



The Abdus Salam  
International Centre for Theoretical Physics



Spring Colloquium on  
**'Regional Weather Predictability and Modeling'**  
**April 11 - 22, 2005**

- 1) *Workshop on Design and Use of Regional Weather Prediction Models, April 11 - 19*
- 2) *Conference on Current Efforts Toward Advancing the Skill of Regional Weather Prediction. Challenges and Outlook, April 20 - 22*

301/1652-11

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**The Eta Model Numerical Design.  
Vertical coordinate**

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USA

# The Eta Model Numerical Design.

## Vertical coordinate

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Spring Colloquium

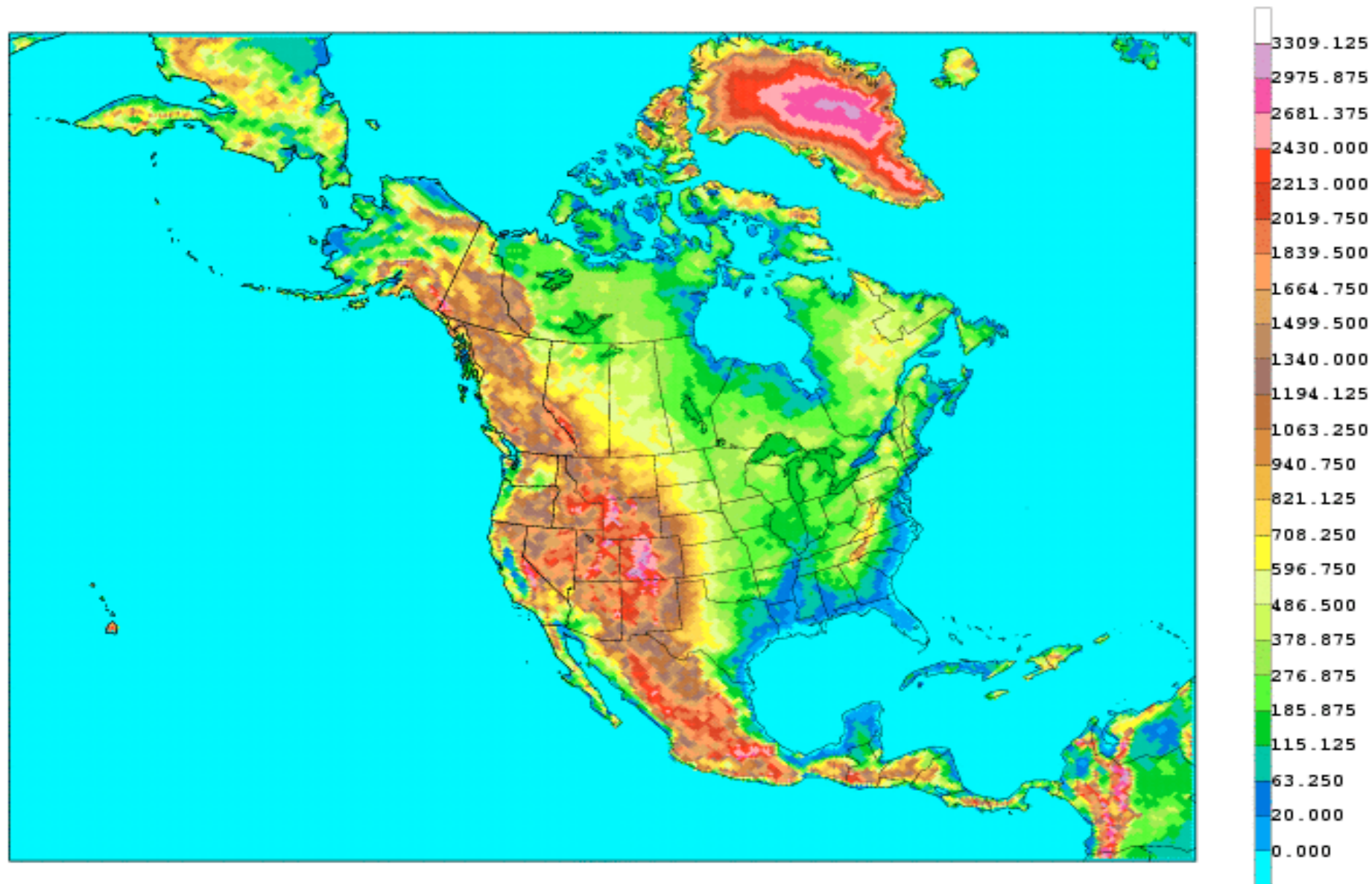
“Regional Weather Predictability and Modeling”  
Abdus Salam ICTP, Miramare, Trieste, 11-22 April 2005

## Vertical coordinate issues:

The Earth has topography !

Domain and topography  
used for NCEP Reg. Reanalysis:

**Eta 32 km/45 layer topography**



## Vertical coordinate choices:

$z, p$ : problems with coordinate surfaces intersecting topography;

N. Phillips (1957) "sigma":

$$\sigma = \frac{p}{p_S} \quad \left( \text{Or, later,} \quad \sigma = \frac{p - p_T}{p_S - p_T} \right)$$

Isentropic:

attractive, but problems with topography not addressed;

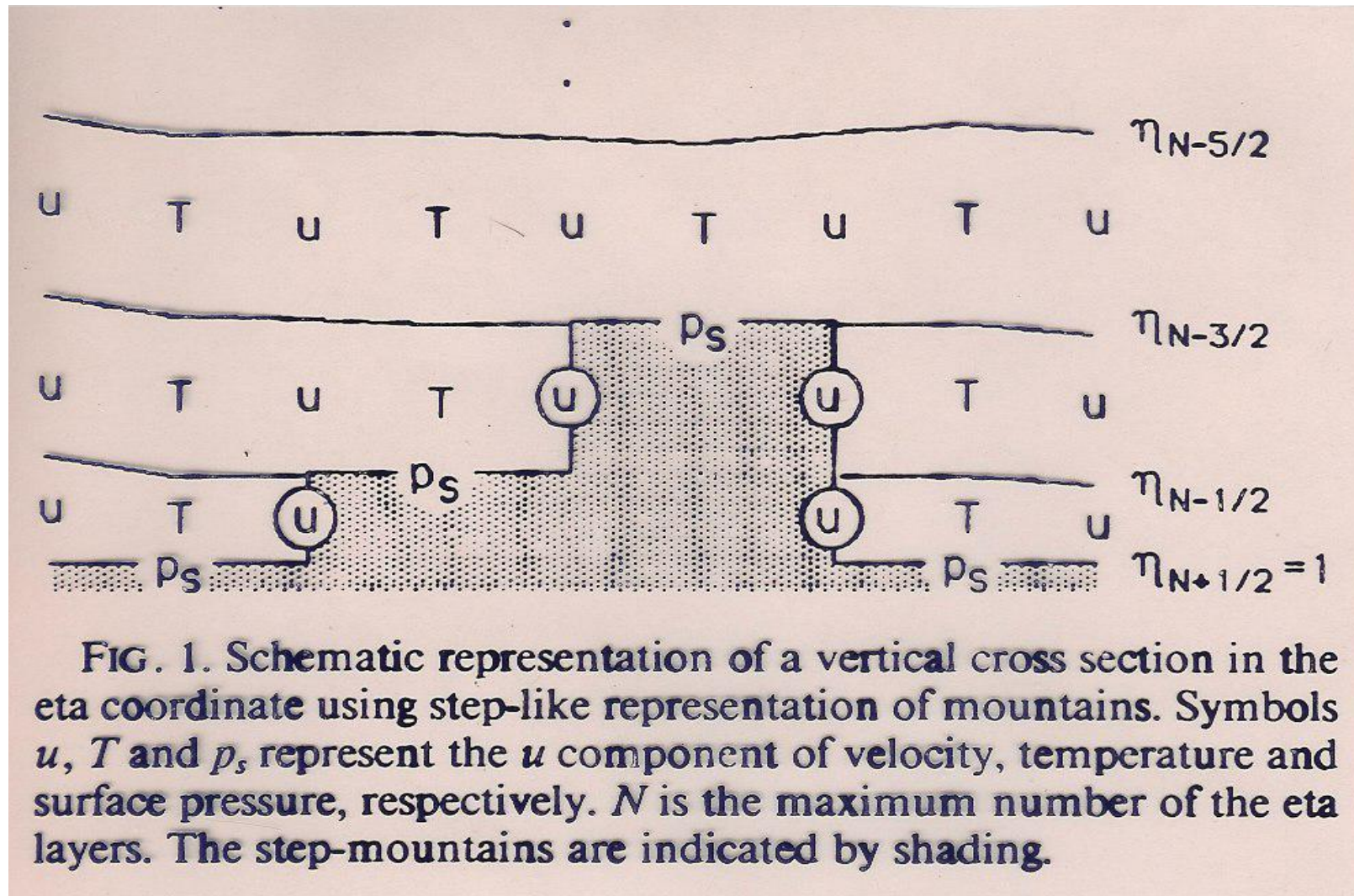
Problems with sigma (PGF, and others, later), thus,

Mesinger (1984) "eta":

$$\eta = \frac{p - p_T}{p_S - p_T} \eta_S, \quad \eta_S = \frac{p_{rf}(z_S) - p_T}{p_{rf}(0) - p_T}$$

Note: can be  
used as a switch,  
eta/ sigma

## Step-topography discretization (Mesinger 1984):





Equations:

Generalization of Simmons, Burridge (1981); just as simple;

