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Quantum Hall droplets on CP (k) and edge effective actions

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QHE on CP^k and edge effective actions

(with V.P. Nair)

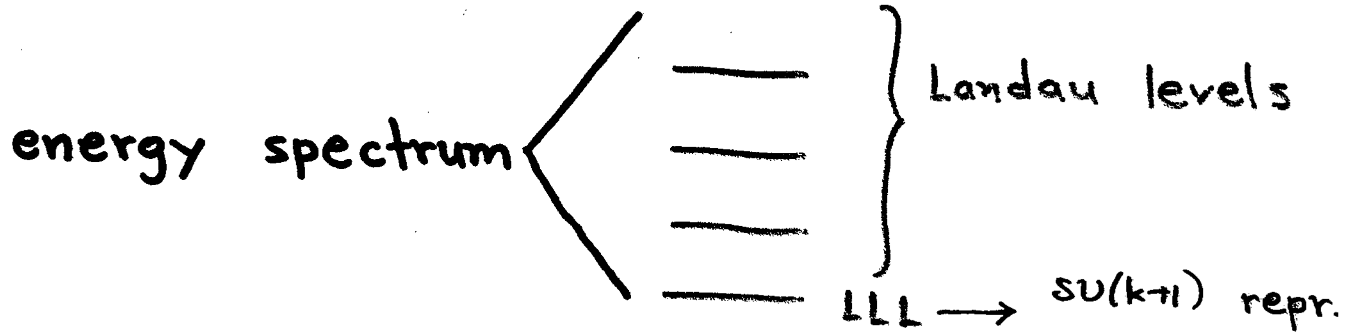
Generalize Zhang and Hu idea to arbitrary even dimensions.

$$CP^k = \frac{SU(k+1)}{U(k)}$$

Uniform treatment for abelian ($U(1)$) and nonabelian ($U(k)$) background magnetic fields

Overview

1) Solve Landau problem of a charged particle moving on CP^k with constant $U(1)$ magnetic field



2) Repeat for $U(1)$ and $SU(k)$ magnetic fields

3) LLL $\equiv$ noncommutative phase space
Provides physical realization of fuzzy CP^k

4) many-body problem in the presence of confining potential $\longrightarrow$ quantum Hall droplet

excitations = suface deformations which preserve volume of droplet

<u>edge effective action</u>

<u>$U(1)$ magnetic field</u>

$$S \sim \int_{\partial D} dt \left[\frac{\partial \phi}{\partial t} \mathcal{L} \phi + \omega (\mathcal{L} \phi)^2 \right]$$

ϕ: scalar field $\quad\quad$ $\mathcal{L}$: derivative along droplet boundary

ω: strength of confining potential

$CP^3 \longrightarrow$ Zhang, Hu model (S^4 with $SU(2)$ instanton)

U(k) magnetic field

S = gauged WZW action on ∂D
(higher dim. generalization of 2d WZW)

Definition of CP^k

CP^k : $2k$ dim manifold

parametrized by Z_α $\quad \alpha = 1, \ldots, k+1$

$$Z_\alpha \sim \lambda Z_\alpha \qquad \lambda \neq 0$$

- $\bar{u}_\alpha u_\alpha = 1 \qquad u_\alpha \sim e^{i\theta} u_\alpha$

- $$u_\alpha = \frac{1}{\sqrt{1+\bar{\xi}\cdot\xi}} \begin{pmatrix} \xi_1 \\ \vdots \\ \xi_k \\ 1 \end{pmatrix}$$

U(1) magnetic field

Haldane (1983) : QHE on $S^2 = CP^1$

gauge field : $A = -in\, \bar{u}\cdot du$

$$F = -in\, d\bar{u}\cdot du \quad = n\,\Omega$$

Kahler form

$$\int F = 2\pi n = 4\pi B R^2 \implies n = 2BR^2$$

integer

radius of CP^k

Wave functions on CP^k

$$\psi \sim u_{\alpha_1} \dots u_{\alpha_p}\, \bar{u}^{\beta_1} \dots \bar{u}^{\beta_q}$$

under $u \to e^{i\theta} u$

$$A \to A + n\, d\theta$$

$$\psi \to e^{i(p-q)\theta}\psi$$

$D\psi = (d - iA)\psi$ inv.

