Spatial Bose-Einstein Condensation on Inhomogeneous Networks

Andrea Trombettoni
(University of Perugia)

With: P. Sodano, G. Giusiano, F. P. Mancini

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Outlook

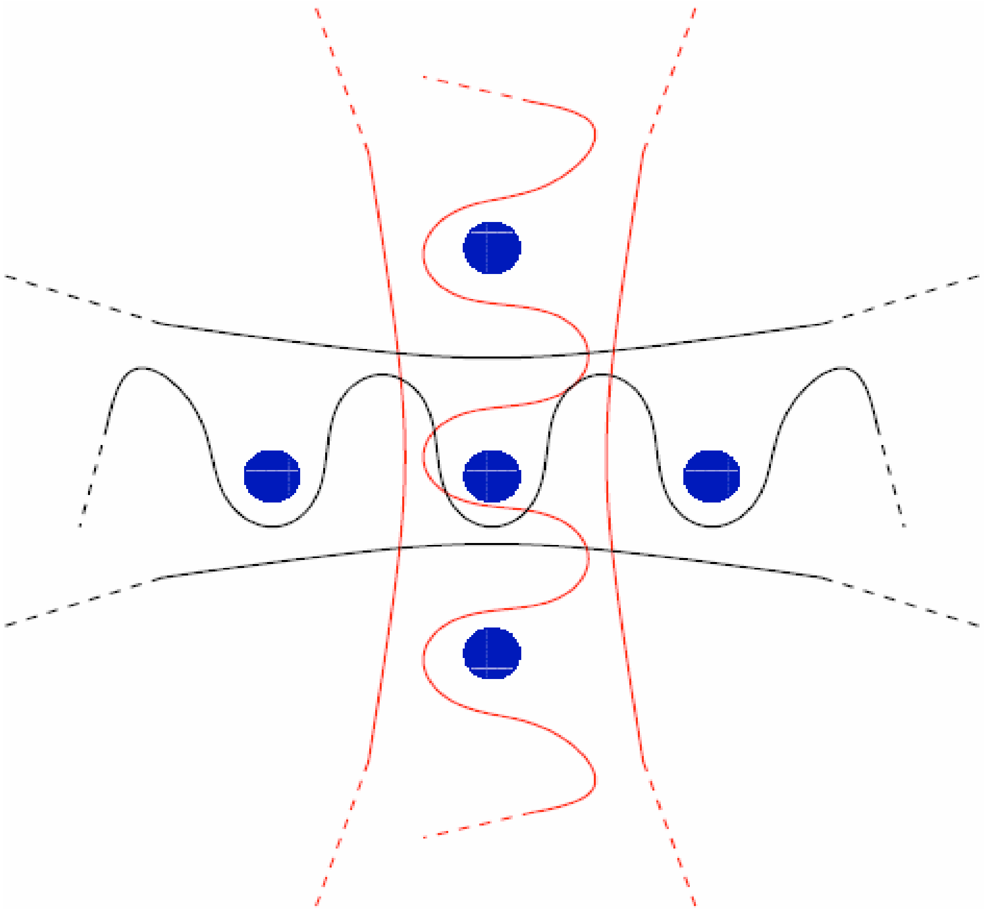
1) Experimental systems:

- Ultracold bosons on inhomogeneous optical lattices
  - Superconducting Josephson junctions on inhomogeneous insulating supports

2) Thermodynamical properties:

- Macroscopic occupation of the localized ground-state (emergence of spatial Bose-Einstein condensation)

