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Non-Commutative Geometry in Condensed Matter Physics and Field Theory
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Emergent gravity as the nematic dual of Lorentz-invariant Elasticity

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These are preliminary lecture notes, intended only for distribution to participants.

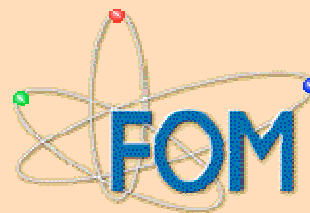
Emergent gravity as the nematic dual of Lorentz-invariant Elasticity

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Zohar Nussinov
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Universiteit
Leiden

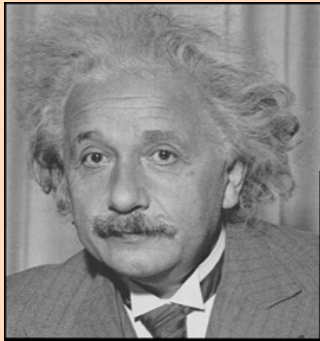
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Emergent Einstein Gravity



$$S = -\frac{c^3}{16\pi G} \int dV \sqrt{-g} R + S_{matter}$$

Einstein's space time = the Lorentz invariant topological nematic superfluid (at least in 2+1D)

A medium characterized by:

- emergent general covariance
- absence of torsion- and compressional rigidity
- presence of curvature rigidity (topological order)

Plan of talk



1. Plasticity (defected elasticity) and differential geometry
2. Fluctuating order and high T_c superconductivity
3. Dualizing non-relativistic quantum elasticity
 - 3.a Quantum nematic orders
 - 3.b Superconductivity: dual Higgs is Higgs
4. The quantum nematic world crystal and Einstein's space time

Quantum-elasticity: basics



Quantum elastic action, isotropic medium

$$S = \mu \left[u_{xx}^2 + 2u_{xy}^2 + u_{yy}^2 + \frac{\nu}{1-\nu} (u_{xx} + u_{yy})^2 + (\partial_\tau u_x)^2 + (\partial_\tau u_y)^2 \right]$$

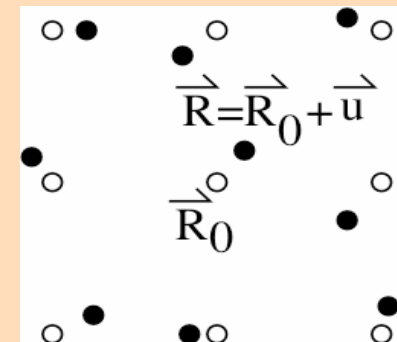
Shear modulus μ , Poisson ratio ν , compression modulus κ

$$\kappa = \mu(1 + \nu)/(1 - \nu)$$

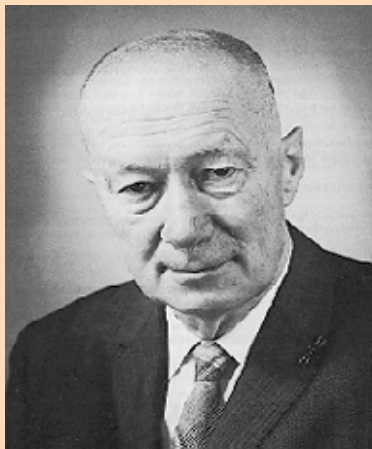
$$c_{ph}^2 = 2\mu/\rho = 1, \hbar = 1, u_{ab} = (\partial_a u_b + \partial_b u_a)/2$$

Describes transversal- (T) and longitudinal (L) phonon,

$$S = \mu \left[\left(\frac{q^2}{2} + \omega^2 \right) |u^T|^2 + \left(\frac{q^2}{1-\nu} + \omega^2 \right) |u^L|^2 \right]$$



Elasticity and topology: the dislocation



J.M. Burgers (Delft):

Discovery of the
topological excitation



War time needs (Peierls, Mott,
Friedel, ...)

The Singularities



Dislocation:

Restores translational invariance

Destroys shear rigidity

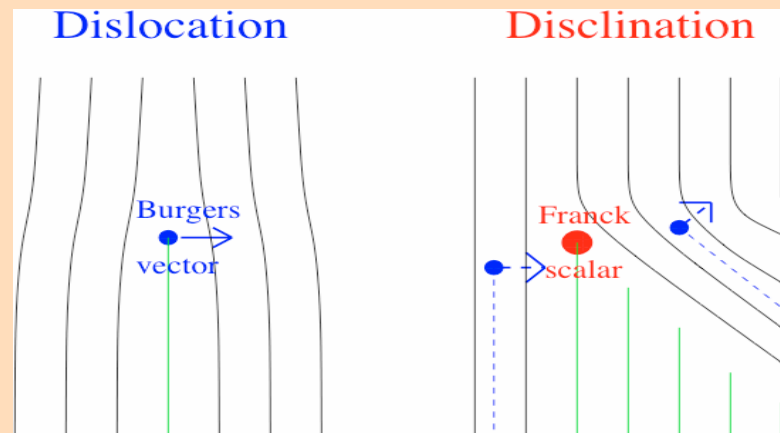
Topological charge: Burgers vector

Disclination:

Restores rotational invariance

Destroys *curvature* rigidity, like mass source in gravity (!)

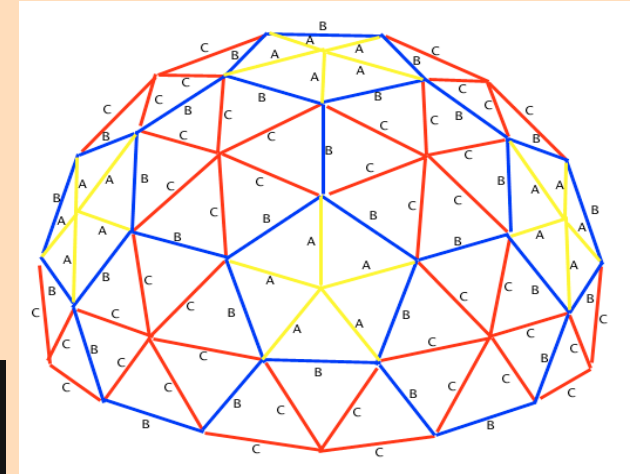
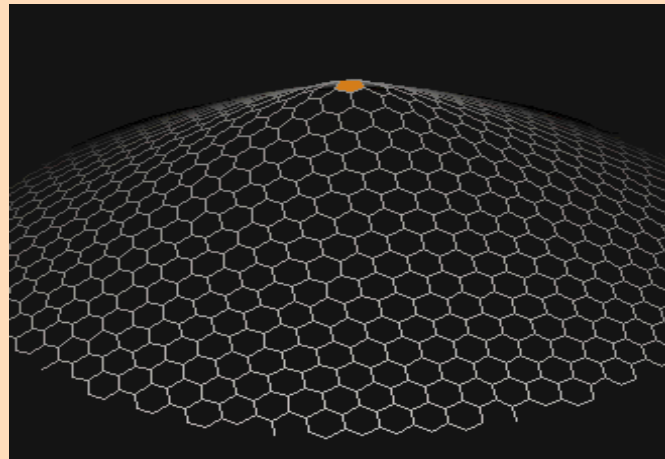
Topological charge: Frank 'scalar'



Disclinations are 'bad' (difficult)

Disclinations 'liberate' **non-abelian** nature of the euclidean group (diffeomorphism)

Engineering curvature



Disclination in ‘buckystuff’: like **conical singularity** in 2+1 D gravity
Fluctuating geometry = simplexes with fluctuating edge-lengths “**Regge Calculus**”

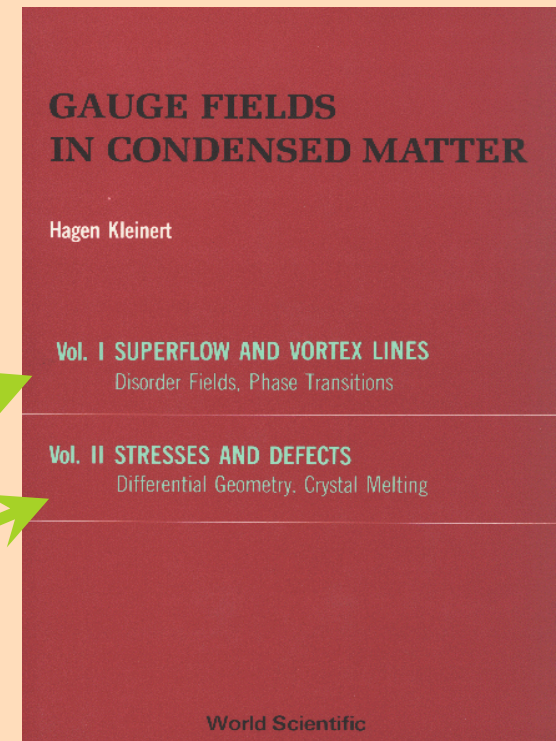
The mathematical machine



Hagen Kleinert
(FU Berlin)

