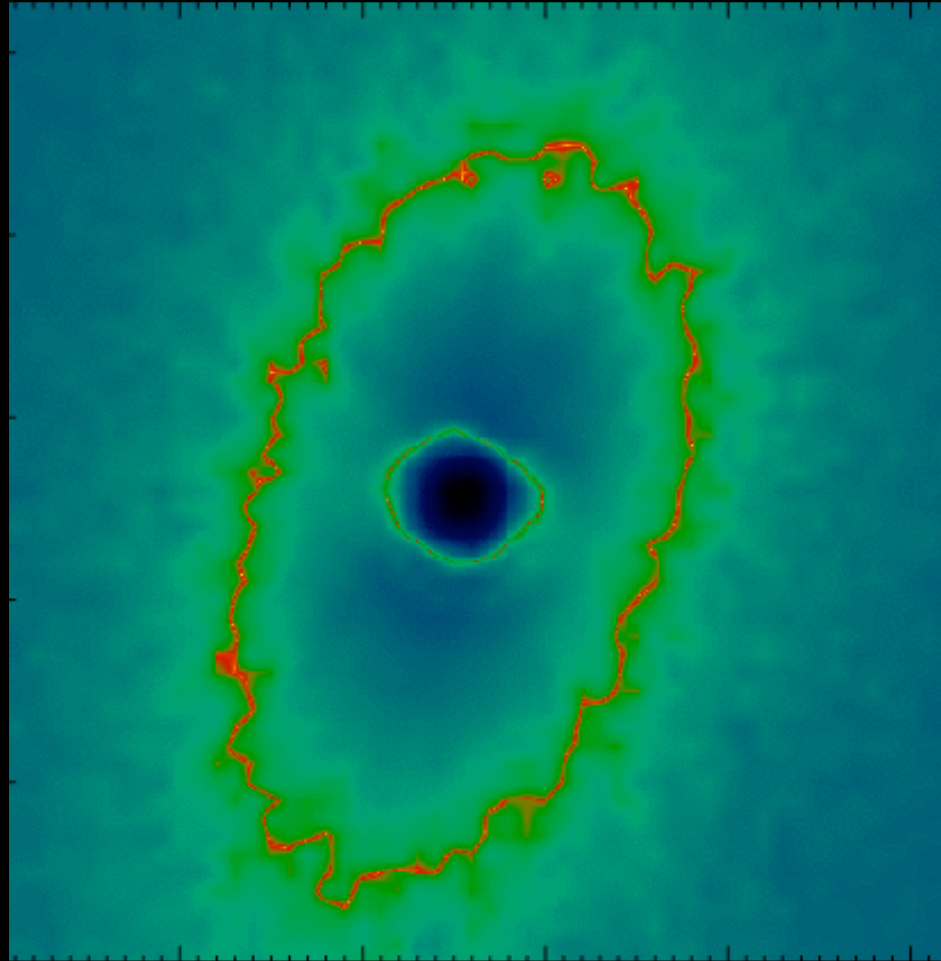


# Simulating Strong Gravitational Lenses and their Small-Scale Structure



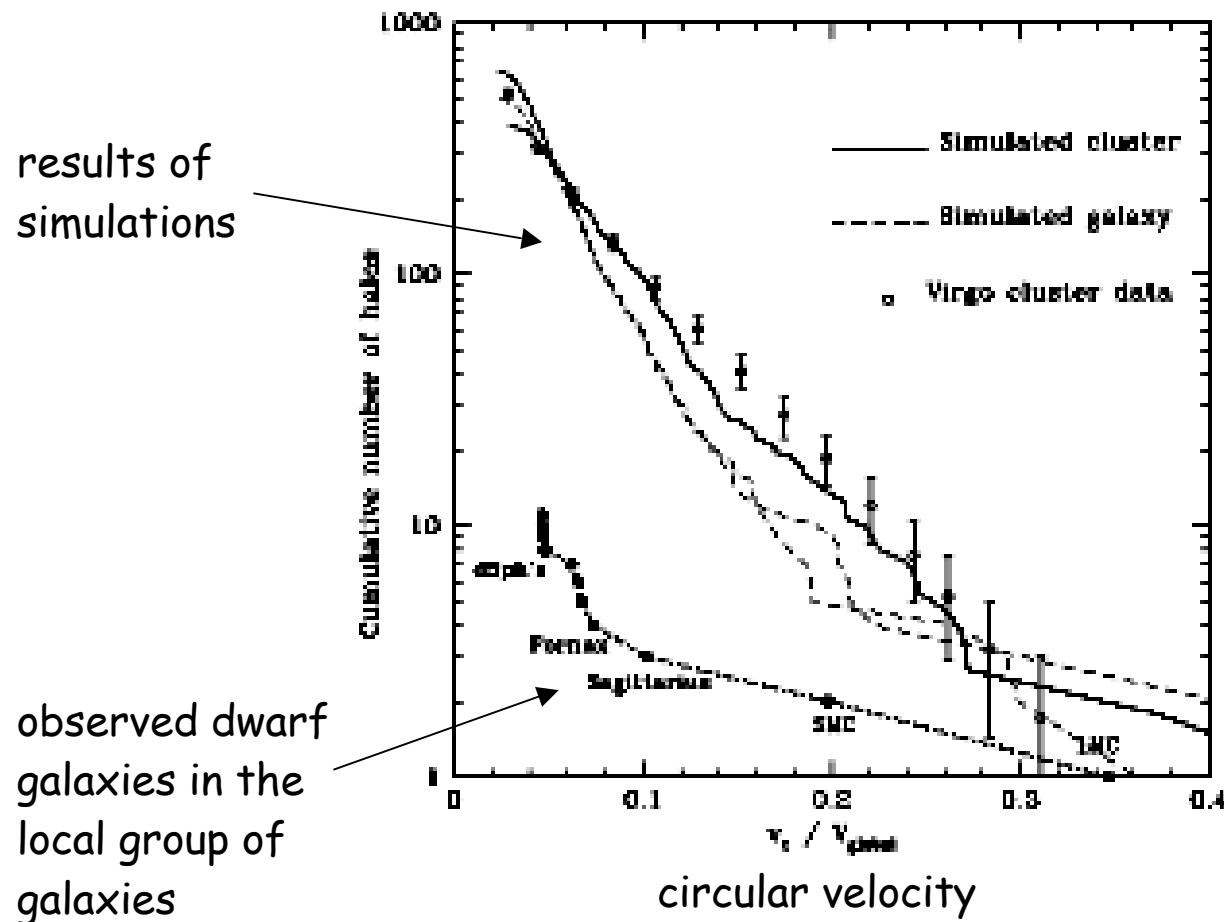
R. Ben Metcalf  
University of California, Santa Cruz

## Outline:

- very brief review of observations  
substructure problem
- lens modeling and data analysis
- simulating gravitational lenses from cosmological  
simulations
- Do the data and the simulations match?

# under abundance of dwarf galaxies in the local group of galaxies

Moore, et al. '99



# Quasar Lensing with Substructure

Small halos are very weak lenses by themselves. But ...

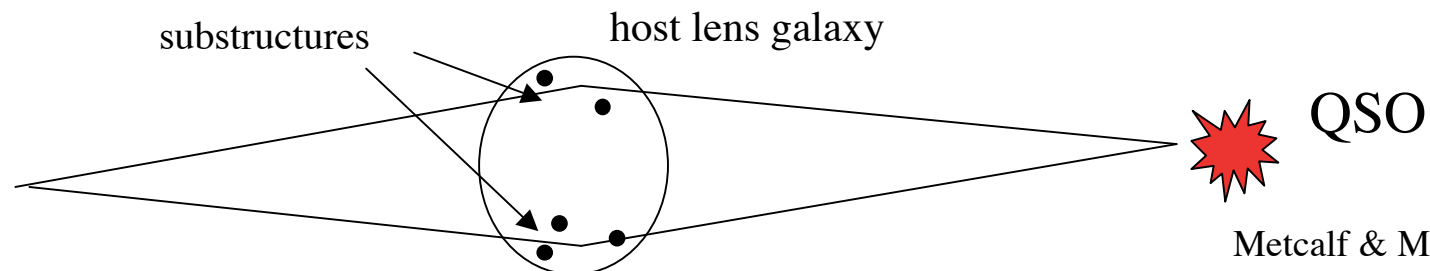
The effects of small halos on lensing are greatly enhanced if they are embedded in a larger strong lens. This makes it possible to detect even a very small amount of substructure.

The magnification is strongly affected near the critical lines where the primary lens' magnification is large.

$$\Delta\mu_{\text{image}} \approx (\mu_{\text{host}})^2 \frac{\delta\Sigma}{\Sigma_{\text{critical}}}$$

change in magnification      host lens magnification      perturbation in surface density      critical surface density

The image magnifications are affected without substantial changes in the image positions. Magnification is more sensitive to small scale mass distribution.



Metcalf & Madau, 2001

# Quasar Lensing with Small-Scale Structure

Small halos are very weak lenses by themselves. But ...

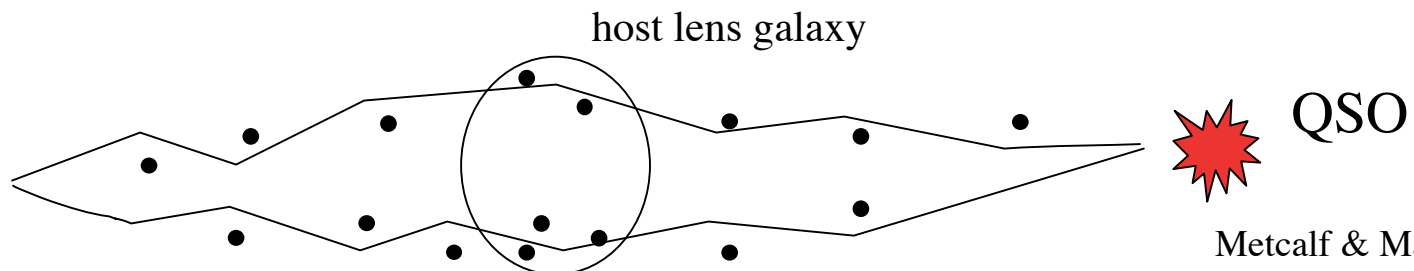
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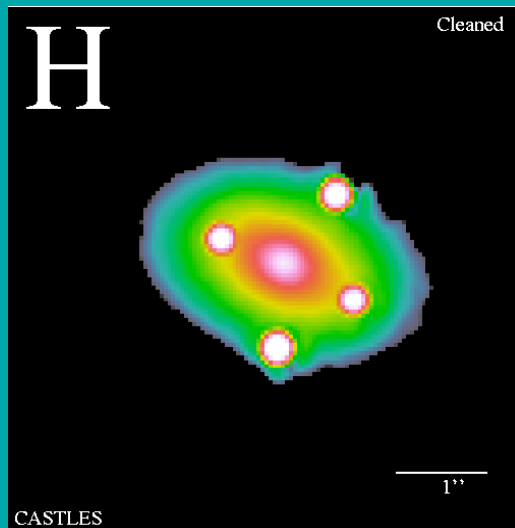
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change in magnification      host lens magnification      perturbation in surface density      critical surface density

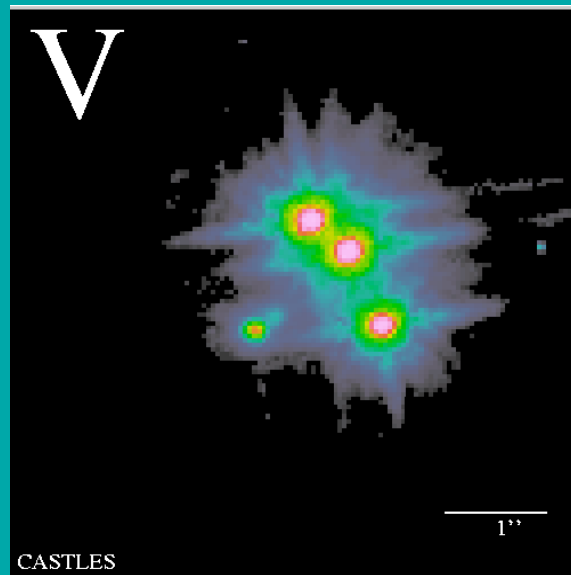
The image **magnifications** are affected without substantial changes in the image positions. Magnification is more sensitive to small scale mass distribution.



