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2) Conference on **Current Efforts Toward Advancing the Skill of Regional Weather Prediction. Challenges and Outlook,** *April 20 - 22*

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Model error dynamics and regional Modeling

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Model error dynamics and regional modeling

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RMI

Introduction

- Our ability to predict the evolution of the atmosphere is affected by presence of two different types of errors:

  - initial condition errors

    ⇨ sensitivity to initial conditions

  - model errors

Following the pionnering works of Thompson (1957) and Lorenz (1963), and the development of chaos theory, the S.I.C. of atmospheric flows has been deeply investigated and is now quite well understood (see e.g. Predictability seminars of the ECMWF).

⇨ Ensemble predictions, data assimilation

Although model error is known to affect prediction since the early development of atmospheric predictions, its dynamics has been poorly investigated.

Not surprising? Sources of model error are quite diverse.

- subgrid scale physics
- numerical approximations
- boundary conditions (surf+horizontal limits)
- hydro vs non-hydro

....

Necessity of better forecasts ? → Knowledge of the evolution of model errors

One of the first attempts to look at the dynamics of this error (Leith, 1978)

$$\frac{dE}{dt} = \lambda E + S$$

S: error source
E: the error (quadratic or RMS)

Sol: $$E(t) = E(0)e^{\lambda t} + \int_0^t S e^{\lambda(t-\tau)} d\tau$$

$E(0) = 0$ → $$E(t) = \frac{S}{\lambda}(e^{\lambda t} - 1)$$

$S = 0$ → $$E(t) = E(0)e^{\lambda t}$$

Leith assumed S constant
No distinction between quadratic or rms errors

Important: without initial error, the error evolves for short times linearly

Aim of the present work

- Discuss recent results on model error dynamics in simple systems
- Results on model error dynamics in a state-of-the-art regional model: the ETA model

Model error dynamics

Formalism of Nicolis (2004)

$\vec{x}$ Model variables

$\vec{x}_N, \vec{y}_N$ Real variables

The model $$\frac{d\vec{x}}{dt} = \vec{F}(\vec{x}, \{\vec{\mu}\})$$

The real system $$\frac{d\vec{x}_N}{dt} = \vec{F}_N(\vec{x}_N, \vec{y}_N, \{\mu'\})$$

$$\frac{d\vec{y}_N}{dt} = \vec{G}_N(\vec{x}_N, \vec{y}_N, \{\mu'\})$$

⇨ Variables not described by the model

$\vec{x}$ and $\vec{x}_N$ span the same phase space

One can distinguish between two types of model errors:

1. Parametric errors (in the same phase space)
2. Phase space truncation errors

1. Parametric errors

1st possibility $\mu = \mu' + \delta\mu$

2nd possibility

$$\frac{d\vec{x}_N}{dt} = \vec{F}_N(\vec{x}_N, \vec{y}_N, \{\mu'\})$$

$$\vec{y}_N = \vec{H}(\vec{x}_N, \{\mu'\})$$

Diagnostic relation

$$\frac{d\vec{x}_N}{dt} = \vec{F}_N(\vec{x}_N, \vec{H}(\vec{x}_N, \{\mu'\}), \{\mu'\}) = \vec{F}_N'(\vec{x}_N, \{\mu'\})$$

In this case and provided that parametric error is small,
One gets for the model error evolution

$$\frac{d\delta\vec{x}}{dt} = \left(\frac{\partial\vec{F}}{\partial\vec{x}}\right)_{\vec{x}_N(t)} \delta\vec{x} + \delta\vec{\phi}$$

$$\delta\vec{x} = \vec{x} - \vec{x}_N$$

$\delta\phi(t)$ is the model error source term

Without
Initial error

$$\left\langle E^2(t)\right\rangle = \int_0^t d\tau' \int_0^t d\tau \left\langle \delta\vec{\phi}^*(\tau') M^*(t,\tau') M(t,\tau) \delta\vec{\phi}(\tau)\right\rangle$$

where M is the fundamental matrix and <> is the average
over the different initial conditions.

2. Truncation errors

Truncation error is a bit more involved. Let us integrate formally the equations for x and x_N:

$$\vec{x}(t) = \vec{x}(0) + \int_0^t d\tau \vec{v}(\vec{x}(\tau), \{\mu\}) \qquad \vec{v} = \frac{d\vec{x}}{dt}$$

$$\vec{x}_N(t) = \vec{x}_N(0) + \int_0^t d\tau \vec{v}_N(\vec{x}_N(\tau), \vec{y}_N(\tau), \{\mu\})$$

Making the difference and averaging over different initial conditions, one gets

$$\langle E^2(t) \rangle = \int_0^t d\tau' \int_0^t d\tau \langle (\vec{v}_N(\tau') - \vec{v}(\tau')) * (\vec{v}_N(\tau) - \vec{v}(\tau)) \rangle$$

In both cases, when developping in Taylor series and assuming that the model perturbation is not delta correlated, one gets for short times

$$\langle E^2(t) \rangle = t^2 \langle (\vec{v}_N(0) - \vec{v}(0)) * (\vec{v}_N(0) - \vec{v}(0)) \rangle \qquad \tau_v = 1/|2\mu_-|$$

This regime is only valid for times smaller that the one associated with the largest negative Lyapunov exponent .

