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Spring Colloquium on
'Regional Weather Predictability and Modeling'
April 11 - 22, 2005

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2) Conference on **Current Efforts Toward Advancing the Skill of Regional Weather Prediction. Challenges and Outlook,** *April 20 - 22*

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Regional NWP Modeling and Predictability, Introduction.
The Eta Model Numerical Design
Lecture 1, part 1: Introduction

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Regional NWP Modeling and Predictability, Introduction. The Eta Model Numerical Design

Lecture 1, part 1: Introduction

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Spring Colloquium
"Regional Weather Predictability and Modeling"
Abdus Salam ICTP, Miramare, Trieste, 11-22 April 2005

A few words on history:

How did the idea of weather prediction,
using equations of motion, and as an initial value problem,
started?

Equations of motion well understood already about 1800.
Leonhard Euler: 1707-1783;

"Predictability": Can this be done, in principle?

Weather prediction via the solution of fundamental atmospheric equations?

Vilhelm Bjerknes (1862-1951)

V. Bjerknes, 1904:

If it is true, as every scientist believes,
that subsequent atmospheric states develop
from the preceding ones according to physical law,
then it is apparent that the necessary and sufficient conditions
for the rational solution of forecasting problems
are the following:

1. A sufficiently accurate knowledge
of the state of the atmosphere at the initial time.

2. A sufficiently accurate knowledge
of the laws according to which one state of the atmosphere
develops from another.

(Translation: Yale Mintz)

Bjerknes embarked, most systematically, on work on his point 1.
However he never seemed to have doubts re the feasibility,
in principle, of an accurate prediction (Bjerknes 1919):

“if the initial condition … and if the equations …
with sufficient accuracy, then the state of the atmosphere
could be determined completely by some super-mathematician
at any subsequent time”

At the same time, Max Margules (1856-1920)

(student of L. Boltzmann
and J. Stefan)
understood the/ a difficulty,

Margules (1904):

wind measurements are not nearly as accurate as needed to calculate pressure changes using the continuity equation!

(Reference:
Peter Lynch, 2004, 50 years of NWP Symposium,
Abstracts book)

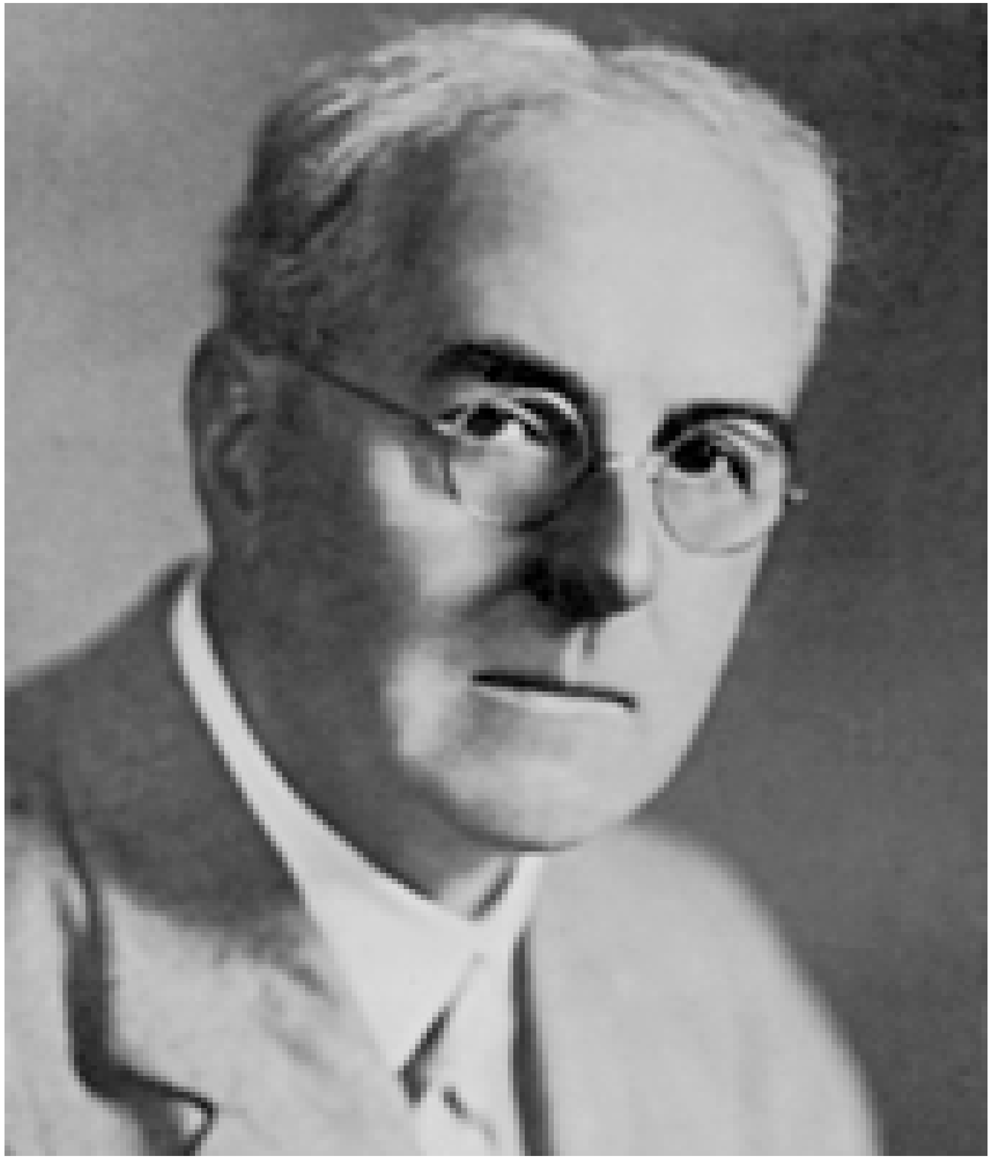
A little later, during World War One (published 1922)
Lewis Fry Richardson (1883-1953)

went ahead and performed a numerical integration of a full set of governing equations (well, did one 6 h time step)

A most unreasonable result

Yet: a charming and visionary book!