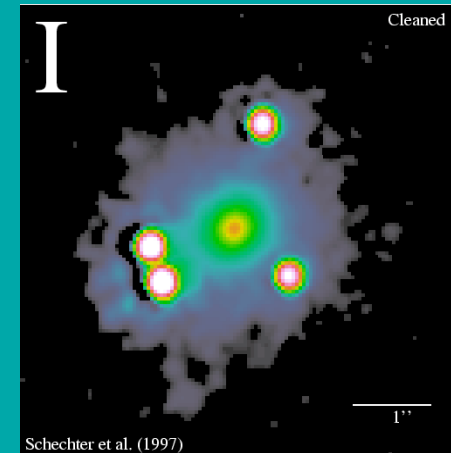
Metcalf & Madau, 2001



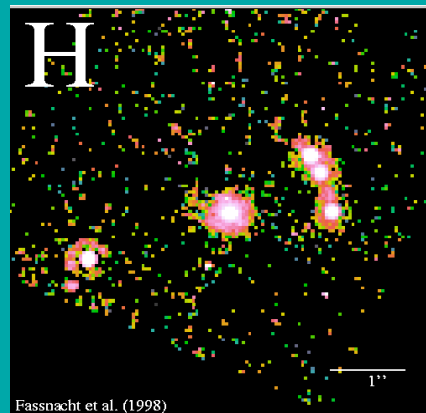
Q2237+0305



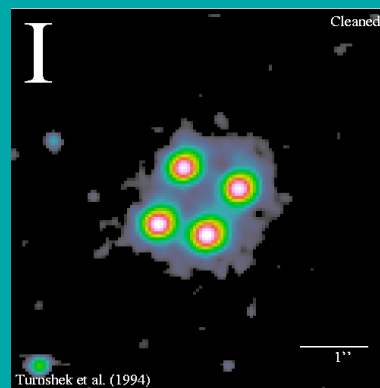
B 1422+231



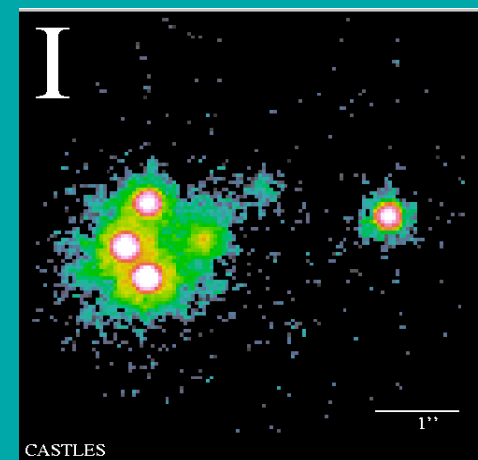
PG 1115+080



B 2045+265



H 1413+117



XRJ 0911.4+0551

# Modeling Gravitational Lenses

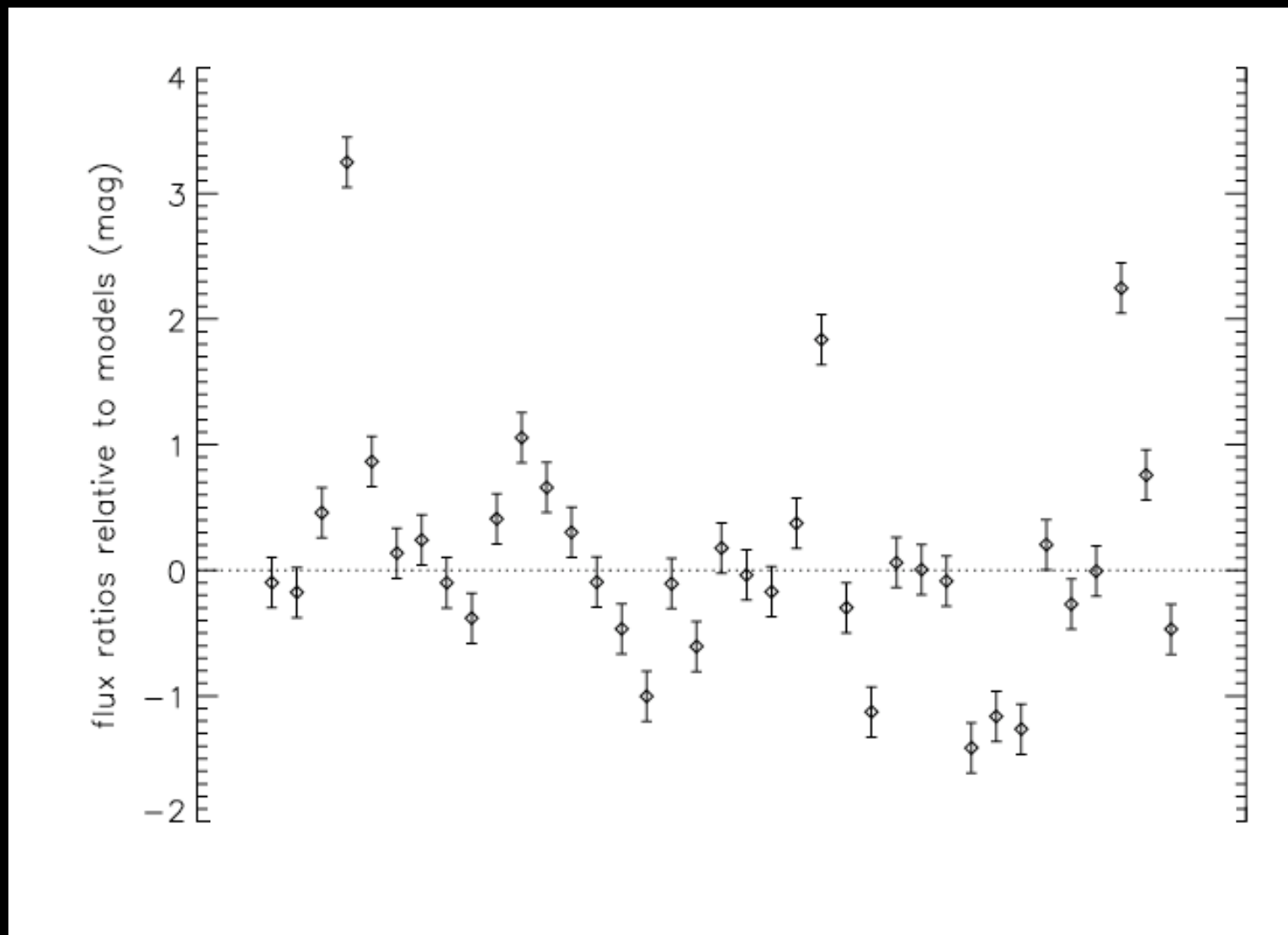
The lens models are inevitably degenerate.

The lens is decomposed into normal modes.

Using the image positions and the position of the lens center, the modes and the space of degenerate lenses in mode-space can be found through a system of linear equations.

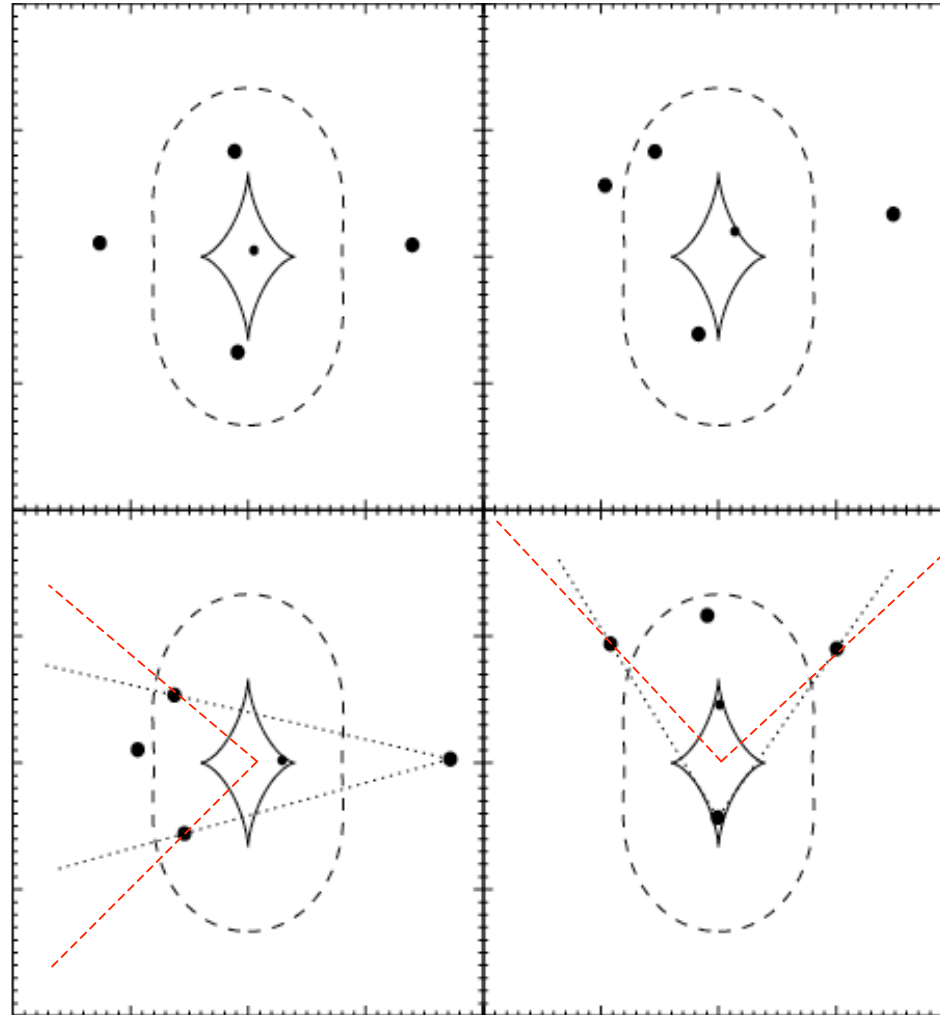
Moving in the degenerate region of mode-space a model can be found that is as much like an idealized model of a galaxy + halo as possible. For example the *Most Singular Isothermal Ellipsoidal model* (MSIE) that fits the positions perfectly.

# flux anomalies in radio relative to the MSIE



## classification of 4-image lens configurations

Einstein Cross  
configuration



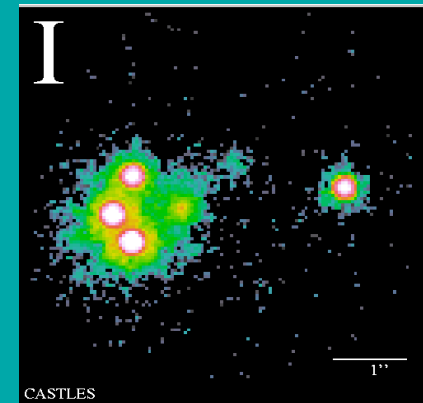
fold caustic  
configuration

short axis  
cusp caustic  
configuration

long axis  
cusp caustic  
configuration

# Cusp Caustic Lenses

In the case of a cusp caustic the lens configuration the quantity  $R = (\mu_1 + \mu_2 + \mu_3) / (|\mu_1| + |\mu_2| + |\mu_3|) \rightarrow 0$  as the source approaches the cusp for any lens.



XRJ 0911.4+0551

## Observed Cusp Caustic Lenses

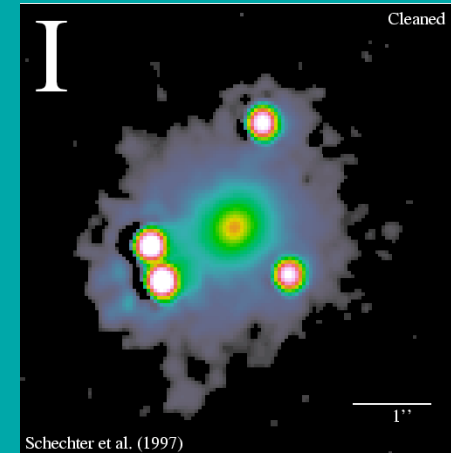
| lens          | R               |            |
|---------------|-----------------|------------|
| B1422+231     | $0.19 \pm 0.01$ | radio      |
| B0712+472     | $0.26 \pm 0.02$ | radio      |
| RX J0911+0551 | $0.19 \pm 0.01$ | optical/IR |
| B2045+265     | $0.50 \pm 0.01$ | radio      |
| RX J1131-1231 | $0.36 \pm 0.01$ | optical/IR |

The cusp caustic relation is violated in every known case.

- Optical & IR continuum ratios could be effected by microlensing by ordinary stars in the lens galaxies.

# Fold Caustic Lenses

In the case of a fold caustic configuration the ratio of the close pair of images should be close to 1.0.



## Observed Fold Caustic Lenses

| lens        | ratio           |        | model |
|-------------|-----------------|--------|-------|
| MG 0414+053 | $1.05 \pm 0.03$ | radio  | 1.19  |
| B0712+472   | $0.84 \pm 0.03$ | radio  | 1.03  |
| B1555+375   | $0.62 \pm 0.03$ | radio  | 1.07  |
| PG1115+080  | $0.93 \pm 0.03$ | mid-IR | 0.85  |

# Measuring Small-Scale Structure with Gravitational Lenses

Monte Carlo simulations

add substructures at random positions  
in this case tidally truncated NFW profiles

construct a host lens model that reproduces the  
image positions *with* the substructure and is  
as near to SIE as possible - fully self-consistent  
lens model

measure the magnifications of the images

20,000 simulations for each parameter set

# Simulations of Lensing by Substructure

The goal is to solve the lensing equation:

$$\text{source position} \longrightarrow \mathbf{y} = \mathbf{x} - \alpha_{\text{host}}(\mathbf{x}) - \sum \alpha_i(\mathbf{x})$$

image position      deflection caused by host galaxy      sum over substructures

For a finite size source the magnification is proportional to the size of the image for a finite size source.

For a point source the magnification can be calculated directly once the images position is found.

With random distribution of substructures that are truncated by the tidal forces of the host galaxy.

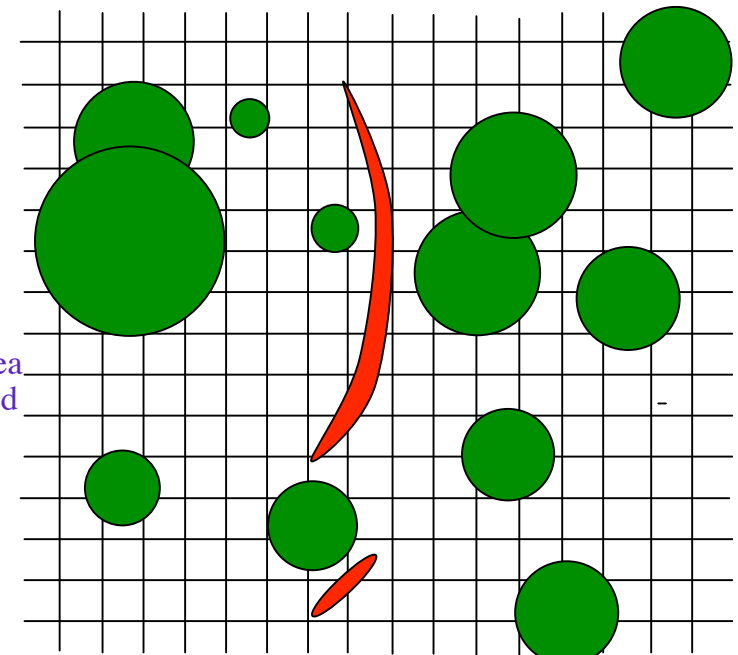
A simple cross section type approximation is not sufficient because we are interested common events where the interaction between clumps can be important not rare events where the influence of one clump dominates the magnification.

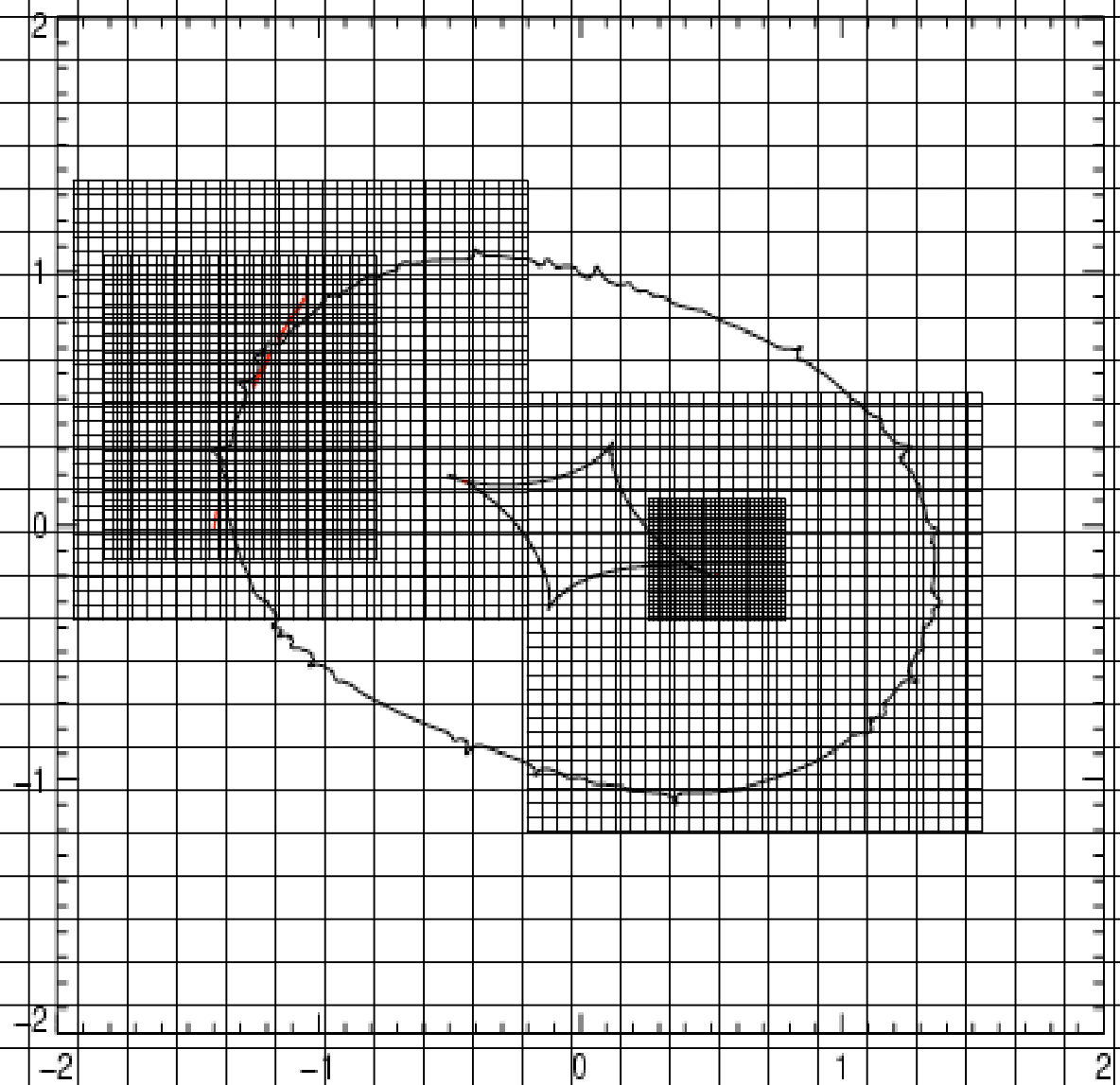
The internal structure of the substructures, most importantly the cutoff radius, is important so an untruncated SIS model is not sufficient.

