Beyond *The Million-Body Problem*: Simulating Galactic Nuclei

RIT

David Merritt

Stefan Harfst

Andras Szell

Amsterdam

Simon Portegies Zwart

Alessia Gualandris

Heidelberg

Rainer Spurzem

Peter Berczik

Drexel

Steve McMillan

Also:

Seppo Mikkola (Tuorla Observatory)

Ioannis Kevrekidis (Princeton)

“Throughout, the term *N-body simulations* is used exclusively for methods based on direct summation, in keeping with tradition.”

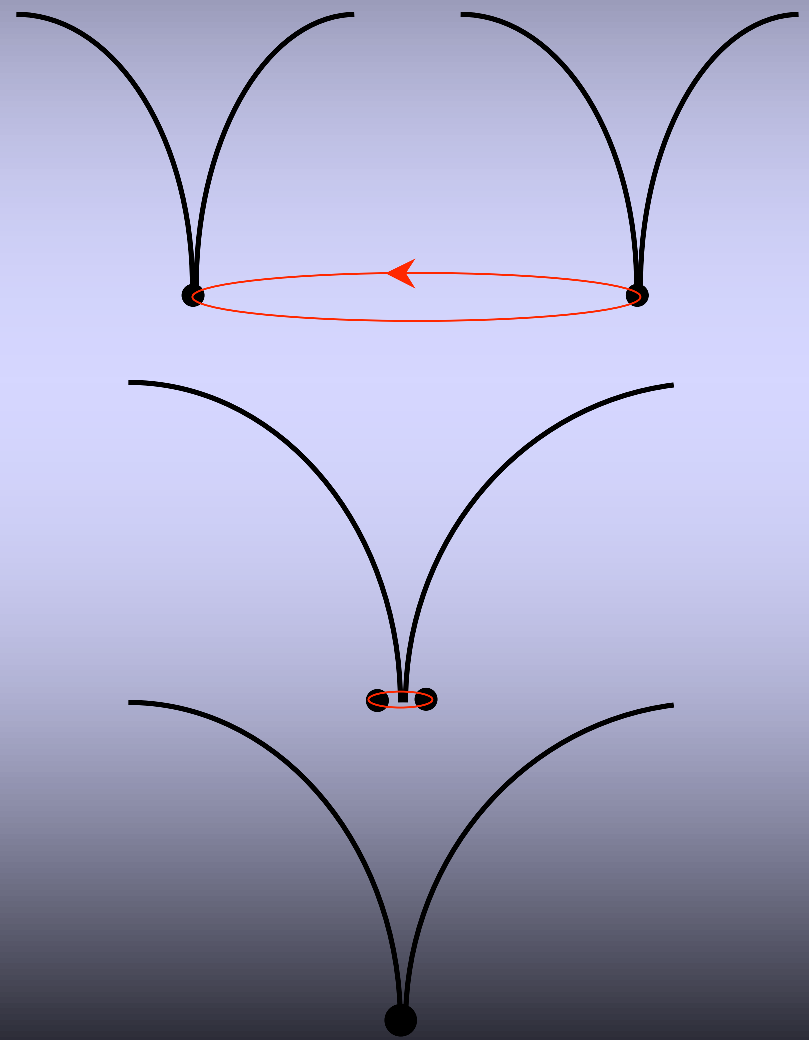
S. J. Aarseth, “Gravitational N-Body Simulations”

Binary Black Holes

Galaxies merge

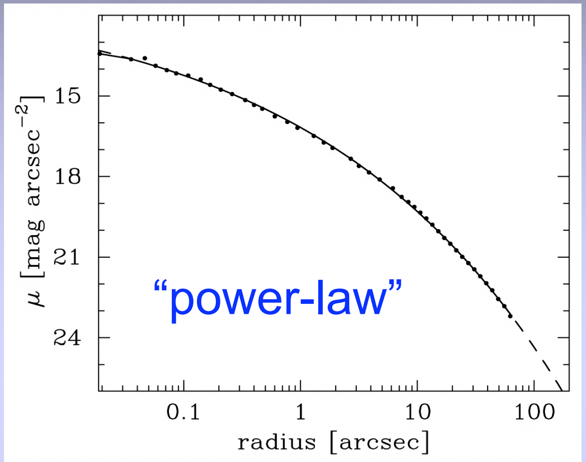
Binary forms

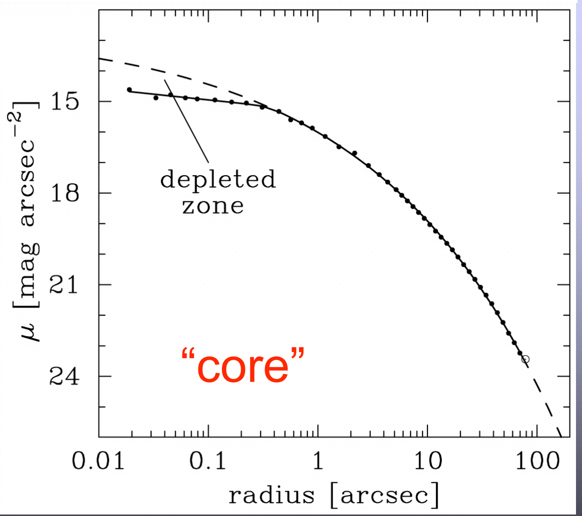
Binary decays, via:
-- ejection of stars
-- interaction with gas
-- gravitational radiation



Radio-quiet AGN are only hosted by "power-law" galaxies.

Radio-loud AGN are only hosted by "core" galaxies, even at galaxy luminosities well below that of classical radio galaxies.





[FIGURE REMOVED]

Capetti et al. 2005

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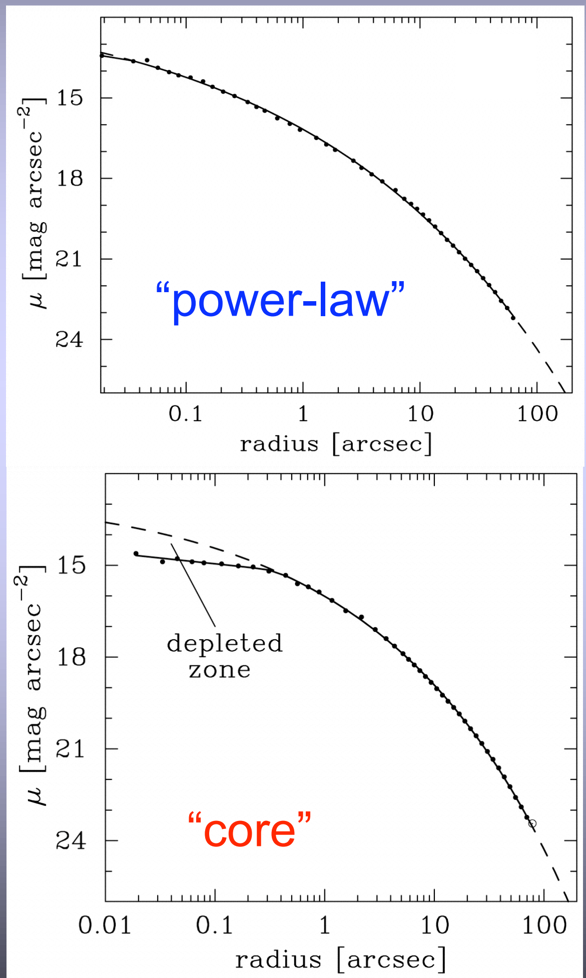

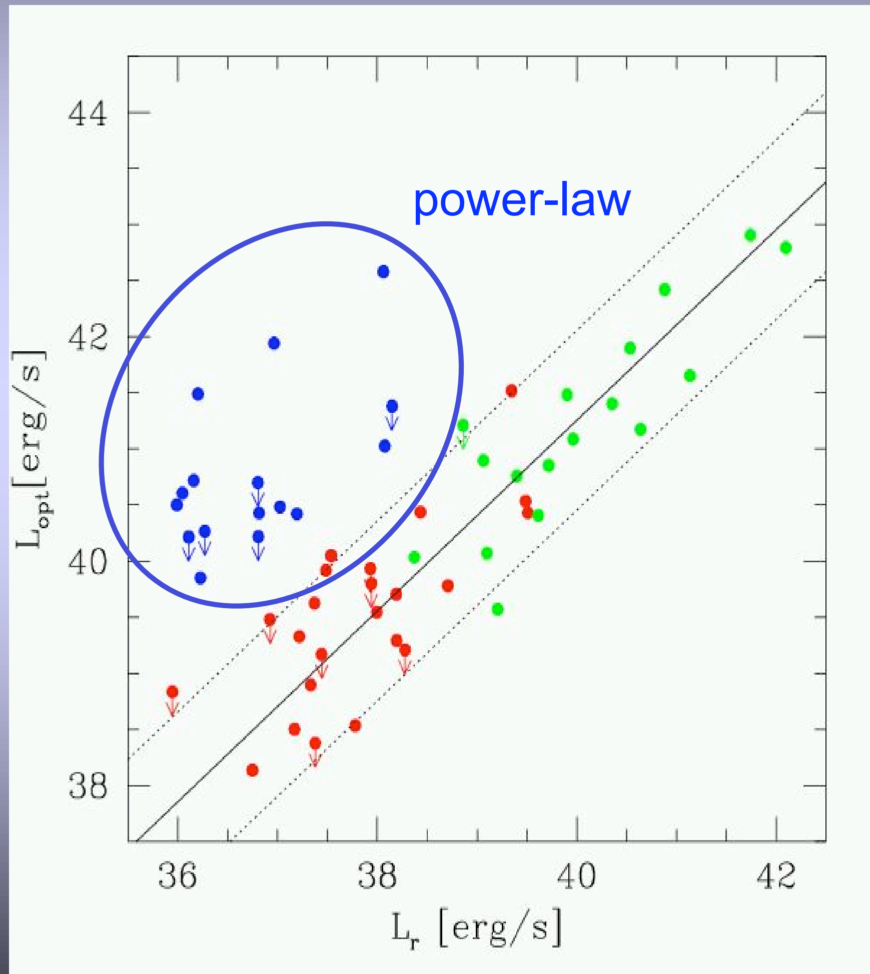

Capetti et al. 2005

What Values of *N* are Required to Simulate Nuclei?

N fixes the ratio of **relaxation time** to **crossing time**:

$$T_{relax} \approx \frac{0.1N}{\ln N} T_{cross}$$

N	T_{relax}/T_{cross}
10^2	2.2
10^3	14.5
10^4	109
10^5	870
10^6	7250
10^{11}	3.9×10^8

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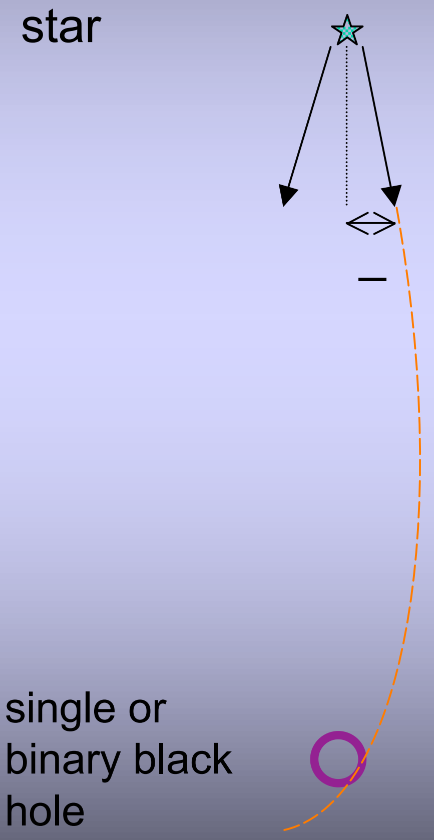
The "separation of time scales" requires $N \approx 10^4$.

Easy!

In **loss-cone** problems, this requirement is more severe.



Stars are scattered by other stars into the loss cone, where they can interact with the central object(s).

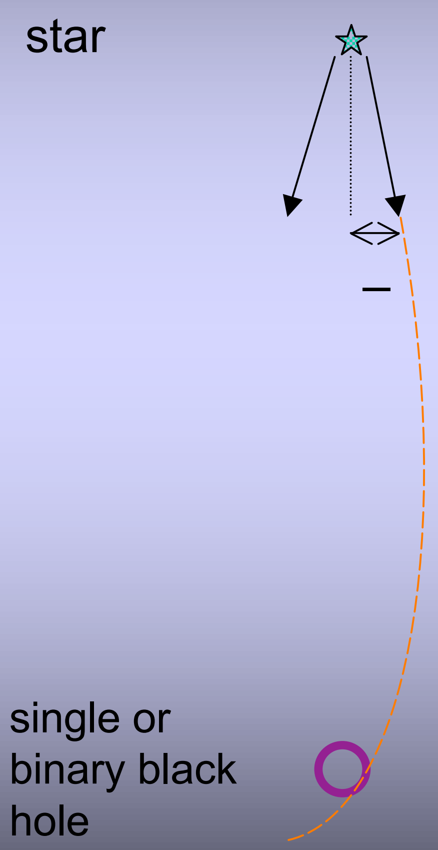
Scattering time is

$$\sim \_^{2} T_{relax} << T_{relax}$$

and separation of the two time scales requires

$$T_{relax} >> \_^{-2} T_{cross}$$

and a much larger *N*.

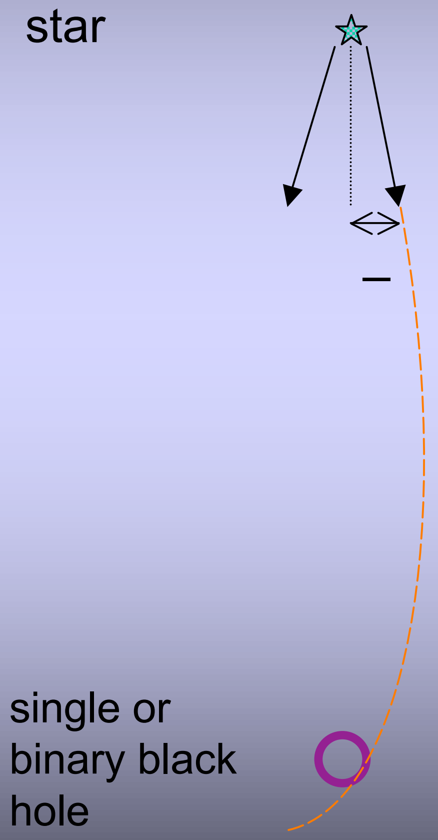


In one (radial) period, a star experiences a change:

$$(\delta J)^2 \approx \frac{T_{orb}}{T_{relax}} J_c^{\,2}(E)$$

J_c = ang. mom. of circular orbit

in its angular momentum.



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$$(\delta J)^2 \approx \frac{T_{orb}}{T_{relax}} J_c^{\,2}(E)$$

J_c = ang. mom. of circular orbit

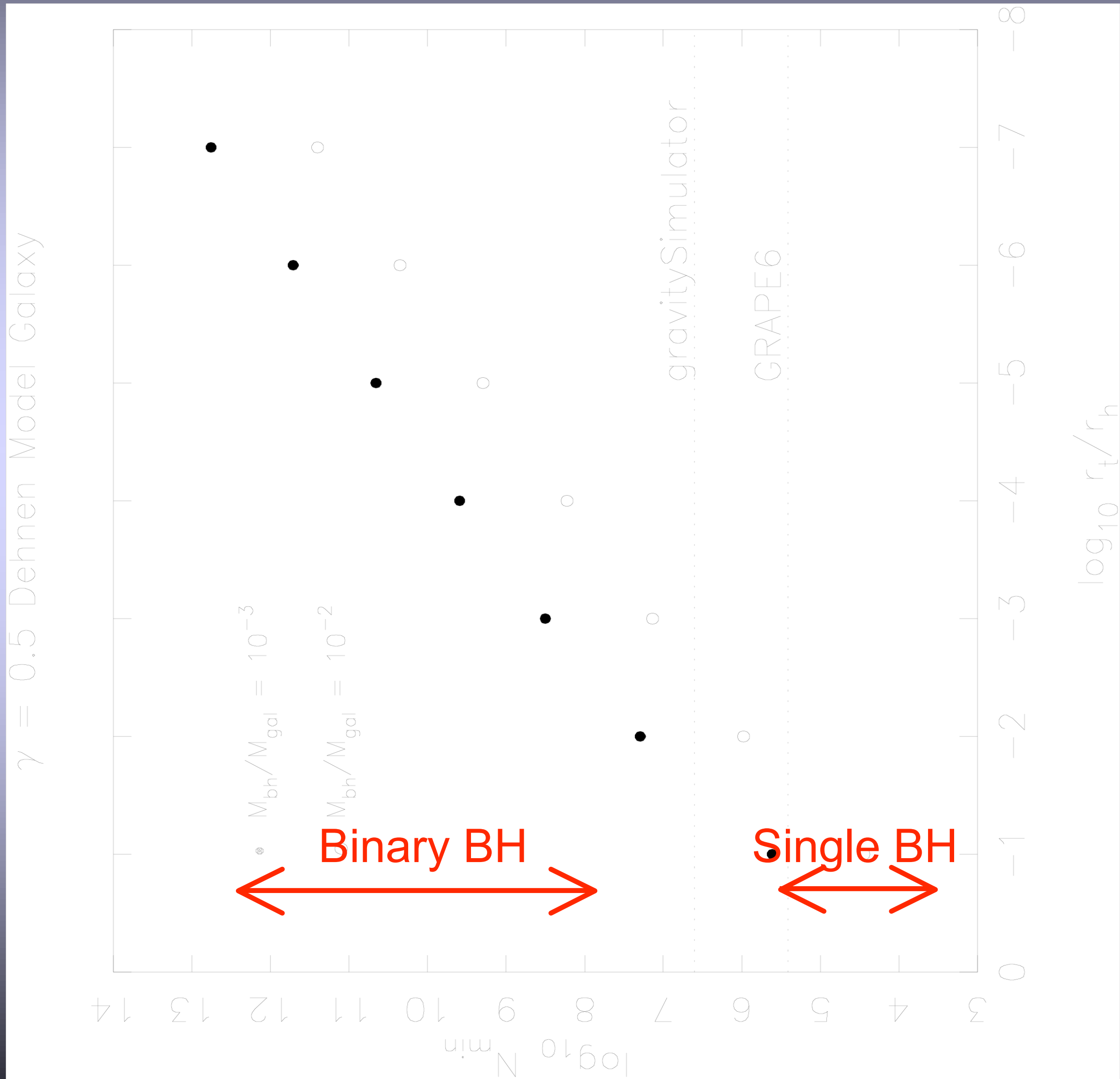
in its angular momentum. Define:

$$q(E) \equiv \frac{\delta J^2}{J_{lc}^{\,2}}$$

$$J_{lc}^{\,2} = 2r_t^{\,2}\left[E - \Phi(r_t)\right]$$

An "empty" loss cone has $q \ll 1$.