Abelian Higgs duality

Theory of plasticity in 3D (classical):
similarity with **euclidean gravity!**



Elasticity and general covariance



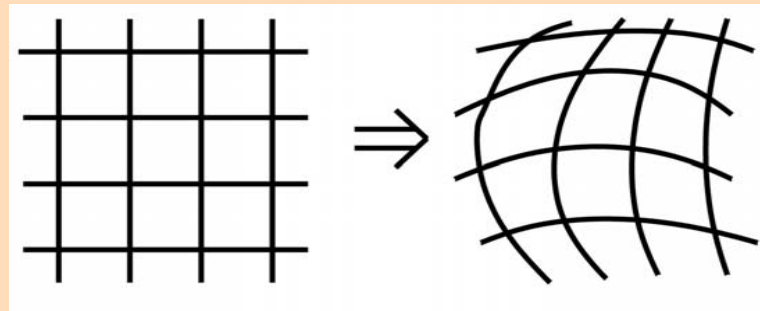
General covariance: infinitesimal coordinate transformation

$$e_a{}^\mu = \delta_a{}^\mu \rightarrow \delta_a{}^\mu - \partial_a \xi^\mu$$

$$g_{\mu\nu} = e_\mu{}^a e_{a\nu} = \delta_{\mu\nu} \rightarrow \delta_{\mu\nu} + (\partial_\mu \xi_\nu + \partial_\nu \xi_\mu)$$

General covariance = gauge invariance under elastic deformations

$$g_{ij} = \delta_{ij} \rightarrow \delta_{ij} + (\partial_i u_j + \partial_j u_i)$$



Plasticity and differential geometry



Covariant derivative:

$$D_\nu v^\mu = \partial_\nu v^\mu + \Gamma_{\nu\lambda}^\mu v^\lambda$$

Connection, linearized:

$$\Gamma_{\mu\nu}^\lambda = \partial_\mu \partial_\nu \xi^\lambda$$

Curvature tensor:

$$R_{\mu\nu\lambda\kappa} = (\partial_\mu \partial_\nu - \partial_\nu \partial_\mu) \partial_\lambda \xi_\kappa$$

Associate: $\xi^\lambda \rightarrow u^\lambda$

==> Einstein tensor corresponds with disclination current!

$$\Theta_{\mu\nu} \equiv G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^\kappa_\kappa$$

‘Bad’ news: torsion = dislocation currents

$$S_{\mu\nu\lambda} = \frac{1}{2} (\partial_\mu \partial_\nu - \partial_\nu \partial_\mu) \xi_\lambda \quad J_{\mu a} = \varepsilon_{\mu\nu\lambda} S_{\nu\lambda a}$$

**Crystal = geometry
with curvature and
torsion.**

Non-linear plasticity



Use gravity techniques to solve problems associated with large defect densities/plastic deformations (K. Kondo, 1952)

Substitute covariant derivatives for derivatives

$$\partial_\nu v^\mu \rightarrow D_\nu v^\mu = \partial_\nu v^\mu + \Gamma_{\nu\lambda}^\mu v^\lambda$$

Use full curvature, torsion tensors instead of linearized tensors

Program not greatly successful when it matters (e.g. glasses).

The universe as a strange crystal



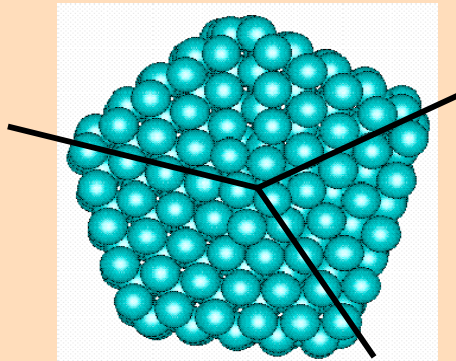
...

Special ‘crystal’: isotropic elastic medium in space and time directions (Euclidean signature)

(a) Lorentz-invariance

(b) Curvature is not quantized, Frank vectors are ...

$\frac{2\pi}{3}$ disclination in a triangular lattice

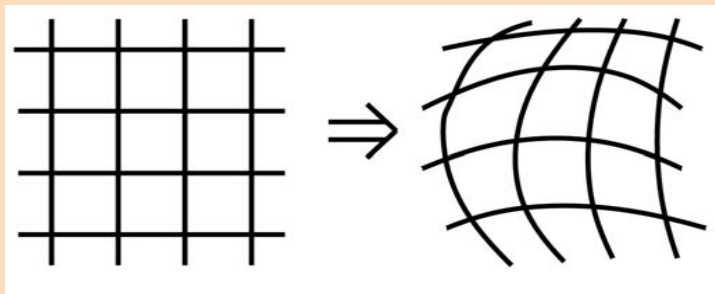


Deep geometrical/topological similarity with space-time (also: Bais et al, ‘quantum doubles’: crystal ==> Witten’s 2+1 D gravity)

Why the universe is not a 'Lorentz' crystal



- (a) Crystal 'geometry' is characterized by **torsion**, the universe is not ...
- (b) Disclinations are (quadratically) confined: **curvature costs infinite energy!**
- (c) Deadly: crystals have **no general covariance**, the flat metric is preferred !!!



← Elastic deformation costs
energy: action is not gauge
invariant ...

These problems seem to be cured in the quantum
NEMATIC world crystal ... (at least in 2+1D)

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Correlated superconductors



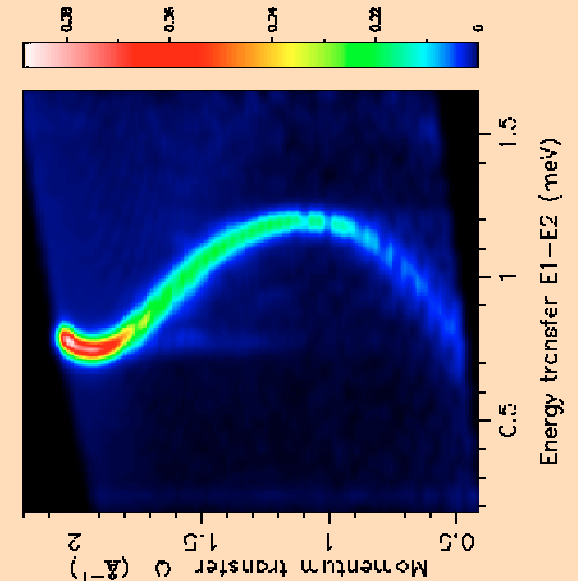
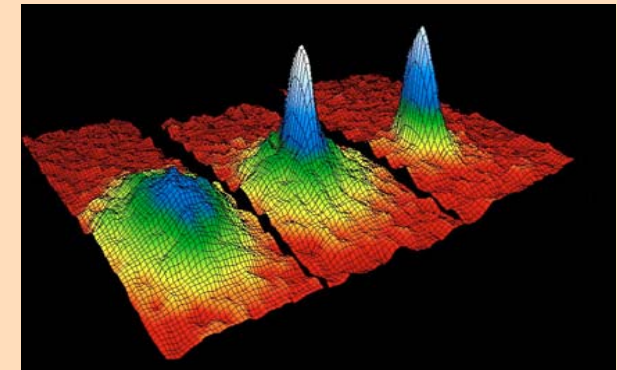
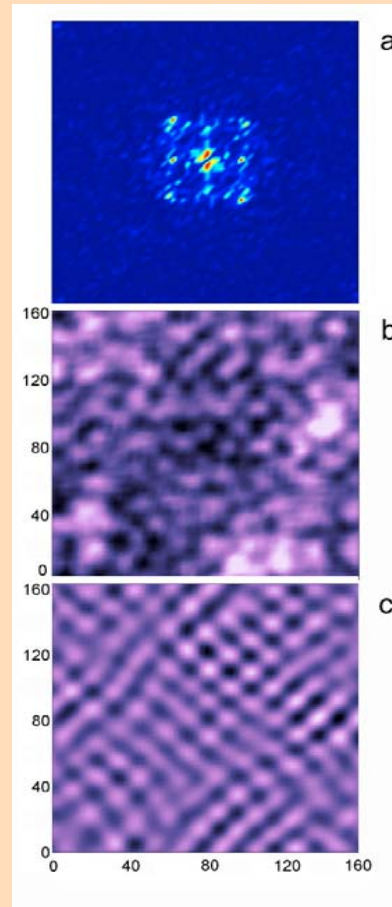
Ideal Bose-Einstein gas

BEC cold atomic gas, BCS metallic superconductor

Helium 4 superfluid

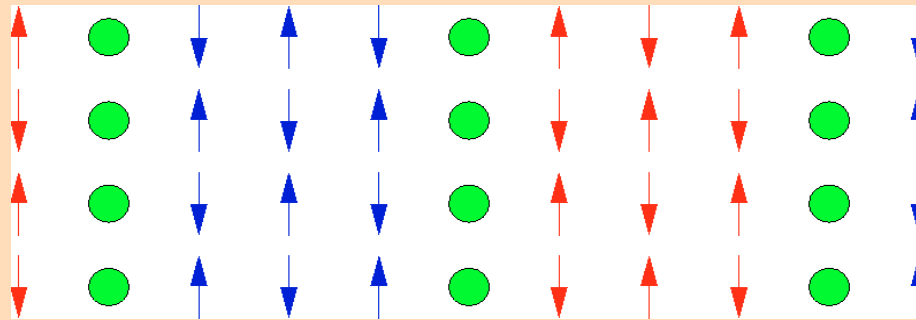
High T_c superconductors

Strongly correlated fluid: locally like a solid.