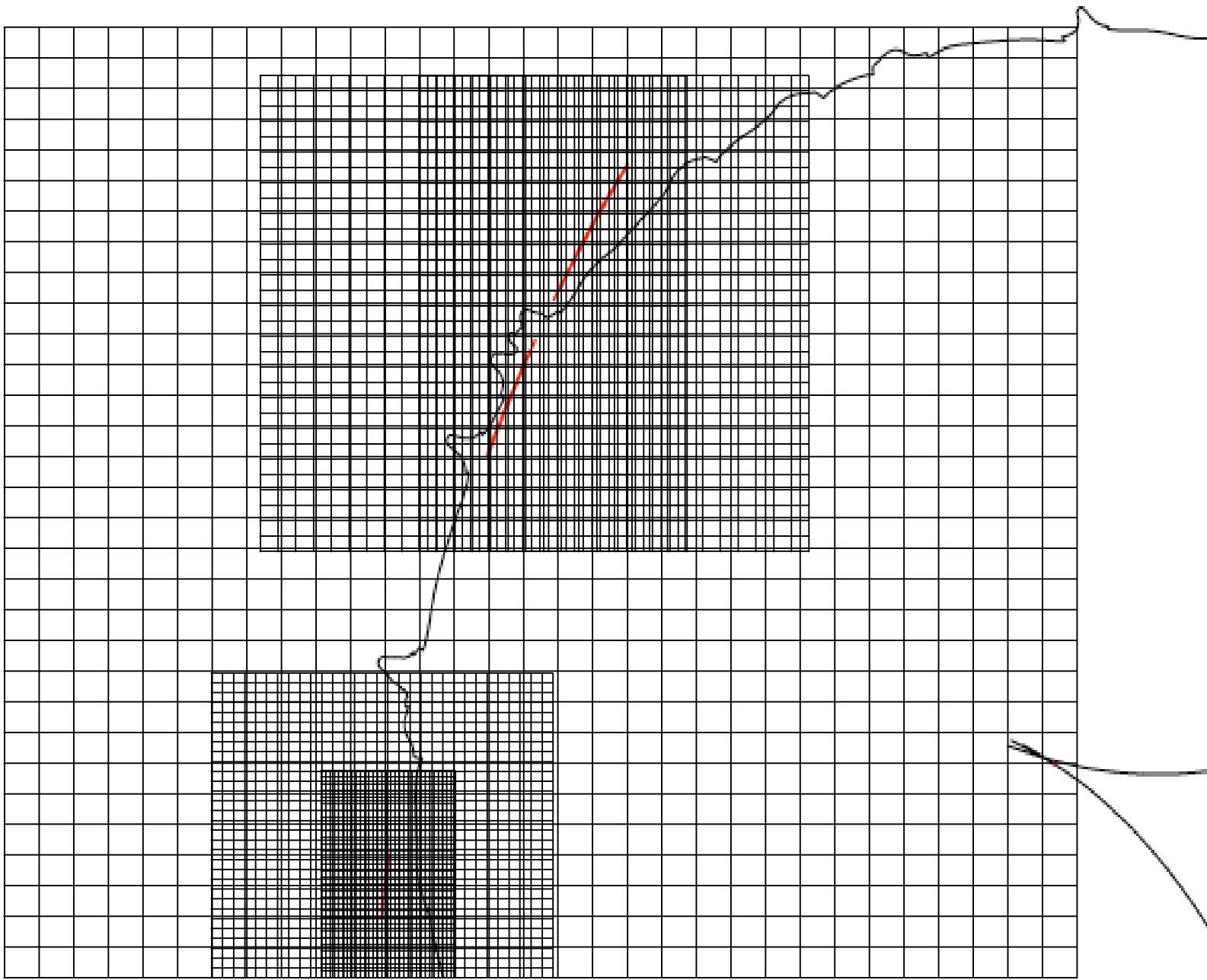
We also want to calculate the magnification for finite sized sources with different sizes over a range that encompasses the size of the substructures.

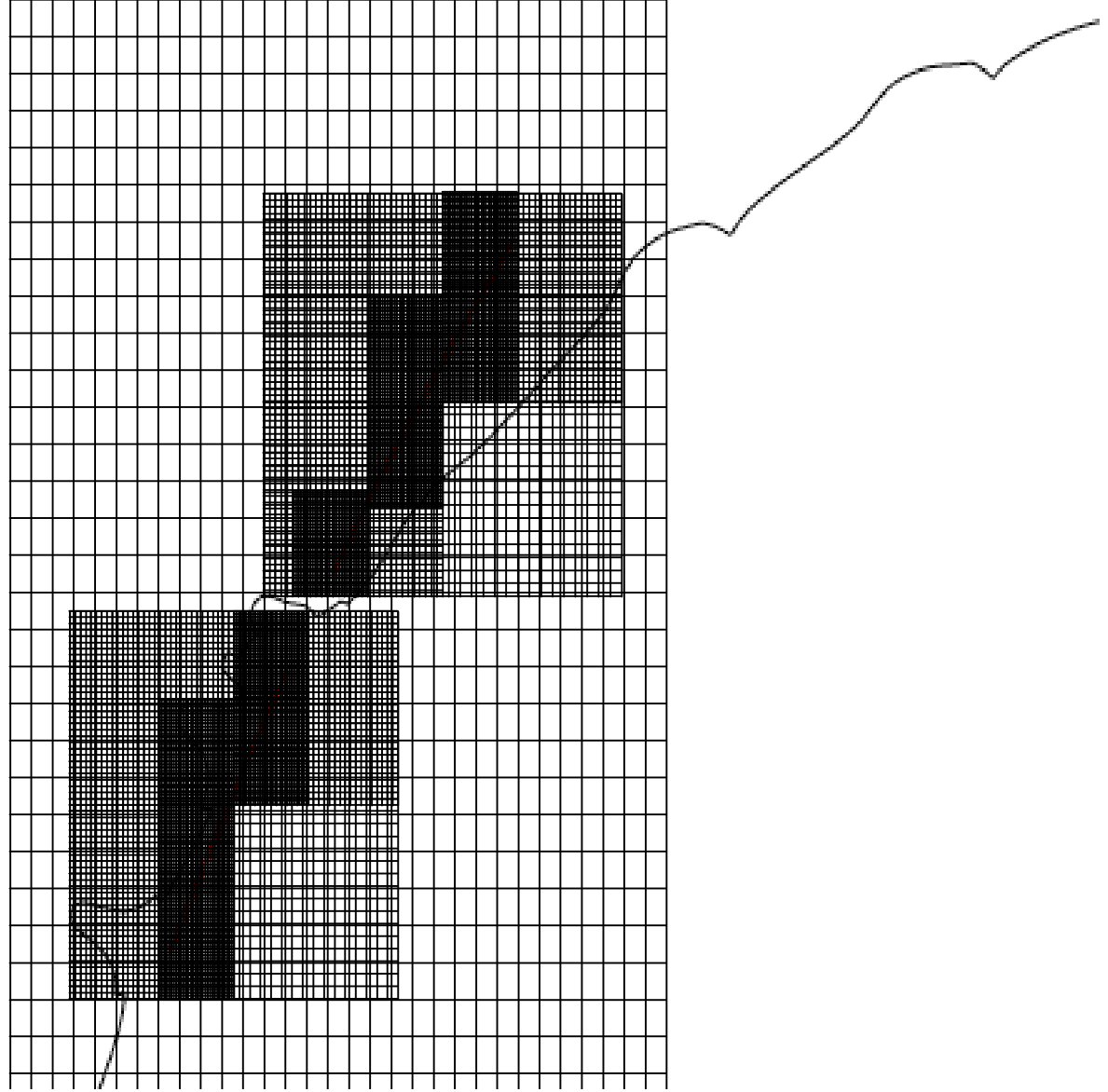
A large dynamical range is required: source sizes from 1 to  $10^3$  pc,  $10^6$  in area typically several hundred clumps included

Use an adaptive grid refinement technique which tracks the image position as it shrinks the grid region and increases the number of cells until it has the required resolution to determine the area of the image.







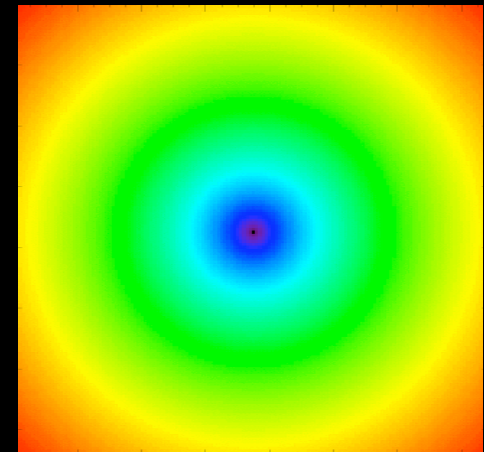
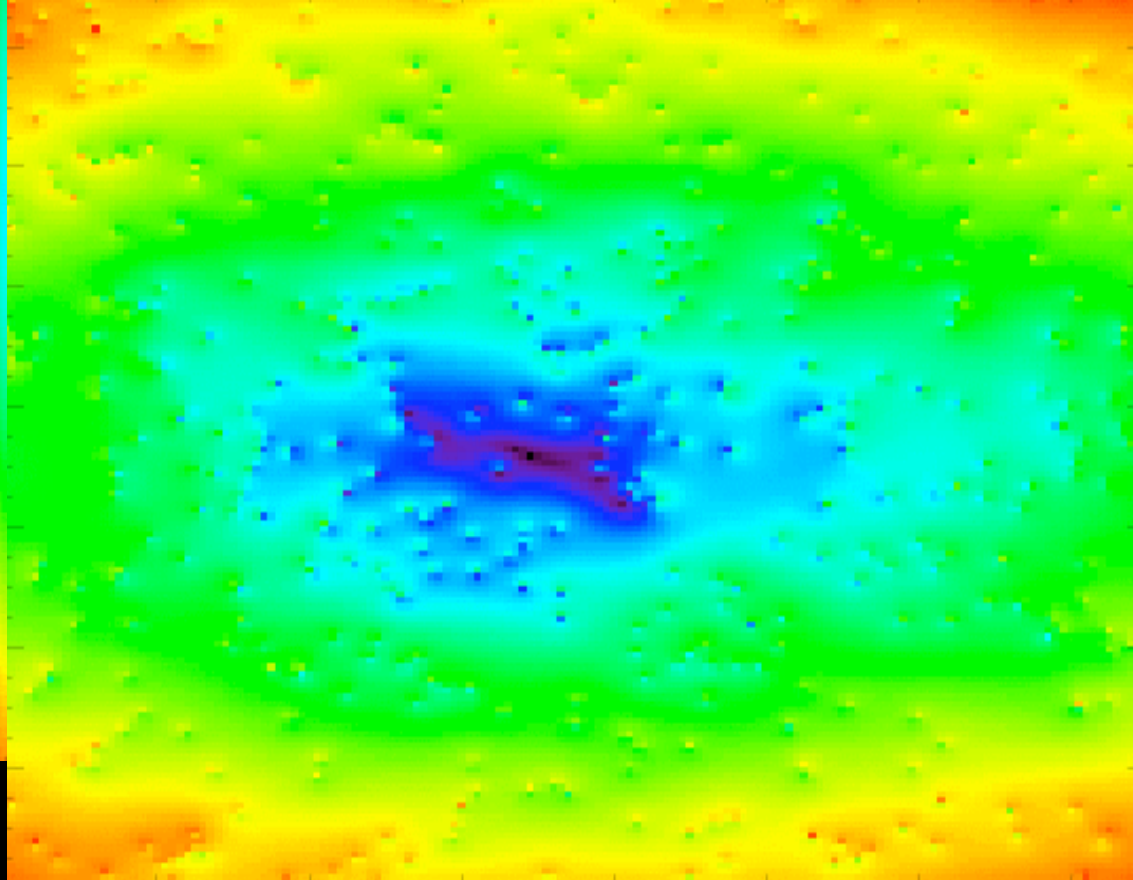
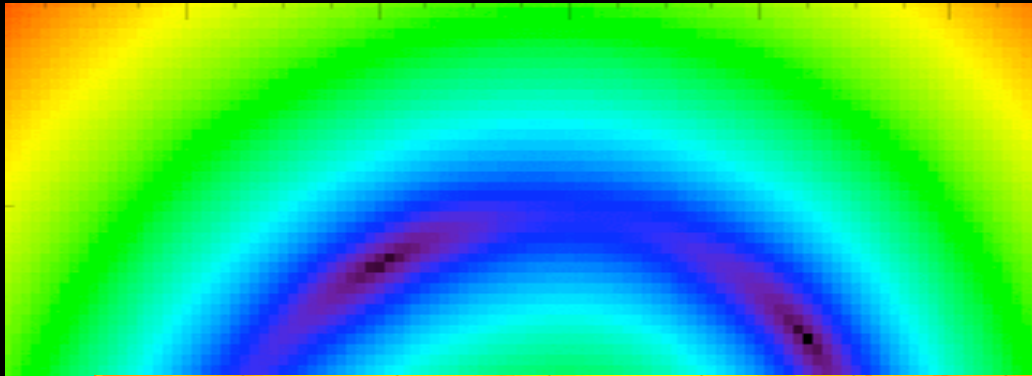


