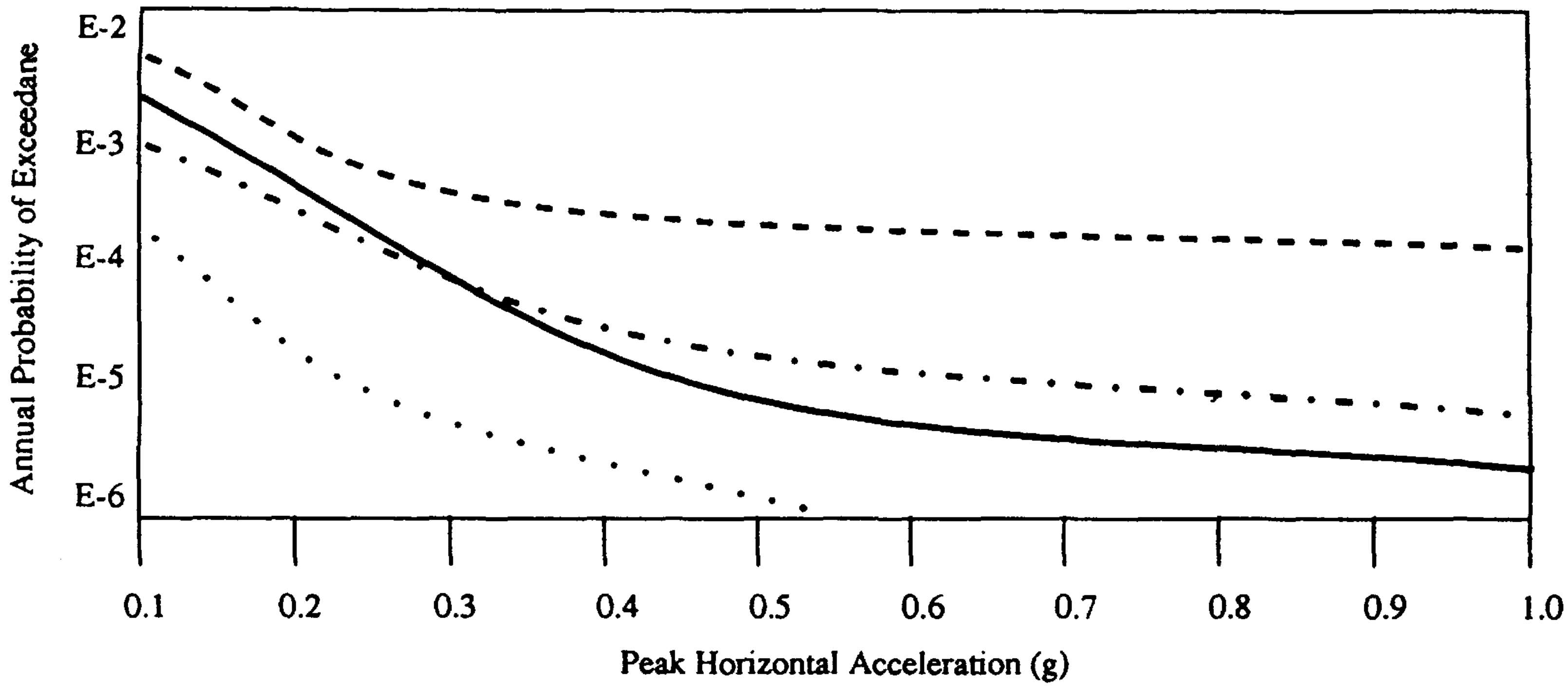


Figure 1: Inter-expert variation in expected seismic hazard curves for a site with sparse regional data

Nature of inter-expert variability when providing parameters for a seismic hazard model