$\Rightarrow p - q = n$

ψ : irr. rep. of $SU(k+1)$ $\qquad T^{q}_{q+n}$

$$CP^k = \frac{SU(k+1)}{U(k)}$$

g : group element of $SU(k+1)$

$g_{\alpha,k+1} = u_\alpha$

$$g = \underbrace{\begin{pmatrix} & & u_1 \\ & & \vdots \\ & & u_{k+1} \end{pmatrix}}_{(k+1)\times(k+1) \text{ matrix}}$$

$$\underbrace{g \to gh}_{\text{same point in } CP^k}, \ h \in U(k) \longleftrightarrow u_\alpha \to e^{i\theta} u_\alpha$$

$$\Longrightarrow \quad CP^k = \frac{SU(k+1)}{U(k)}$$

t_A : generators of $SU(k+1)$ in fundamental rep.

t_A :
- $t_a \quad a = 1, \dots, k^2-1 \qquad \in SU(k)$
- $t_{k^2+2k} \qquad \in U(1)$
- t_α coset gener. $\alpha = 1, \dots, 2k$
  - t_{+i} raising, $i = 1, \dots, k$
  - t_{-i} lowering

$\mathrm{Tr}(t_A t_B) = \frac{1}{2}\delta_{AB}$

$$t_{k^2+2k} = \frac{1}{\sqrt{2k(k+1)}} \begin{bmatrix} 1 & & & \\ & \ddots & & \\ & & 1 & \\ & & & -(k+1) \end{bmatrix}$$

$\underline{U(1) \text{ gauge field}}$: $A = in\sqrt{\frac{2k}{k+1}}\, Tr\left(t_{k^2+2k}\, g^{-1}dg\right)$

$= -in\, g^*_{k+1,\alpha}\, g_{\alpha,k+1} = -in\, \bar{u}\cdot du$

<u>wave functions</u> : functions on $SU(k+1)$

$$\psi \sim \underbrace{\mathcal{D}^J_{L,R}(g)}_{\text{Wigner } \mathcal{D}\text{-function}} = \langle L \mid \hat{g} \mid R \rangle \qquad \hat{g} = e^{iT\cdot\theta}$$

L, R quantum numbers characterizing states within J rep.

$$\hat{L}_A\, g = T_A\, g$$
$$\hat{R}_A\, g = g\, T_A$$

left / right } rotations on g

Under right $U(1)$ rotation : $g \to gh \qquad h \in U(1)$

$$A \longrightarrow A - \frac{nk}{\sqrt{2k(k+1)}}\, d\theta$$

$$\psi \to \psi\, e^{iR_{k^2+2k}\,\theta}$$

$$D\psi = (d - iA)\psi \quad \text{invariant}$$

$$R_{k^2+2k} = \frac{-nk}{\sqrt{2k(k+1)}}$$

"right hypercharge"

$\psi \sim$ singlet under right $SU(k)$ rotations

$$\Psi^J_{m,-n} = \sqrt{N}\, \langle \underbrace{m}_{m=1,\ldots,\dim J = N} \mid \hat{g} \mid \underbrace{R_a = 0,\ R_{k^2+2k} = -\frac{nk}{\sqrt{2k(k+1)}}}_{SU(k)_R \text{ singlet with fixed } U(1)_R \text{ charge}} \rangle$$

