The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

Spring Colloquium on
'Regional Weather Predictability and Modeling'
April 11 - 22, 2005

1) Workshop **on Design and Use of Regional Weather Prediction Models,** *April 11 - 19*

2) Conference on **Current Efforts Toward Advancing the Skill of Regional Weather Prediction. Challenges and Outlook,** *April 20 - 22*

301/1652-12

Numerical Developments for Nonhydrostatic Mesoscale Models

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Numerical Developments for Nonhydrostatic Mesoscale Models

J. Steppeler, DWD

Trieste 2005

Plan of Lecture

- The Nonhydrostatic Model LM
- Runge Kutta and Semi-Lagrangian methods (provide sufficient accuracy for practical purpooses)
- Z-coordiantes with step and shaved element boundaries
- Direct implicit solvers
- Global nonhydrostatic modelling on isocahedral or similar grids

Features of the LM - Design Aspects

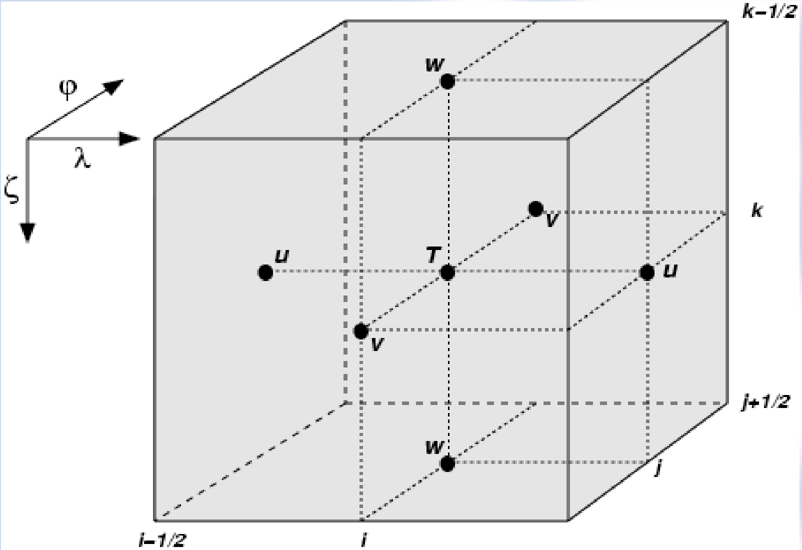
- Modelling Scales from 50 m to 50 km
- Nonhydrostatic Compressible Dynamics
- Efficient Numerics
- Comprehensive Physics
- Nudging Data Assimilation
- Code Portability
- Mesh Refinement
- Use for Both Research and Operations

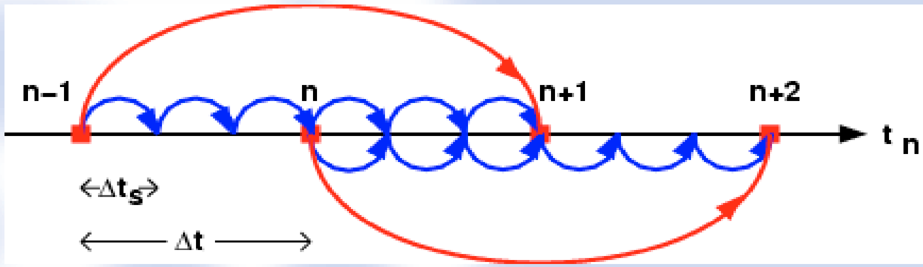
Features of the LM - Overview

Dynamics and Numerics

- Advection Form
- Prognostic Variables: u,v,w,T,q_v,q_c, (q_i,q_{tke},q_r,q_s)
- Coordinates: Rotated Lat-Lon, Generalized Terrain Following
- Arakawa C / Lorenz Grid Staggering
- 2nd Order Centred Differencing
- Leapfrog / Split explicit Time Integration (HE-VI)
- Numerical Smoothing: 4th Order Horizontal Diffusion, Rayleigh Damping, 3-D Divergence Damping
- Optional Time Integration Schemes: 2-TL RK with 3rd Order Advection, 3-TL SI Scheme

Grid Structure and Time Integration





$$\partial\phi / \partial t = F + S$$

Features of the LM - Overview

Physical Parameterization

- Vertical Diffusion by
  - Diagnostic K-Closure
  - Prognostic TKE
- Grid Scale Clouds by Saturation Adjustment and
  - Warm Rain- / EM-DM- / Cloud Ice - Scheme
- Moist Convection
  - Tiedtke Mass Flux Scheme
  - Kain Fritsch Scheme with CAPE-Closure (untested)
- Radiation: Two-Stream Scheme (Ritter, Geyleyn, 1992)
- Soil Processes
  - Two Layer Extended Force Restore Soil Model
  - New Multi Layer Soil Model Including Melting and Freezing

Features of the LM - Overview

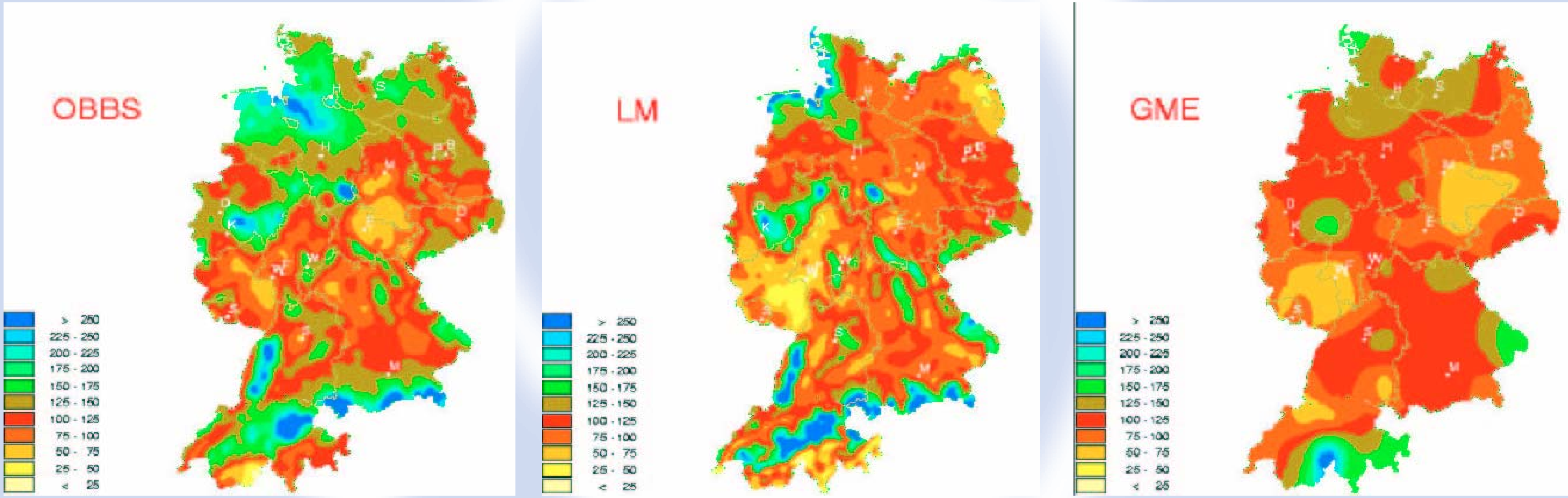
Boundary Conditions

- Periodic, Wall or Relaxation Boundary Conditions for Idealized Cases
- One-Way Nesting by Davies Relaxation for Real Cases Interpolated from GME (GME2LM), IFS (IFS2LM), LM (LM2LM, Current Work)
- Two Way Interactive Self Nesting (Current Work)

Initial Conditions

- Artificial Data for Idealized Cases (User Defined!)
- Interpolated from GME or IFS; DFI Initialization
- Continuous Data Assimilation
  Nudging (u,v,p,T,q_v); External Analysis of SST; Variational Soil Moisture Analysis (00UTC); Latent Heat Nudging for radar reflectivities

Monthly Precipitation Sum for September 2001



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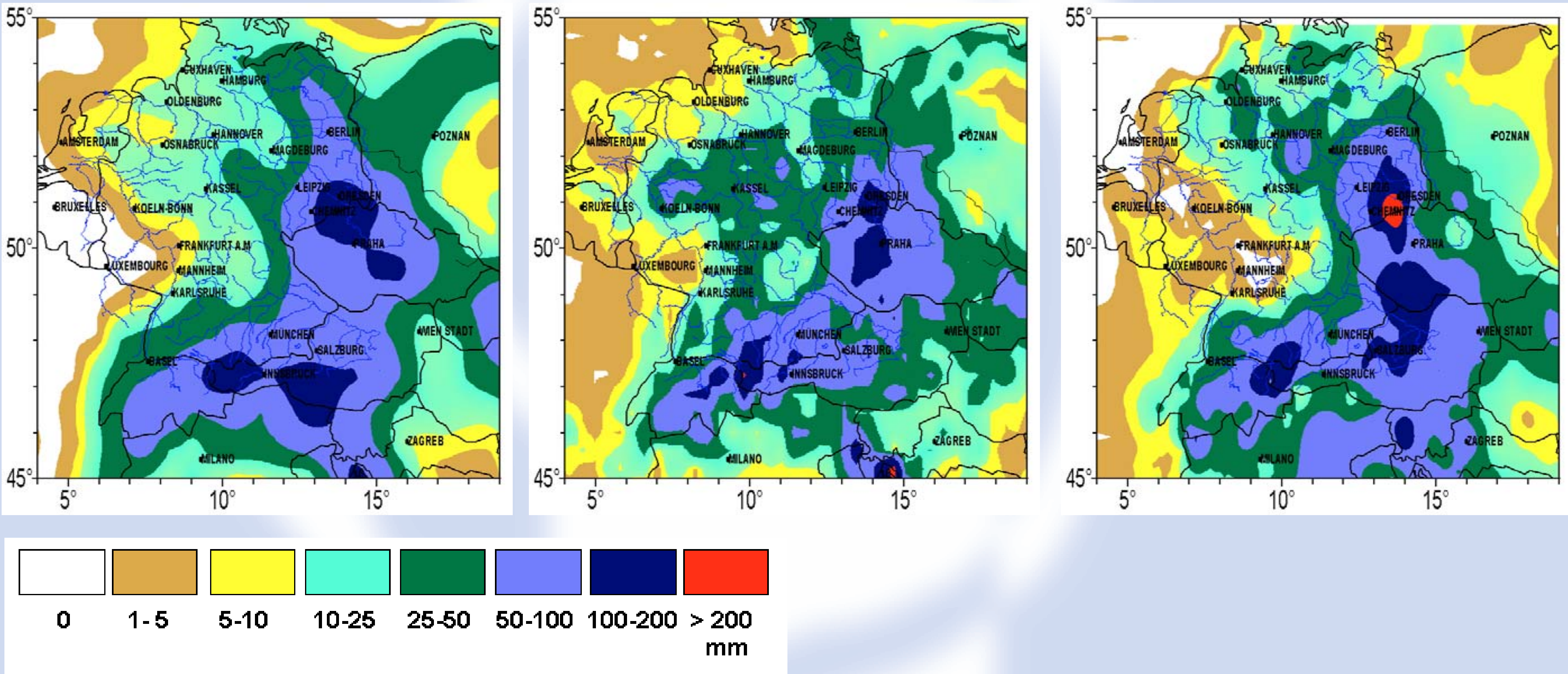
DWD

The Elbe Flood Disaster, August 2002

Precipitation accumulated for the period 10 Aug 06 UTC - 13 Aug 06 UTC

GME forecast **LM forecast** **Observation**



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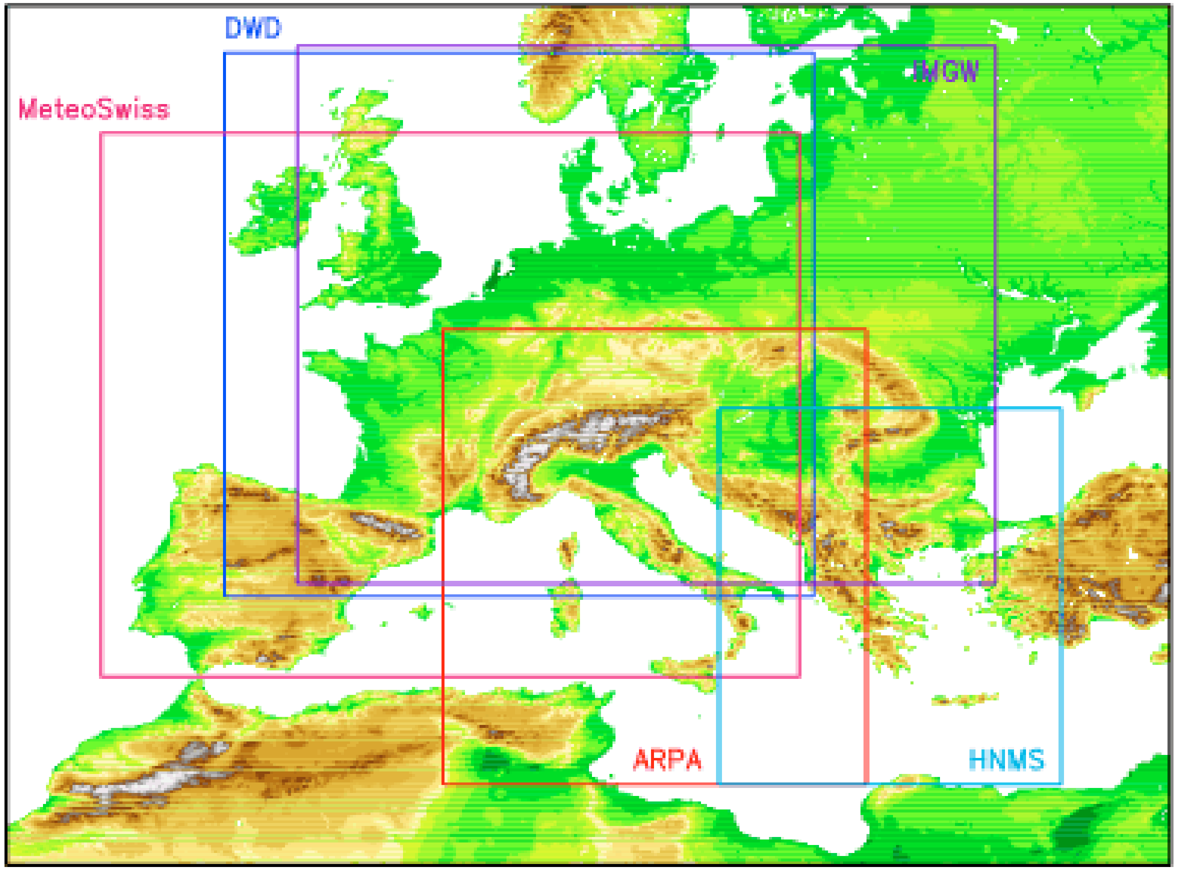
DWD

COSMO - Integration Domains



LM - Configurations at COSMO Centres

Configurations	ARPA–SMR	DWD	HNMS	MeteoSwiss	IMGW
Domain Size	234 x 272	325 x 325	95 x 113	385 x 325	385x321, 193x161
Grid Spacing	0.0625°	0.0625°	0.1250°	0.0625°	0.0625°, 0.125°
Number of Layers	35	35	35	45	35
Time Step	40 sec	40 sec	80 sec	40 sec	40 sec, 80 sec
Forecast Range	48 hrs	48 hrs	48 hrs	48 hrs	36 hrs, 72 hrs
Model Runs	00, 12 UTC	00, 12, 18 UTC	00 UTC	00, 12 UTC	00, 12 UTC
Boundaries	GME	GME	GME	GME	GME
LBC Updates	1 hr	1 hr	1 hr	1 hr	1 hr
Initial State	GME	Nudging	GME	Nudging	GME
Initialization	Digital Filtering	None	Digital Filtering	None	Digital Filtering
External Analyses	None	SST, S–Depth, Soil Moisture	None	SST, S–Depth	None
Hardware	IBM SP (Pwr3)	IBM SP (Pwr3)	CONVEX	NEC SX–5	SGI 3800
No. of Processors	32 / 64	160 / 1280	14 / 16	12 / 16	96 / 100