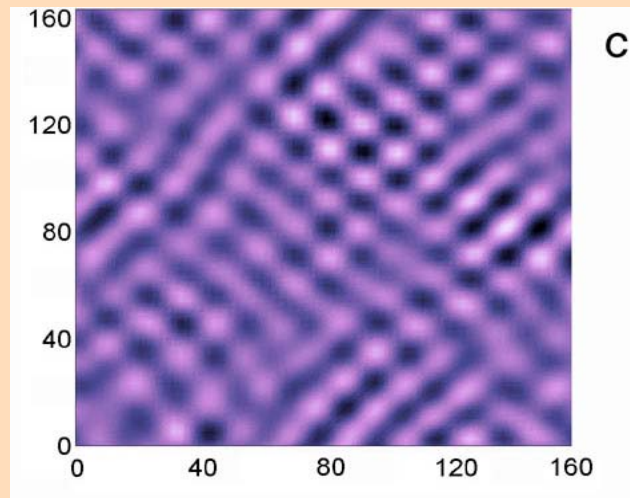
Roton

Stripe order: charge, spin, domain walls



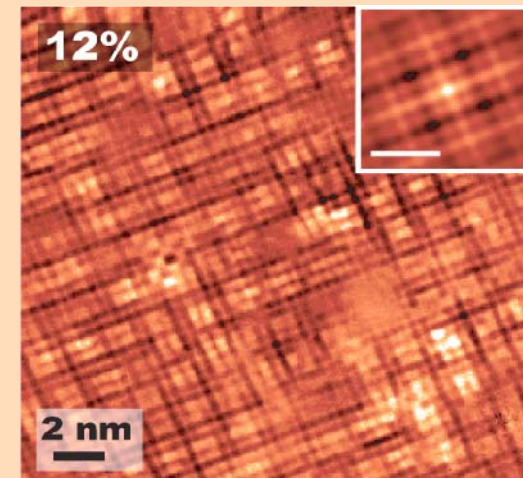
- Charge order (e.g. STM)
- Spin order (neutrons, NMR)
- ‘Topological order’, ‘domainwall-ness’, ‘antiphaseboundariness’

Electrons coming to a standstill



Kapitulnik et al (Stanford)

STM



Davis et al (Cornell)

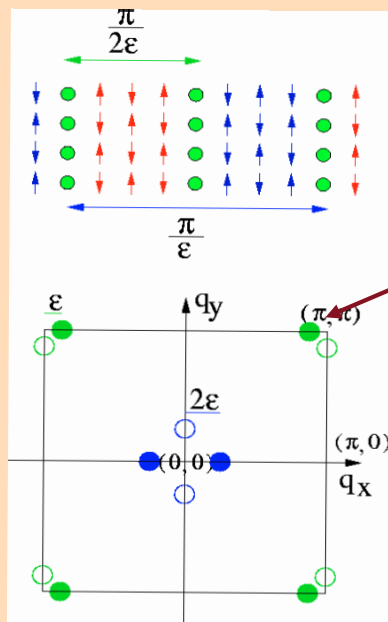
These 'stripes' are **ubiquitous** in doped Mott insulator (nickelates, manganites, ...)

Cuprates: stripes are extremely quantum mechanical ...

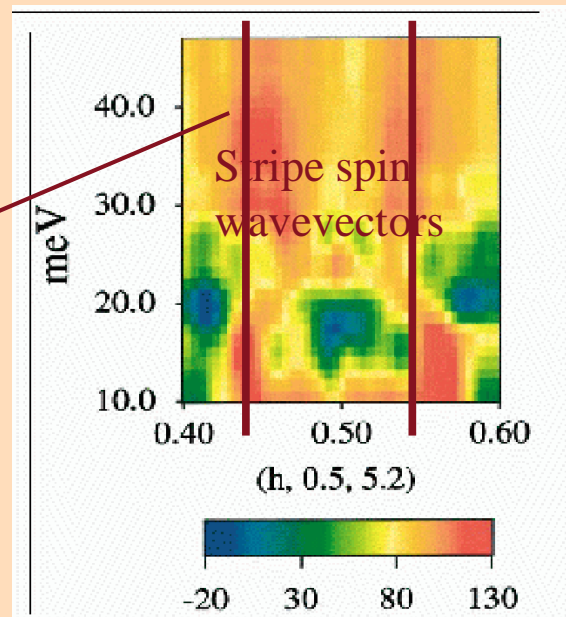
Quantum Fluctuating stripe order



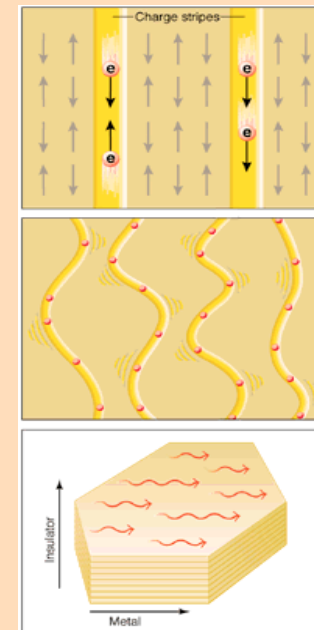
Fourier transform



Neutron scattering: spin fluctuations, subpicosecond!



YBCO $T_c=60K$: Mook et al, Oak Ridge

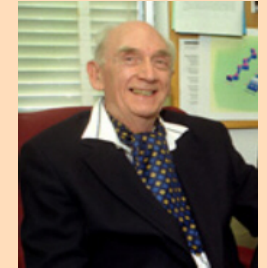


JZ, Science 286, 251 (1999)

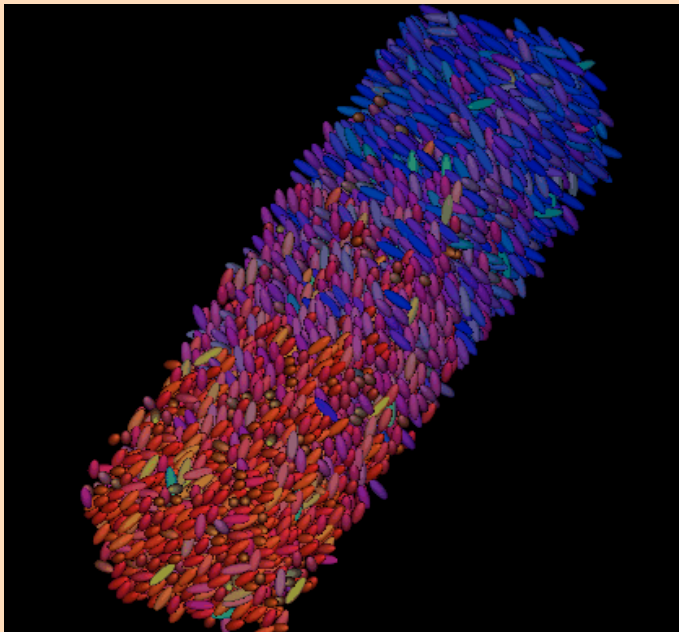
Nematic superfluids



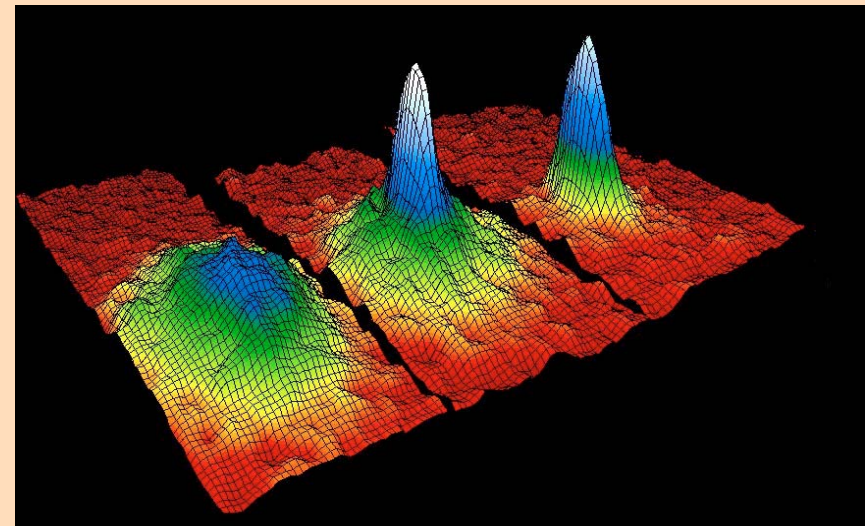
Kivelson, Fradkin, Emery, Nature 393,
550, 1998