Minimum Number of Stars Required to “Resolve” Central Object



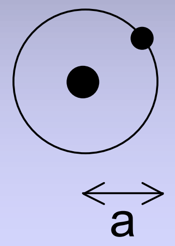
I.e., minimum *N* to maintain an empty loss cone.

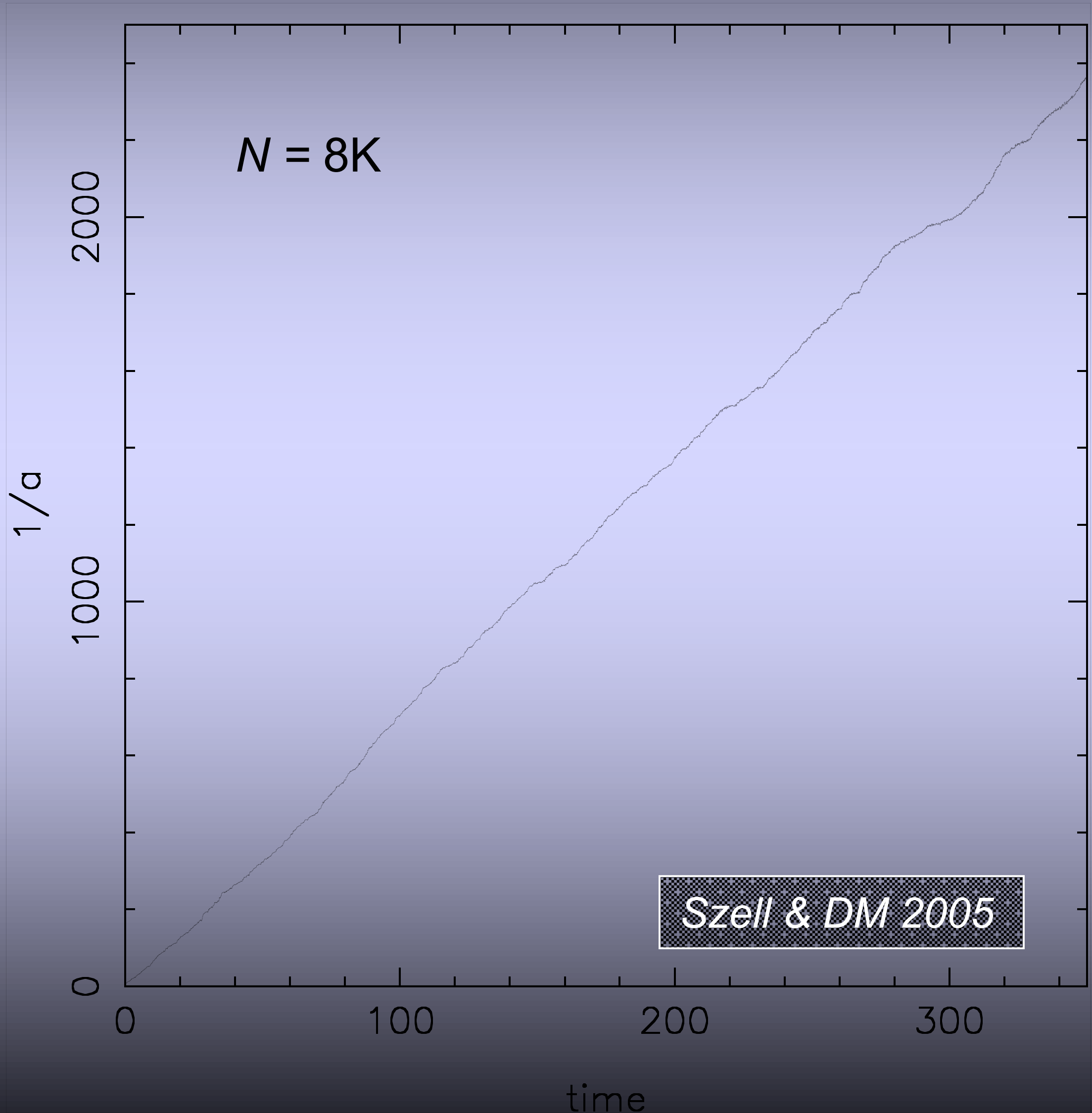
r_t: radius of capture sphere

r_h: BH influence radius

$log_{10} r_t / r_h$

N-Body Decay of a Massive BH Binary





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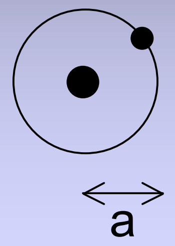

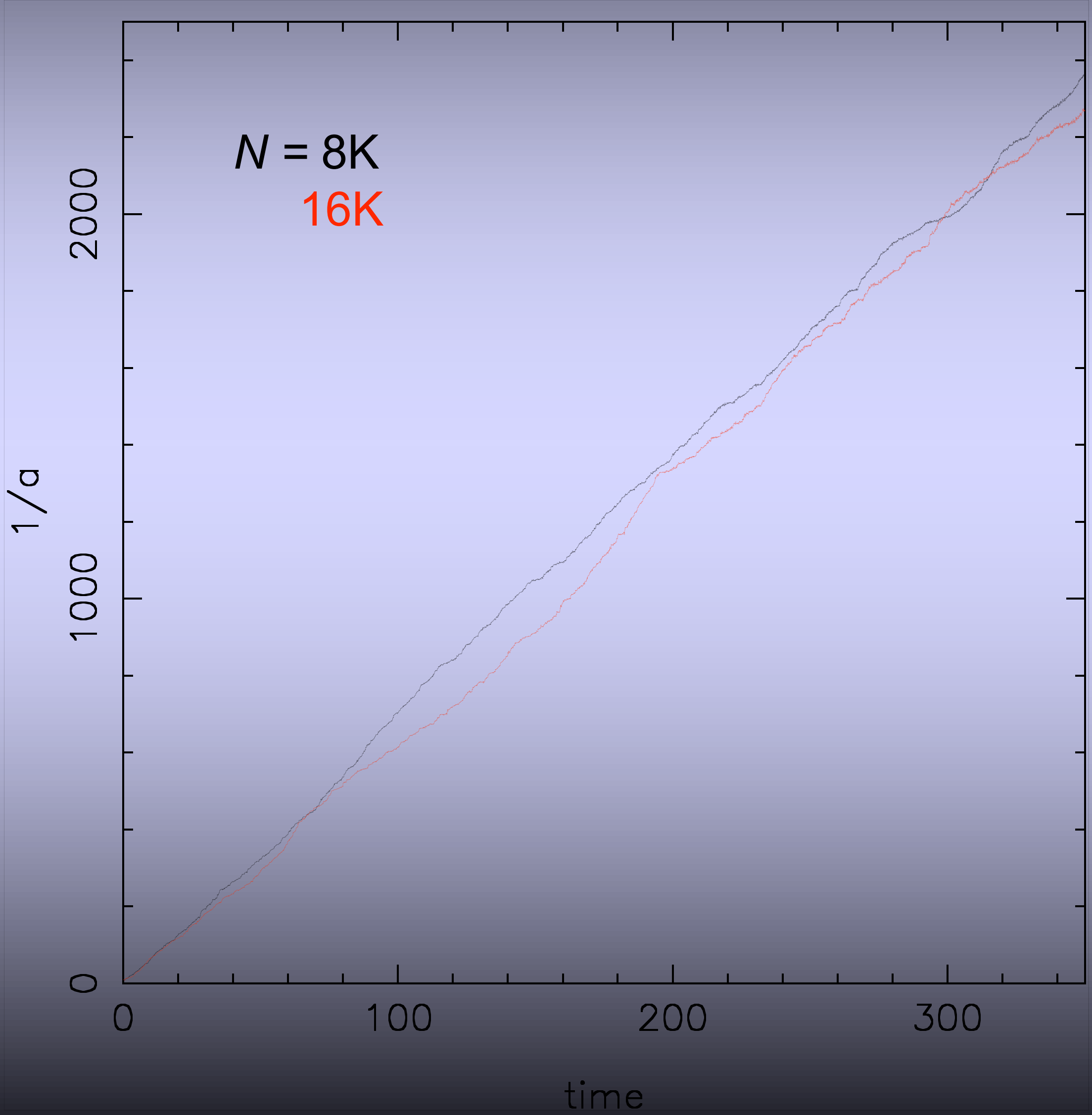

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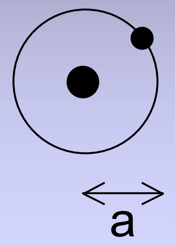

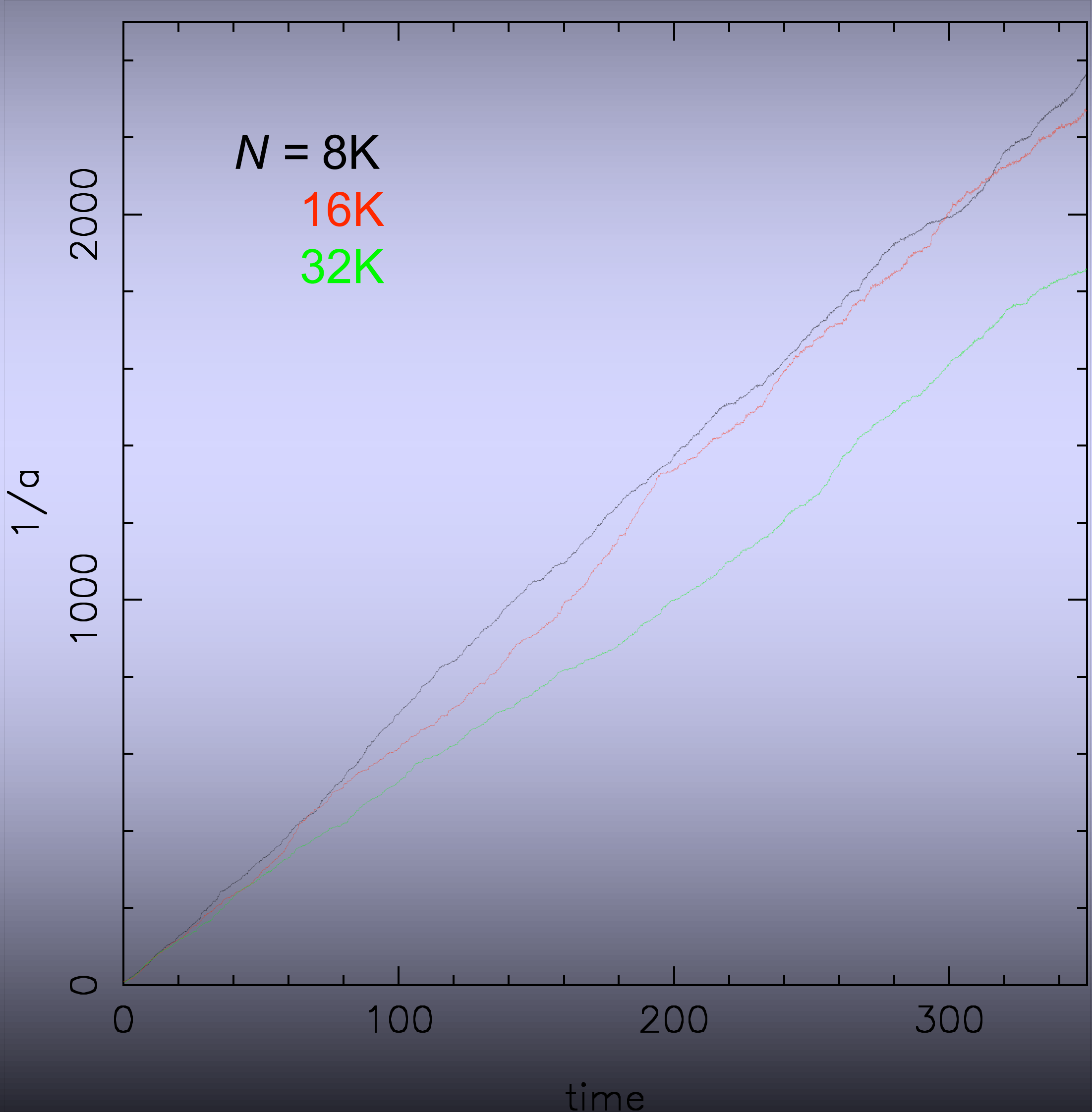

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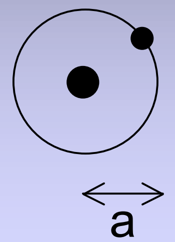

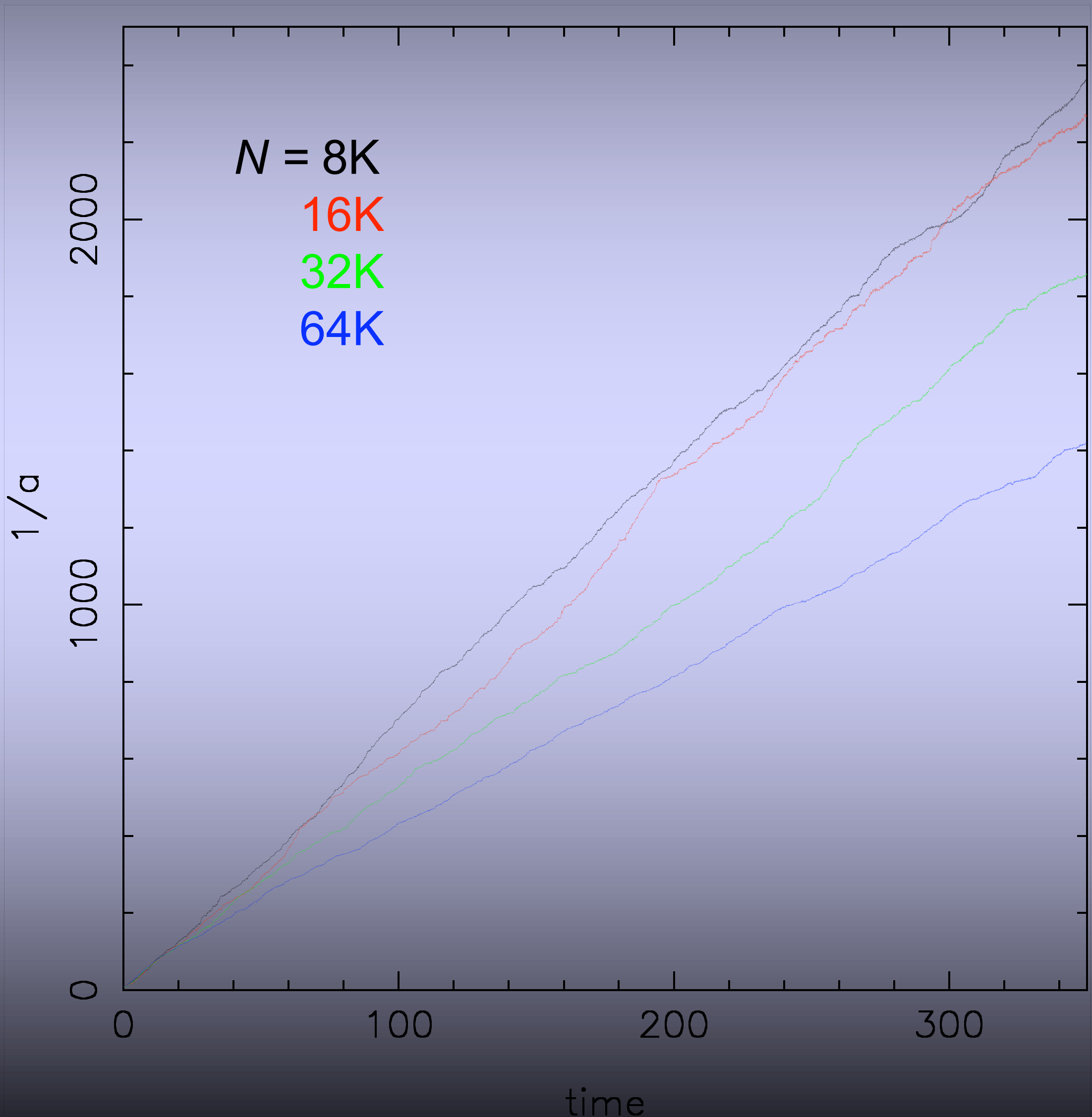

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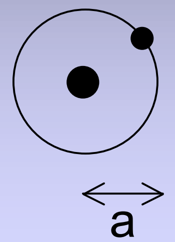