Hamiltonian

$$H = -\frac{1}{4M} \sum_{i=1}^{k} \left(D_{+i} D_{-i} + D_{-i} D_{+i} \right)$$

$$[D_{+i}, D_{-i}] = 2B$$

$$[t_{+i}, t_{-j}] \in U(k)$$

$$\left[\hat{R}_{+i}, \hat{R}_{-j}\right] = i f_{ij}^{\alpha} \hat{R}_{a} + \delta_{ij} \sqrt{\frac{2(k+1)}{k}} \hat{R}_{k^2+2k}$$

$$= 0 \qquad - \delta_{ij}\, n = -(2BR^2)\, \delta_{ij}$$

$$\boxed{D_{\pm i} = i \frac{\hat{R}_{\pm i}}{R}}$$

left rotations $\hat{L}_A$: $\left[\hat{L}_A, \hat{R}_B\right] = 0 \qquad \left[\hat{L}_A, H\right] = 0$

$\boxed{L_A : \text{magnetic translations}}$

$$H = \frac{1}{2MR^2} \left(\sum_A R_A^2 - R_{k^2+2k}^2 \right)$$

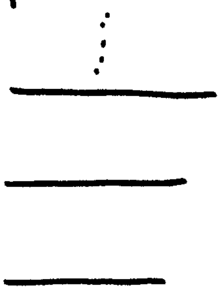
$$= \frac{1}{2MR^2} \left(C_2(J) - \frac{n^2 k}{2(k+1)} \right)$$

$SU(k+1)$ rep. $\mathcal{J}$: $T_p{}^q$ $\qquad p-q=n$

energy: $$E = \frac{B}{2M}(k+2q) + \frac{q(q+k)}{2MR^2}$$

$$\left(B \sim \frac{n}{2R^2} \right)$$

q : Landau index

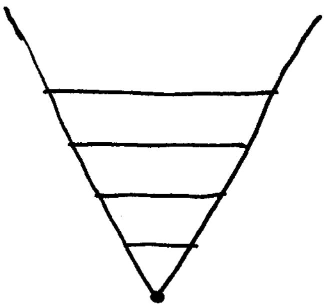


$$\left.\begin{matrix} q=1 \\ q=0 \quad \text{LLL} \end{matrix}\right\} \sim \frac{B}{M}$$

large $B \longrightarrow$ restrict to LLL

LLL : $R_{-i}\Psi = 0$ (holomorphicity condition

$\left| R_a = 0 ,\ R_{k^2+2k} = -\frac{nk}{\sqrt{2k(k+1)}} \right\rangle$ is lowest weight state



$$\text{degeneracy} = N = \dim \mathcal{J} = \frac{(n+k)!}{n!\,k!}$$

Hilbert space of LLL $\longrightarrow$ symmetric rank n $SU(k+1)$ rep.

LLL: CP^k with $U(1)$ $\longleftrightarrow$ fuzzy CP^k

U(1) and SU(k) magnetic fields

SU(k) magnetic fields : $A^a = 2i\,\mathrm{tr}\left(t^a g^{-1} dg\right)$

$a = 1, \ldots, k^2 - 1$

$$\psi \sim \langle \ldots \underset{\swarrow \atop m}{L} \mid \hat{g} \mid \underbrace{R \qquad \rangle}_{}$$

$\swarrow \alpha$ particular $SU(k)_R$ rep. $= \tilde{J}$ with fixed $U(1)_R$ charge

$\psi^{J}_{m,\alpha}$

$m = 1, \ldots, \dim J = N$ $\qquad J = SU(k+1)$ rep.

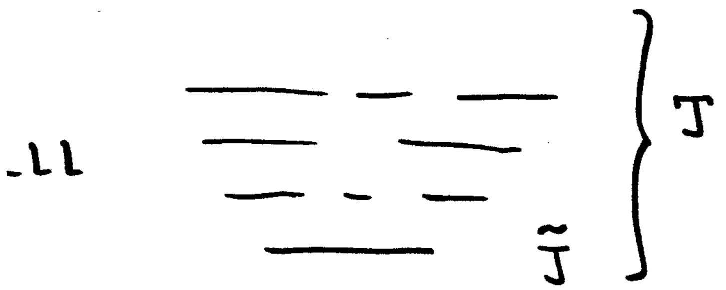
$\alpha = 1, \ldots \dim \tilde{J} = N'$ $\qquad \tilde{J} : SU(k)_R$ rep.

