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Integrable spin chains and string/gauge theory duality

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Integrable Spin Chains and String/Gauge Theory Duality

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Motivation: Holography

- **The Gauge/String Theory Duality**
  (t'Hooft; Polyakov: 1970's)
- **The Holographic Principle**
  (t' Hooft; Susskind:1993)
- **The AdS/CFT(SCYM) Correspondence**
  (Maldacena, 1997)

The key to understand holography:
to find strings in gauge theory!

Gauge/String Theory Duality (History)

- **'t Hooft large-N Limit: (say, for U(N) group)**

  $N \to \infty$, $g \to 0$, with $\lambda = g_{YM}^{2} N$ fixed

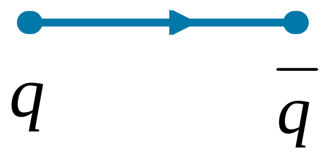
  Planar diagram are dominant

  Summation of planar diagrams

  = Emergence of string worldsheet

- **In (confining) QCD,**

  Color flux tube = QCD string

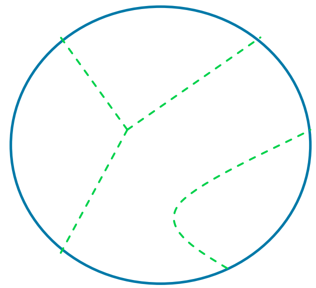
  or in loop equation formalism,

  Wilson loop $\oint A_\mu dx^\mu$

  = string-like dynamical variable

Maldacena Conjecture: AdS/SCYM (AdS/CFT) Duality

- **IIB string theory on** $AdS_5 \times S^5$ **(w/ N 5-flux)**
  = D=4, Ñ=4 SYM (w/ U(N) gauge group)
- **Share same symmetry groups**
- **A realization of holography:**
  Bulk propagation $\Longleftrightarrow$
  Boundary correlations



- **Plane-wave (BMN) limit:**
  Boosting along an angular direction on S^5
  $\Longrightarrow$ **Plane gravitational wave background**
- **IIB string theory in plane wave limit is solvable**
- **New way to test AdS/SCYM duality**
  Calculate anomalous dimension in Ñ=4 SYM
  Compare with string spectrum in plane wave

BMN limit in Ñ=4 SYM

- **Bosonic Field content:**

$$A_\mu, \phi^i (i = 1, \cdots, 6)$$ **adjoint**

- **J(=L) rotates**

$$Z = \phi^5 + i\phi^6$$

Δ measures the dimension

- **The BMN limit**

$$g_{YM}{}^2 N \to \infty, J \to \infty, J^2 / g_{YM}{}^2 N$$ **fixed**

- **So**

$$N \to \infty, J \to \infty, J^2 / N$$ **fixed**

and

$$g_{YM} \ll 1$$ **fixed is allowed**

(Perturbative regime in SYM is now allowed!)

Spin Chain in SYM (I)
(Minahan and Zarembo, 2002)

Consider long composite operators

$$\hat{O}_I(x) = Tr\{\phi^{i_1}(x)\phi^{i_2}(x)\ldots\phi^{i_L}(x)\}$$

They get mixed under one-loop renormalization:

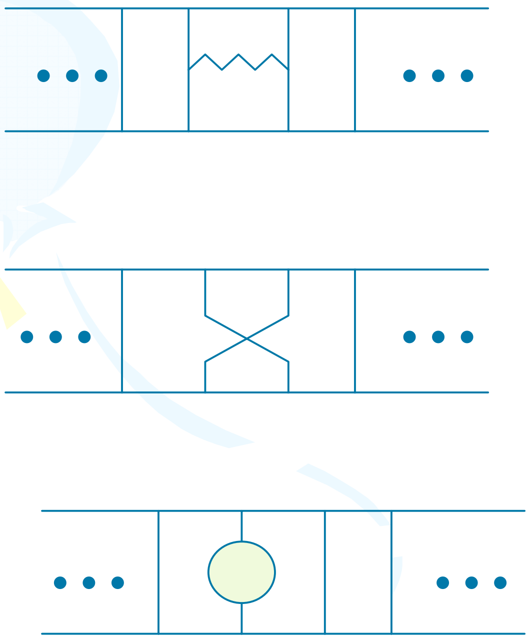
$$\hat{O}_I^{(bare)}(x) = \sum_J Z_{IJ}\,\hat{O}_J^{(ren)}(x)$$