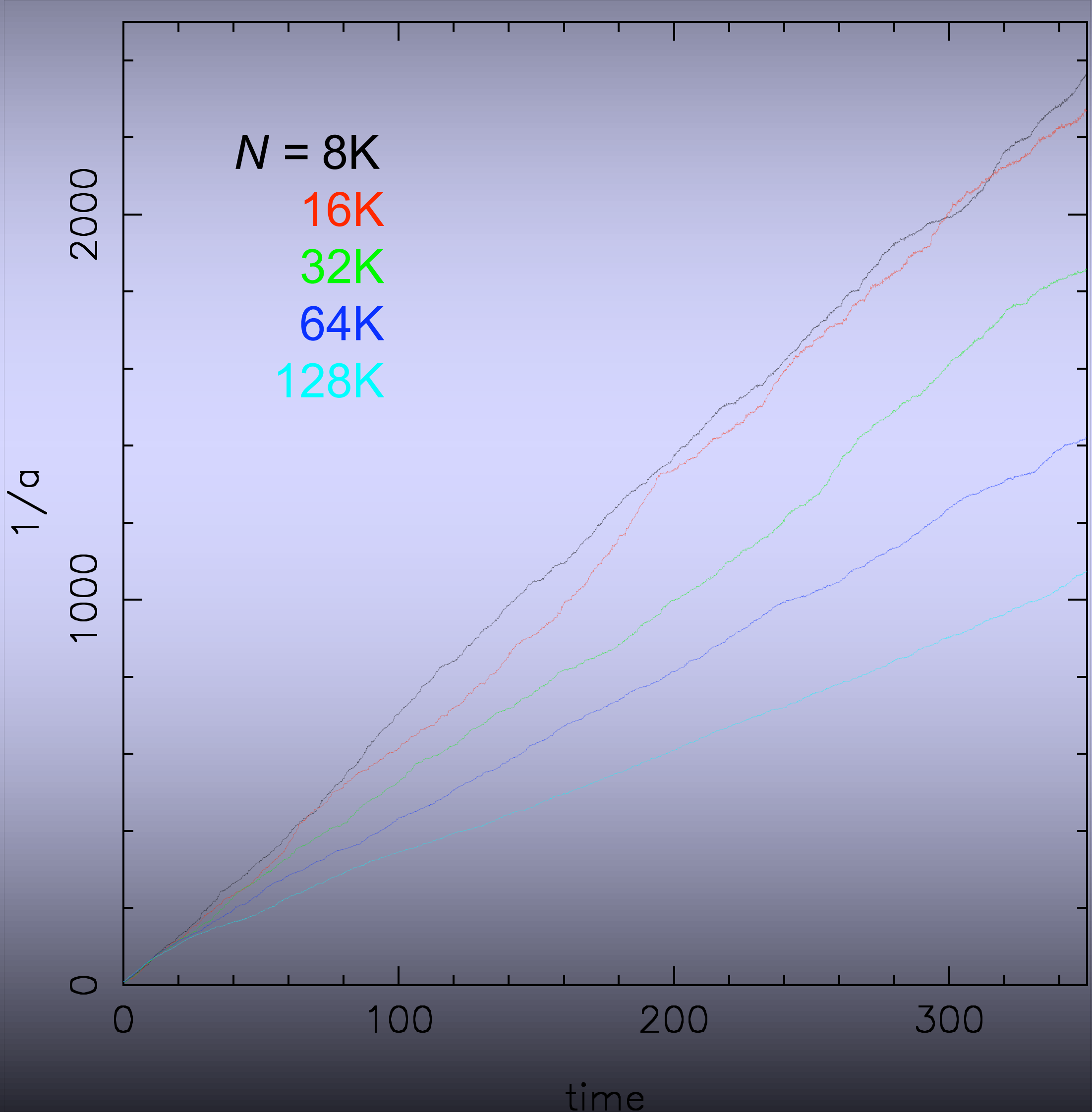

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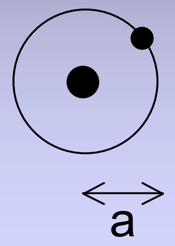

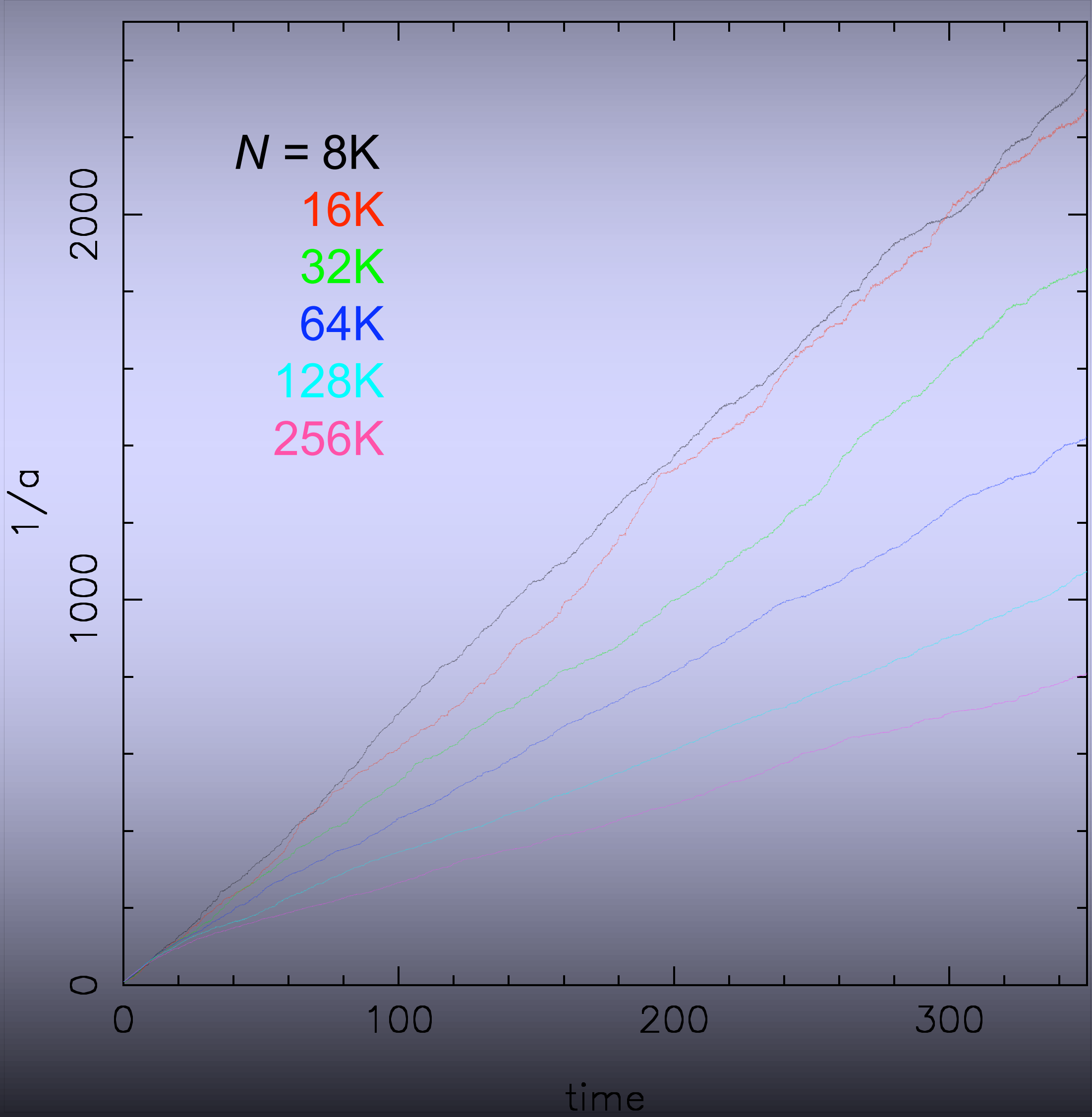

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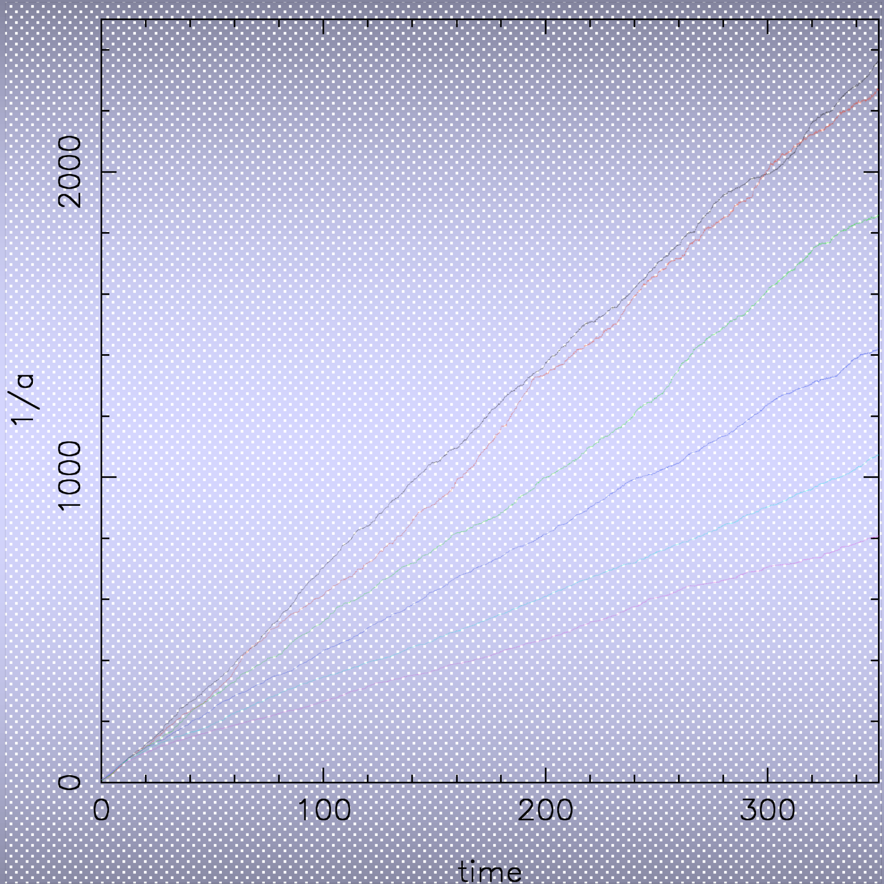

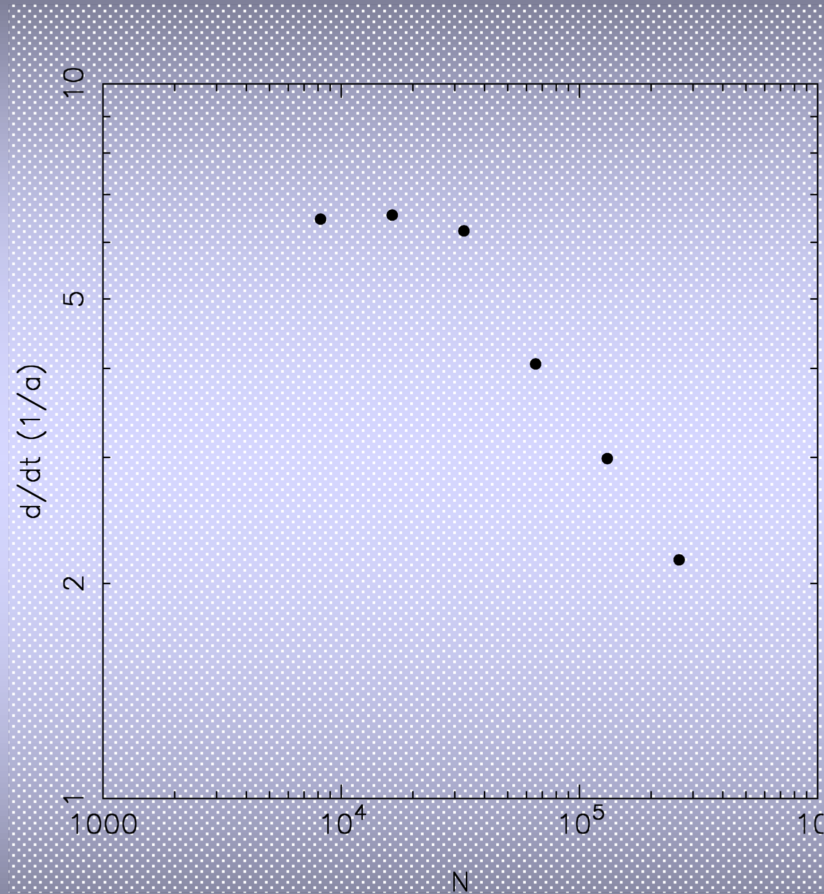

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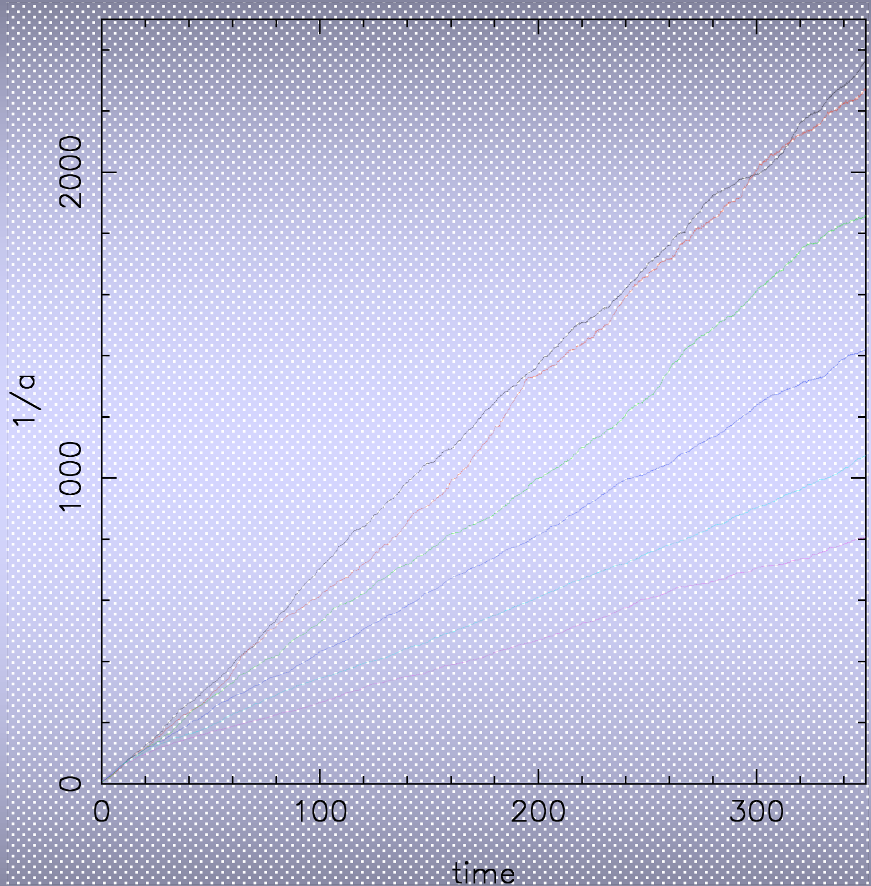
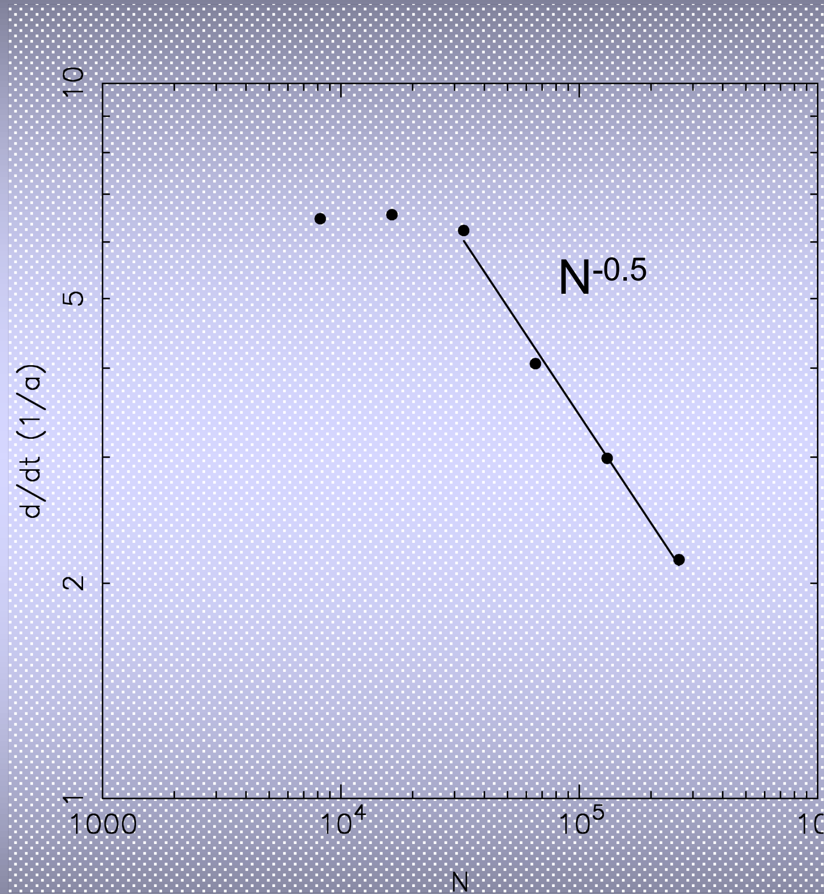
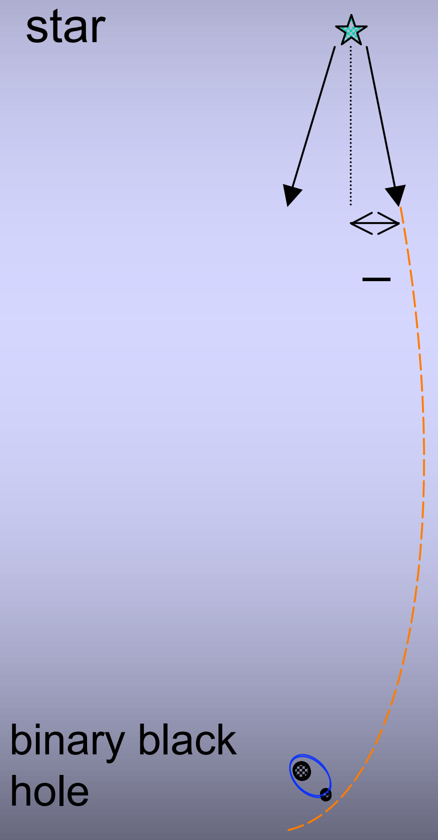






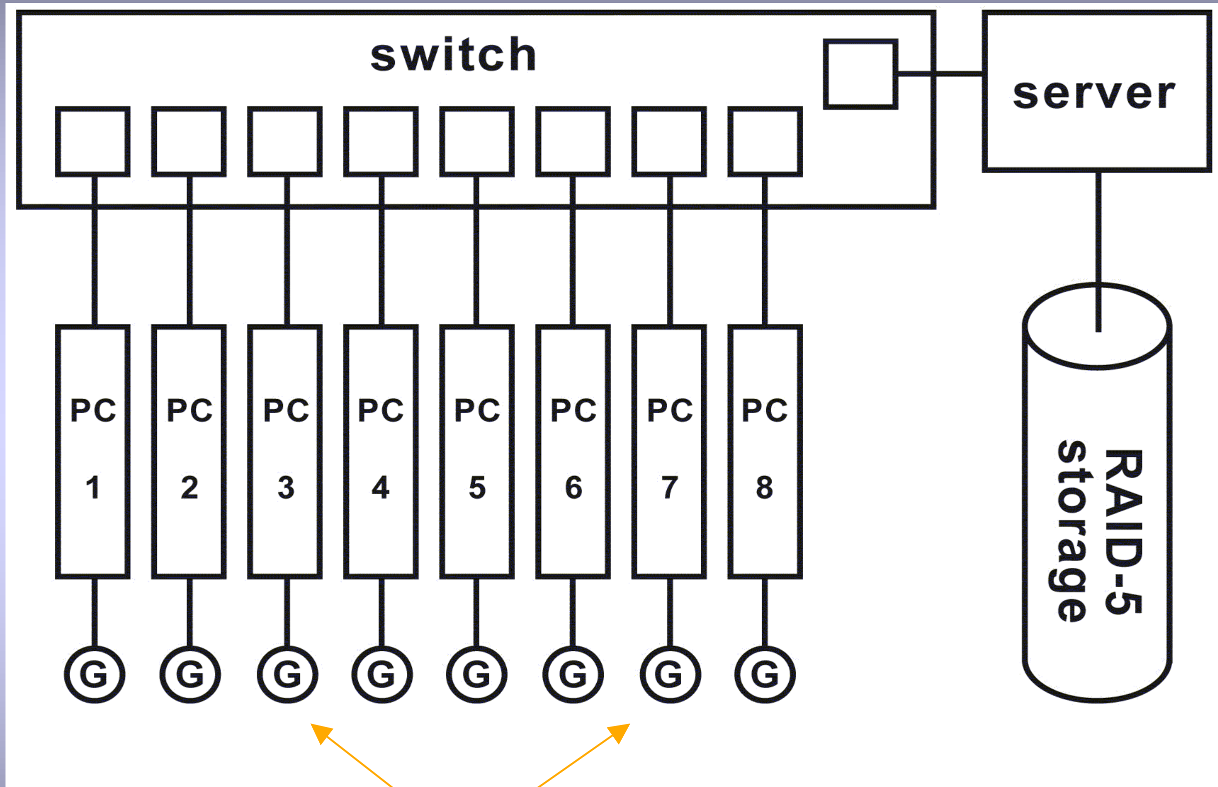
In the empty-loss-cone regime, you expect the decay rate to scale inversely with *N*:

$$\frac{d}{dt}\left(\frac{1}{a}\right) \propto T_r^{-1} \propto N^{-1}$$

This requires $N \approx 10^{6\text{-}7}$.

