

**Theoretical and Observational Studies of
Topology of Large Scale Structure**
**Study of Space and Structure through Shapes of Large S
cale Structures**

2005. 6. 1

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Why is topology study useful?

1. Gaussianity of the **linear (primordial) density field** predicted by simple inflationary scenarios
2. Topology of galaxy distribution at NL scales sensitive to **cosmological parameters &**
o galaxy formation mechanism
3. Direct Intuitive meaning

Large Scales

Primordial Gaussianity

Small Scales

Galaxy Formation

Cosmological Parameters

Genus – A Measure of Topology

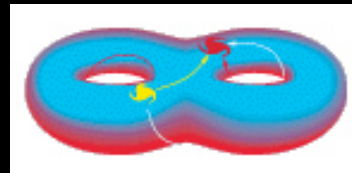
- **Definition**

G = # of holes - # of isolated regions

in iso-density contour surfaces

$$= 1/4\pi \int_S \kappa \, dA \quad (\text{Gauss-Bonnet Theorem})$$

[ex. $G(\text{sphere})=-1$, $G(\text{torus})=0$,



1

: 2 holes – 1 body = +1

- **Gaussian Field**

Genus/unit volume

$$g(\lambda) = A (1 - \lambda^2) \exp(-\lambda^2/2)$$

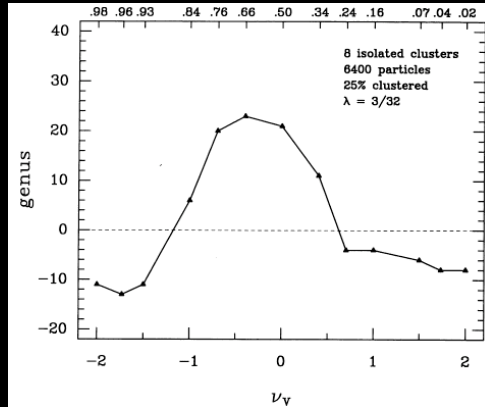
where $\lambda = (r - r_b) / r_b$ &

$$A = 1/(2\pi)^2 \langle k^2/3 \rangle^{3/2}$$

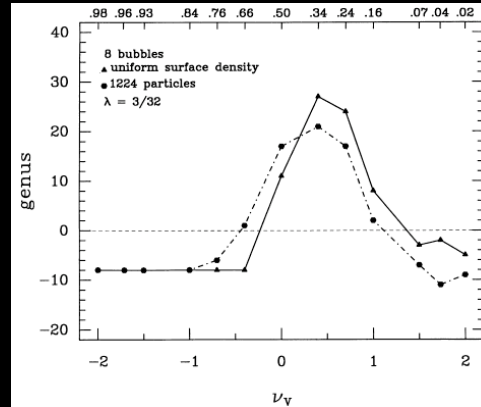
$$\text{if } P(k) \sim k^n, \quad A R_G^3 = [8\sqrt{2\pi^2}]^{-1} * [(n+3)/3]^{3/2}$$

● Non-Gaussian Field (Toy models)

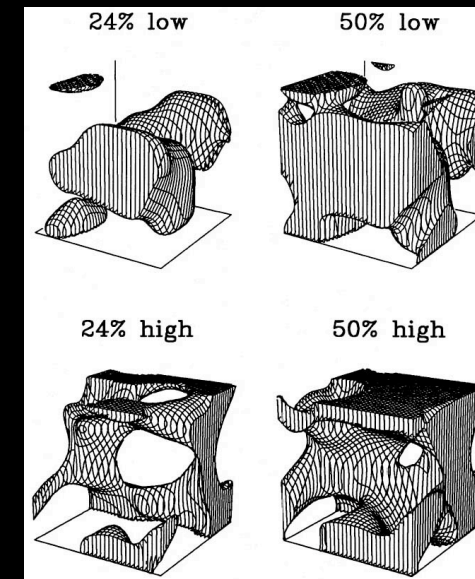
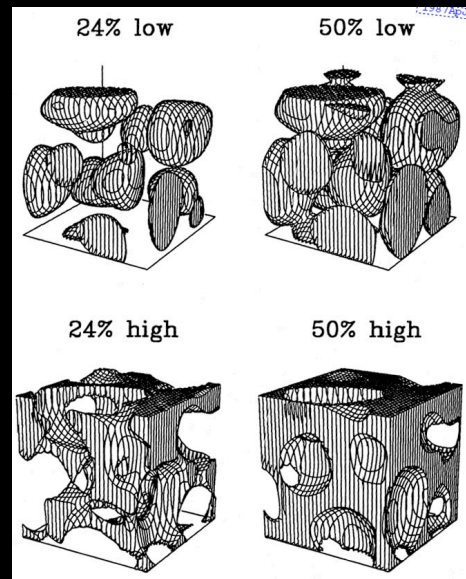
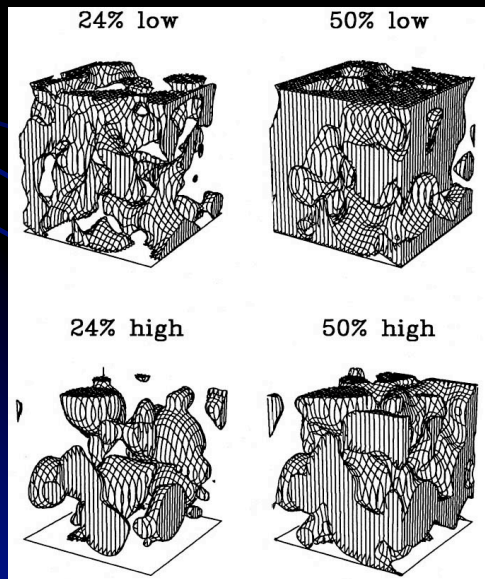
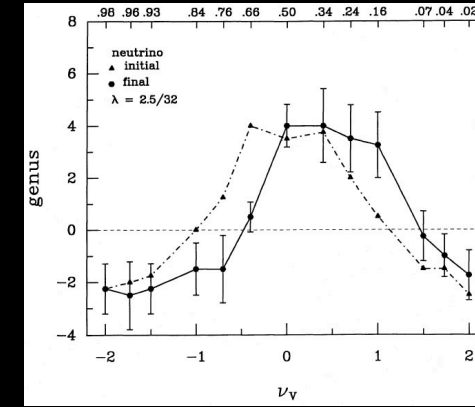
Clusters



Bubbles



HDM



(Weinberg, Gott & Melott 1987)

History of LSS Topology Study

I. Early Works

- **1986: Hamilton, Gott, Weinberg; Gott, Melott, Dickinson**
– smooth small-scale NL clustering to recover initial topology
- **1987-8: GWM, WGM, MWG, Gott et al.**
– cosmological & toy models. $R_c > 3r_c$ to recover initial topology
- **1989: Gott et al.** – observed galaxies, dwarfs, clusters
- **1991: Park, Gott** – gravitational & biasing effects
- **1992: Weinberg, Cole** – PS, initial skewness, biasing effects
- **1994: Matsubara** – 2nd order perturbation in weakly NL regime
- **1996: Matsubara** – redshift space distortion in L regime
- **1996: Matsubara, Suto** – gravitational & z-space distortion
- **Etc....**

II. Recent Works

- 2000: Colley et al. – Simulation of SDSS
- 2001, 2003: Hikage, Taruya & Suto – dark halos (analytic & numerical)
- 2003: Matsubara – 2nd order perturbation theory
- [Minkowski functionals (Mecke, Buchert & Wagner 1994; Schmalzing & Buchert 1997 etc.)]

III. 3D genus analysis of observational data

1989: Gott et al.	- CfA 1 etc.
1992: Park, Gott, & da Costa	- SSRS 1
1992: Moore et al.	- IRAS QDOT
1994: Rhoads et al.	- Abell Clusters
1994: Vogeley et al.	- CfA 1+2
1997: Protogeros & Weinbergs	- IRAS 1.2Jy
1998: Springel et al.	- IRAS 1.2Jy
1998: Canavezes et al.	- IRAS PSCz
2002: Hikage et al.	- SDSS EDR
2003: Hikage et al.	- SDSS LSS Sample 12
2004: Canavezes & Efsthious	- 2dFRGS

IV. 2D Genus (LSS)

- **2D genus before SDSS**

- Suggested by Melott (1987)
- Coles & Plionis (1991): Lick Galaxy Catalogue
- Plionis, Valdarnini, & Coles (1992): Abell and ACO cluster catalogue
- Park et al. (1992): CfA Slice
- Colley (2000): Simulated SDSS
- Park, Gott, & Choi (2001): HDF
- Hoyle, Vogeley & Gott (2002): 2dFGRS

- **2D genus with SDSS**

- Hoyle, Vogeley & Gott (2002): weak evidence for variation in the genus with galaxy type

Current status of LSS topology study

1. Large scales ($\gg 10 h^{-1}\text{Mpc}$)

Small survey size \rightarrow No strong constraints on primordial Gaussianity

2. Small scales ($< 10 h^{-1}\text{Mpc}$)

Little study so far

Effects of gravitational evolution,
galaxy biasing & internal physical properties of galaxies,
redshift-space distortion ...

Effects of Gravitational Evolution, Biasing, & Redshift Space Distortion on Topology

• Λ CDM Simulations

(Kim & Park 2004. 7)

(Ki

TreePM code GOTPM (Dubinski, Kim, Park 2003)

2048³ mesh (initial condition)

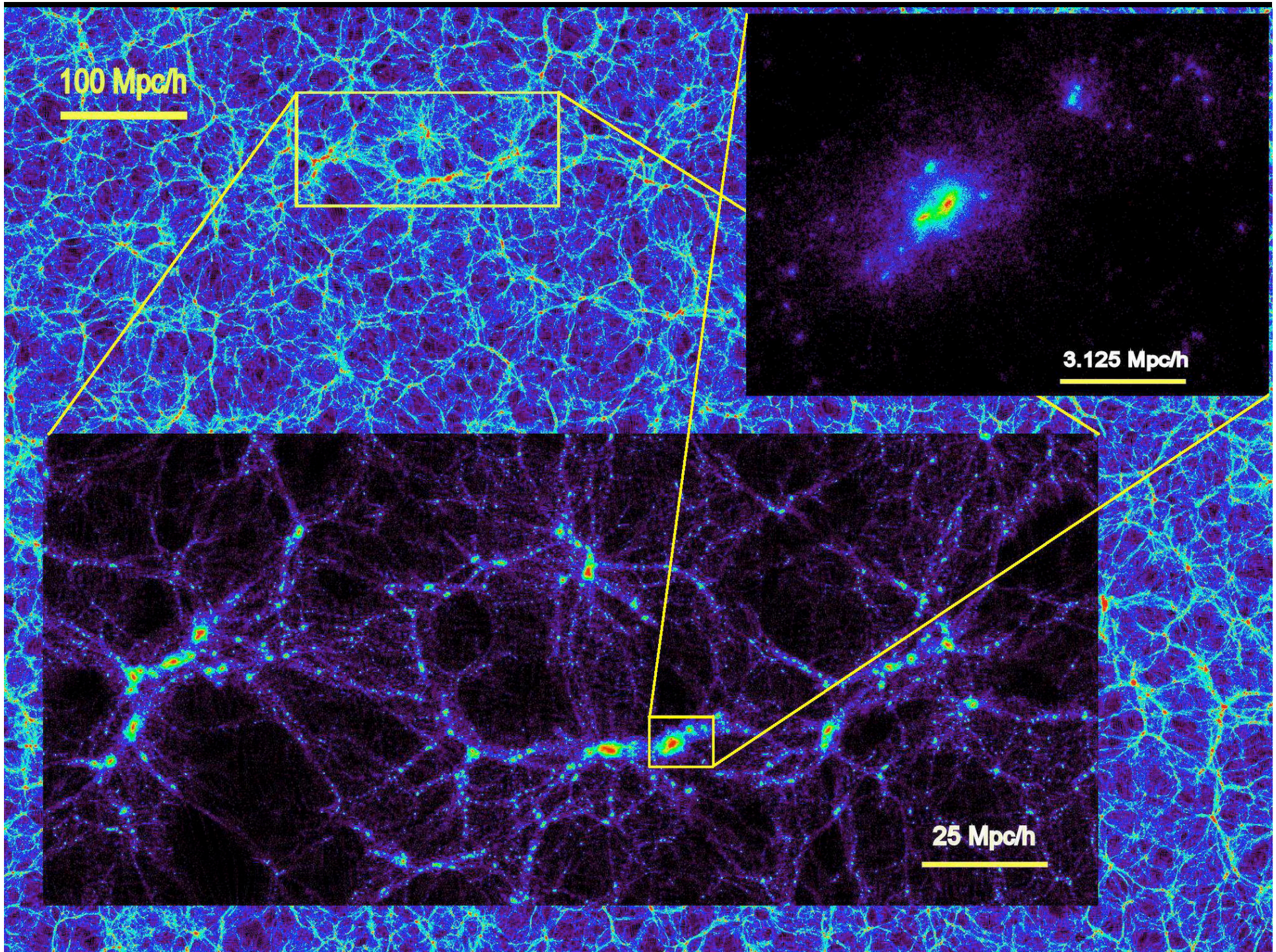
2048³ CDM particles

1024 & 5632 h⁻¹Mpc size boxes (>30 maxR_G)

50 & 275 h⁻¹kpc force resolutions

FOR PRECISION COMPARISON

between cosmological models with the real universe



Dark Halo Identification

(Kim & Park 2004:

• $\Omega_{DM} 1024 h^{-1} Mpc$)

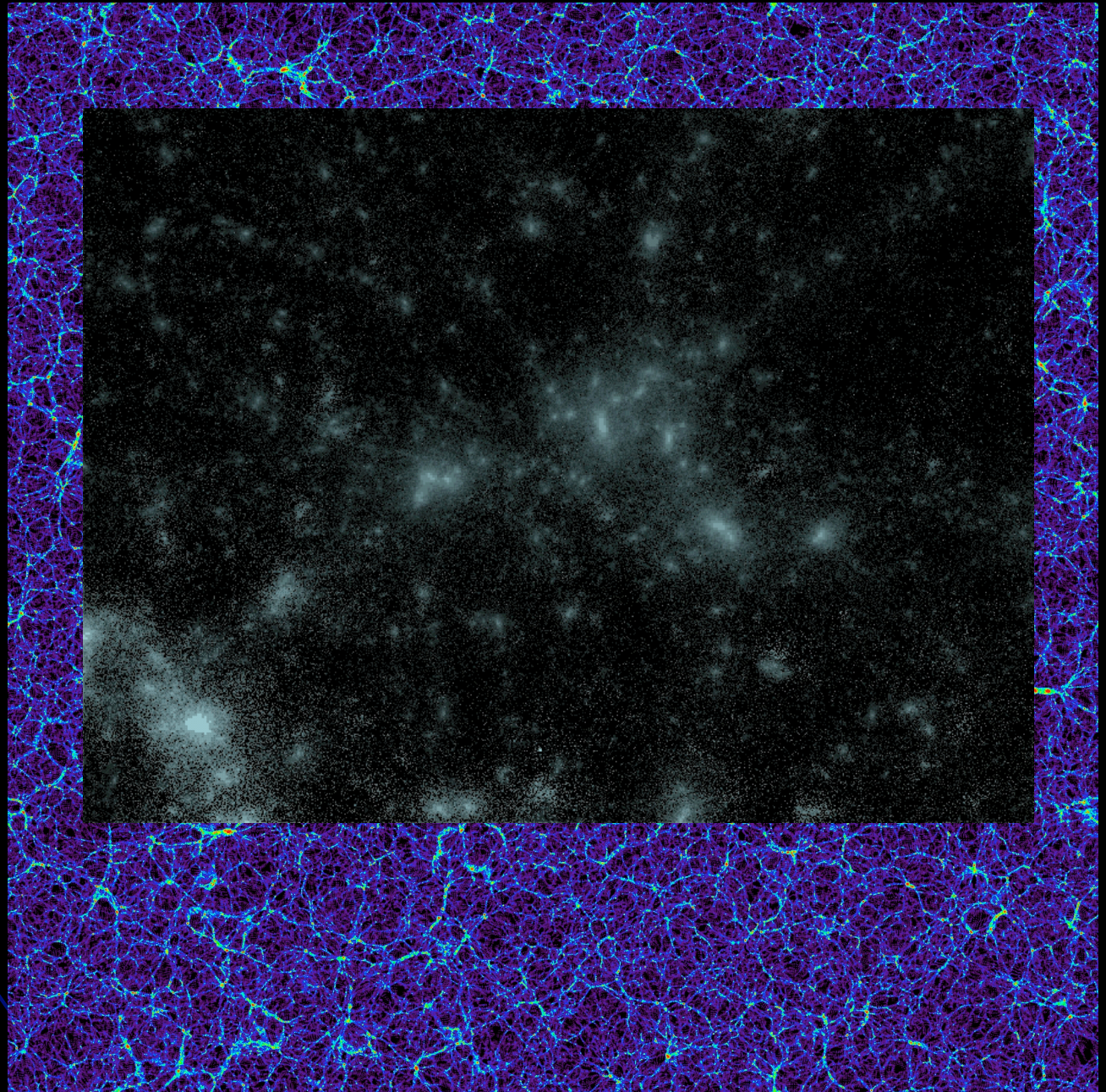
Physically Self-Bound

Halo centers
- local density peaks

Binding E wrt local halo centers

Tidal radii of subhalos wrt bigger halos

Halos with ≥ 53 particles ($5 \times 10^{11} M_{\odot}$)



[Kim & Park 2004 : 5632 & 1024 $h^{-1} Mpc$]

N-body simulation parameters

Table 1: Simulation Characteristics

	model	Ω_m	Ω_Λ	h	b	N_m^a	N_p	$L(h^{-1}\text{Mpc})$	z_i	N_{step}	code
a)	→ Λ CDM	0.27	0.73	0.71	1.11	2048^3	2048^3	1024	17	680	PMTree
	→ Λ CDM	0.27	0.73	0.71	1.11	2048^3	2048^3	5632	17	170	PMTree
b)	→ Λ CDM	0.3	0.7	0.7	1.11	512^3	512^3	128	23	980	PMTree
c)	→ Λ CDM	0.3	0.7	0.7	1.11	2048^3	1024^3	409.6	47	470	PM
	→ SCDM	1.00	0.00	0.5	1.5	2048^3	1024^3	1024	23	230	PM
	→ SCDM	1.00	0.00	0.5	1.5	2048^3	1024^3	409.6	47	470	PM

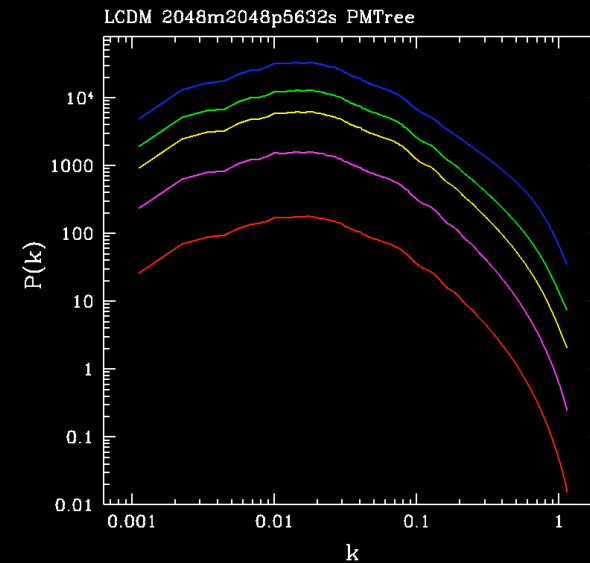
^aSize of mesh on which initial conditions are defined.