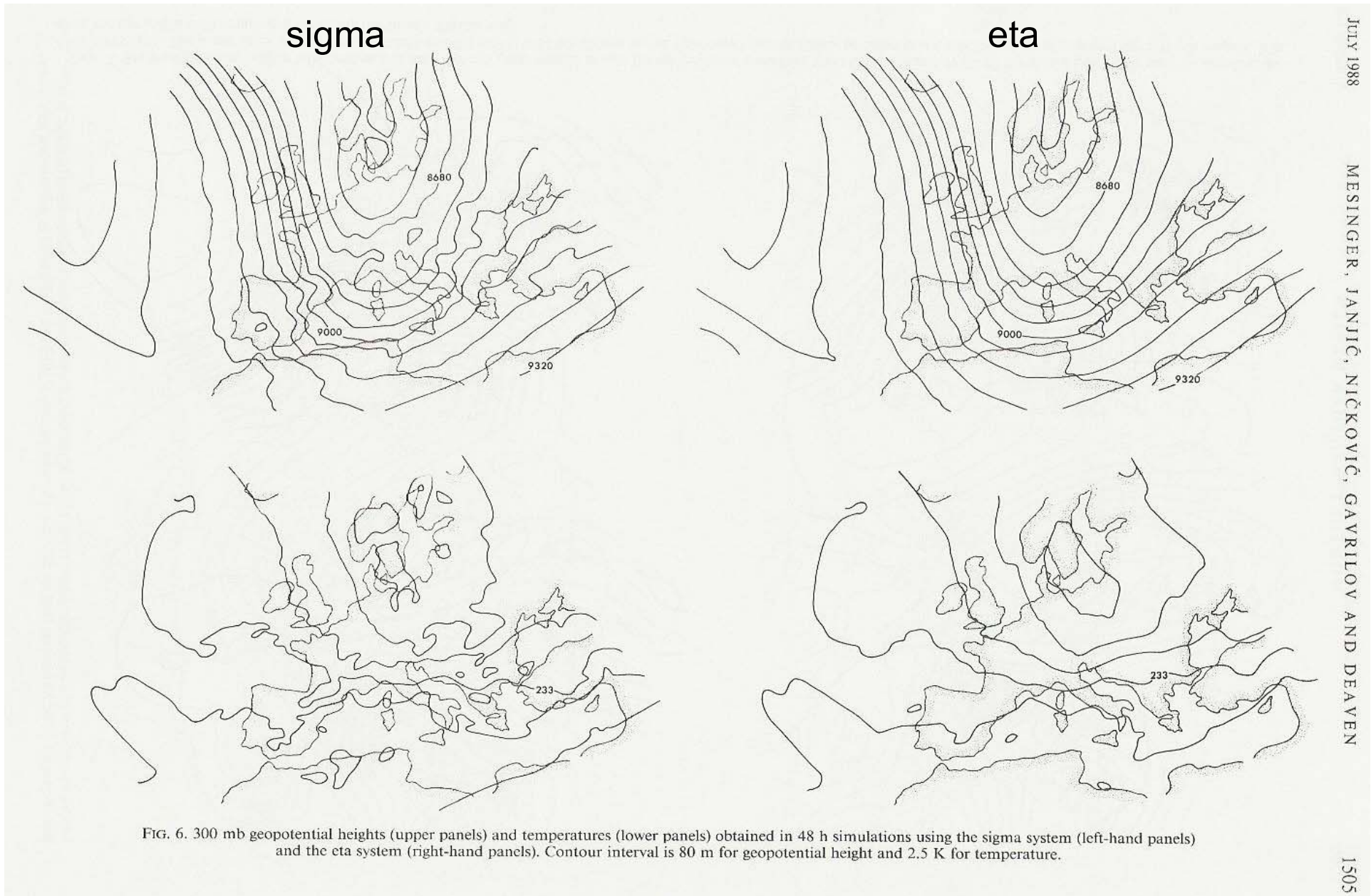
Moreover:

- Conservation of **angular momentum** (PGF), as done in Simmons, Burridge, doable;
- Conservation of **energy** in transformation between **potential and kinetic** (“ $\omega\alpha\Box\Box$ ”) doable as well

(Both, Mesinger 1984 in 2D,

energy: Dushka Zupanski, Appendix of Mesinger et al. 1988, 3D)

# The very first result, 1984, using the switch eta/ sigma:





In NCEP's "Eta Model",  
**eta** did extremely well:  
 tests during the early nineties  
 using the eta/ sigma switch,  
 on cases, and samples of  
 forecasts,  
 very favorable for the  
 eta, e.g.:

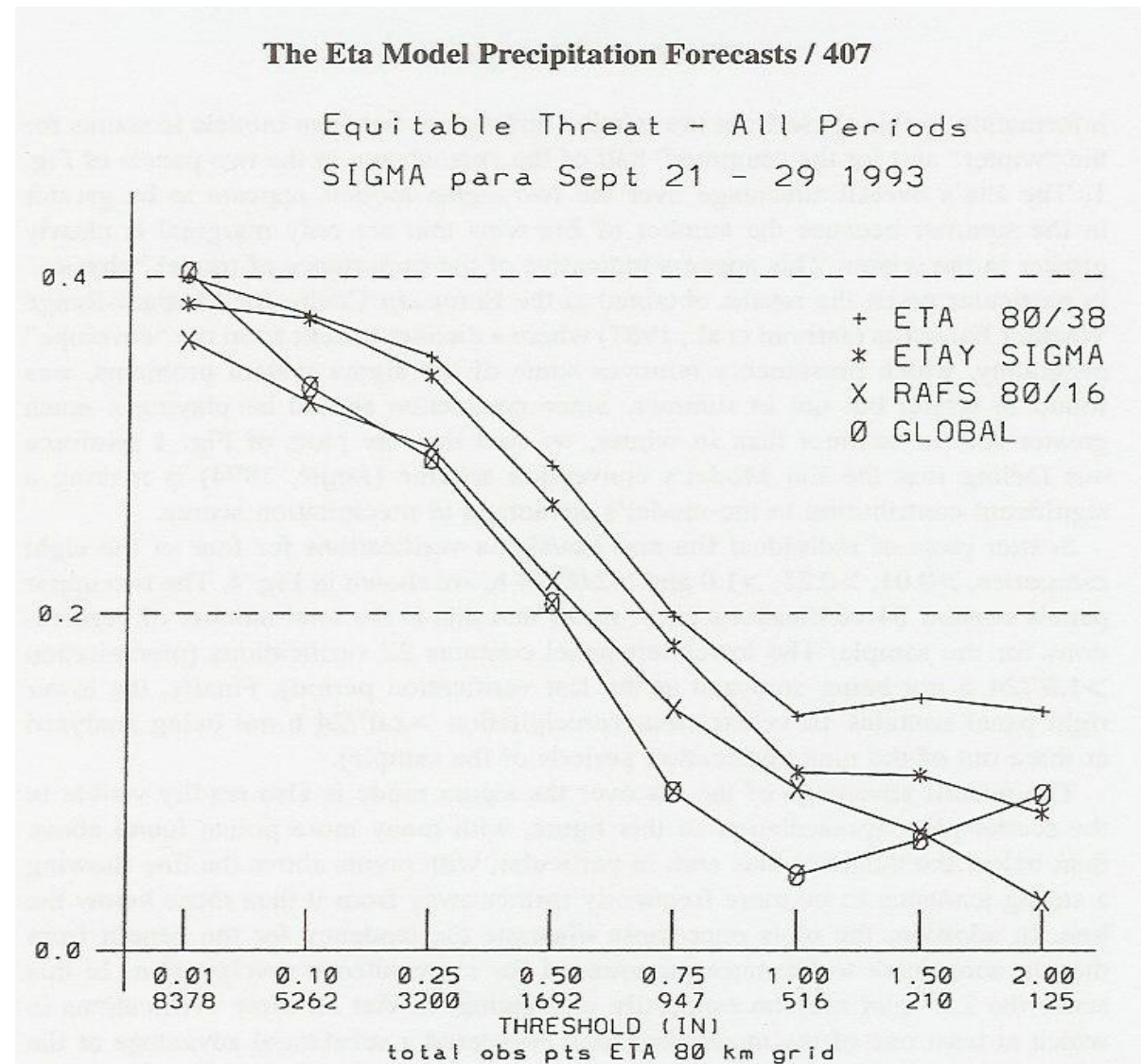


Fig. 3 Equitable precipitation threat scores for two versions of the Eta Model: Eta 80 km/38 layers ("ETA"), and the same version of the Eta Model but run using sigma coordinate ("ETAY"), and for the NGM (RAFS), and the Avn/MRF ("global") Model; for a sample of 16 forecasts verifying 1200 UTC 21 September through 1200 UTC 29 September 1993. Eight forecasts are each verified once, for 12-36 h, and the remaining eight each twice, for 00-24 and for the 24-48 h accumulated precipitation.

However,

a 10-km Eta in 1998 did a poor job on a case of so-called Wasatch downslope windstorm, while a sigma system MM5 did well;

Eta: bad press ever since:

“ill suited for high resolution prediction models”

Schär et al., *Mon. Wea. Rev.*, 2002;

Janjic, *Meteor. Atmos. Phys.*, 2003;

Steppeler et al., *Meteor. Atmos. Phys.*, 2003;

Mass et al., *Bull. Amer. Meteor. Soc.*, 2003;

Zängl, *Mon. Wea. Rev.*, 2003;

more ??

Is sigma a good way to go after all?

Let us just look at what the sigma problem is,  
and at some recent results!