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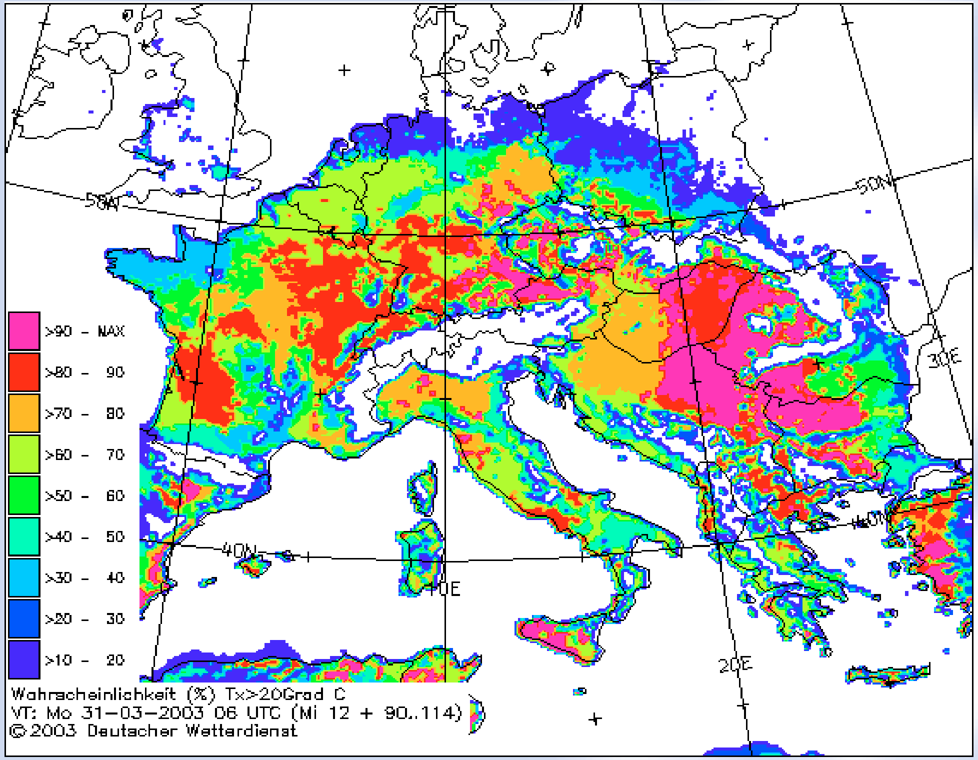
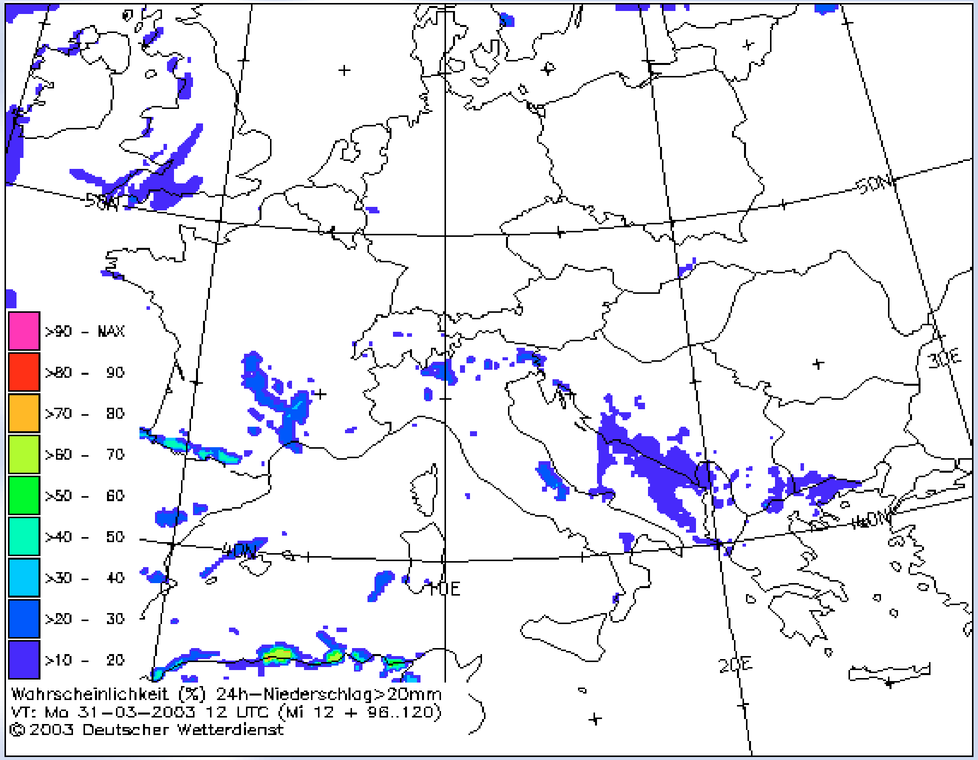
COSMO Limited Area Ensemble Prediction System

(Regionalization of the ECMWF Ensemble using LM)

Probability for 2m-Temperature exceeding 20 C



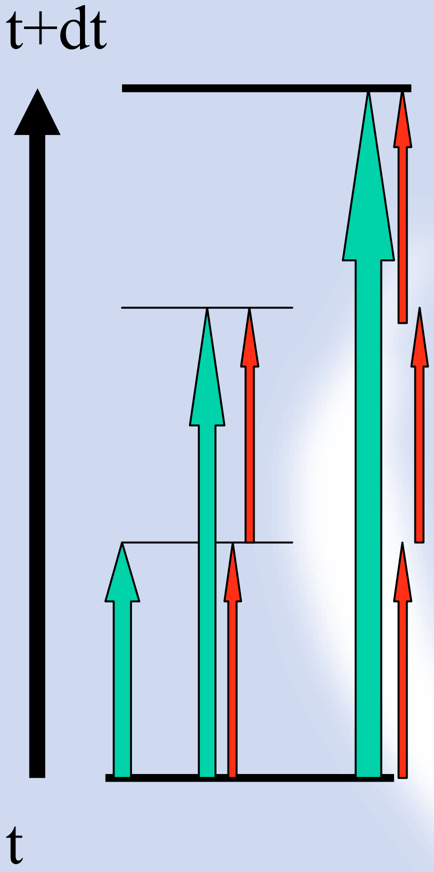
Probability for 24-h precipitation exceeding 20 mm



Recent and Current Numerical Developments

- Mesoscale ensemble
- LM_K: operational cloud prediction model at a scale of dx=2.8 km (inludes supporting developments in analysis, physics model interpretation)
- LM-Z
- Runge Kutta
- Implicit and SL
- Next generation model and global mesoscale model: ICON project

The Runge Kutta scheme (NCAR)



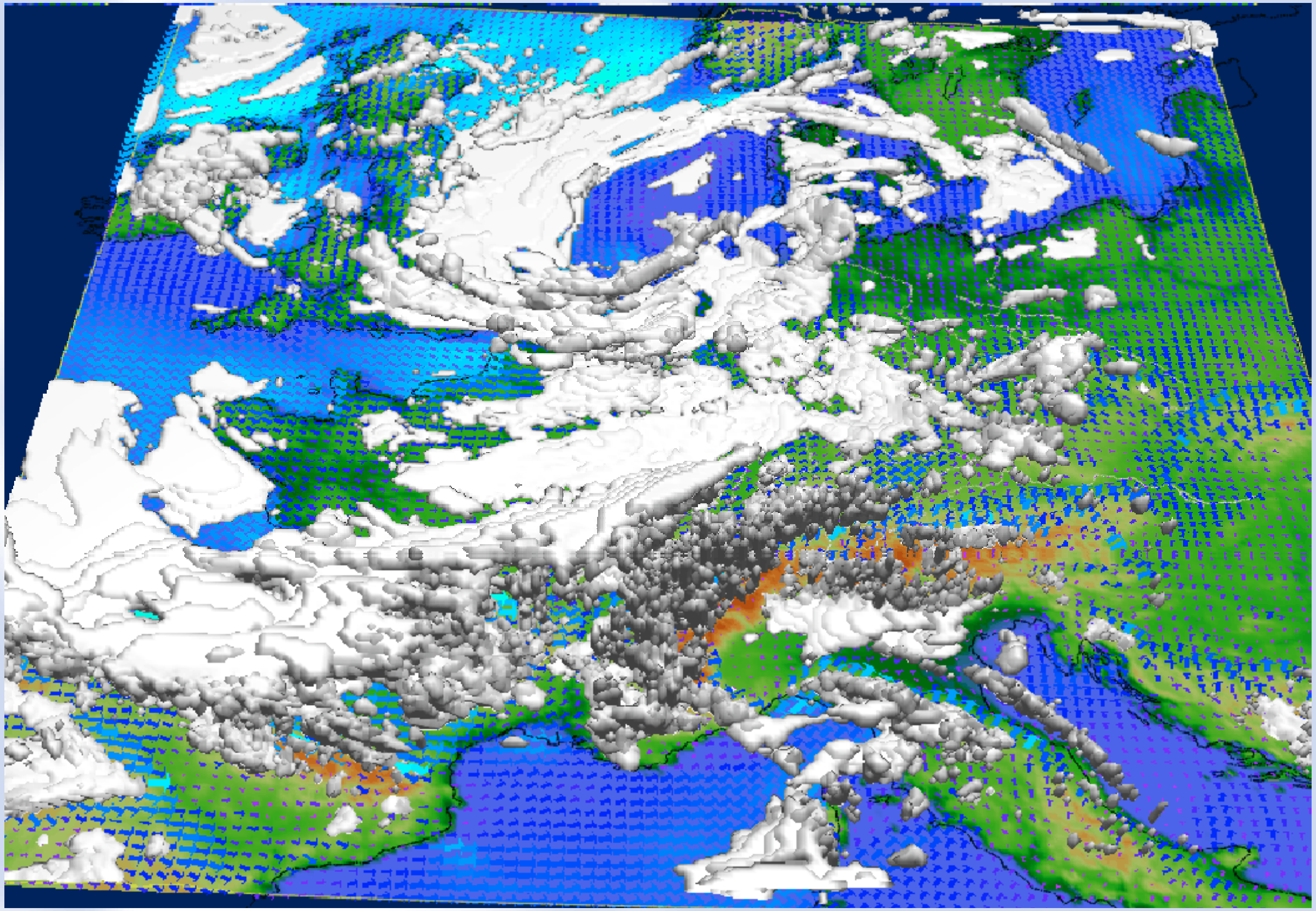
- RK is a two time level 3rd order in time scheme, involving substepping for fast waves
- Spacial order is 3 or 5 (upstream differencing)
- Approximation conditions concern vert. coordiante and phys. interface
- Semi-lagrange: 2nd order in time, 3rd order in space, could be easier to achieve efficiency with large dt

Expected advantages of the Z-coordinate

- The atmosphere at rest can be represented in Z-coordinates, but not in terrain following coordinates
- Stratiform clouds and low stratus are predicted better in LM-Z
- Mountain and valley winds are better with LM-z
- Precipitation amplitudes should be better with LM-Z, in particular maxima and minima near mountains

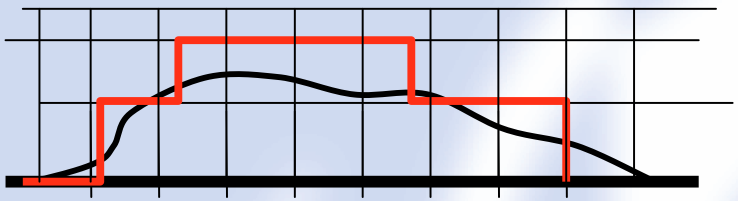
3-D Cloud-Picture

18 January1998



The mountain related bias of convectional clouds and precipitation is supposed to disappear with the Z-coordinate

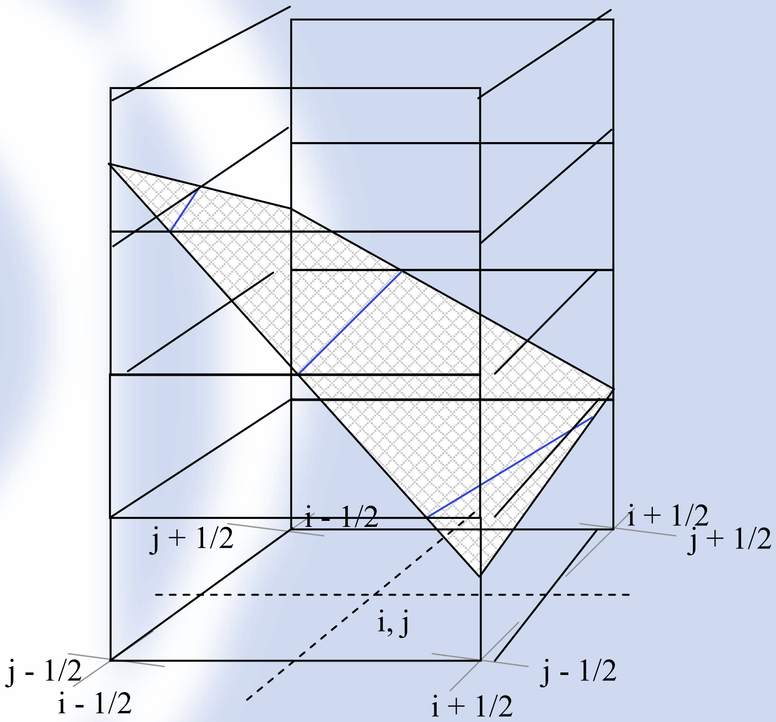
The step-orography



•The shaved elements are mathematically more correct than step boundaries

•By shaved elements the z-coordinate is improved such that the criticism of Gallus and Klemp (2000), Mon. Wea. Rev. **128**, 1153-1164 no longer applies