Anomalous Dimension Matrix

$$\Gamma \equiv (\gamma_{IJ}) \qquad \gamma_{IJ} = \mu \frac{\partial \ln Z_{IJ}(\mu)}{\partial \mu}\Big|_{\lambda^{(ren)}}$$

Spin Chain in SYM (II): Planar One-Loop Results

Large-N ⟹ Planar ⟹ Nearest-neighbor interactions



$$\Gamma = \frac{\lambda}{16\pi^2}\sum_l \left\{ K_{l,l+1} - 2P_{l,l+1} + 2 \right\}$$

$$K_{l,l+1} = \delta_{i_l i_{l+1}} \delta_{j_l j_{l+1}}$$

$$P_{l,l+1} = \delta_{i_l}^{j_{l+1}} \delta_{i_{l+1}}^{j_l}$$

Γ is the Hamiltonian of an integrable SO(6) spin chain!

A Gauge Theory Dual for Open Strings (I)

Trick: Adding D-branes: where open string ends

Doing orientifolding: un-oriented strings

AdS^5xS_5/Z_2 Orientifold: (BGMNN)

O7-Plane + 4 D7-branes: at $x^7 = x^8 = 0$

2N D3-branes: in $x^1, x^2 \quad x^3$ **rections**

Near-Horizon Limit of D3-branes:

Neumann: x^1 **to** $x^6, \quad x^9; \; (x^5, x^6) \Rightarrow (Z, \bar{Z})$

Dirichlet: $(x^7, x^8) \Rightarrow (W, \bar{W})$

A Gauge Theory Dual for Open Strings (II)

Gauge Theory Dual: D=4, Ñ=2 Sp(N) theory

Vector multiplet: (V,W) adjoint

Hypermultiplet: (Z,Z') anti-symmetric

(3,3) strings

4 Hypermultiplets: $(\tilde{q}_A, q_A)$ fundamental

(A=1,..,4) (3,7) strings

In Ñ=1 language, V: vector supermultiplet

All others: chiral supermultiplets

Superpotential: $\hat{W} \sim \underline{q_A W \tilde{q}_A} + tr\{(\Omega W)(\Omega Z)(\Omega Z')\}$

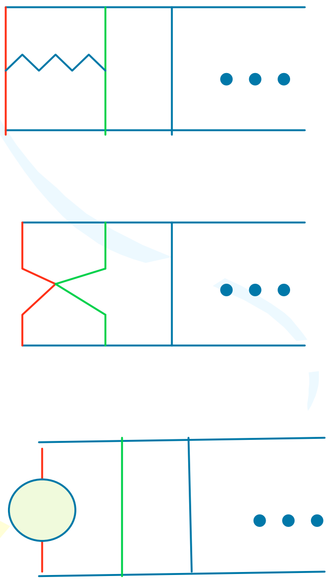
Anomalous Dimension and Open Spin Chain

Composite operators: $\hat{O}_I^{(open)} = \lambda_{pq} Q^p \Omega (\Phi_{i_1} \Omega) \cdots (\Phi_{i_L} \Omega) Q^q$

(Φ=Z,Z',W; invariant 2-tensor) **"quarks"** **Ω=**

Bulk Interactions: the same as in D=4, Ñ=4 SYM (A Theorem)

Bdry Interactions: additional Feynman diagrams!



$$\Gamma_{open} = \frac{\lambda}{4\pi^2} \sum_{l=1}^{L-1} (1 - P_{l,l+1}) + \frac{\lambda}{4\pi^2} (\Sigma_1 + \Sigma_2)$$

$$\Sigma_1 = \Sigma \otimes I \otimes \cdots \otimes I$$ **integrable**

$$\Sigma_2 = I \otimes \cdots \otimes I \otimes \Sigma$$ **bdry terms!**

$$\Sigma = diag(0,0,1)$$

Γ is the Hamiltonian of an integrable open SU(3) spin chain!