lensing simulations

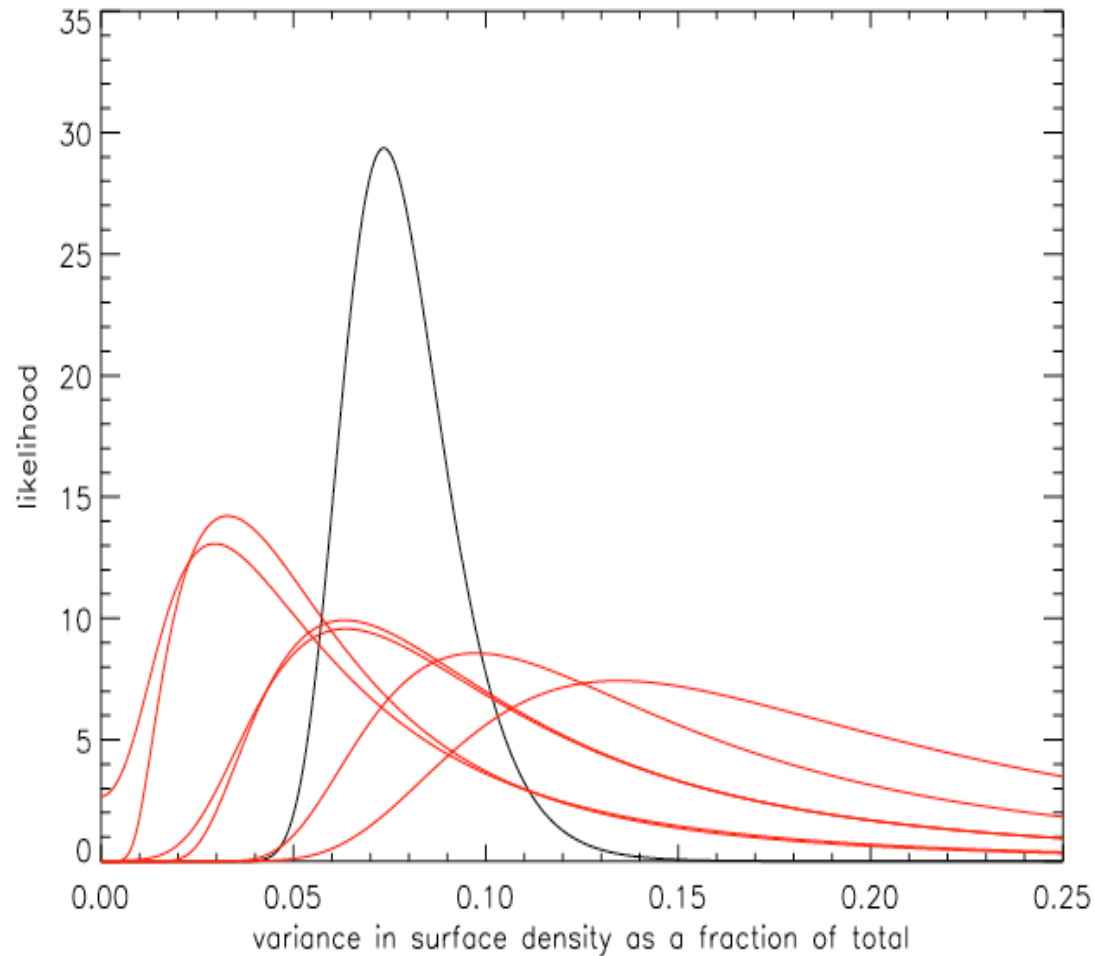
simulated  
image

blowup of simulated  
image with stars

source before lensing



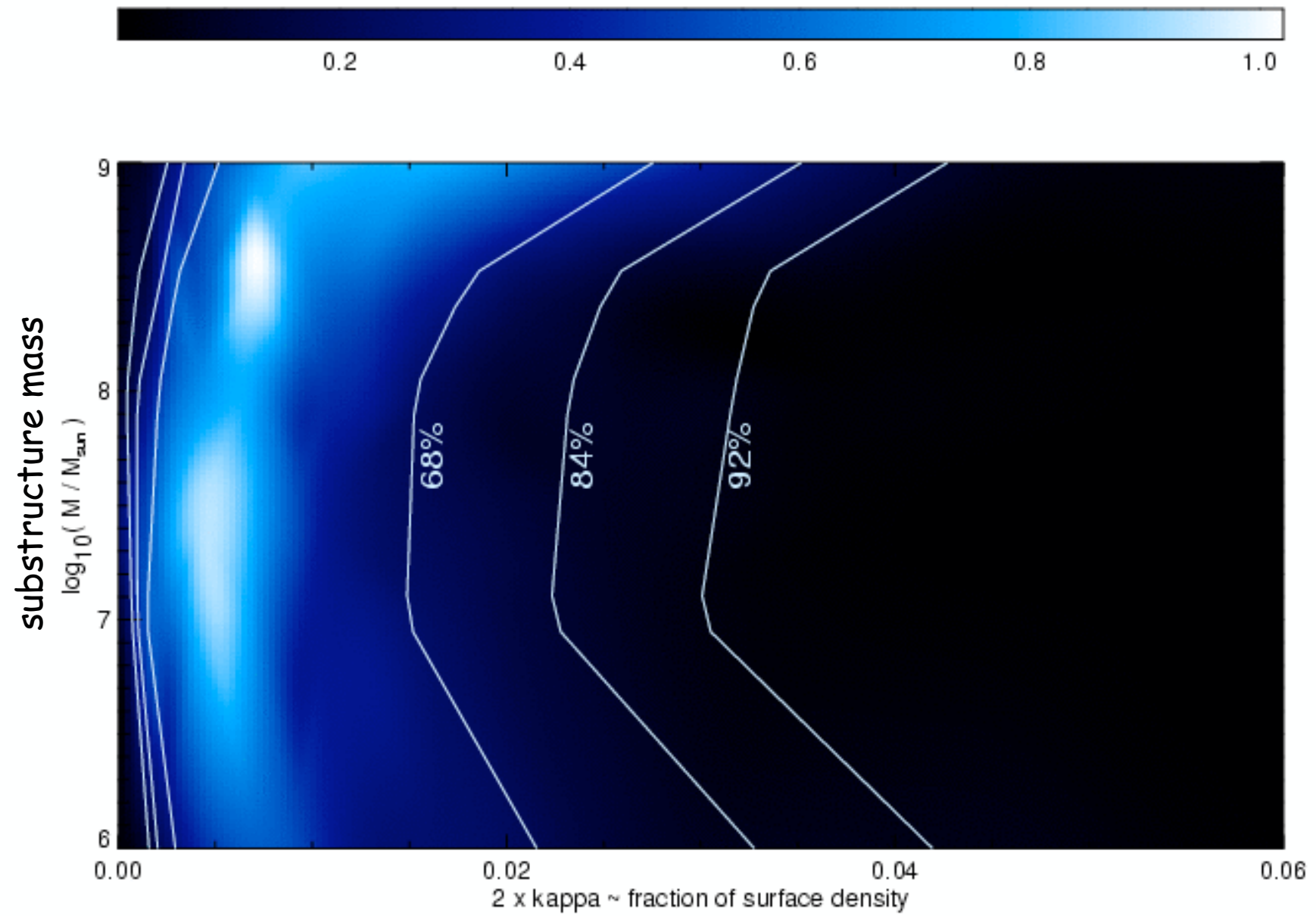
# Likelihood constraints using ALL flux ratios



~ 10% of mass in substructure !!

But:

- there is no constraint on the size scale of the discrepancies: large scale asymmetry in galaxy/halo or small scale structure?



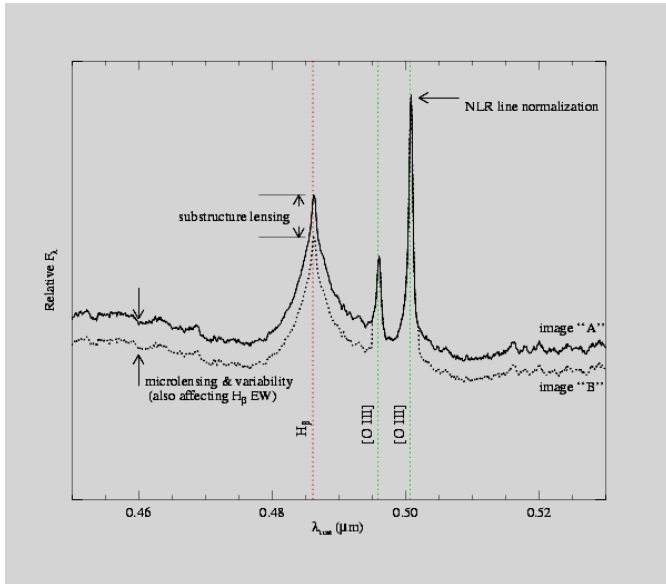
2-d confidence region

# Spectroscopic Gravitational Lensing

To avoid problems with model degeneracy, differential extinction and plasma scattering we are developing a different technique.

Irrespective of lens model, the magnification of different emission regions of the QSO will be the same unless there is substructure within the size scales spanned by the regions.

The narrow line region (NLR) is effected by only the very large substructures. The radio and broad line region (BLR) are effected by substructures and large enough that microlensing by stars in the lens galaxy is unlikely. The visible emission region is very small and susceptible to microlensing by stars.

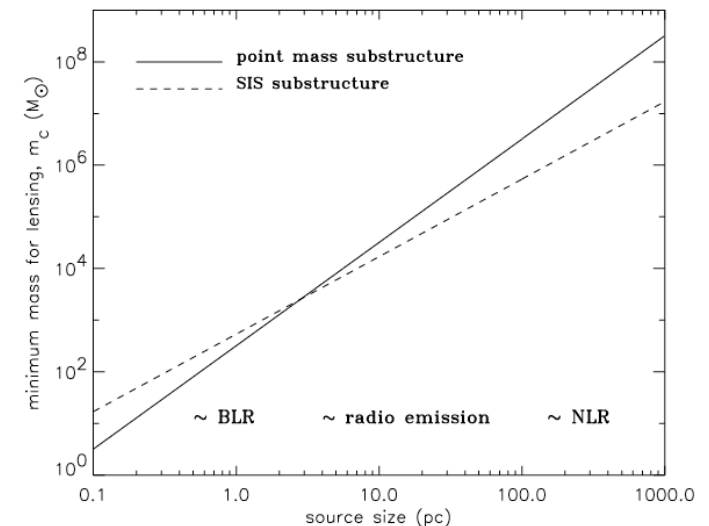
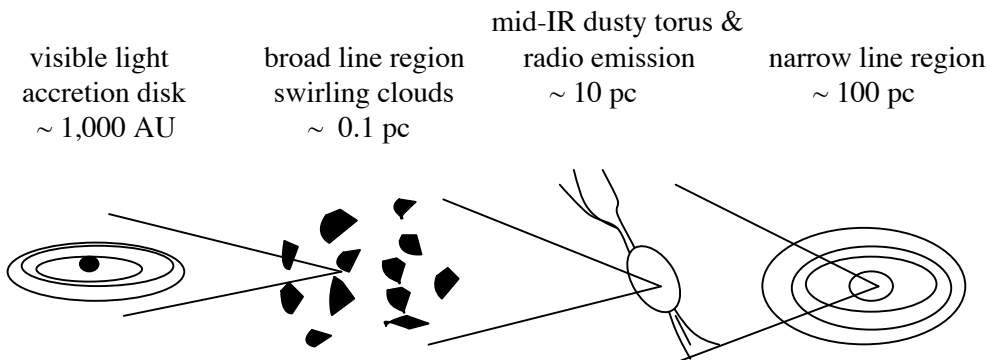


the differential magnification ratio:

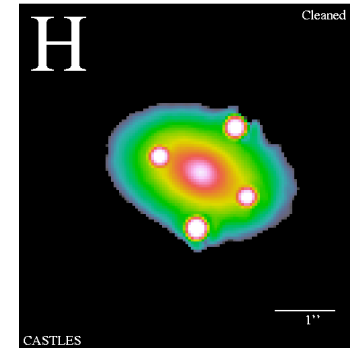
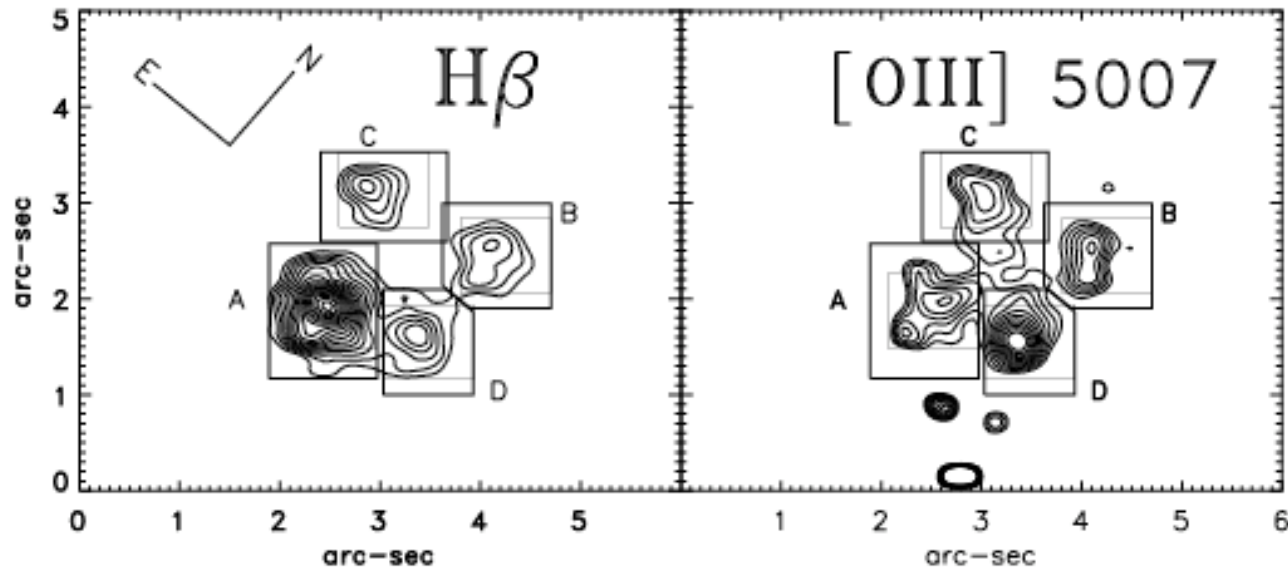
$$\text{DMR}_{AB} = \frac{f^A(\text{BLR})}{f^B(\text{BLR})} \frac{f^B(\text{NLR})}{f^A(\text{NLR})} = \frac{\mu^A(\text{BLR})}{\mu^B(\text{BLR})} \frac{\mu^B(\text{NLR})}{\mu^A(\text{NLR})}$$

approximate lower lens mass cutoff for sources of a given size

In the absence of substructure  $\text{DMR}_{AB} = 1$ .

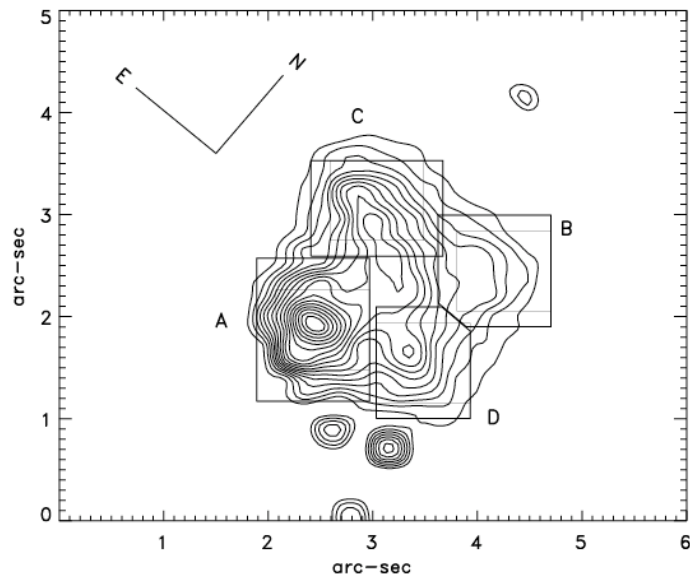


images in the broad and narrow lines



The rectangles are the apertures used to measure magnification ratios.

total flux

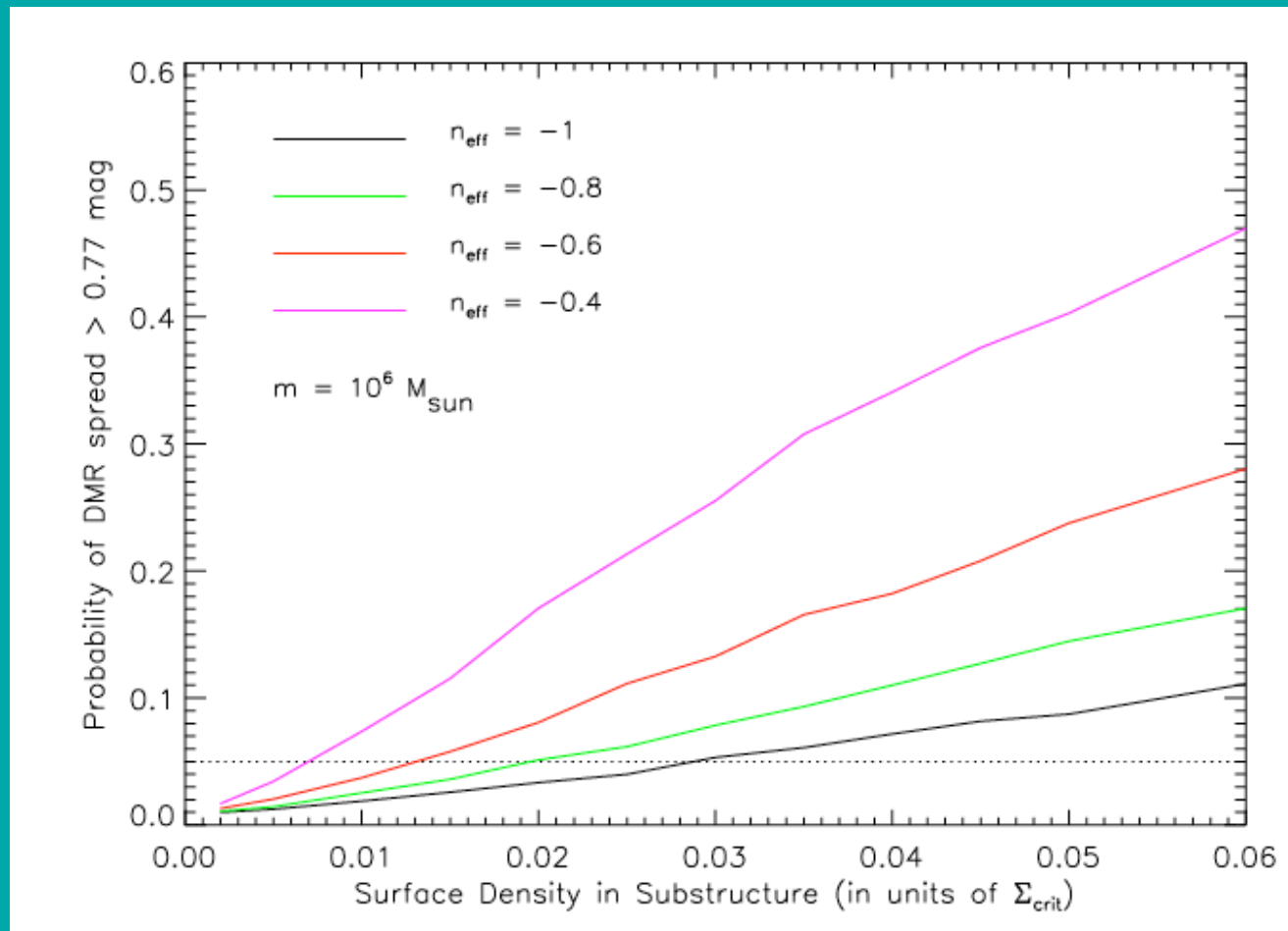


In the line emission the QSOs are consistent with being point sources. The lens galaxy is well subtracted.