Shaved elements



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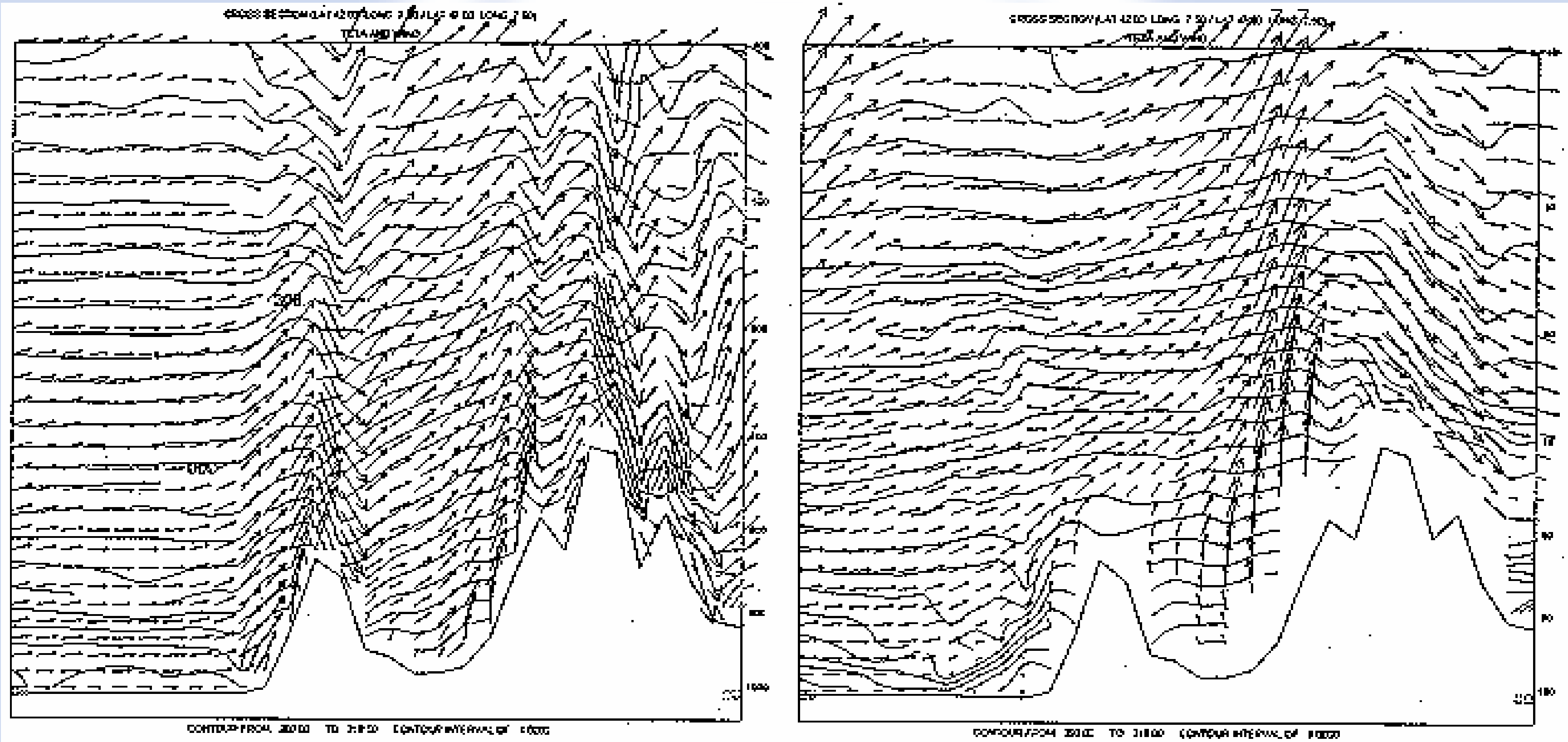
DWD

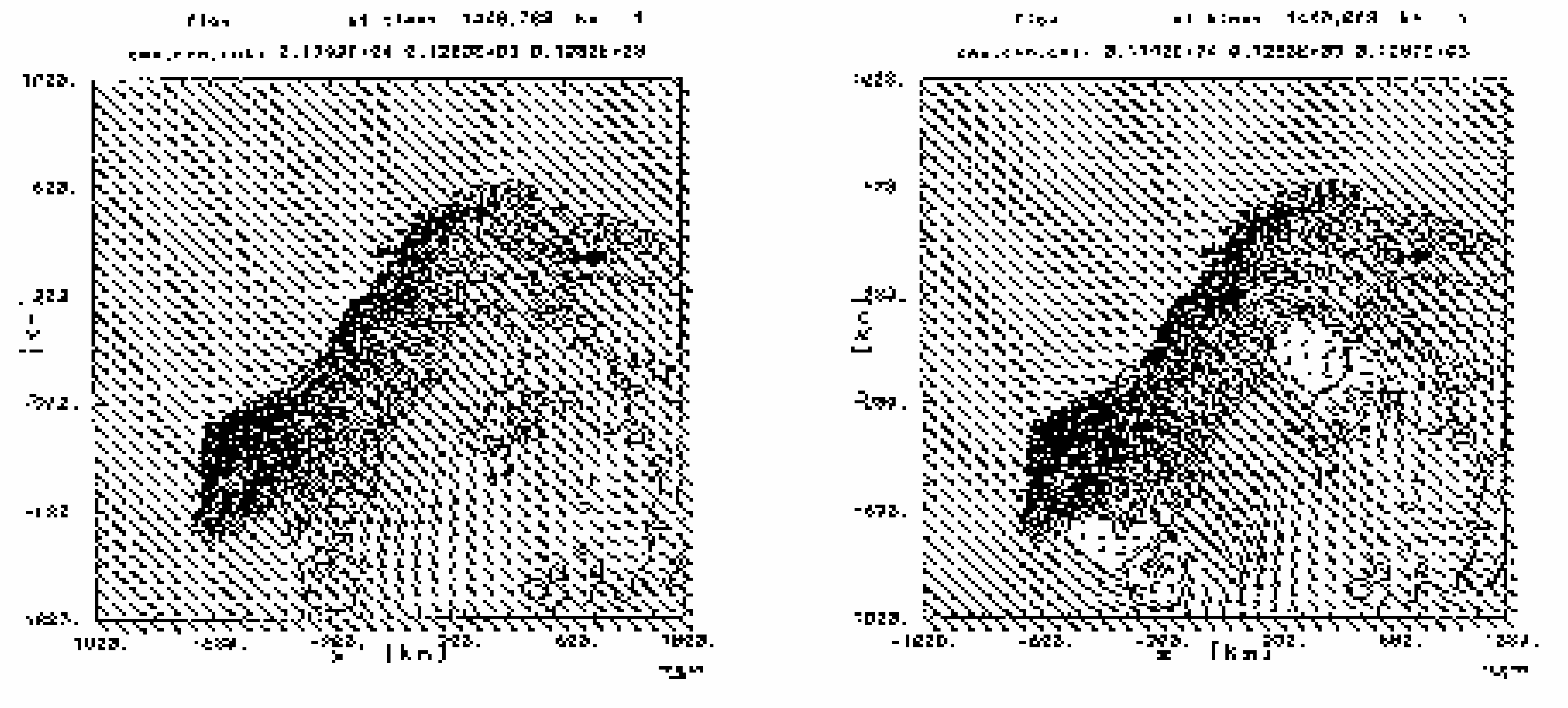
Cross Section for Flow Over the Alps, Forecasted by Eta Model with Step Orography (Right) and with Terrain Following Orography (Left)



The finite volume method for the treatment of the fast waves in LM-Z

- The computation of the fast waves is based on the the evaluation of fluxes into a cell
- The evaluation of the fluxes requires weights associated with the cell surfaces, which depend on their open part
- Advection terms (slow waves) are computed by finite difference methods

SKANIA Test: Korrect (left) faulty (right)



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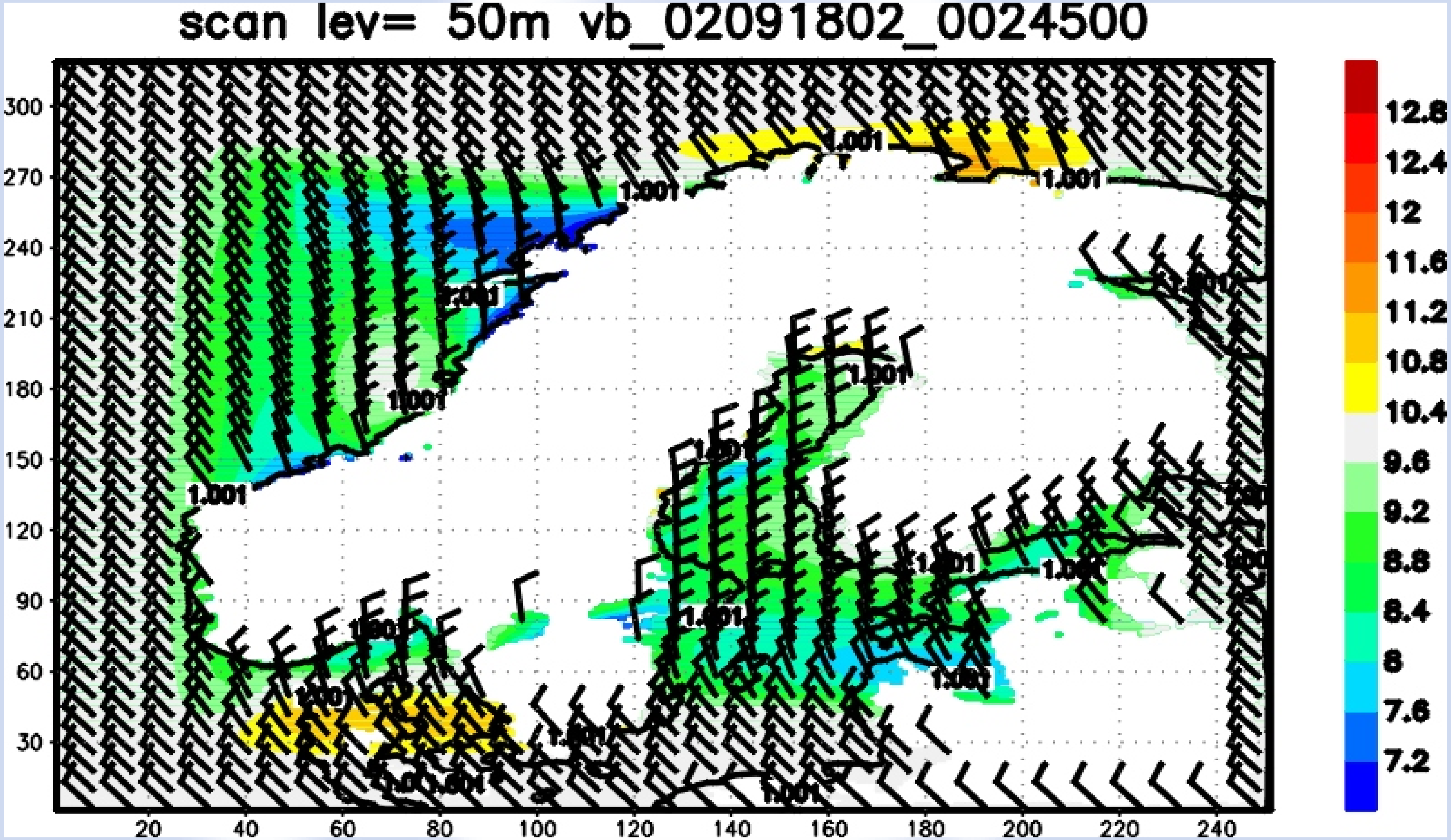

SKANIA Test for Z_LM

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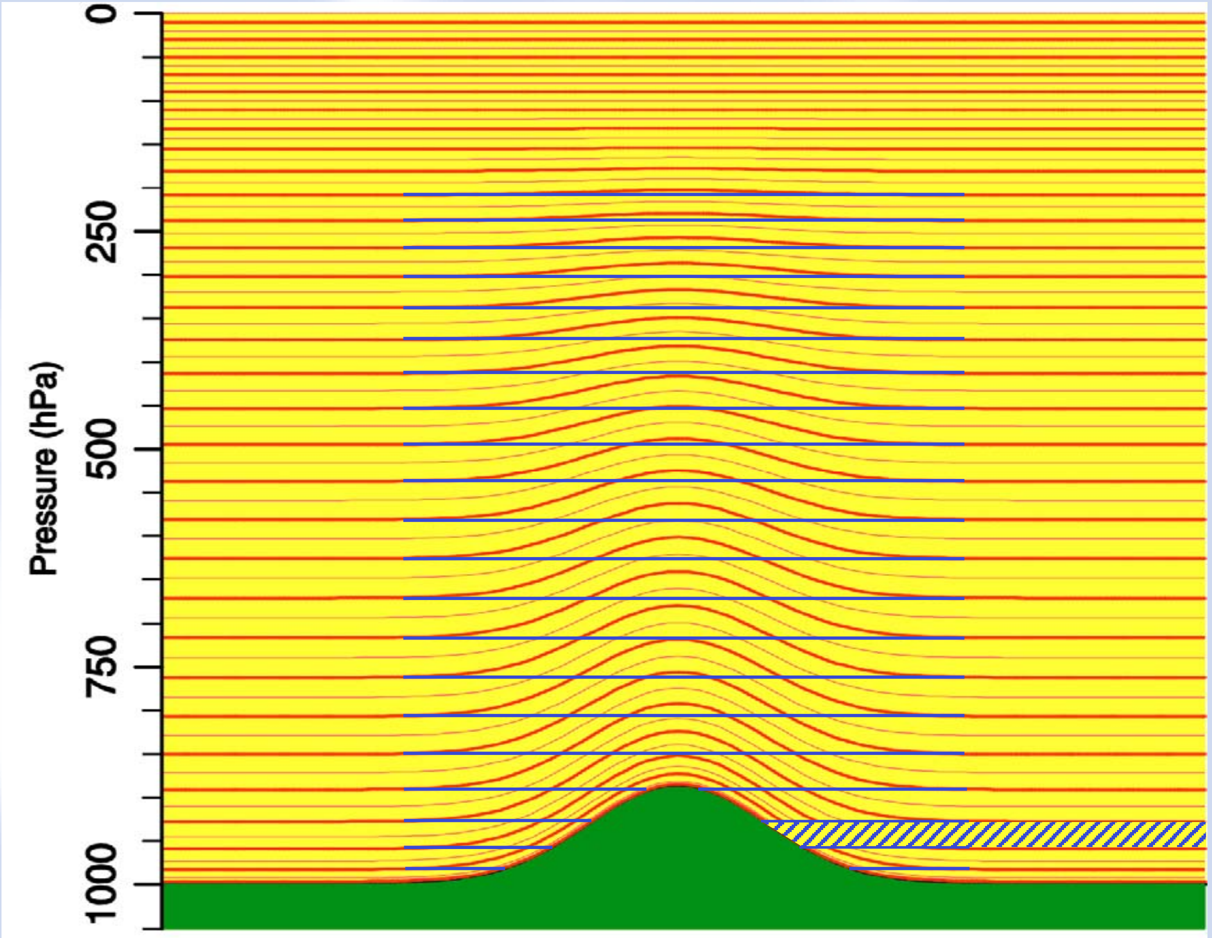
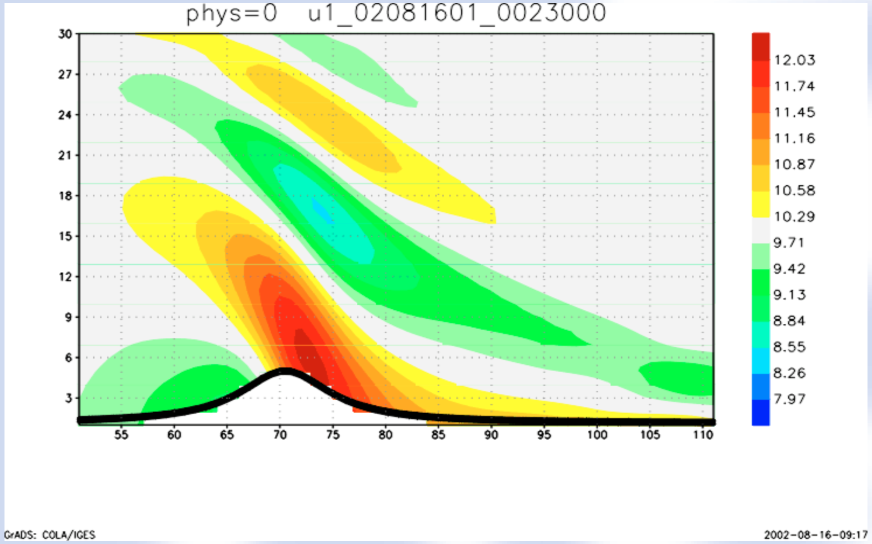
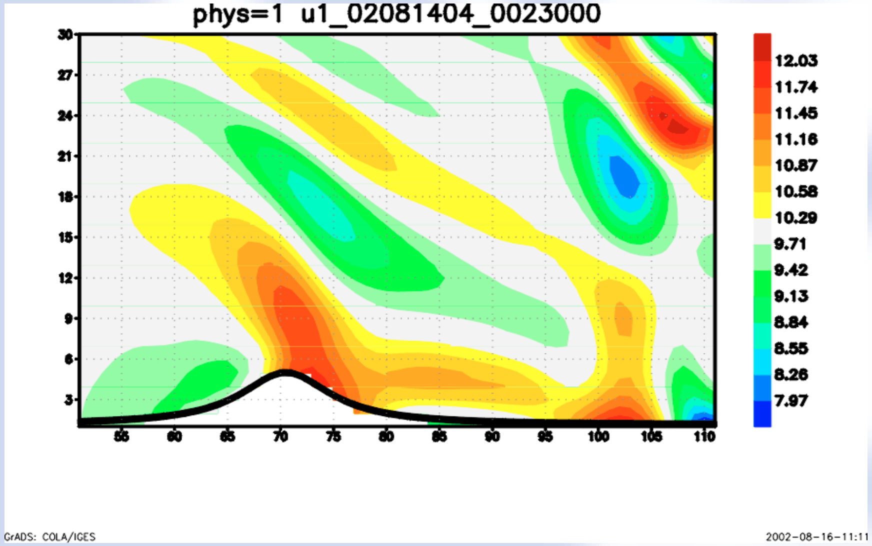