Creating a star network with ultracold bosons



Temperature ~ 0-500 nK
Number of particles ~ 1000-10000
Number of wells ~ 100

$$V(x) \approx V_0 \cos^2(kx)$$

$$k = 2\pi/\lambda \qquad \lambda \approx 800 \text{ nm}$$

$$E_R = \hbar^2 k^2 / 2m$$

$$V_0 = s \cdot E_R \qquad s \approx 10 - 30$$

Creating a comb-shaped network with superconducting Josephson junctions

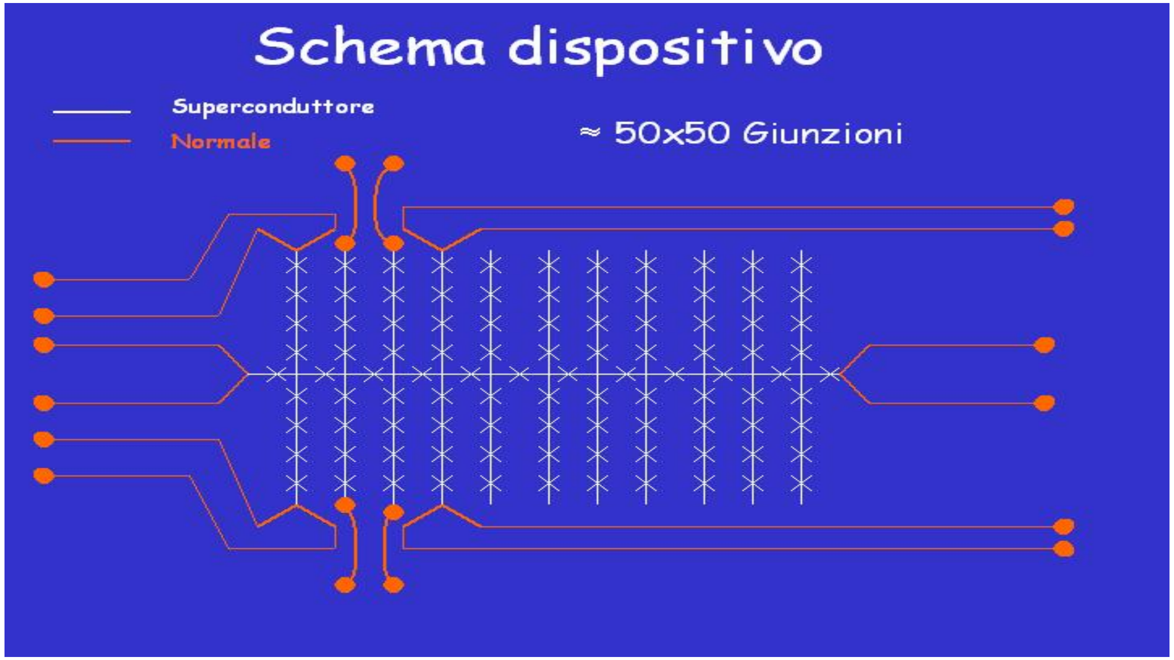


Figure of the experimental setup (courtesy of M. Cirillo *et al.*)

Critical Josephson energy $E_J = \phi_0 I_C$

Non-interacting bosons on a graph

$$\hat{\mathrm{H}} = -t \sum_{i,j} A_{ij} \hat{\mathbf{a}}_i^{+} \hat{\mathbf{a}}_j$$

$$\left[\hat{\mathbf{a}}_{\mathrm{i}}, \hat{\mathbf{a}}_{\mathrm{j}}^{+}\right] = \delta_{\mathrm{ij}} \qquad \hat{n}_j = \hat{\mathbf{a}}_j^{+} \hat{\mathbf{a}}_j$$

In the following: bosons on discrete structures which are not necessarily regular lattices:

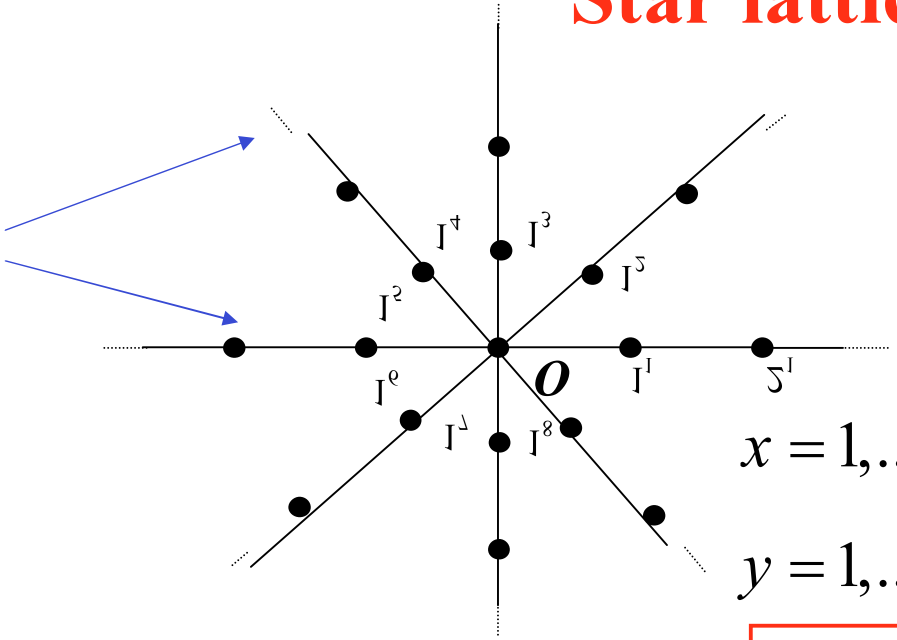
Graphs: Adjacency Matrix $A_{ij} = \begin{cases} 1 \text{ if } i - j \text{ is a link} \\ 0 \qquad \text{otherwise} \end{cases}$

B.E.C. of non interacting bosons on inhomogeneous low-dimensional structures: d<2.