Hard!

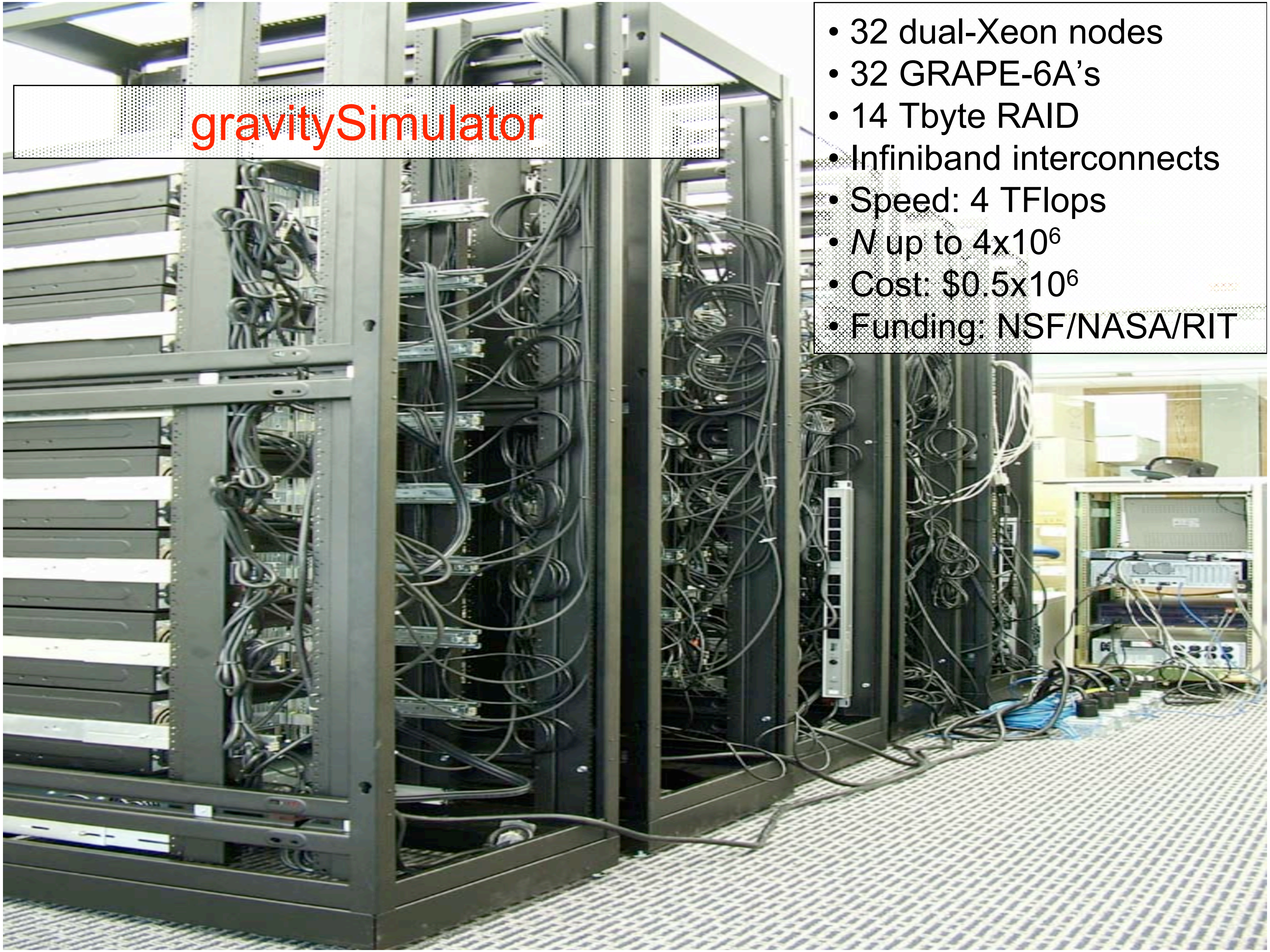
The GRAPE Cluster



mini-GRAPEs
(GRAPE-6A)

N < 131,072

gravitySimulator



- 32 dual-Xeon nodes
- 32 GRAPE-6A's
- 14 Tbyte RAID
- Infiniband interconnects
- Speed: 4 TFlops
- *N* up to 4×10^6
- Cost: \$0.5x10^6
- Funding: NSF/NASA/RIT

Algorithms

Basic algorithm is a parallel, direct-summation code (NBODY1) with fourth-order (“Hermite”) integrator; individual, block time steps; and (optional) force softening.

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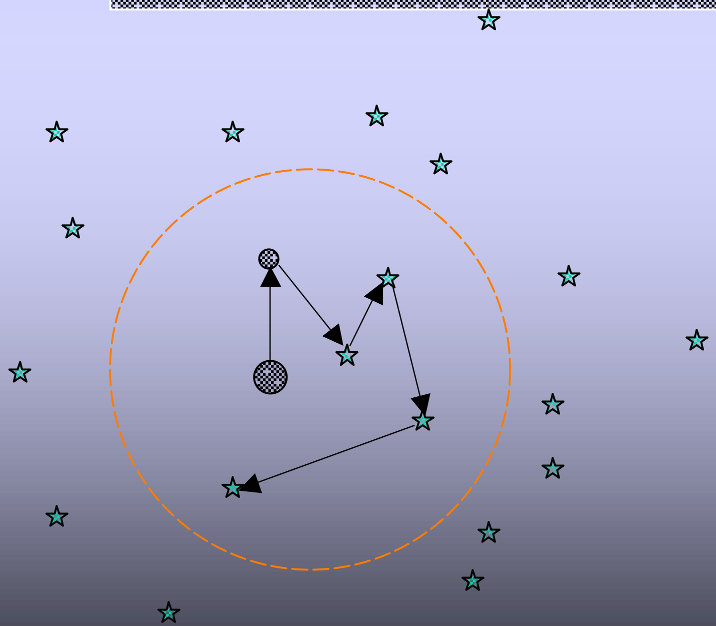
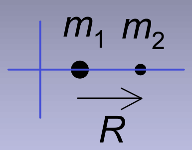
Close interactions near the black hole(s) are handled with a *chain regularization* algorithm (*Mikkola & Aarseth 1990, 1993*):



Regularization of the 1D Problem

Euler 1737



$$\left(\frac{m_1 m_2}{m_1 + m_2}\right)\ddot{R} = -\frac{G m_1 m_2}{R^2}$$

Time transformation : $dt = R(t)d\tau$

$$\frac{d^2 R}{d\tau^2} = \frac{1}{R}\left(\frac{dR}{d\tau}\right)^2 - G(m_1 + m_2)$$

Energy integral : $h = \frac{1}{2}\dot{R}^2 - \frac{(m_1 + m_2)}{R}$

$$= \frac{1}{2R^2}\left(\frac{dR}{d\tau}\right)^2 - \frac{(m_1 + m_2)}{R}$$

$$\frac{d^2 R}{d\tau^2} = 2hR + (m_1 + m_2)$$

Coordinate transformation : $u^2 = R$

$$\frac{d^2 u}{d\tau^2} = \frac{1}{2}hu$$

3D: Kustaanheimo & Stiefel 1965

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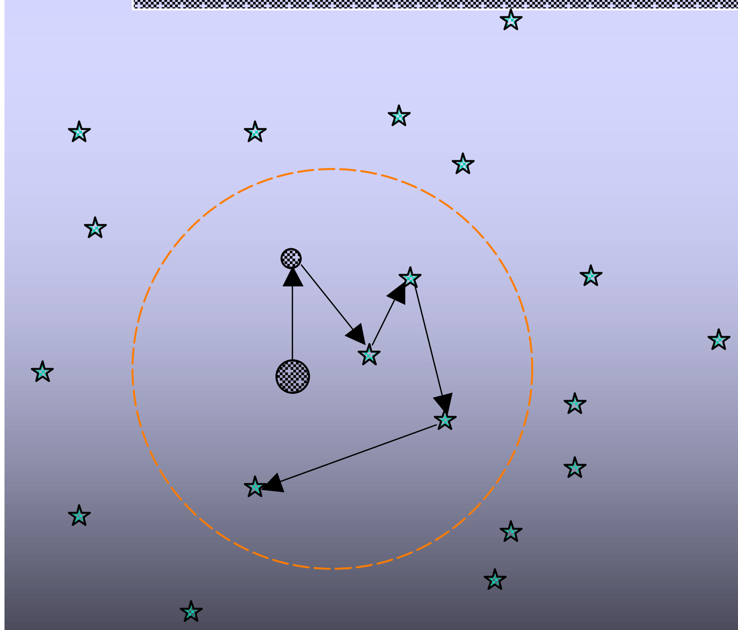
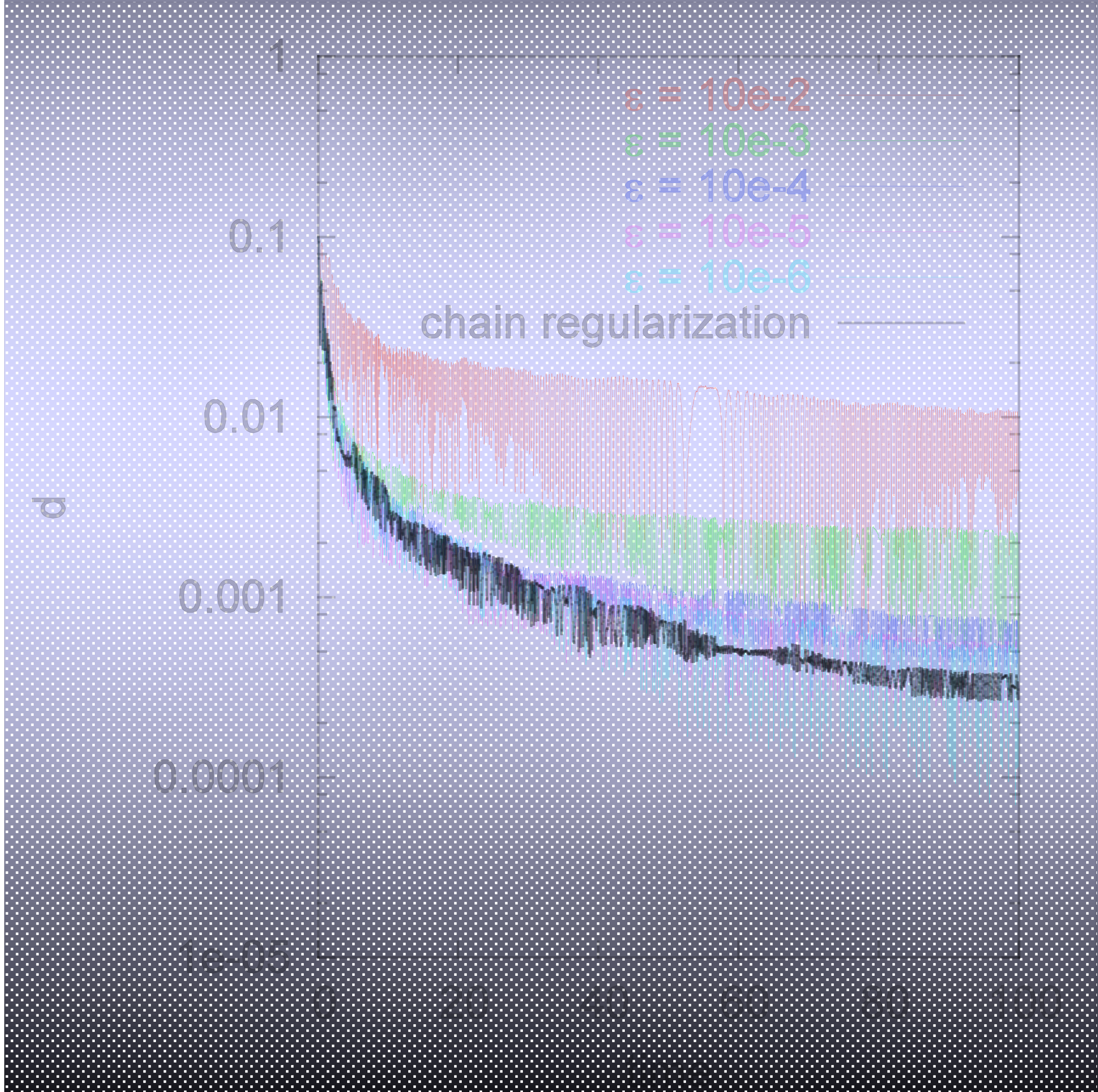


- KS regularization applied to neighboring particles in chain
- Zare-Szebehely time transformation
- External particles added as perturbers
- External particles see chain as resolved pseudo-particle
- Implementation by A. Szell, S. Mikkola

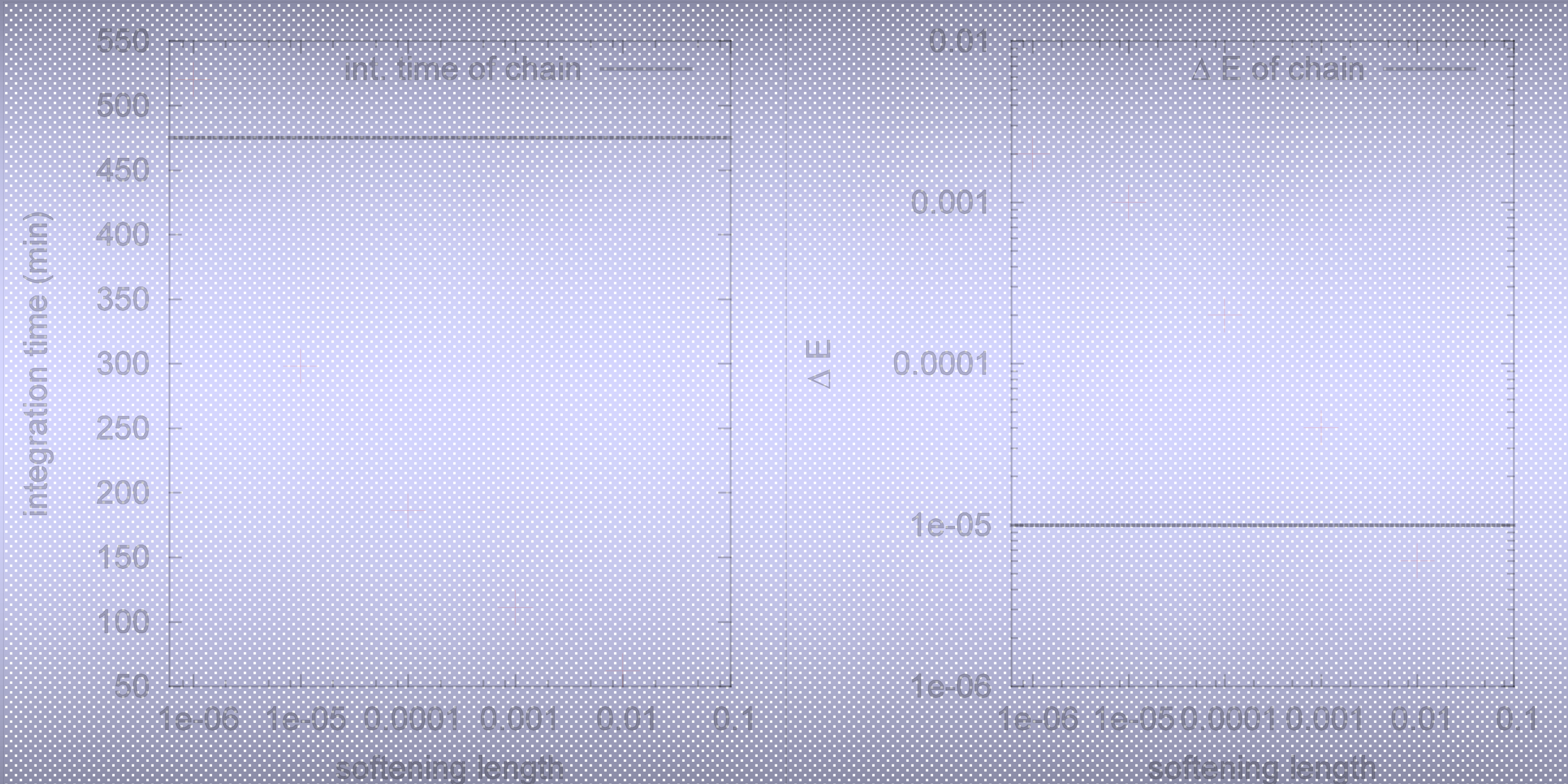
Performance: Chain Regularization Algorithm



Binary evolution, with and without chain.