Without Physics



Dry Physics

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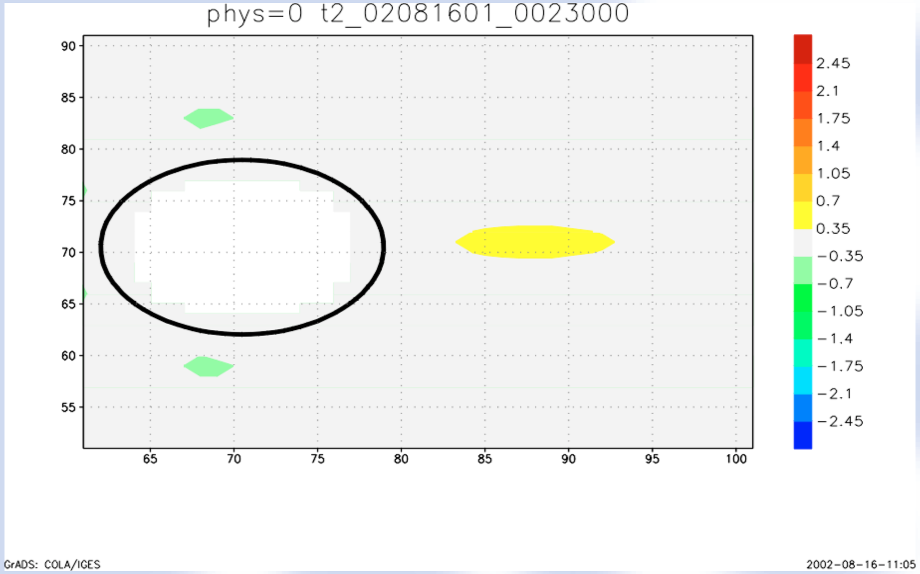
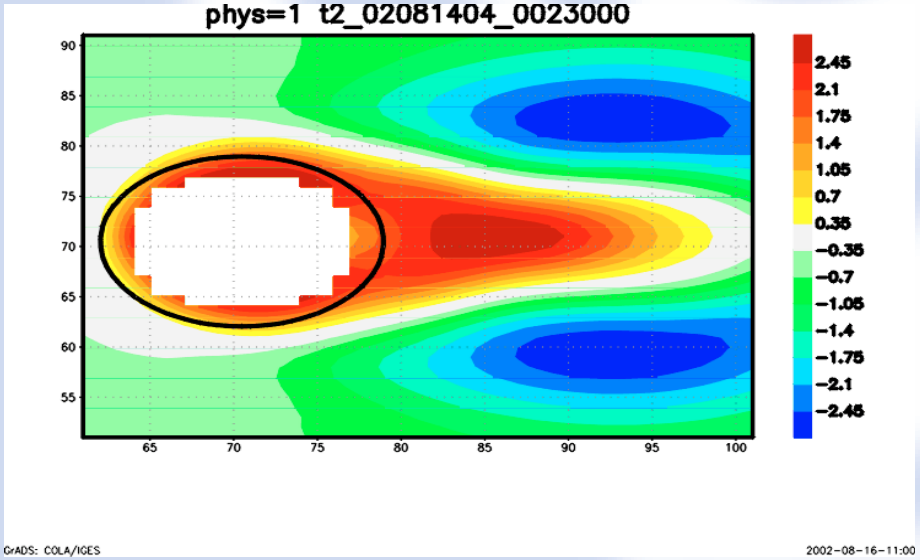


NO physics



„dry“ Physics

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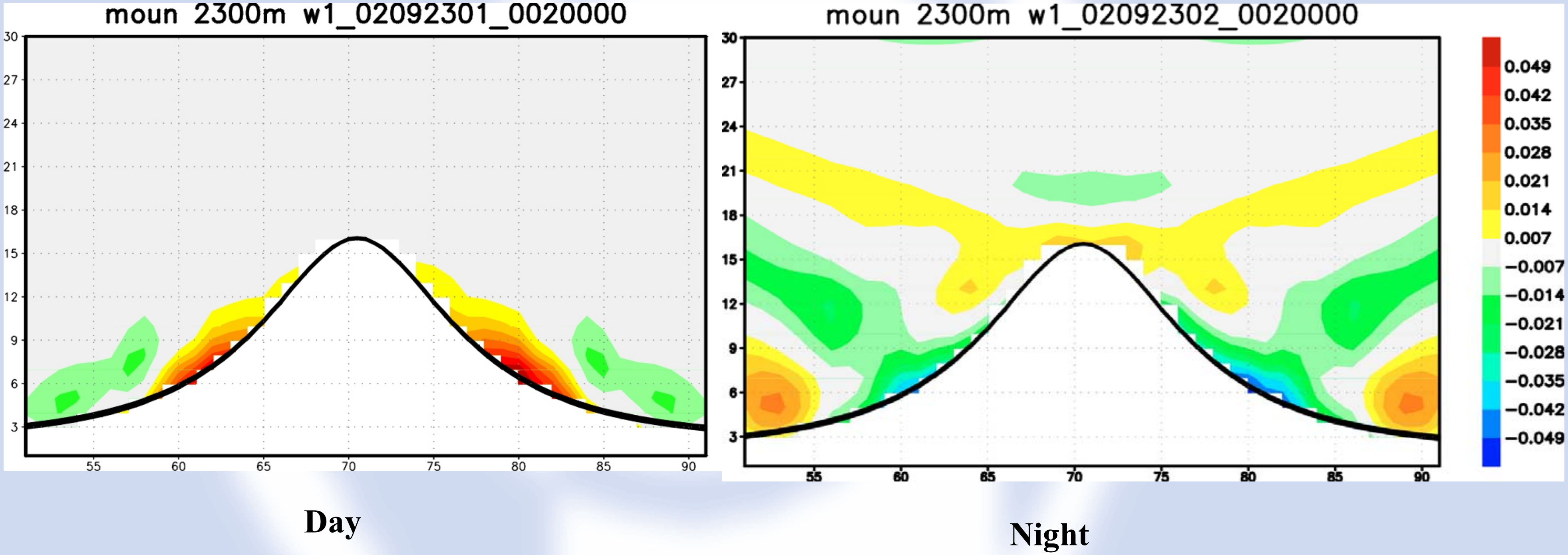

The Atmosphere at Rest Computed with the Z_LM

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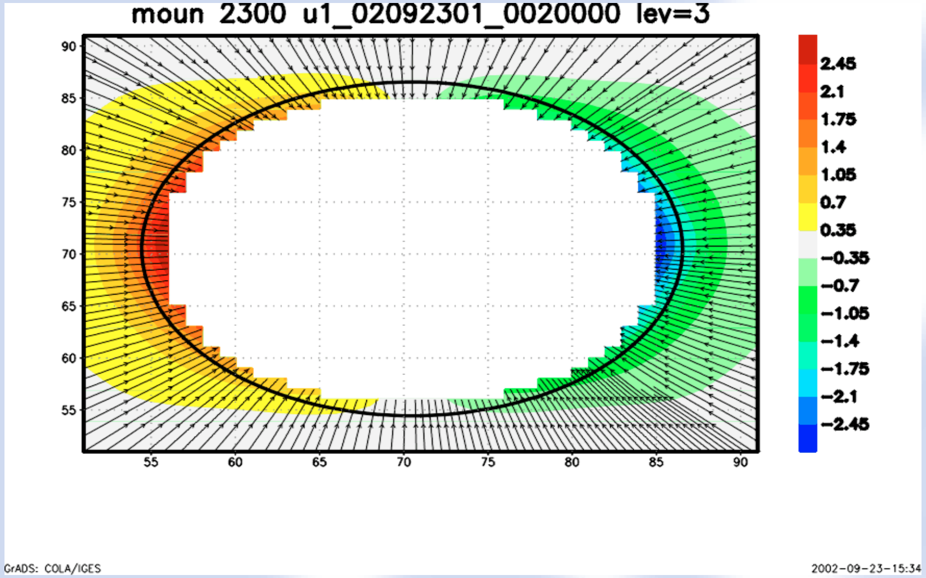
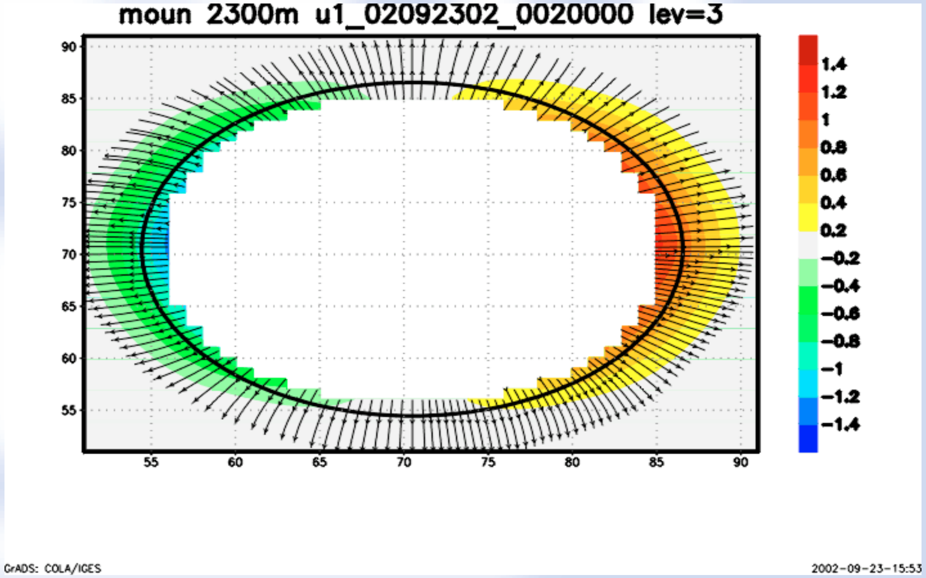
Test