"… errors increase with the number of steps"



Milutin Milanković (1979-1958), in 1913, having accepted a professorship in applied mathematics in Belgrade, was looking for a field in which he could use his mathematical talents …

“I enjoyed working methodically, without haste, and this was then for me possible. … I was searching for the main orientation of all of my future work. …

… On an old, toiled on since long ago soil, … it is hard to arrive at a reach harvest. … Already Varićak [his high school math teacher] was telling me that in the Kingdom of Science there are unpopulated and uncultivated lands beyond or between densely populated science settlements. …

… a schematic … three concentric circles. Mathematics … symbol of a Sun, in the center. … Shining upon all of the exact sciences inside the nearest circular region … But barely entering the descriptive natural sciences of the one beyond. … I decided to have a look into these borderline sciences and started with Meteorology. …

Asked Vujević, any papers with considerable use of mathematics? He gave me several of those.”

One: distribution of solar energy on the Earth’s surface

The initial equation: erroneous!

Weather prediction?

“I was struck looking at the difficulty of the task.
The variety of Earth climates bewildered me, clouds of the sky would make be frown, every rain would make me depressed, and when a gust of wind would come by,
in particular the Belgrade's koshava, I would ask myself
"Who could capture all the whims of Eole into mathematical formulas?”

“... all that evolves in such a complicated manner that, at least for the time being, it seems impossible to subject these phenomena to a mathematical analysis to a degree which would enable one to foresee their succession”.

However, Milanković found comfort in the idea that

“... every region on Earth ... has its average climate which … has not changed much over the centuries. This ... can be the subject of mathematical analysis.”

"at least *for the time being*": most appropriate!

Many milestones. A few of the major ones:

- Courant, Friedrichs, Lewy (1928): time step requirement for numerical stability;
- Understanding (Carl-Gustaf Rossby, Jule Charney, Arnt Eliassen, ..): atmospheric equations contain fast types of motion, which in real atmosphere are never present to a significant degree

  One way around the problem: use "filtered" equations;
- First multi-purpose, programmable, electronic computer;
- First successful NWP effort:
  Charney, Fjortoft, von Neumann (1950);
- First operational numerical forecast: 1954;
- . . .

However, "at least": How predictable *is* the weather?

Earliest work on atmospheric "predictability": Phil Thompson (1957)

Points out that an accurate description of the initial state is simply impossible. Consequences?

"… two solutions … initial states that differ by a random error field …"

Tools of homogenous turbulence,
"predictability time limit":
a bit more than a week

Breakthrough towards full understanding:
Ed Lorenz (1963)

"chaos theory"

Small scale errors
will grow also !

From:
“The Essence of Chaos” (Lorenz 1993):

“Chaos”

1. The property that characterizes a **dynamical system** in which most orbits exhibit **sensitive dependence**; full chaos