- a) → IBM SP3 at KISTI, 128 CPUs, 900 Gbytes,
- b) → IBM SP3 at SNU, 16 CPUs,
- c) → QUEST at KIAS, 128 CPUs, 256 Gbytes,

Gravitational Evolution

Initial conditions

- Growth by gravitational instability
- weakly interacting & cold matter
- Present epoch at $\Omega_m = 0.3$



Dependence of Genus on ...

(Park, Kim & Gott 2005)

1. Smoothing Scales

$$R_G = 1.5 \sim 150 h^{-1} \text{ Mpc}$$

2. Tracers

Matter, Peaks in initial density field,
Dark halos, HOD 'galaxies'

3. Redshift space distortion

4. Cosmogony : LCDM vs SCDM

Genus-Related Statistics

- Amplitude drop R_A

$$R_A = A_{\text{obs}} / A_{\text{PS}}$$

- Shift parameter $\Delta\nu$

By fitting $G_{\text{obs}}(\nu)$ over $-1 < \nu < 1$

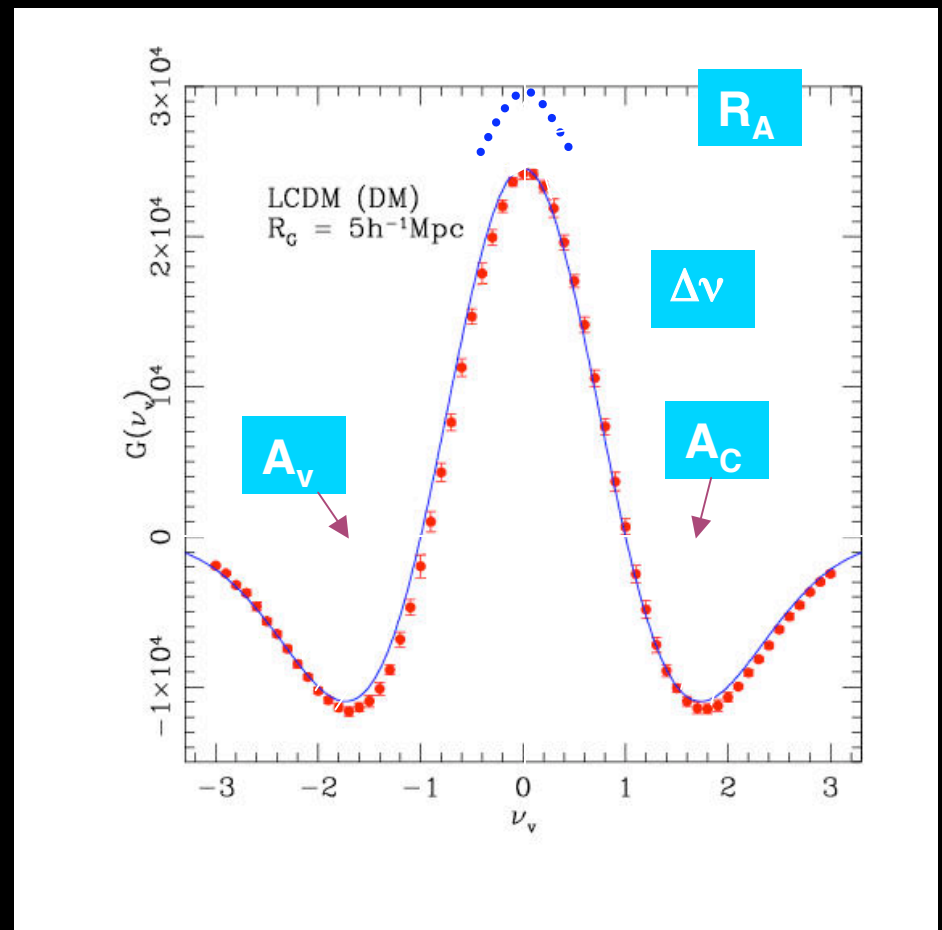
- Asymmetry parameters

A_V & A_C

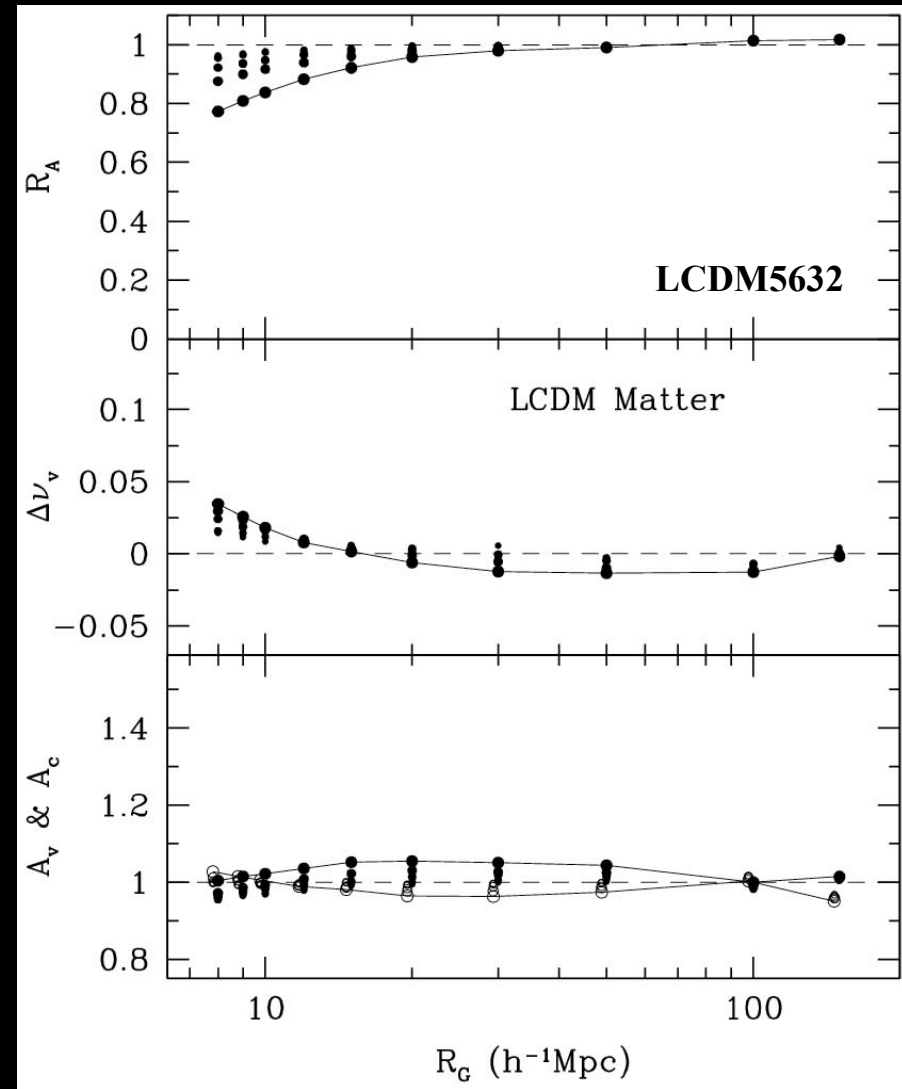
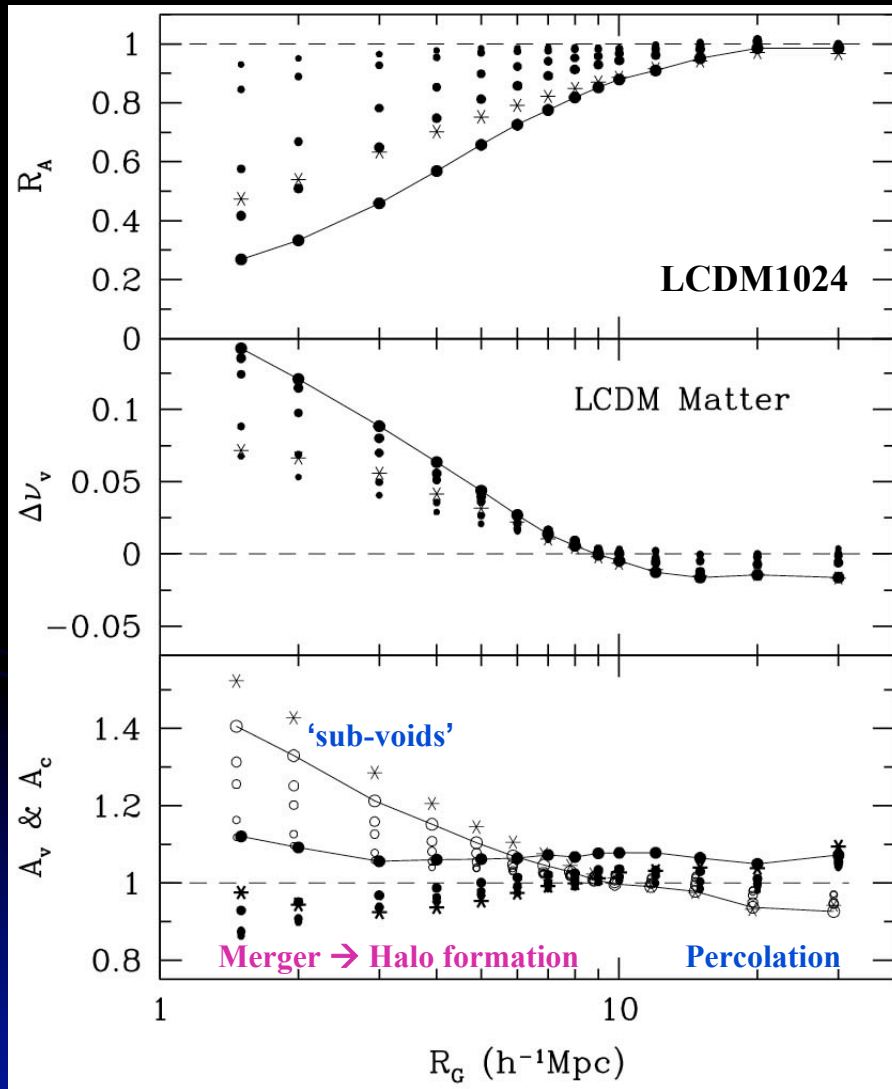
$$A = \int G_{\text{obs}}(\nu) d\nu / \int G_{\text{fit}}(\nu) d\nu$$

where intervals are

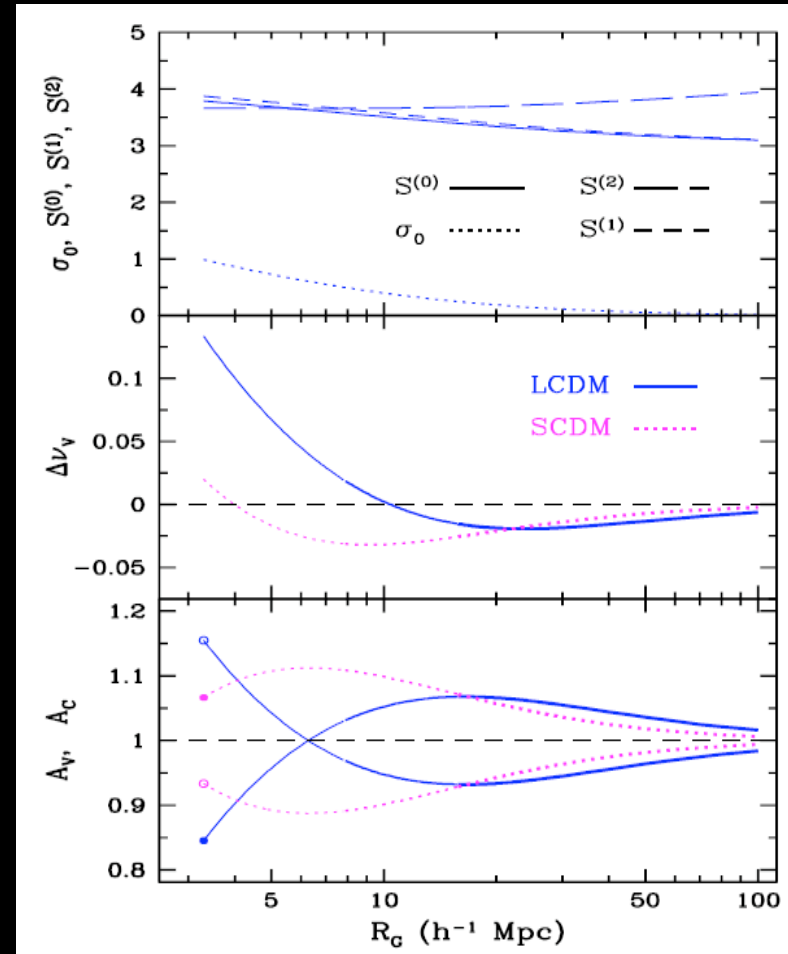
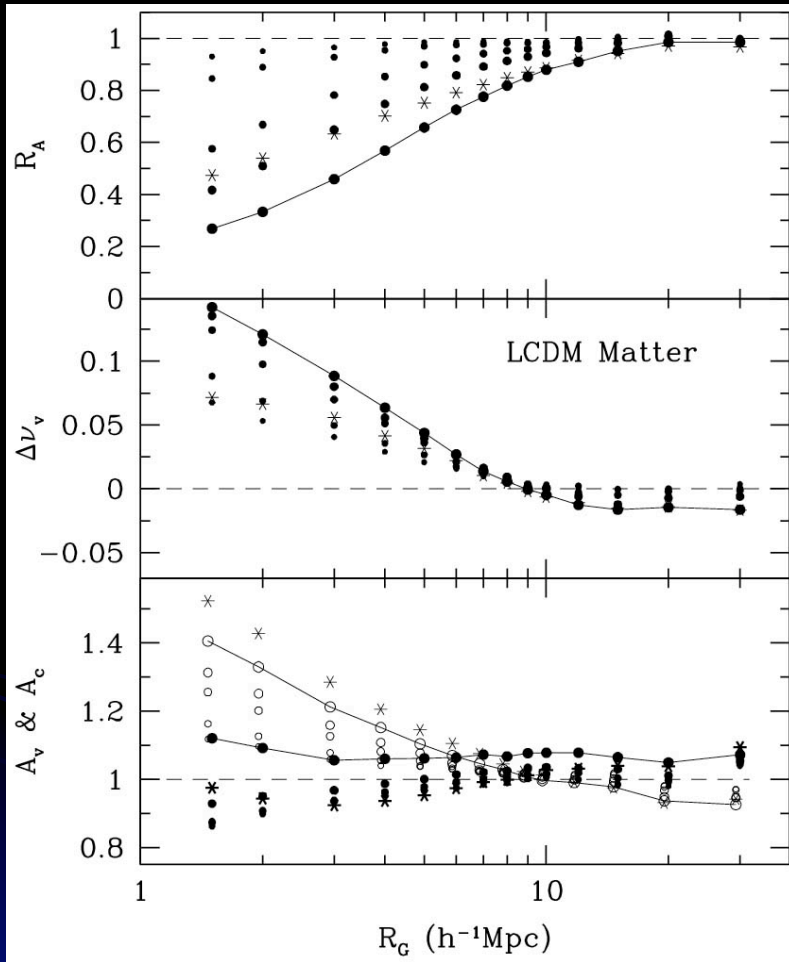
$-1.2 \sim -2.2$ (A_V), $1.2 \sim 2.2$ (A_C)