$R_{\pm i} \longrightarrow$ covariant derivatives

$L_A \longrightarrow$ magnetic translations

$$H = \frac{1}{4MR^2} \sum_{i=1}^{k} \left(R_{+i} R_{-i} + R_{-i} R_{+i}\right)$$
$$= \frac{1}{2MR^2} \left(\sum_A R_A^2 - \sum_a R_a^2 - R_{k^2+2k}^2\right)$$
$$= \frac{1}{2MR^2} \left[C_2(J) - C_2(\tilde{J}) - \frac{n^2 k}{2(k+1)}\right]$$

LLL : $R_{-i}\, \psi = 0$



degeneracy $= N = \dim J$

$n \to \infty$

$\dim J = \dim \tilde{J}\ \dfrac{n^k}{k!}$

$(\tilde{J} = (0, jk) \quad j = 1, 2, \ldots)$

$$\hat{R}_a \, \psi^{J}_{m,\alpha} = (T_\alpha)^{\tilde{J}}_{\alpha\beta} \, \psi^{J}_{m,\alpha}$$

$$\hat{R}_{k^2+2k} \, \psi^{J}_{m,\alpha} = -\frac{nk}{\sqrt{2k(k+1)}} \, \psi^{J}_{m,\alpha}$$

J rep. of $SU(k+1)$: $T^{q,\ell'}_{p,\ell}$

$U(1)$ $SU(k)$

$$\left.\begin{array}{l} \sqrt{2k(k+1)} \; R_{k^2+2k} = -k(p-q) + \ell - \ell' = -nk \\ \text{lowest weight} \longrightarrow q=0 \quad \ell=0 \end{array}\right\} \begin{array}{l} \ell' = jk \\ j=1,2,\ldots \end{array}$$

$$SU(k)_R \text{ repr} = (0, jk) = \tilde{J}$$

$$\dim \tilde{J} = \frac{(k+kj-1)!}{(k-1)!\,(jk)!} = \text{finite as } R^2 \to \text{large}$$

$$\dim J = \dim \tilde{J} \; \frac{(n-j+jk+k)\,(k+n-j-1)!}{k!\;(n-j)!}$$

$$\xrightarrow[R^2 \to \infty]{n \to \infty} \dim \tilde{J} \; \frac{n^k}{k!}$$

Edge states

LLL : N degenerate states

K fermions $K<N$

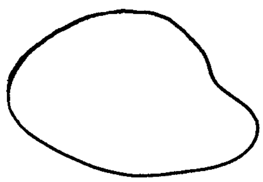
confining potential $\longrightarrow$ fermions localize near minimum of potential $\longrightarrow$ **incompressible quantum Hall droplet**

excitations = volume preserving deformations of the boundary

edge states $\rightarrow$ dynamics confined on boundary

2d case: edge effective action

$$S \sim \int dt\, d\theta \left(\frac{\partial \Phi}{\partial t} + \omega \frac{\partial \Phi}{\partial \theta} \right) \frac{\partial \Phi}{\partial \theta}$$



confining potential $V \sim \omega z \bar{z}$

S = $(1+1)\,d$ chiral action

How does this generalize in higher dimensions ?

Generalize method by Sakita

droplet characterized by $\hat{\rho}_0 = \sum_{i=1}^{K} |i\rangle\langle i|$

$$\hat{\rho}_0 = \begin{pmatrix} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & \ddots & & & & \\ & & & & 1 & & & \\ & & & & & 0 & & \\ & & & & & & 0 & \\ & & & & & & & \ddots \\ & & & & & & & & 0 \end{pmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}1\\1\\1\\ \ddots\\1\end{matrix}}\right\} K \text{ occupied states} \\ \left.\vphantom{\begin{matrix}0\\0\\ \ddots\\0\end{matrix}}\right\} N-K \text{ empty states} \end{matrix}$$