PGF/resolution: in hydrostatic systems

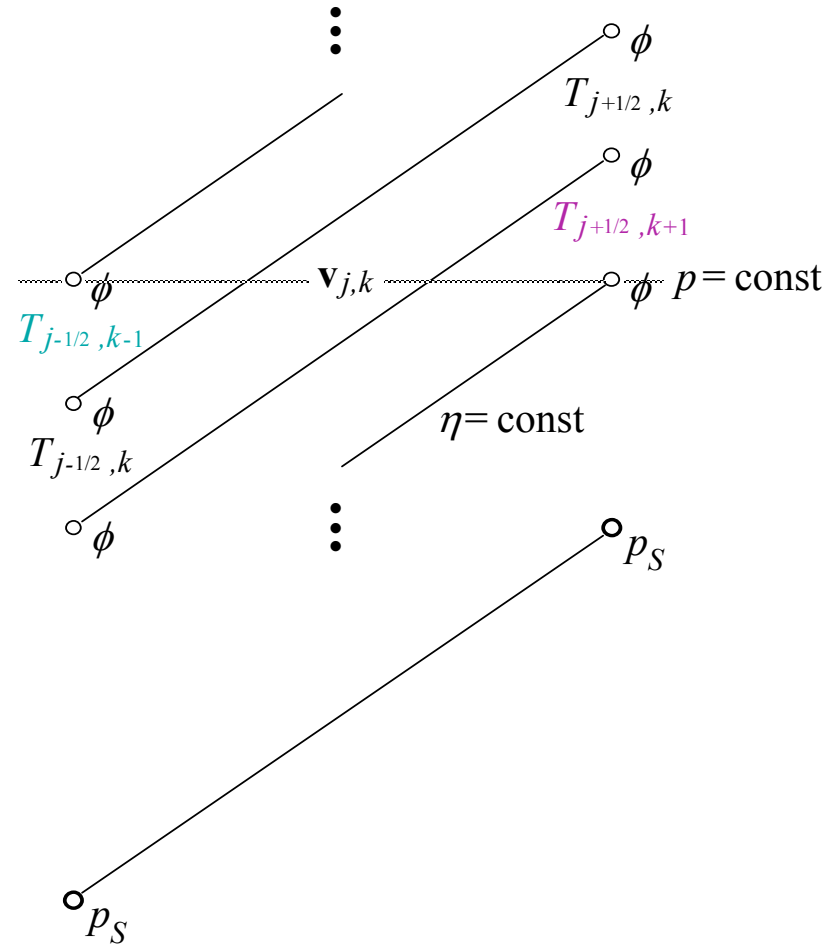
$$\phi = \phi_S - R_d \int_{p_S}^p T_v d \ln p \quad (1)$$

Thus: PGF depends only on variables from the ground up to the considered  $p=\text{const}$  surface !

From this point of view, all PGF/ hydrostatic equation sigma system schemes, three groups:

- a. Those with hydrostatic eq. analog that relates geopotentials used for PGF to temperatures both below and above the considered level;
- b. “Level schemes”: geopotentials used for PGF obtained by vertical integration of temperatures from the ground only up to the considered coordinate surface (e.g., straightforward isentropic coordinate schemes);
- c. “Layer schemes”: using layer temperatures to define geopotential increments through layers (best from the point of view of (1))

Continuous case:  
 PGF should depend on,  
 and only on,  
 variables from the ground  
 up to the  $p=\text{const}$  surface



The **best** type of scheme:

will depend on  $T_{j+1/2, k+1}$ , which *it should not*;  
 will *not* depend on  $T_{j-1/2, k-1}$ , which *it should*.

The problem **aggravates with resolution !!**

Thus, PGF problem of terrain-following coordinates:

**Not** one of “two large terms”

$$-\nabla_p \phi \rightarrow -\nabla_\sigma \phi - RT \nabla \ln p_S$$

(Easy to make them much smaller, subtract “reference” atmosphere” while having the error the same or about the same)

**Not** one of the “truncation error”;

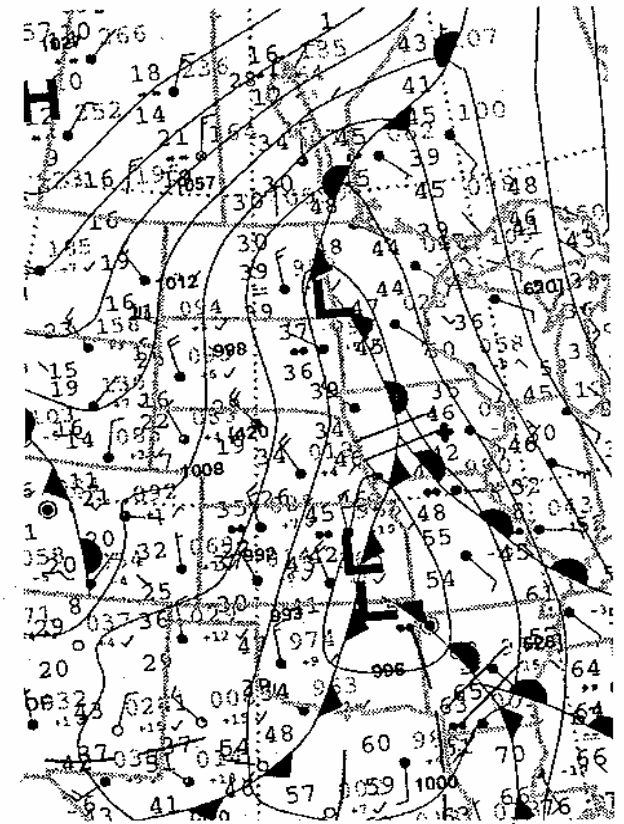
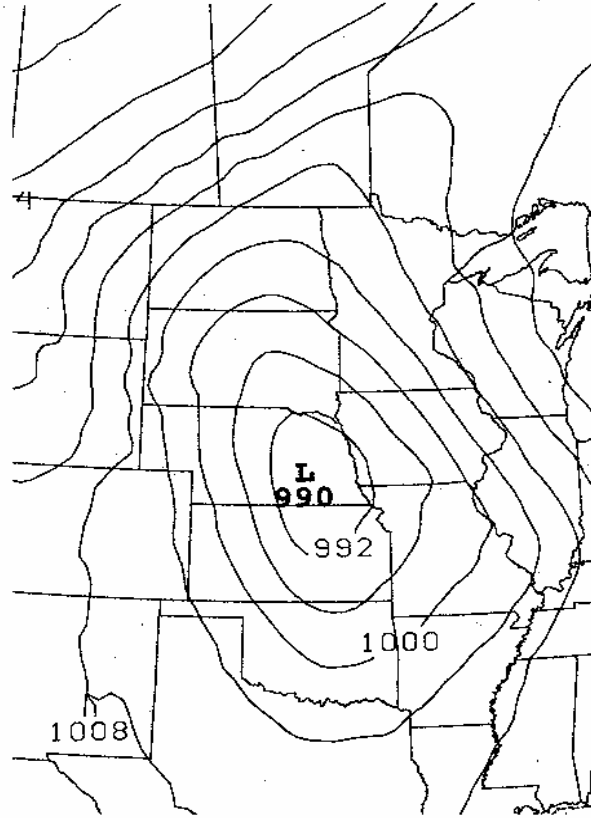
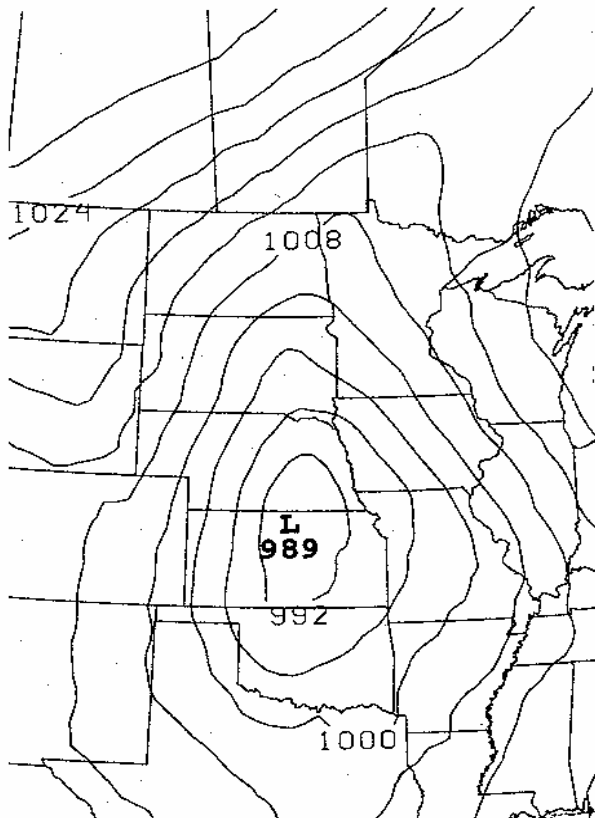
The error is likely to **increase** with increased Taylor-series accuracy;

It is likely to **increase** with increased resolution



# Any signs of an impact?

One experiment: Eta (left), 22 km, switched to use sigma (center), 48 h position error of a major low increased from 215 to 315 km