Dependence of genus on smoothing scale



Perturbation theory (Matsubara 2003)



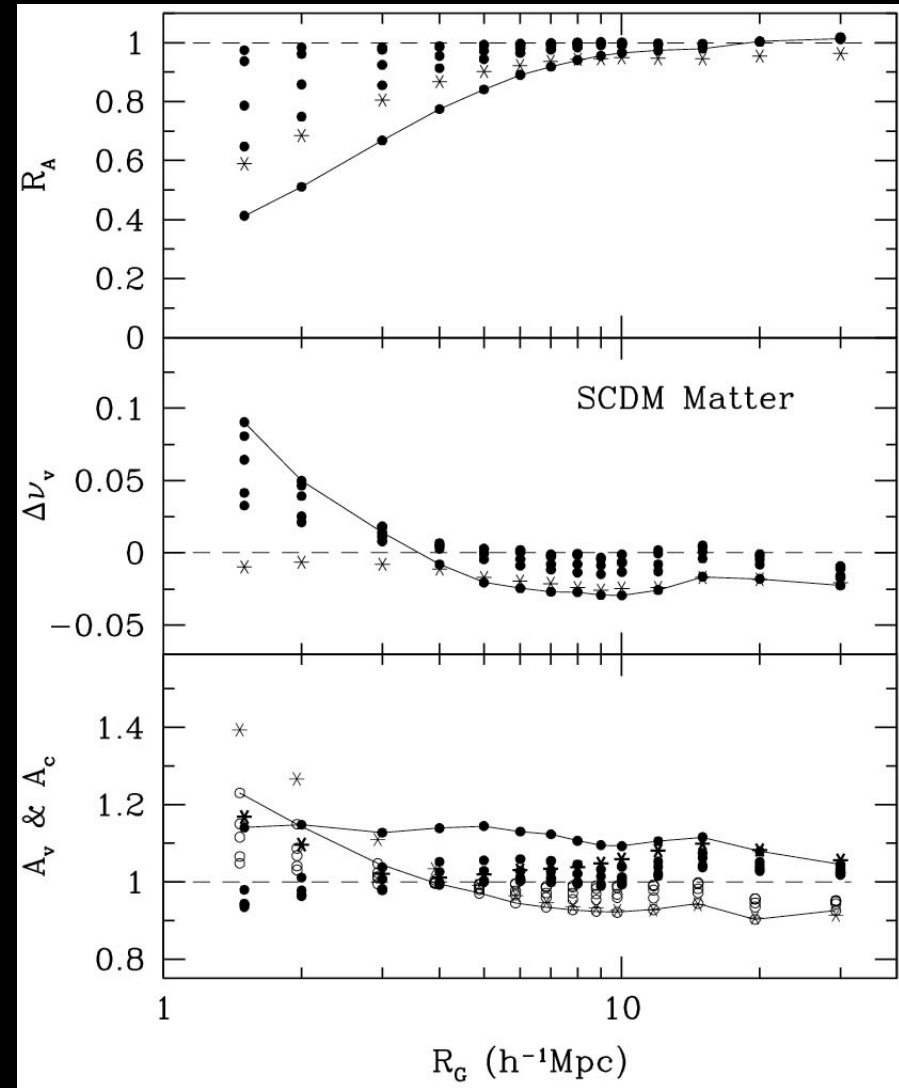
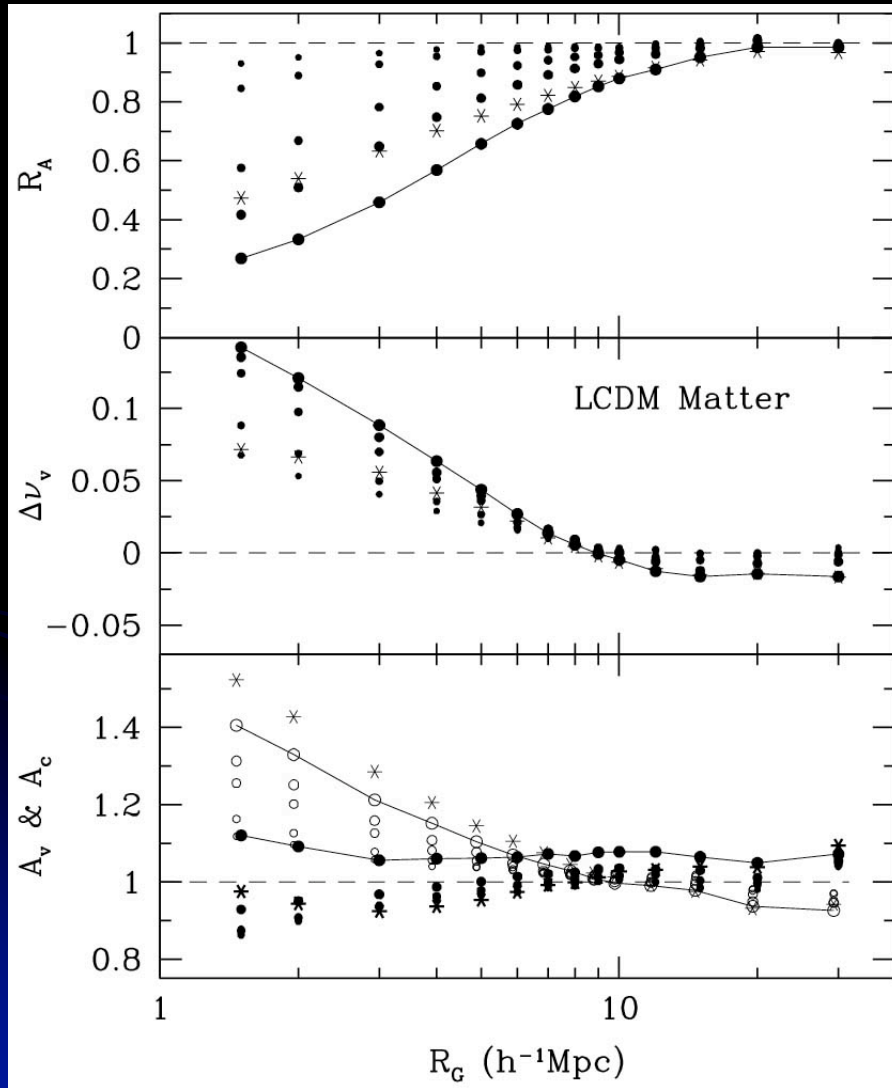
$$G(\nu_f) = \frac{1}{(2\pi)^2} \left(\frac{\sigma_1}{\sqrt{3}\sigma_0} \right)^3 e^{-\nu_f^2/2} (1 - \nu_f^2 - [(S^{(1)} - S^{(0)})(\nu_f^3 - 3\nu_f) + (S^{(2)} - S^{(0)})\nu_f] \sigma_0)$$

$$S^{(0)} = \langle \delta^3 \rangle / \sigma_0^4 = \frac{1}{\sigma_0^4} \int \frac{d^3 k_1}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} B(k_1, k_2, k_{12}),$$

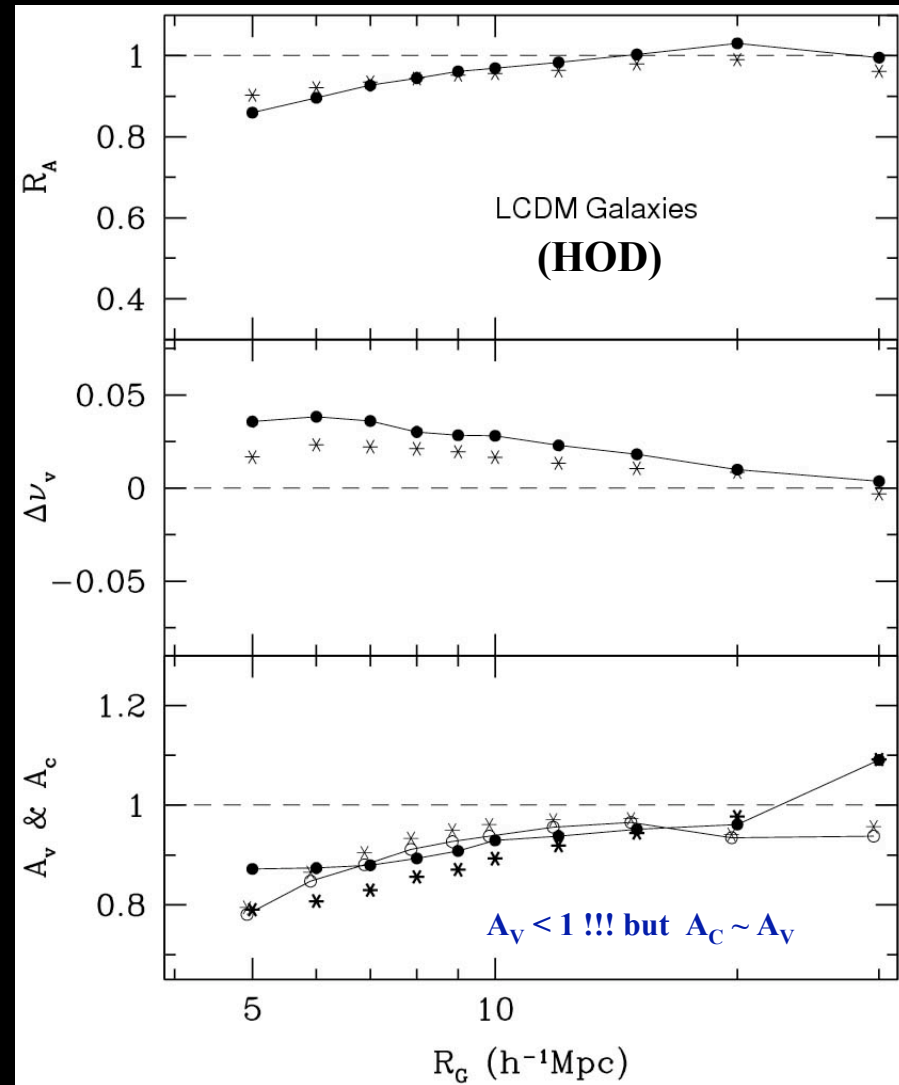
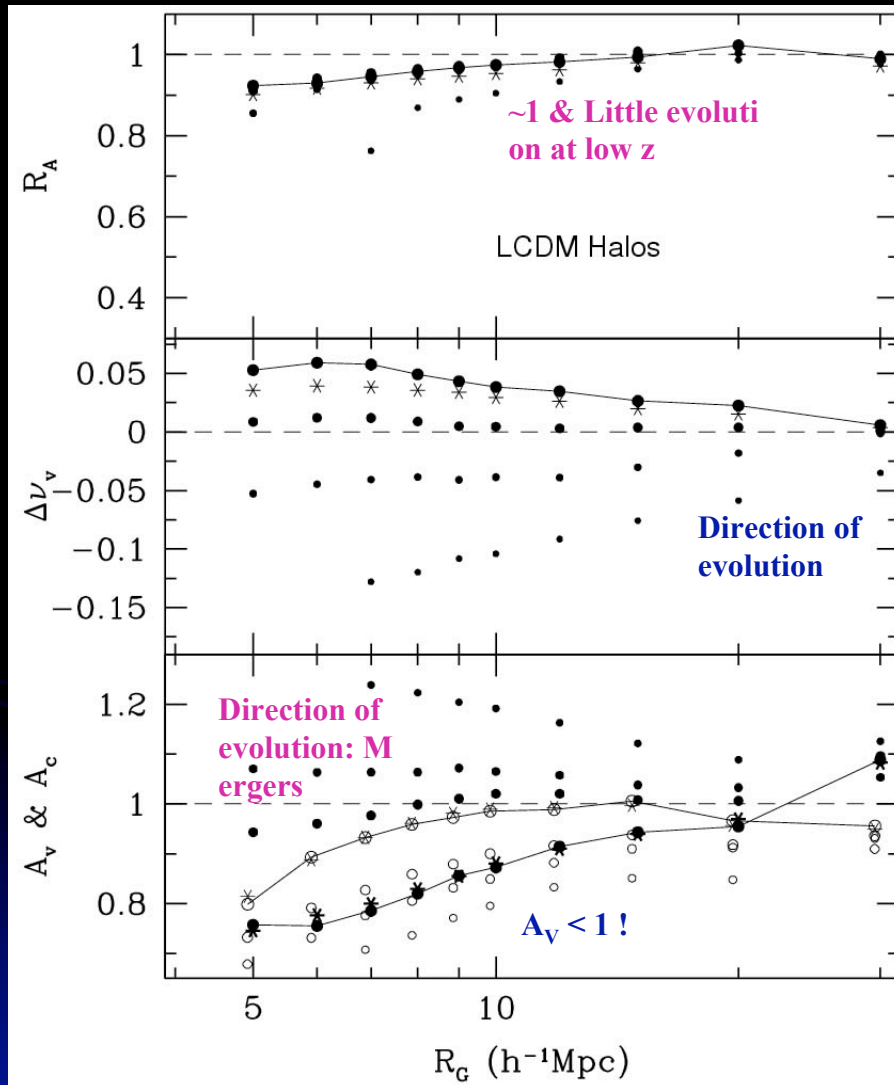
$$S^{(1)} = -\frac{3}{4} \langle \delta^2 \nabla^2 \delta \rangle / \sigma_0^2 \sigma_1^2 = \frac{3}{4\sigma_0^2 \sigma_1^2} \int \frac{d^3 k_1}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} k_{12}^2 B(k_1, k_2, k_{12}),$$

$$S^{(2)} = -\frac{9}{4} \langle (\nabla \delta \cdot \nabla \delta) \nabla^2 \delta \rangle / \sigma_0^4 = \frac{9}{4\sigma_0^4} \int \frac{d^3 k_1}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} \mathbf{k}_1 \cdot \mathbf{k}_2 k_{12}^2 B(k_1, k_2, k_{12})$$

Dependence of genus on cosmogony



Dependence of genus on LSS tracers



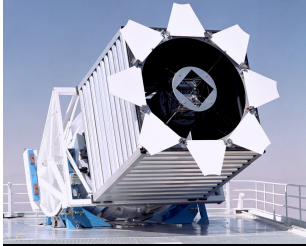
$$\langle N_{\text{sat}} \rangle = (M/M_1)_{-} \text{ for } M > M_{\text{min}} \text{ where } \log M_{\text{min}} = 11.76, \log M_1 = 13.15, _ = 1.13$$

Findings ...

(Park, Kim & Gott 2005)

1. Strong dependences on scale, time & tracers
2. R_A freezes at $z \sim < 3$ (insensitivity of amplitude of G to time)
3. Gravitation evolution makes A_V increase & Observed $A_V < 1$
→ proper **biasing mechanism** needed
- (4. Problem with HOD: $A_C \sim A_V$?)

Voids are not the places where there is nothing, but places where history of the universe is better kept.