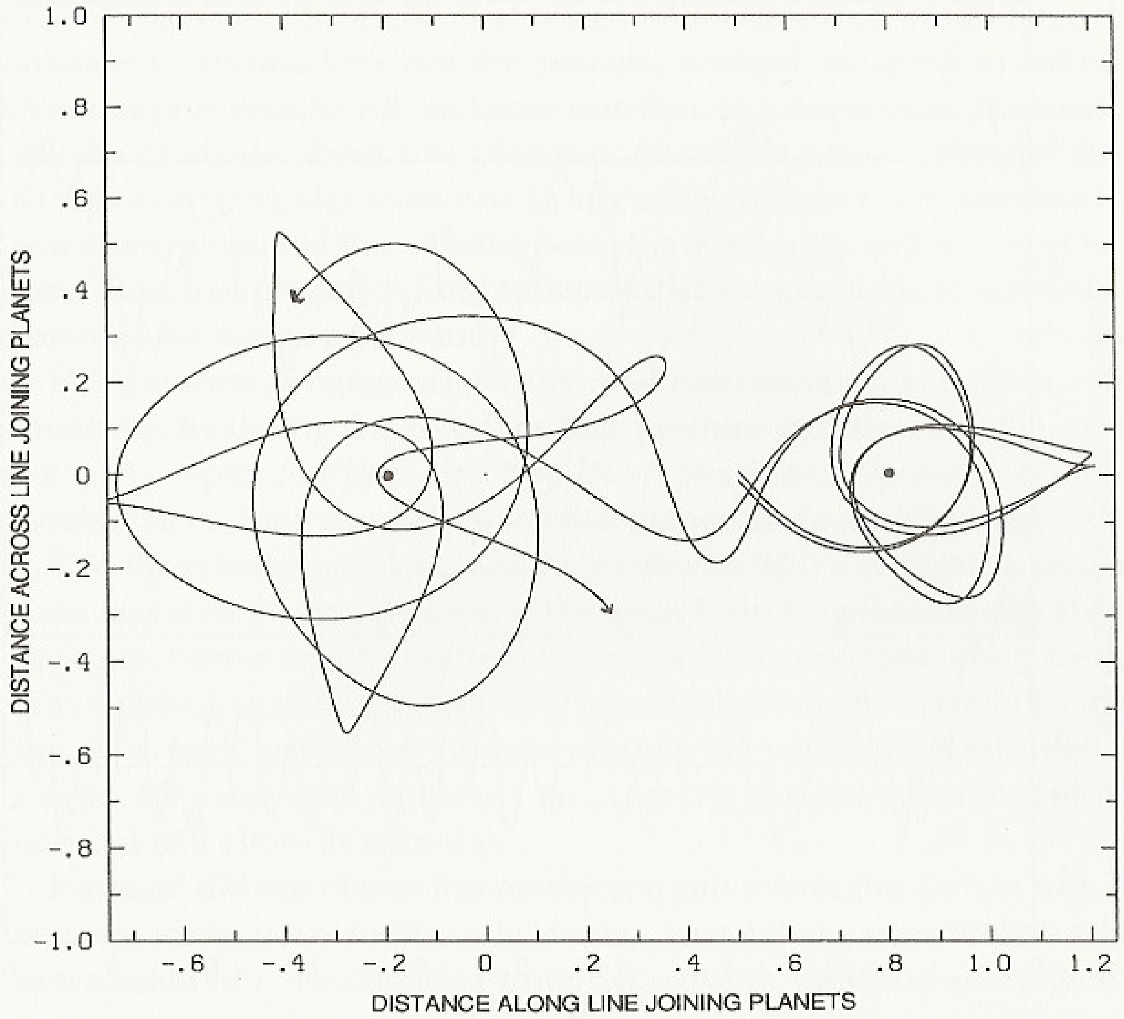


Figure 35. Two possible orbits of a satellite, starting with nearly identical conditions, as given by numerical solutions of Hill’s reduced equations, extending for two years. The frame of reference from which the satellite is viewed rotates so as to make the planets, which are located 0.2 units to the left and 0.8 units to the right of the origin, and which are indicated by the dots, appear stationary.

Regional/ limited area modeling:

Purpose: obtain a better result, due to the ability to use higher resolution ("value added")

Some history:

The first operational implementation of a LAM using forecast boundary condition: apparently at SMHI (Bengtsson and Moen 1971)

After some efforts in looking at available records, Bengtsson and Moen became "convinced that [the system] actually was put into operation in 1969" (Bengtsson, personal communication; see Mesinger 2001)

(3-level quasi-geostrophic model, used at two resolutions)

Forecast BC for the "rectangle" version of the UK Met Office model, "Bushby-Timpson 10 level primitive equation model", August 1972;

U.S. Nat'l Met. Center (NMC): 1973, "LFM" model;

JMA, Météorologie National, ...

Yugoslavia: January 1978, manually prepared BCs, off DWD fcst charts

However:

What kind of “value added” can we hope to achieve?

Is it just more detail (e.g., topography, land surface, …)?

Or, one may be able to simulate additional, more demanding, physical processes?

More detail: is one only hoping to improve “small scales” (“downscaling”)?

What about “upscaling”?

Two meanings however:

- Improve also largest scales a nested model can accommodate;
- Have nested model impact the “driver model” (so-called “two way nesting”)

But also, **other reasons** to run a nested model:

- Have data in your system for various applications;
- Use the model for research/ experiments

NWP/ numerical methods today ?

A “World of Models”; groups of people form around a model;

~20-30, maybe ~40, highly respected “global”, limited area, and also “variable resolution” models;

The textbook by Pielke (2002): Appendix B, an extensive description of 10 “mesoscale” models, with a list/ references of 13 more!

One of the 10, “the Eta Model”,

- strong emphasis on the numerical design aspect of the problem

Approaches used

Some years ago a standard classification:

finite-difference, spectral, finite element;

Now perhaps better:

Finite-difference Eulerian, Lagrangian; finite-volume; regional spectral; movable nested grids; variable resolution; unstructured grids (e.g., OMEGA)

Some regional spectral: have larger scales from the driver model !

(Some regional climate models (RCMs): spectral nudging !)

The Eta Model

Early history: first (ancestor) code written in 1973

Aim: use "the Arakawa approach"

- Maintenance of chosen integral properties;
- Avoidance of computational modes;
- . . .

The very first code: some of each

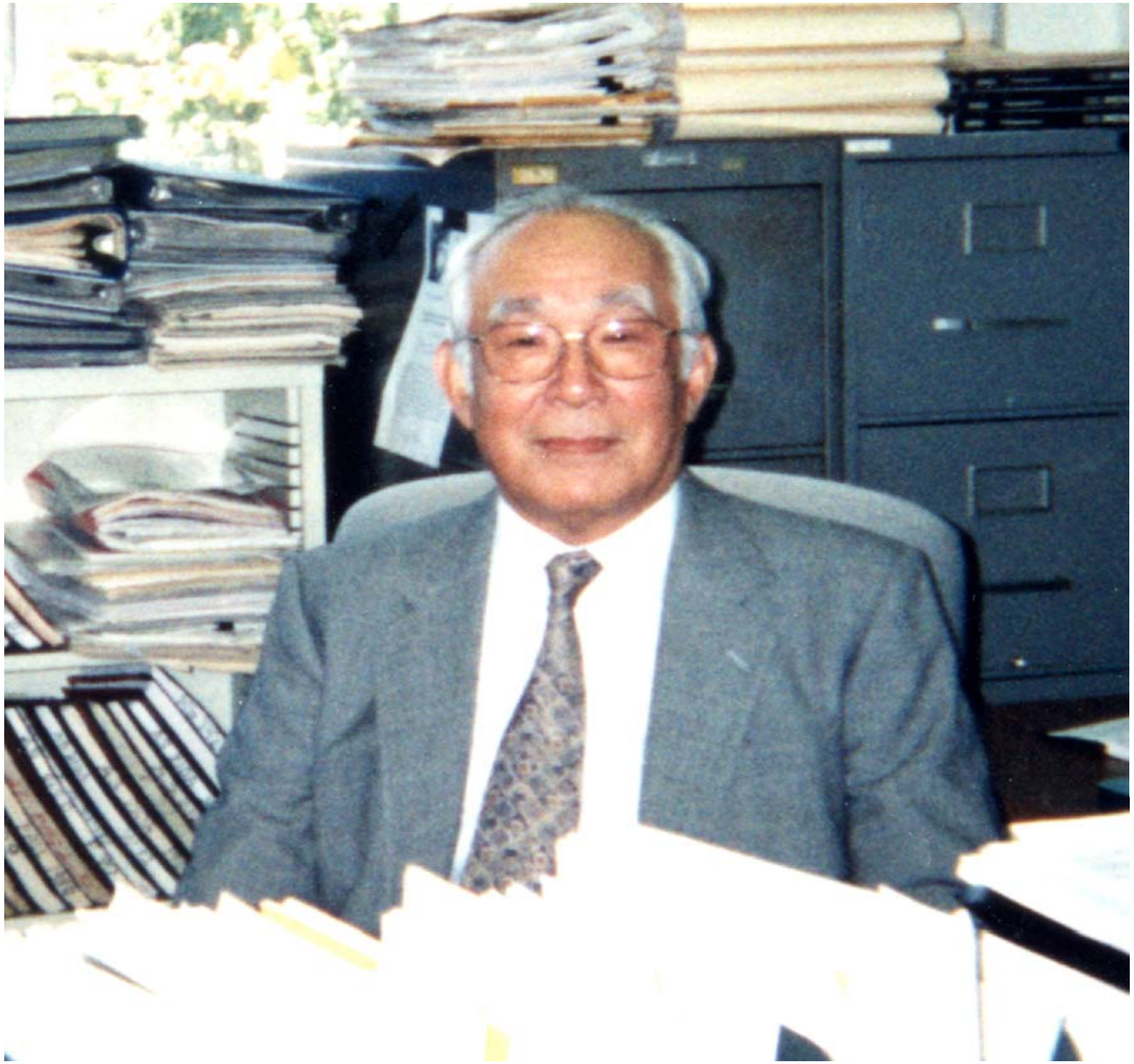
Later (1984, 1988):