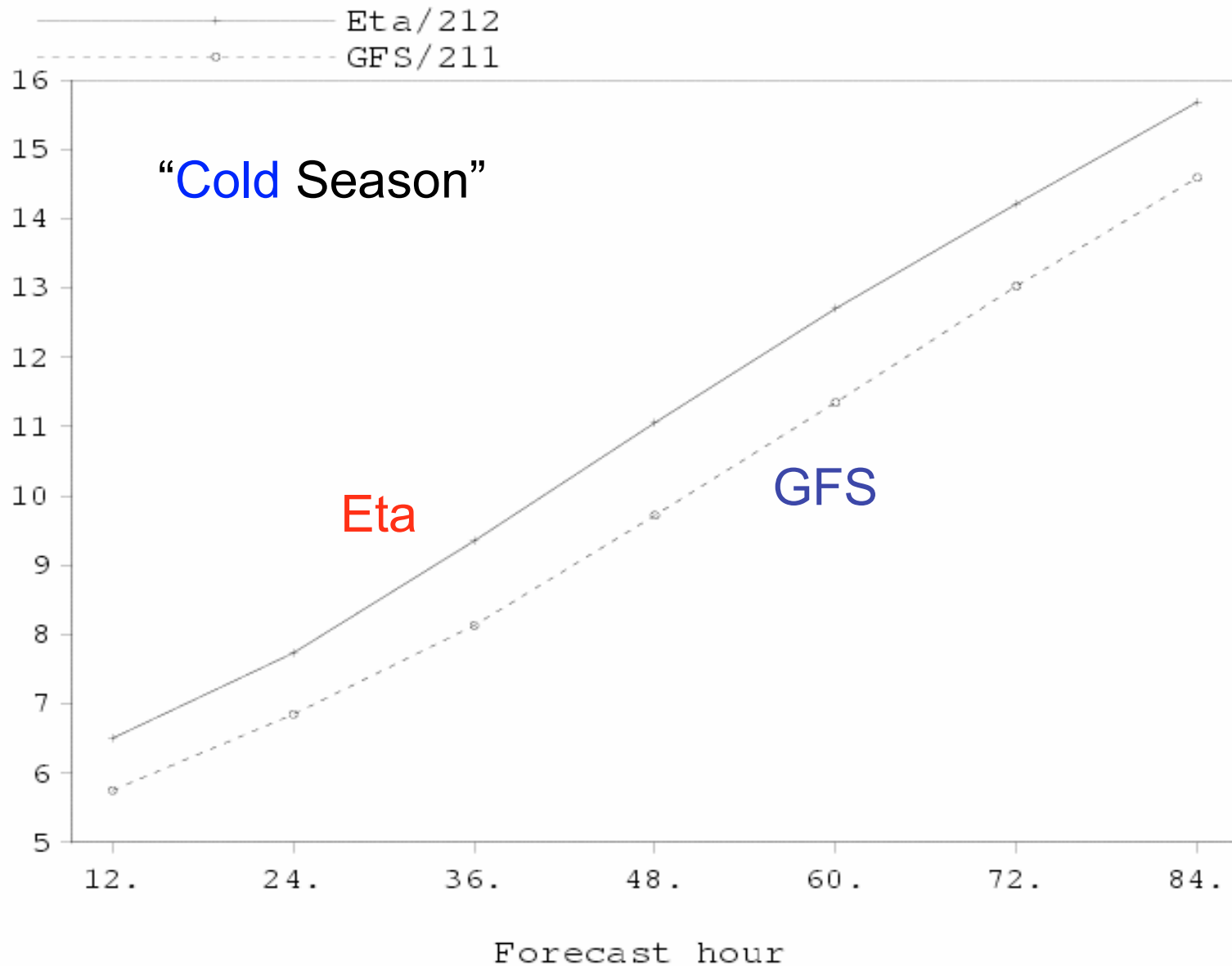
# Recent performance results

Three-model precipitation scores,  
on NMM ConUS domains ("East" ,..., "West"),  
available since Sep. 2002

- Operational **Eta**: 12 km, driven by **6 h old GFS** forecasts;
- **NMM**: “Nonhydrostatic Mesoscale Model” nonhydrostatic, **8 km**, most other features same or similar to Eta, but switched back to **sigma**, driven by the Eta;
- **GFS** (Global Forecasting System) as of the end of Oct. 2002 T254 (55 km) resolution, **sigma**

## 6 h old GFS LBCs ?

250 mb wind rms fits to raobs, m/s, Nov 2003–Apr 2004



Back to the three models:

NOAA-wide e-mail of 19 July 2002

announcing the **operational implementation of the NMM**,  
referring to the choice of the vertical coordinate:

"This choice will avoid the problems encountered **at high resolution** (10 km or finer)

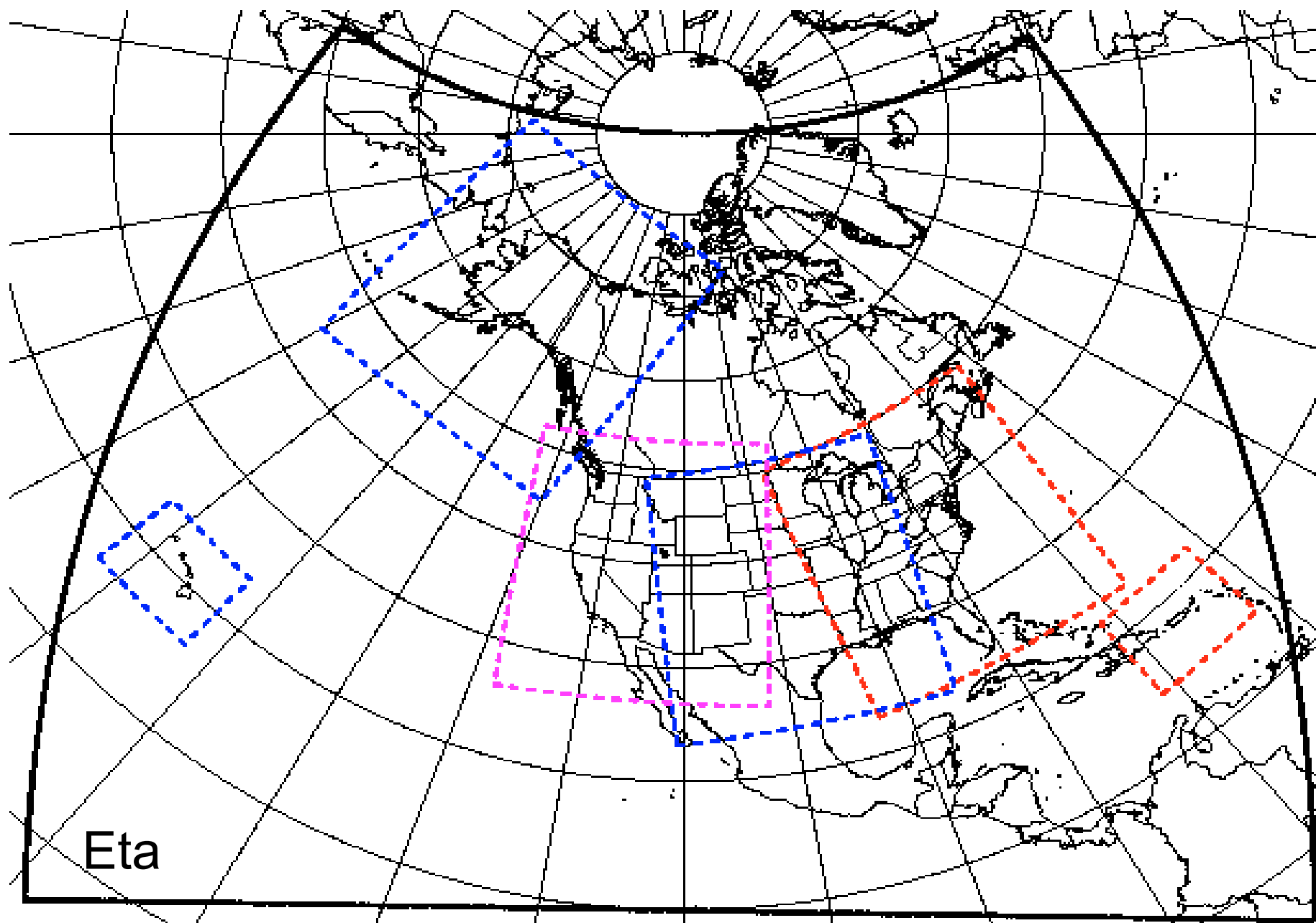
**with the step-mountain coordinate**

**with strong downslope winds**

**and will improve**

**placement of precipitation in mountainous terrain".**

Did this indeed happen?



Eta

Five "high-res windows"





A very large number of performance measures !

However: are any of them "equitable",  
in the sense of Gandin and Murphy (MWR 1992)?  
(No reward for over- or underforecasting the event !)

Equitable threat score:

equitable with respect to random forecasting;  
- not in the sense of Gandin and Murphy :-)

Marzban (WF 1998):

looked at 14 measures and found none equitable !

Baldwin and Kain (WF 2005, in preparation):

looked at 6 performance measures

Of these, two:

Odds ratio skill score;

Heidke skill score;

[1926; originally proposed by Doolittle (1988)] !

symmetric with respect to two types of correct forecast;

Equitable threat score, and Threat score

emphasize correct forecasts of rain (yes, yes)

more than correct forecasts of no rain (no, no)

But neither of them is equitable :-)

J12.6

17th Prob. Stat. Atmos. Sci.; 20th WAF/16th NWP

# BIAS NORMALIZED PRECIPITATION SCORES

Fedor Mesinger<sup>1</sup> and Keith Brill<sup>2</sup>

<sup>1</sup>NCEP/EMC and UCAR, Camp Springs, MD

<sup>2</sup>NCEP/HPC, Camp Springs, MD



# Motivation

- **Equitable threat scores**: commonly used to assess the performance of model precipitation forecasts. Purpose (hoped for): assess **placement of precipitation**
- However: sensitive to bias
- E.g.: Common wisdom has it that bias somewhat greater than 1 tends to benefit equitable threat score.
- Thus: can we “normalize” the equitable threat score, to remove the impact of bias? (Also, standard threat score. Acknowledgment: Joe Schaefer).



## Two Methods of Bias Normalization

1.  $dH/dF$  method: Assume the incremental change in hits per incremental change in bias is proportional to the “unhit” area,  $O-H$
2. Odds Ratio method: Assume that the odds ratio remains unchanged as the hit and forecast areas are changed to satisfy the condition  $\text{bias} = 1$

## dH/dF Method

Assumption:  $\frac{dH}{dF} = a(O - H), \quad a = \text{const.}$  (1)

Solve (1) to get  $H(F) = be^{-aF} + O, \quad b = \text{const.}$  (2)

Since  $H=0$  for  $F=0$ :  $b = -O \rightarrow H(F) = O(1 - e^{-aF})$  (3)

Solve for a to get  $a = -\frac{1}{F} \ln\left(1 - \frac{H(F)}{O}\right)$  (4)

If  $H_b$  and  $F_b$  are known values of  $H$  and  $F$  with  $O$  given,  $a = -\frac{1}{F_b} \ln\left(1 - \frac{H_b}{O}\right)$  (5)

Insert (5) into (3) to get  $H(F) = O\left[1 - \left(\frac{O - H_b}{O}\right)^{\frac{F}{F_b}}\right]$  (6)

Bias = 1 implies  $F = O$ , and adjusted  $H$ ,  $H_a$  is given by  $H_a = O\left[1 - \left(\frac{O - H}{O}\right)^{\frac{O}{F}}\right]$  (7)

Note that the subscript  $b$  has been dropped from (7) as the distinction is no longer needed as it is in (6).