Sloan Digital Sky Survey



1. Imaging of North Galactic Cap

2.5m APO telescope with a mosaic CCD camera

u, g, r, i, z photometric bandpasses → objects for spectroscopy

2. Spectroscopy

~ 10^6 galaxies & 10^5 quasars with rms z-error ~ 30 & 300 km/s

3. Samples

Main Galaxies: $r_{\text{Pet}} < 17.77$ → Recent Samples 14 & 15 (~DR3 & DR4)

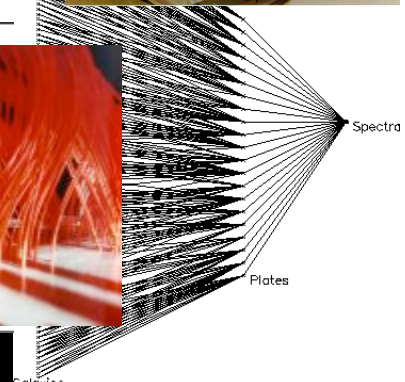
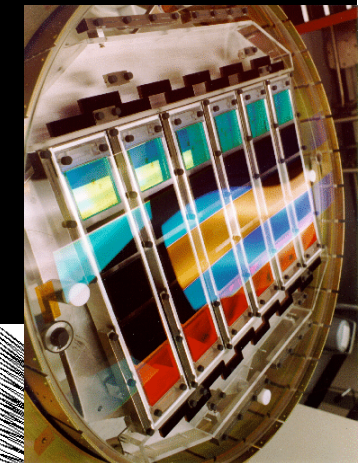
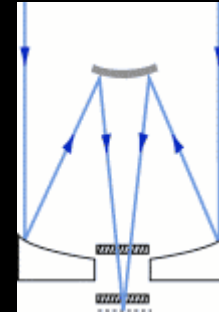
Quasars

Luminous Red Galaxies (LRG): $z < 0.4$ & > 0.4 samples

Survey characteristics

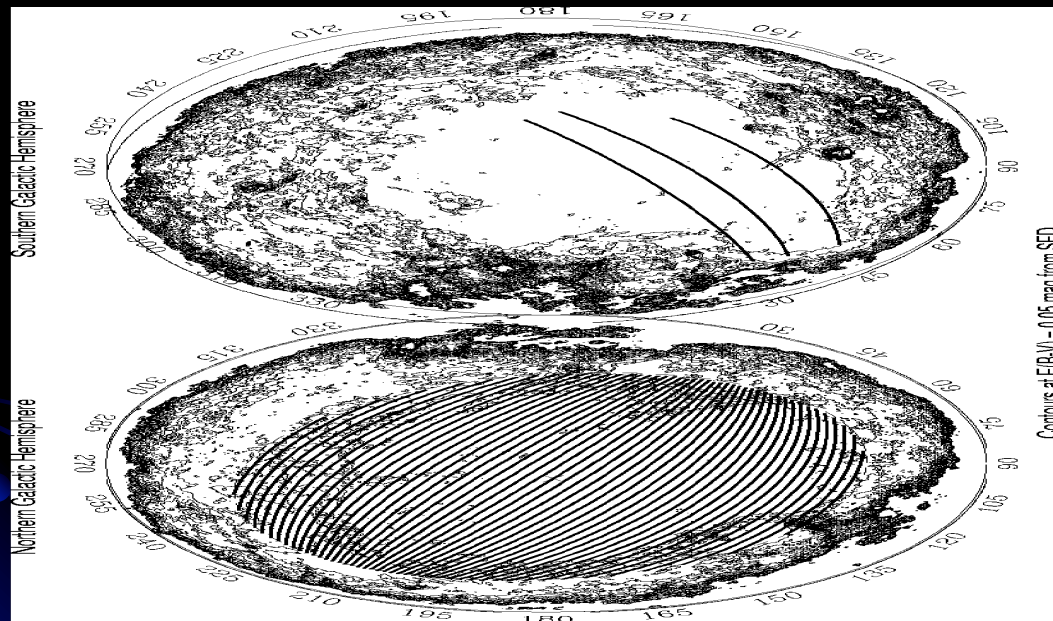
TABLE 1
SDSS EQUIPMENT SUMMARY

Parameter	Value
Telescope and Site: Apache Point Observatory	
Latitude and longitude	N32°46'49".3, W105°49'13".5
Elevation	2788 m
Survey telescope	2.5 m diameter, modified Ritchey-Chrétien design; 27% central obscuration
Survey area	North Galactic cap, 10,000 deg ² , minimal Galactic extinction, plus three stripes in south Galactic cap
Instruments	Imaging camera and two double spectrographs
Photometric telescope	20 inch (0.5 m), with one CCD camera, filter wheel, and shutter
Imaging Camera	
Photometric CCDs	30, 2048 × 2048, SITe/Tektronix, 49.2 mm square
CCD read noise	<5 e ⁻ pixel ⁻¹ (overall system is sky limited)
Image frame size	2048 × 1361 pixels (13'.52 × 8'.98)
Image column separation	25'.17
Detector separation along column	17'.98
Focal-plane image scale	3.616 mm arcmin ⁻¹
Detector image scale	3.636 mm arcmin ⁻¹
Pixel size and scale	24 μm; 0".396 pixel ⁻¹
Filters	<i>riuzg</i> scanned in that order, 71.7 s apart
Integration time	54 s
Operating mode	Time-delay and integrate ("drift scan")
Field distortion	<0".1 over entire field
Field size	2°.5
Flux calibration	Standard-star fields at 15° intervals along scans, tied to BD +17°4708, atmospheric extinction determined by PT
Astrometric CCDs	22, 0.25 × 2 inches, above and below CCD columns; <i>r</i> filter plus 3 mag neutral density filter, 10.5 s integration time
Spectrographs	
Channels	One red, one blue for each spectrograph
CCDs	SITe/Tektronix (as for imager)
Coverage	3800–6150 Å (blue), 5800–9200 Å (red), λ/Δλ ≈ 1800
Number of fibers	320 × 2
Fiber diameter	3"
Flux calibration	Standard stars in each field, tied to colors observed with camera
Integration time	45 minutes, in three exposures [nominal (S/N) ² > 15 pixel ⁻¹ at <i>g</i> * = 20.2]
Pixel size	69 km s ⁻¹
Wavelength calibration	Hg, Cd, and Ne arc lamps, rms error of 0.07 pixels (10 km s ⁻¹)
Flat field	Quartz lamps

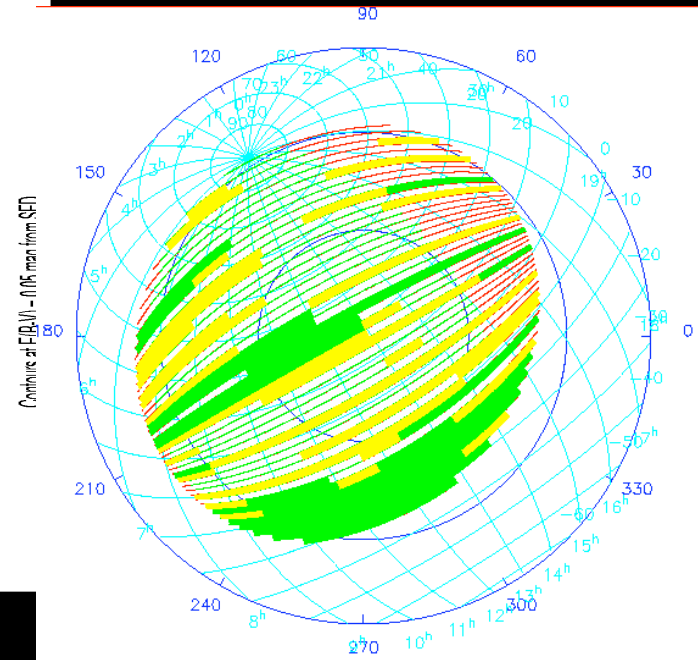


Sky Coverage

- Minimize Galactic foreground extinction



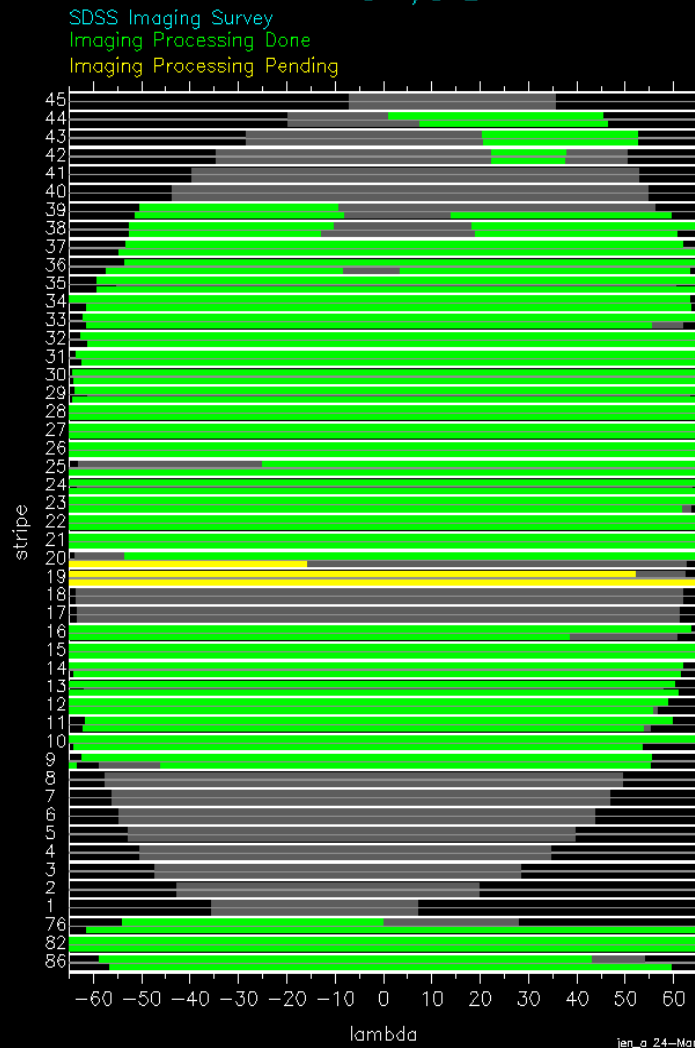
Projection on the sky (Galactic coordinates) of the Northern and Southern SDSS surveys. The lines show the individual stripes to be scanned by the imaging camera. These are overlaid on the extinction contours of Schlegel, Finkbeiner and Davis (1998). The Survey pole is marked by the 'X' (Fig 2. York et al. 2000)



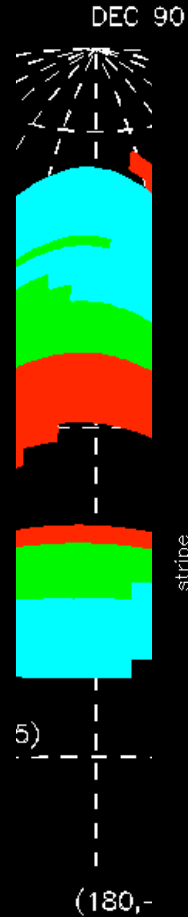
Both equatorial and galactic coordinates are plotted.

SDSS Sky Coverage

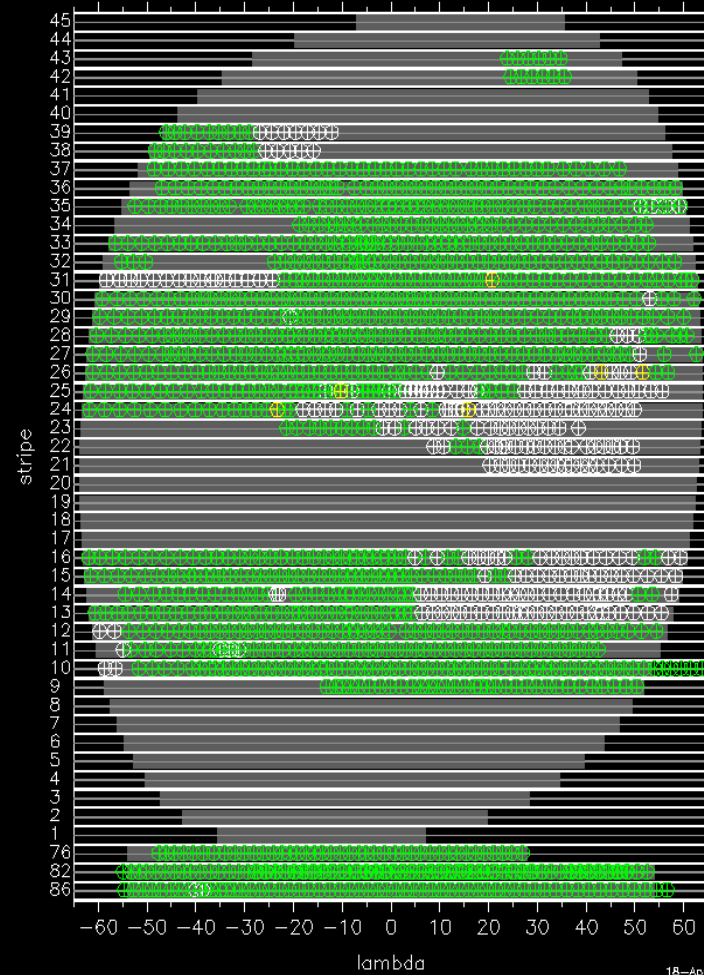
SDSS Imaging coverage as of DR4
DR1/DR2





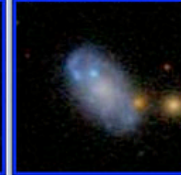
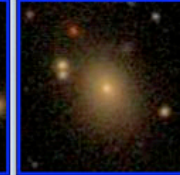
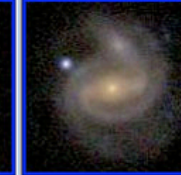



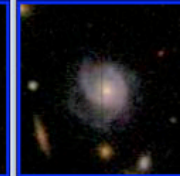
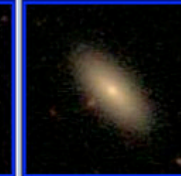



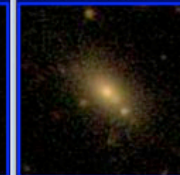
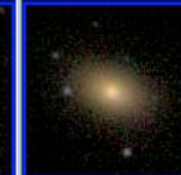

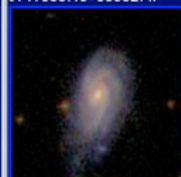
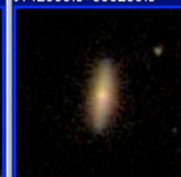



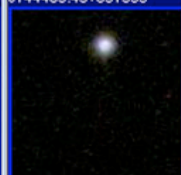

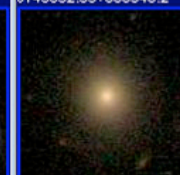

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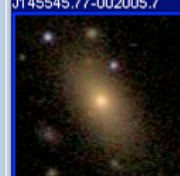









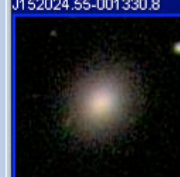


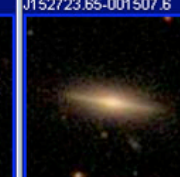








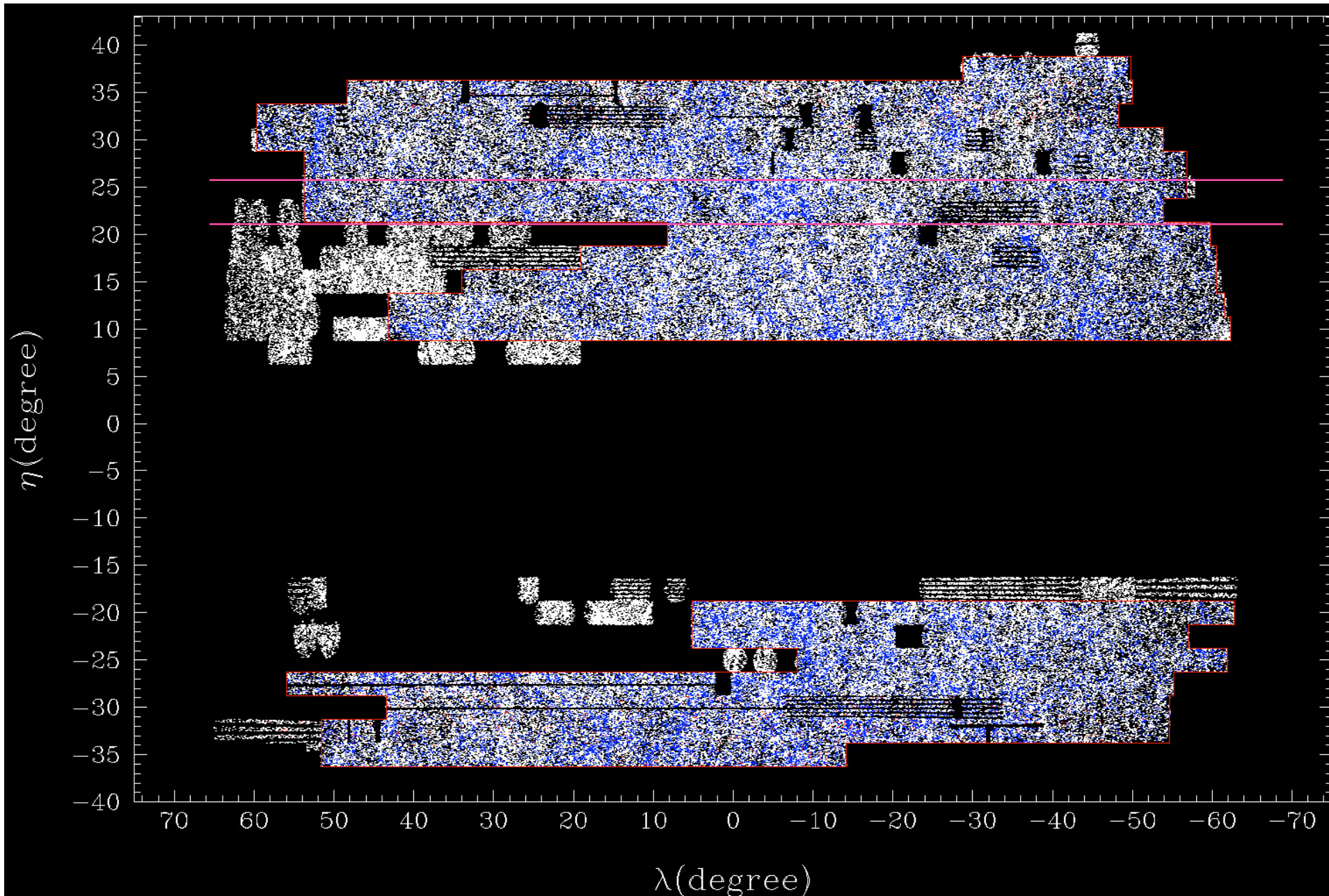
Spectroscopic Survey rerun 23
Tiles Done
Tiles defined not obs Tiles observed not complete