Nematic order



Bose condensation



The Singularities



Dislocation:

Restores translational invariance

Destroys shear rigidity

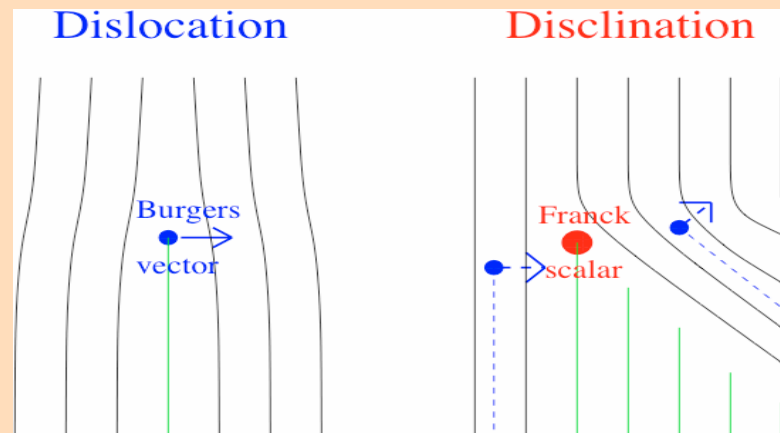
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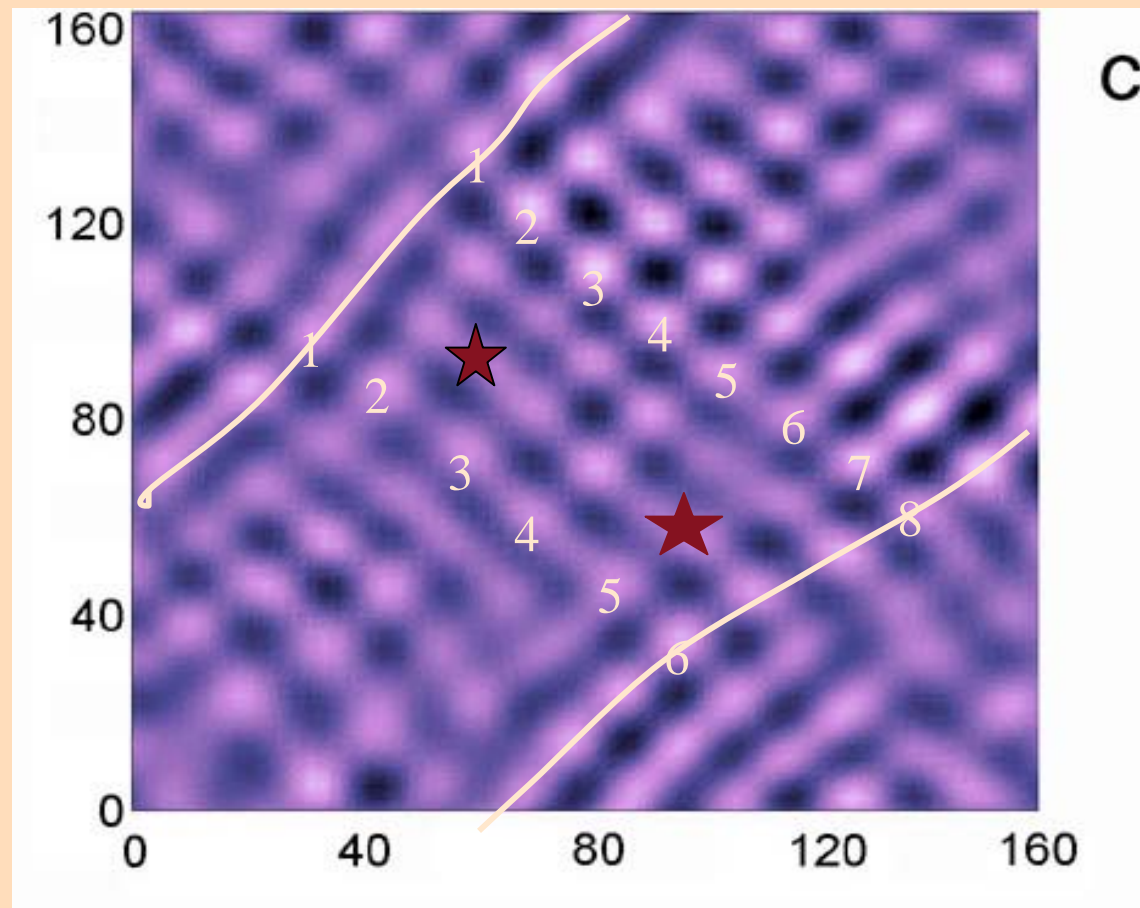
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Disclinations are 'bad' (difficult)

Disclinations 'liberate' **non-abelian** nature of the euclidean group (diffeomorphism)

Dislocations: example



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XY-electromagnetism duality in 2+1D



Relativistic short hand (magnon),

Consider quantum phase dynamics (XY in 2+1D),

$$S = \frac{1}{g} (\partial_\mu \varphi)^2 \mod(2\pi) \quad \boxed{H = \sum_i (n_i)^2 - J \sum_{\langle ij \rangle} \cos(\varphi_i - \varphi_j)}$$

Hubbard-Stratanovich auxiliary field ξ_μ

$$S = g \xi_\mu \xi_\mu + i \xi_\mu \partial_\mu \varphi$$

Divide in smooth and multivalued field configurations, $\varphi = \varphi_{sm} + \varphi_{MV}$

$$S = g \xi_\mu \xi_\mu + i \xi_\mu \partial_\mu \varphi_{MV} + i \xi_\mu \partial_\mu \varphi_{sm} \quad g \xi_\mu \xi_\mu + i \xi_\mu \partial_\mu \varphi_{MV} - i \varphi_{sm} (\partial_\mu \xi_\mu)$$

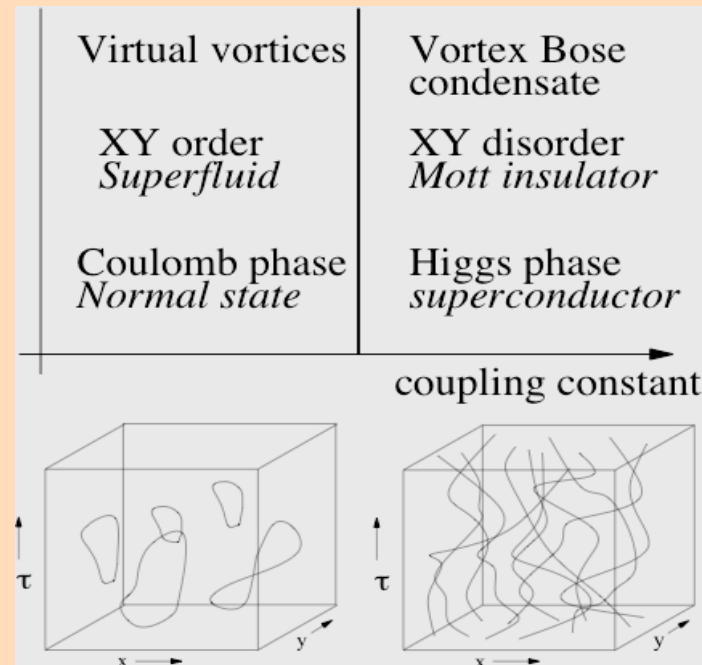
φ_{sm} acts like Lagrange multiplier $\Rightarrow \xi_\mu$ conserved \Rightarrow imposed by gauge field A_μ

$$\partial_\mu \xi_\mu = 0 \quad \Rightarrow \quad \xi_\mu = \varepsilon_{\mu\nu\lambda} \partial_\nu A_\lambda$$

