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Quantum physics with time-space non commutativity

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I WILL DEVELOP GOVINDARAJAN'S TALK FURTHER IN 2 NEW DIRECTIONS:

. SYMMETRIES & SPIN-STATISTICS CONNECTION. (BOSE & FERMI STATISTICS ARE REPLACED BY NONABELIAN BRAID STATISTICS).

. QUANTISED EVOLUTIONS. (TIME EVOLUTION BECOMES DISCRETE.)

BOTH HAVE STRIKING PHENOMENOLOGICAL IMPLICATIONS.

SYMMETRIES, SPIN & STATISTICS

IF V A VECTOR SPACE AND A GROUP $G = \{g\}$ ACTS ON V, ITS ACTION ON $V \otimes V$ IS GIVEN BY A <u>COPRODUCT</u> $\Delta : G \to G \times G$.

USUALLY

$$\Delta(g) = g \times g.$$

$$\Delta(g)(\xi \otimes \eta) = (g\xi) \otimes (g\eta), \quad \xi \otimes \eta \in V \otimes V.$$

BUT Δ NOT UNIQUE. BASIC PROPERTY WE REQUIRE:

Δ A REPRESENTATION,

$$\boxed{\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2).}$$

NOW CHAICHIAN, TUREANU, PRESNAJDER

POINT OUT ADDITIONAL CONDITIONS IF

$V = \mathcal{A}$, AN ALGEBRA.

IN THAT CASE, THERE IS A MULTIPLICATION MAP m:

$$m : V \otimes V = \mathcal{A} \otimes \mathcal{A} \to \mathcal{A},$$

$$m(\xi \otimes \eta) = \xi\eta .$$

Δ HAS TO BE COMPATIBLE WITH THIS MAP:

$$m\left[\Delta(g)\ \xi \otimes \eta\right] = g(\xi \cdot \eta).$$

CONSIDER 4 DIMENSIONS AND TAKE

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}.$$

THEN $\mathcal{A}$ IS ALGEBRA OF FUNCTIONS ON $\mathbb{R}^4$ WITH MULTIPLICATION = $*$-PRODUCT:

$$\alpha * \beta\,(x) = \alpha\, e^{i\theta^{\mu\nu}\overleftarrow{\partial}_\mu \overrightarrow{\partial}_\nu}\,\beta\,(x).$$

SO IF $P_\mu = -i\partial_\mu$, MULTIPLICATION MAP m_θ FOR $\theta \neq 0$ IS:

$$m_\theta\,[\alpha \otimes \beta] = m_0\,[\underbrace{e^{-i\theta^{\mu\nu} P_\mu \otimes P_\nu}}_{F_\theta} \times \alpha \otimes \beta]$$

$$= m_0\,[\,F_\theta\ \alpha \otimes \beta\,],$$

$m_0\,[\alpha \otimes \beta] = \alpha\beta$: USUAL POINT-WISE PRODUCT.

IT FOLLOWS: POINCARÉ GROUP G ACTS ON $\mathcal{A}$ IF COPRODUCT Δ_θ IS

$$\Delta_\theta(g) = F_\theta^{-1}\,(g \otimes g)\,F_\theta .$$

$\boxed{\text{MOYAL } * \text{-ALGEBRA ADMITS POINCARÉ INVARIANCE WITH DEFORMED COPRODUCT!}}$

$\boxed{\text{CONSEQUENCES}}$

A.P.B., G. MANGANO, A.PINZUL, B.QURESHI, S. VAIDYA.

SCATTERING AND ANGULAR MOMENTUM

FOR 2 IDENTICAL PARTICLES, UNDER ROTATIONS [SPINLESS SAY]:

$$\boxed{\Delta(R)\,|p_1,p_2\rangle = e^{-i(Rp_1)\cdot\theta\cdot(Rp_2)} \times e^{+i\,p_1\cdot\theta\cdot p_2}\,|Rp_1, Rp_2\rangle}$$

NO REASON WHY THIS STATE HAS ONLY EVEN ANGULAR MOMENTA; IN CENTRE-OF-MASS:

$$\int d\Omega\; Y^{*}_{LM}\;\Delta(R)\,|p_1,p_2\rangle \neq 0, \quad \text{L }\underline{\text{ODD}}, \text{ L ODD}.$$

$\Rightarrow$ ANGULAR MOMENTA FORBIDDEN BY PAULI PRINCIPLE APPEAR IN SCATTERING.