Accuracy parameter _ of integrator was fixed.

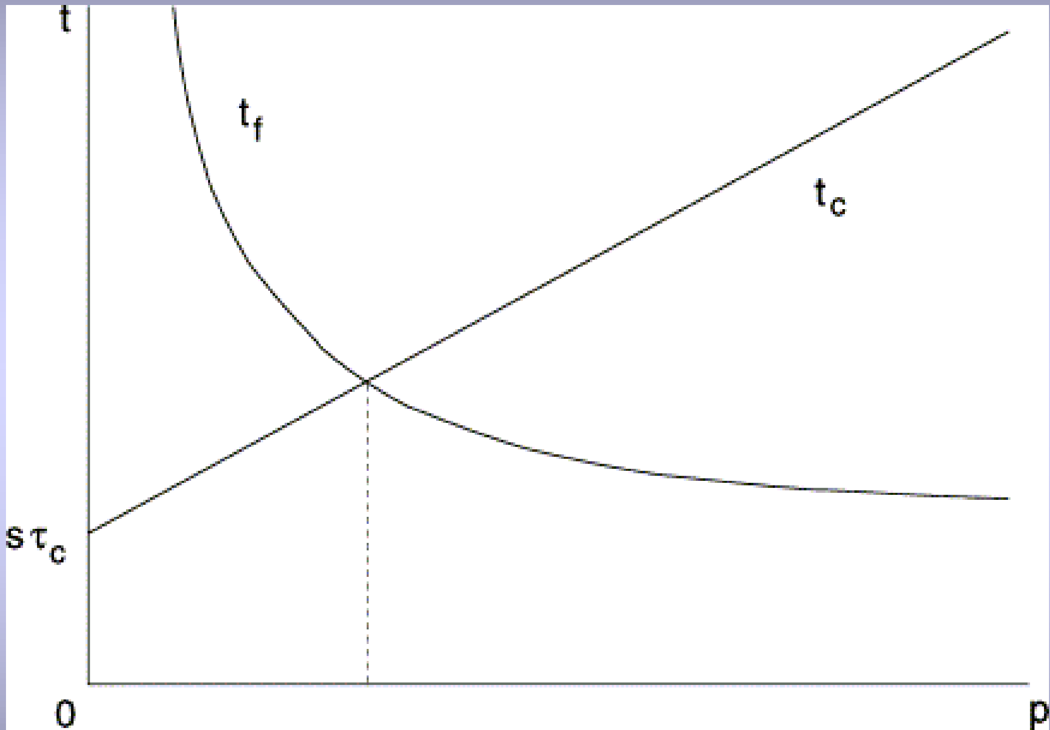
Performance: Chain Regularization Algorithm



Integration time

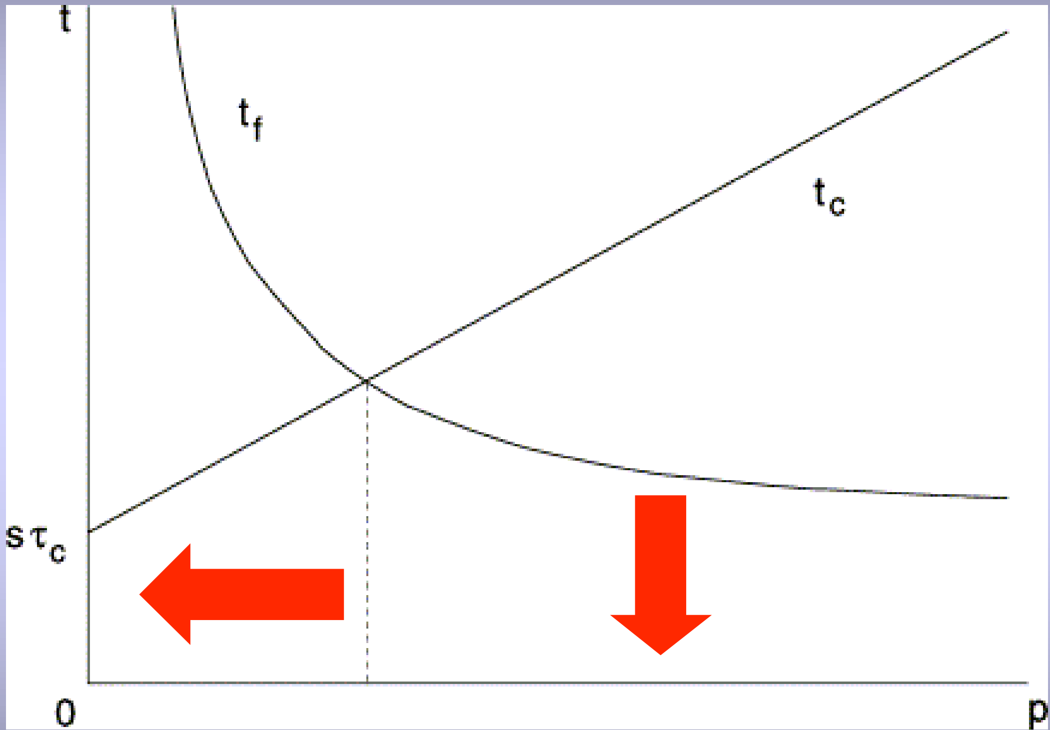
Energy error

Communication vs. Computation



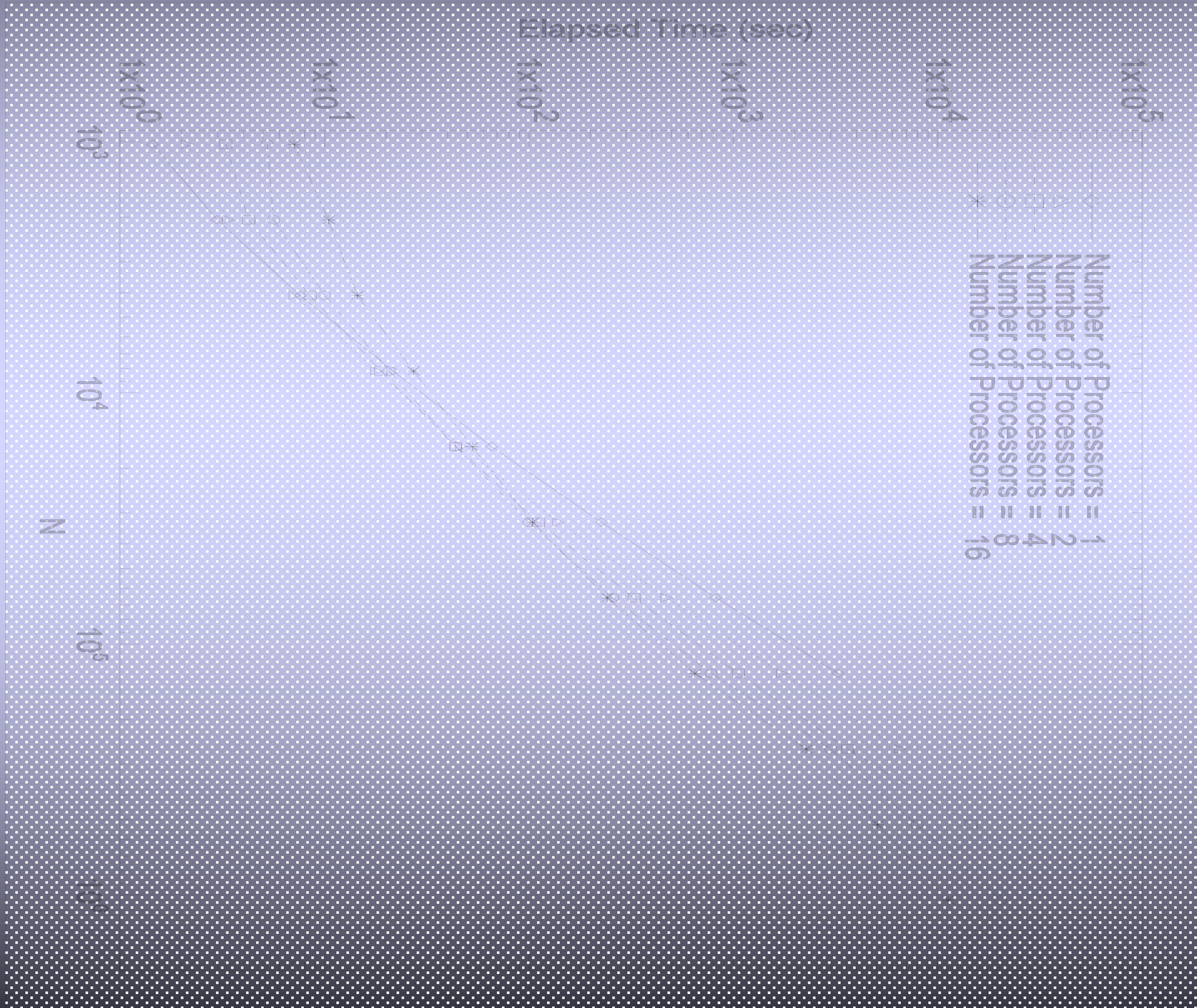
Communication (t_c) and force calculation (t_f) times as a function of processor number *p*, for fixed *N*.

Communication vs. Computation



Communication (t_c) and force calculation (t_f) times as a function of processor number *p*, for fixed *N*.

Performance: Parallel GRAPE Code

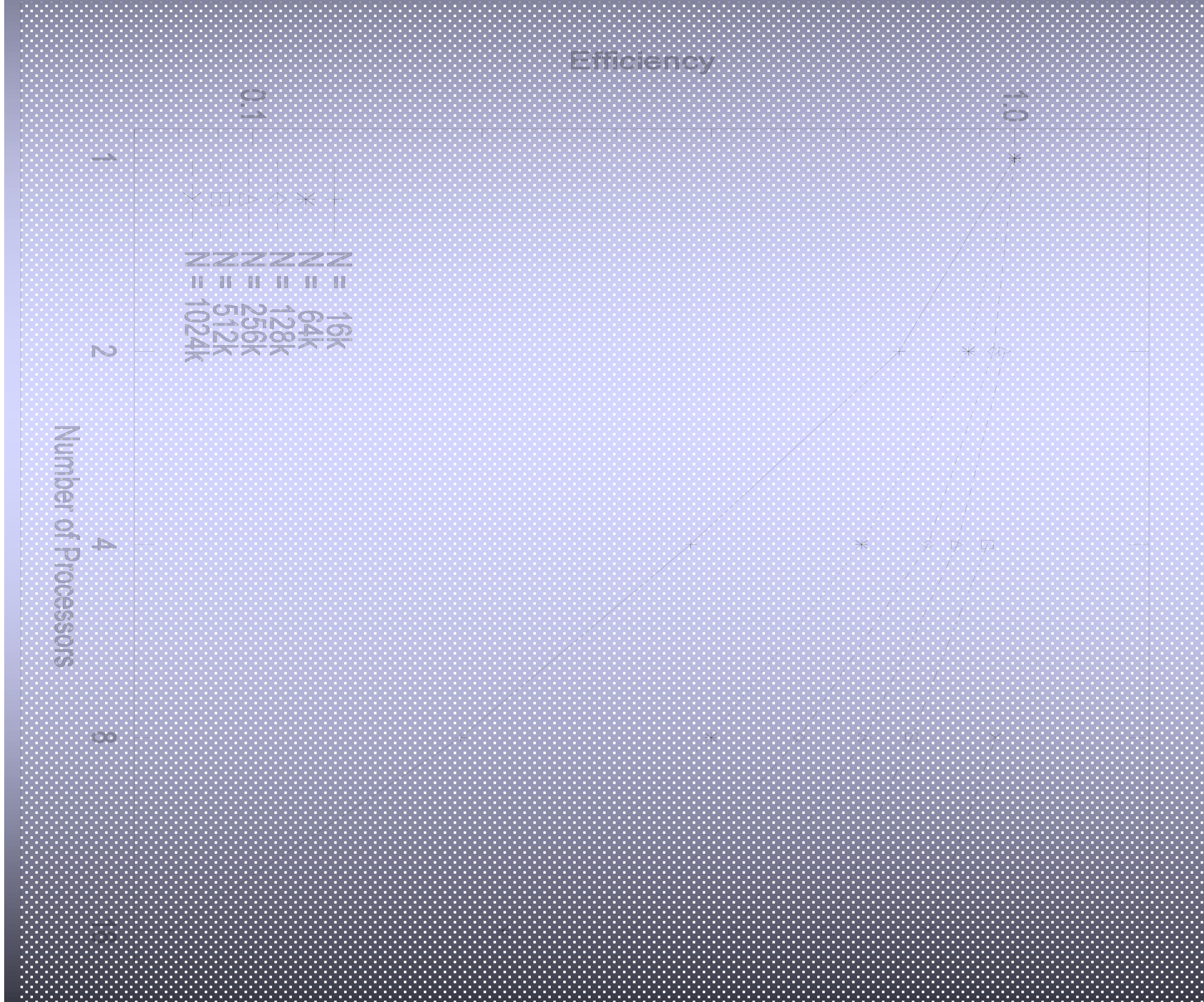


Wall clock time for integration of Plummer model for one *N*-body time unit.

(No chain)

S. Harfst

Performance: Parallel GRAPE Code



"Efficiency," defined as:

$$E = T_1/(pT_p)$$

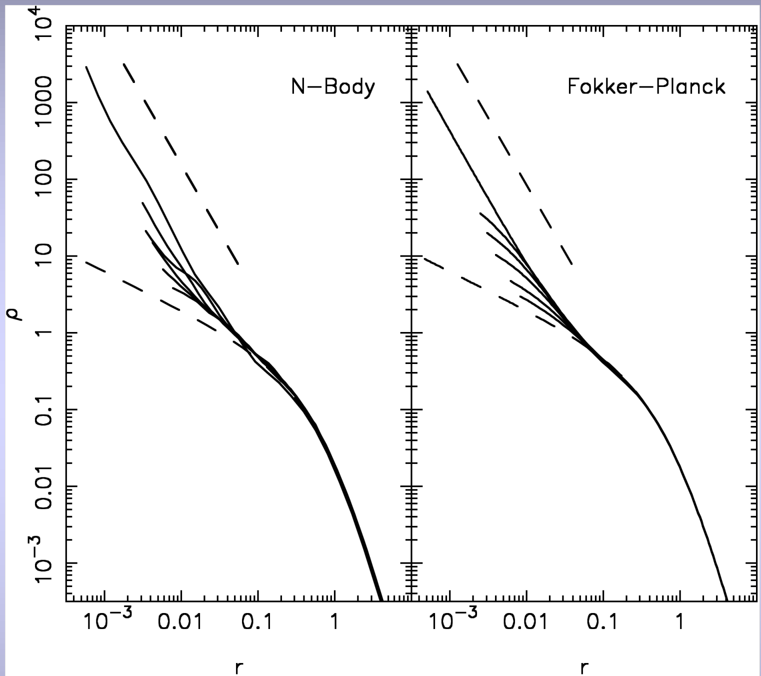
where:

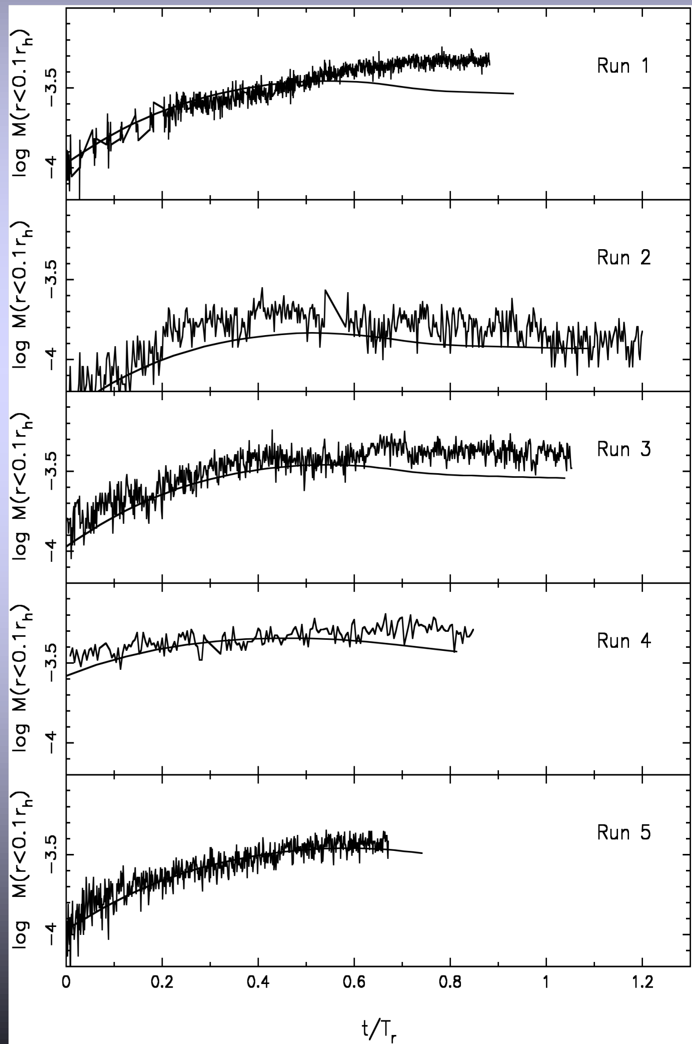
T_1 = time on one processor

T_p = time on p processors

S. Harfst

Growth of a Bahcall-Wolf Density Cusp





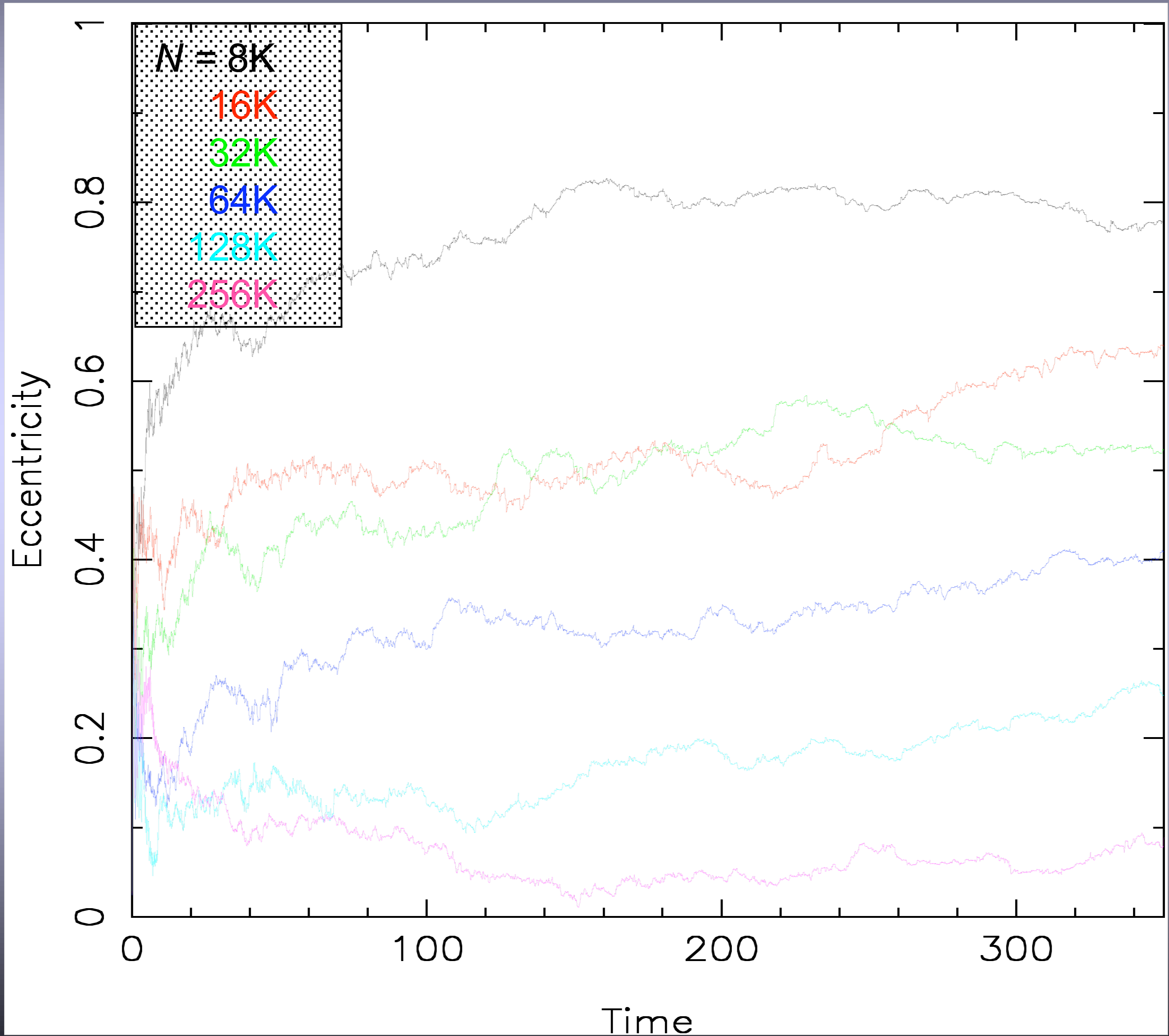
- GRAPE-6
- NBODY1h
- Chain
- 100K<*N*< 250K

A collisional, _ ~ $r^{-7/4}$ density cusp forms around a black hole on a time scale of ~T_r

(*Bahcall & Wolf 1976, 77*)

Preto, DM & Spurzem 2004

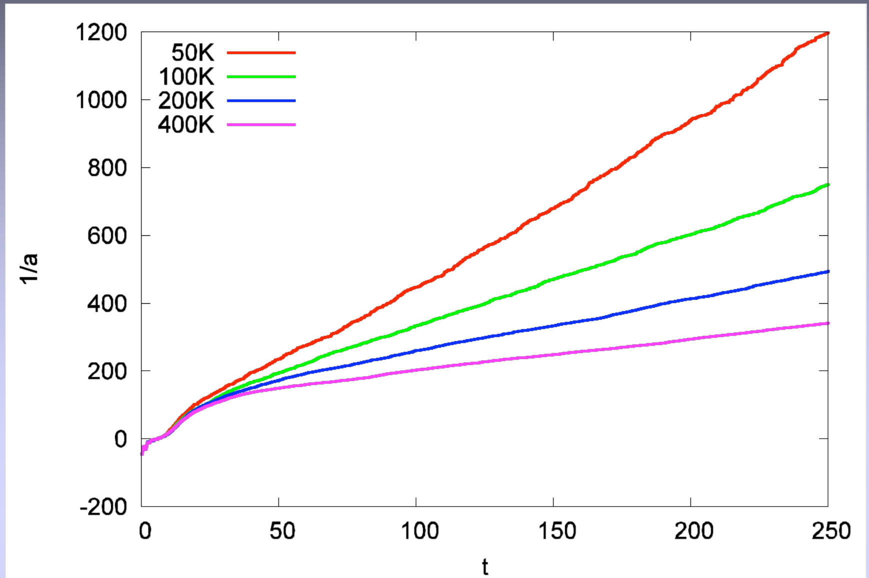
Eccentricity Evolution



- GRAPE-6
- NBODY1h
- Chain

Szell & DM 2005

Binary Black Hole Evolution with a GRAPE Cluster

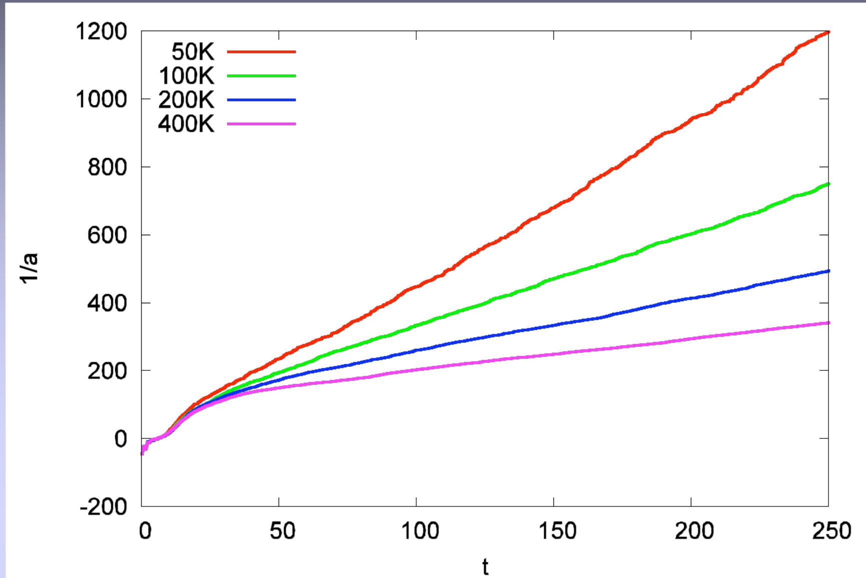


Initial conditions: two equal-mass black holes near center of Plummer-model galaxy.

- GRAPE cluster
- No chain
- 50K<*N*< 400K

Berczik & DM 2005

Binary Black Hole Evolution with a GRAPE Cluster



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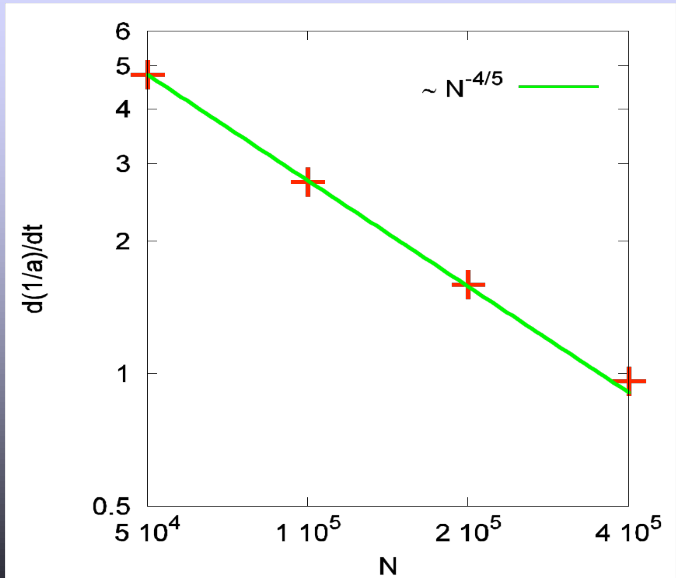
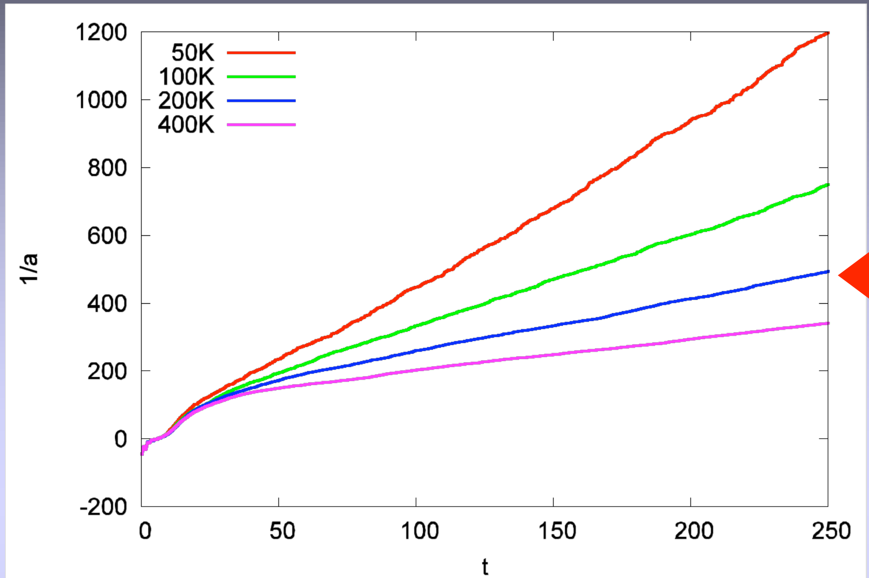
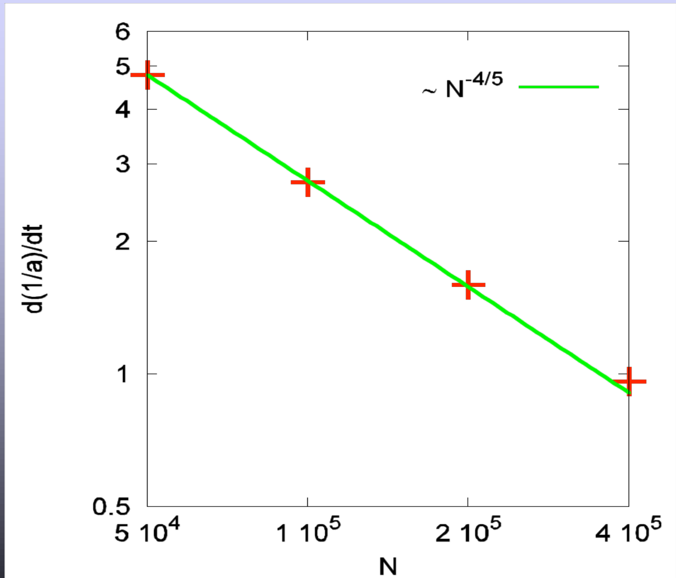


Result: Decay rate scales nearly as 1/*N* !

Berczik & DM 2005

Binary Black Hole Evolution with a GRAPE Cluster



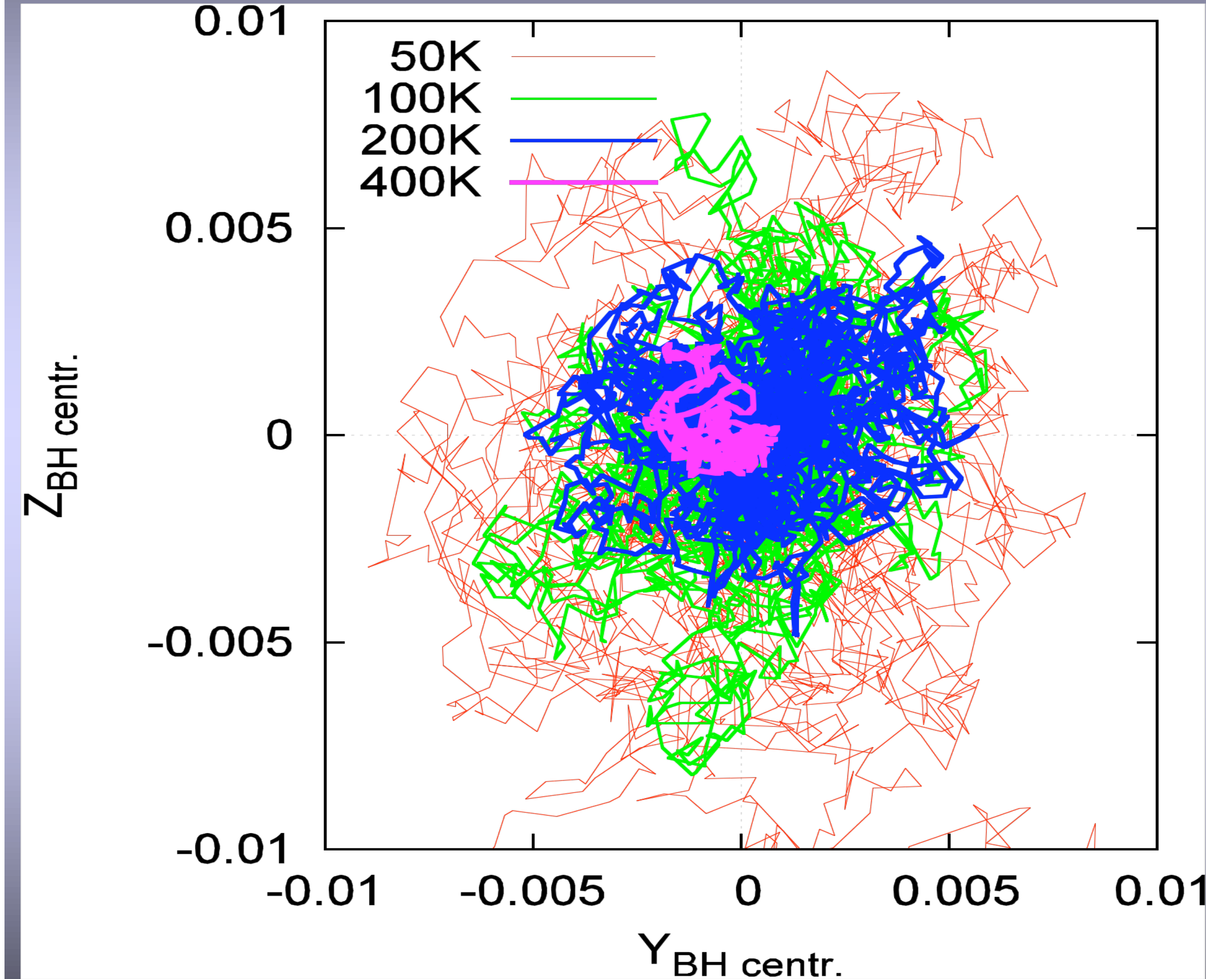


For the same initial conditions, Chatterjee, Hernquist & Loeb (2003) found that the decay rate *stabilizes* at *N* ~ 200*K*.

We do not reproduce their result.

CHL03 used a "hybrid" code, in which the large-scale force was computed from a basis-function expansion.

Binary Black Hole Evolution with a GRAPE Cluster



Wandering of the binary with respect to the density center of the galaxy.

Consistent with predictions of classical Brownian motion theory.

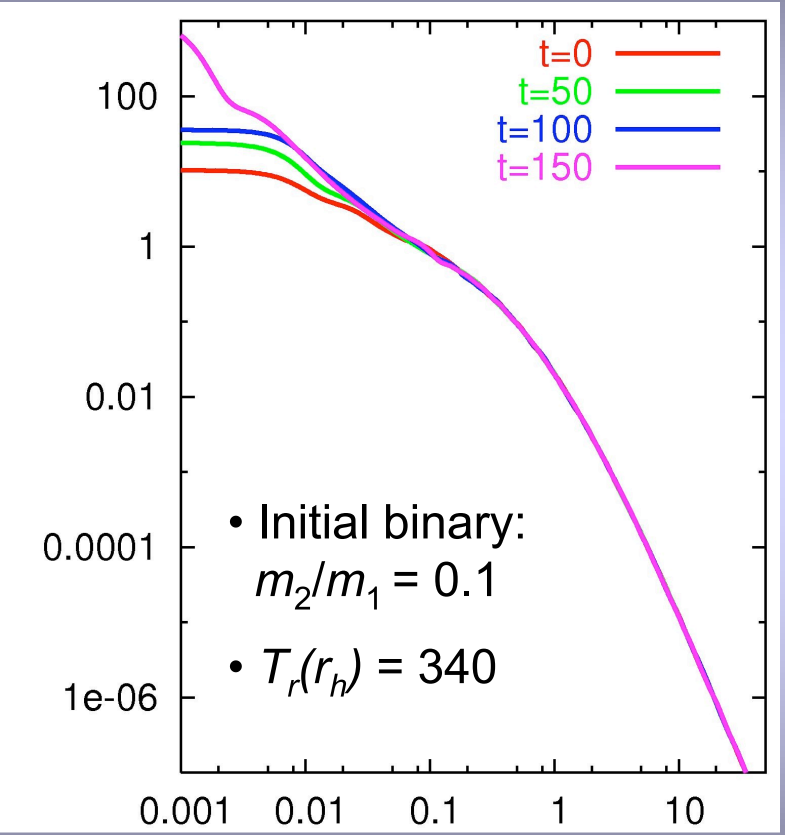
Smaller than reported by CHL03.