Dual action: $S = g F_{\mu\nu} F^{\mu\nu} + i A_\mu J_V^\mu,$

Vortex current: $J_V^\mu = \varepsilon_{\mu\nu\lambda} \partial_\nu \partial_\lambda \varphi \Leftrightarrow \oint d\varphi = 2\pi n$

Disorder field theory



Theory describing vortex **tangle**:

Long range interaction Core vs kinetic energy Hard cores

$$S = \int dx^d d\tau \left[\left| (\partial_\mu - iA_\mu) \Psi \right|^2 + m^2 |\Psi|^2 + w |\Psi|^4 + F_{\mu\nu} F^{\mu\nu} \right]$$

Dualize elasticity: stress gauge fields (Kleinert)



Quantum elasticity: $S = C_{\mu\nu ab} \partial_\mu u^a \partial_\nu u^b$

Hubbard-Stratanovich = stress (σ_μ^a) - strain ($\partial_\mu u^a$) duality,

$$S = \sigma_\mu^a C_{\mu\nu ab}^{-1} \sigma_\nu^b + i \sigma_\mu^a \partial_\mu u_{sm}^a + i \sigma_\mu^a \partial_\mu u_{MV}^a$$

Integrate over smooth displacement fields u_{sm}^a

$$\partial_\mu \sigma_\mu^a = 0 \quad \Rightarrow \quad \sigma_\mu^a = \varepsilon_{\mu\nu\lambda} \partial_\nu B_\lambda^a$$

Conservation of stress imposed by 'stress photons' B_μ^a (pseudo tensors)

Dual action: $S = [(\varepsilon \partial B) C^{-1} (\varepsilon \partial B)] + i B_\mu^a J_\mu^a$

$$J_\mu^a = \varepsilon_{\mu\nu\lambda} \partial_\nu \partial_\lambda u_{MV}^a \quad \Leftrightarrow \quad \oint du^a = b^a$$

J_μ^a are **dislocation** currents, $\vec{b} = (b_x, b_y)$ **Burgers vector**.

Phonons as stress photons



‘Phonon’ action (elastic medium) in stress photons:

$$S = \frac{\hbar c_{ph}}{4\mu} \int dq^2 d\omega \left[\boxed{2(\omega^2 + q^2/2) |B_{-1}^T|^2} + \frac{1}{1+\nu} \boxed{((1-\nu)\omega^2 + q^2) |B_{+1}^T|^2} + \boxed{(\omega^2 + q^2) |B_{-1}^L|^2} \right]$$

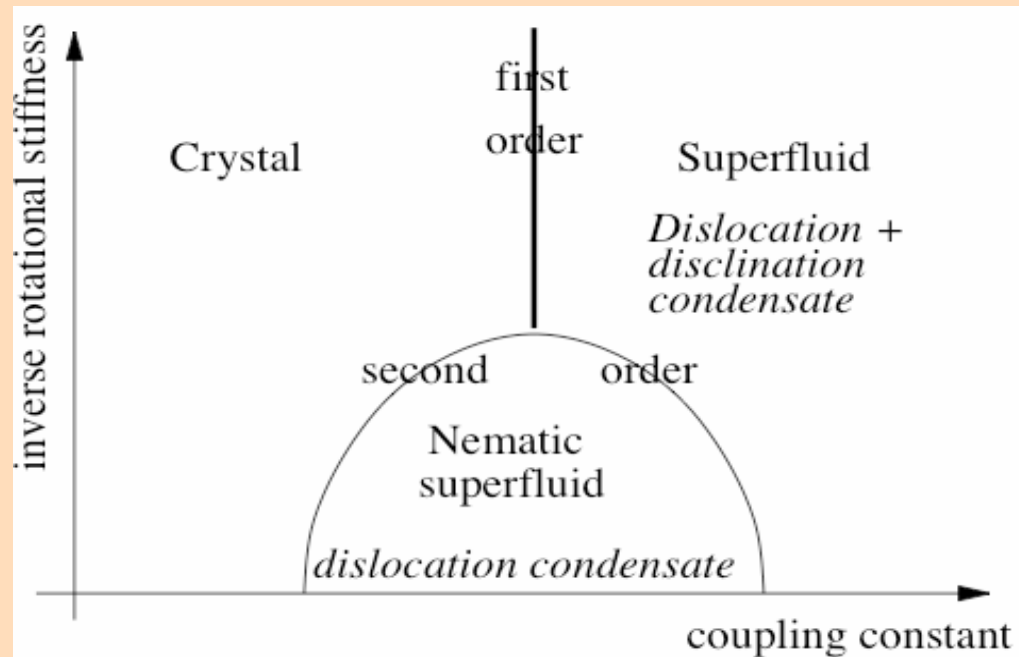
Transversal phonon
Longitudinal phonon
Instantaneous stress

Strain (phonon) propagator recovered through dual relation:

$$\langle \langle \partial_\mu u^a | \partial_\nu u^b \rangle \rangle_{\vec{p}} = C_{\mu\mu aa}^{-1} \delta_{\mu\nu} \delta_{ab} - C_{\mu\lambda ac}^{-1} C_{\nu\kappa bd}^{-1} \langle \langle \sigma_\lambda^c | \sigma_\kappa^d \rangle \rangle_{\vec{p}}$$

$$\sigma_\mu^a = \varepsilon_{\mu\nu\lambda} \partial_\nu B_\lambda^a$$

Nelson-Halperin-Young + \hbar

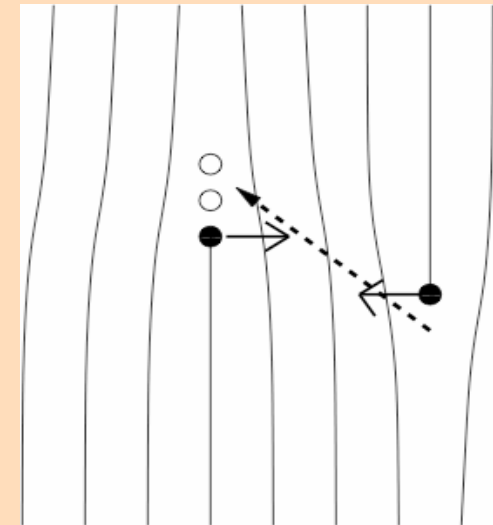


Nematic ('hexatic') quantum order:

Dislocations bose condense

Disclinations stay massive

Idealization: interstitials non-existent.



Particle transport at
dislocation collisions



The dual (dislocation) condensate

Dislocation Meissner Phase: $\Psi = |\Psi|e^{i\psi}, \langle |\Psi|^2 \rangle \neq 0 \Rightarrow L_{eff} = L_{Meissner} + L_{Maxwell} + L_{Director}$

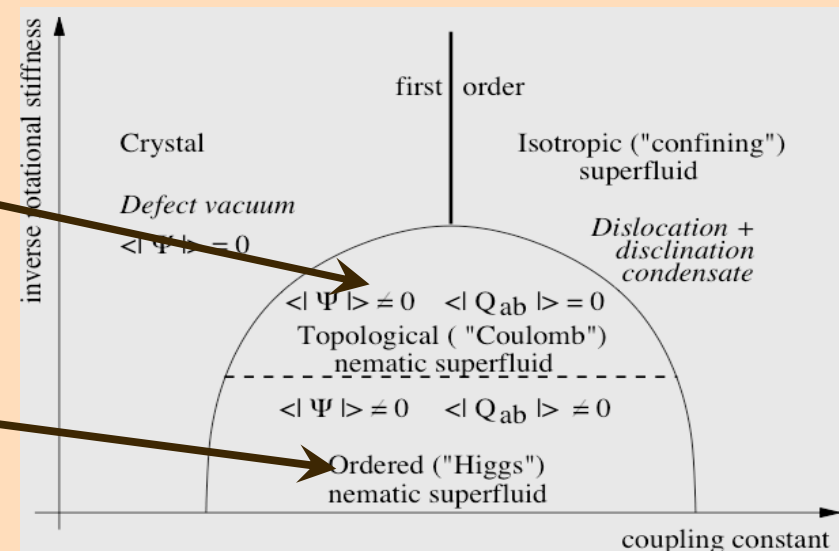
$$L_{Meissner} = \frac{q_0^2 |n_T(Q_{ab})|^2}{2\mu} \left[|B_{-1}^T|^2 + 4 \frac{\hat{\omega}^2}{1 + \hat{\omega}^2} |B_{-1}^L|^2 - 2 \frac{\hat{\omega} (1 - \hat{\omega}^2)}{1 + \hat{\omega}^2} (B_{+1}^{T*} B_{-1}^L + h.c.) \right]$$

$$L_{Maxwell} = "(\epsilon \partial B) C^{-1} (\epsilon \partial B)" \quad L_{Director} = "(\partial Q)^2 + m_Q^2 Q^2 + w_Q Q^4"$$