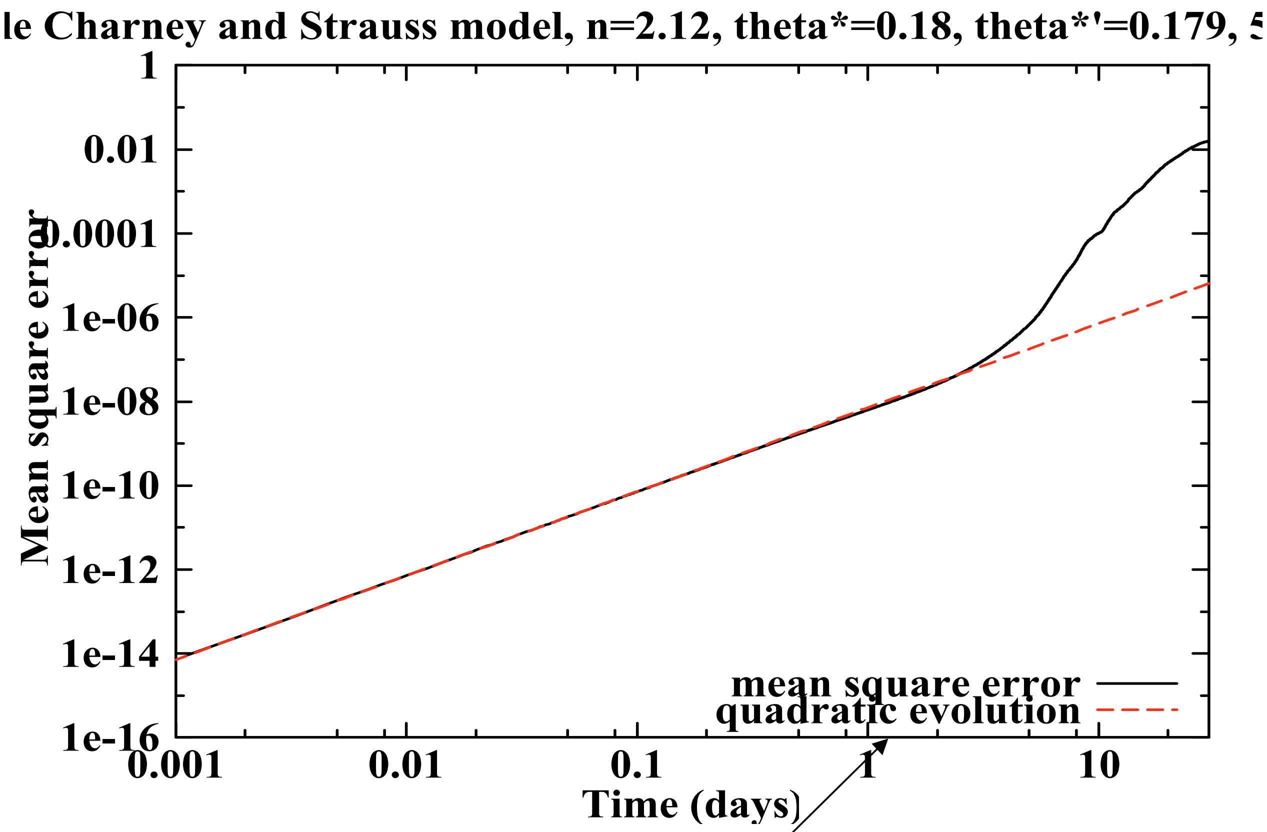
Beyond this period, higher order terms are necessary involving the whole spectrum of Lyapunov exponents (and their respective time scales).

(see Nicolis, 2004)

More specifically, one expects to see the impact of the most stable directions before the other ones.

Charney-Strauss Model

12 variables



Time scale associated
With the largest amplitude
Lyapunov exponent

AND FOR MORE COMPLEX SYSTEMS?

A central quantity for quantifying model error

$$\vec{\varepsilon}(t_0) = \frac{d\vec{x}}{dt} - \frac{d\vec{x}_N}{dt}$$

Or if one does not have access to the velocity and taking into account the fact that the initial error is 0 at t_0.

$$\vec{\varepsilon}_{approx}(t_0) = (x(t_0 + \Delta t) - x_N(t_0 + \Delta t)) / \Delta t$$

The regional model

- Eta regional model implemented at RMI
  0. Hydrostatic version
  1. 0.32° on the rotated lat x lon grid
  2. Time step: 120 s.
  3. 45 levels
  4. BCs and Ics provided by T170 Avn
  5. Target region centered over France