Berczik & DM 2005

Cusp Regeneration

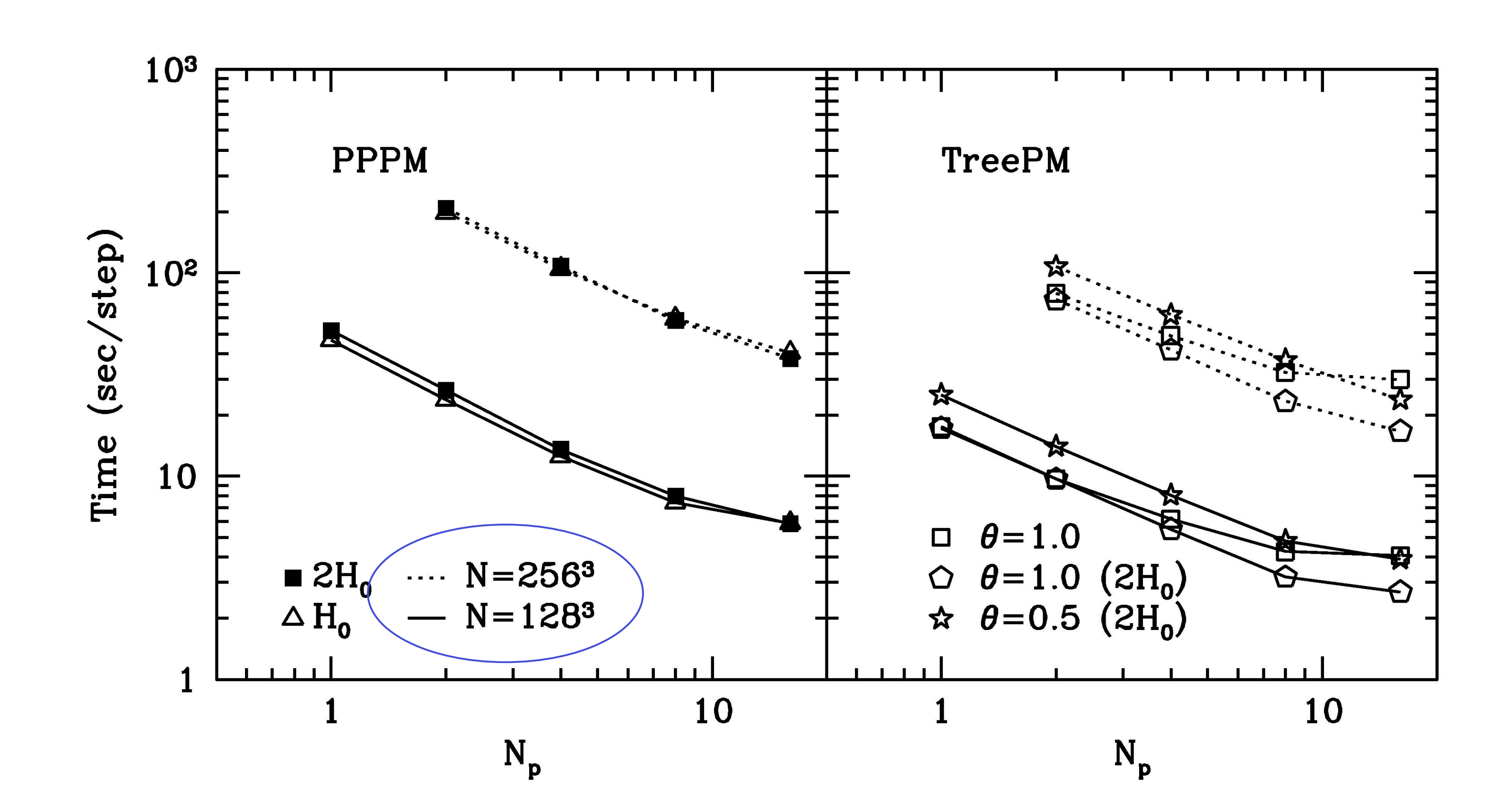
On a time scale of $T_r(r_h)$, a cusp that was destroyed by a binary BH can re-generate itself.

This could have happened in the case of the MW nucleus.



Szell & DM 2005

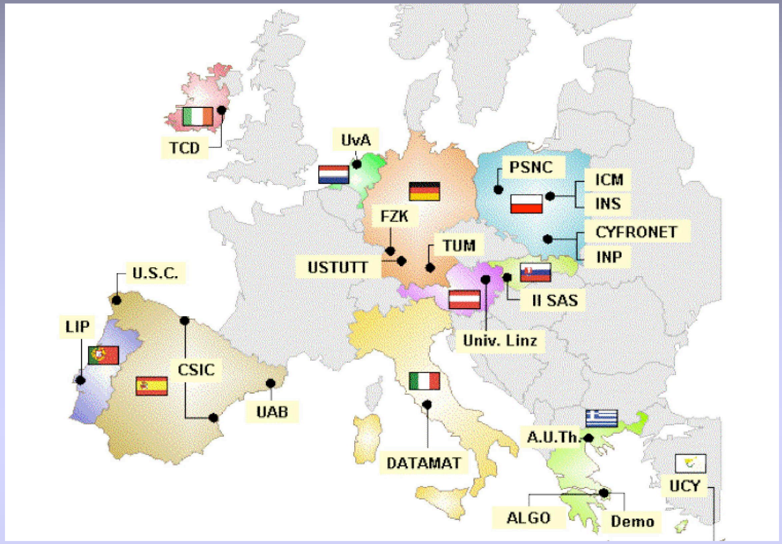
Going to Larger *N*. I. Approximate Algorithms



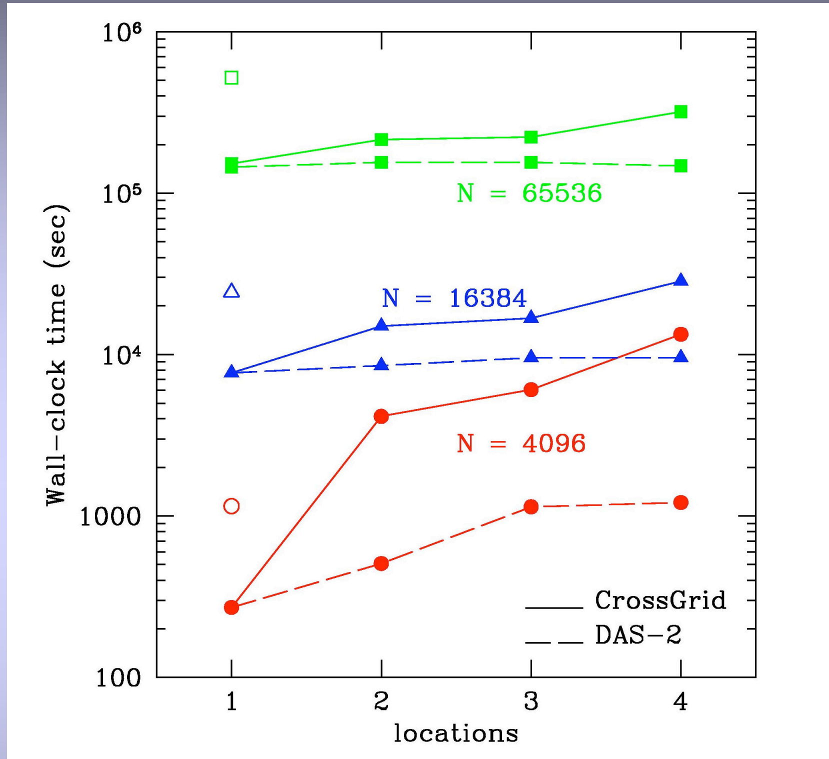
Yoshikawa & Fukushige (2005)
PPPM and TreePM methods on a GRAPE cluster.

Main worry: accuracy *vs*. speed

Going to Larger *N*. II. Grid Computing



Cross-Grid network

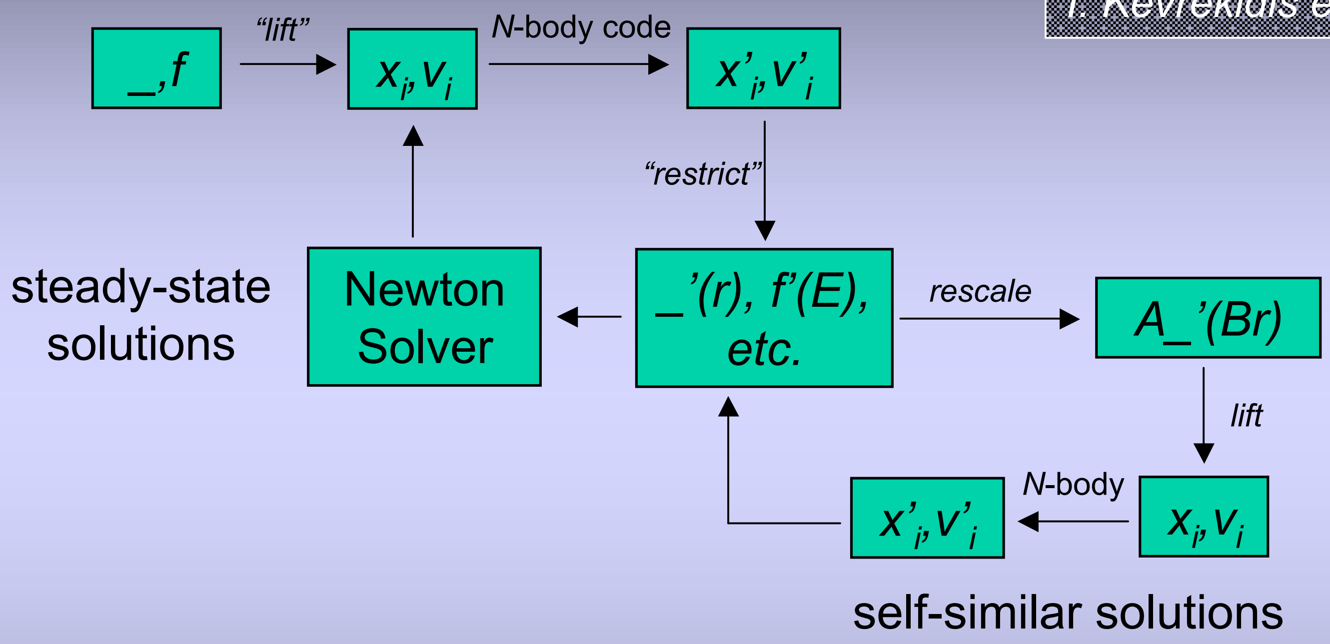


Tirado-Ramos, Gualandris & Portegies Zwart (2005)
N-body codes on the Pan-European CrossGrid network.

Main worry: communication losses

Going to Larger *N*. III. Dynamic Renormalization

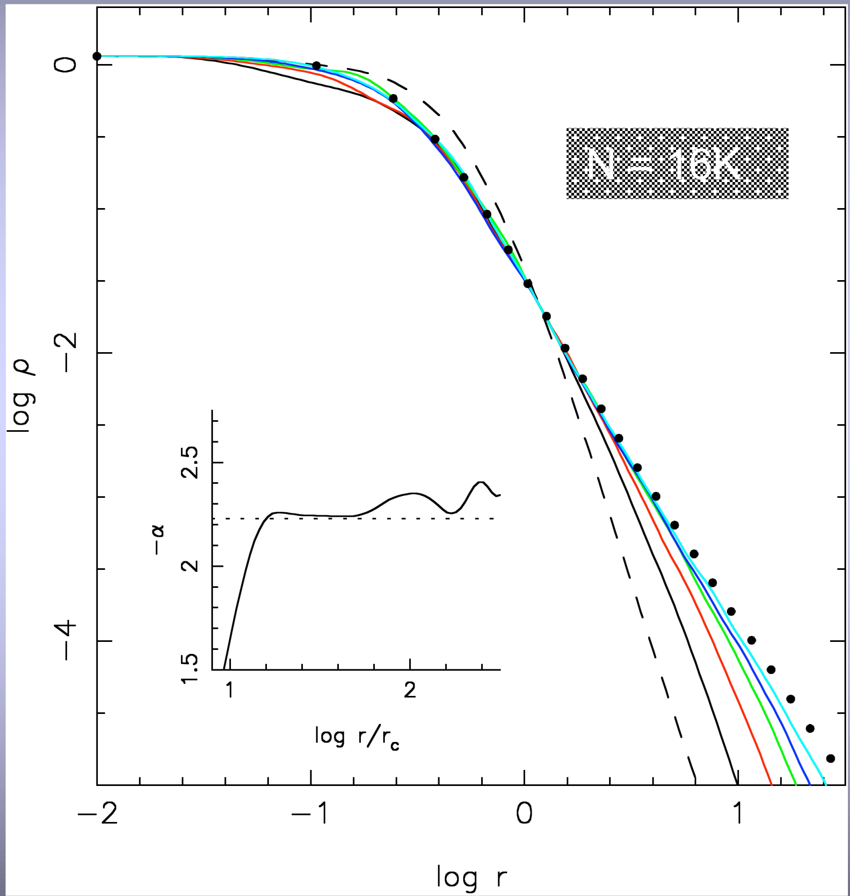
I. Kevrekidis et al.

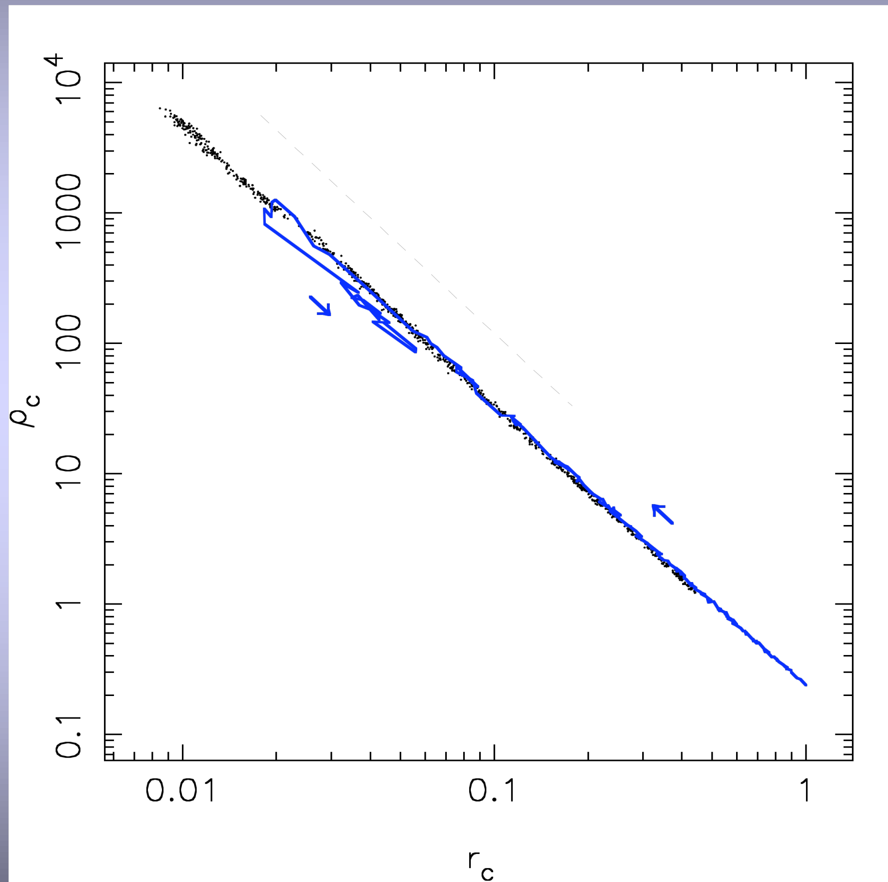


Useful for:

- Self-similar solutions *(core collapse)*
- Oscillatory solutions *(gravothermal oscillations)*
- Acceleration of *N*-body evolution *(almost everything else!)*

Core Collapse – Without the Binaries!





Szell, Merritt & Kevrekidis (2005)

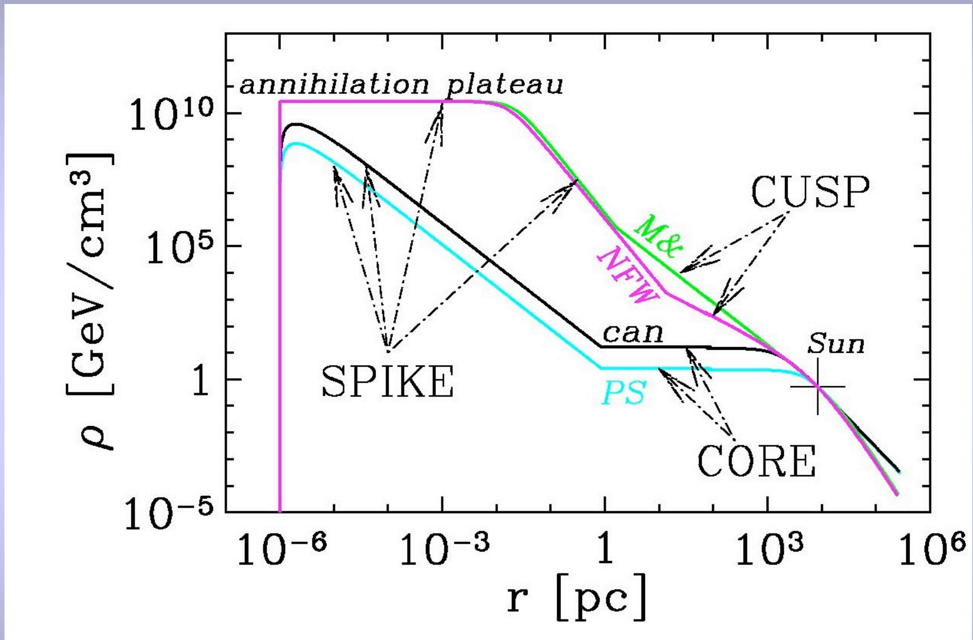
CONCLUSIONS

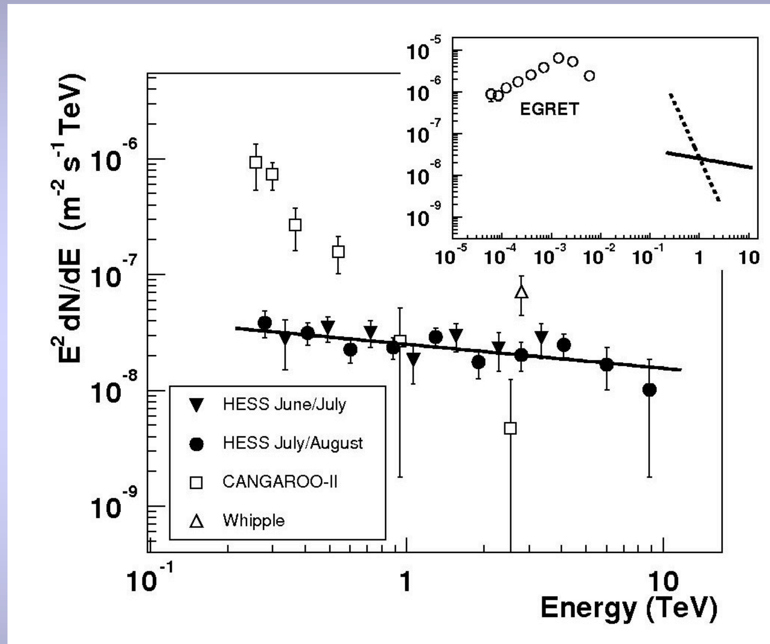
- Realistic simulation of dynamical processes in galactic nuclei requires particle numbers in excess of $\sim 10^6$.
- Such high particle numbers are now accessible via a combination of special-purpose hardware, parallel processing, and new algorithms.
- Progress should be made in the near future on problems including:

  -- Evolution of binary supermassive black holes
  -- Evolution due to encounters of galactic nuclei
  -- The loss-cone problem of single and binary black holes
  -- The interplay of dark and luminous matter
  -- ….. !