SPIN & STATISTICS

ABOVE TRANSFORMATION LAW COMPATIBLE ONLY WITH BRAID STATISTICS :

$$\boxed{|p_2, p_1\rangle = \pm e^{+2i\, p_2 \cdot \theta \cdot p_1} |p_1, p_2\rangle = \pm F_\theta^{-2} |p_1, p_2\rangle}$$

$\pm$ = BOSON / FERMION.

WITH $|p_1, p_2\rangle = \pm a^+_{p_1} a^+_{p_2} |0\rangle$,

$$\boxed{a^+_{p_2} a^+_{p_1} = \pm e^{+2i\, p_2 \cdot \theta \cdot p_1} a^+_{p_1} a^+_{p_2}}$$

$$\boxed{\begin{array}{l} \text{WITH} \quad c^+_p = a^+_p \big|_{\theta = 0} \\ \quad a^+_p = c^+_p e^{-i p \cdot \theta \cdot P} \end{array}}$$

a^+_p IS A COMPOSITE OF $c^+_{p'}, c_q$.

CONSEQUENCES

PAULI - FORBIDDEN TRANSITIONS BEING CALCULATED.

ALSO MARYLAND EXP. $\lesssim 10^{-25}$.

SUPERKAMIOKANDE, BOREXINO (GRAN SASSO, hep-ph/0406252) QUOTE FOR FORBIDDEN TRANSITIONS IN C^{12}, O^{16}, LIFETIMES $\gtrsim 10^{27}$ YEARS.

IF

$$|p_1, p_2\rangle = \pm\, |p_2, p_1\rangle$$

THEN

$$\Delta(R) \times \text{LHS} \neq \Delta(R) \times \text{RHS}.$$

QUANTUM HALL EFFECT (QHE)

HERE IF ONLY LOWEST LANDAU LEVEL FILLED,

$$[\hat{x}_a, \hat{x}_b] = i\theta_{ab}$$

SO (EXCITONS) EXCITATIONS LIKE SPIN EXCITATIONS IN THIS SPACETIME BACKGROUND OBEY BRAID STATISTICS:

$$\boxed{a_p^{s\,+}\, a_q^{s'\,+} = -\, e^{i\, p\cdot\theta\cdot q}\, a_q^{s'\,+}\, a_p^{s\,+}.}$$

s, s' = SPIN COMPONENTS

AFFECTS PROPERTIES OF KIVELSON-SONDHI SKYRMIONS ?

QUANTISED EVOLUTIONS

WE NOW CONSIDER MODELS (WHICH DEFORM SPACETIME ALGEBRAS $C^\infty(\mathbb{R} \times M)$) WHICH ALLOW ONLY DISCRETE TIME TRANSLATIONS BY AMOUNTS $n\theta$ ($n \in \mathbb{Z}$, θ : NONCOMMUTATIVE PARAMETER).

SUCH MODELS EXIST IN ALL DIMENSIONS.

THEY LEAD TO STRIKING PHYSICS LIKE CONSERVATION OF ENERGY ONLY MOD $2\pi/\theta$.

THE NONCOMMUTATIVE CYLINDER

REVIEW OF COMMUTATIVE CYLINDER

CYLINDER IS $S^1 \times \mathbb{R}$ WITH COORDINATES

$$(e^{i x_1}, x_0) = p.$$

ITS ALGEBRA IS GENERATED BY

COORDINATE FUNCTIONS

$$e^{i\hat{x}_1}, \quad \hat{x}_0 \quad , \quad e^{i\hat{x}_1}(p) = e^{ix_1}, \quad \hat{x}_0(p) = x_0 .$$