18-Apr-2005 14:59

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J135506.96+000455.7 	J135508.79+000837.6 	J135922.11+001223.8 	J140137.99+000402.5 	J140407.39+000838.4 
J140535.4+001105.9 	J141302.55+000846.9 	J141330+001109 	J141452.57+001146.3 	J141718.67+000540.4 
J141722.17+000125.3 	J141835.45+000927.7 	J142636.5+000230.9 	J143019.7+000708.9 	J143540.07+001217.7 
J144329.47+001213.9 	J144403.49+031939 	J144418.52+000238.6 	J145052.59+000540.2 	J145123.41+000025.4 

J143546.66-002216.4 	J143743.83-001857.7 	J143753-002342.7 	J145216.06-001737 	J145544.92-001822.1 
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Sample 15 SDSS galaxies on Survey coordinate plane

blue: BEST, red: bright galaxies (14.0–15.0)

2005.5.9 quest ~/yychoi/Sam15/Type/145150/survey.i

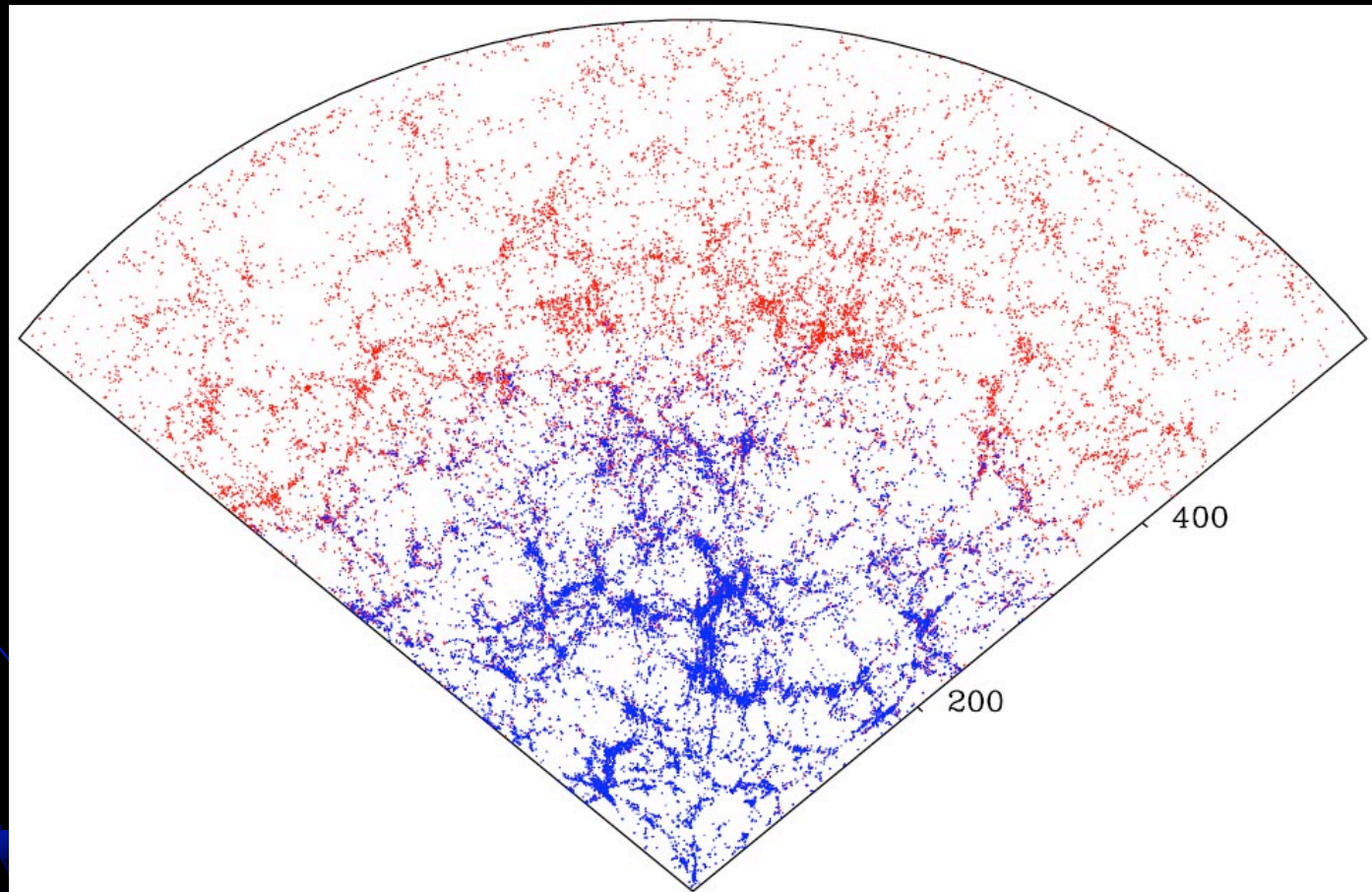
**SDSS LSS Sample 15 in
survey coor. (410K galaxies)**

Horizon of human knowledge is expanding

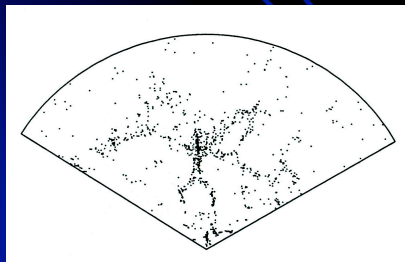
SDSS V ~ 10^2 CfA V ~ 1/5000 Horizon V

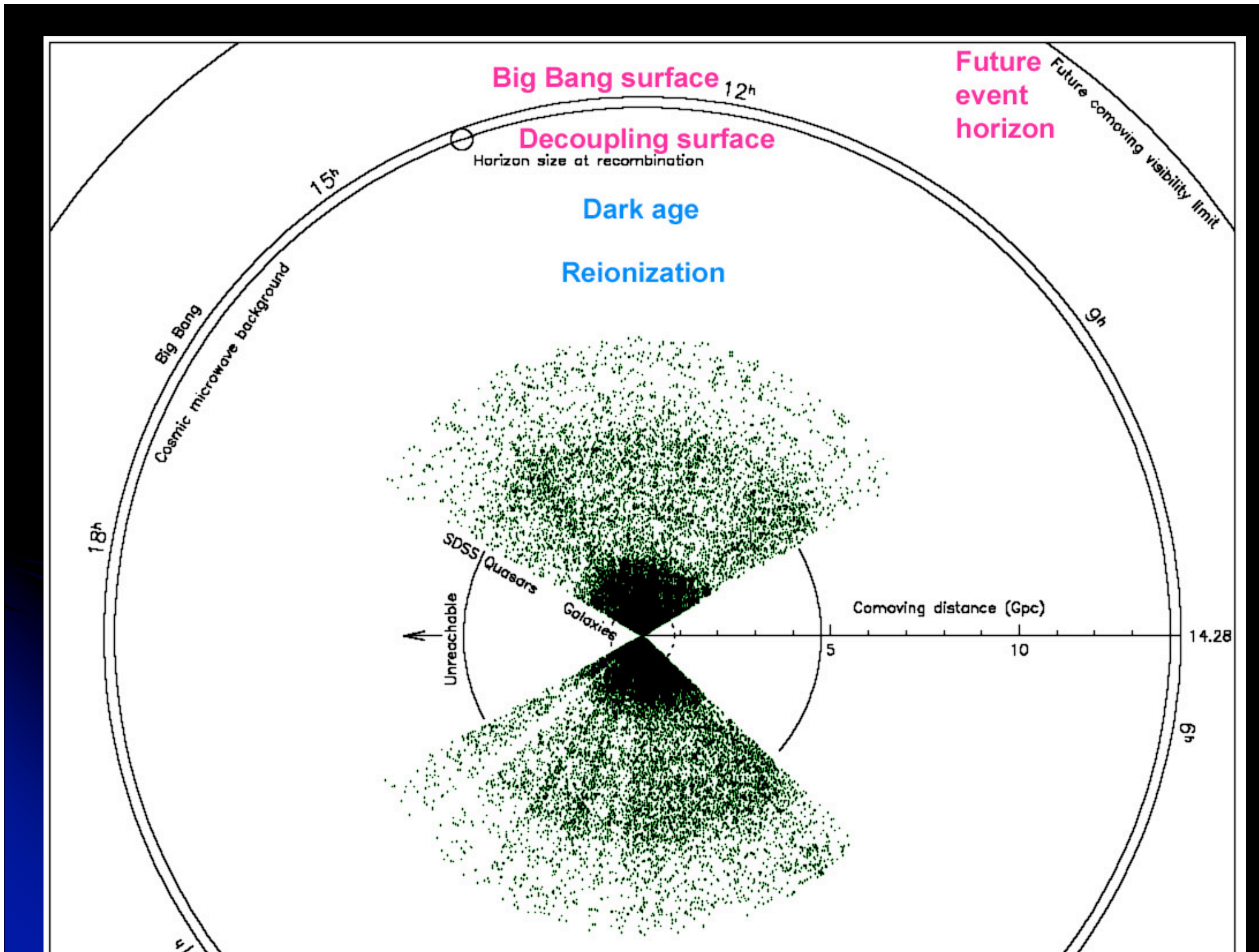
Italy A ~ 1/1700 Earth Surface

2005 SDSS

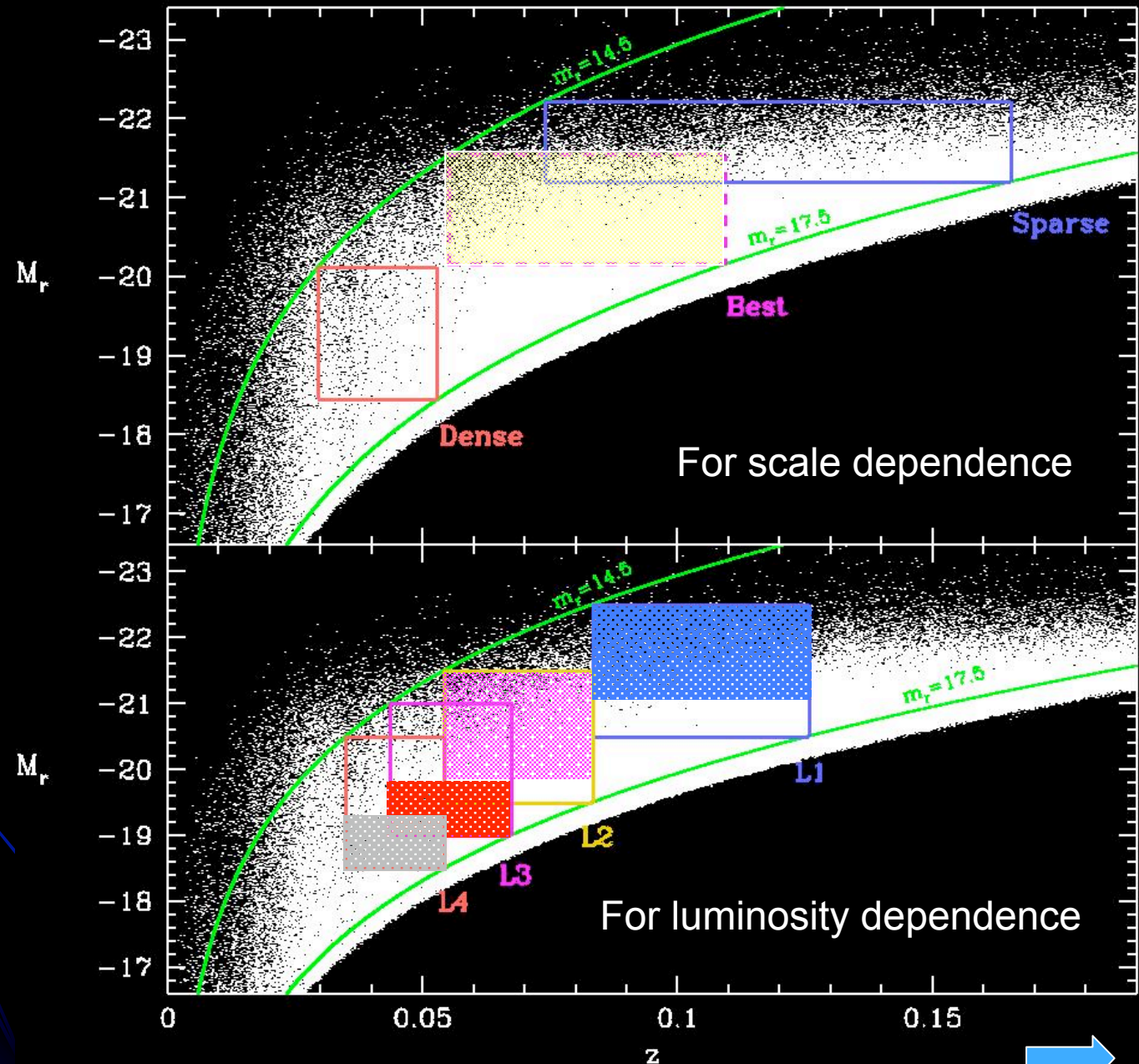


1986 CfA



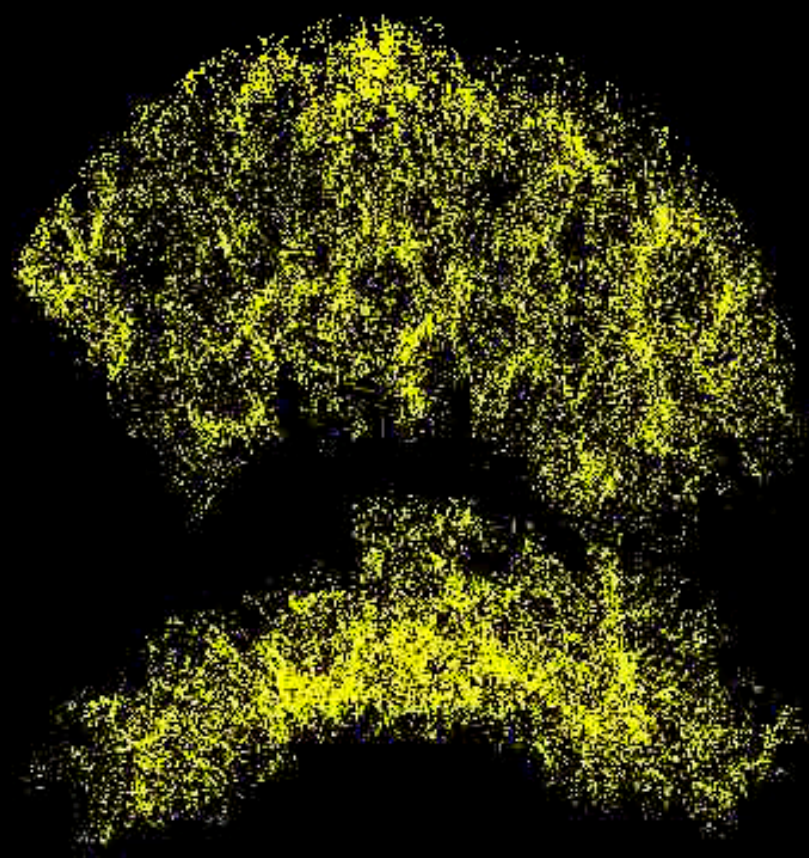


Sample Definition for Genus Analysis



BEST Sample
-20.15 < M_r < -21.53
162.9 < r < 319.0 h^{-1} Mpc
0.0550 < z < 0.1091
36,000 galaxies
 $d = 6.14 h^{-1}$ Mpc