- Avoid sigma system PGF errors: quasi horizontal coordinates

Note: formal (Taylor series) accuracy not emphasized

Tends not to help towards achieving some of these objectives, might even hurt (?)

Akio Arakawa:



(Some of Akio's
early wisdom:
Taroh Matsuno,
today Director General,
FRCGC, Yokohama)

Some of the numerical design features of the Eta, that each led to a very clear improvement in the results, and/or deserve to be noted ("Workstation Eta" code, "ICTP 2005" version, roughly in chronological order):

- Formulation of the lateral boundary conditions, including an effort to minimize separation of solutions on two subgrids of the model's so-called E grid (Mesinger 1977);
- A scheme to couple the gravity waves on these two subgrids of the E grid (Mesinger 1973, 1974);
- Arakawa-type momentum advection scheme of Janjic (1984), that maintains average wavenumber in case of the horizontal nondivergent flow, and very effectively suppresses the generation of noise in the advection processes;
- The eta vertical coordinate, that achieves approximately horizontal coordinate surfaces, and thus removes pressure gradient and other problems of the almost universally used terrain-following systems;
- A scheme to achieve exact conservation of energy in space differencing in transformations between potential and kinetic energy;
- A nonhydrostatic add-on option (Janjic et al. 2001);
- Removal of a problem that had been identified with the eta "step-topography" discretization, that of separation of flow in the lee of mountains in cases of strong downslope windstorms;

Regional NWP Modeling and Predictability, Introduction. The Eta Model Numerical Design

Lecture 1, part 2

Lateral boundary conditions, time differencing

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• Lateral boundary condition scheme(s)

The problem:
Considered already in Charney (1962):

Linearized shallow-water eqs., one space dimension, characteristics;

"at least two conditions have to be specified at inflow points and one condition at outflow".

Sundström (1973);

Davies (1976): "boundary relaxation scheme"

Res. Activities ..., 1999:

A TEST OF THE ETA LATERAL

Thomas L. Black, Geoffrey
U.S. National Centers for Environme

Over the years considerable degree of concern has been expressed by various investigators regarding the non well-posedness of the one-way boundary conditions of hydrostatic limited-area models. To aggravate the feelings, it is perhaps universally considered that "A common and essential ingredient of limited-area strategies is the introduction of an adjustment region immediately adjacent to the lateral boundaries, where one or both of the techniques of blending and diffusion, either explicit or implicit, are applied" (Côté et al. 1998). As a summary, Côté et al. cite as many as ten papers stating that they "all indicate that lateral boundary condition error can, depending upon the meteorological situation, importantly contribute to the total error." This assessment seems to have played a crucial role in their favoring a global variable resolution as opposed to a limited-area strategy.

Warner, T. T., R. A. Peterson, and R. E. Treadon, 1997: A tutorial on lateral boundary conditions **as a basic and potentially serious limitation to regional numerical weather prediction.** Bull. Amer. Meteor. Soc., 78, 2599-2617.

(Emphasis FM)

The Eta LBC scheme:

LBCs needed along a single outer bndry line of grid points

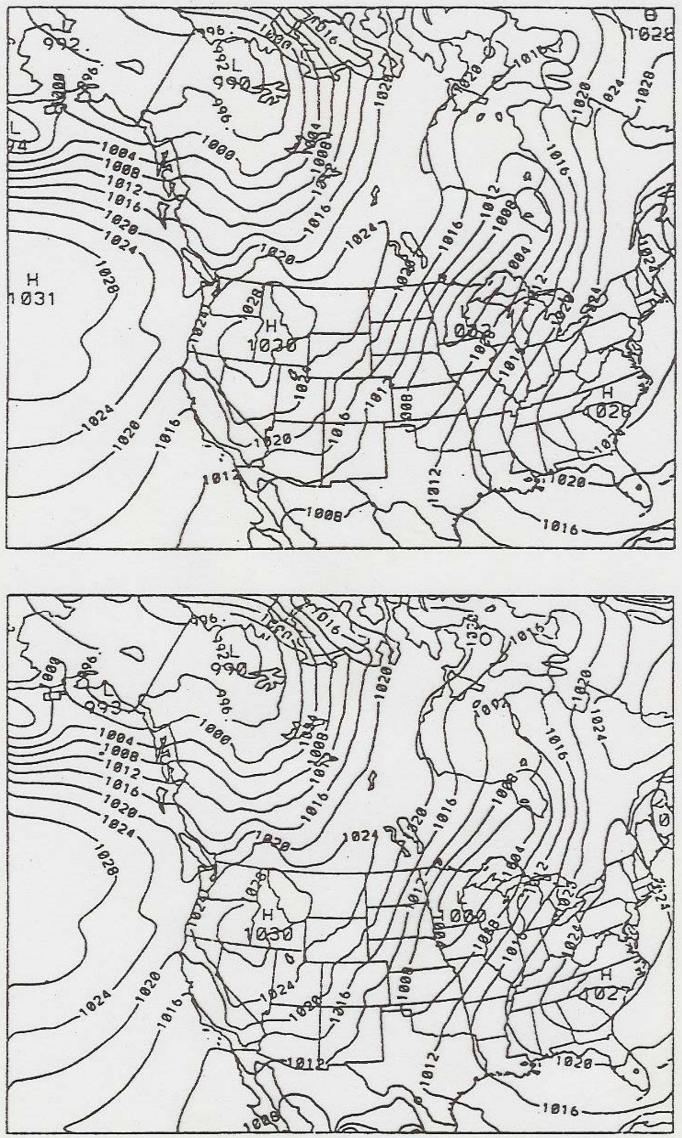
(as required by the mathematical nature of the initial-boundary value problem we are solving)

The scheme (Mesinger 1977)

- At the inflow boundary points, all variables prescribed;
- At the outflow boundary points, tangential velocity extrapolated from the inside (characteristics!);
- The row of grid points next to the boundary row, "buffer row"; variables four-point averaged (this couples the gravity waves on two C-subgrids of the E-grid, will be explained as the next item)

Thus: No "boundary relaxation" !

"Control"



"using "control" the lateral bnd. condition"
Driven

No space "interpolation errors at the lateral boundary

Black at al., 99; RAAOM,
50th Anniv. of NWP, 2001

Figure 4: A section of the then operational 32-km Eta 48-h sea level pressure forecast, valid at 1200 UTC 17 October 1998, top panel; same except for a run over a smaller domain, done using the operational forecast to supply its boundary conditions, bottom panel. Boundaries of the plots shown are the outermost boundaries of the smaller domain, thus, in the bottom panel, all of the forecast domain of the nested run is shown.

“limitation“:

Near inflow boundaries, LA model cannot do better -
it can only do worse - that its driver model

- Time differencing

A variety of time differencing schemes

Gravity-inertia terms, linearized shallow-water equations:

The forward-backward scheme:

(Richtmyer ?)

$$u^{n+1} = u^n - g\Delta t\, \delta_x h^{n+1},$$

$$v^{n+1} = v^n - g\Delta t\, \delta_x h^{n+1},$$

$$h^{n+1} = h^n - H\Delta t\,(\delta_x u + \delta_y v)^n.$$

Stable, and neutral, for time steps twice those of the leapfrog scheme;

No computational mode

Coriolis terms: trapezoidal scheme

$$u^{n+1} = \ldots \qquad + \tfrac{1}{2} f\Delta t\,(v^n + v^{n+1})$$

$$v^{n+1} = \ldots \qquad - \tfrac{1}{2} f\Delta t\,(u^n + u^{n+1})$$

Unconditionally neutral

Elimination of u,v from pure gravity-wave system leads to the wave equation, (5.6):

$$\frac{\partial^2 h}{\partial t^2} - gH\frac{\partial^2 h}{\partial x^2} = 0. \qquad (5.6)$$

We can perform the same elimination for each of the finite difference schemes.

The forward-backward and space-centered approximation to (5.5) is

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} + g\frac{h_{j+1}^n - h_{j-1}^n}{2\Delta x} = 0,$$
$$\frac{h_j^{n+1} - h_j^n}{\Delta t} + H\frac{u_{j+1}^{n+1} - u_{j-1}^{n+1}}{2\Delta x} = 0, \qquad (5.7)$$

We now substract from the second of these equations an analogous equation for time level $n-1$ instead of n, divide the resulting equation by Δt, and, finally, eliminate all u values from it using the first of Eqs. (5.7), written for space points $j+1$ and $j-1$ instead of j. We obtain

$$\frac{h_j^{n+1} - 2h_j^n + h_j^{n-1}}{(\Delta t)^2} - gH\frac{h_{j+2}^n - 2h_j^n + h_{j-2}^n}{(2\Delta x)^2} = 0. \qquad (5.8)$$

This is a finite difference analogue of the wave equation (5.6). Note that although each of the two equations (5.7) is only of the first order of accuracy in time, the wave equation analogue equivalent to (5.7) is seen to be of the second order of accuracy.

If we use a leapfrog and space-centered approximation to (5.5), and follow an elimination procedure like that used in deriving (5.8), we obtain

$$\frac{h_j^{n+1} - 2h_j^{n-1} + h_j^{n-3}}{(2\Delta t)^2} - gH\frac{h_{j+2}^{n-1} - 2h_j^{n-1} + h_{j-2}^{n-1}}{(2\Delta x)^2} = 0. \tag{5.9}$$

This also is an analogue to the wave equation (5.6) of second-order accuracy. However, in (5.8) the second time derivative was approximated using values at three consecutive time levels; in (5.9) it is approximated by values at every second time level only, that is, at time intervals $2\Delta t$. Thus, while the time step required for linear stability with the leapfrog scheme was half that with the forward-backward scheme, (5.9) shows that we can omit the variables at every second time step, and thus achieve the same computation time as using the forward-backward scheme with double the time step.