Kain-Fritsch vs Betts-Miller-Janjic convective schemes

BMJ

Adjustment scheme to reference vertical
Moisture and temperature profiles

KF

Mass-flux scheme: parameterize the
rearrangement of mass in a vertical column

Convective schemes constitute important sources of uncertainties

First experiment with BMJ

The central scheme of the BMJ

$$\Delta T = (T_{ref} - T)\Delta t / \tau$$

$$\Delta Q = (Q_{ref} - Q)\Delta t / \tau$$

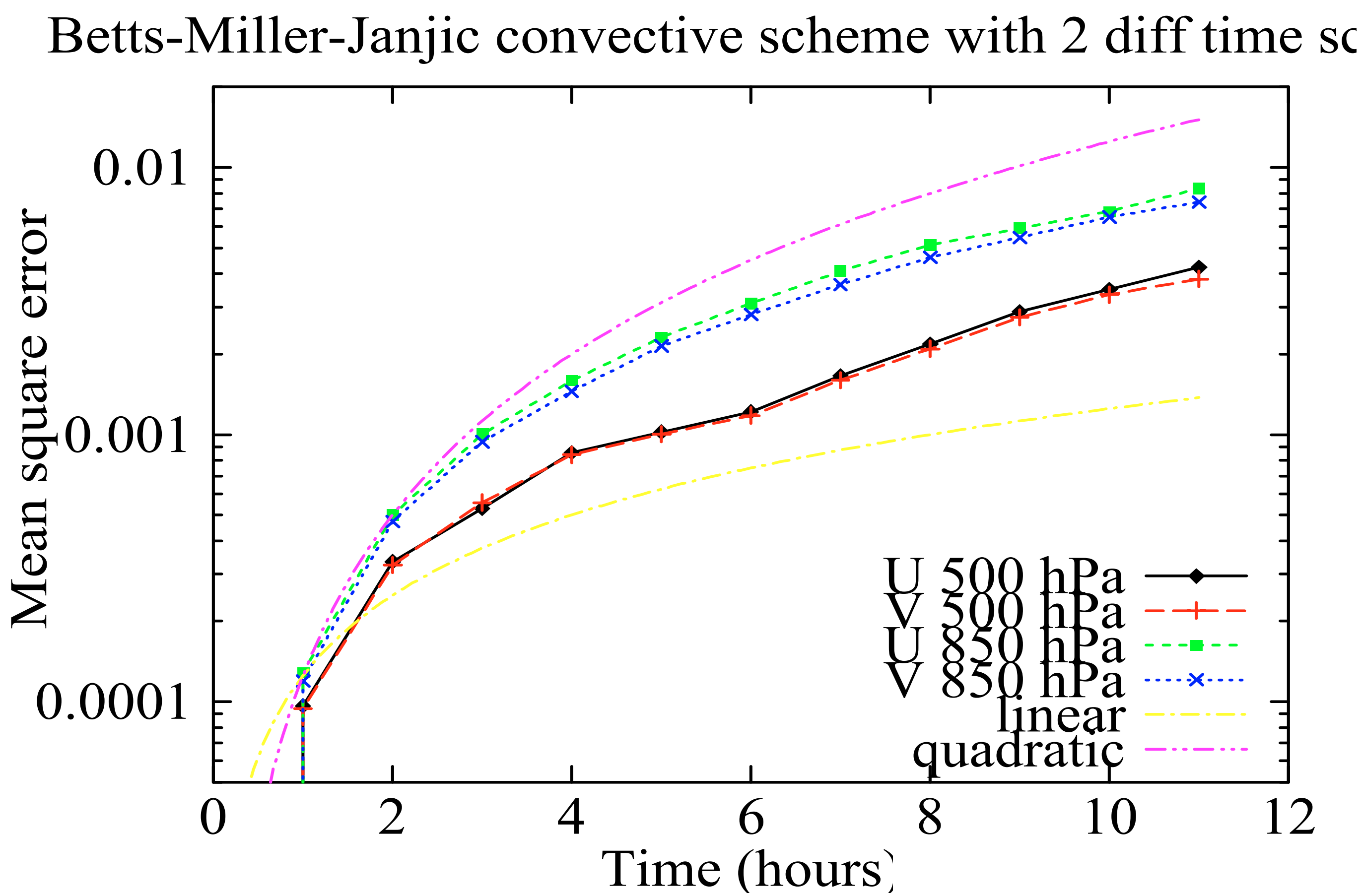
Let us modify the relaxation time scale τ

$$\tau' = 2500 \text{ s}$$

$$\tau = 2400 \text{ s}$$

⇨ Parametric error

Parallel runs are made with these time scales and with the same initial conditions.