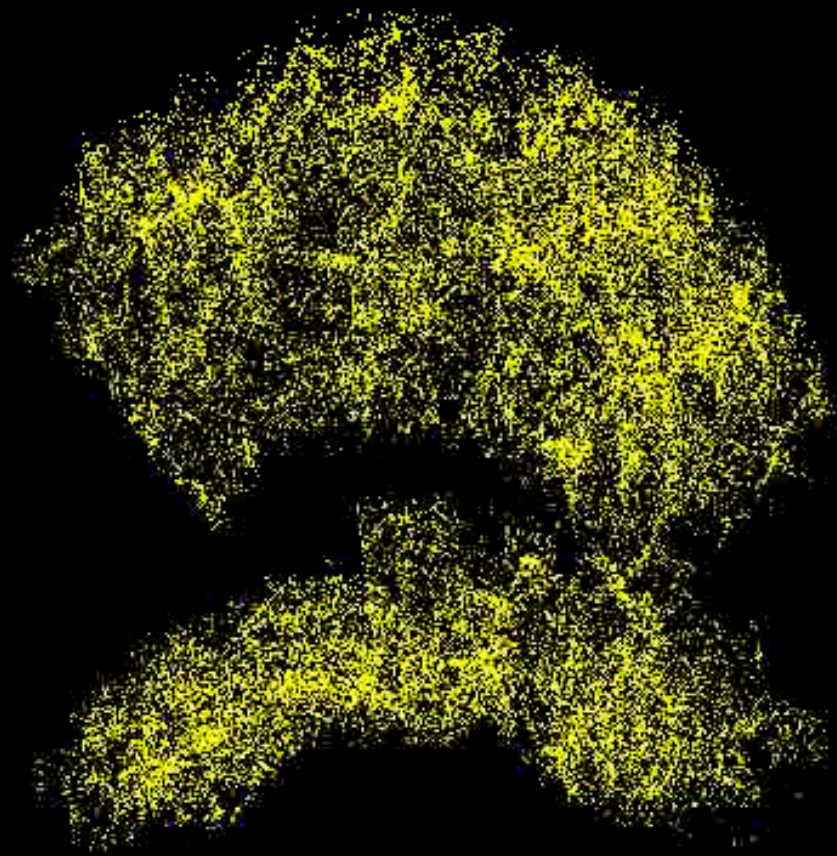




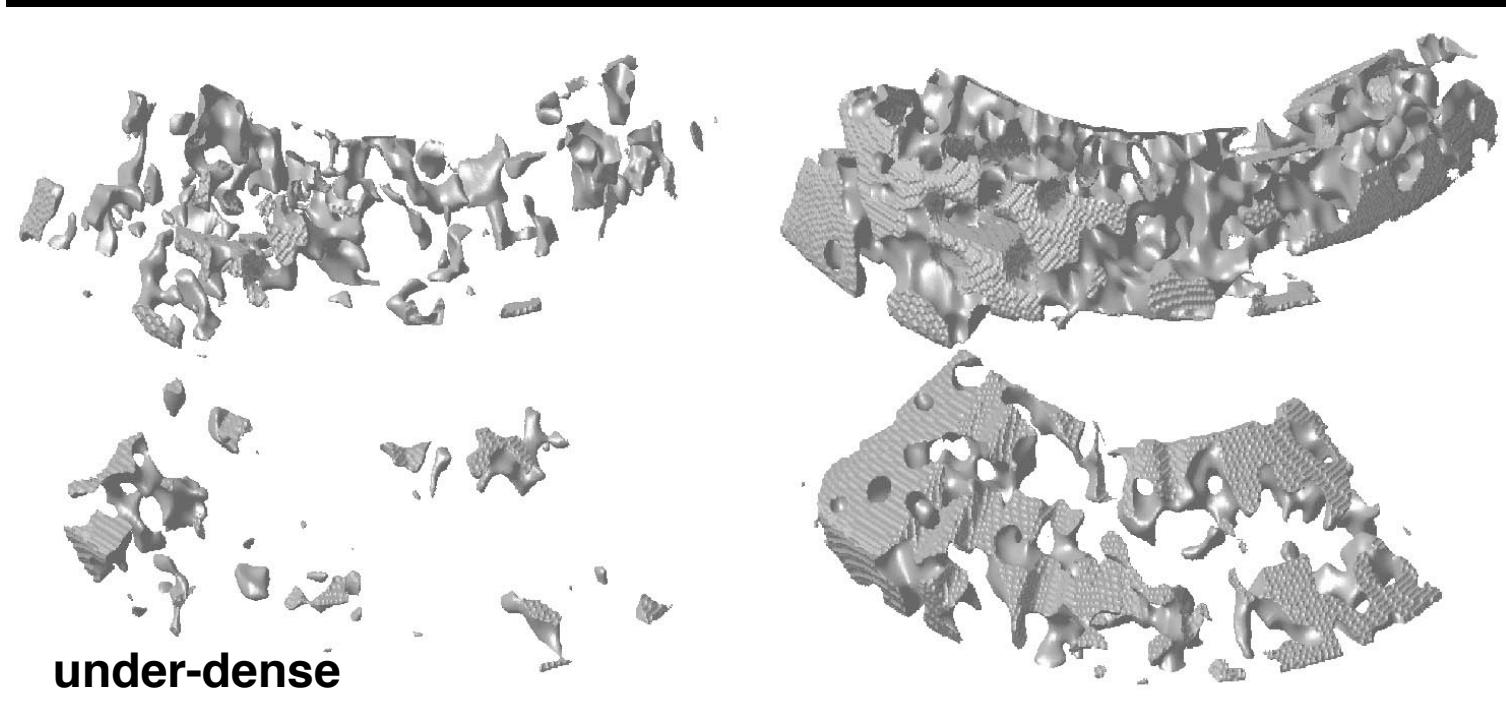
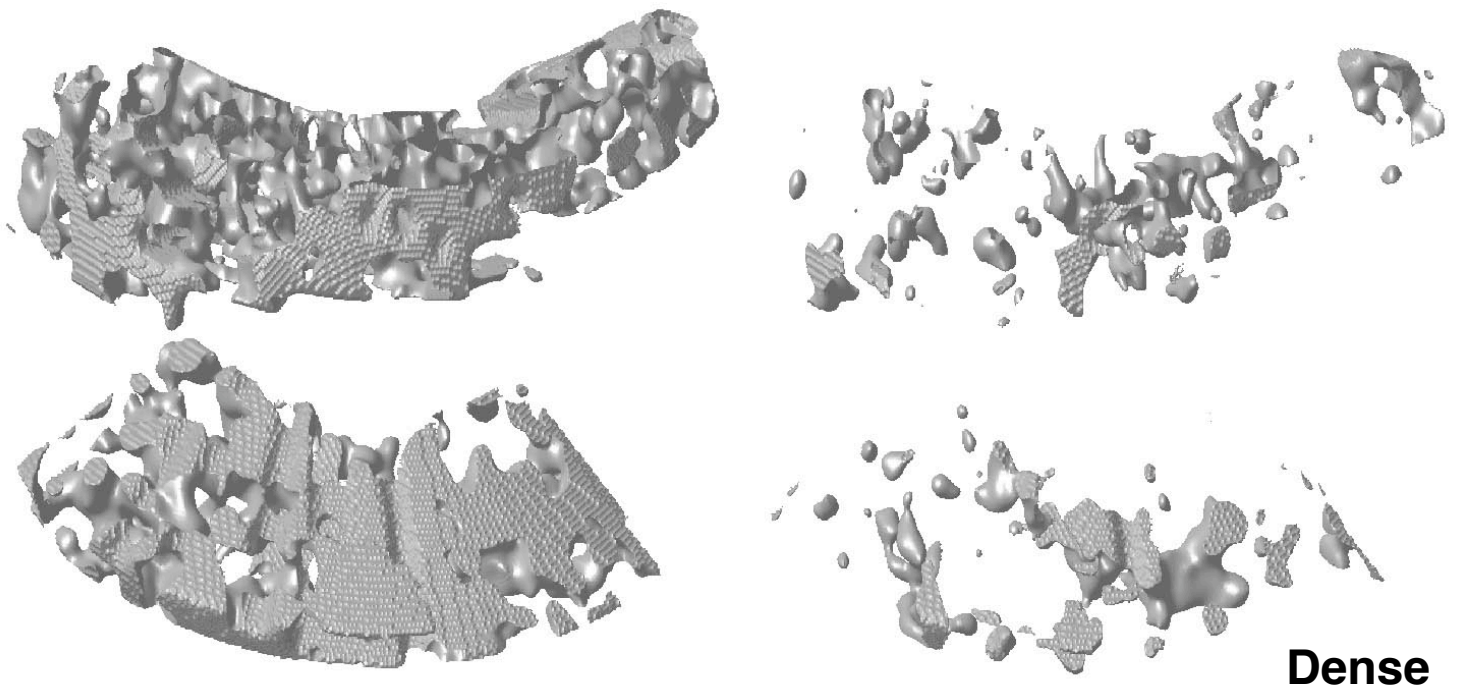
3D View of S DSS

3D View of a mock SDSS

HOD galaxy for
mation prescript
ion

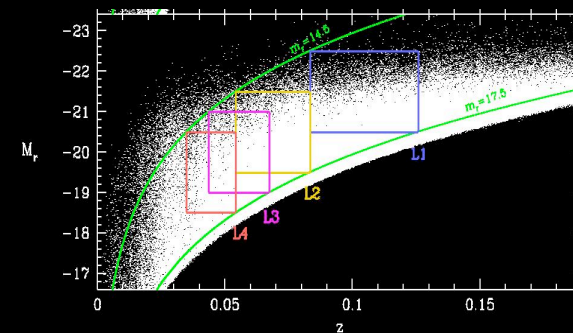
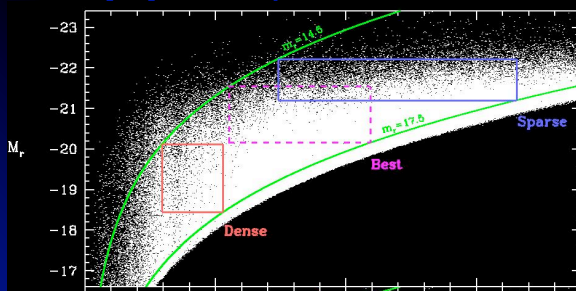
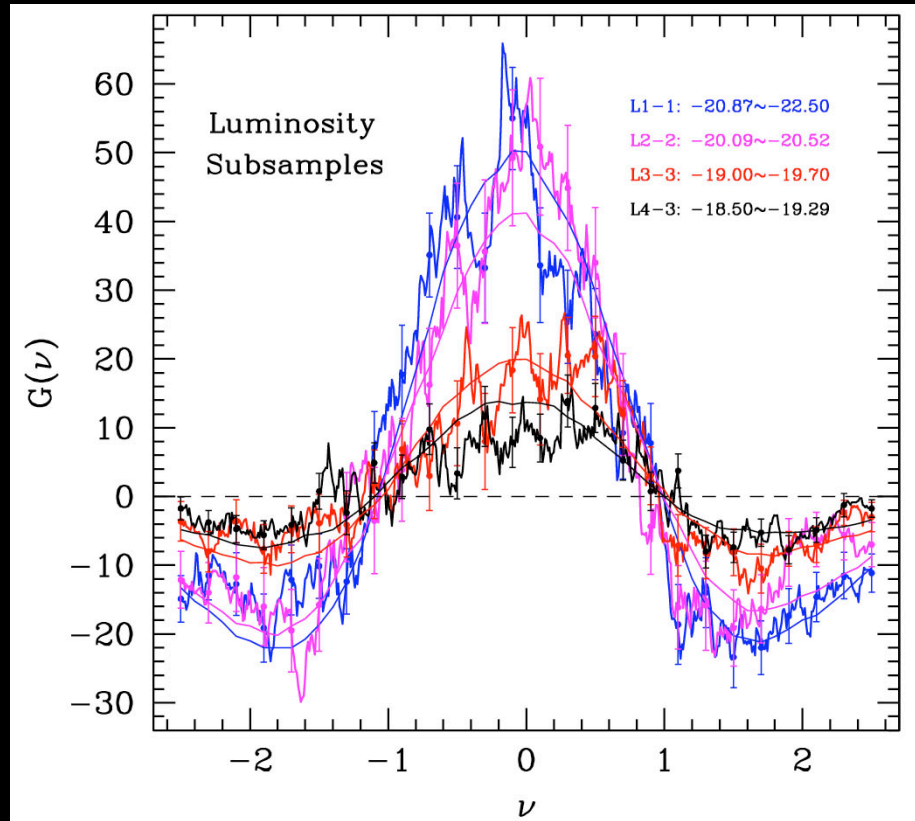
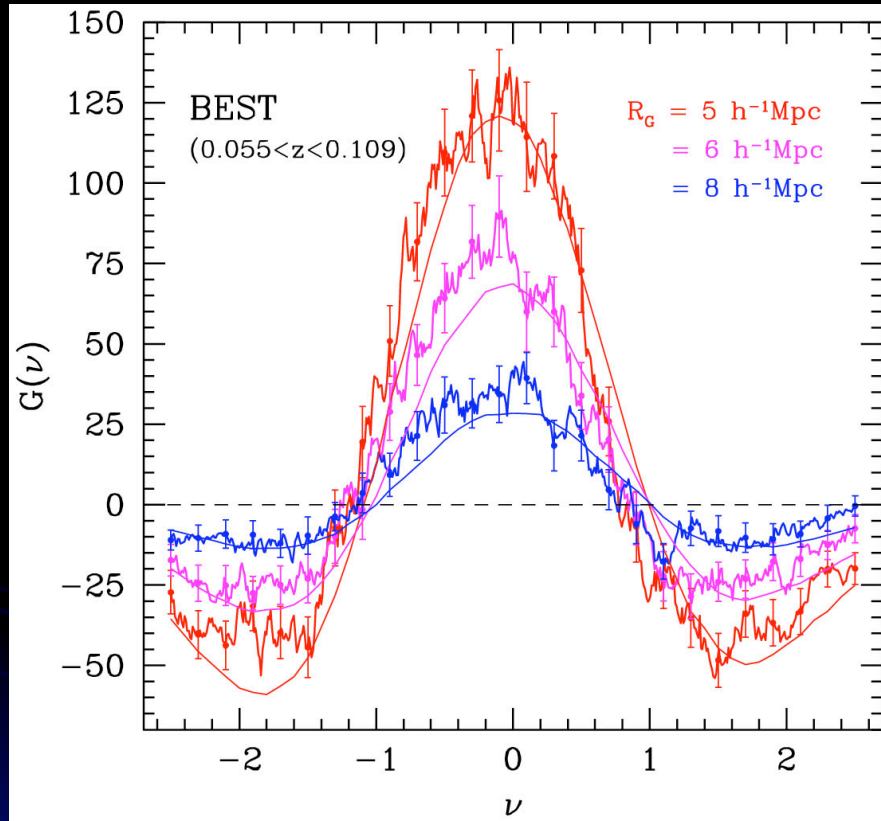


**Galaxy
Density
Field**

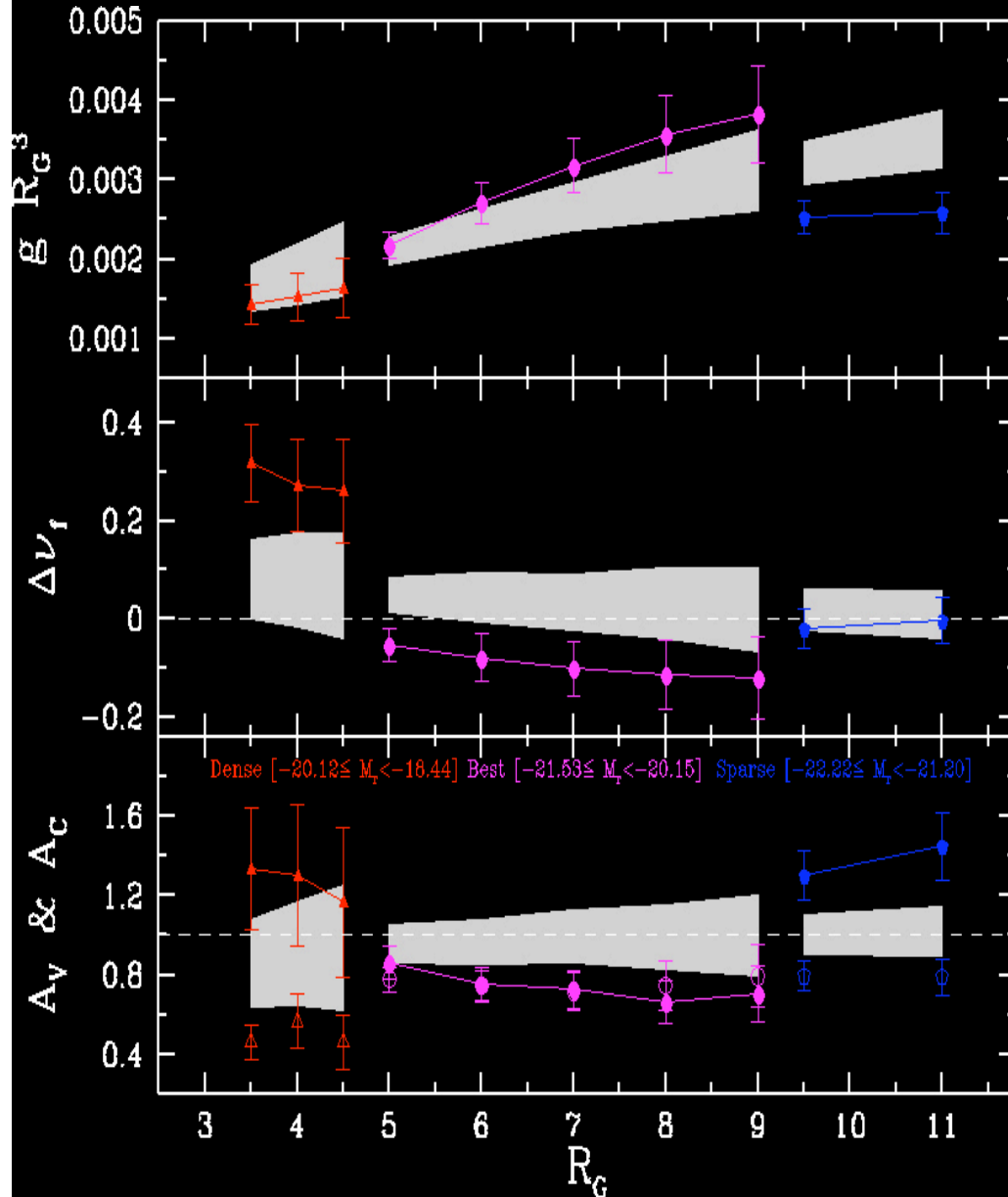


SDSS Sample 14: Genus Analysis

(Park, Choi, Vogeley, Gott, Kim, Suto, et al. 2005)