Under time evolution

$$\hat{\rho}_0 \to \hat{\rho} = \hat{U}\,\hat{\rho}_0\,\hat{U}^+$$ ← unitary matrix

$$S = \int dt \left[i\, \mathrm{Tr}\left(\hat{\rho}_0\, \hat{U}^+ \partial_t \hat{U}\right) - \mathrm{Tr}\left(\hat{\rho}_0\, \hat{U}^+ \hat{\mathcal{H}}\, \hat{U}\right)\right]$$

$$i\,\frac{d\hat{\rho}}{dt} = \left[\hat{\mathcal{H}}, \hat{\rho}\right]$$

$\hat{U}$: "collective" variable describing all particle-hole excitations

Express S in terms of bosonic fields on the droplet boundary

$N \times N$ matrices $\longrightarrow$ functions on CP^k (symbols

matrix multiplication $\longrightarrow$ $*$-product

$\mathrm{Tr} \longrightarrow N \int d\mu\,(CP^k)$

$$\mathrm{Tr}(\hat{A}\,\hat{B}\,\hat{C}) = \mathcal{N}\int d\mu\; A(\xi,\bar{\xi}) * B(\xi,\bar{\xi}) * C(\xi,\bar{\xi})$$

CP^k with $U(1)$ field

$$\hat{A} \longrightarrow A(\xi,\bar{\xi}) = \langle \xi | \hat{A} | \xi \rangle = \sum_{s,m} \langle \xi | s \rangle A_{sm} \langle m | \xi \rangle$$

$$= \mathcal{D}_{s,-n}\, A_{sm}\, \mathcal{D}^{*}_{m,-n}$$

$$\hat{A}\hat{B} \longrightarrow (AB)(\xi,\bar{\xi}) = \sum \langle \xi | s \rangle \underbrace{A_{sr}\, B_{rm}}_{A_{sr}\,\delta_{rr'}\,B_{r'm}} \langle m | \xi \rangle$$

$$\delta_{rr'} = \sum_{\xi} \langle r | \xi \rangle \langle \xi | r' \rangle = \sum_{p} \mathcal{D}_{r,p}\, \mathcal{D}^{*}_{r',p}$$

$$\mathcal{D}_{r,p} = \sqrt{\frac{(n-i_1 - \cdots i_k)!}{n!\, i_1! \cdots i_k!}}\; \hat{R}_{+1}^{i_1}\, \hat{R}_{+2}^{i_2} \cdots R_{+k}^{i_k}\, \mathcal{D}_{r,-n}$$

$$(AB)(\xi,\bar{\xi}) = \sum_{s = i_1 + \cdots i_k} (-1)^s \frac{(n-s)!}{n!\, i_1! \cdots i_k!} R_{-1}^{i_1} \cdots R_{-k}^{i_k} A \; R_{+1}^{i_1} \cdots R_{+k}^{i_k} B$$

$$\equiv A(\xi,\bar{\xi}) * B(\xi,\bar{\xi})$$

$$A * B = AB - \frac{1}{n} \sum_{i=1}^{k} R_{-i} A\, R_{+i} B + \mathcal{O}\left(\frac{1}{n^2}\right)$$

$$\Big([A,B]\Big)(\xi,\bar{\xi}) = -\frac{1}{n}\sum_{i=1}^{k}\left(R_{-i}AR_{+i}B - R_{-i}BR_{+i}B\right) + \mathcal{O}\left(\frac{1}{n^2}\right)$$

Poisson bracket on CP^k

$$\{A,B\} = (\Omega^{-1})^{ij}\left(\frac{\partial A}{\partial \xi^i}\frac{\partial B}{\partial \bar{\xi}^j} - \frac{\partial A}{\partial \bar{\xi}^i}\frac{\partial B}{\partial \xi^j}\right)$$

$$\Omega = -i\left[\frac{d\bar{\xi}\cdot d\xi}{1+\bar{\xi}\cdot\xi} - \frac{\xi\cdot d\bar{\xi}\;\;\bar{\xi}\cdot d\xi}{(1+\bar{\xi}\cdot\xi)^2}\right]$$