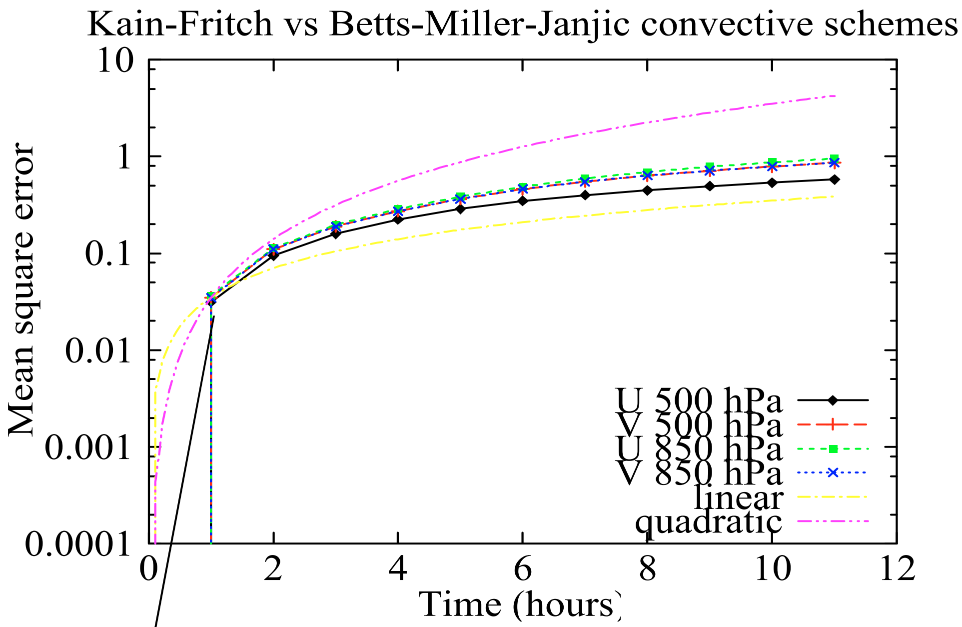


Obtained with 140 realizations (summer 2004)

BMJ vs KF

The difference could be qualified as a parametric error

Experiment: -two runs performed with different schemes
-same initial condition
-140 runs from july 2004 to september 2004
-runs at 00Z and 12Z