Scale dependence



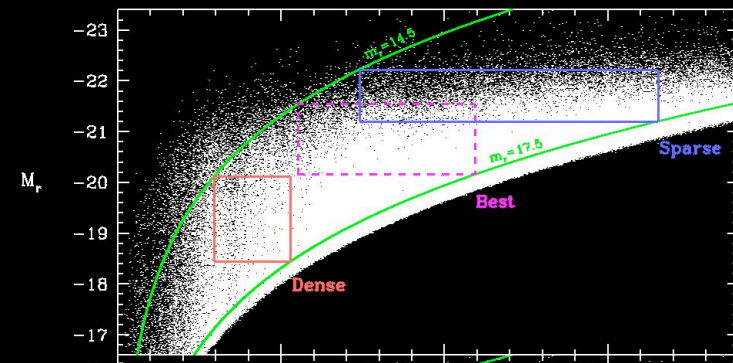
gR_G^3 – genus density per smoothing volume

$\bullet f \cdot \dot{\nu}$ Shift

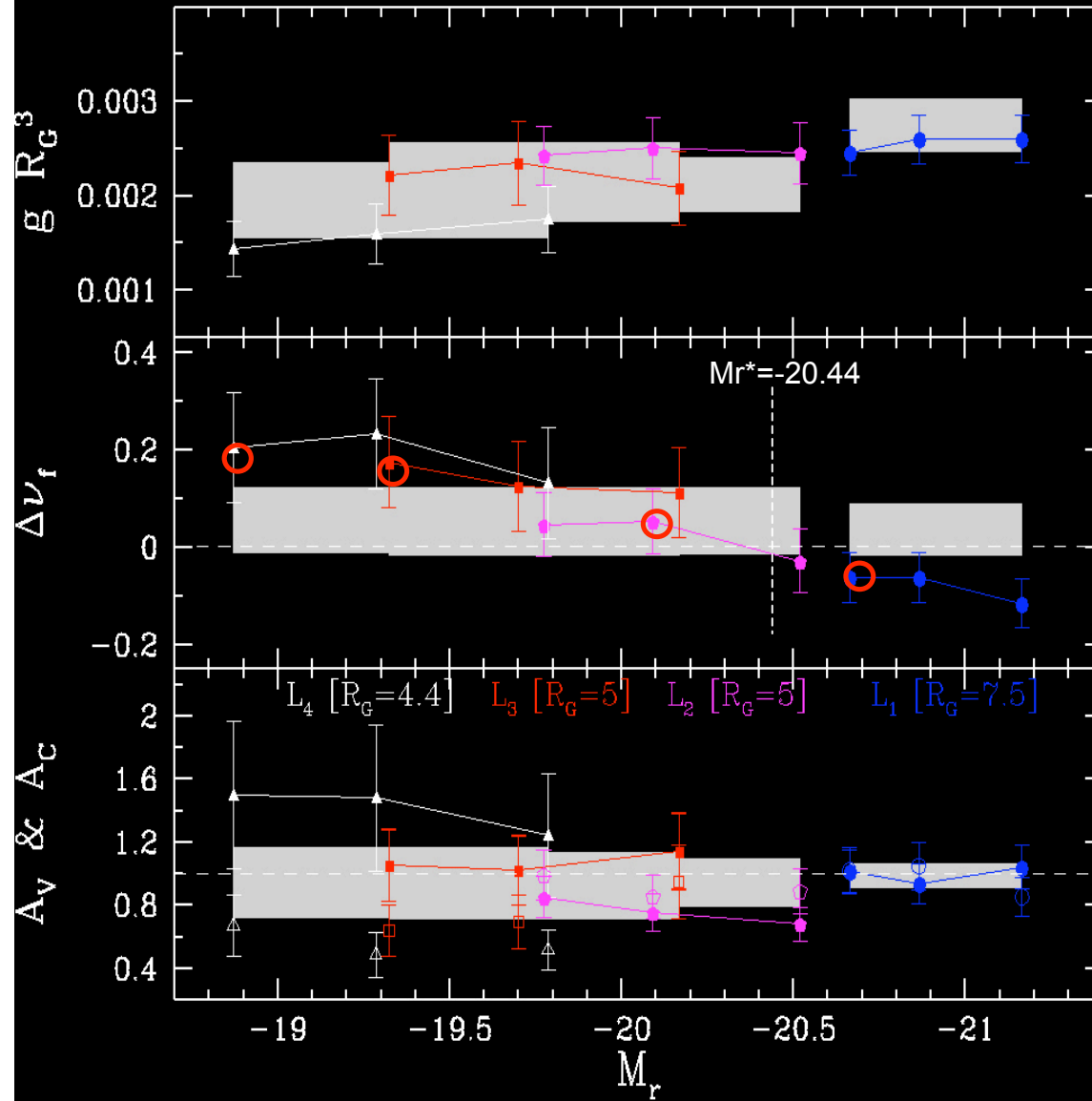
A_C & A_V – clusters & voids multiplicity

Shaded area – 1σ limits from 100 Mock surveys

- Strong evidence for **biased galaxy formation** in low density environments ($A_V < 1$)
- **Effects of the Sloan Great Wall**



Luminosity dependence



gR_G^3 – genus density / smoothing volume

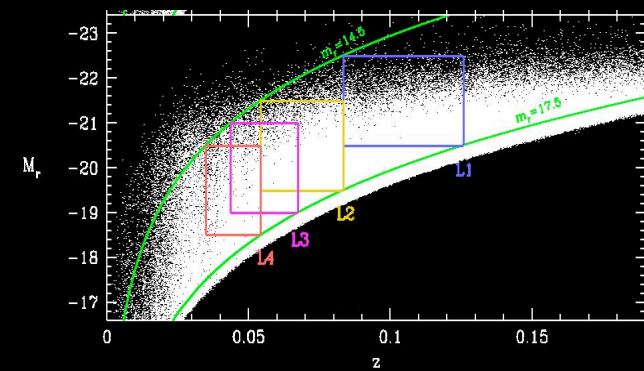
$\cdot f \cdot \dot{V}$ Shift

A_C & A_V – clusters & voids multiplicity S

haded area - 1σ from 100 Mock surveys

L-dependence of topology !!!

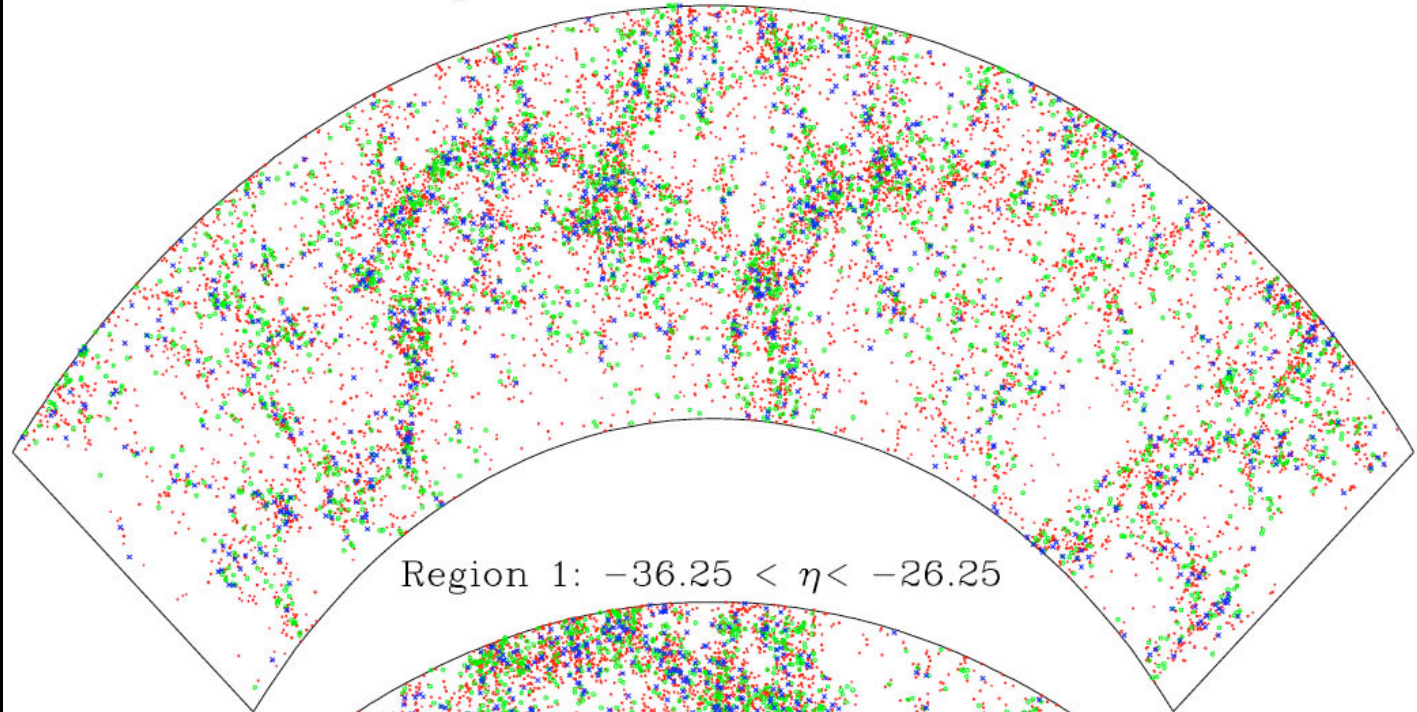
- Faint galaxies
- more positive shift
- lower void multiplicity



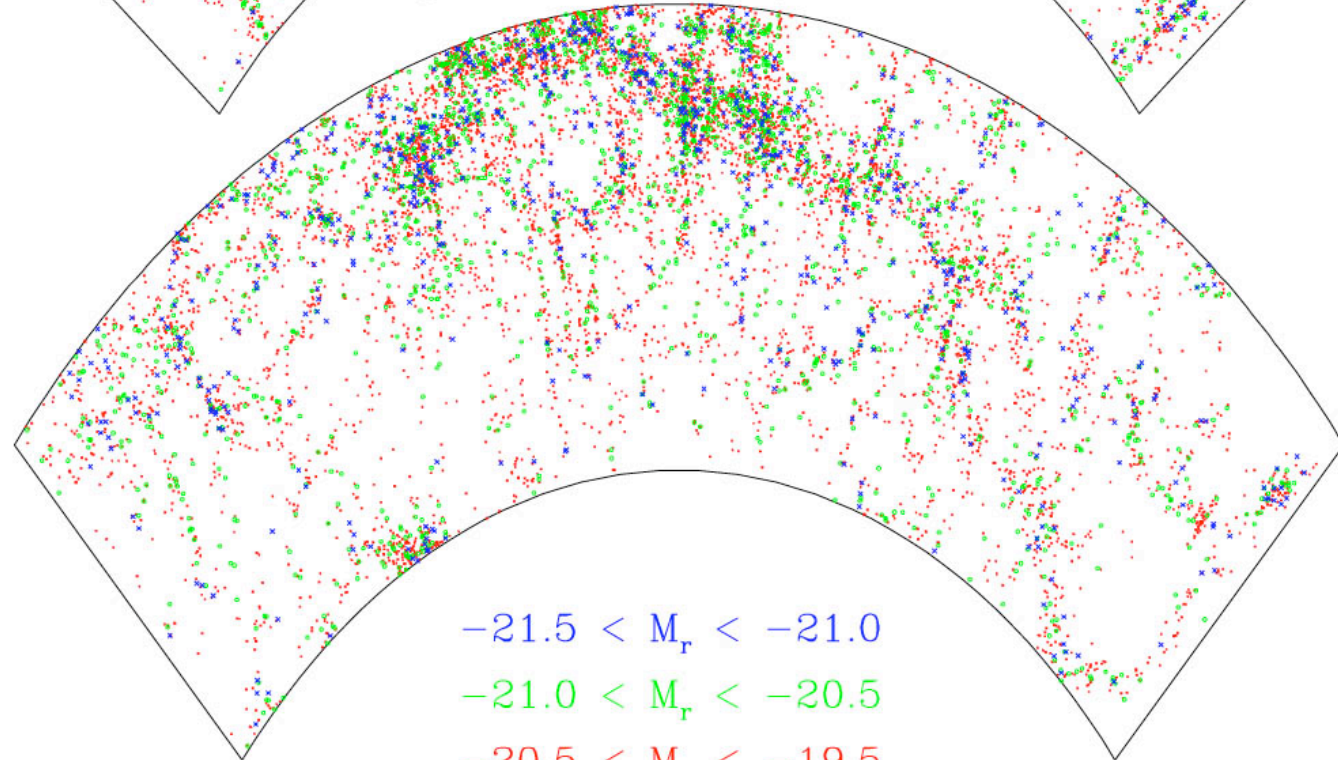
Biased Formation of Galaxies

1 & 2 point distribution, but also topology!

Region 2: $21.25 < \eta < 36.25$



Region 1: $-36.25 < \eta < -26.25$



$-21.5 < M_r < -21.0$

$-21.0 < M_r < -20.5$

$-20.5 < M_r < -19.5$

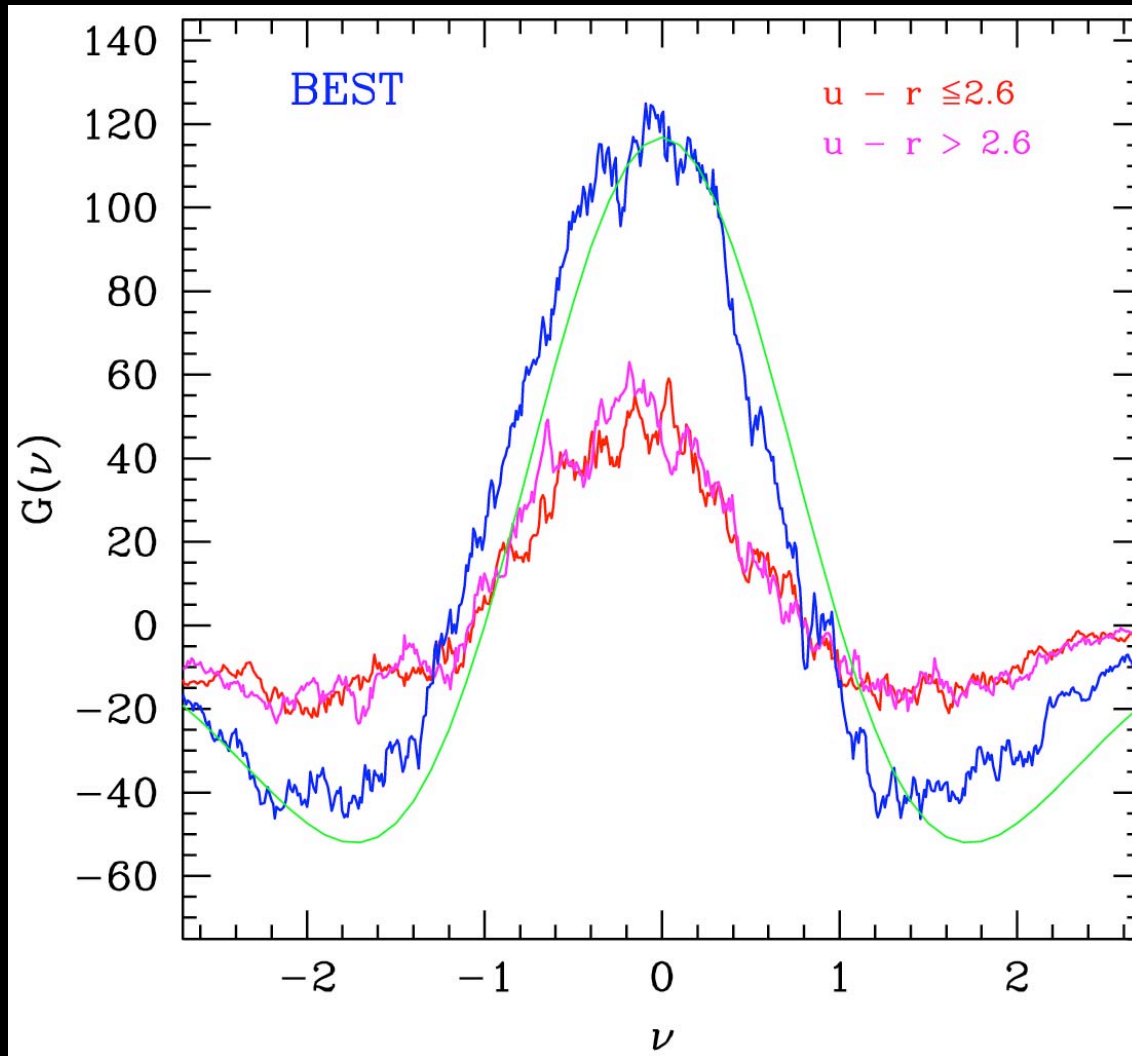


On going works

(Choi, Gott, Kim, Park, Suto, Vogeley, Yahata 2005)

1. Color & Morphology Dependence
2. R_A parameter
3. LRG, quasars, clusters
4. Comparison with Cosmological & galaxy formation models
5. Minkowski functionals

Dependence of $G(\nu)$ on color



(Vogeley, Choi, Park, et al. 2005)