Matching Open String Spectrum

Open String Spectrum:

$$\Delta - L = 1 + \sum_{n=-\infty}^{\infty} N_n \sqrt{1 + \frac{\pi g_s N n^2}{L}}$$

Algebraic Bethe Ansatz:

Ground state: (Ferromagnetic pseudo-vacuum)

$$|0> \sim Q^p \Omega (Z\Omega)^L Q^q \qquad (\Delta - L = 1; E_0 = 0)$$

Single Z' excitation (in Neumann direction;Δ-L=2)

$$\gamma_{Z'} = \frac{\lambda}{\pi^2} \sin^2 \frac{n\pi}{2L} \rightarrow \frac{n^2 \lambda}{4L} = \frac{\pi g_s N n^2}{2L^2}$$

Single W excitation (in Dirichlet direction; Δ-L=2)

$$\gamma_W = \frac{\lambda}{\pi^2} \sin^2 \frac{n\pi}{2(L+1)} \rightarrow \frac{n^2 \lambda}{4L} = \frac{\pi g_s N n^2}{2L^2} = \gamma_{Z'}$$

Semi-classical spinning strings: Beyond BMN limit

- Gauge theory: Spin chain excitations with
  Bethe roots of finite density

✓ SO(6): rank 3, allowing at most
three commuting (J_1, J_2, J_3)

✓ String side: Spinning strings as
classical solutions (eqs of motion + Virasoro)

✓ Gauge theory: Spin chain excitations with
Bethe roots of finite density (Frolov,Tseytin et al.)

✓ Open spinning strings and open spin chains
(Chen, Wang and Wu; Stefanski)

Non-Linear Sigma Model as Continuum Limit of Spin Chain

- ✓ **Well-Known limiting procedure in Condensed matter theory**
- ✓ **Non-linear sigma model describes first quantized strings**
- ➢ **Spin chain induced non-linear sigma model reproduces a sector of string theory in $AdS_5 \times S^5$ (Kruczenski and et al.)**

? String excitations = spin waves

? How about string interactions

Further Developments

- ✓ **Higher orders and non-planar diagrams**
  - **two-, three loops: numerical**
  - **Non-planar: NNN-interactions (several groups)**
- ? **To all orders, a long-ranged spin chain?**

- ✓ **Incorporating fermions: PSU(2,2|4) spin chain?**
  (Beisert, Staudacher, et al.)
- ✓ **Other Susy scalar or gauge field theories:**
  Orbifolded gauge theories, Wess-Zumino models:
  (Wang and Wu; Rioban)
- ✓ **Spinning strings as spin chain excitations**
- ✓ **Nonlinear sigma model as continuum limit of spin chain:**
- ? **Toward a precise formulation of gauge/string duality?**

Summary

- ✓ Integrable spin chain emerges in perturbative gauge theories, out of Feynman diagrams!

- ✓ Gauge/string duality can now be tested
  at string level (beyond supergravity)
  for plane-wave limit and spinning strings

Implications: The door is just open for having

- ➢ A profound relation between
  Yang-Baxter and Yang-Mills
- ➢ better understanding of Holography, namely
  AdS/CFT or gauge/string theory duality

Speculation on Directions to Go

- There seems to be interesting **integrable** or **quantum group (e.g. Yangian) symmetry structure** hidden in (**perturbative**) gauge or string theory (related to **twistors?**)
- There should be **new way** to **formulate or construct** string theory (using **gauge theory** or **discrete spin chain**)
- **Condensed matter** theory may **inspire ideas** for **missing links in string theory:**

  ? Rich phases in Strongly correlated systems

  ? Effects of Non-commutative geometry