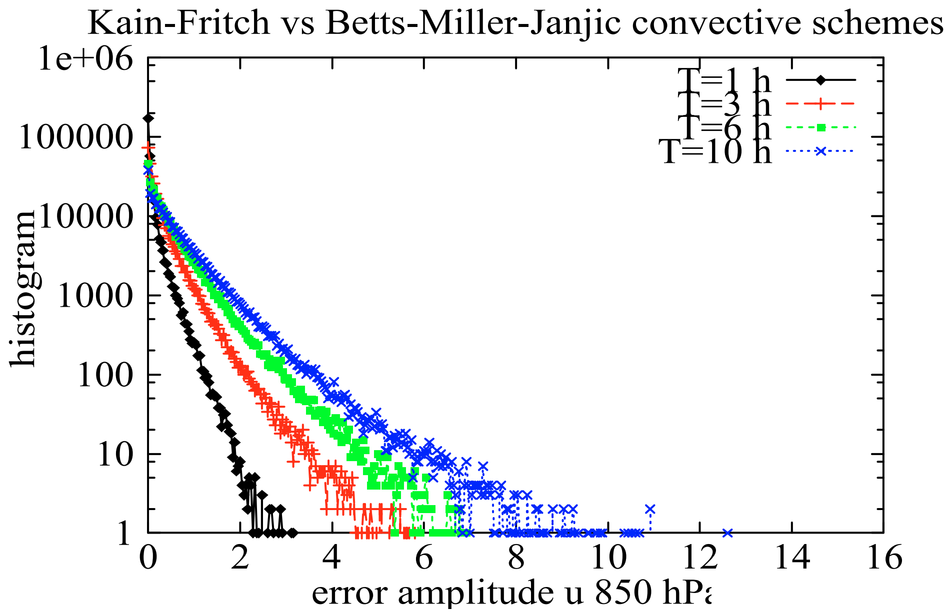


Mean square error evolution

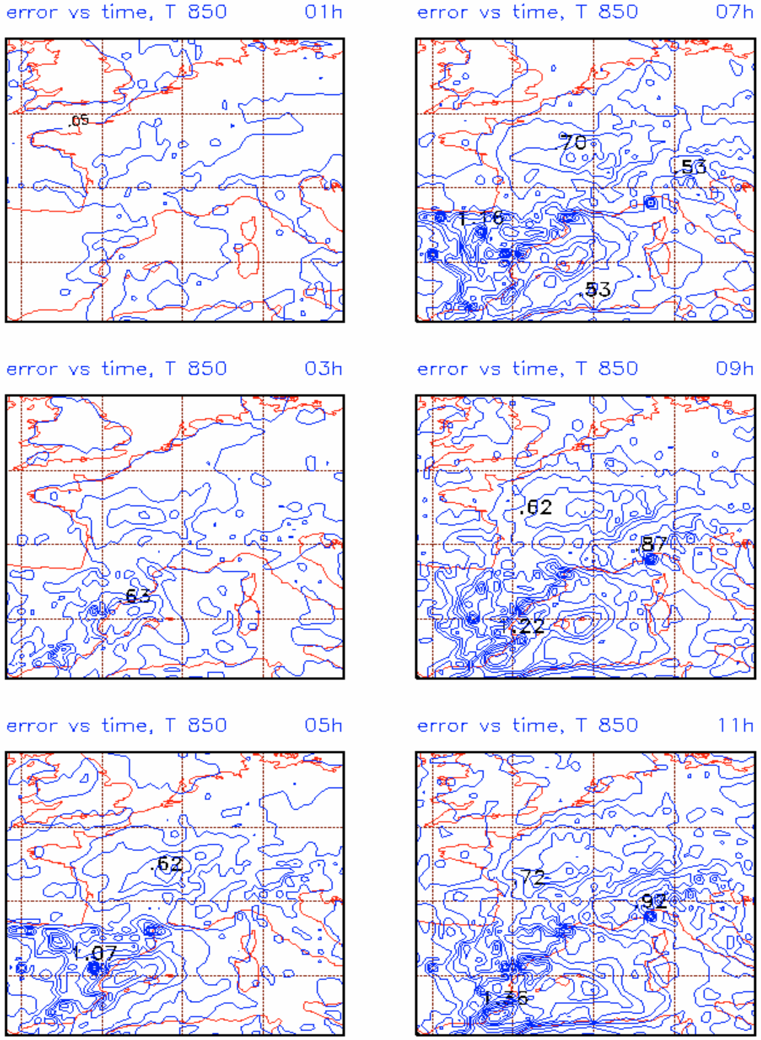
-slower than quadratic law

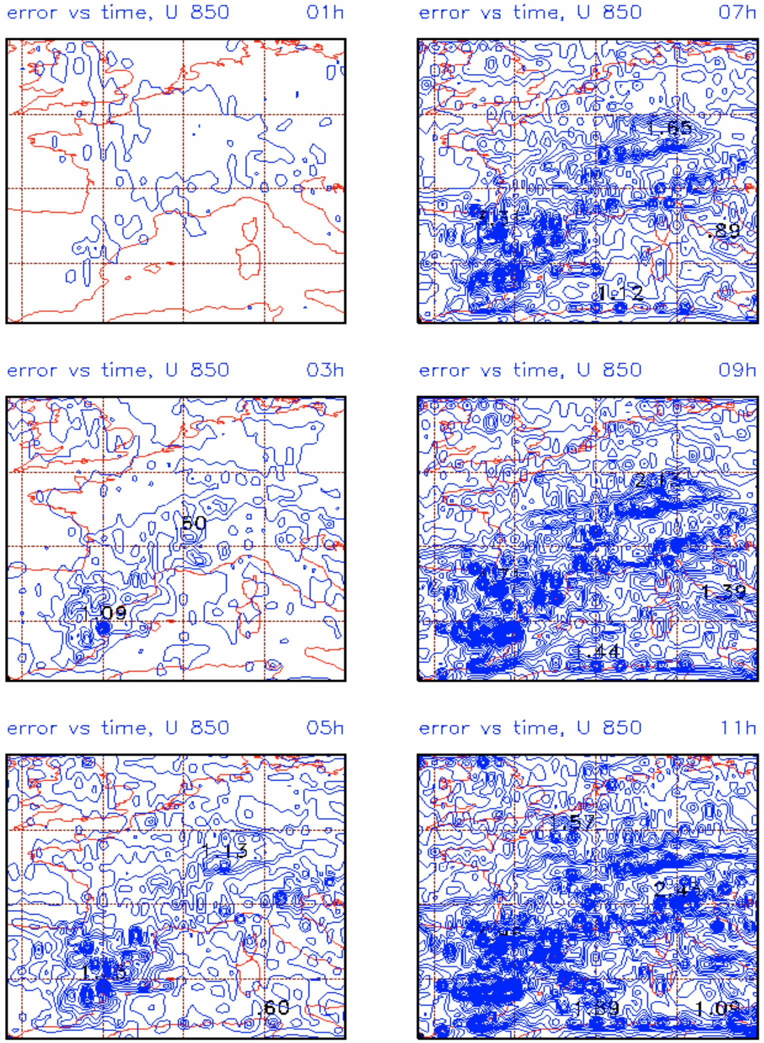
!!!! The error is NOT small

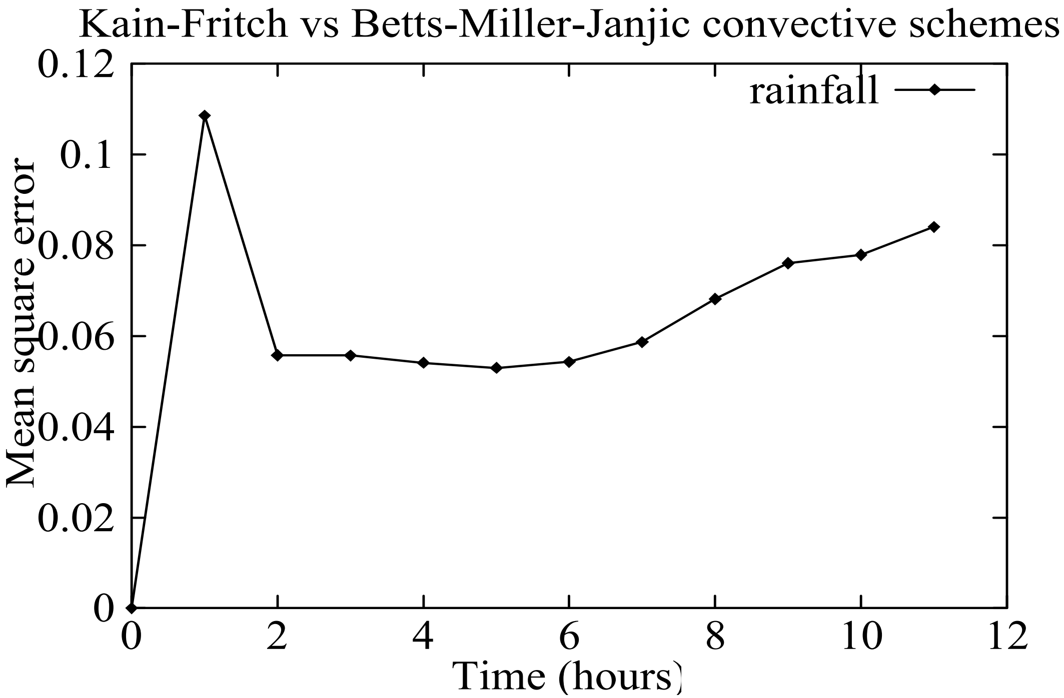
An approximation
of the amplitude of model error