OF PARTICULAR IMPORTANCE IS

MOMENTUM OPERATOR

$$\hat{p} = \text{"} -i \frac{\partial}{\partial x_1} \text{"}, \quad [\hat{p}, e^{i\hat{x}_1}] = e^{i\hat{x}_1},$$

$$[\hat{p}, \hat{x}_0] = 0 .$$

WE HAVE

$$e^{i2\pi\hat{p}} \; e^{i\hat{x}_1} \; e^{-i2\pi\hat{p}} = e^{i(\hat{x}_1 + 2\pi)} = e^{i\hat{x}_1},$$

$$e^{i2\pi\hat{p}} \; \hat{x}_0 \; e^{-i2\pi\hat{p}} = \hat{x}_0 .$$

$\Rightarrow$ $e^{\pm i 2\pi \hat{p}}$ AND ITS POWERS GENERATE CENTRE OF ALGEBRA.

SO IN IRREDUCIBLE REPRESENTATION, (IRR),

$e^{i 2\pi \hat{p}}$ = A PHASE $e^{i\varphi} \times \mathbb{1}$.

$\Rightarrow$ SET OF EIGENVALUES OR SPECTRUM SPEC $\hat{p}$ OF $\hat{p}$ = $\frac{\varphi}{2\pi} + \mathbb{Z}$

EIGENFUNCTIONS ψ_n:

$$\psi_n(x_1) = e^{i x_1 \left(n + \frac{\varphi}{2\pi}\right)}$$

<u>NONCOMMUTATIVE</u> <u>CYLINDER</u>

ITS ALGEBRA $\mathcal{A}_\theta(S^1 \times \mathbb{R})$ GENERATED BY $e^{i\hat{x}_1}$, $\hat{x}_0$, $[\hat{x}_0, \hat{x}_1] = i\theta$:

$$\mathcal{A}_\theta(S^1 \times \mathbb{R}) = \langle e^{i\hat{x}_1}, \hat{x}_0 : [\hat{x}_0, e^{i\hat{x}_1}] = -\theta e^{i\hat{x}_1} \rangle$$

SO AS BEFORE : POWERS OF

$\exp\left(i \frac{2\pi}{\theta} \hat{x}_0\right) \in$ CENTRE OF $\mathcal{A}_\theta(S^1 \times \mathbb{R})$.

SO IN AN IRR,

$e^{i\frac{2\pi}{\theta}\hat{x}_0} =$ PHASE $e^{i\varphi} \times \mathbb{1}$, SPEC $\hat{x}_0 = \theta\left(\frac{\varphi}{2\pi} + \mathbb{Z}\right)$

IT LOOKS LIKE TIME MAY GET QUANTISED WITH SPACING θ.

THE ALGEBRA $\mathcal{A}_\theta(S^1 \times \mathbb{R})$ IS GENERATED BY $e^{i\hat{x}_1}$, $e^{i\omega\hat{x}_0}$

NOW ON AN EIGENSTATE OF $\hat{x}_0$,

$$e^{i\omega\hat{x}_0} = e^{i\omega\theta\left(\frac{\varphi}{2\pi} + n\right)}.$$

$\Rightarrow$ $e^{i\omega\hat{x}_0}$ IS QUASIPERIODIC IN ω:

$$e^{i\left(\omega + \frac{2\pi}{\theta}\right)\hat{x}_0} = e^{i\varphi}\, e^{i\omega\hat{x}_0}$$

AND WE CAN RESTRICT ω TO ITS FUNDAMENTAL PERIOD:

$$-\pi/\theta \le \omega \le +\pi/\theta .$$

$\Rightarrow$ A GENERAL ELEMENT OF ALGEBRA

IS

$$\hat{a} = \sum_n \int_{-\pi/\theta}^{\pi/\theta} d\omega \; \alpha_n(\omega) \, e^{in\hat{x}_1} \, e^{i\omega\hat{x}_0} \, .$$

THESE ARE ALSO WAVE FUNCTIONS.