Network geometry naturally induces topological (quantum) order.

Star lattice

arms



$i \equiv (x, y)$

$x = 1, \ldots, L$; distance from the center

$y = 1, \ldots, p$; labels arms

Total number of sites: $N_S = pL + 1$

z_i coordination number of a given site: **2**

z_O coordination number of the center: ***p***

$$-t \sum_{\mathrm{j}} A_{\mathrm{ij}} \psi_\nu(j) = E_\nu \psi_\nu(i)$$

Eigenvalue equation

Spectrum σ formed by N_S states and divided in 3 parts: $\{ E_0, \sigma_0, E_+ \}$

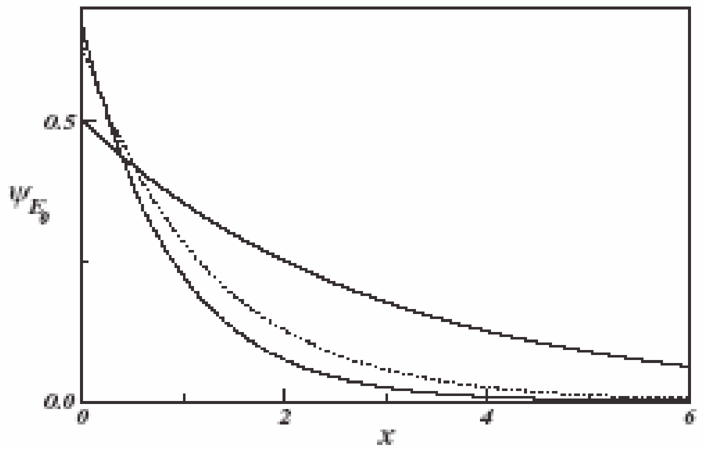
σ_0 → pL-1 delocalized states with $E \in [-2t, 2t]$

two bound states → E_0<-2t *and* E_+>2t

E_0 **: localized ground-state**

$$\psi_{E_0} = \sqrt{\frac{p-2}{2p-2}}\, e^{-x/\xi_0}$$

$$\xi_0 = \frac{2}{\log(p-1)}, \quad p > 2$$



Adding arms enhances localization

$$E_0 = -t\frac{p}{\sqrt{p-1}}$$

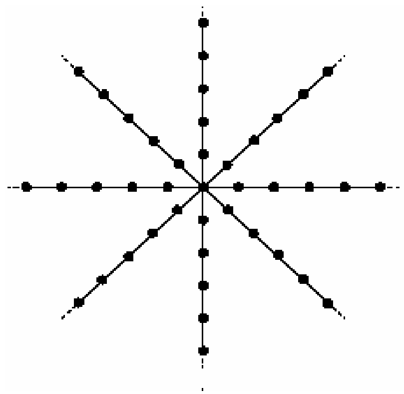
p=2 → $E_0 = -2t$ **Linear chain**

p≠2 → $\Delta(p) = |E_0| - 2t$ **Gapped spectrum**

Thermodynamics for bosons hopping on a star lattice

$$N_T = N_{E_0} + \int_{E \in \sigma_0} \frac{N_S \rho(E)}{z^{-1} e^{\beta(E-E_0)} - 1} + N_{E_+}$$

Ground-state — *delocalized states* — *excited state*



$$T < T_C \quad \rightarrow \quad \frac{N_{E_0}}{N_S} \neq 0$$

$$T_C \approx \frac{p-2}{2\sqrt{p-1}} \frac{E_J}{k_B}$$

Topology effect

$$f = \frac{N_T}{N_S}$$

$$E_J = 2tf$$

$$\frac{N_{E_0}\left(T/T_C\right)}{N_T} \approx 1 - \frac{T}{T_C}$$

I. Brunelli et al., *J. Phys. B* **37**, S275 (2004)

Conclusions and perspectives

-Topology role in inducing new phases for bosons on graphs

-The experimental signature of the Bose-Einstein condensation is given by the inhomogeneous distribution of bosons in the graph below the critical temperature

-Dream: To induce desired macroscopic coherent behaviors by acting on the geometry and topology of networks

Rather new area:

R. Burioni et al., *Europh. Lett.* **52**, 251 (2000)

L. B. Ioffe et al., *Phys. Rev. Lett.* **90**, 107003 (2003)