(From Mesinger, Arakawa, 1976)

Splitting used:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -f\,\mathbf{k}\times\mathbf{v} - g\nabla h,$$

$$\frac{\partial h}{\partial t} + \nabla\cdot(h\mathbf{v}) = 0. \qquad (1)$$

is replaced by

$$\frac{\partial \mathbf{v}}{\partial t} = -f\,\mathbf{k}\times\mathbf{v} - g\nabla h,$$

$$\frac{\partial h}{\partial t} + \nabla\cdot(h\mathbf{v}) = 0. \qquad (2)$$

as the “adjustment step”,

and

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = 0, \qquad (3)$$

as the “advection step”

Note that height advection (corresponding to pressure in 3D case) in is carried in the adjustment step (or, stage), even though it represents advection!

This is a necessary, but not sufficient, condition for energy conservation in time differencing in the energy transformation (“$\omega\alpha$”) term (transformation between potential and kinetic energy). Splitting however, as above, makes exact conservation of energy in time differencing not possible (amendment to Janjic et al. 1995). Energy conservation in the Eta, in transformation between potential and kinetic energy is achieved in space differencing.

Time differencing in the Eta: two steps of (2) are followed by one, over $2\Delta t$, step of (3).

How is this figured out?

To achieve energy conservation in time differencing one needs to replicate what happens in the continuous case. Energy conservation in the continuous case, still shallow water eqs. for simplicity:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -f\,\mathbf{k}\times\mathbf{v} - g\nabla h, \qquad (1.1)$$

$$\frac{\partial h}{\partial t} + \nabla\cdot(h\mathbf{v}) = 0. \qquad (1.2)$$

To get the kinetic energy eq., multiply (1.1) by $h\mathbf{v}$, multiply (1.2) by $\frac{1}{2}\mathbf{v}\cdot\mathbf{v}$, and add,

$$\frac{\partial}{\partial t}\frac{1}{2}h\mathbf{v}\cdot\mathbf{v} + h(\mathbf{v}\cdot\nabla)\frac{1}{2}\mathbf{v}\cdot\mathbf{v} + \frac{1}{2}\mathbf{v}\cdot\mathbf{v}\nabla\cdot(h\mathbf{v}) = -gh\mathbf{v}\cdot\nabla h \qquad (4)$$

For the potential energy eq., multiply (1.2) by *gh,*

$$\frac{\partial}{\partial t}\frac{1}{2}gh^2 + gh\nabla\cdot(h\mathbf{v}) = 0 \qquad (5)$$

Adding (3) and (4) we obtain

$$\frac{\partial}{\partial t}\left(\frac{1}{2}h\mathbf{v}\cdot\mathbf{v} + \frac{1}{2}gh^2\right) + \nabla\cdot\left(\frac{1}{2}\mathbf{v}\cdot\mathbf{v}\,h\mathbf{v}\right) + \nabla\cdot(gh^2\mathbf{v}) = 0. \qquad (6)$$

Thus, the total energy in a closed domain is conserved

For conservation in time differencing terms that went into one and the other divergence term have to be available at the same time;

- Kinetic energy in horizontal advection (the 1st divergence term):

Formed of contributions of horizontal advection of **v** in (1.1), and mass divergence in (1.2)
Not available at the same time with the split-explicit approach;

cannot be done;

- Energy in transformations potential to kinetic (the 2nd divergence term):

Formed of the advection of *h* term on the right side of (4), coming from the pressure-gradient force, and the mass divergence term of (5), coming from the continuity eq.;

Both are done in the adjustment stage with the splitting as in (2) and (3);
cancellation is thus possible if the two are done at the same time

However: they are done separately with the forward-backward scheme;

Thus, with the forward-backward scheme, cannot be done;

Time steps used for the adjustment stage very small;
not considered a serious weakness

(Eta “nest” at 10 km resolution used adjustment time step of 20 s)

Time differencing in the code:

Adjustment stage: cont. eq. forward, momentum backward

(the other way around should be a little better, Misha Rancic might explain)

Do vertical advection over 2 adj. time steps

Repeat (except no vertical advection now)

Do horizontal diffusion;

Do horizontal advection over 2 adj. time steps
("stepped over" Heun scheme);

Do some physics calls;

Repeat;

Do more physics calls;

. . . . .

Mesinger, lecture 1, part 2, cont'd:

- Gravity-wave coupling scheme

Arakawa 1997:

Reviews of various discretization methods applied to atmospheric models include Mesinger and Arakawa (1976), GARP (1979), ECMWF (1984), WMO (1984), Arakawa (1988) and Bourke (1988) for finite-difference, finite-element and spectral methods and Staniforth and Côté (1991) for the semi-Lagrangian method.

7.2 Horizontal computational mode and distortion of dispersion relations

Among problems in discretizing the basic governing equations, computational modes and computational distortion of the dispersion relations in a discrete system require special attention in data assimilation. Here a computational mode refers to a mode in the solution of discrete equations that has no counterpart in the solution of the original continuous equations. The concept of the order of accuracy, therefore, which is based on the Taylor expansion of the residual when the solution of the continuous system is substituted into the discrete system, is not relevant for the existence or non-existence of a computational mode.

geostrophic ajustm. :