What has happened since ?

“Odds ratio method”:

declared not to have a valid basis

(Manuscript on only one method about to be submitted/  
in internal review)

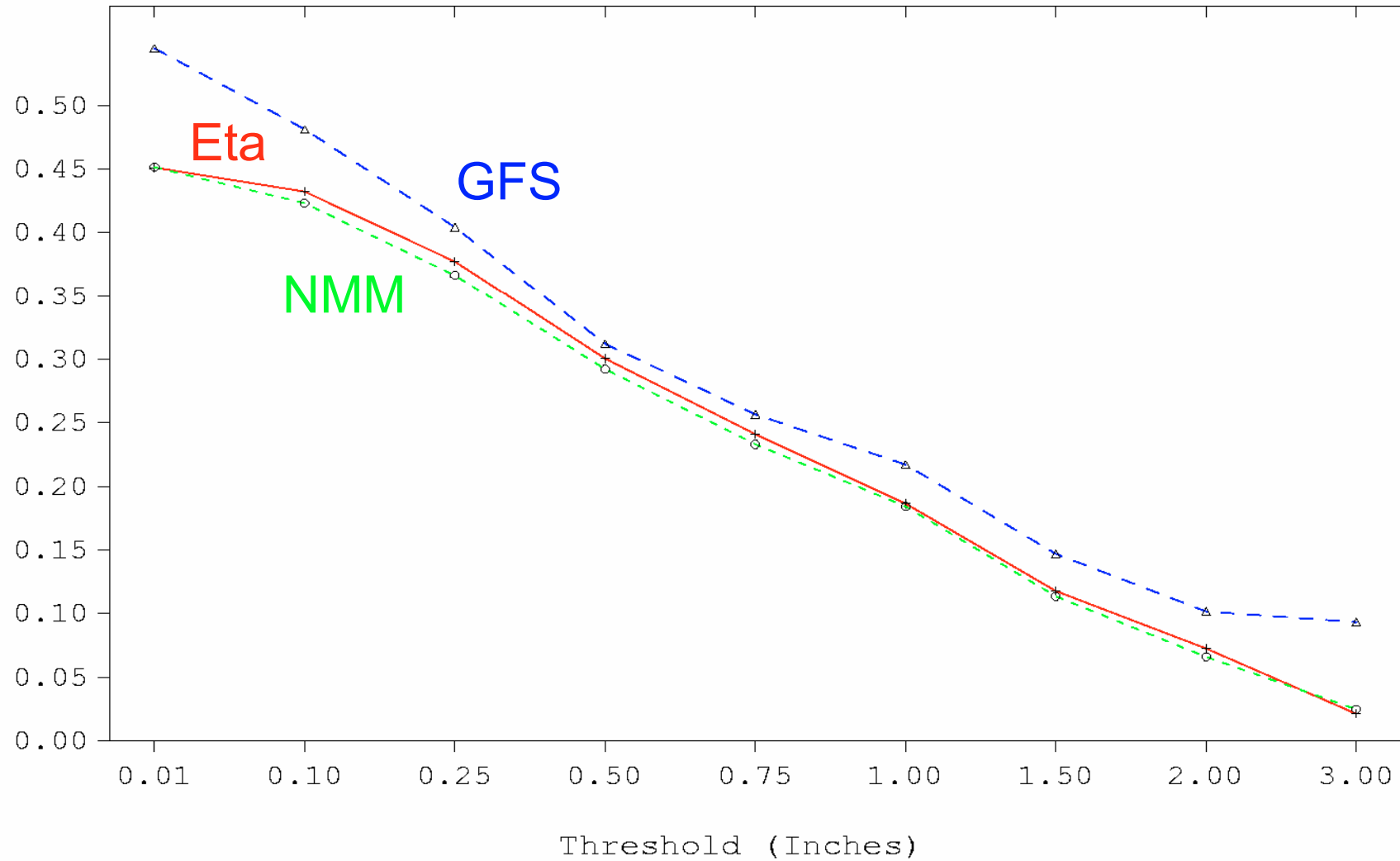
# dH/dF BN Eq Threat, Eastern Nest, Sep 2002-Aug 2003

## Bias normalized eq. threats

—+— Eta  
- - - o - - - NMM  
- - - ^ - - - GFS

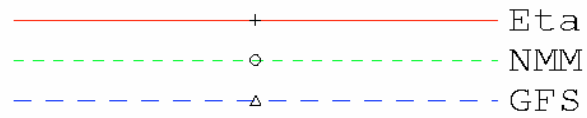
Observation counts:

3498476 2023725 1255316 666577 373246 215337 79835 32875 8260



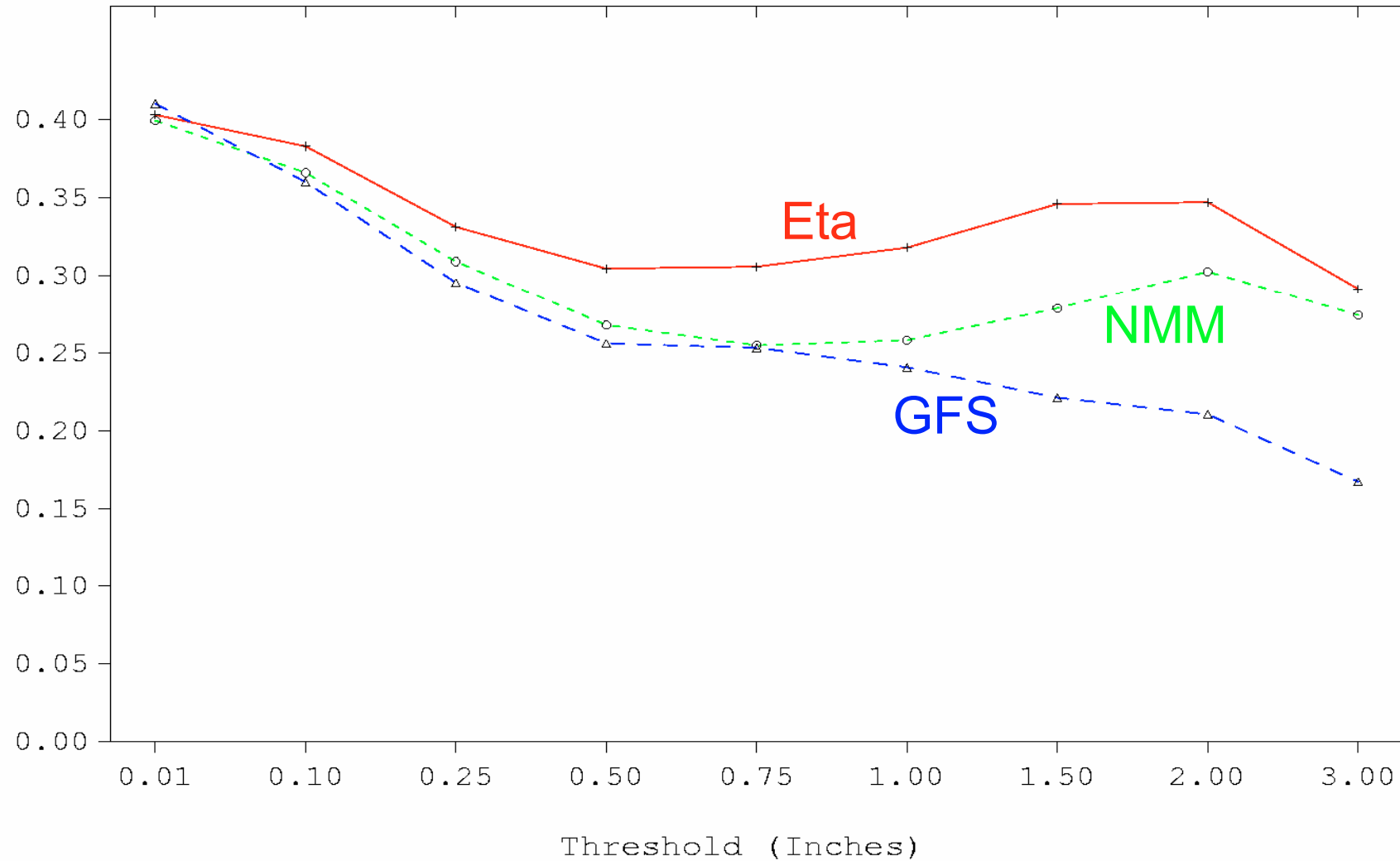
# dH/dF BN Eq Threat, Western Nest, Sep 2002–Aug 2003

(Five very heavy el Niño precip events)



Observation counts:

2958107 1180387 532073 205652 100148 55514 21158 10132 3134

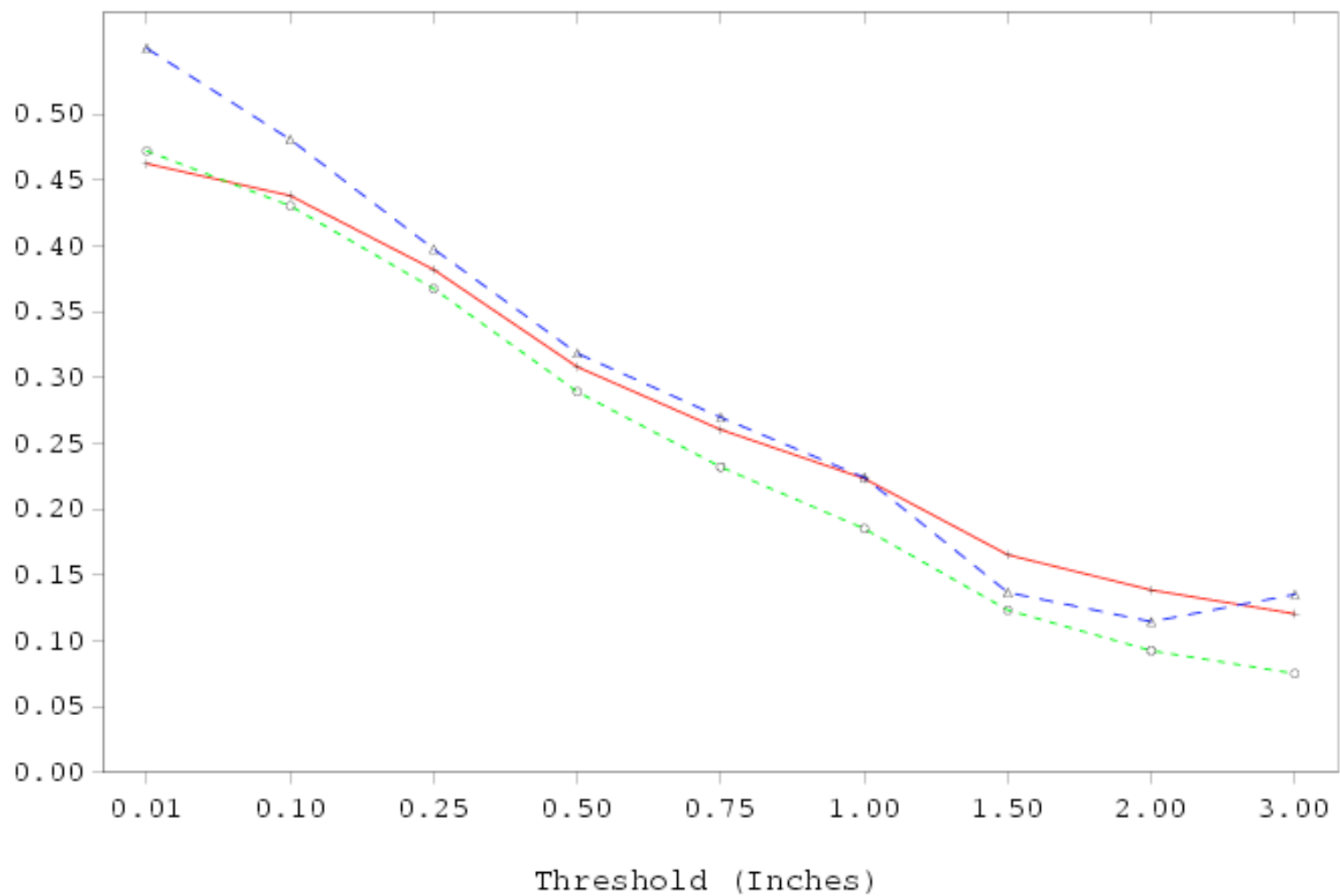


# Bias Normalized Eq. Threat, Eastern Nest, Feb 04-Jan 05

—+— Eta  
- - - - -◇- - - - - NMM  
- - - - -△- - - - - GFS

Observation counts:

3449866 2017237 1265655 676161 386399 232222 88759 38209 11784

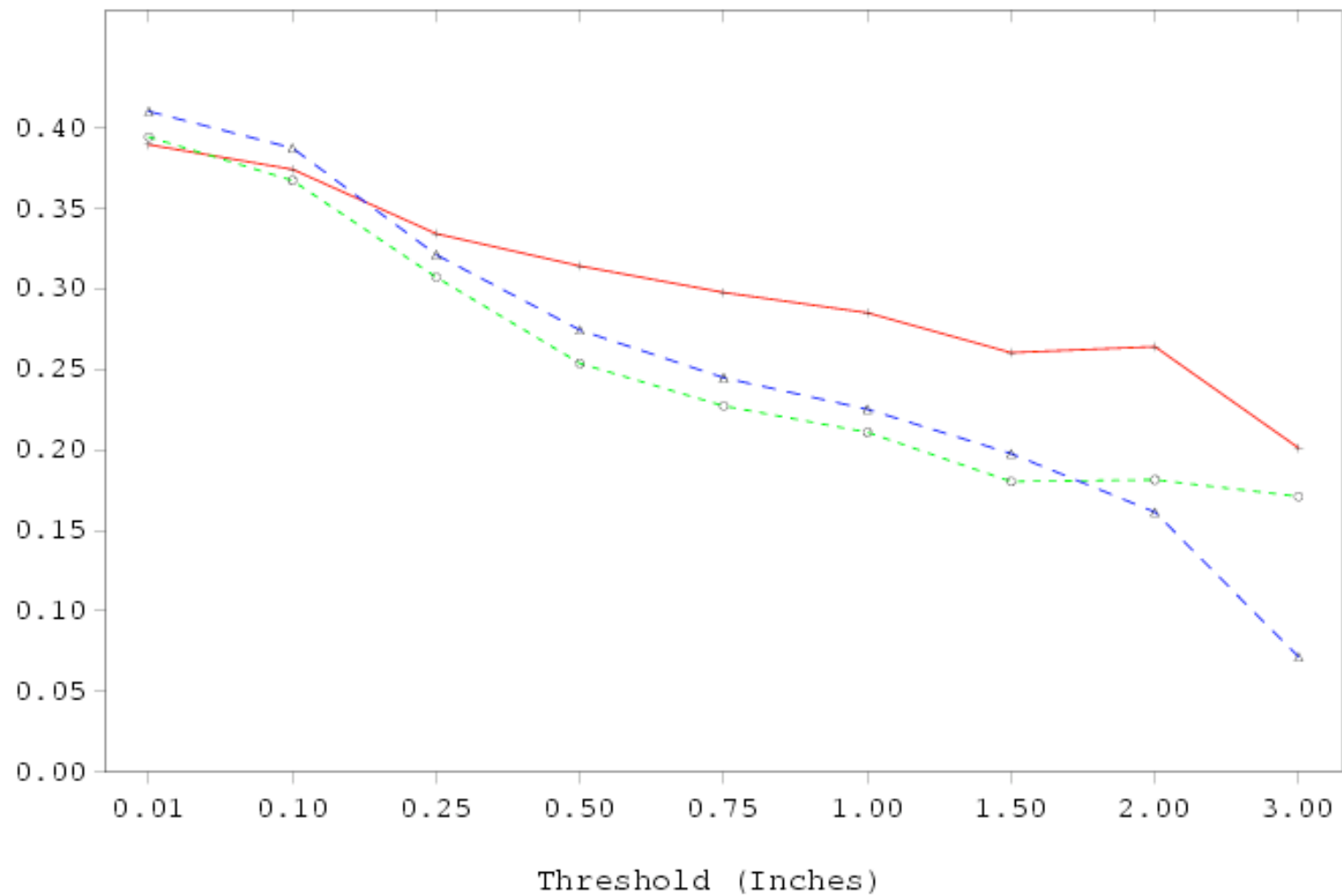


Bias Normalized Eq. Threat, Western Nest, Feb 04-Jan 05

—+— Eta  
- - - - -◇- - - - - NMM  
- - - - -△- - - - - GFS

Observation counts:

3107013 1273120 589141 237658 116838 65419 23617 10361 2516



## Eta vs NMM:

East, no major topography: 12-km Eta about the same as the 8-km NMM, even a tiny bit better;

West, complex topography: 12-km Eta **much better** than the 8-km (sigma system) NMM !!

## GFS vs Eta:

East: **GFS** (when corrected for bias) uniformly better;

West: **Eta** *much* better (overcoming handicaps of the 6 h lateral boundary error, and less successful data assimilation) !



Thus, summary of performance results:

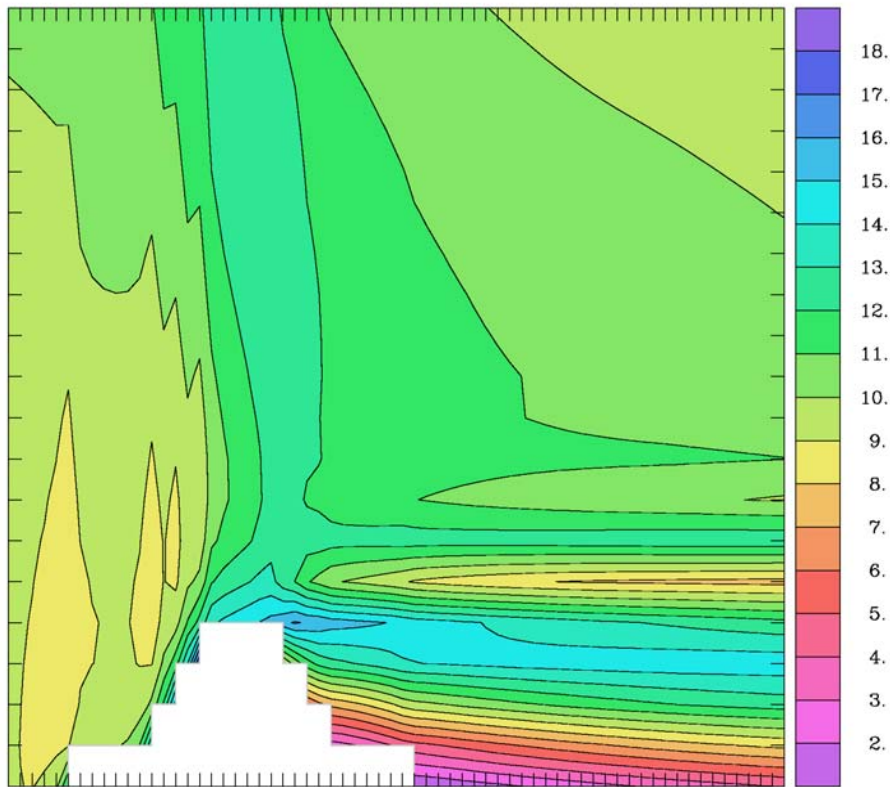
Very strong indication that **the eta works extremely well !**