Dark Matter Distribution at the Galactic Center

Aharonian et al. 2004



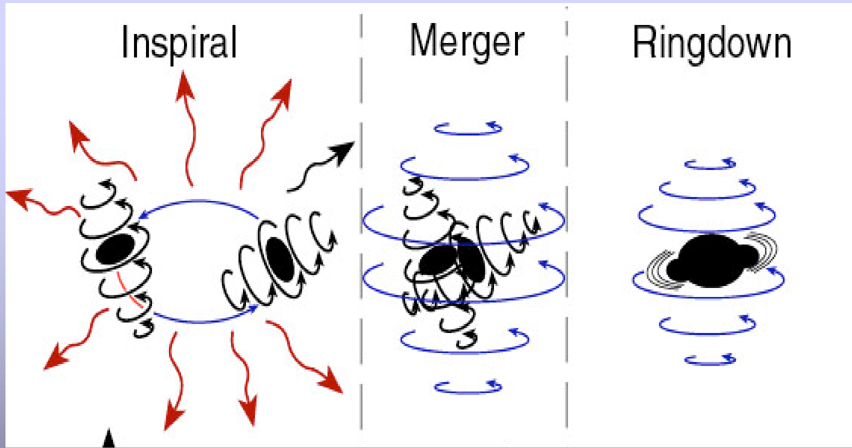


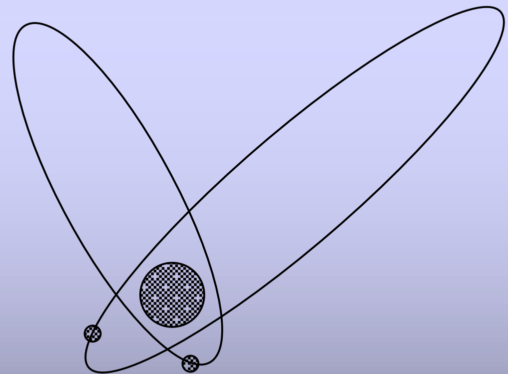
Strength of particle annihilation signal depends on the dark matter distribution within inner pc of Galactic center.

Massive Black Hole Binaries

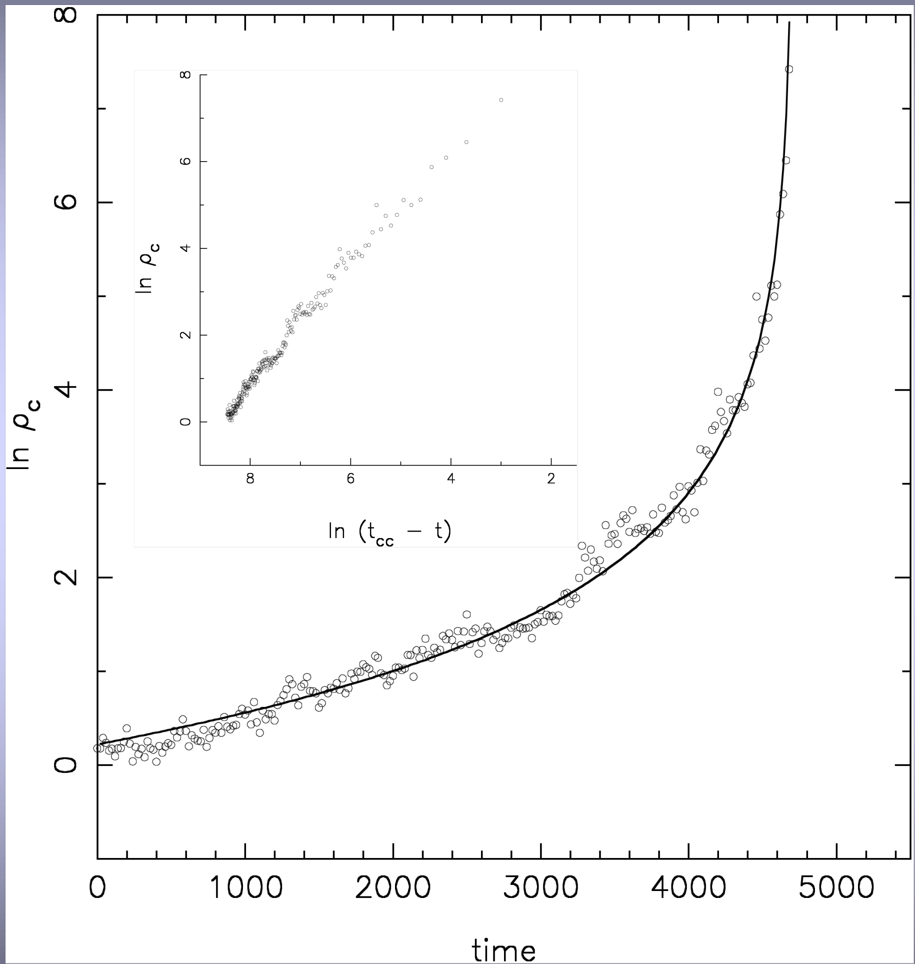
Two of the strongest potential sources of gravitational waves in the low-frequency (LISA) regime are:

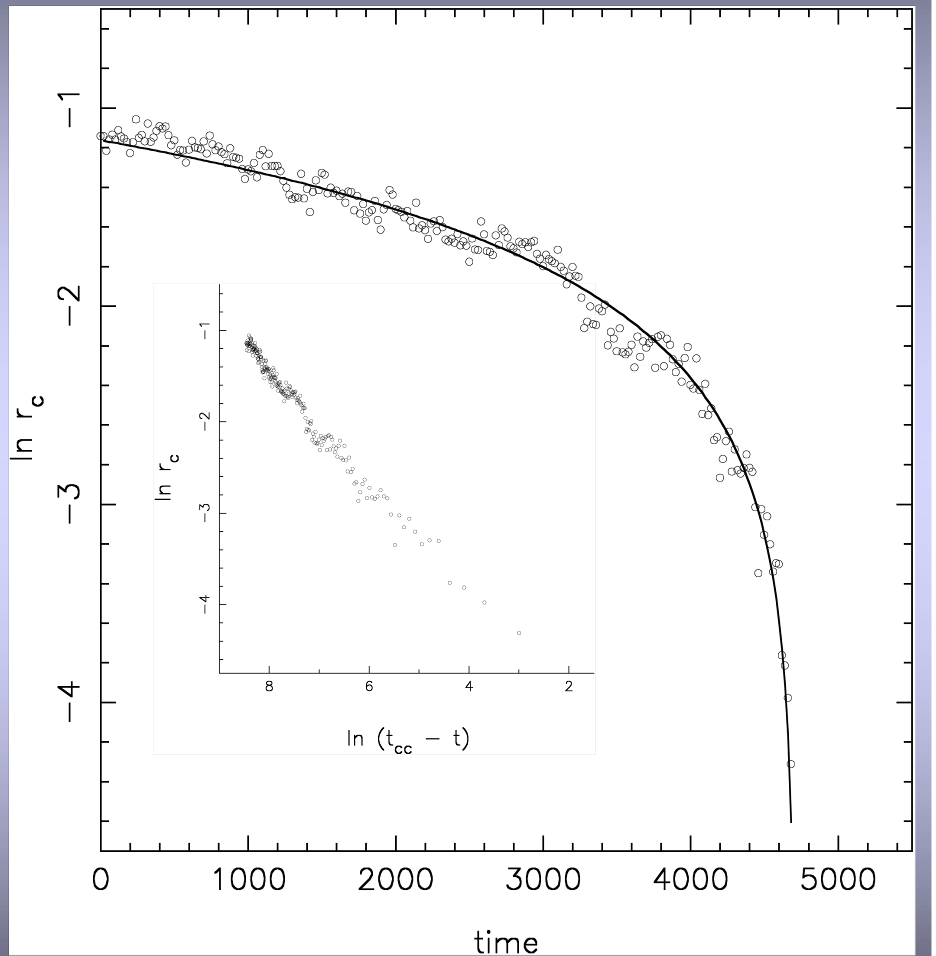
- Coalescence of binary supermassive black holes
- Extreme-mass-ratio inspiral into supermassive black holes





$$\rho(r,t) \approx \rho_c(t)\rho_*\left[\frac{r}{r_c(t)}\right]$$

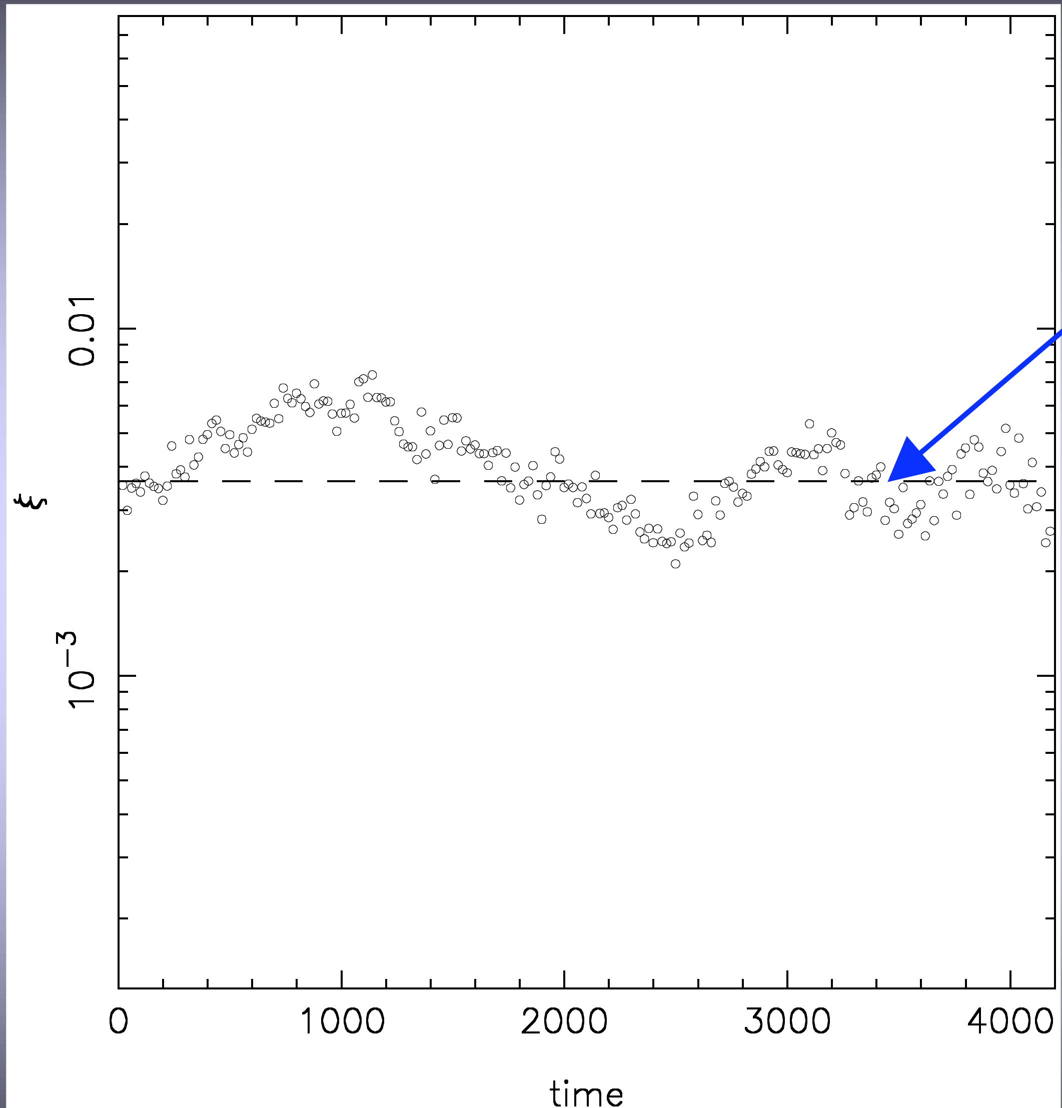




$$\rho_c(t) = \rho_0\left(t_{cc} - t\right)^{\gamma}, \quad \gamma \approx -1.41$$
$$(\text{fluid model}: \gamma = -1.18)$$

$$r_c(t) = r_{c0}\left(t_{cc} - t\right)^{\delta}, \quad \delta \approx 0.63$$
$$(\text{fluid model}: \delta = 0.53)$$

Dimensionless Core-Collapse Rate

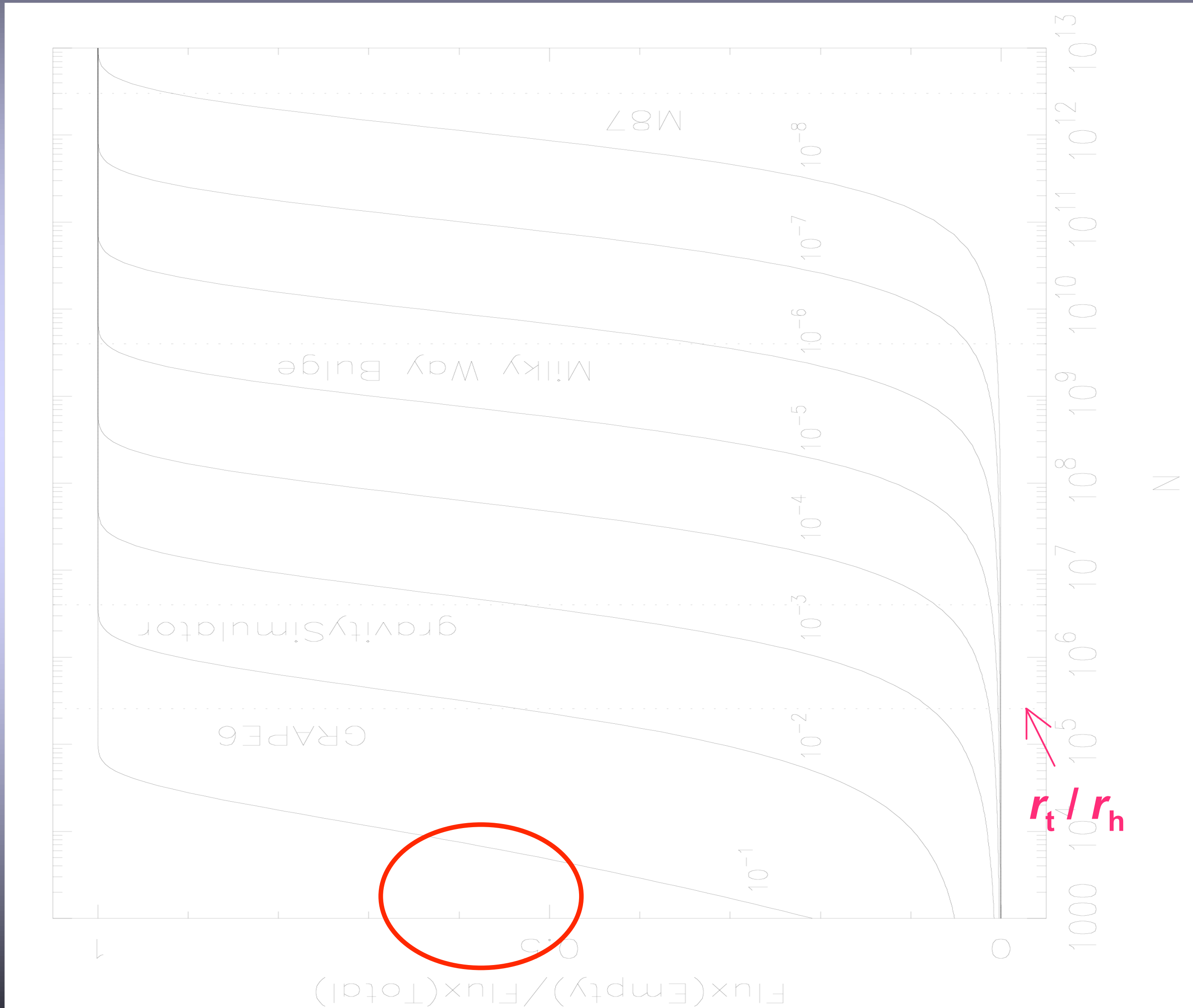


Heggie & Stevenson (1988):

_ ≈ 0.00364

(Fokker-Planck)

Figure-of-Merit for Loss Cone at Center of *N*-Body Galaxy



Vertical axis: fraction of loss-cone flux that comes from the "empty loss cone" region.

r_t: radius of capture sphere

r_h: BH influence radius

$M_\bullet = 0.01\ M_{gal}$