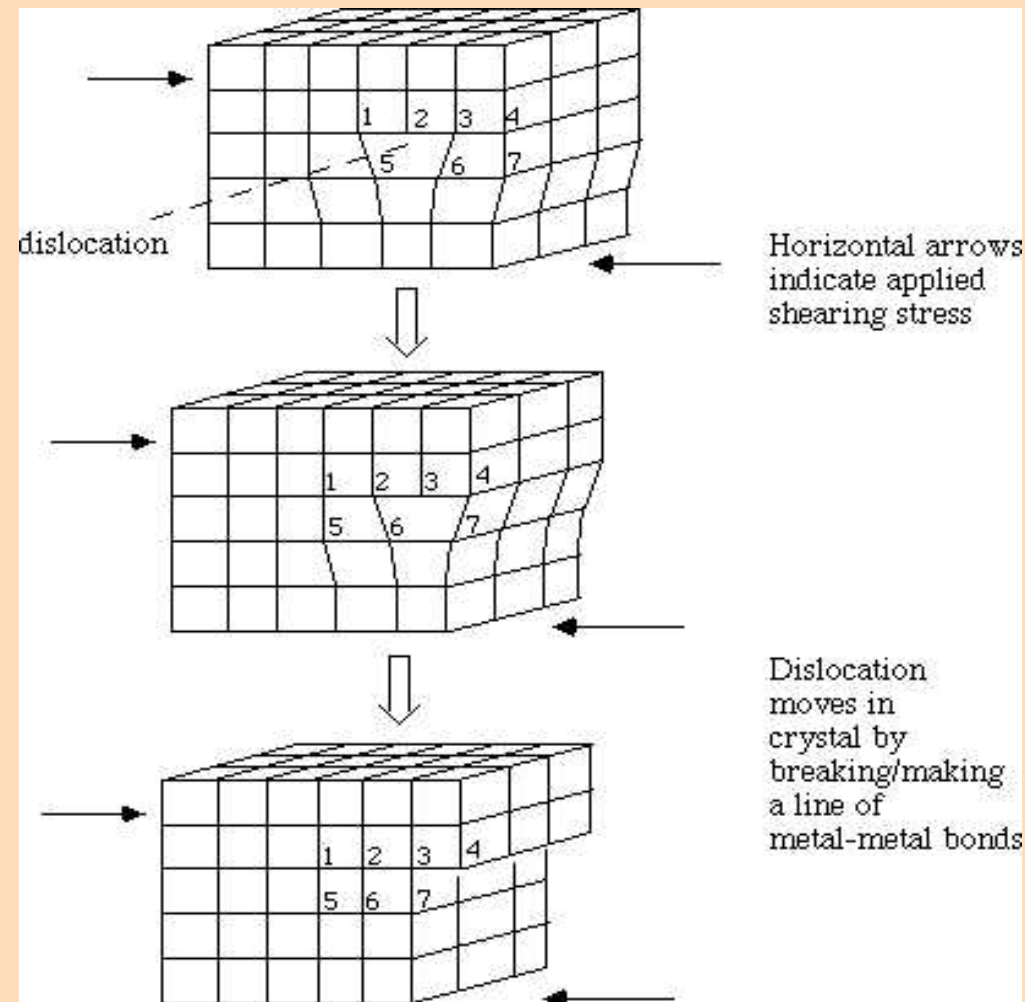
$$\lambda_{shear} = 1/q_0 : \text{ shear penetration depth}$$

“Topological Nematic” (Toner-Lammert-Rokhsar) Burgers vectors disordered **Isotropic, but massive disclinations**

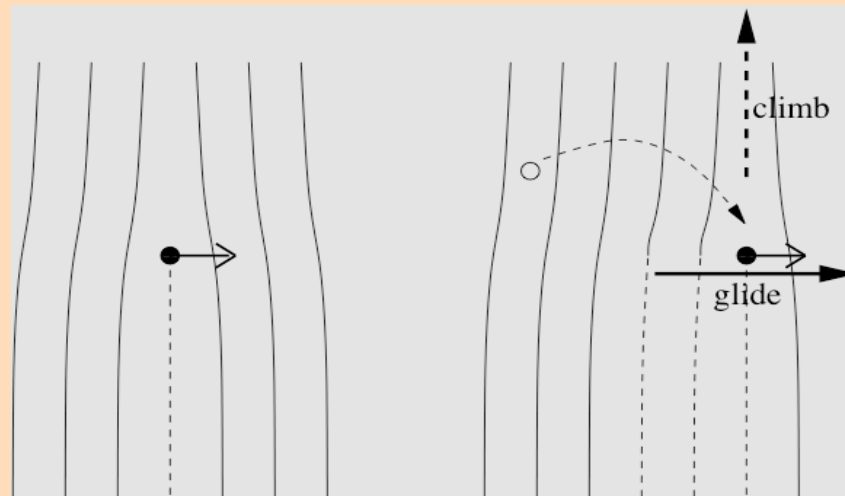
“Glide nematic” Burgers vectors disordered Broken rotational symmetry **Anisotropic** λ_{shear}



Dislocations and shear



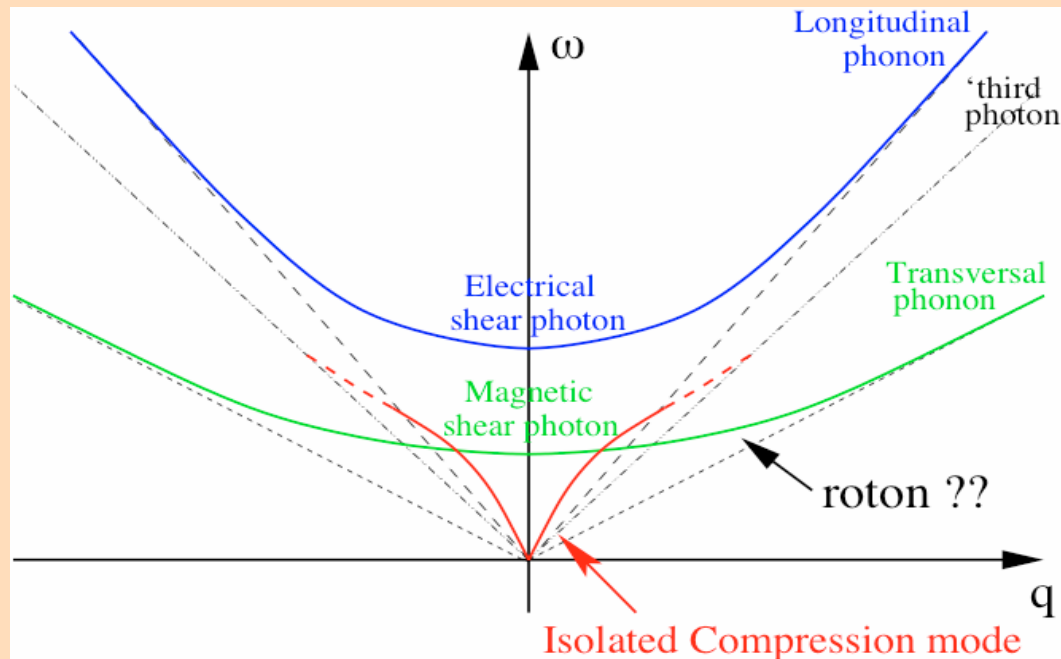
Dynamics: glide principle



‘**topological dynamics**’: propagation only possible along Burgers vector

Field theory: requirement for **finite compression** modulus in liquid

Topological nematic superfluid: excitations



Superfluid hydrodynamics:

Isolated massless compression, massive shear: Euler fluid

Periodicity (vortex quantization): inherited from dislocation condensate

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Superconductivity: photons vs. stress-photons



Charged elastic medium (bosonic Wigner crystal): couple in **EM** gauge fields

$$S = \int dx^2 d\tau \left[n_e e \vec{u} \cdot \vec{E} + F_{\mu\nu} F^{\mu\nu} \right] \quad E_{x,y} = \pm \left(\partial_{x,y} A_\tau - \frac{1}{c} \partial_\tau A_{x,y} \right) \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Dualize in stress photons, focus on **magnetic** sector:

$$L_{tot} = L_{AA} + L_{AB} + L_{BB}$$

$$L_{AA} = \left(\frac{1}{\lambda_L^2} + \frac{\omega^2}{c^2} + q^2 \right) |A_{-1}|^2 + \dots |A_{+1}|^2$$

$$\frac{1}{\lambda_L^2} = \frac{n_e^2 e^2}{\rho c^2}$$

bare London penetration depth,
superconductor??