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x} + fv,$$

$$\frac{\partial v}{\partial t} = -g\frac{\partial h}{\partial y} - fu,$$

$$\frac{\partial h}{\partial t} = -H\,\nabla\cdot\mathbf{v}$$

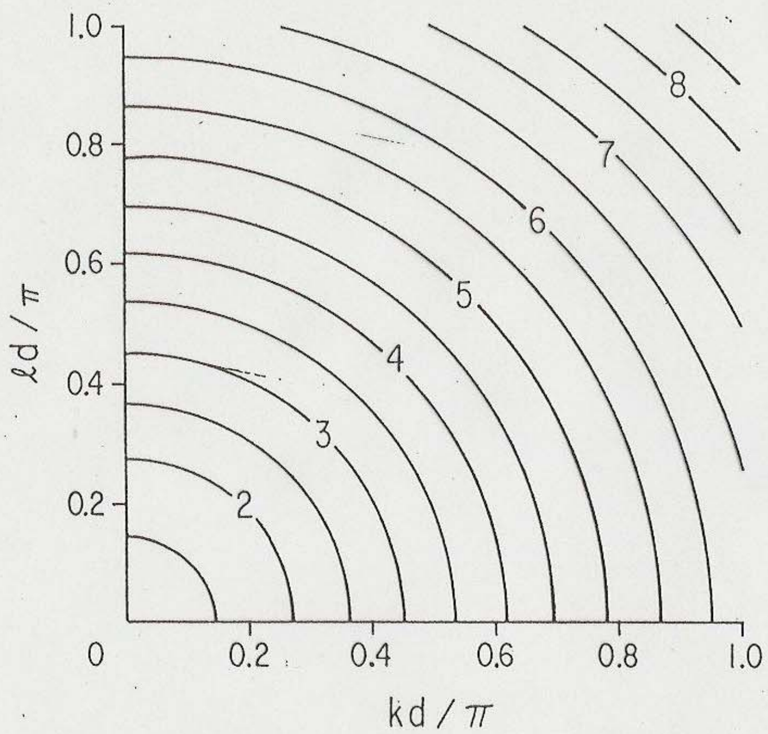


FIG. 9. Contours of the (nondimensional) frequency as a function of the (nondimensional) horizontal wave numbers for the differential shallow water equation for $\lambda/d = 2$, presented for comparison with Fig. 8.

$\lambda \equiv \sqrt{gH}/f$

Arakawa, dynamics:
- geostrophic adjustment
- simulation of slow, quasi-geostrophic motions

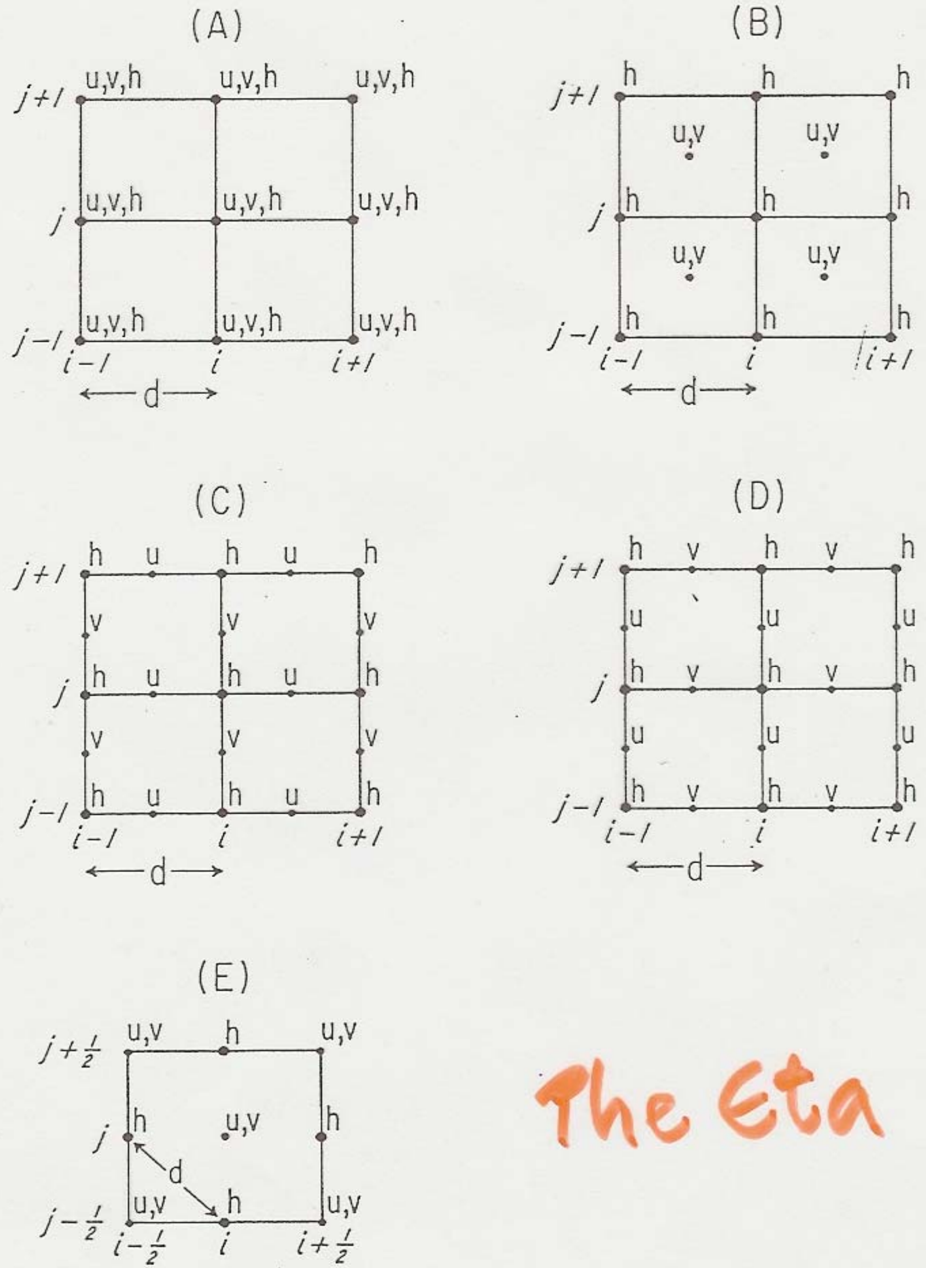


FIG. 3. Spatial distributions of the dependent variables on a square grid.