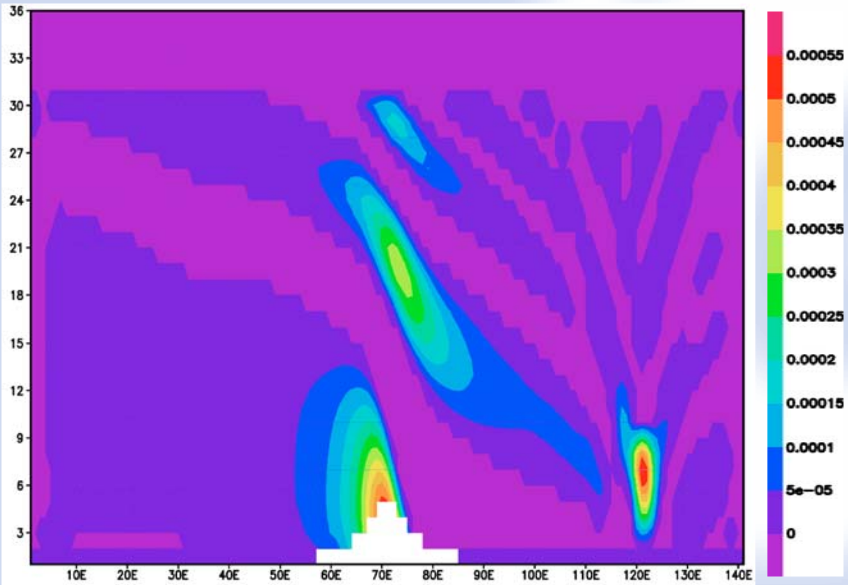


Day

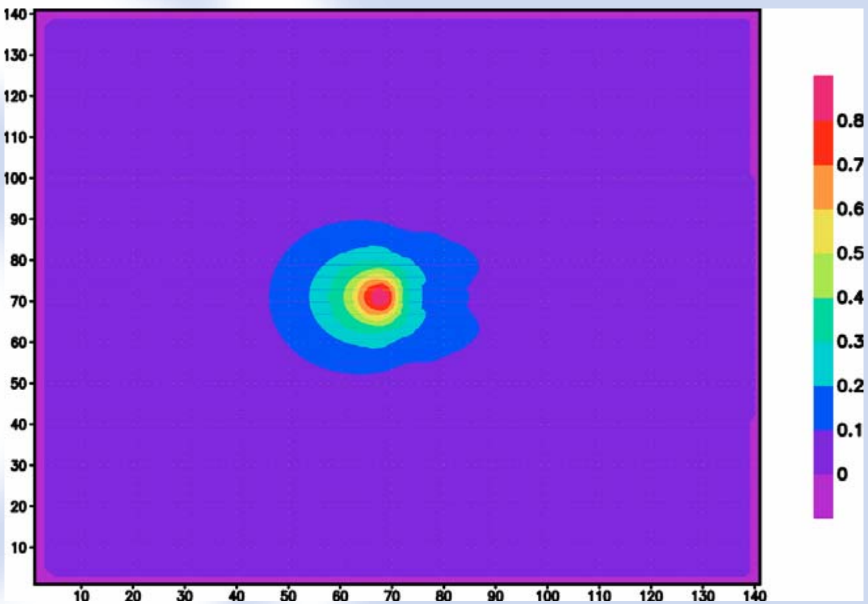


Night

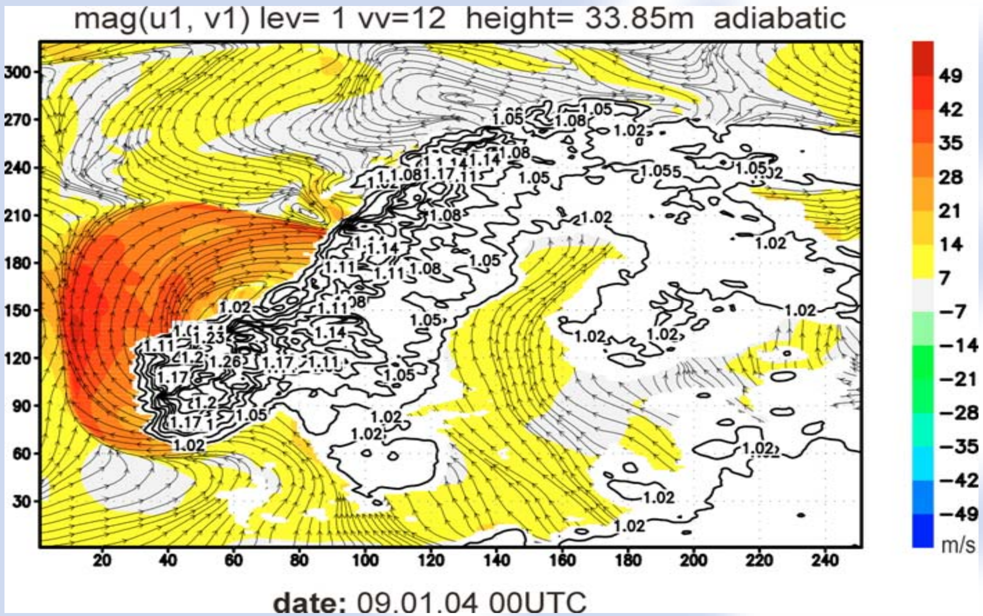
Cloud water

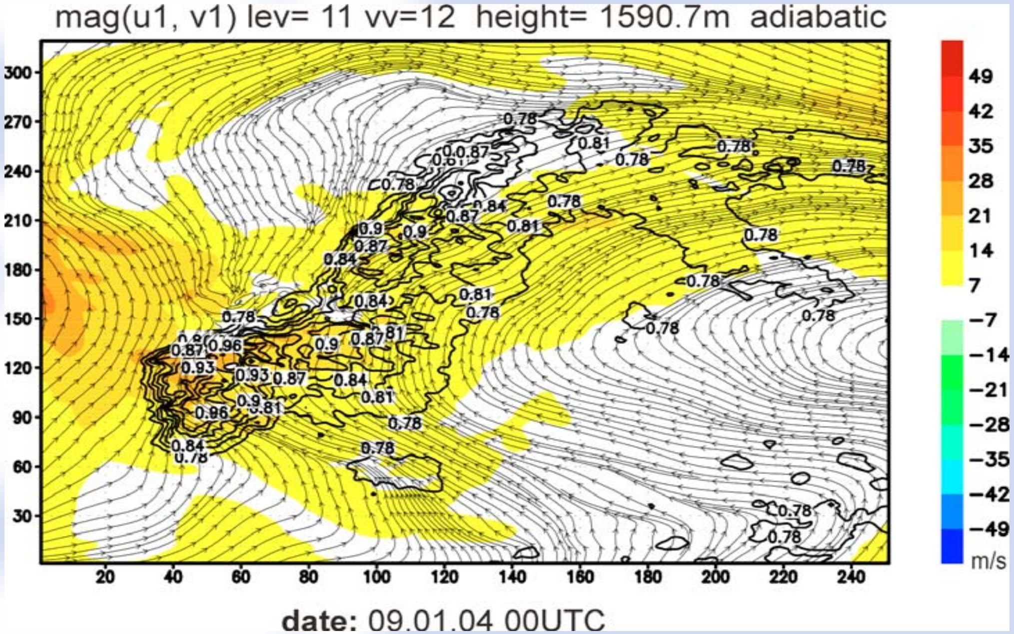


Precipitation

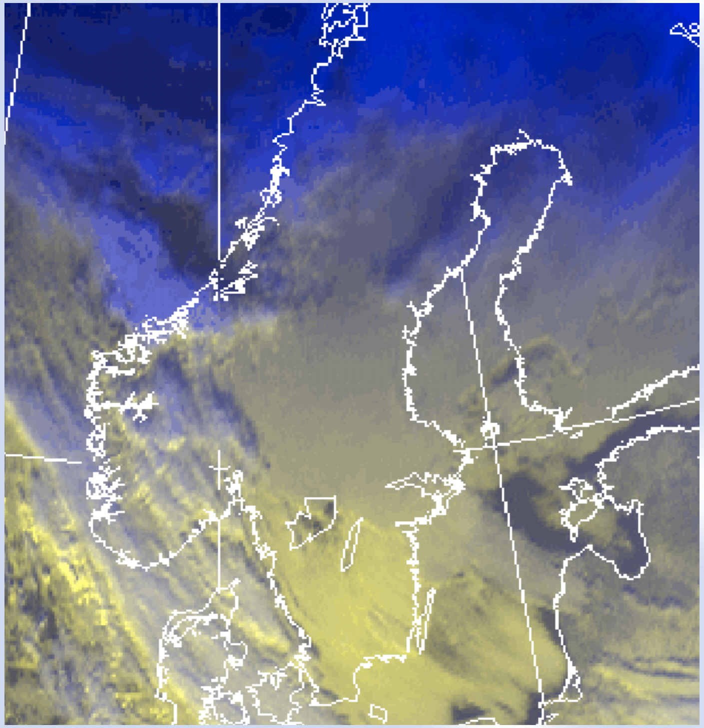


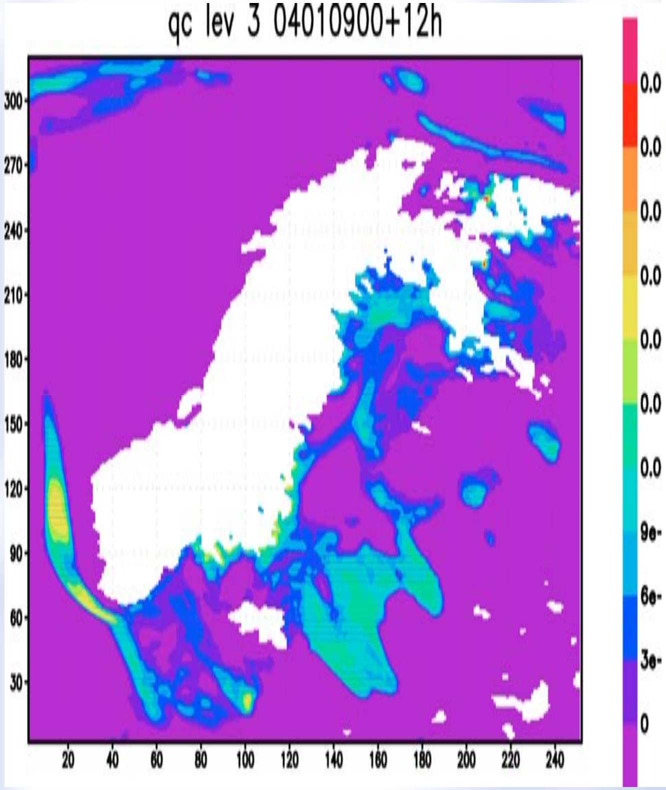
No physics / Physics





Low stratus




qc lev 3 04010900+12h
300
270
240
210
180
150
120
90
60
30
20 40 60 80 100 120 140 160 180 200 220 240
0.0
0.0
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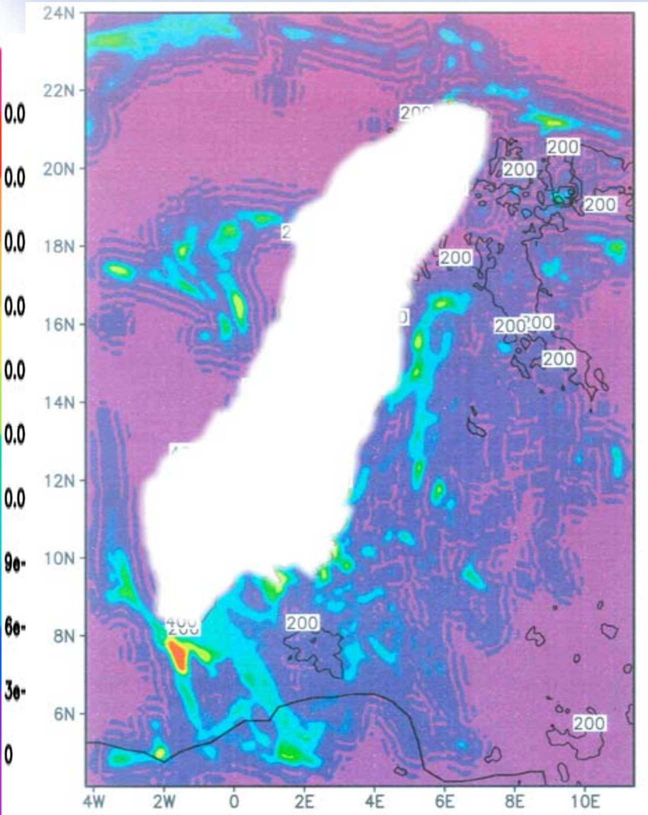
0.0
9e-
6e-
3e-
0


24N
22N
20N
18N
16N
14N
12N
10N
8N
6N
4W 2W 0 2E 4E 6E 8E 10E
200

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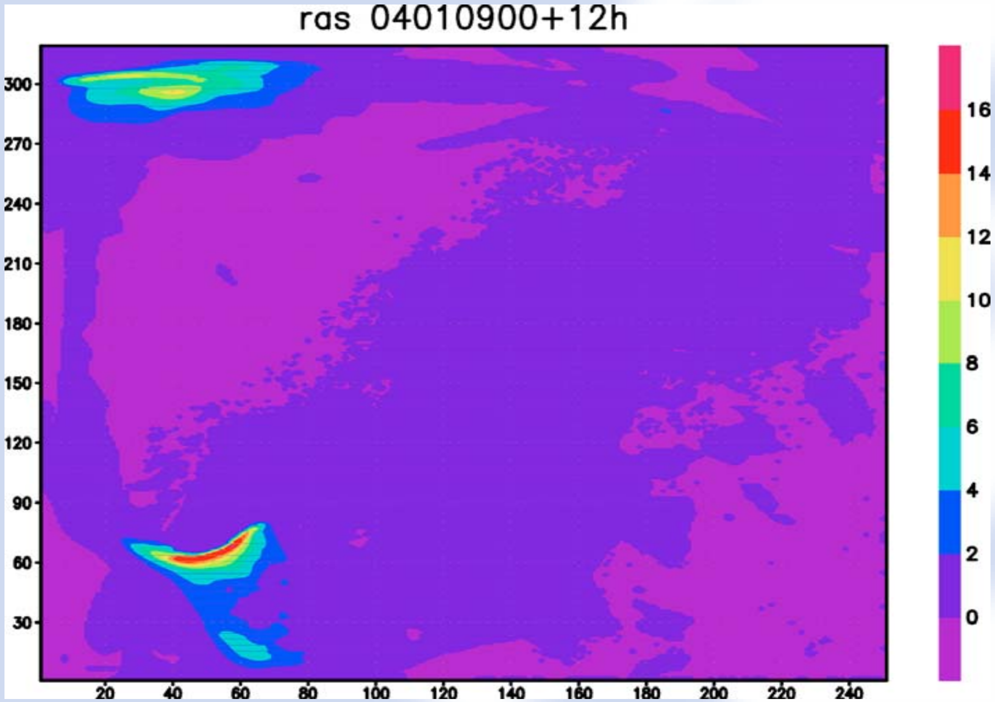
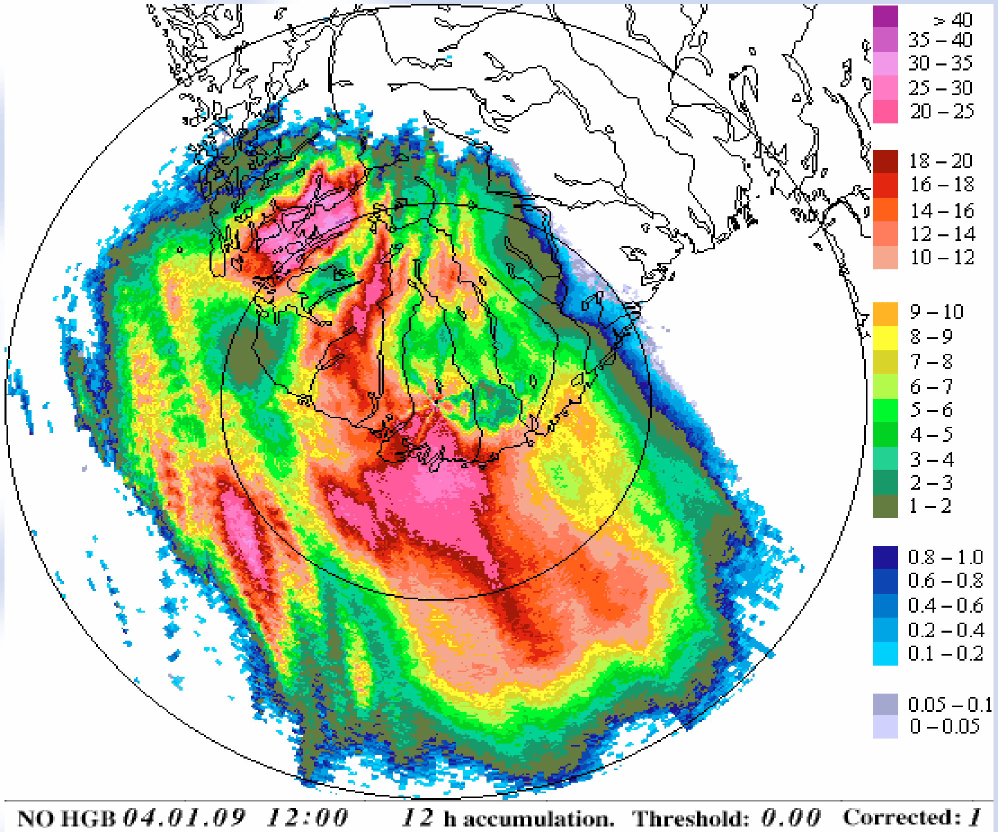
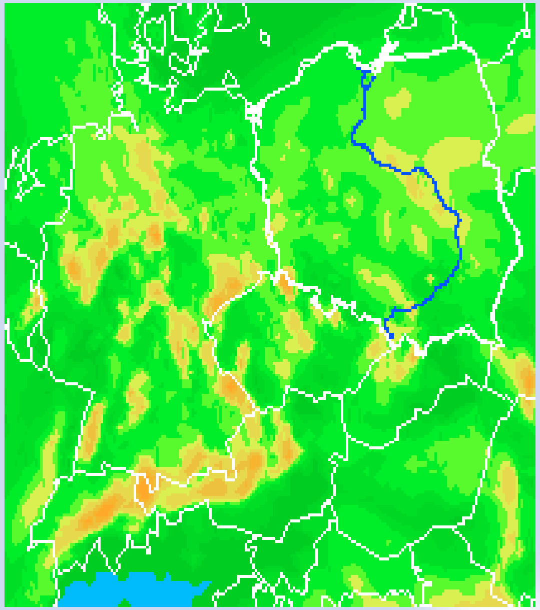
Z-coordinate