Effective EM action: integrate out stress photons

$$L_{AB} = -\frac{n_e e}{\rho c^2} B_{-1}^T A_{-1} + f(A_{+1}, B_{-1}^L, B_{+1}^T)$$

$$L_{BB} = \frac{1}{4\mu} \left[\left(2\omega^2 + q^2 + \frac{2}{\lambda_{shear}^2} \right) |B_{-1}^T|^2 + g(B_{-1}^L, B_{+1}^T) \right]$$

Shear length finite (fluid):
Crystal: Meissner term **eaten**
compensation incomplete,
electromagnetic Meissner
'liberated'!!

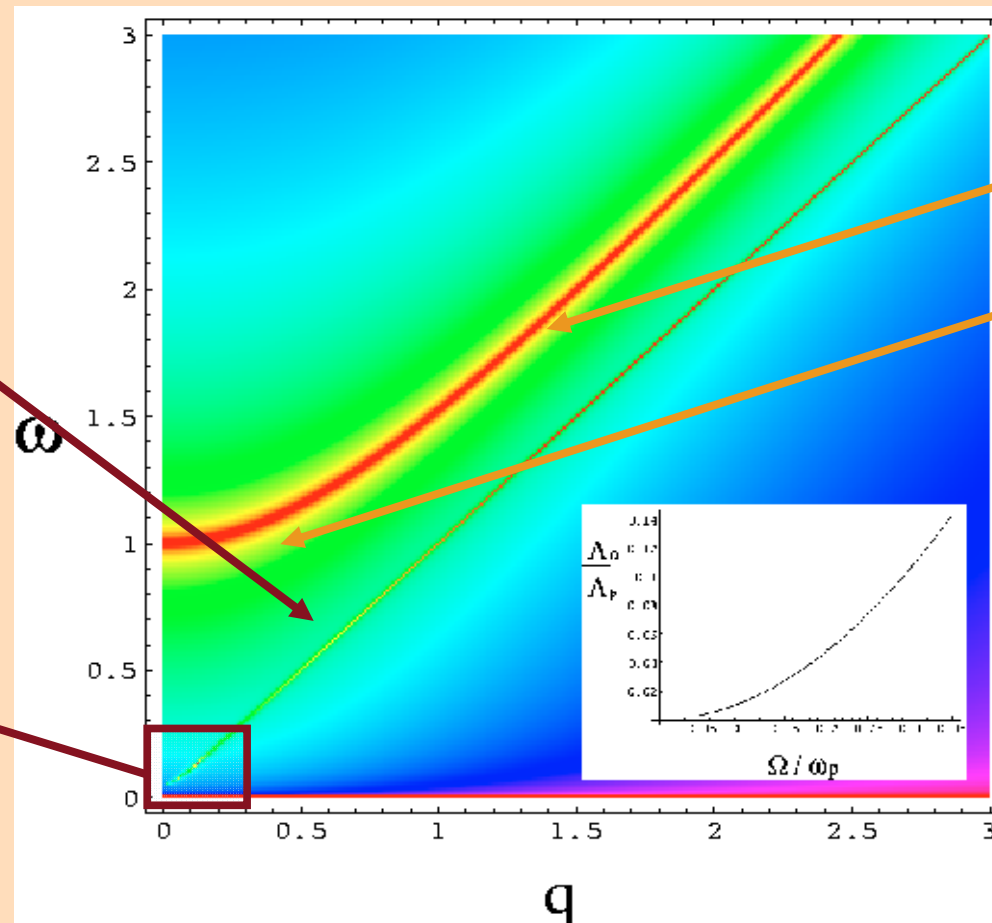
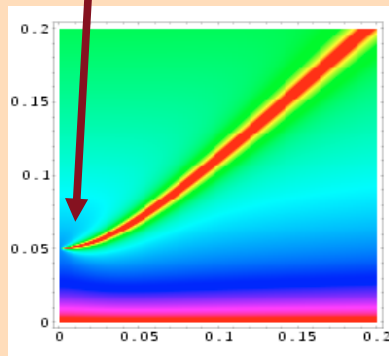
The dual Higgs boson and electron loss



Dual (shear)
Higgs photon

Dual Higgs mass

$\Omega \approx 50 \text{ meV}!$



Plasmon

$\omega_p \approx 1 \text{ eV}$

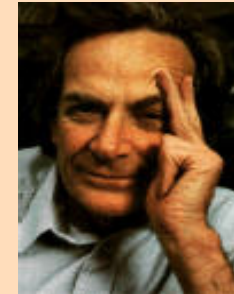
Superfluid hydrodynamics



Landau

Superfluid: quantized Euler fluid

$$\partial_t \vec{u} + (\vec{u} \bullet \vec{\nabla}) \vec{u} = \vec{\nabla} p, \text{"mod}(2\pi)"$$



Feynman

Footnote (this talk): this is the zero temperature hydrodynamics of a solid which has lost its rigidity against long range shear forces.

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Emergent gravity: general covariance



Distances measured by hopping from lattice site to lattice site, metric:

$$g_{\mu\nu} = \delta_{\mu\nu} + (\partial_\mu u_\nu + \partial_\nu u_\mu)$$

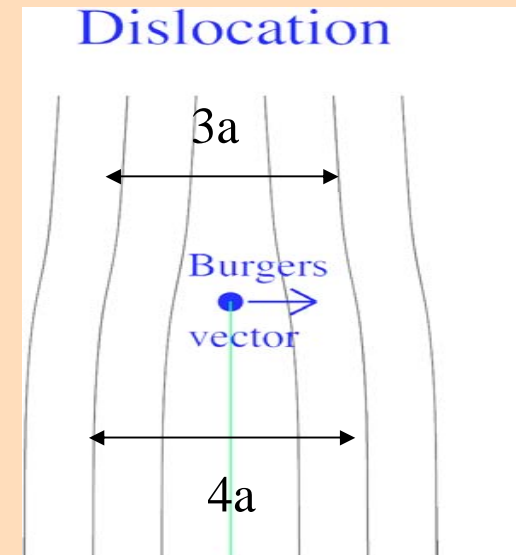
General covariance: metric is defined modulo local translations

Dislocation condensate:

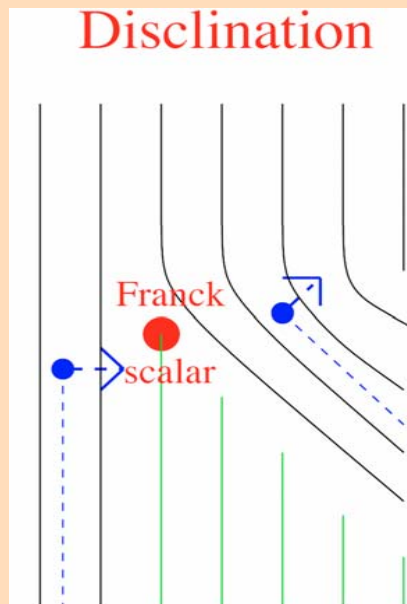
Coherently delocalized dislocations \Rightarrow
distances defined modulo local translations!

Bonus:

Dislocations represent torsion, dislocation condensate = Riemann space without torsion (like general relativity)!



Emergent gravity: curvature



Represent **curvature** (e.g., in 2+1D literally like conical defects)

Disclination current (2+1D):

$$\Theta_{\mu\nu} = \varepsilon_{\mu\kappa\lambda} \partial_{\kappa} \partial_{\lambda} \omega_{\nu}, \quad \omega_{\mu} = \frac{1}{4} \varepsilon_{\mu\nu\lambda} (\partial_{\nu} u_{\lambda} - \partial_{\lambda} u_{\nu})$$

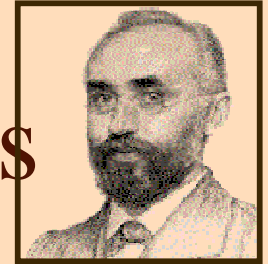
corresponds with the (linearized) Einstein tensor:

$$\Theta_{\mu\nu} \cong G_{\mu\nu} = R_{\nu\mu} - \frac{1}{2} g_{\nu\mu} R$$

In “solid”: disclinations are **confined** (infinite energy)

In nematic superfluid: **deconfined** but massive ...