The Eta

(Two C sub-grids)

Gravity wave terms only:

on the lattice separation problem. If, for example, the forward–backward time scheme is used, with the momentum equation integrated forward,

$$u^{n+1} = u^n - g\Delta t \delta_x h^n, \qquad v^{n+1} = v^n - g\Delta t \delta_y h^n, \tag{2}$$

instead of

$$h^{n+1} = h^n - H\Delta t\left[(\delta_x u + \delta_y v) - g\Delta t \nabla_+^2 h\right]^n, \tag{3}$$

the method results in the continuity equation (Mesinger, 1974):

$$h^{n+1} = h^n - H\Delta t\left[(\delta_x u + \delta_y v) - g\Delta t\left(\frac{3}{4}\nabla_+^2 h + \frac{1}{4}\nabla_\times^2 h\right)\right]^n. \tag{4}$$

Single-point perturbation spreads to both *h* and *h* points !

Eq. (4):

Following a pulse perturbation (height increase) at the initial time, at time level 1 increase in height occurs at four nearest points equal to 2/3 of the increase which occurs in four second nearest points.

This is not ideal, but is a considerable improvement over the situation with no change at the four nearest height points !

In the code: continuity eq. is integrated forward.
"Historic reasons". With this order, at time level 1 at the four second nearest points a decrease occurs, in the amount of 1/2 of the increase at the four nearest points !

Might well be worse? Still:

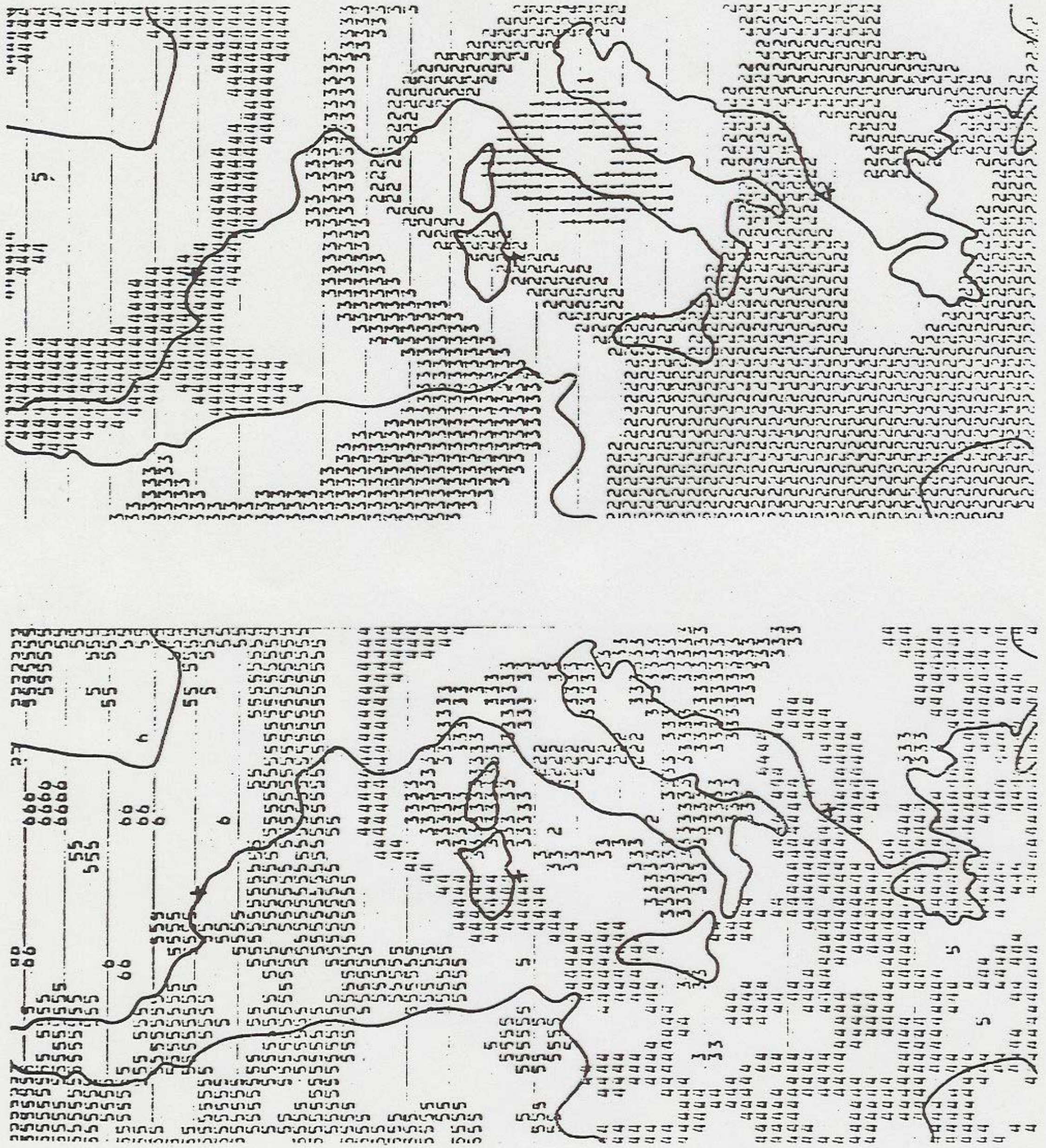


Figure 8 Sea level pressure, 00 GMT 24 August 1975, 24 hr forecast with variable boundary conditions. Above: with w = .25; below: with w = 0.

- Non-hydrostatic option:

Janjic et al. 2001:

$$\left(\frac{\partial w}{\partial t}\right)^{\tau+1/2} \rightarrow \frac{w^{\tau+1} - w^{\tau}}{\Delta t}$$

Some of the references made (more in Matt Pyle's manual)

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