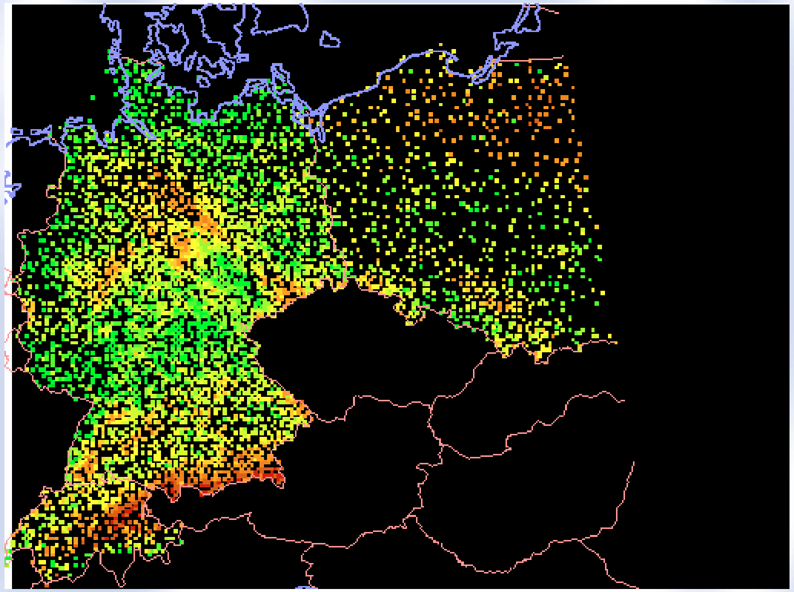


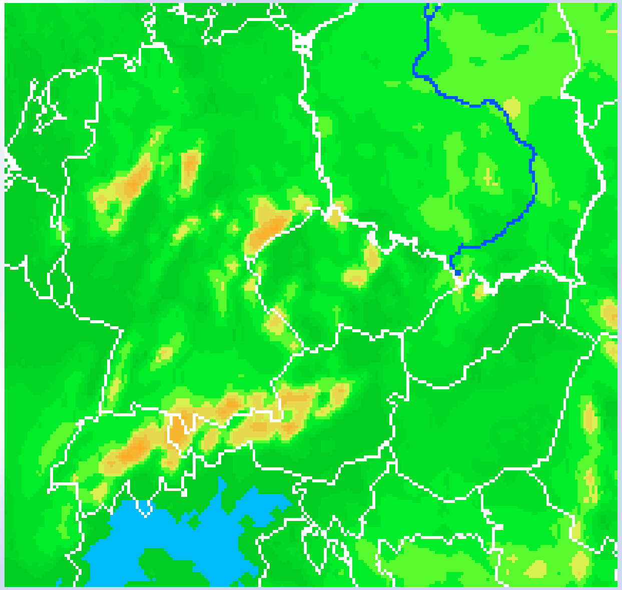


28039700 6 to 6

- LM-Z OBS LM-tf







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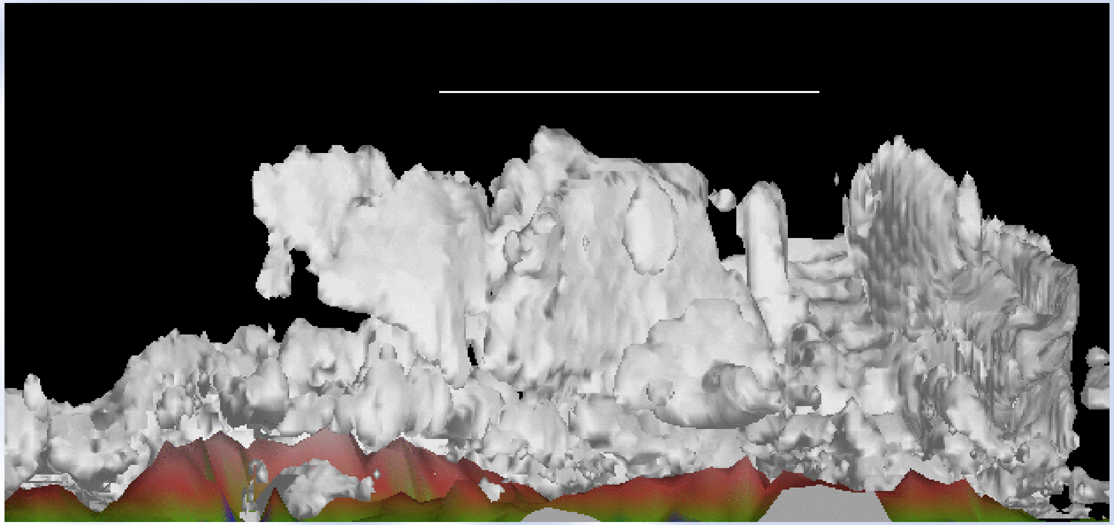
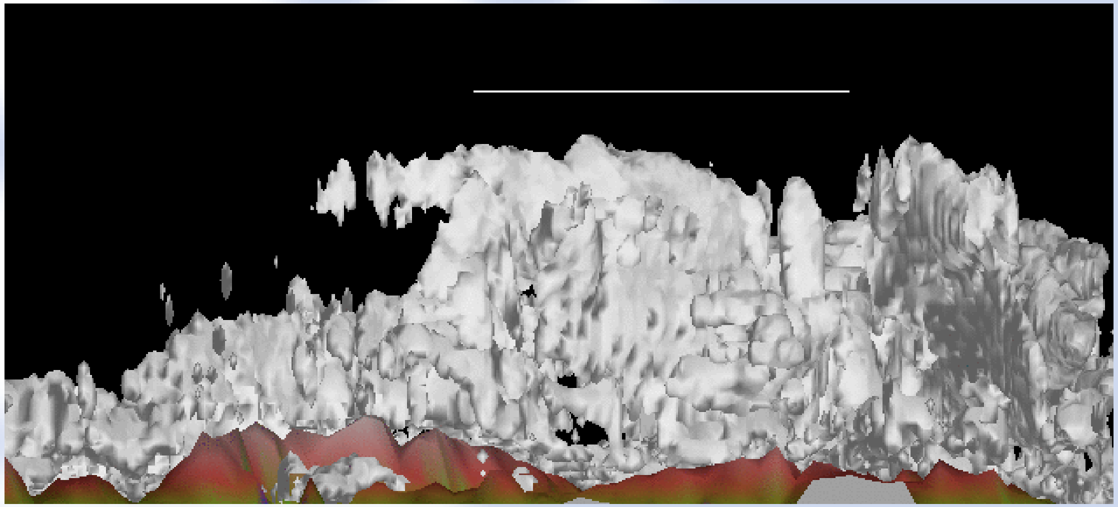
- LM_Z



- LM_tf



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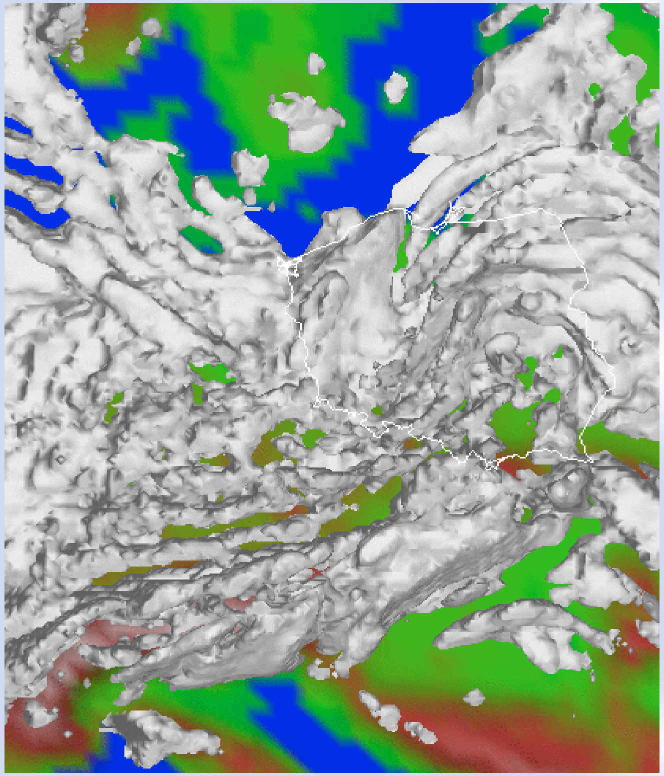
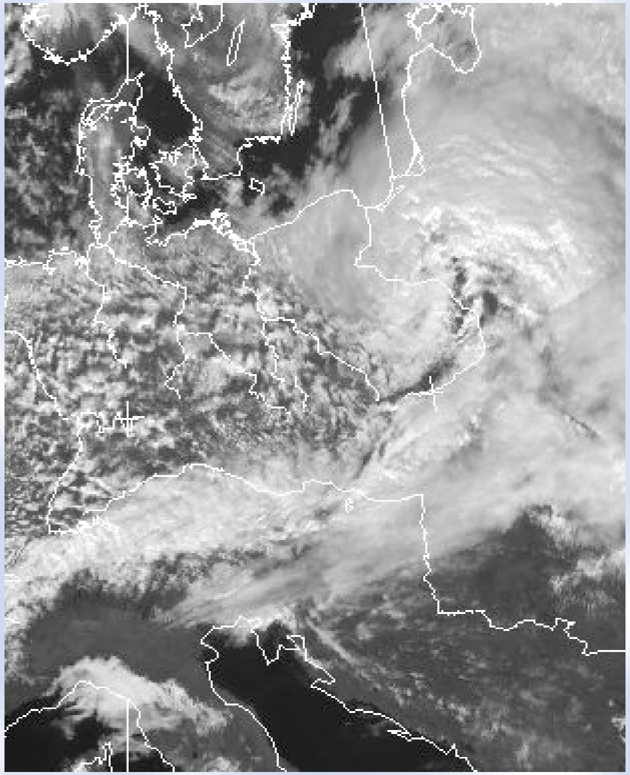
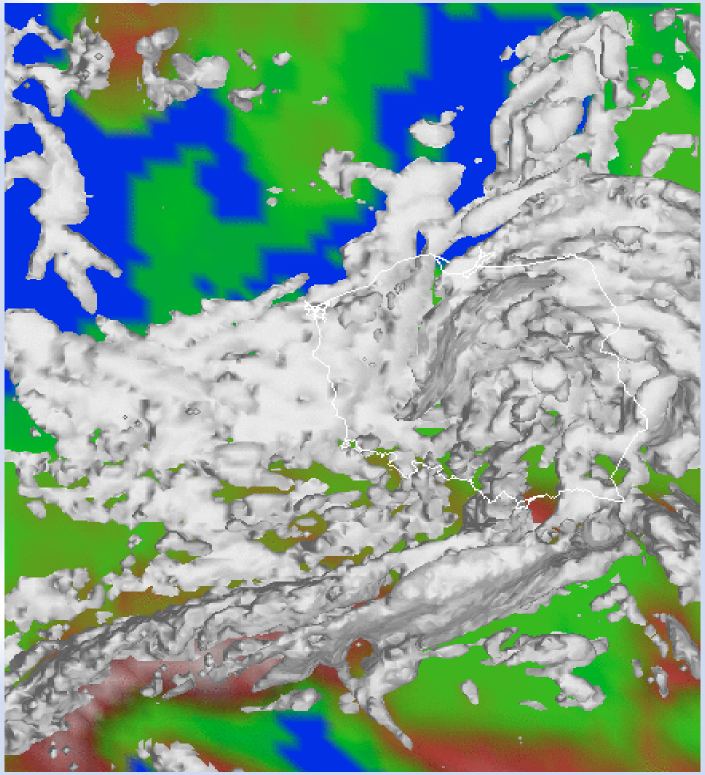
- LM_Z

LM_tf

Current state of LM-Z

- Idealised tests with bell shaped mountain show the expected feature of improved wind, temperatures and precipitation
- SKANIA Test with LM-Z reveals no problems
- First realistic runs are encouraging. LM-Z compared to LM-tf showed large improvements concerning the prediction of clouds and precipitation.

Some Desirable features of Next Generation NH Models

- Sufficient Accuracy/high (Spatial) Approximation order (Achieved with RK and SL)
- A problem is the combination of high order and conservation
- Approximation condition orography: **Z-Coordinate**
- Both RK and SL have currently the computational expense of second order centred differences / Leapfrog
- One of the keys to numerical efficiency is **implicit time integration**
- Global mesoscale models (achieved on Isocaheronlike grids) may be feasible within 15 years

Implicit Approaches

Example : $\partial h / \partial t = \partial (uh) / \partial x$ u :constant field; h : dynamic field

Nonlinear : $h^{n+1} - h^{n} = dt([uh]_x^{\,n} + [uh]_x^{\,n+1})/2$

Tangent linear : $h^{n+1} - h^{n} = dt(u_x h^{n+1} + u_x h^{n} + u^{n+1} h_x + u^{n} h_x)/2$

Locally homogenised : $h^{n+1} - h^{n} = dt(uh_x^{\,n+1} + uh_x^{\,n})/2$

Direct Methods for Locally Homogenized SI

- The Equations of Motion are homogeneously linearised at each Grid Point
- At Each Grid-Point a Problem of Constant Coefficients is Defined
- For Each Grid-Point The Associated Linear Problem Can be Solved Using an FT and a Linear Problem Specific to Each Grid-Point
- The GFT (Generalised FT) Computes the Results of the **Different FTs** Using **One Transform**
- The numerical cost of GFT is Simlar to that of an FT
- A Fast GFT exists similar to Fast FT

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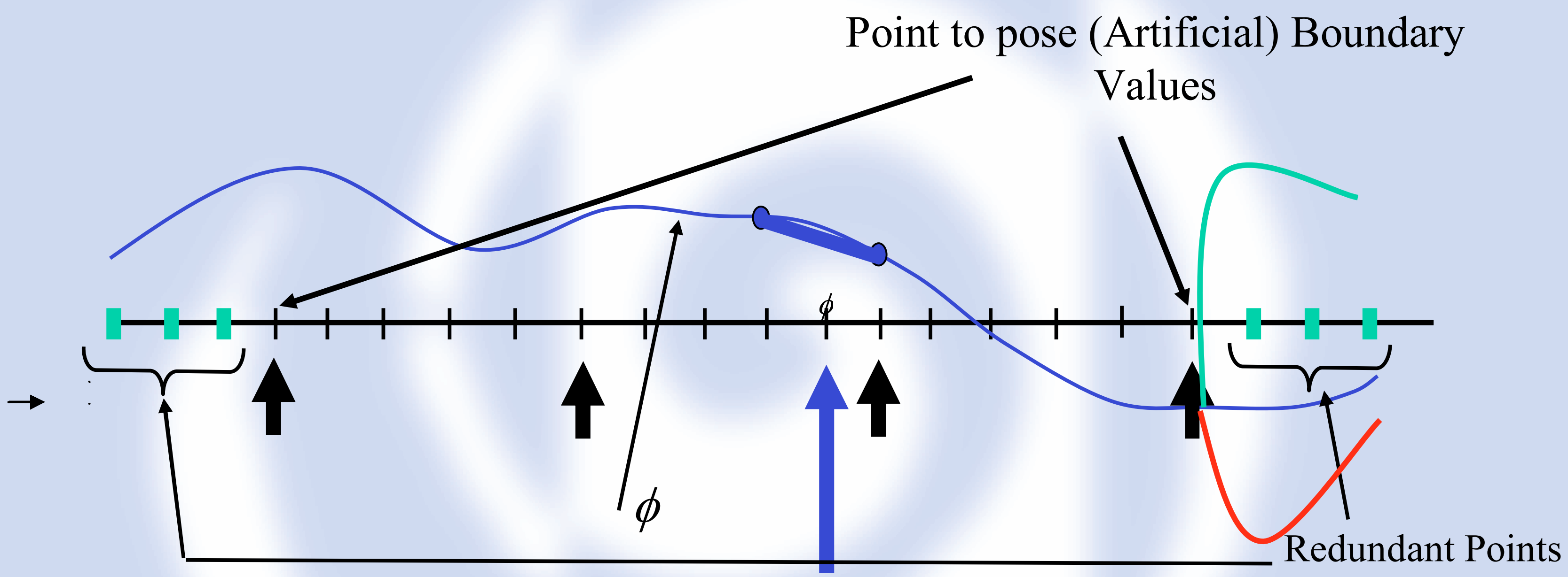
Boundary- and Exterior Points



- Redundant points can be included in the FT
- The result of the time-step does not depend on the continuation of the field to redundant points

Organisation of the Implicit Time-Step

- The Fourier Coefficients are the same for the grid Points of a Subregion
- The Linearised Eqs. are different for each Gridpoint
- In Case of only One Subregion the Support Points of the derivative ϕ_{0x} are the Boundary Values
- ϕ_{0x} does not create time-Step Limitations
- GFT Returns the Grid-Point-Values after doing a Different Eigenvalue Calculation at Each Point

The SI Timestep

Subtract Large Scale Part of each function

Compute Fourier Coefficients

Choose Gridpoint

Compute next time level in Fourier Space

Transform Back

Use Result Only for Chosen Grid-Point

Do the Above for all Other Grid Points

Set Boundary Values as in Finite Difference Methods

Example:1-d Schallow Water Eq

$$\partial u / \partial t = -u \frac{\partial u}{\partial x} - \frac{\partial h}{\partial x}$$
$$\partial h / \partial t = -\partial (hu) / \partial x$$

SI Scheme :

$$u^{n+1} - u^n = -dt(u^n (u_x^{n+1} + u_x^n)/2 + (h_x^{n+1} + h_x^n)/2)$$
$$h^{n+1} - h^n = -dt(h^n (u_x^{n+1} + u_x^n)/2 + u^n (h_x^{n+1} + h_x^n)/2)$$

Definitions:

$$\Delta u = u^{n+1} - u^n ; \Delta h = h^{n+1} - h^n ;$$
$$rsu = -u^n u_x^n - h_x^n ; rsh = u^n h_x^n - h^n u_x^n$$

SI Scheme:

$$\Delta u = rsu - dtu^n \Delta u_x - dt\Delta h_x = rsu - dt(u^n \Delta u^0{}_x - \Delta h^0{}_x) - dtu^n \Delta u'_x - dt\Delta h'_x$$
$$\Delta h = rsh - dtu^n \Delta h_x - dth^n \Delta u_x = rsh - dt\underbrace{(u^n \Delta h^0{}_x - h^n \Delta u^0{}_x)}_{\text{Periodicity Terms (=0in the following)}} - dtu^n \Delta h'_x - dth^n \Delta u'_x$$

Operation in Fourier Space

SI Scheme:

$$\Delta u = rsu - dtu^n \Delta u_x - dt\Delta h_x = rsu - dtu^n \Delta u'_x - dt\Delta h'_x$$

$$\Delta h = rsh - dtu^n \Delta h_x - dth^n \Delta u_x = rsh - dtu^n \Delta h'_x - dth^n \Delta u'_x$$

In the following rsu, rsh, u^n, h^n will be taken at some chosen gridpoint.