The [OIII] line especially noisy in this case because of atmospheric absorption.

The BLR is found to be  $0.65 \pm 0.1$  mag too bright relative to the NLR in image A! The other differential magnification ratios are consistent with each other.

# Constraints on Substructure from Spectroscopic Gravitational Lensing

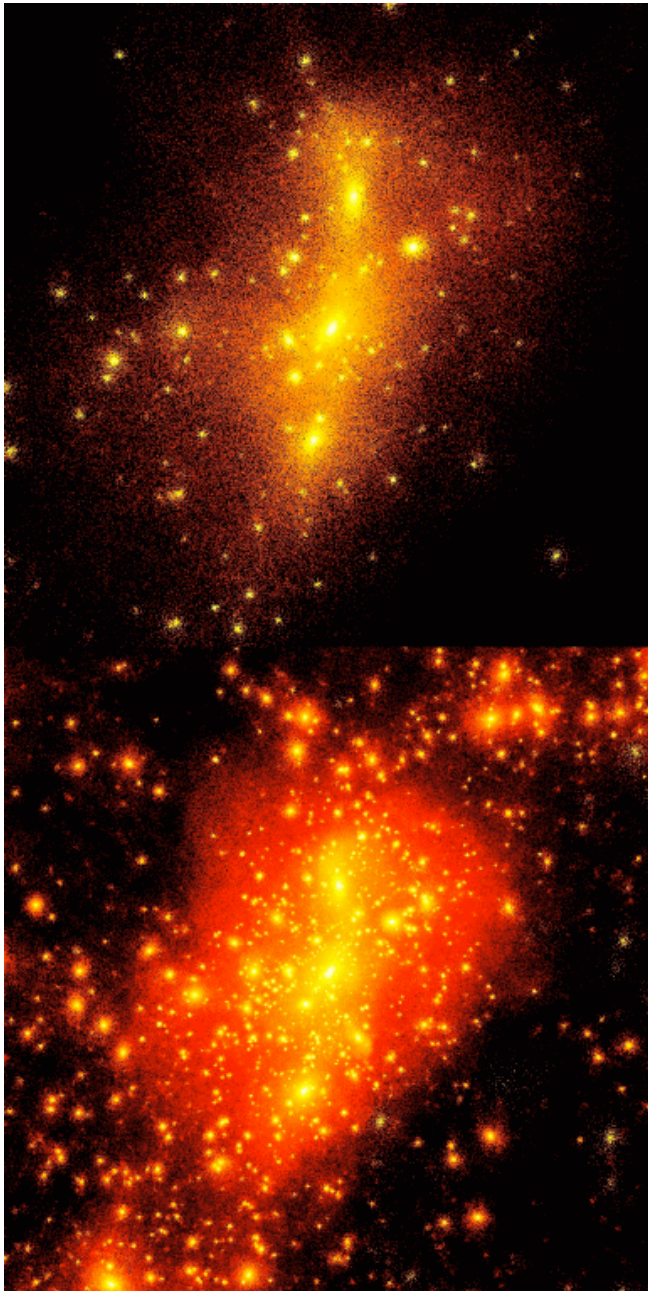


$> 4\%$  in substructure  
 $m \sim 10^6 M_{\text{sun}}$

but only one lens so far

see Metcalf et al. 2004

$\sim 1/2 \times$  fraction of mass in substructure

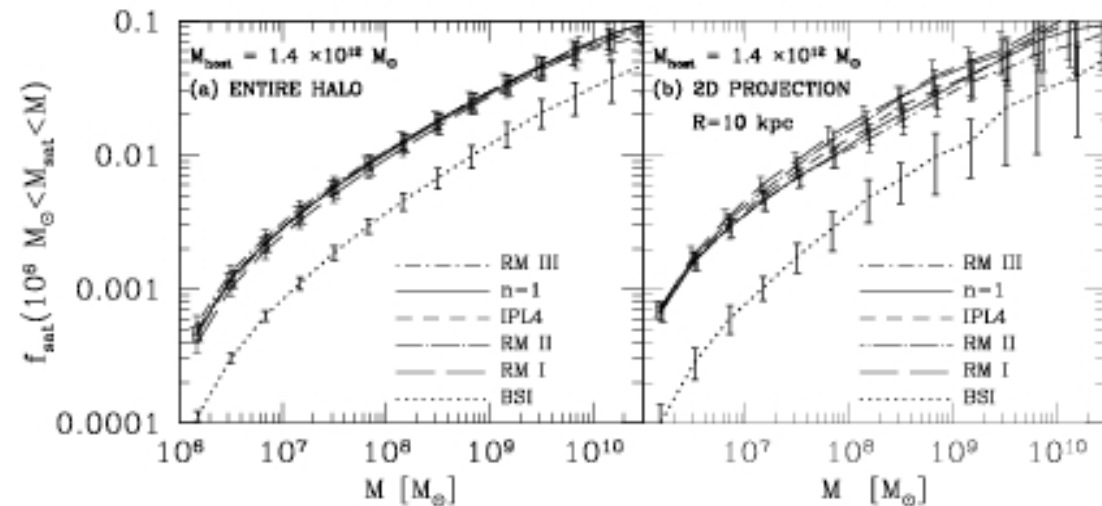


# CDM Substructure Predictions

Moore *et al.* 1999.

Low resolution  
1 million particles

Improved resolution has resulted in more substructure in dark matter halos.



Zentner & Bullock 2003

Numerical effects could be suppressing substructure.

Baryons could increase it or destroy more of it.

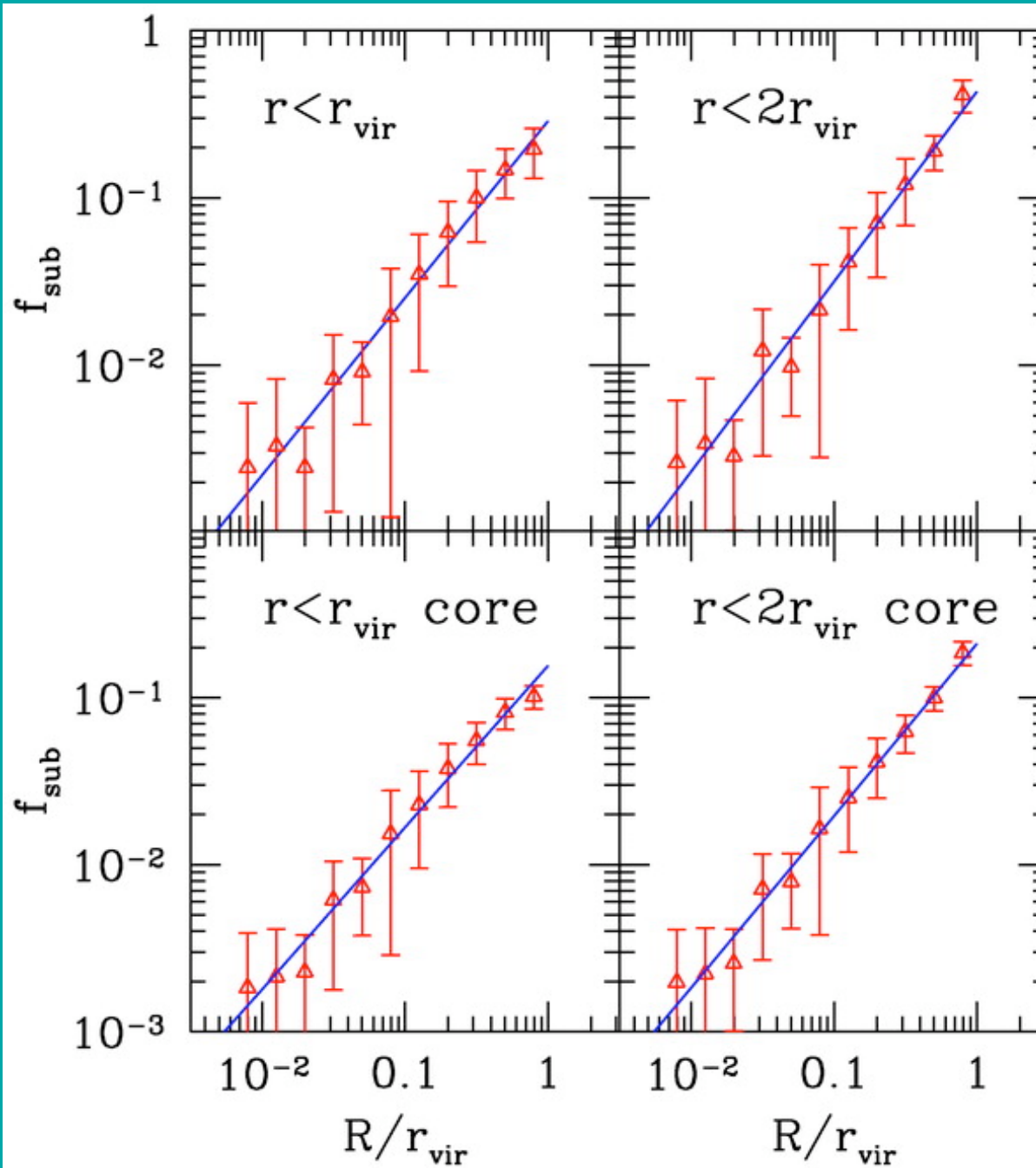
High resolution  
8 million particles

Mass function

$$\frac{dN}{dM} \propto M^{-(2.0-1.6)}$$

For a  $(102 \text{ Mpc})^3$  box  
smallest resolved clump is  $10^7 M_{\text{sun}}$

fraction of surface density in substructure



only the cores  
of the  
substructures

# Simulated Strong Gravitational Lenses

Can N-body and SPH simulations reproduce the lensing data?

The baryons are an essential ingredient in reproducing a realistic gravitational lens.

We use Moore et al. (1999)'s N-body simulation of a galactic halo.

particle mass  $\sim 1.68 \times 10^6 M_{\text{sun}}$

force resolution is 0.5 kpc

$\sim 10^6$  particles

Implant an artificial galaxy

- both an elliptical and a disk + bulge, 50,000 particles

Let system relax for 200 Myr

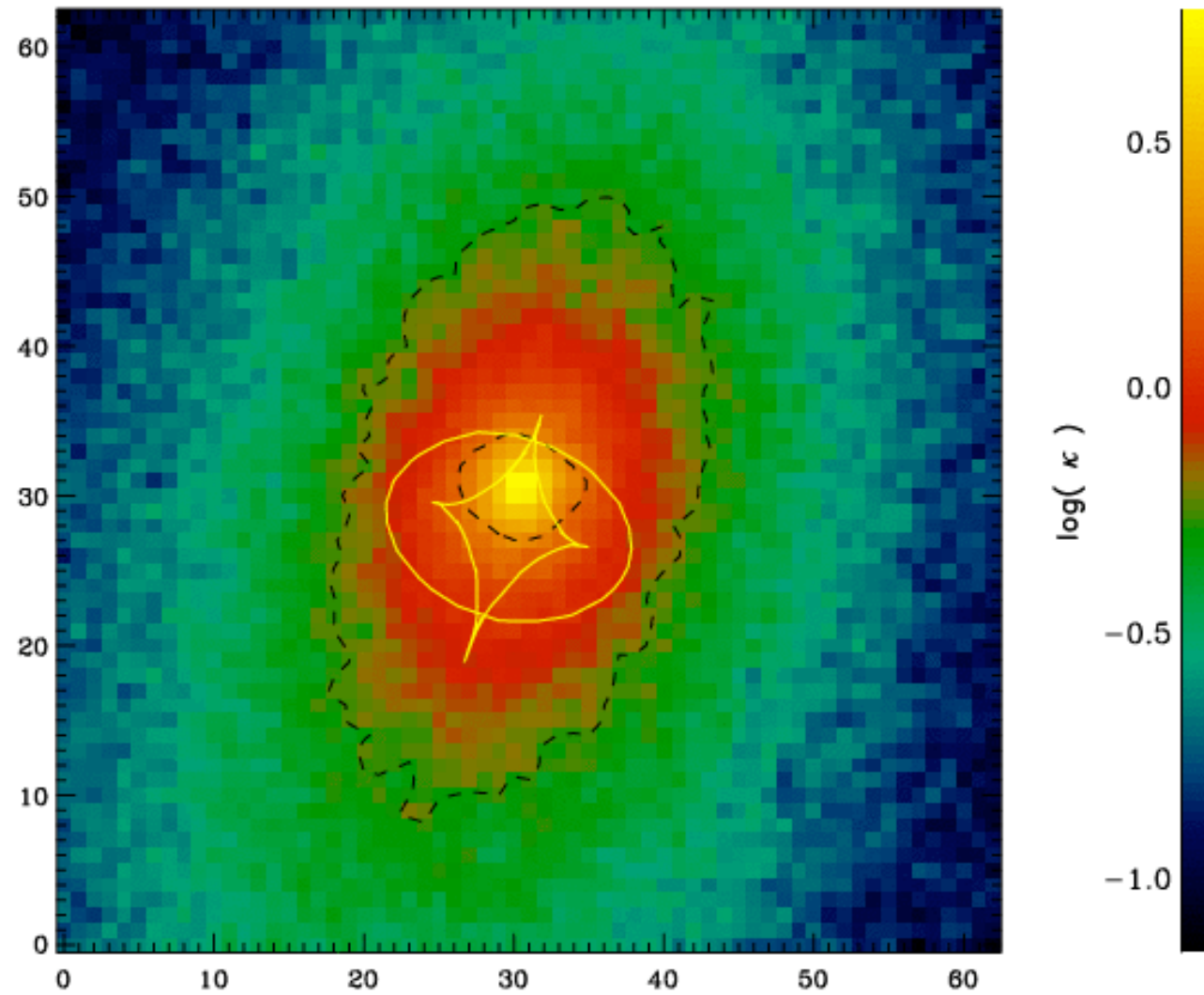
collaborators:

Adam Amara (Cambridge), TJ Cox (Harvard/SantaCruz)