THE SYMBOL a OF $\hat{a}$ IS A

FUNCTION

$$a : \; S^1 \times \theta\left(\frac{\varphi}{2\pi} + \mathbb{Z}\right) \to \mathbb{C} ,$$

$$a\left(e^{ix_1}, \; \theta\left(\frac{\varphi}{2\pi} + m\right)\right) = \sum_n \int d\omega \; \alpha_n(\omega) \, e^{inx_1} \, e^{i\omega\theta\left(\frac{\varphi}{2\pi} + m\right)}$$

SCALAR PRODUCT GIVEN AS BEFORE BY

$$\langle \hat{b} | \hat{a} \rangle = \int_0^{2\pi} dx_1 \; \bar{b}\left(e^{ix_1}, \theta\left(\frac{\varphi}{2\pi} + m\right)\right) \times$$

$$\times \, a\left(\quad \text{"} \quad \right)$$

IF b SYMBOL OF $\hat{b}$.

TIME TRANSLATION & SCHRÖDINGER EQ.

LET ∂_0 ACT LIKE DIFFERENTIATING TIME:

$$\partial_0 e^{i\omega \hat{x}_0} = i\omega e^{i\omega \hat{x}_0}.$$

THEN $e^{\tau \partial_0}$ TRANSLATES $\hat{x}_0$ TO $\hat{x}_0 + \tau$.

STANDARD SCHRÖDINGER EQ (IF IT EXISTED) WOULD BE

$$(i\partial_0 - \hat{H})\,\hat{a} = 0.$$

BUT ∂_0 DOES NOT EXIST !

PROOF $e^{i\frac{2\pi}{\theta}\hat{x}_0}$ ON AN EIGENSTATE OF $\hat{x}_0 = e^{i\frac{2\pi}{\theta}\theta\left(\frac{\varphi}{2\pi}+n\right)} \Rightarrow$

$$e^{i\frac{2\pi}{\theta}\hat{x}_0} = e^{i\varphi}\,\mathbb{1}.$$

APPLY ∂_0 : $i\frac{2\pi}{\theta} e^{i 2\pi/\theta\, \hat{x}_0} = 0$! CONTRADICTION.

SO TRY DISCRETE TIME TRANSLATION

$e^{\tau \partial_0}$ BY AMOUNT τ.

APPLY TO $e^{i\,2\pi/\theta\; \hat{x}_0} = e^{i\varphi}\, \mathbb{1} \Rightarrow$

$e^{i\,2\pi/\theta\,(\hat{x}_0+\tau)} = e^{i\varphi}\,\mathbb{1}$ OR

$$\boxed{e^{i\,2\pi/\theta\;\tau} = \mathbb{1} \Rightarrow \tau = n\theta \quad n \in \mathbb{Z}.}$$

TIME TRANSLATION QUANTISED: ALLOWED ONLY IN MULTIPLES OF θ.

SCHRÖDINGER CONSTRAINT (EQ.)

$\exp[-i\,\theta\,(i\partial_0]\;\hat{a} = \exp[-i\,\theta\,\hat{H}\,]\;\hat{a}$

$\hat{H} = \hat{H}\left(\hat{p}_1, e^{i\hat{x}_1^L}\right)$ = NO EXPLICIT $\hat{x}_0$ DEPENDENCE.

HILBERT SPACE $\mathcal{H} = \langle \hat{a} : \hat{a}$ FULFILLS SCHRÖDINGER CONSTRAINT, SCALAR PRODUCT $\langle\cdot|\cdot\rangle\rangle$.

$$\boxed{\begin{array}{l} \text{IF } \hat{a} \in \mathcal{H}, \\ \hat{a} = e^{-i\theta \hat{H} \hat{x}_0^R} \; \chi(e^{i\hat{x}_1}) \end{array}}$$

SO IN VECTOR STATES, ENERGY IS DEFINED ONLY MOD $\frac{2\pi}{\theta}$, VIA TIME-DEPENDENCE $\exp(-i\hat{H}\hat{x}_0^R)$.