Definition: $\tilde{\phi} = F\phi$

Linear Equations at the chosen gridpoint:

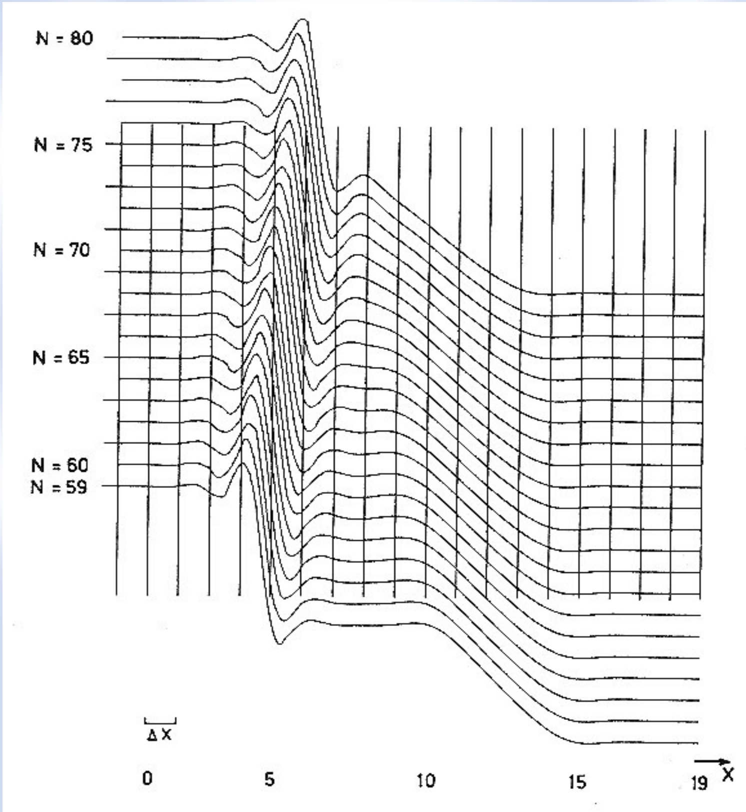
$$\tilde{\Delta u} - dtu^n \tilde{\Delta u}_x - dt\tilde{\Delta h}_x = \tilde{rsu}$$

$$- dtu^n \tilde{\Delta h}_x + \tilde{\Delta h} - dth^n \tilde{\Delta u}_x = \tilde{rsh}$$

$\tilde{\Delta u}, \tilde{\Delta h}$ are obtained from the solution of the linear equation.

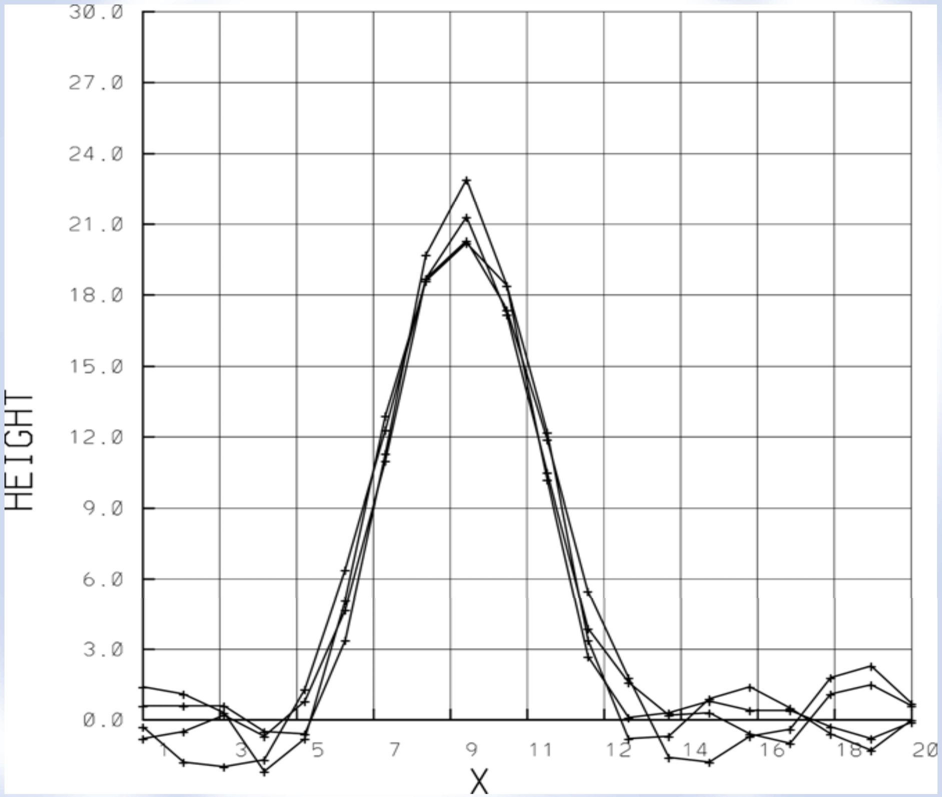
The back transformed fields $\tilde{\Delta u}, \tilde{\Delta h} = F^{-1}\tilde{\Delta u}, F^{-1}\tilde{\Delta h}$ will be taken at the chosen gridpoint only.

Computational Example: 1-d Schock Wave

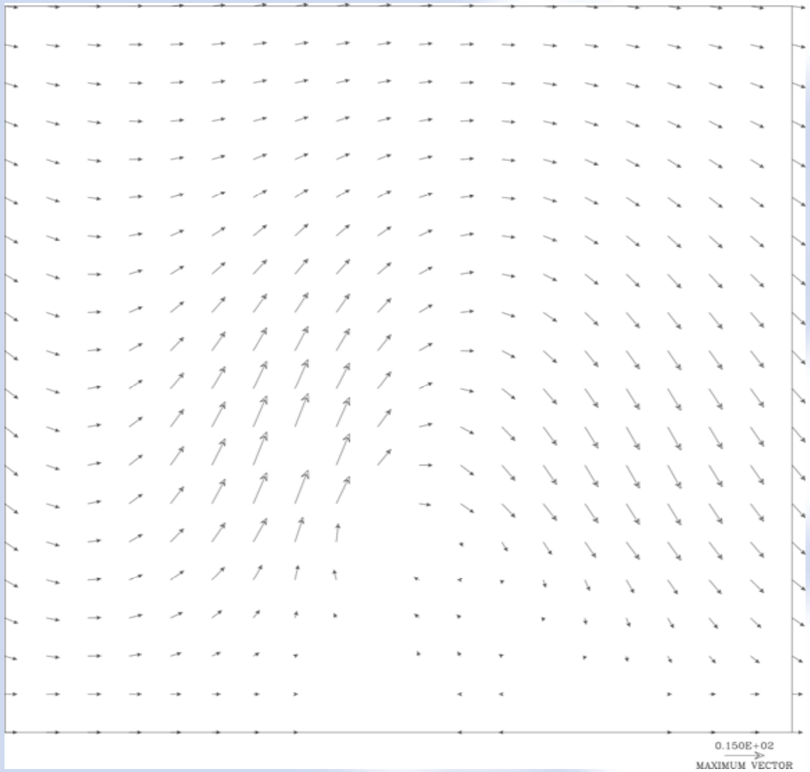


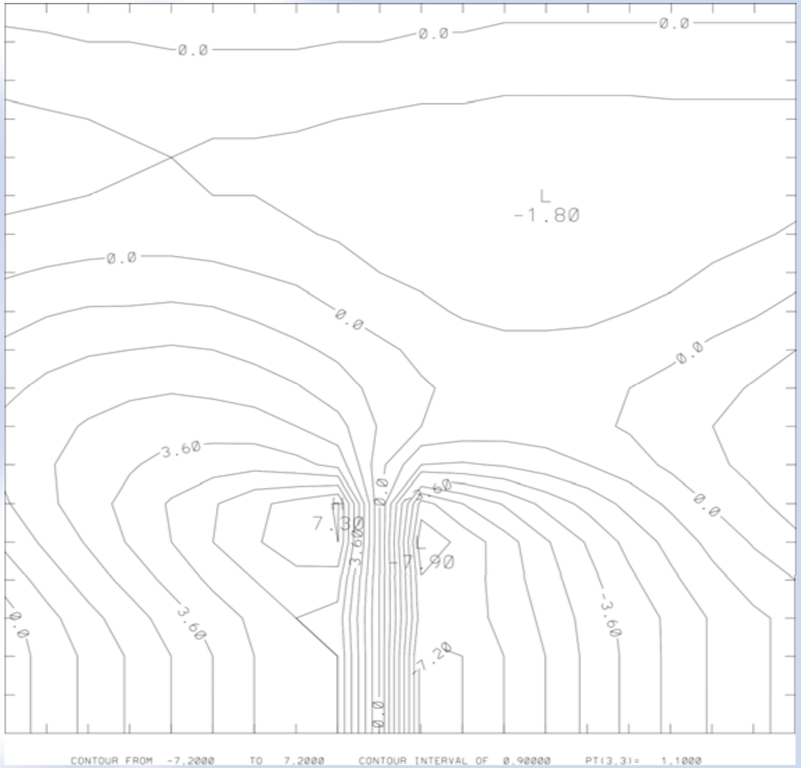
The time Step could be increased up to the CFL of advection (10 m/sec)

1-d Shallow Water Equs. Periodic boundaries with bell shaped initial disturbance



2-d Shallow Water Equs. with Barrier





Implicit Conclusions

- A direct si- method was proposed
- The method is based on a generalised Fourier Transform
- The generalised FT is potentially as efficient as the FT (fast FT)
- 1-d and 2-d tests have been performed

A family Grids on the sphere

- Current approach: lat lon and Kurihara will not be discussed
- The Baumgardner principle
- Great circle grids for triangles and rhomboids/ Isocahedron, cube and 8 surface body grid stencils
- Computation of directional derivatives of a given order
- Overview of numerical concepts
- Interpolation
- Rooftile grid
- Dual grid and conservation
- Choice of options and modular workplan
- Conclusions

Points of concern in this lecture

- Is o3 important? (“next generation Dynamics”? Can issues addressed in WRF and LMK be incorporated? Can current nh-solutions/ solutions under reserach be incorporated: i.e. FT Preconditioners?)
- Model development with limited resources? (Are current developments incorporated easily? Fallback positions? Baumgardner doctrine?)

The Baumgardner principle

(Rules of good behaviour on triangels
Proven for o2, not yet for o3

Supported by the success of Skamarock nesting)

- No global coordinate
- Keep approximation order at grid interfaces
- The faithful are rewarded by having no problems carrying plane discretisations to the sphere

Desirable features of discretisations on the sphere

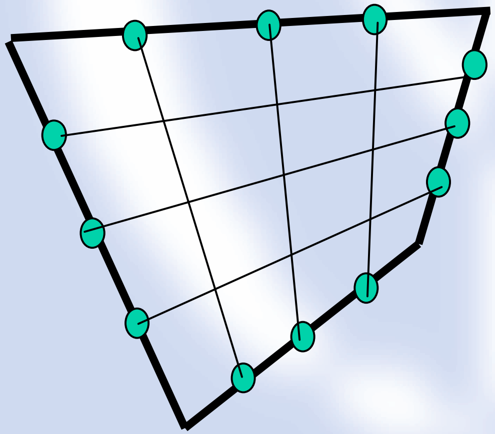
- **Nonhydrostatic**
- **Accuracy: Order 3 or higher in space [and time]**
- (Observation of approximation conditions: smoothness $\left(\partial/\partial x\right)^4$ (for third order schemes), Smooth physics interface, smooth orography (dh<dz) or z-coordinate)
- **Conservation: mass, energy**
- **Efficiency (computer time and development time)**
- (Positivity of advection: flux correction)
- (Nesting option: Skamarock method)
- **Ability to incorporate developments for nh models**

Quasi regular grids

- Structured (index i,j)
- Each line of points j,i j,i+1 j,i+2............ is on a great circle
- Obtained by projecting bilinear grids to the sphere
- Projection of any vector r to the sphere with image ra: $r\vec{a} = \vec{r} \,/\, |\,\vec{r}\,|$

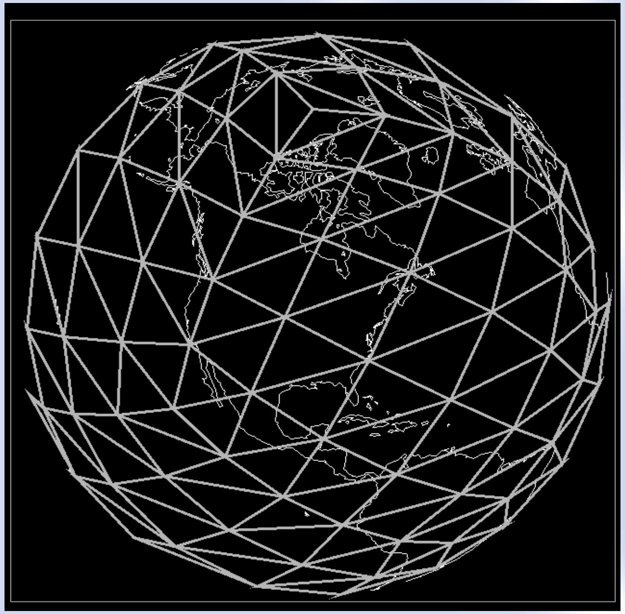
Bilinear grids

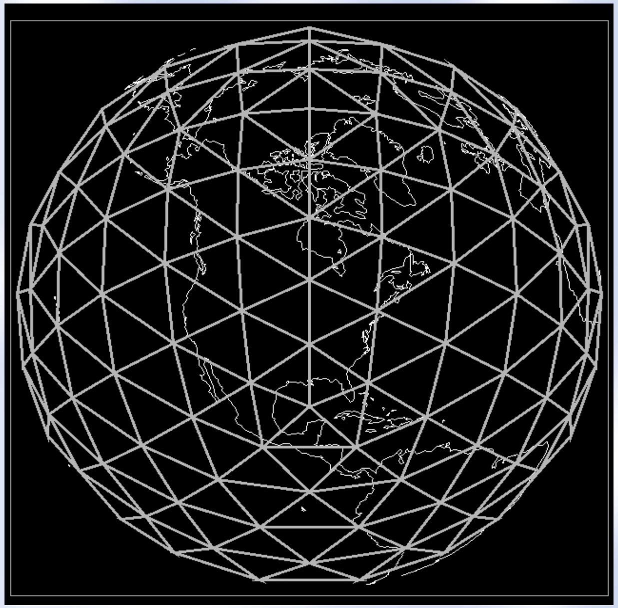
- Four points r1,r2,r3,r4 may have any position in space
- Divide the sides of the rhomboid equally and connect opposite points
- Bilinear grid theorem: each coordinate line intersects each line of the crossing coordinate line family. The grid is regular in each direction.