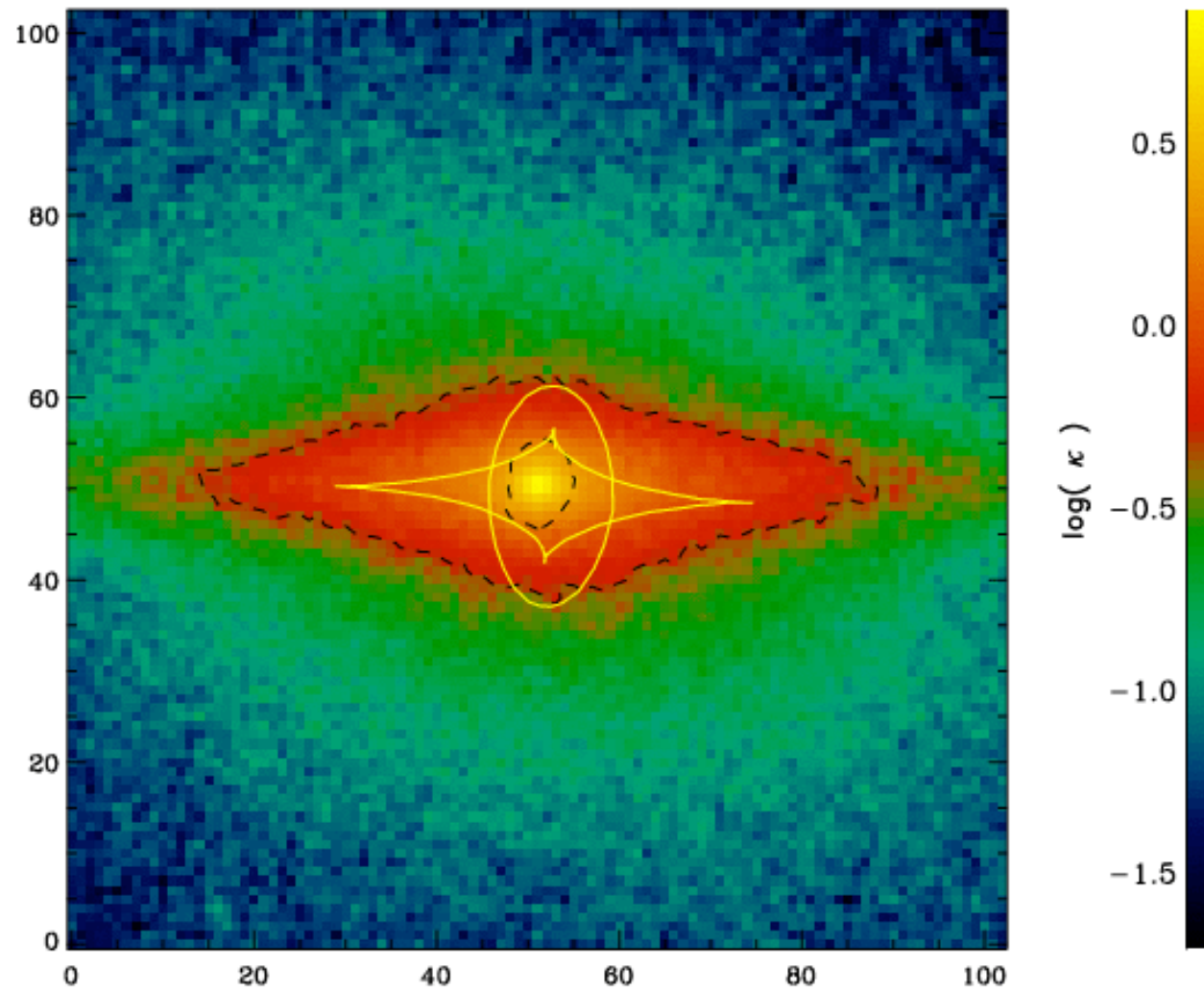
J. Ostriker (Princeton/Cambridge)



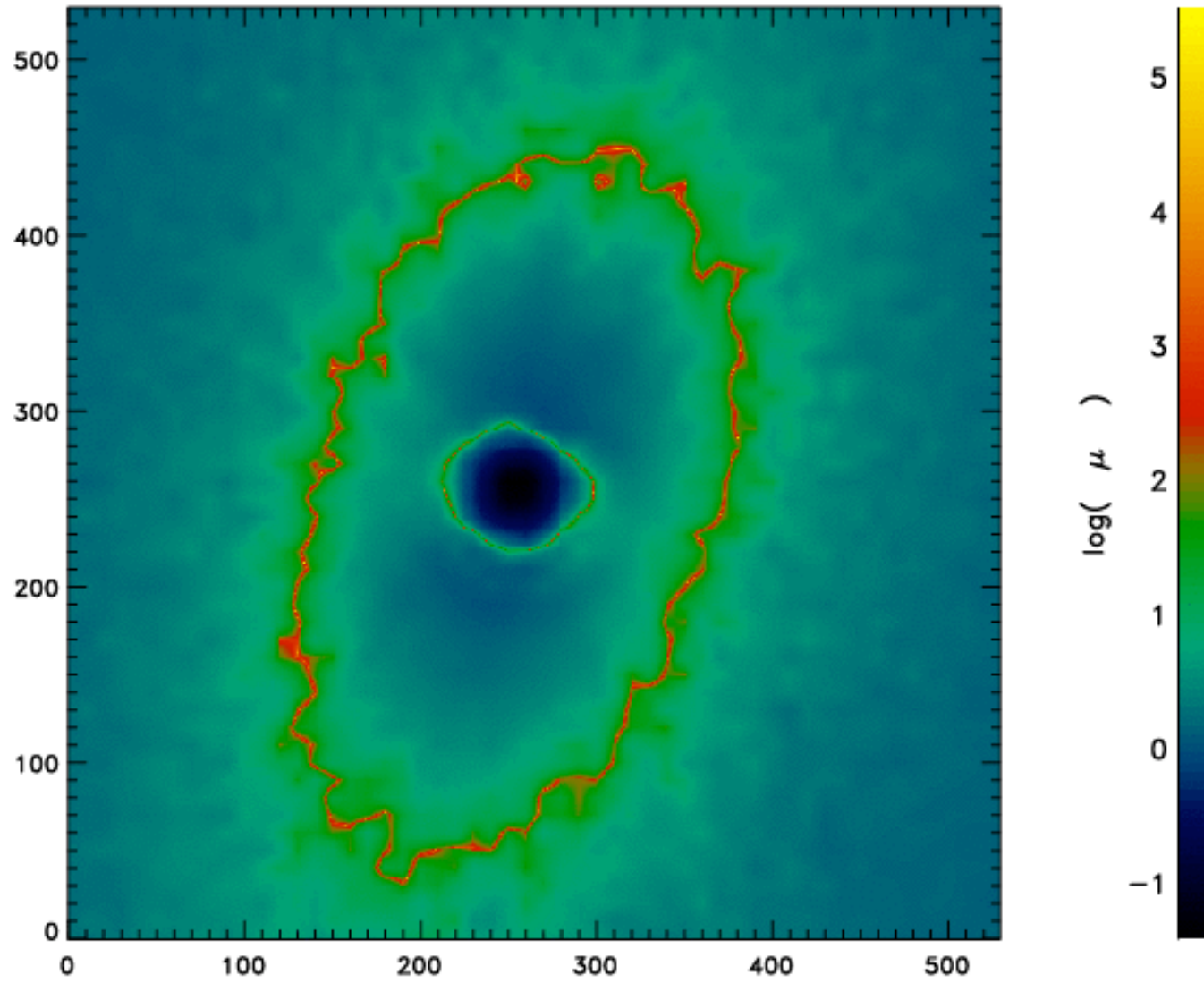
simulations of gravitational lenses :  
critical curves, caustic curves & surface density

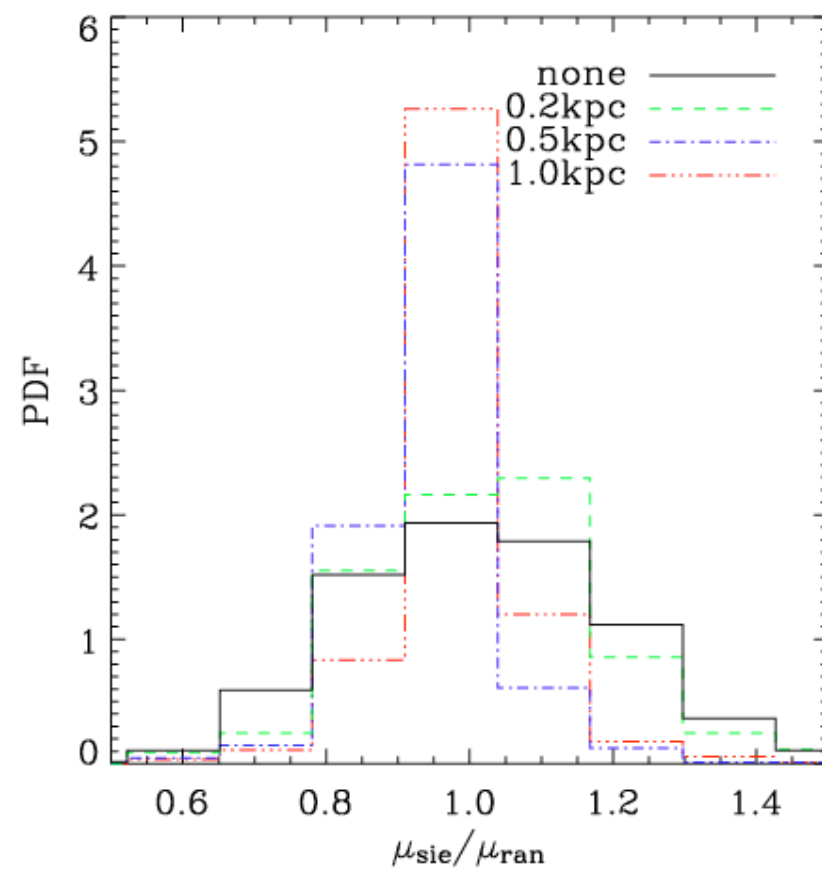


simulations of gravitational lenses :  
critical curves, caustic curves & surface density

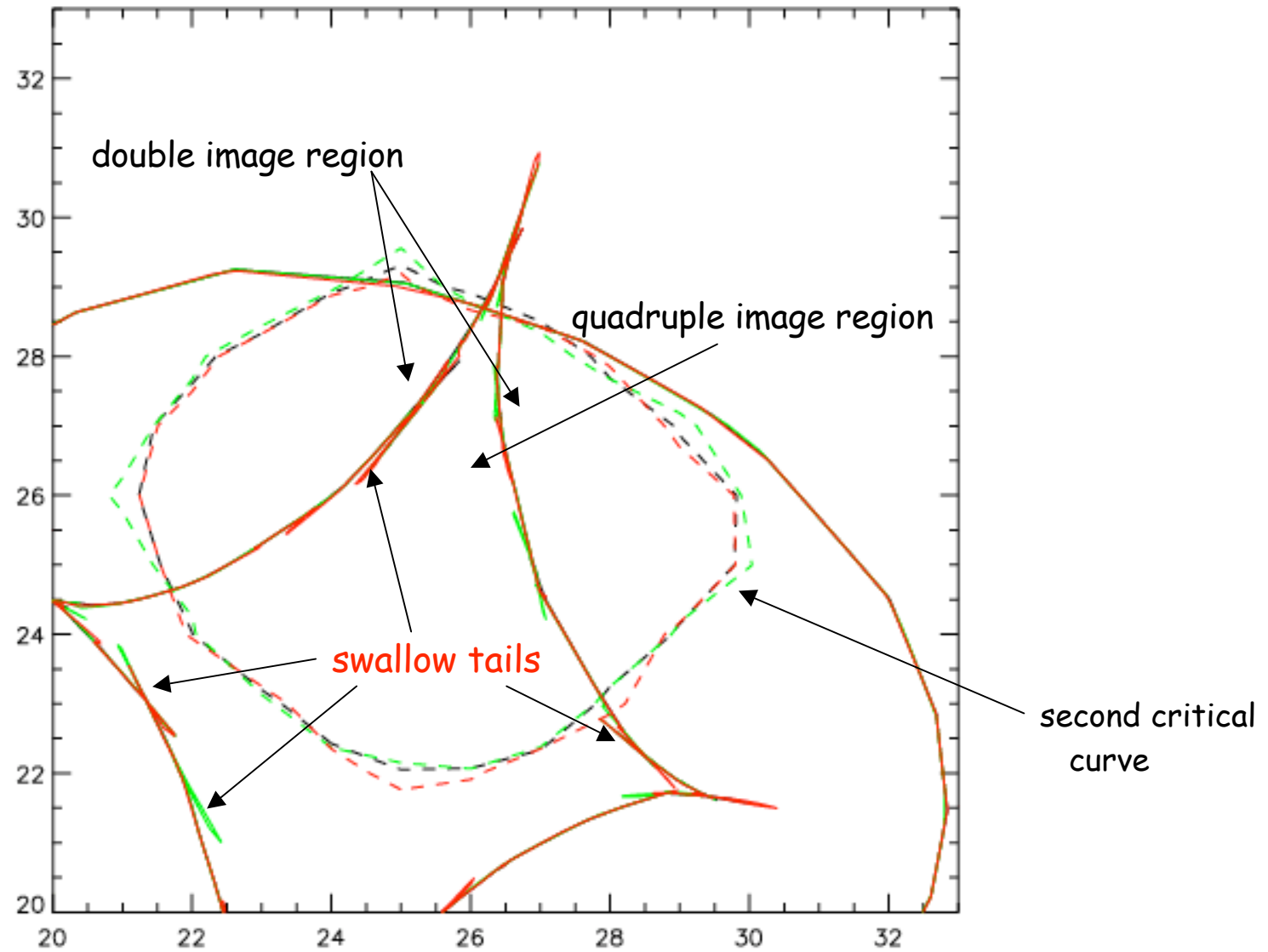


## simulations of gravitational lenses : magnification





## caustic structure for several realizations of the noise



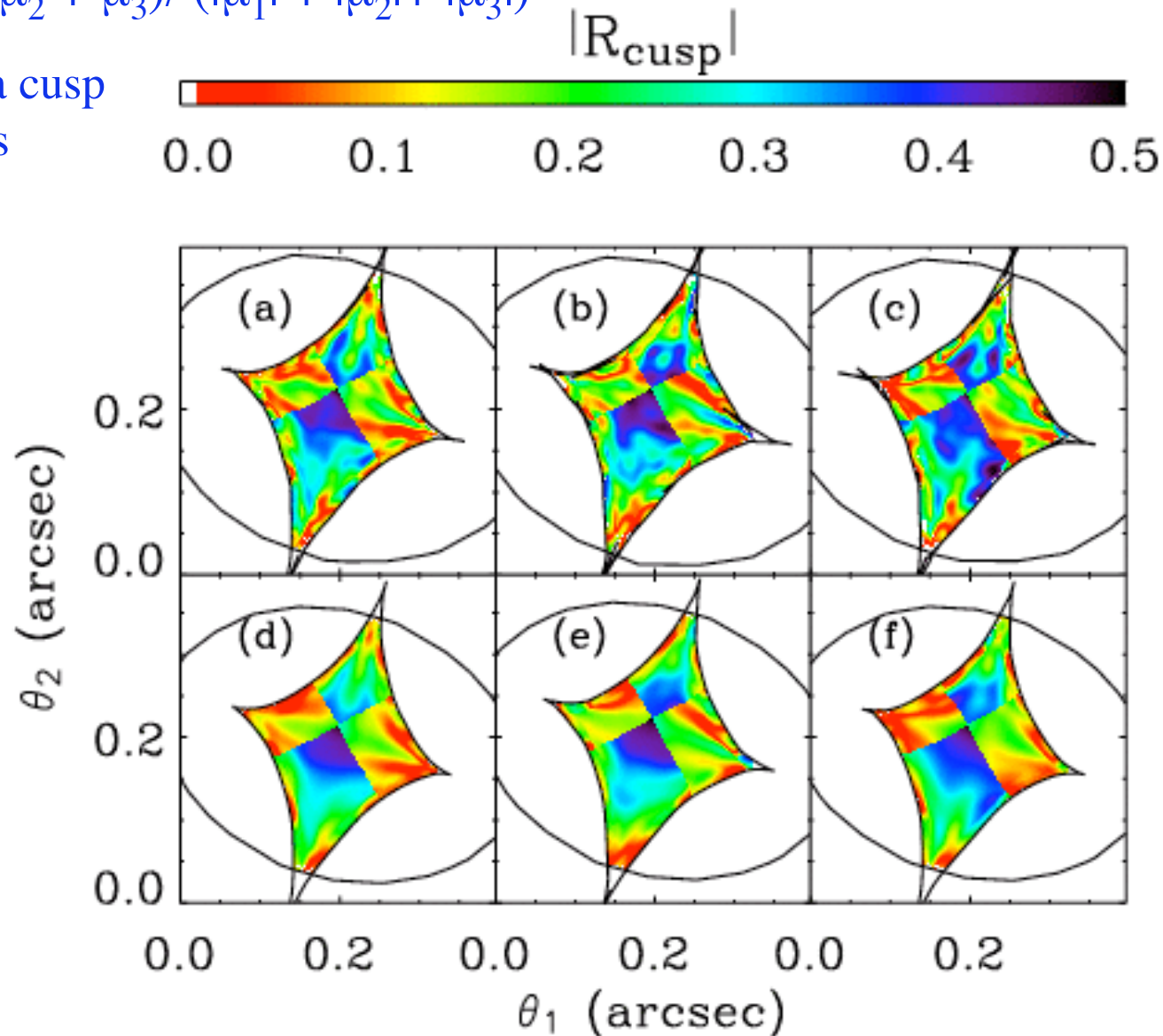
# Cusp caustic relation as a function source position

$$R_{\text{cusp}} = (\mu_1 + \mu_2 + \mu_3) / (|\mu_1| + |\mu_2| + |\mu_3|)$$

In the case of a cusp  
caustic the lens  
configuration,  
 $R_{\text{cusp}} \rightarrow 0$  for a  
smooth lens.