**But what about its downslope windstorm problem ?**

# The Eta Problem:

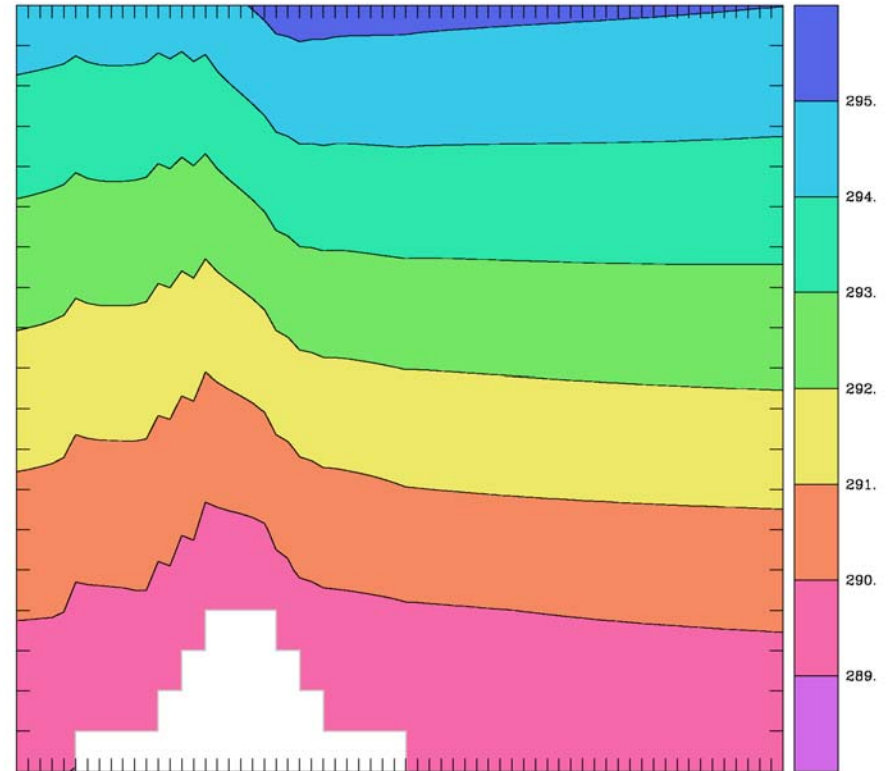
Flow separation on the lee side (à la Gallus and Klemp 2000)

Horizontal velocity (m/s) at t = 6.00 h



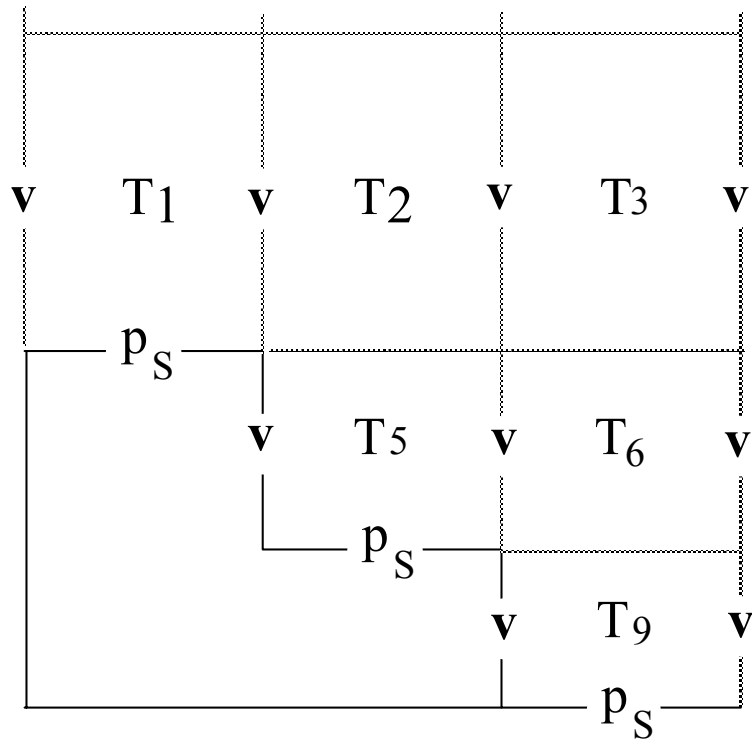
CONTOUR FROM 2 TO 18 BY 1

Potential temperature (K) at t = 6.00 h



CONTOUR FROM 289 TO 295 BY 1

# Suggested explanation



Flow from left: from the box 1 the flow enters box 2 to the right of it. When conditioned to move downward, it will move downward via the interface between boxes 2 and 5. Some of the air that entered box 2 will continue to move horizontally into box 3.

**Missing: the flow directly from box 1 into 5 !** (It would have existed had the discretization accounted for the terrain slope !) As a result: **some of the air which should have moved slantwise from box 1 directly into 5 gets deflected horizontally into box 3.**

# Refined (sloping steps) eta

(Mesinger and Jović)

Discretization accounting for slopes. Continuity equation (at  $\eta$  points not zero):

$$\frac{\partial p_s}{\partial t} = - \int_0^{\eta_s} \nabla \cdot \left( \mathbf{v} \frac{\partial p}{\partial \eta} \right) d\eta - \left( \dot{\eta} \frac{\partial p}{\partial \eta} \right)_s \quad (3)$$

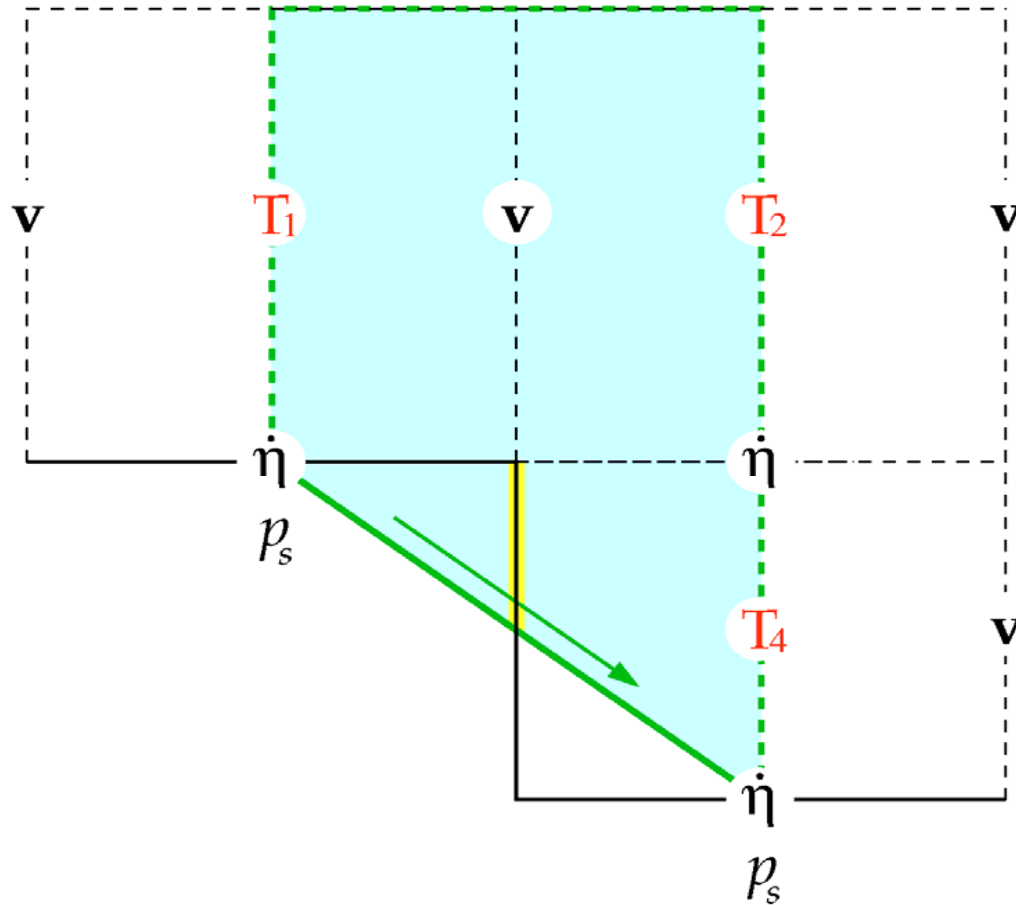
Approach:

Define **slopes at v points**, based on four surrounding  $h$  points. Slopes discrete, valid on halves of the sides of  $h$  points, and halves of the eta layers. **Slantwise transports** calculated within the 1st term on the right of (3), and in other equations as appropriate.