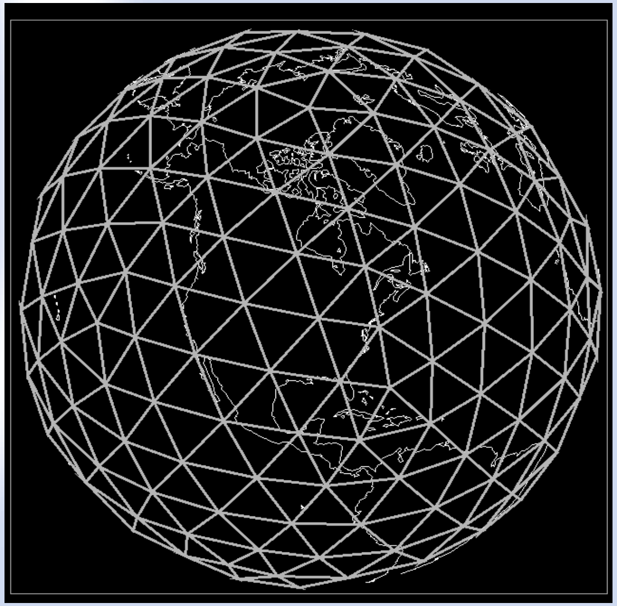


Orange cut grids

- NP=3　　　　NP=4　　　　NP=5



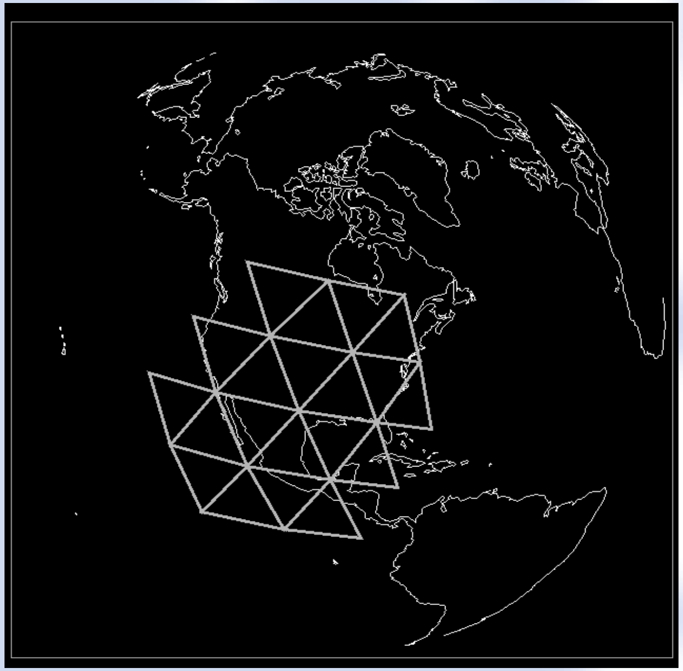




Grid Stencils Baumgardner

- Edges grid
- Order3

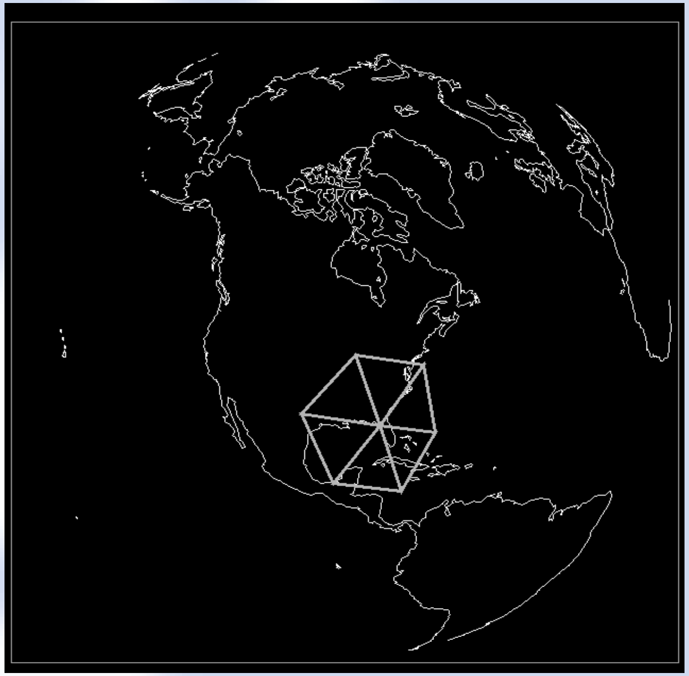
Redundancy 19:9



Edges grid

Order 2

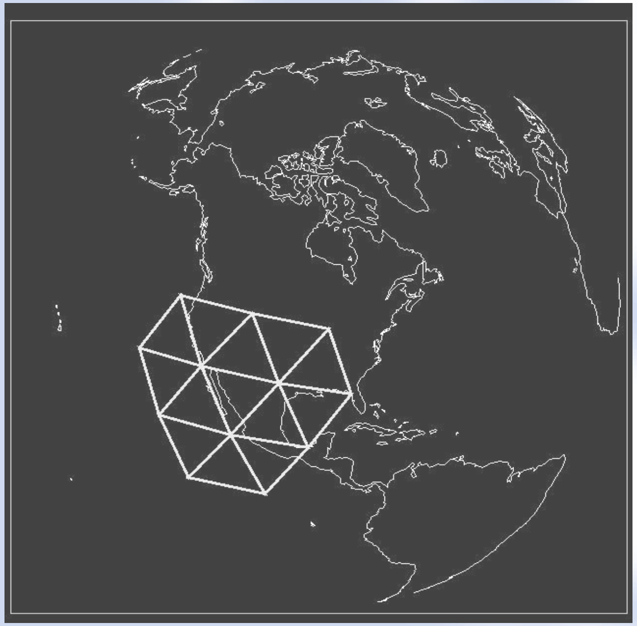
Redundancy 6:5 or 5:5



Grid Stencils Baumgardner

- Area grid
- Order3

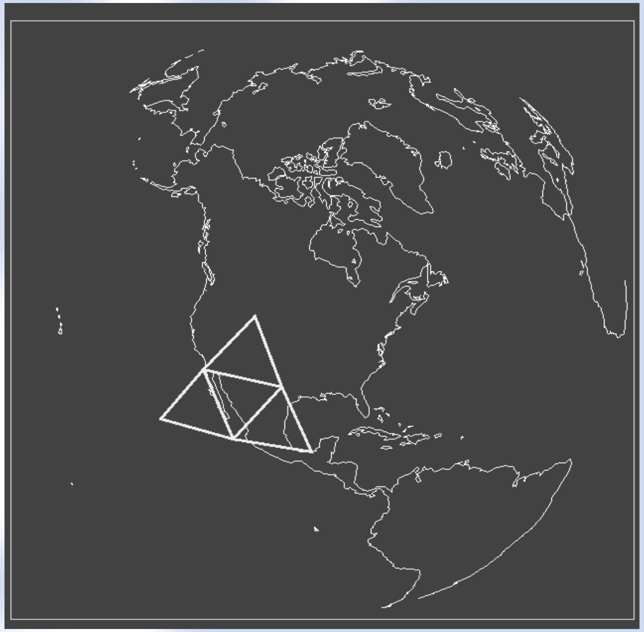
Redundancy 13:9



Area grid

Order 1 (Finite Volume)

Redundancy 4:3



Great Circle Grid Stencils

- Edges grid
- Order 3
- Local coordinate, for example local geographic
  - Possibility 1:

    irregular, but locally nearly regular grid

    Non orthogonal grid
  - Possibility 2:

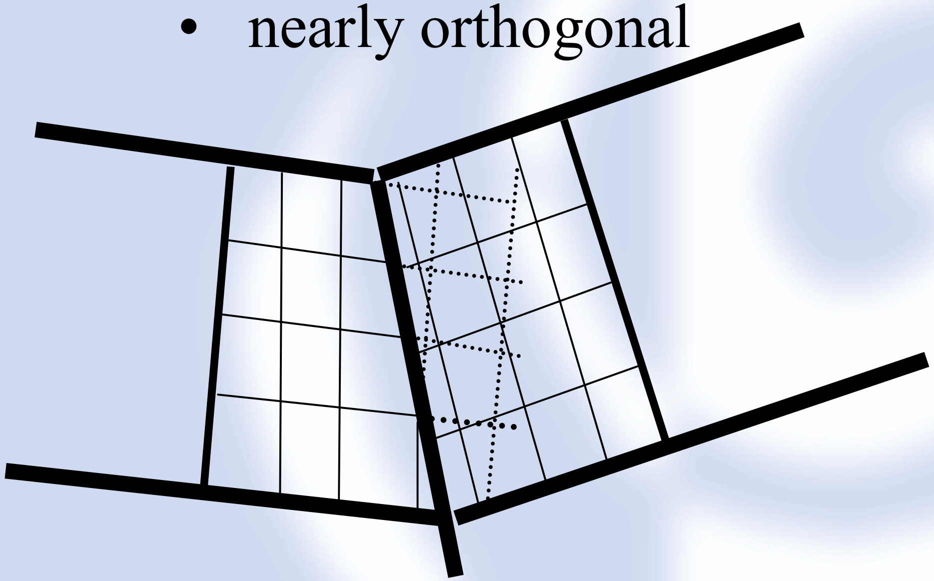
    Rooftile grid: regular and nearly orthogonal

Interpolation

- Grid redundancy is an issue for all methods relying on interpolation
- Cascade interpolation for regular grids
- Serendipidity interpolation: the part going into 2d and 3d look like linear.
- Serendipidity grids replace forecasts of some points by order consistent interpolation

Rooftile Grids

- Grid matching at most boundaries
- nearly orthogonal



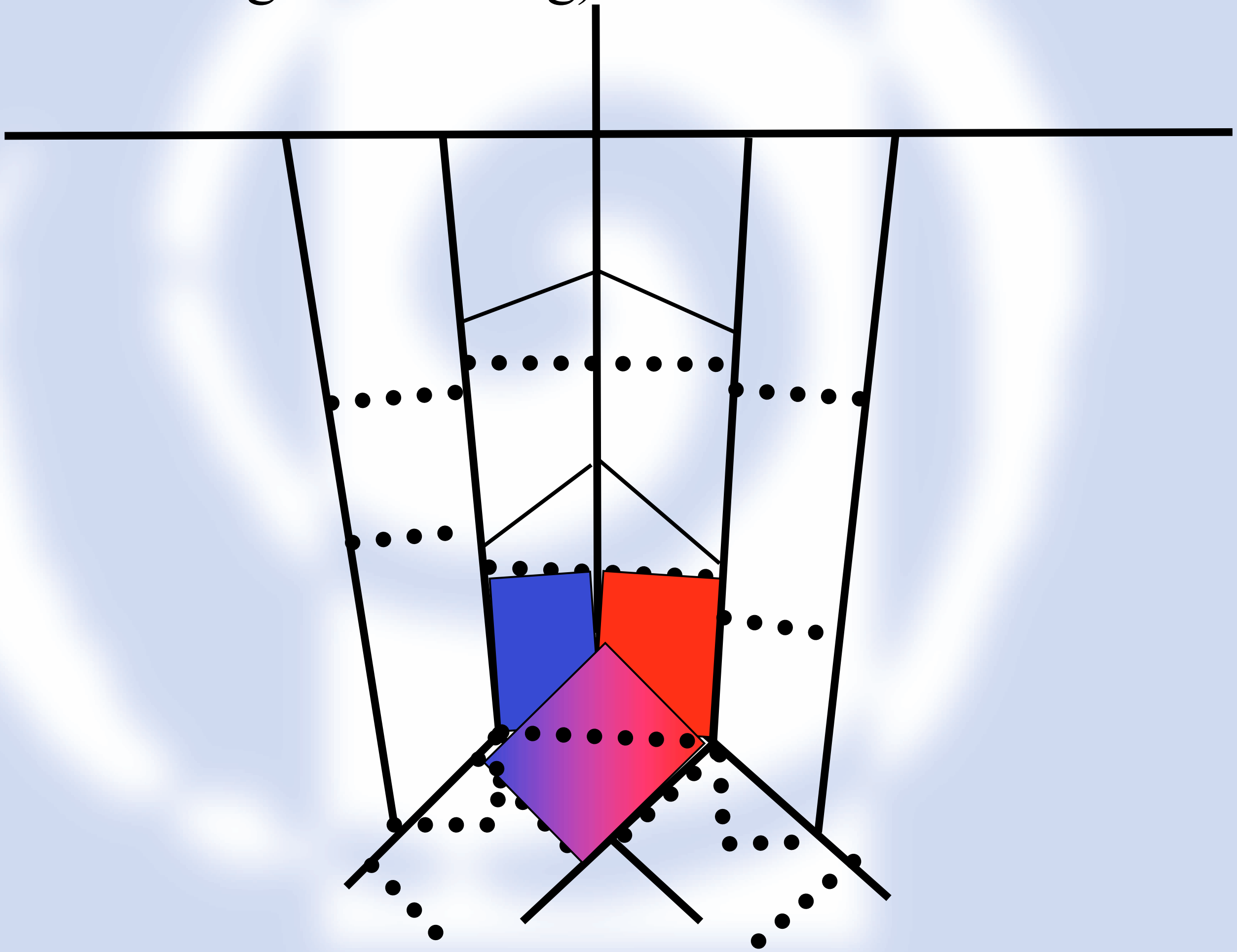
•Interpolation O3 for boundary values

•for 100 points per tile grid irregularity is 1 %

Deutscher Wetterdienst

Rooftile Grids, 4-Body

- Triangles are used to match areas (implying irregular shaped cells or double grid covering). All other boundaries match



Discretisation Options, Based on Interpolation

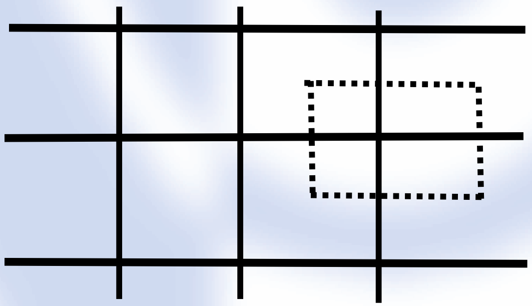
- Finite Volumes: Bonaventura choice, best on regularised grids, conservation possible, often low order, tested by Ringler and Steppeler
- Baumgardner: suitable for somewhat irregular grids, tested for order 2
- Baumgardner Order2: Amplitudes on edges; small grid redundancy
- Baumgardner Order3: Amplitudes on triangle surfaces, very irregular grid for plane waves, yet untested, (some grid redundancy)
- Great circle grids: very similar to limited area discretisations, order 2,3 easily possible, RK, SI, SL, adaptation of all local developments easy (grid redundancy no problem)
- Tiled grids: very uniform grids (~1%), less elegant look, spectral elements possible)
- Serendipidity grids (can be derived as a further development of SE)

Saving factors of Discretisations

• Finite Volumes:	1	
• Baumgardner Order2:	1	
• Baumgardner Order3:	1	
• Great circle grids: RK, SI, SL	1 now	3 seem possible
• Tiled grids:	1.5	
• Serendipity grids	3 1.3^{-1}	
• Unstructured	2^{-1}	
• Conservation		

Dual grid and conservation

- Use conservation form, compute fluxes
- 1st possibility: WRF-method
- 2nd: Flux correction
- Issue: order of representation of the conserved quantity



Serendipidity grids

- Grid interpolation: 3rd order
  - In a square grid grid redundancy is so large (64:15) that it becomes a problem
  - Some of the redundant grid points can be interpolated from the non redundant ones in an order consistent way, resulting in:
    - efficient interpolation
    - Saving of 27:7 from redundantly forecasted points
    - Easier for posing boundary values and theoretically more satisfying in this respect
  - The field is known to 4 rth order error at all points, meaning that c-grid structures for fast waves can be generated.
- Spectral elements:
  - Some of the basis functions can be shown to contribute in a neglegible way
  - avoiding these contributions leads to the same grid structures and saving as above

Conclusions

- RK achieves sufficient accuracy, all schemes discussed are about equal in efficiency
- Increase of efficiency by a factor of 10 is possible by implicit methods and serendipidity grids
- Combined Order 3 and conservation is possible using the dual grid method
- Great circle grids based on the isocahedron allow to make local models global