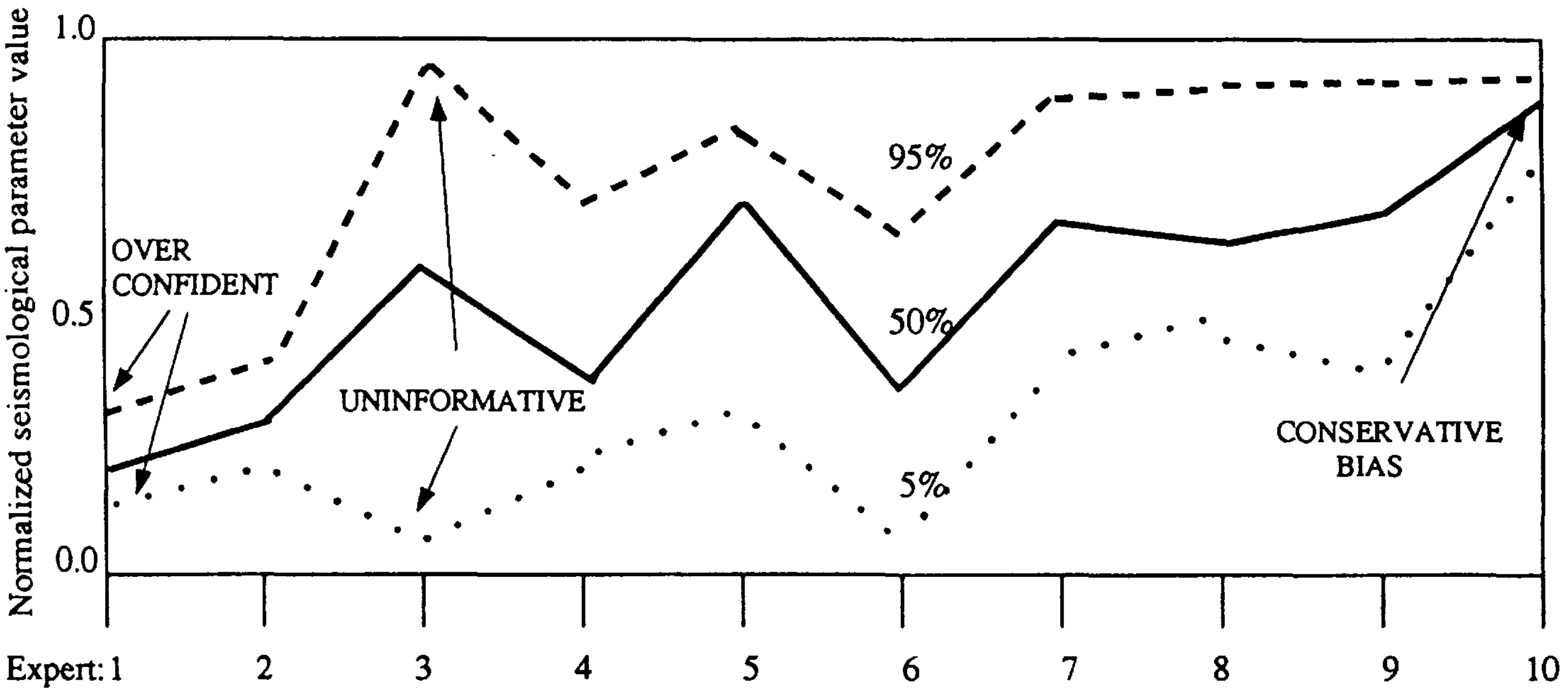


Figure 2. Inter-expert variation in 5%, 50% and 95% estimates of seismological parameters

Note the variations in individual confidence limits – or "informativeness".

In this lecture, we have tried to emphasise that there are several statistical approaches can be used to provide a degree of objectivity about inferences from incomplete, inaccurate and inadequate data.

Such results can provide important guidance for PSHA models, but not always…..

This afternoon, in Unit 19, we will discuss a structured and rational procedure for eliciting expert opinion for those parameters and variables which are not amenable to robust analysis…..

….and illustrate applications with a variety of recent case histories.

Thank you.