Histograms of the absolute
error at different lead times


error vs time, T 850 01h
error vs time, T 850 07h
error vs time, T 850 03h
error vs time, T 850 09h
error vs time, T 850 05h
error vs time, T 850 11h


error vs time, U 850 01h
error vs time, U 850 07h
error vs time, U 850 03h
error vs time, U 850 09h
error vs time, U 850 05h
error vs time, U 850 11h



Kain-Fritch vs Betts-Miller-Janjic convective schemes
rainfall
Mean square error
0.12
0.1
0.08
0.06
0.04
0.02
0
0
2
4
6
8
10
12
Time (hours)

Errors due to the presence of boundaries

The impact of boundaries on predictability in regional Models has already been deeply investigated

e.g. Anthes, 1985; Vukicevic and Errico, 1990;.....

An important source of errors (see Warner et al, 1997)

Here we are trying to quantify the model error associated with the boundaries

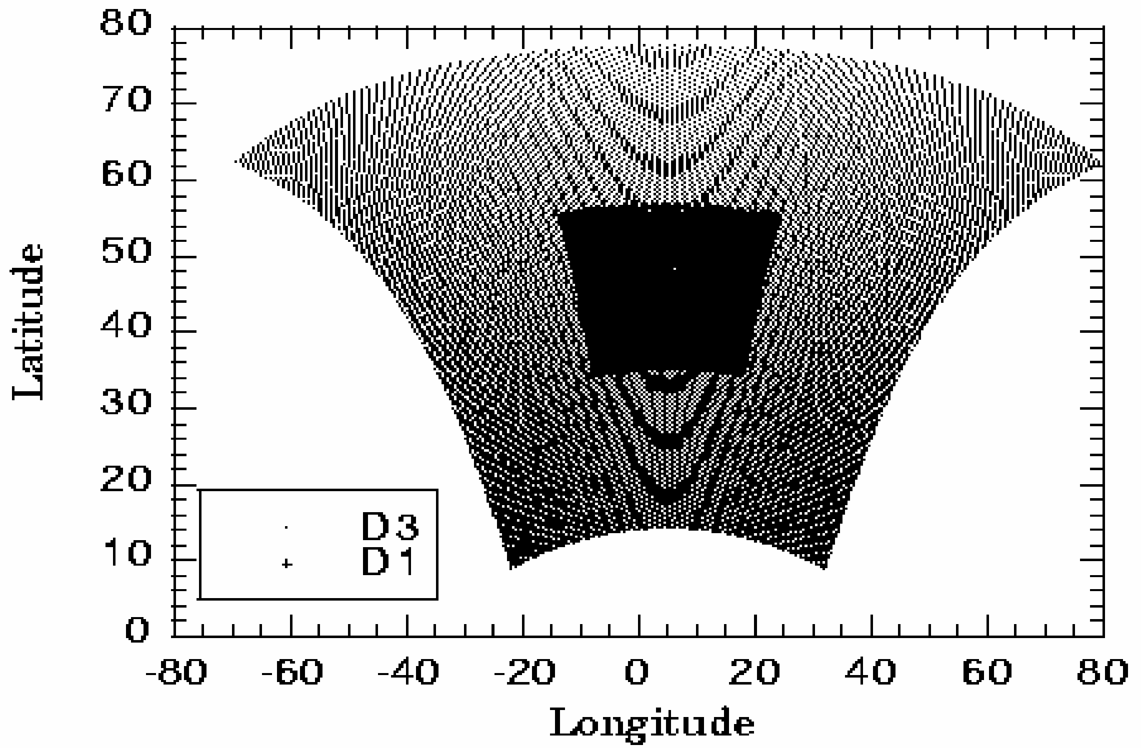
What experiment?

We cannot compare with a high resolution global model

Eta domains

1 very large domain:
A proxy of reality

1 smaller domain of interest



Same resolution, same physics, nearly the same location of the grid points (error 10^{-3}).

The boundaries of both domains are fed with the AVN predictions and the initial values are given by the AVN analyses

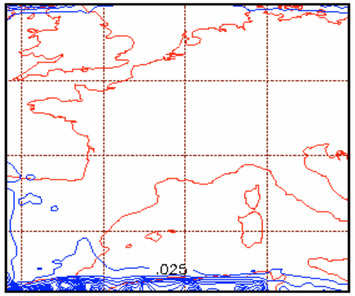
The difference between the two ETA model versions is in the domain size. The largest one being considered as the best representation of reality,

the model error is due to the boundaries fed with the AVN integrations.