(1) $$\boxed{\Big([A,B]\Big)(\xi,\bar{\xi}) = \frac{i}{n}\{A,B\}}$$ for large n

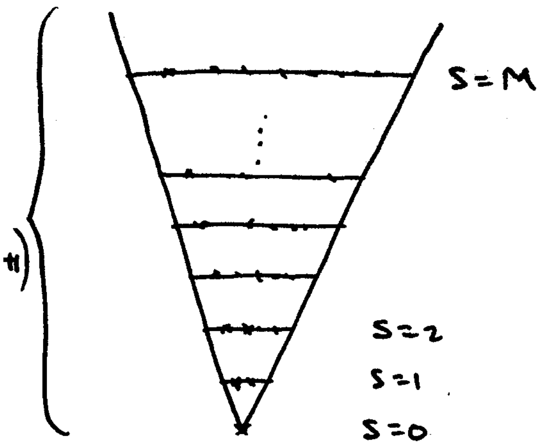
Pick ρ_0

K fermions densely packed around minimum of confining potential

(2) ρ_0 = constant over the phase volume occupied by droplet

$$\rho_0 \sim \Theta\left(R_D^2 - \bar{z}\cdot z\right) \qquad R_D = \text{droplet radius}$$

$\partial\rho_0 \longrightarrow$ δ-function with support at the droplet boundary convenient choice for $\hat{\rho}_0$: fill up all SU(k) multiplets up to a maximum "hypercharge" M



$$\hat{\rho}_0 = \sum_{s=0}^{M} |s\rangle\langle s|$$

$$\rho_0 = \sum_{s=0}^{M} \frac{n!}{s!\,(n-s)!} \frac{(\bar{\xi}\cdot\xi)^s}{(1+\bar{\xi}\cdot\xi)^n}$$

$$\xrightarrow[n,\,M]{\text{large}} \Theta\left(1 - n\frac{r^2}{M}\right)$$

spherical droplet of radius $R_D \sim \sqrt{\frac{M}{B}}$

corresponding confining potential $V = \omega n \frac{\bar{\xi}\cdot\xi}{1+\bar{\xi}\cdot\xi} \to \omega \bar{z} z$

$\langle s|\hat{V}|s\rangle = \omega s$

$$S = \int dt \; i\left(\mathrm{Tr}\,\hat{\rho}_0\,\hat{U}^+\partial_t\hat{U}\right) - \mathrm{Tr}\left(\hat{\rho}_0\,\hat{U}^+\,\hat{V}\,\hat{U}\right)$$

$$\hat{U} = e^{i\hat{\Phi}}$$

$$[\hat{\Phi}, \hat{\rho}_0](\xi,\bar{\xi}) = \frac{i}{n}\{\Phi, \rho_0\} = \frac{i}{n}\underbrace{(\Omega^{-1})^{ij}\frac{\partial\Phi}{\partial\xi_i}\hat{e}_j}_{=\ \mathcal{L}\Phi}\frac{\partial\rho_0}{\partial r^2}$$

$\mathcal{L}\Phi =$ tangential derivative along the boundary of the droplet

$$\mathcal{L} = i\left(\xi\cdot\frac{\partial}{\partial\xi} - \bar{\xi}\cdot\frac{\partial}{\partial\bar{\xi}}\right)$$

$$S \sim \int_{D} dt \, \frac{\partial \rho_0}{\partial r^2} \left(\frac{\partial \phi}{\partial t} + \omega \mathcal{L} \phi \right) \mathcal{L} \phi$$

$$S \sim \int_{\partial D} dt \left(\frac{\partial \phi}{\partial t} + \omega \, \mathcal{L} \phi \right) \mathcal{L} \phi$$

Higher dim. chiral action $\Big(\underbrace{(2k-1)}_{\partial D} + \underset{t}{1} \Big)$ dim