$\Rightarrow$ IN SCATTERING AMPLITUDE, ENERGY IS CONSERVED ONLY MOD $\frac{2\pi}{\theta}$. NOTE:

SCATTERING AMPLITUDE $\langle \cdot, \text{OUT} | \cdot, \text{IN} \rangle$ IS OVERLAP OF VECTOR STATES, LEADS TO PERIODIC δ-FUNCTIONS $\delta_{S^1}(E_f - E_i) = \delta_{S^1}(E_f - E_i \pm \frac{2\pi}{\theta})$.*

BUT $\hat{H}$ COMMUTES WITH $(i\partial_0 - \hat{H})$, PRESERVES SCHRÖDINGER CONSTRAINT, IS AN OBSERVABLE. ENERGY MEASURABLE.

UNRESOLVED QUESTION: PROBABILITY DISTRIBUTION OF ENERGY CHANGE IN SCATTERING. HAVE PRELIMINARY ANSWERS.

*

INSTEAD OF USUAL

$$\int dt\, e^{-i(E_f - E_i)t} = 2\pi\, \delta(E_f - E_i),$$

$E_{i,f}$: INITIAL & FINAL ENERGIES,

WE GET

$$\sum_{n \in \mathbb{Z}} e^{-i(E_f - E_i)\theta n} \cong 2\pi\, \delta_{S^1}(E_f - E_i).$$

QUANTISED EVOLUTION WORK WITH

T. R. GOVINDARAJAN, A. G. MARTINS, P. TEOTONIO-SOBRINHO (hep-th/0410067).

CONSEQUENCES

. ENERGY NONCONSERVATION, IN SCATTERING.

. MICROCANONICAL ENSEMBLE IS AFFECTED : HENCE SO IS CANONICAL ENSEMBLE :

THE PROBABILITY $P_1\left(E + j\frac{2\pi}{\theta}, E\right)$ TO SCATTER FROM INITIAL ENERGY E TO $E + j\frac{2\pi}{\theta}$ CALCULABLE FROM MICROSCOPIC MODEL.

IN MULTIPLE SCATTERING, PROBABILITY $P_{n+1}\left(E + \frac{2\pi}{\theta}k, E\right)$ OF FINDING $E + \frac{2\pi}{\theta}k$ AFTER $n+1$ SCATTERINGS IF INITIAL ENERGY E :

$$P_{n+1}\left(E + \frac{2\pi}{\theta}k, E\right) = \sum_j P_1\left(E + \frac{2\pi}{\theta}k, E + \frac{2\pi}{\theta}j\right) P_n\left(E + \frac{2\pi}{\theta}j, E\right)$$

MARKOV PROCESS. TRYING TO CALCULATE

$$P_\infty\left(\chi E + \frac{2\pi}{\theta}\gamma, E\right).$$

WILL AFFECT BLACK BODY SPECTRUM ? COSMIC MICROWAVE BACKGROUND [CMB] ?

FINAL REMARKS

- CONSISTENT QUANTUM THEORIES ARE POSSIBLE WITH TIME-SPACE NONCOMMUTATIVITY.

- WE MENTIONED SEVERAL PHENOMENOLOGICAL SIGNALS. HERE IS ONE MORE: $Z^0 \to 2\gamma$, STRICTLY FORBIDDEN FOR $\theta = 0$ BY POINCARÉ INVARIANCE ALONE (YANG'S THEOREM) ALLOWED FOR $\theta \neq 0$ (BAL & A. PINZUL, hep-th/0410199).

AND MORE APPLICATIONS:

- LOOP QUANTUM COSMOLOGY

(BOJOWALD, DATE, ...) IS BASED ON $\mathcal{A}_\theta(S^1 \times \mathbb{R})$. OUR SCHRÖDINGER EQ IS WHEELER- DEWITT EQ.

- WITH TIME- SPACE NONCOMMUTATIVITY, SPATIAL LOCALISATION NOT POSSIBLE BY UNCERTAINTY PRINCIPLE: BIG BANG SINGULARITY SHOULD DISAPPEAR. [DOES DISAPPEAR ?]

MUCH INTERESTING AND NOVEL PHYSICS IS POSSIBLE IF $\theta \neq 0$. AFFECTS

- SPIN & STATISTICS.
- QUANTUM HALL EFFECT.
- STATISTICAL MECHANICS, CMB
- $Z^0 \to 2\gamma$
- QUANTUM COSMOLOGY, ...