no smoothing

smoothed at  
0.5 kpc



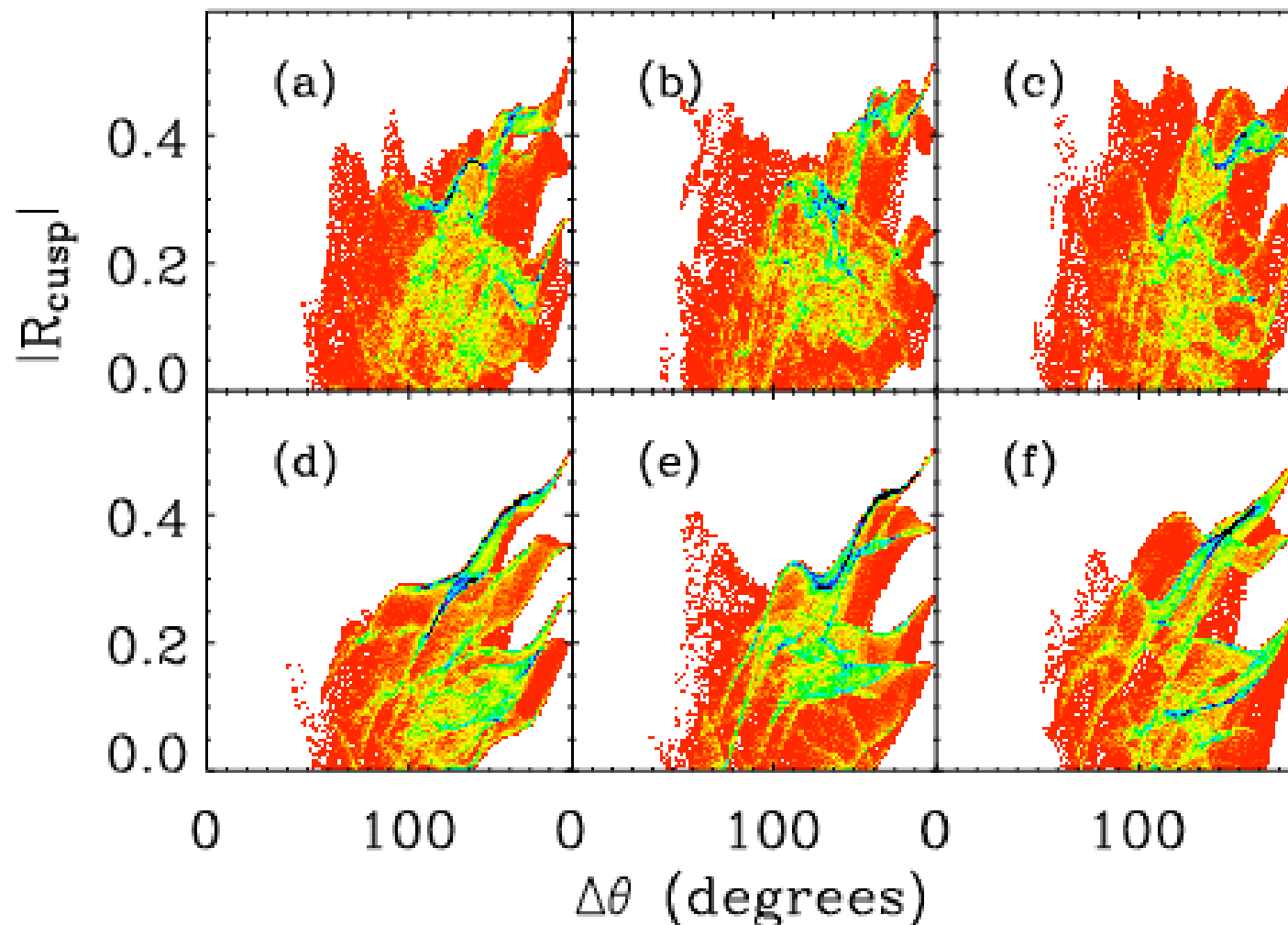
2D histograms of

$R_{\text{cusp}}$

Number per pixel



no smoothing



smoothed at  
0.5 kpc

Opening angle of image triplet

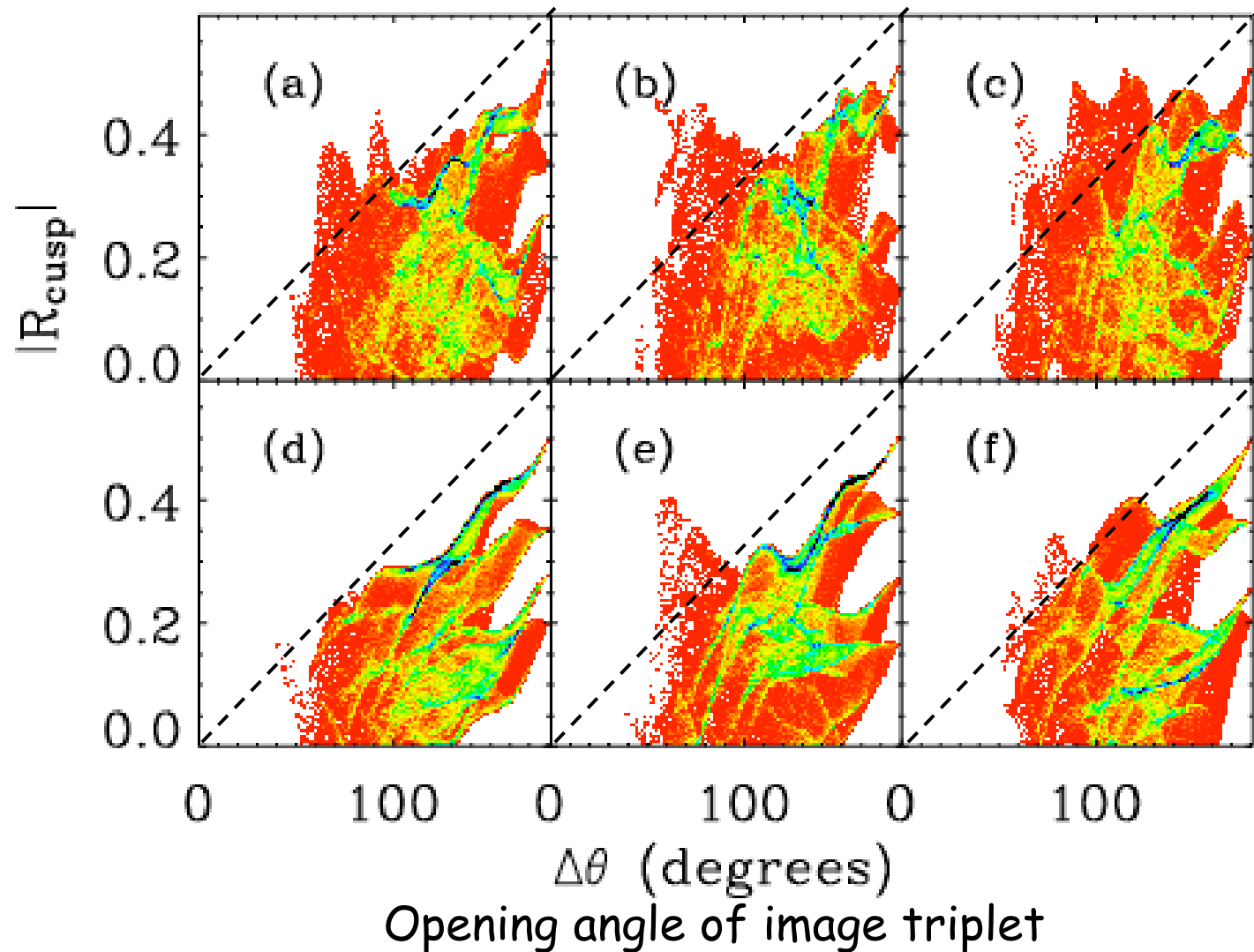
2D histograms of

$R_{\text{cusp}}$

Number per pixel



Set criterion  
for what a  
cusp caustic  
violation.



## 2D histograms of

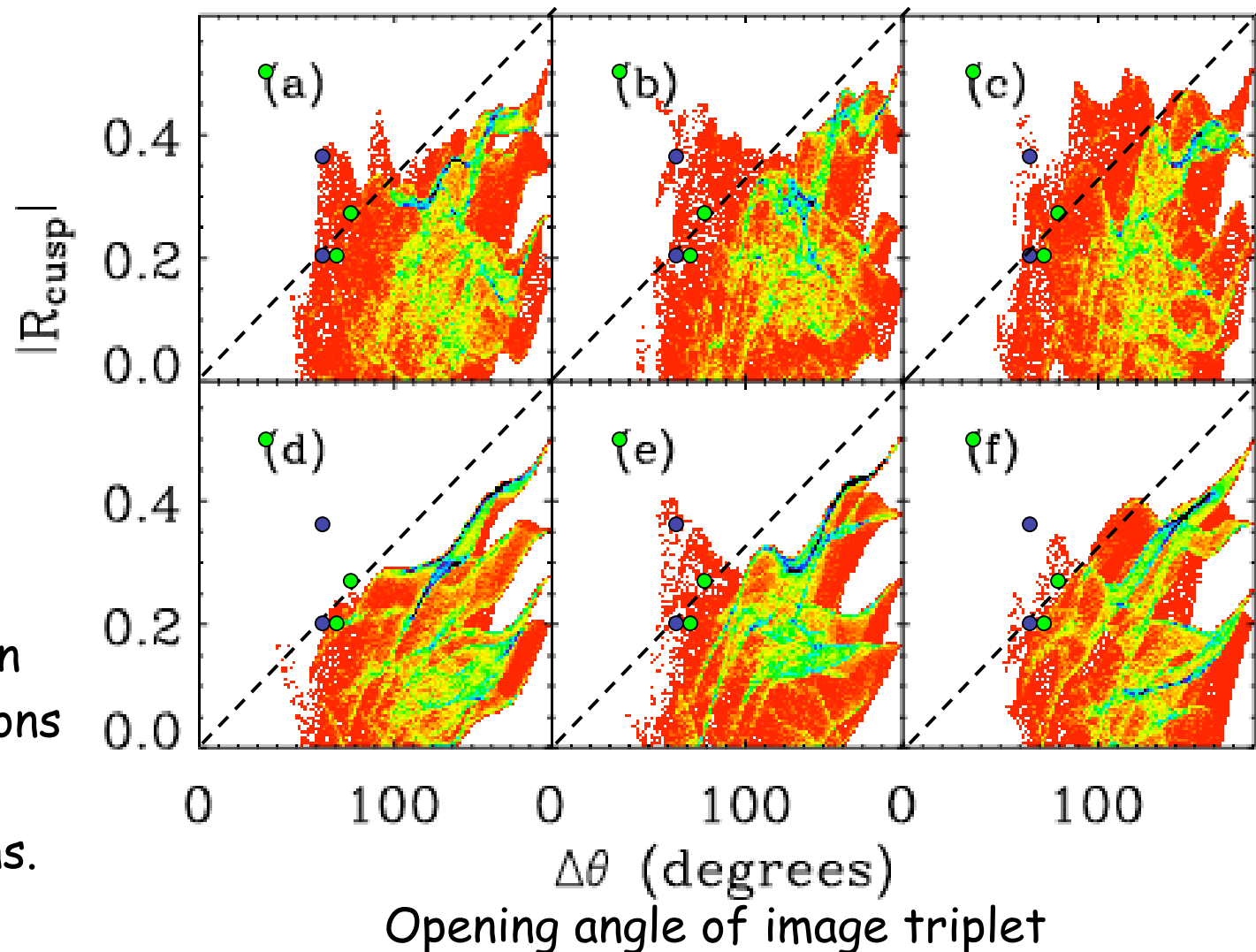
$R_{\text{cusp}}$

Number per pixel

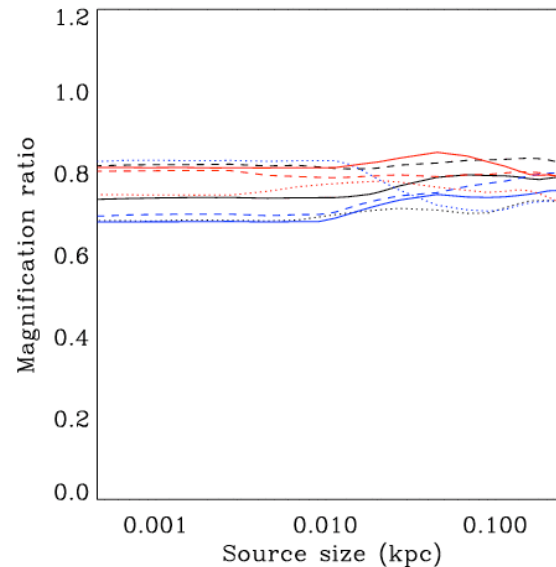
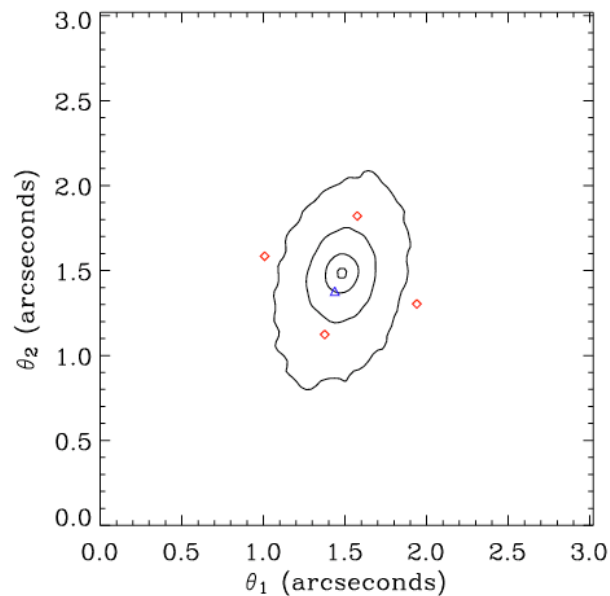


The observed values of  $R_{\text{cusp}}$  are clearly not drawn from the same distribution.

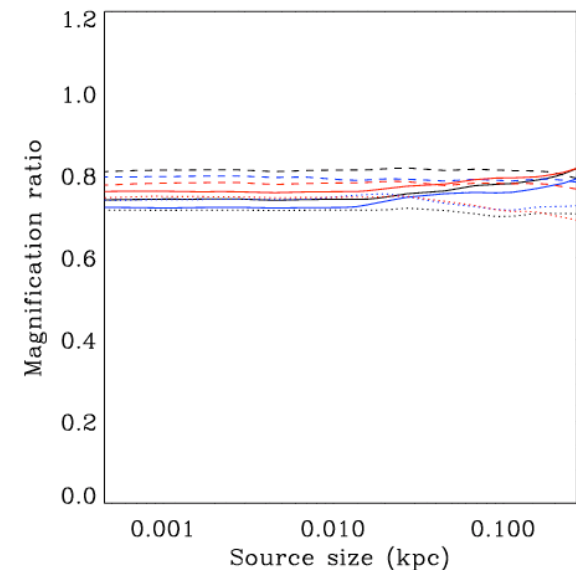
There is not enough substructure in these simulations to account for these violations.



The magnification ratios as a function of source size for a simulated Einstein Cross.