<u>Special case</u> : CP^3 $\left(S^2 \text{ bundle over } S^4 \right)$

CP^3, $U(1)$ $\xrightarrow[\text{choosing } \rho_0]{\text{by appropriately}}$ edge effective action for Zhang, Hu S^4 with $SU(2)$

<u>Twistor connection</u>

$$Z_\alpha \sim \lambda Z_\alpha \qquad \alpha = 1, \ldots, k+1$$

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{pmatrix} = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \pi_1 \\ \pi_2 \end{pmatrix} \qquad \omega_A = \underbrace{(x_4 - i \vec{\sigma} \cdot \vec{x})}_{x\text{'s : } S^4\text{-coordinates}} \pi_A$$

Kahler 2-form on CP^3 decomposes into

$$\Omega_{CP^3} = \Omega_{CP^1} - iF$$

F : $SU(2)$ instanton field

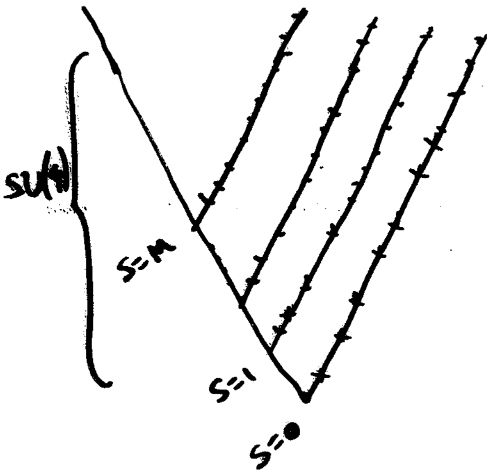
$$A_\mu = i \, \frac{N^a \eta^a_{\mu\nu} x^\nu}{(1+x^2)} \qquad \eta^a_{\mu\nu} = \epsilon_{a\mu\nu 4} + \delta_{a\mu} \delta_{4\nu} - \delta_{a\nu} \delta_{4\mu}$$

$$N^a = \frac{\bar{\pi} \sigma^a \pi}{\bar{\pi} \cdot \pi}$$

- $$\{A,B\}_{CP^3} = (1+x^2)\, K^{\mu\nu} \frac{\partial A}{\partial x^\mu} \frac{\partial B}{\partial x^\nu} + (1+x^2)\, \{A,B\}_{CP^1}$$

$$K^{\mu\nu} = -\frac{1}{2} N^a \eta^a_{\mu\nu} \quad \left(\text{local complex structure on } S^4\right)$$

- choose V_{conf} such that ρ_0 defines a droplet on S^4 (no S^2 dependence)



$$\rho_0 = \sum_{s=0}^{M} \frac{n!}{s!\,(n-s)!} \frac{(\bar{z}_3 z_3 + \bar{z}_4 z_4)^{n-s} (\bar{z}_1 z_1 + \bar{z}_2 z_2)^{s}}{(\bar{z}\cdot z)^n}$$

$$= \sum_{s=0}^{M} \frac{n!}{s!\,(n-s)!} \frac{(x^2)^s}{(1+x^2)^s}$$

s ~ SU(2) isospin

$$\longrightarrow \quad \Theta\left(1 - \frac{n x^2}{M}\right)$$

$$S \sim \int dt \underbrace{d\mu(CP^1)}_{\sim n} \int_{\partial D} \left(\frac{\partial \phi}{\partial t} + \omega\, \mathcal{L}\phi\right) \mathcal{L}\phi$$

$$\mathcal{L}\phi = 2x^\nu K^{\mu\nu} \partial_\mu \phi$$ = derivative along the droplet boundary

Edge dynamics for CP^k with $U(k)$ background

1. $$\hat{A} \longrightarrow \underbrace{A_{\alpha\beta}(\xi,\bar{\xi})}_{\text{matrix valued function}} = \langle \xi_\alpha | \hat{A} | \xi_\beta \rangle = \mathcal{D}_{m,\alpha} \, A_{ms} \, \mathcal{D}^*_{s,\beta}$$