Dependence on Morphology of Galaxies

Existing Morphology Classifier

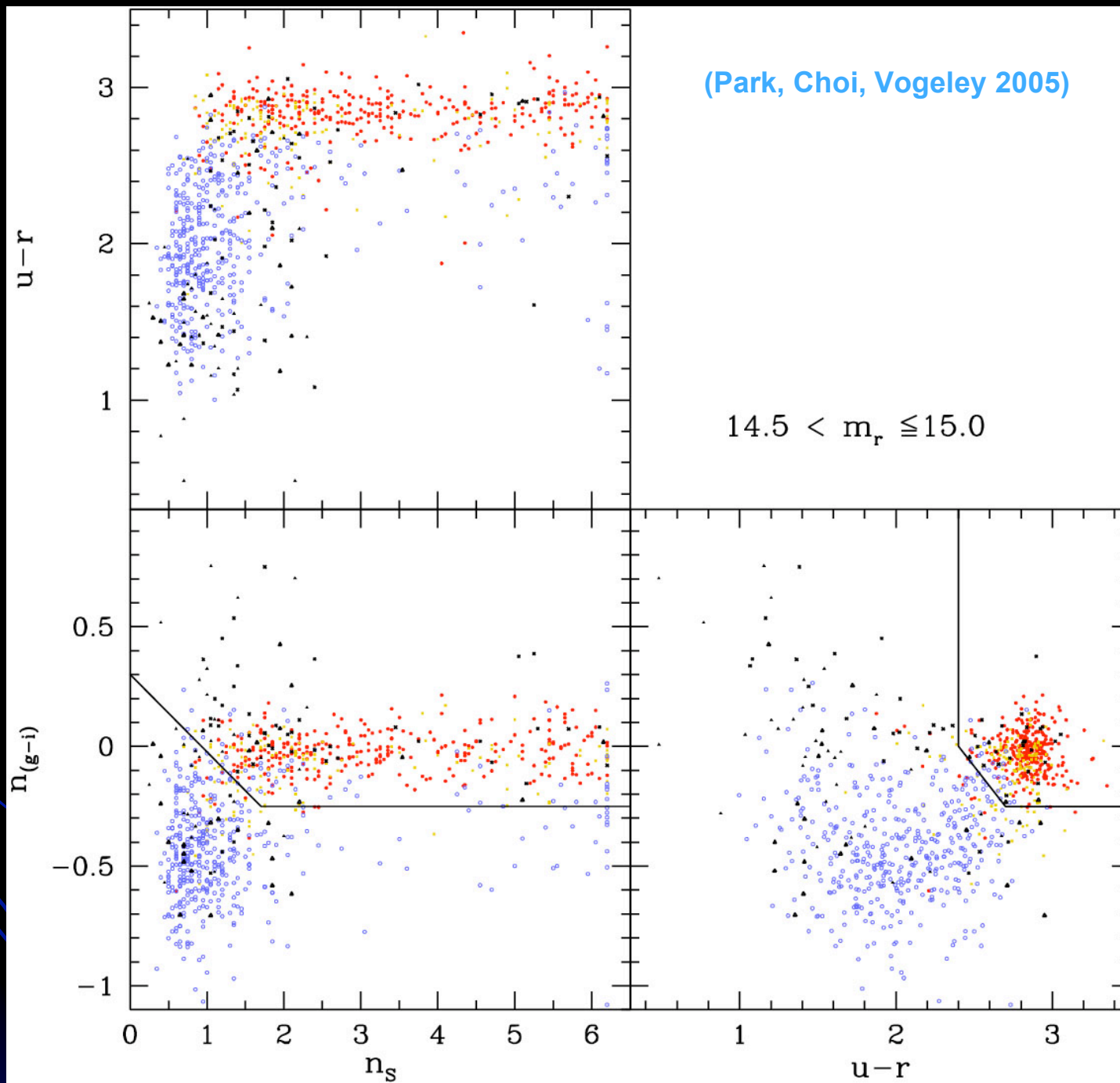
1. Structure Parameter : Sersic index n , concentration index C , profile likelihoods
2. Star formation : $u-r$ color, emission/absorption lines, PCA of spectra

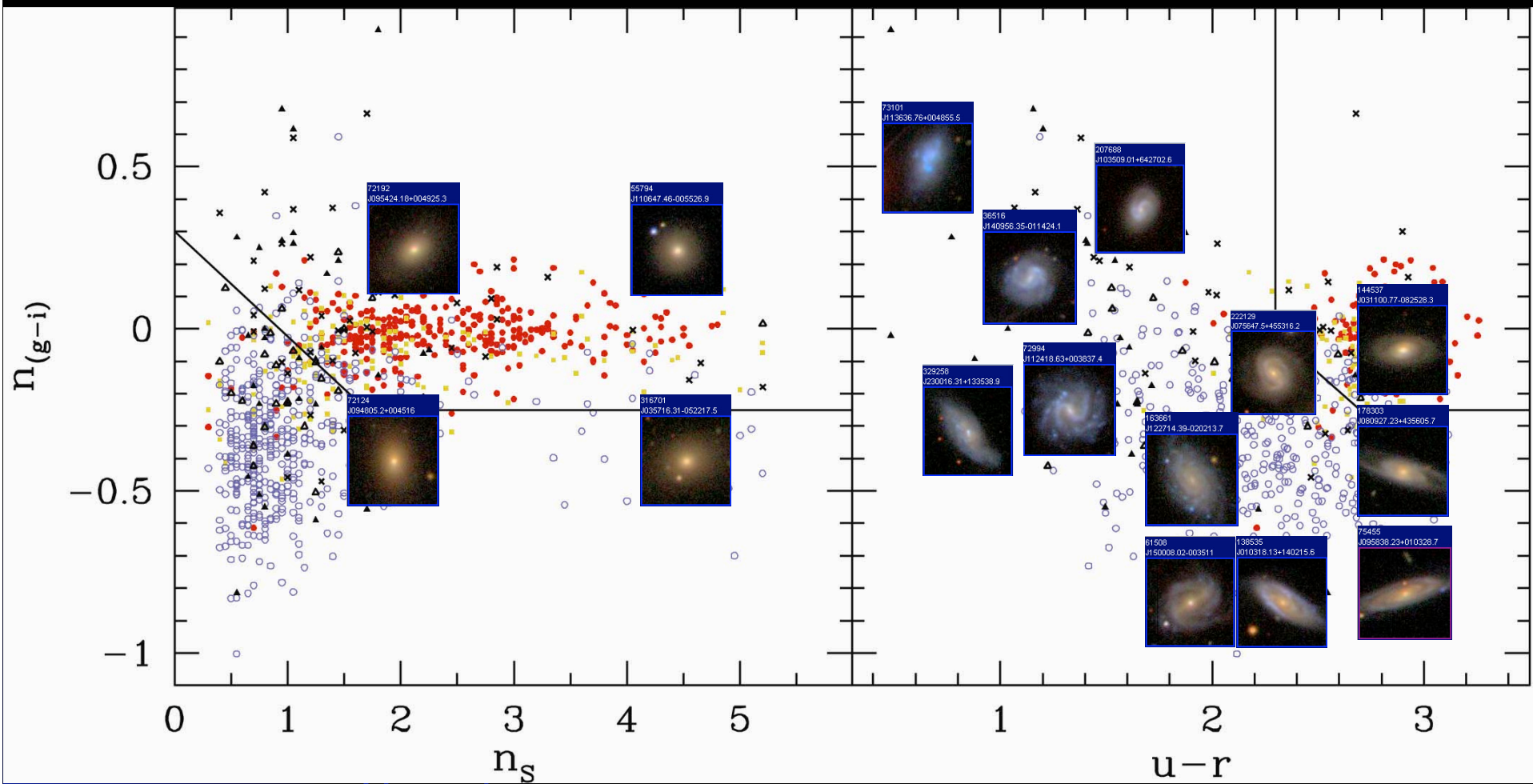
Automated Morphology Classifier

Three Parameter Classification

1. Star formation history : $u-r$ color
2. Structure + Star formation : color gradient $\sigma(g-i)$
3. Structure Parameter : Sersic index n

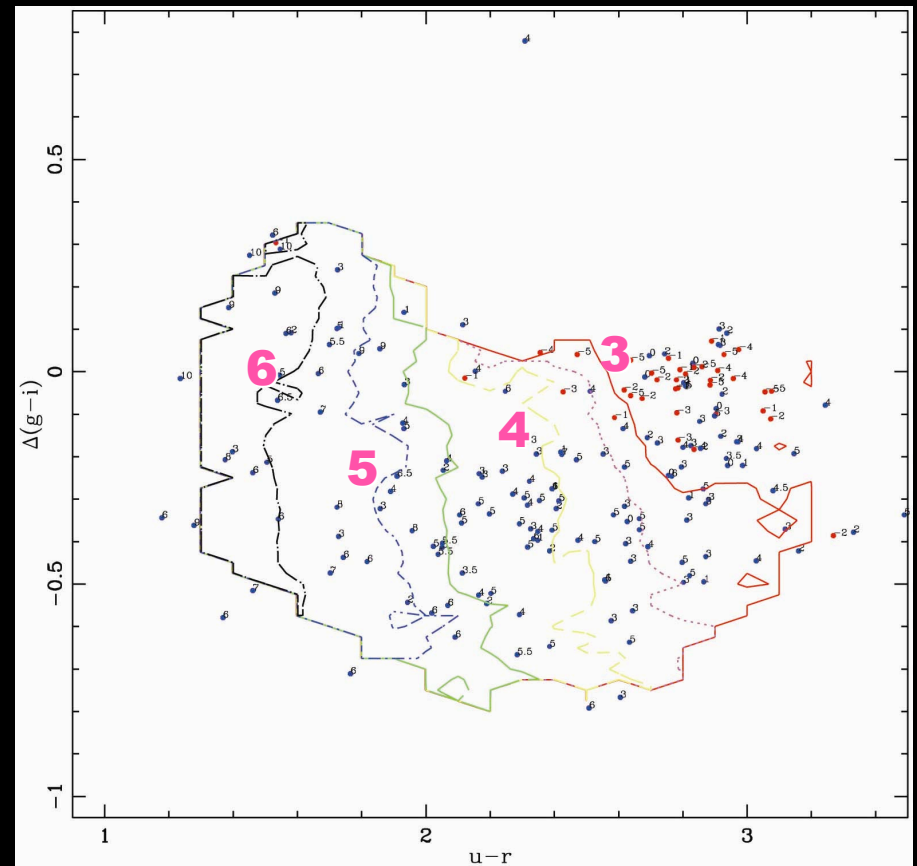
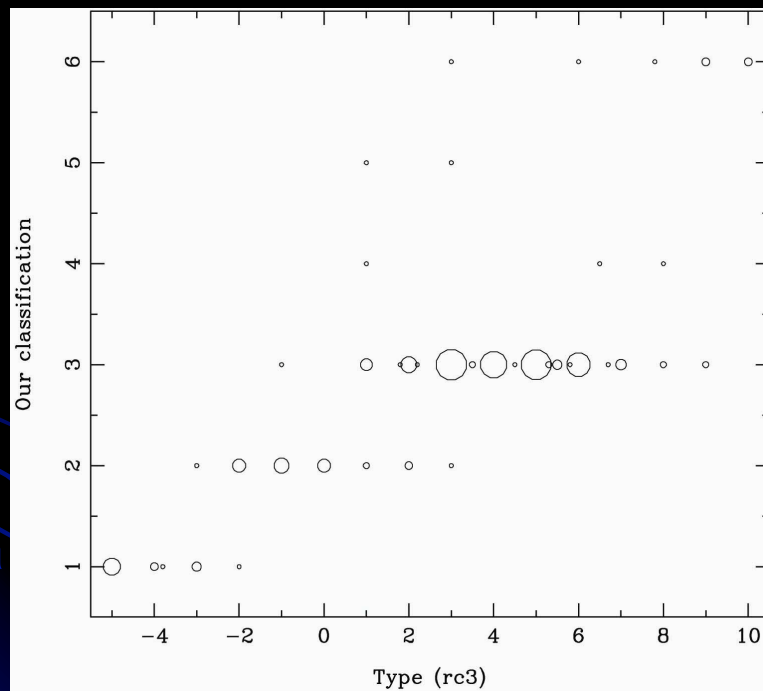
Morphology Classification of Galaxies



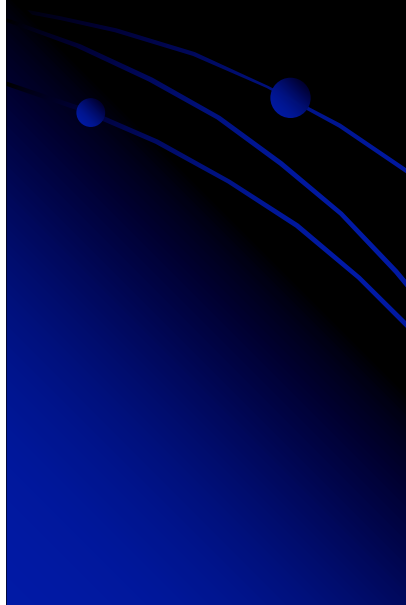
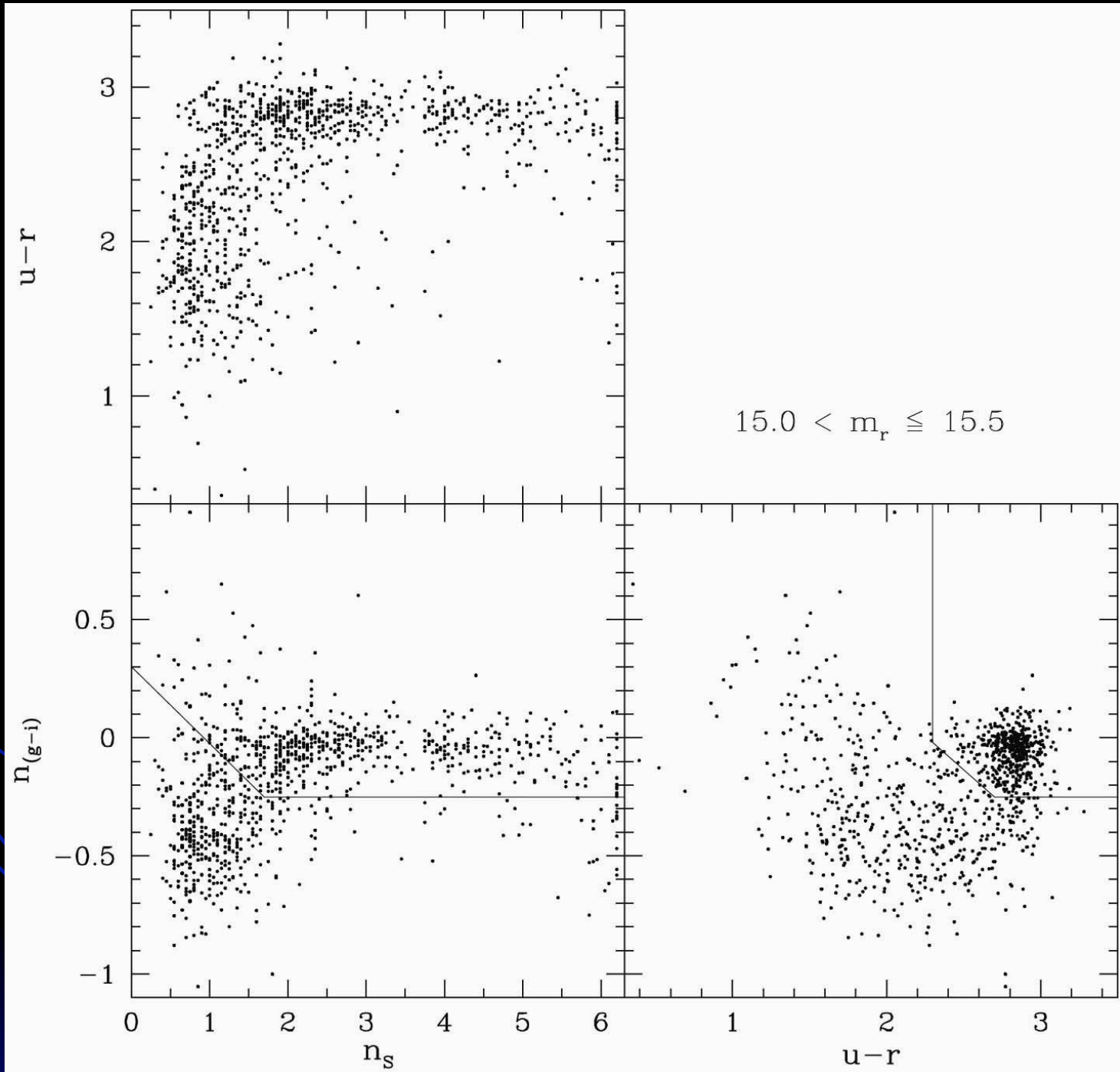


200 RC3 galaxies with $m_{pg} < 13.2$

I
AS
AE
S
S0/Sa
E/S0



(Park, Choi, Vogelely 2005)



Internal Physical Parameters of Galaxies

Morphology, Surface Brightness, Luminosity, Velocity Dispersion, Color, Spectral Type, SFR, etc

Collective Physical Properties of Galaxy Subsets

Galaxy Clustering Properties (z): Correlation Function, Power Spectrum, Count in Cell, Topology, etc

Velocity Field

Halo Mass Distribution, Luminosity Function (z), Color-Magnitude Relation, etc.

Galaxy Biasing, Gaussianity of Initial Density Fluctuation, δ , δ^2 etc.

δ_m , δ_g , b , etc.

Galaxy Formation & Evolution

Environment: Local Density & Cosmic Epoch

Different Tracers of Structure Formation: Galaxy, Cluster, Group, Void, Quasar, etc.