Dynamics: double curl gauge fields



Recall: ‘conventional’ stress photons \Rightarrow only sources are disclinations

$$Q^{\kappa\lambda} = \varepsilon^{\kappa\gamma} g^{\kappa} B^{\gamma\lambda} \rightarrow B^{\kappa\lambda} \gamma^{\kappa\lambda}, \quad \gamma^{\kappa\lambda} = \varepsilon^{\kappa\gamma} g^{\kappa} g^{\gamma} n_b^{\lambda}$$

Disclination currents: one ‘order’ higher in duality

$$\Theta_{\mu\nu} = \varepsilon_{\mu\kappa\lambda} \partial_{\kappa} \partial_{\lambda} \omega_{\nu}, \quad \omega_{\mu} = \frac{1}{4} \varepsilon_{\mu\nu\lambda} (\partial_{\nu} u_{\lambda} - \partial_{\lambda} u_{\nu})$$

Kleinert: double curl stress gauge fields (two forms)

$$\sigma_{\mu\nu} = \varepsilon_{\mu\kappa\lambda} \varepsilon_{\nu\alpha\beta} \partial_{\kappa} \partial_{\alpha} h_{\lambda\beta}$$

$$\Rightarrow S_{WC} = \int d^2x d\tau \left[\frac{1}{4\mu} \left(\sigma_{\mu\nu}^2 - \frac{\nu}{1+\nu} \sigma_{\mu\mu}^2 \right) + i h_{\mu\nu} \eta_{\mu\nu} \right]$$

Sources are truly conserved ($\partial_{\mu} \eta_{\mu\nu} = 0$) ‘defect currents’

$$\eta_{\mu\nu} = \theta_{\mu\nu} + \varepsilon_{\lambda\mu\kappa} \partial_{\lambda} J_{\nu\kappa}$$

The nematic ‘Lorentz’ crystal



Dislocation melting of the 2+1D ‘Lorentz’ (space-time isotropic) crystal:

(a) Consider topological nematic \Rightarrow space-time isotropy.

(b) No preferred time direction \Rightarrow glide constraint is impossible \Rightarrow

Dislocations turn into sources of compressional ‘photons’ \Rightarrow
sound acquires a Higgs mass!

Symmetric = simple version of non-relativistic nematics

The nematic ‘Lorentz’ crystal



Input: $S_{WC} = \int d^2x d\tau \left[\frac{1}{4\mu} \left(\sigma_{\mu\nu}^2 - \frac{\nu}{1+\nu} \sigma_{\mu\mu}^2 \right) + i h_{\mu\nu} \eta_{\mu\nu} \right]$

$$\sigma_{\mu\nu} = \varepsilon_{\mu\kappa\lambda} \varepsilon_{\nu\alpha\beta} \partial_\kappa \partial_\alpha h_{\lambda\beta}$$

$$\eta_{\mu\nu} = \theta_{\mu\nu} + \varepsilon_{\lambda\mu\kappa} \partial_\lambda J_{\nu\kappa}$$

Condense dislocations ‘isotropically’, $S_{dislo} = \int d^2x d\tau \left[\frac{m_d^2}{2} J_{\mu\nu}^2 + i J_{\mu\nu} \varepsilon_{\mu\kappa\lambda} \partial_\kappa h_{\lambda\nu} \right]$

$$S_{eff,space} = \int d^2x d\tau \left[\frac{1}{4\mu} \left(\sigma_{\mu\nu}^2 - \frac{\nu}{1+\nu} \sigma_{\mu\mu}^2 \right) + \frac{1}{2m_d^2} \sigma_{\mu\nu} \frac{1}{\partial^2} \sigma_{\mu\nu} \right]$$

$$S_{discl} = \int d^2x d\tau \left[\frac{m_\theta^2}{2} \theta_{\mu\nu}^2 + i h_{\mu\nu} \theta_{\mu\nu} \right]$$

Geometrical meaning



Consider dual 'stress' geometry: $h_{\mu\nu} = g_{\mu\nu} - \delta_{\mu\nu}$

Stress tensor turns into **Einstein** tensor: $\sigma_{\mu\nu} \equiv G_{\mu\nu}$

$$S_{eff,space} = \int d^2x d\tau \left[\frac{1}{4\mu} \left(G_{\mu\nu}^2 - \frac{\nu}{1+\nu} G_{\mu\mu}^2 \right) + \frac{1}{2m_d^2} G_{\mu\nu} \frac{1}{\partial^2} G_{\mu\nu} \right]$$

Non-linear generalization $\partial_\mu \rightarrow D_\mu$, etc; identify $m_d^2 = 8\pi G$

$$\rightarrow \int d^2x dt \sqrt{-g} \frac{1}{2m_d^2} G_{\mu\nu} \frac{1}{-D^2} G_{\mu\nu} \rightarrow -\frac{c^3}{16\pi G} \int d^2x dt \sqrt{-g} R$$

At long distances this becomes exactly the Einstein action

Incompressible (2+1 D): shear and compression massive, curvature rigidity is still present.

Gravitating matter



Normal matter: gravity is uniformly attractive

$$S_{matter} = \int d^2x d\tau [h_{\mu\nu} T_{\mu\nu}]$$

T - symmetric Belinfante energy-momentum tensor

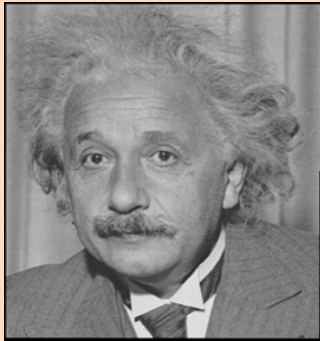
No condensed matter analogy !

Disclinations: massive excitations ‘guarding’ the curvature rigidity of space

$$S_{discl} = \int d^2x d\tau \left[\frac{m_\theta^2}{2} \theta_{\mu\nu}^2 + i h_{\mu\nu} \theta_{\mu\nu} \right]$$

Anti-dislocations do antigravity ... Startrek!

Emergent Einstein Gravity



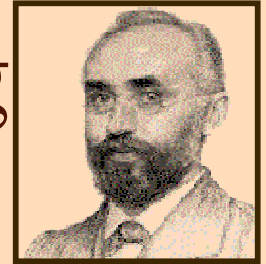
$$S = -\frac{c^3}{16\pi G} \int dV \sqrt{-g} R + S_{matter}$$

Einstein's space time = the Lorentz invariant topological nematic superfluid (at least in 2+1D)

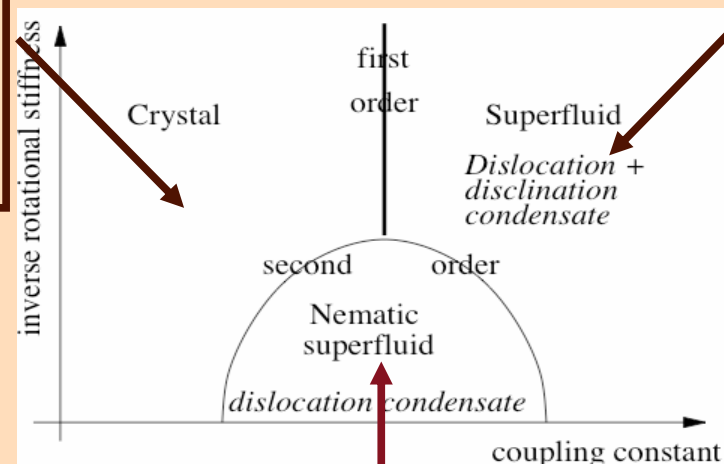
A medium characterized by:

- emergent general covariance
- absence of torsion- and compressional rigidity
- presence of curvature rigidity (topological order)

Plastic Cosmology: the competing phases



The Lorentz-crystal: non-gravitating torsion and vacuum pressure dominated universe (curvature confined)



Space-time does not resist curvature: matter causes catastrophic deflation

Our place
Fortunately!

Conclusions



- Condensed matter physics: **strongly correlated superconductors**
 - The quantum nematic orders
 - The Meissner phase as the dual dislocation condensate
- **Waiting for experiments**
- Relativistic generalization: **emergent gravity**.
 - Makes sense in 2+1D, 3+1D generalization?
- Allegory or the holy truth? **Subject for the Theology Department!**

J. Zaanen, Z. Nussinov and S.I. Mukhin, Ann. Phys. (NY) 310, 181 (2004) (cond-mat/0309397) ; H. Kleinert and J. Zaanen, Phys. Lett. A 324, 361 (2004) (cond-mat/0309379); V. Cvetkovic, S.I. Mukhin, J. Zaanen, in preparation.