$\alpha, \beta = 1, \dots, \dim \tilde{J} = N'$

2. $$\hat{A}\hat{B} \longrightarrow (AB)_{\alpha\beta}(\xi,\bar{\xi}) \equiv A_{\alpha\gamma} * B_{\gamma\beta}$$
$$= A_{\alpha\gamma} B_{\gamma\beta} - \frac{1}{n} \sum_{i=1}^{k} \hat{R}_{-i} A_{\alpha\gamma} \hat{R}_{+i} B_{\gamma\beta} + \mathcal{O}\left(\frac{1}{n^2}\right)$$

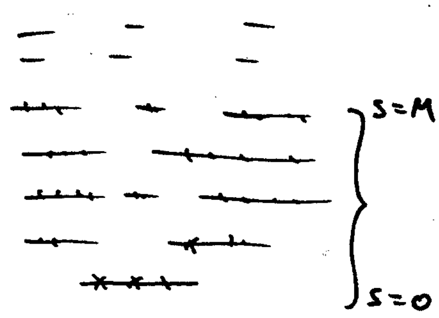
3. $$[\hat{A},\hat{B}] \longrightarrow [A,B] - \frac{1}{n}\left(R_{-i} A \, R_{+i} B - R_{-i} B R_{+i} B \right) + \mathcal{O}\left(\frac{1}{n^2}\right)$$
$$= [A,B] + \frac{i}{n} (\Omega^{-1})^{jm} \left(\mathbf{D}_j A \mathbf{D}_m B - \mathbf{D}_j B \mathbf{D}_m A \right)$$
$$\mathbf{D}_j = \partial_j A + [\mathcal{A}_j, A]$$

$SU(k)$ gauge field

4. choose ρ_0 :

all $SU(k)$ multiplets filled up to a maximum "hypercharge" number

$$(\rho_0)_{ab} \sim \Theta\left(1 - \frac{n\bar{\xi}\cdot\xi}{M}\right) \delta_{ab}$$



Edge effective action can be written in terms of unitary field $G \in U(\dim \tilde{J})$

$$KE = \int dt \; i \; Tr\left(\hat{\rho}_0 \hat{U}^+ \partial_t \hat{U}\right)$$

$$= \frac{i}{4\pi} \int_{D\times R} \Big[-d\rho_0 \; G^+ \dot{G} \, dt \; G^+ D G$$

$$-d\rho_0 \left(\mathcal{L} \, dG \, G^+ + \mathcal{L} \, G^+ dG + \mathcal{L} \, G \mathcal{L} \, G^+ \right) + \frac{1}{3} \rho_0 \left(G^+ dG\right)^3 \Big] \wedge \left(\frac{i\Omega}{\pi}\right)^k$$

Higher dimensional ($2k$ dim) gauged WZW model

(Kahler-Chern-Simons
Nair + Schiff)

radial variable (r^2) = extra dimension for WZW-term

$$S = \frac{1}{4\pi} \int_{\partial D} dt \; tr \Big[\left(G^+ \dot{G} \; G^+ \mathcal{L} G + \omega \left(G^+ \mathcal{L} G\right)^2 \right)$$

$$- i \left(\bar{\xi} \cdot \bar{\mathcal{L}} - \xi \cdot \mathcal{L} \right) \left(\dot{G} G^+ + G^+ \dot{G} \right) \Big]$$

$$+ \frac{i}{4\pi} \int_D tr \left[G^+ \dot{G} \left(G^+ dG\right)^2 \right] \wedge \left(\frac{i\Omega}{\pi}\right)^{k-1}$$

$$\mathcal{L} = i \left(\xi \cdot D - \bar{\xi} \cdot \bar{D} \right)$$

QHE in higher dimensions

- physical realization of fuzzy spaces
- related to dynamics of droplets of higher dimensional incompressible fluids
- interesting class of edge field theories
  higher dim. generalization of WZW actions
- higher dimensional bosonization