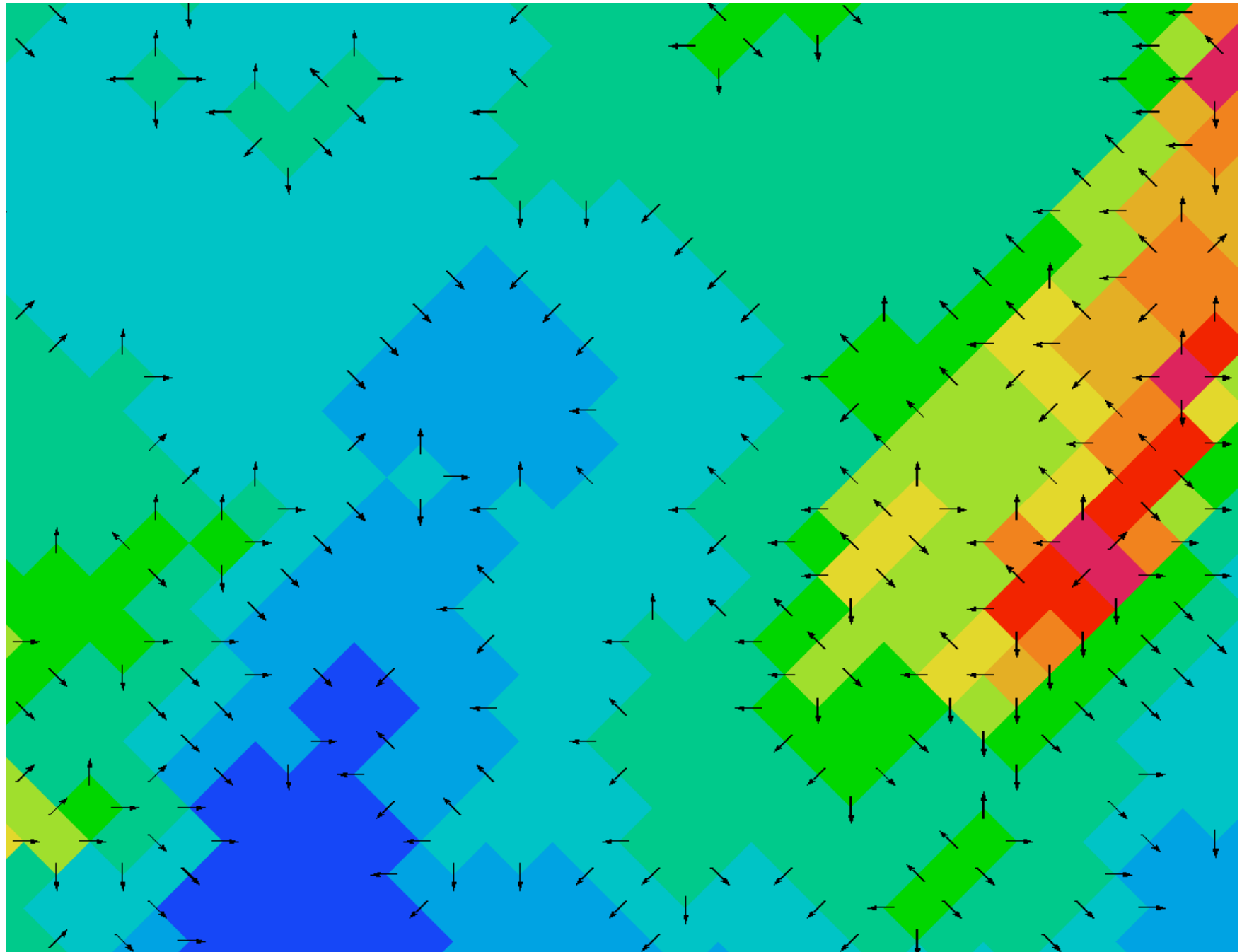
Other possibilities available. However: keep the eta feature of having **cells in horizontal of about equal volume** (difference compared to Adcroft et al. 1997, shaved cells) ! This makes Arakawa-type conservation schemes, used in the Eta, very nearly finite-volume schemes. Also, robust in the CFL sense.

## The sloping steps, vertical grid

The central  $\mathbf{v}$  box exchanges momentum, on its right side, with  $\mathbf{v}$  boxes of **two** layers:

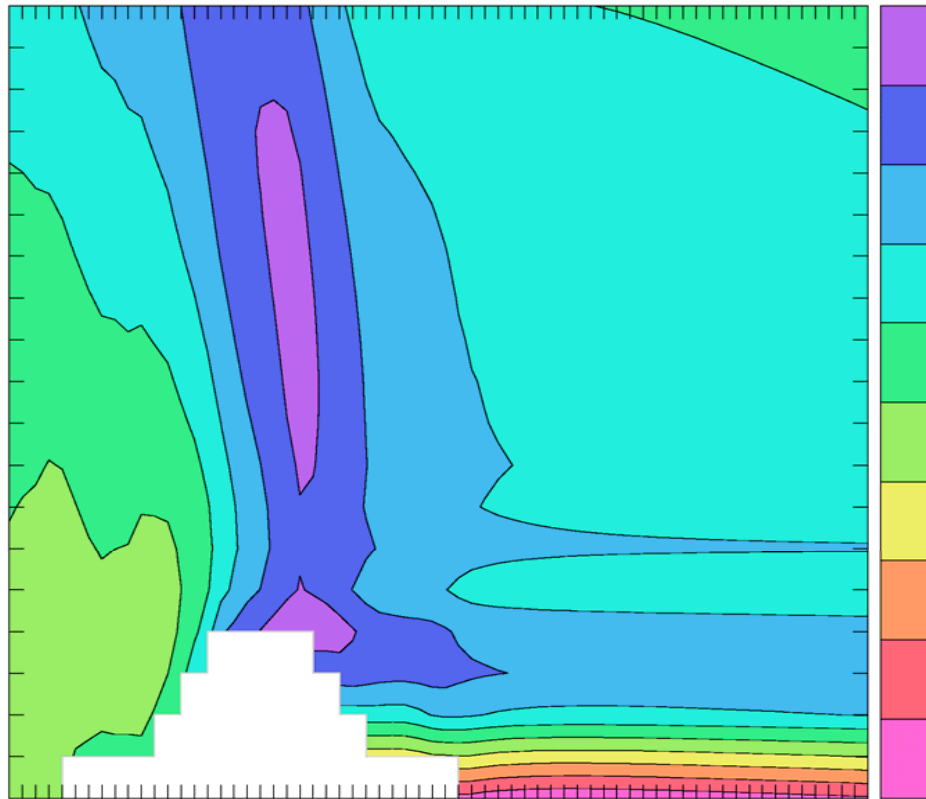






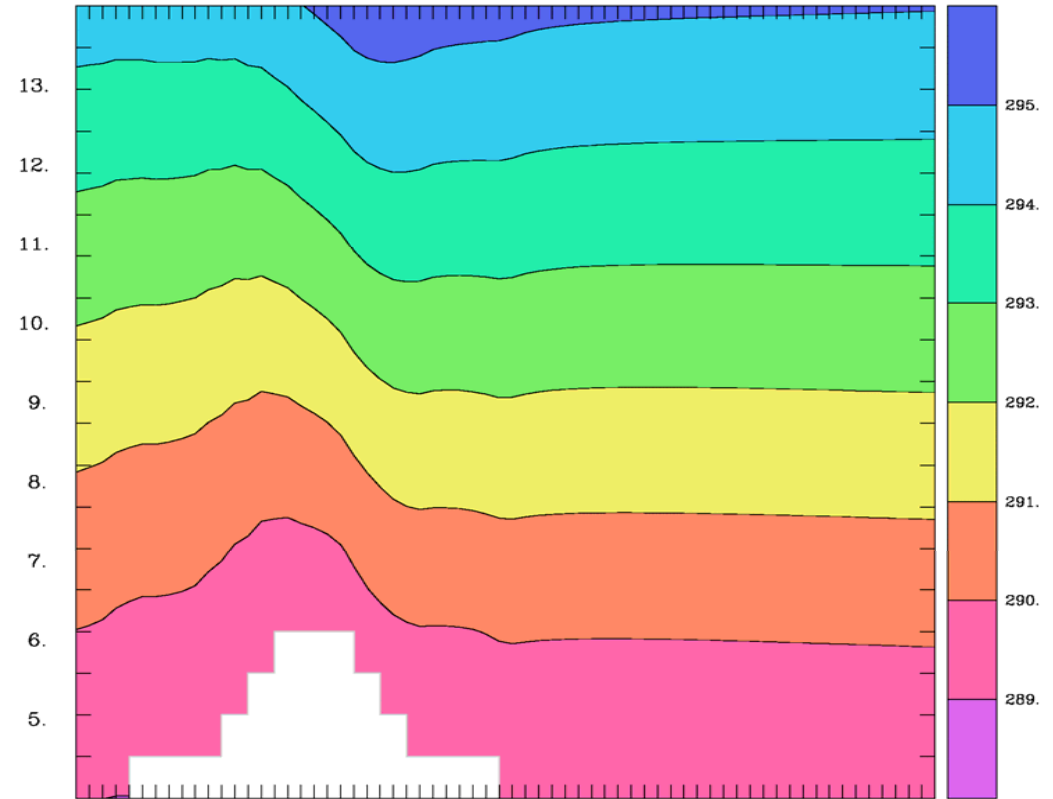
# Slantwise advection of mass, momentum, and temperature, and “ $\omega\alpha$ ”:

Horizontal velocity (m/s) at t = 6.00 h



CONTOUR FROM 5 TO 13 BY 1

Potential temperature (K) at t = 6.00 h



CONTOUR FROM 289 TO 295 BY 1

Velocity at the ground immediately behind the mountain increased from between 1 and 2, to between 4 and 5 m/s. “lee-slope separation” removed.  
Zig-zag features in isentropes at the upslope side removed.



# Conclusions (re eta)

12-km **Eta**: excellent QPF performance over complex topography ! Better than the sigma system 8-km NMM, and better than the GFS;

The Eta **downslope windstorm problem**: correctible/corrected, while keeping favorable Eta features:

- quasi horizontal coordinates (PGF !);

- very nearly finite-volume**;

- robustness in the CFL sense.

## Some of the references made

Baldwin, M. E., and J. S. Kain, 2005: Sensitivity of several performance measures to displacement error, bias, and event frequency. *Wea. Forecasting* (in preparation).

Simmons, A. J., and D. M. Burridge, 1981: An energy and angular-momentum conserving vertical finite-difference scheme and hybrid vertical coordinates. *Mon. Wea. Rev.*, **109**, 758-766.