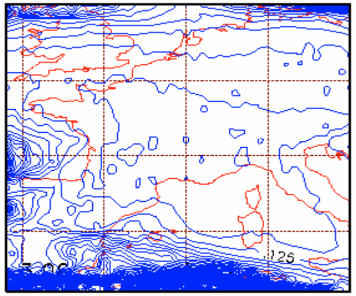
-boundary coupling scheme
-prediction error of the global model

+ small location error between the grids.

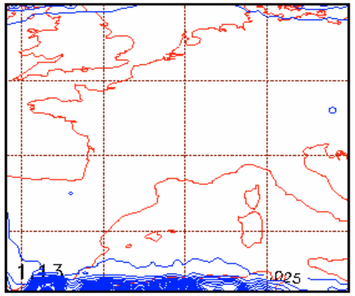
error vs time, U 850 01h



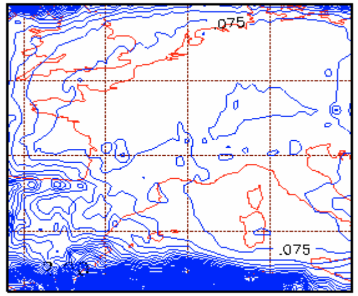
error vs time, U 850 07h



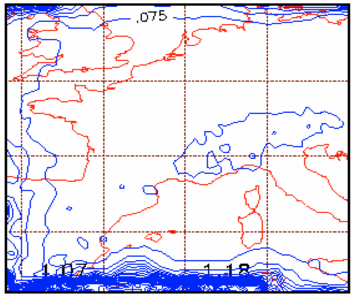
error vs time, V 850 01h



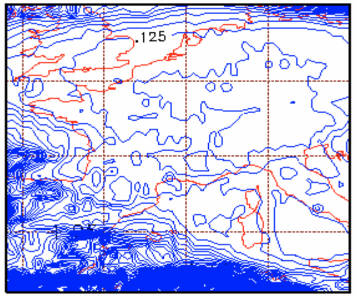
error vs time, V 850 07h



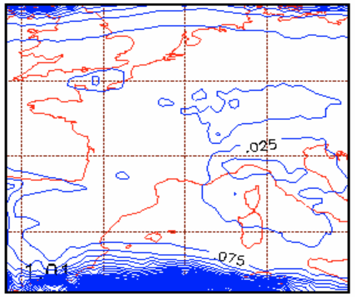
error vs time, U 850 03h



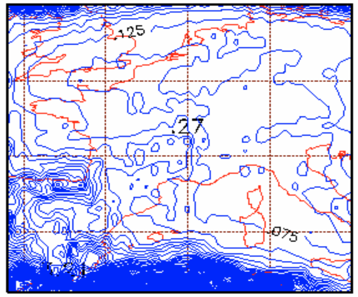
error vs time, U 850 09h



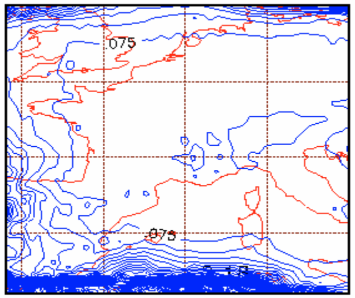
error vs time, V 850 03h



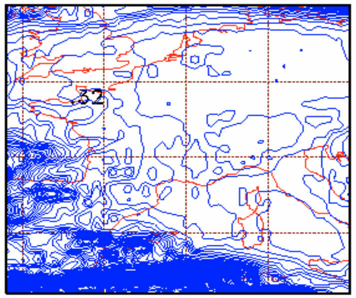
error vs time, V 850 09h



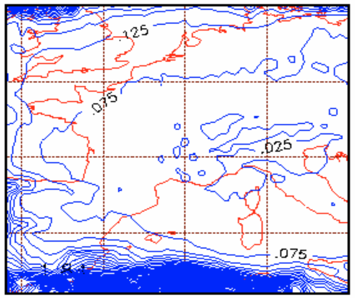
error vs time, U 850 05h



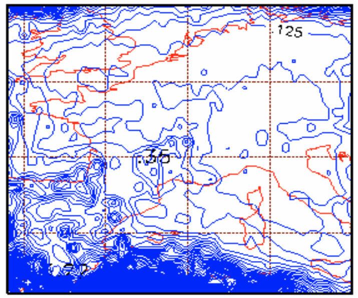
error vs time, U 850 11h



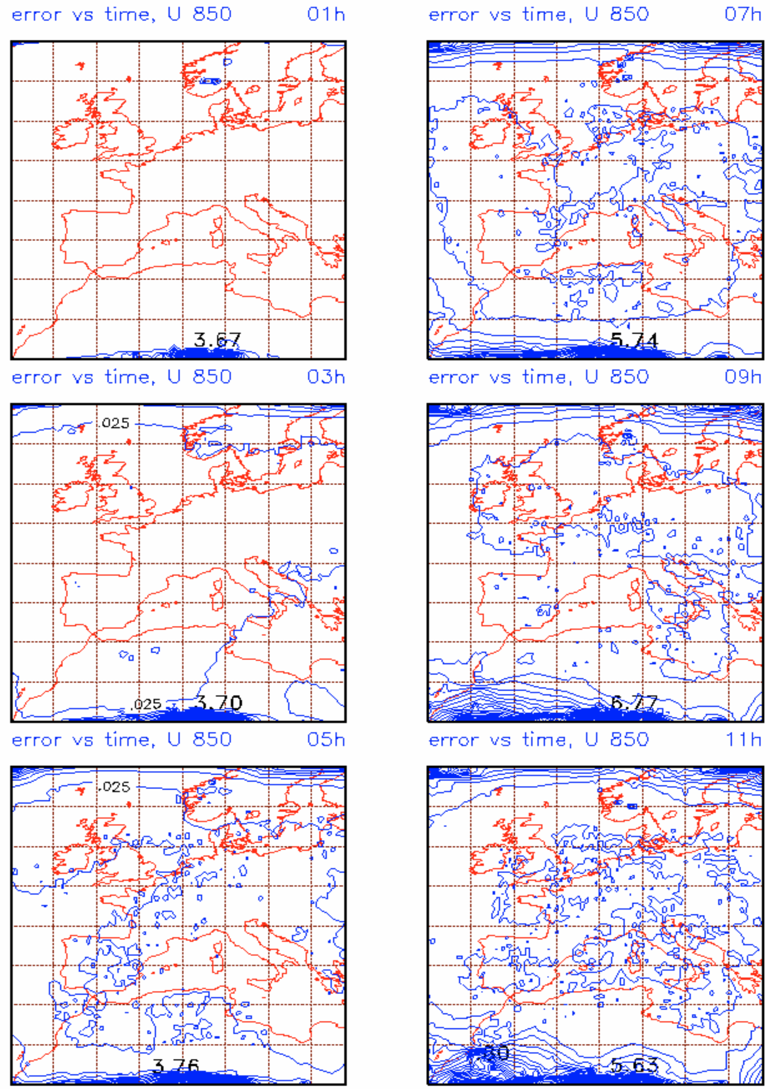
error vs time, V 850 05h



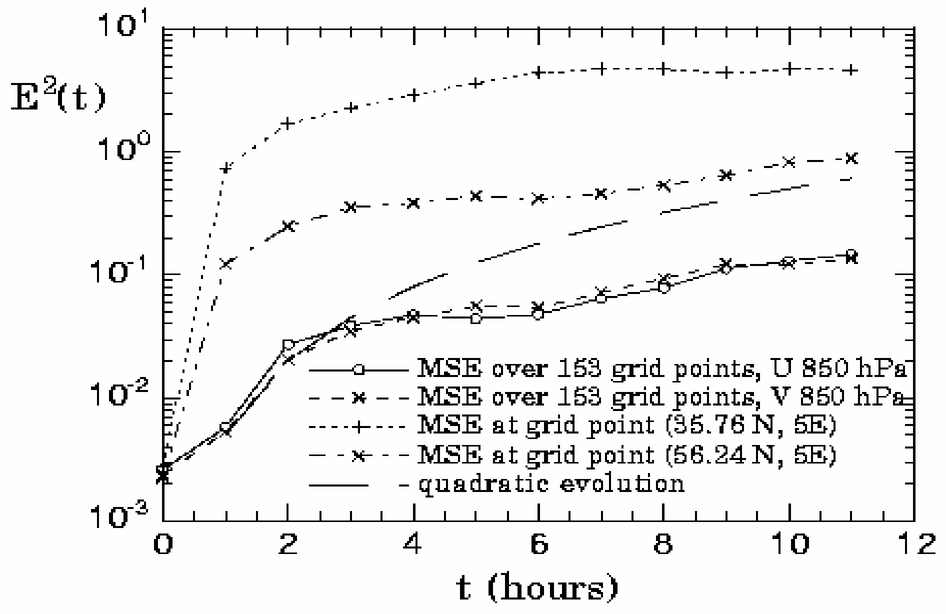
error vs time, V 850 11h



Similar effect with a larger domain



70 realizations



There is an initial error

- The error dominates at the boundaries
- At the central grid points, an error is present due to the slight discrepancy in the grid point location
-Interestingly, this error behaves in a similar way as the one found for the difference between KF and BMJ

Conclusion

- Model error is a key component of the predictability problem.
- The understanding of the dynamics of this error is essential for evaluating the quality of predictions

necessity of theoretical investigations of its properties

This error has been investigated in the Eta model.

- KF vs BMJ quite important errors after a short period (12 h)
- Impact of one-way boundaries
  affect considerably a layer of a few degrees close to the boundaries

THE ERROR IS NOT SMALL

Conclusion

Necessity of quantifying the impact of model error

A few key recent papers:

-Buizza et al, 1999, QJRMS Ensemble predictions
-Palmer, 2001, QJRMS general overview of the concept
-Nicolis, 2003, 2004, JAS theoretical investigation
-Vannitsem & Toth, 2002, JAS idem
-Barkmeijer et al, 2003, QJRMS Ensemble predictions + theor
-D Zupanski et al, 2002, MWR data assimilation
........growing literature....