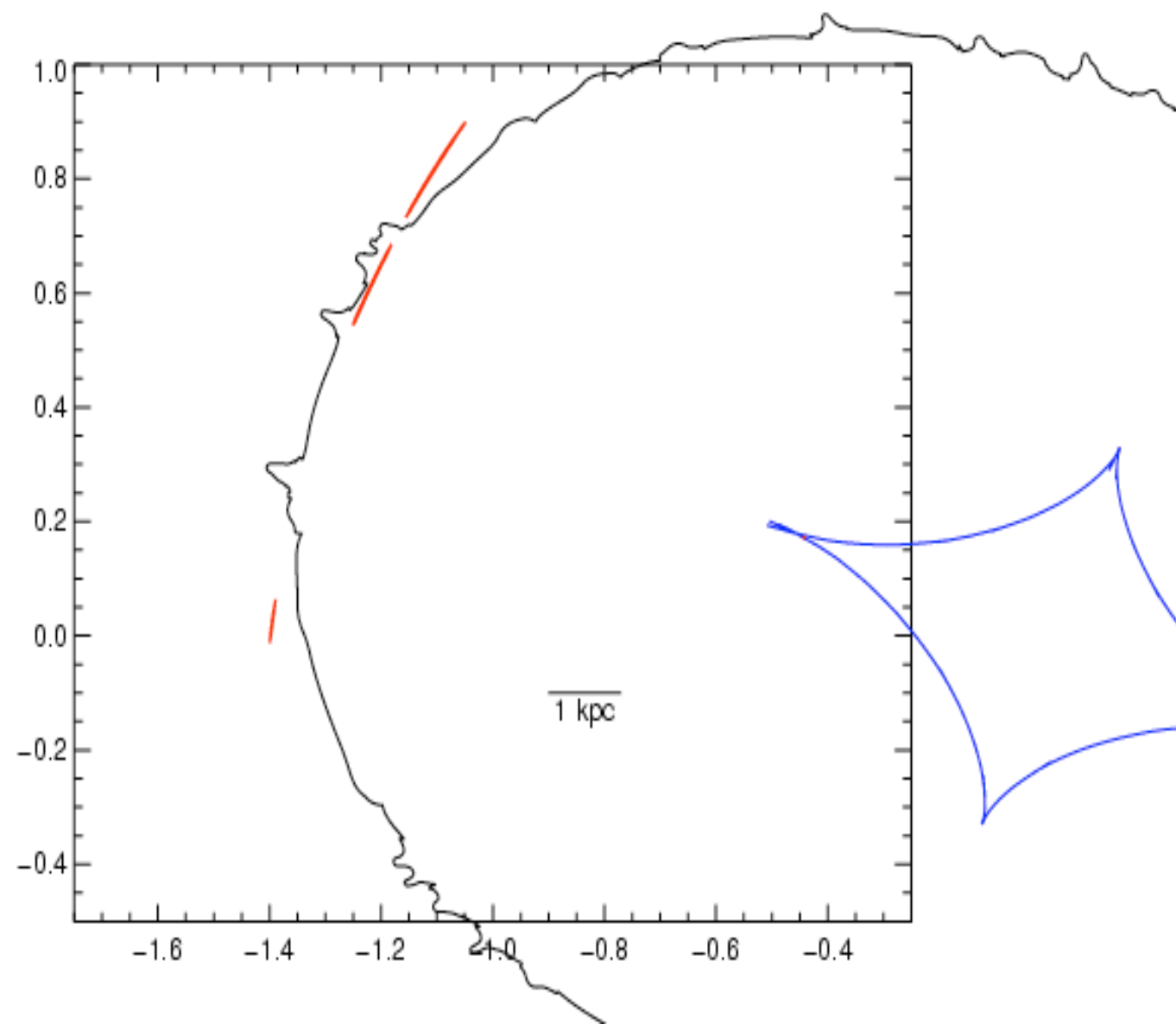


No smoothing



0.5 kpc smoothing.

For an Einstein Cross the configuration the magnification ratios are generically independent of source size up to  $\sim 1$  kpc if there is no substructure below 0.5 kpc.

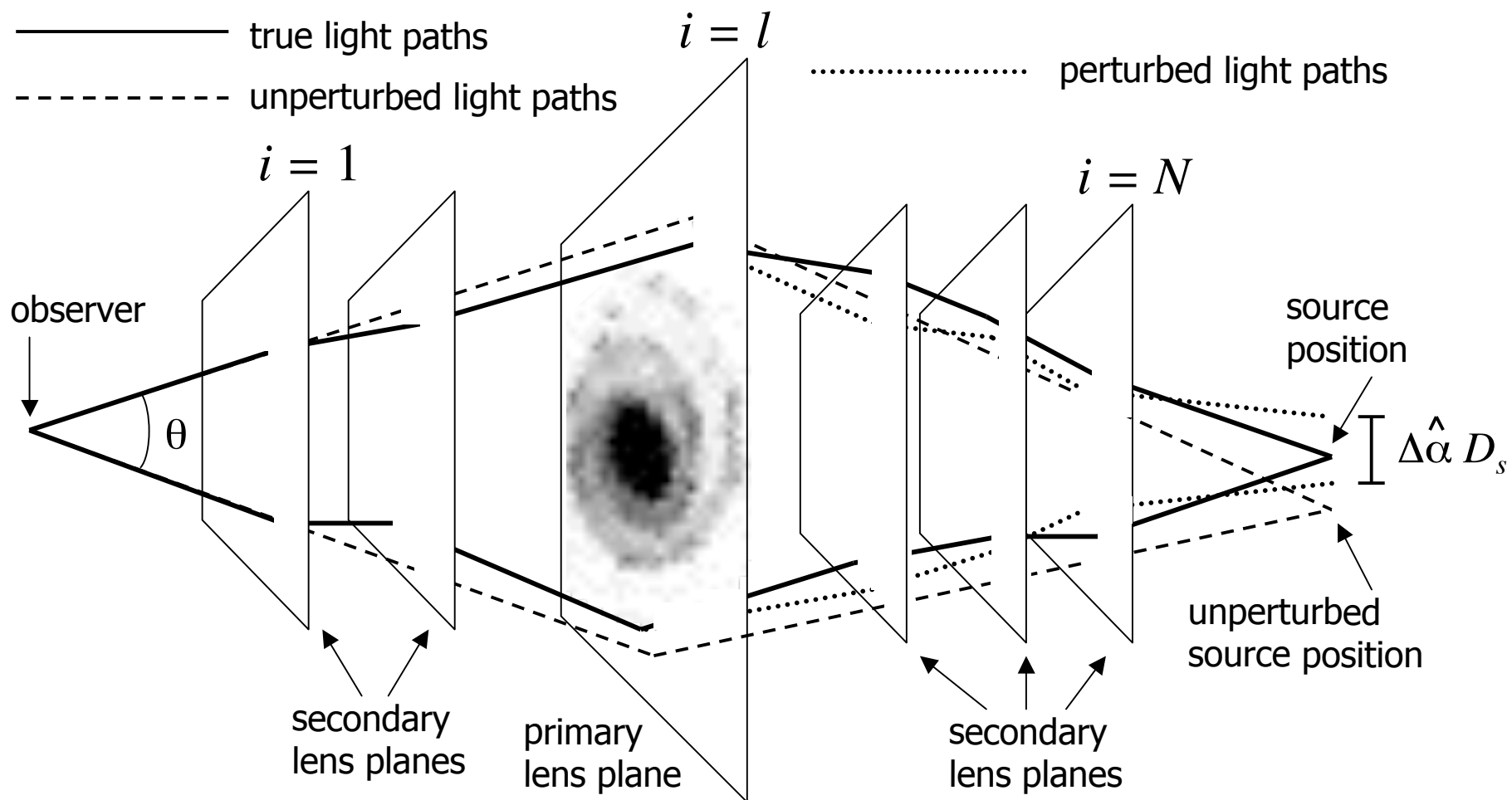


# Intergalactic Small Scale Structure

It was believed that the halos and galaxies outside of the primary lens would not contribute enough to the lensing to account for the observed magnification anomalies.

This conclusion was based on an analytic cross section type calculation (Chen, Kravtsov & Keeton 2003) where only one intergalactic halo is included.

Numerical simulations have shown that in fact intergalactic dark matter halos could account for all of the monochromatic magnification ratio anomalies.



multi-plane  
lens equation

$$\mathbf{D}_s \vec{\beta} = \vec{x}_{N+1}(\vec{\theta}) \quad ; \quad \vec{x}_{k+1} = \mathbf{D}_{k+1} \vec{\theta} - \sum_{i=1}^k \mathbf{D}_{i,k+1} \vec{\alpha}_i(\vec{x}_i)$$

# Importance of Intergalactic Structure

perturbations to the deflection angle

rms deflection  
perturbation in  
milliarcseconds

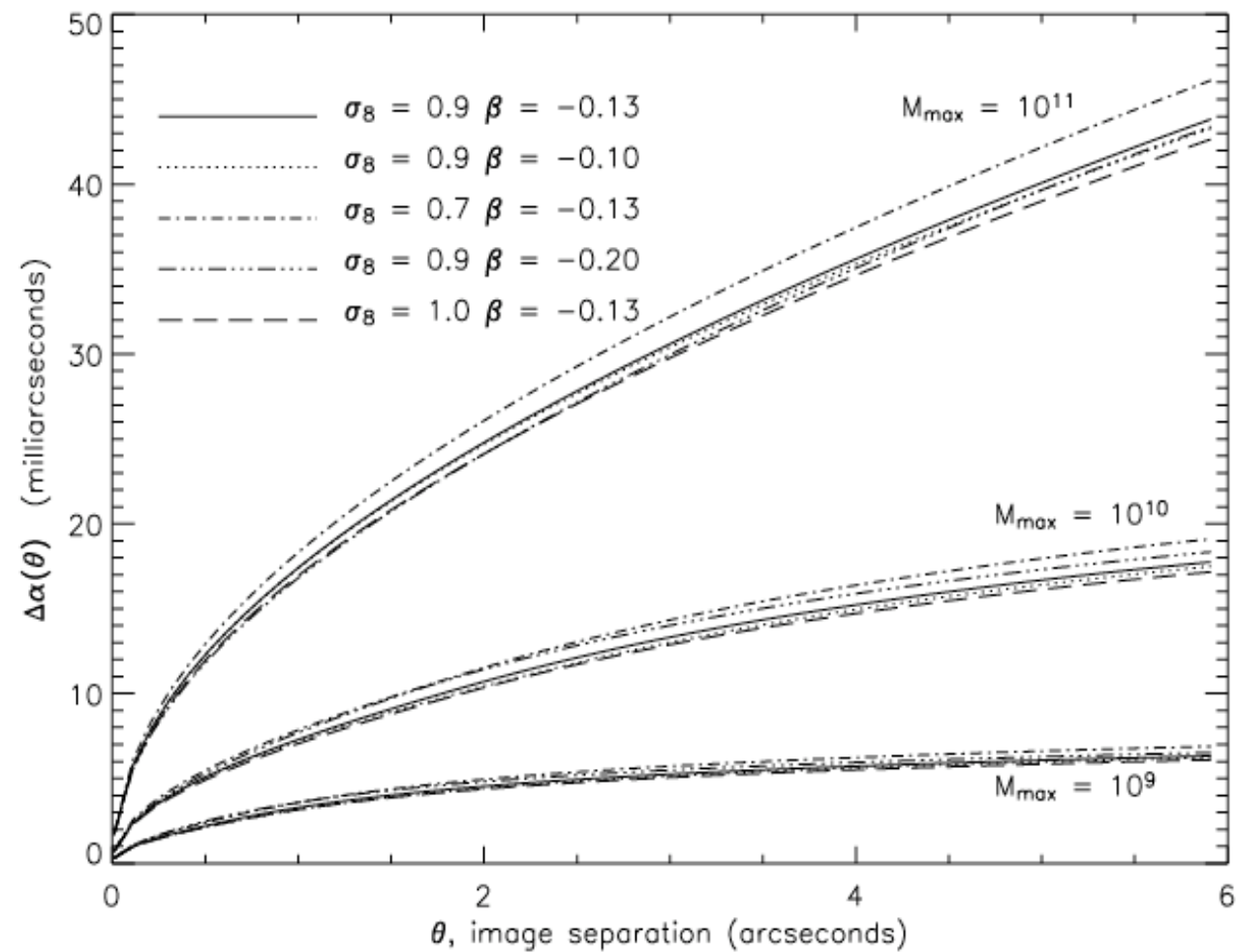
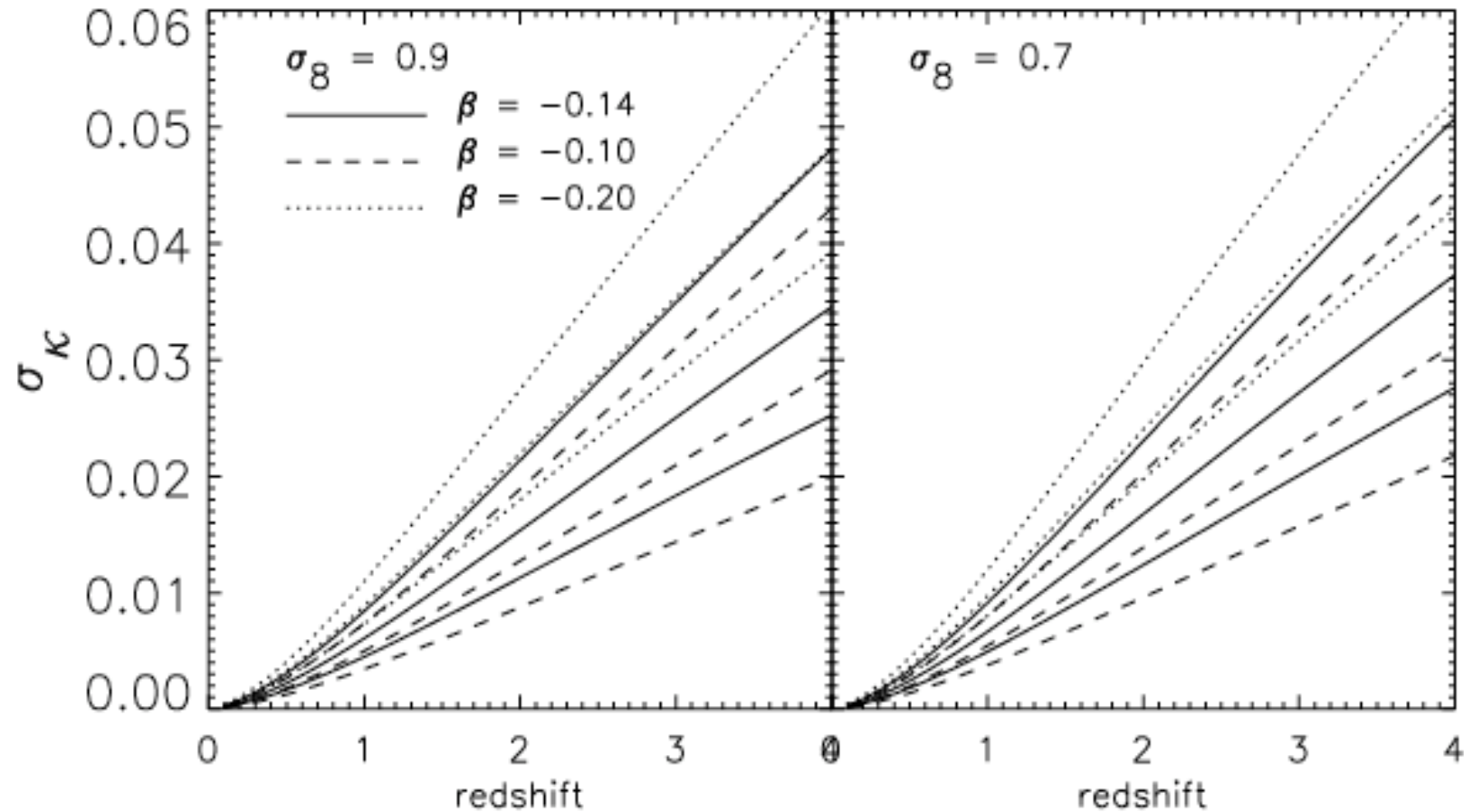


Image separation

## Importance of Intergalactic Structure

standard deviation in the convergence ( $\sim$  surface density)  
of the universe

perturbation to magnification  $\Delta\mu \sim \mu^2 \delta\kappa$

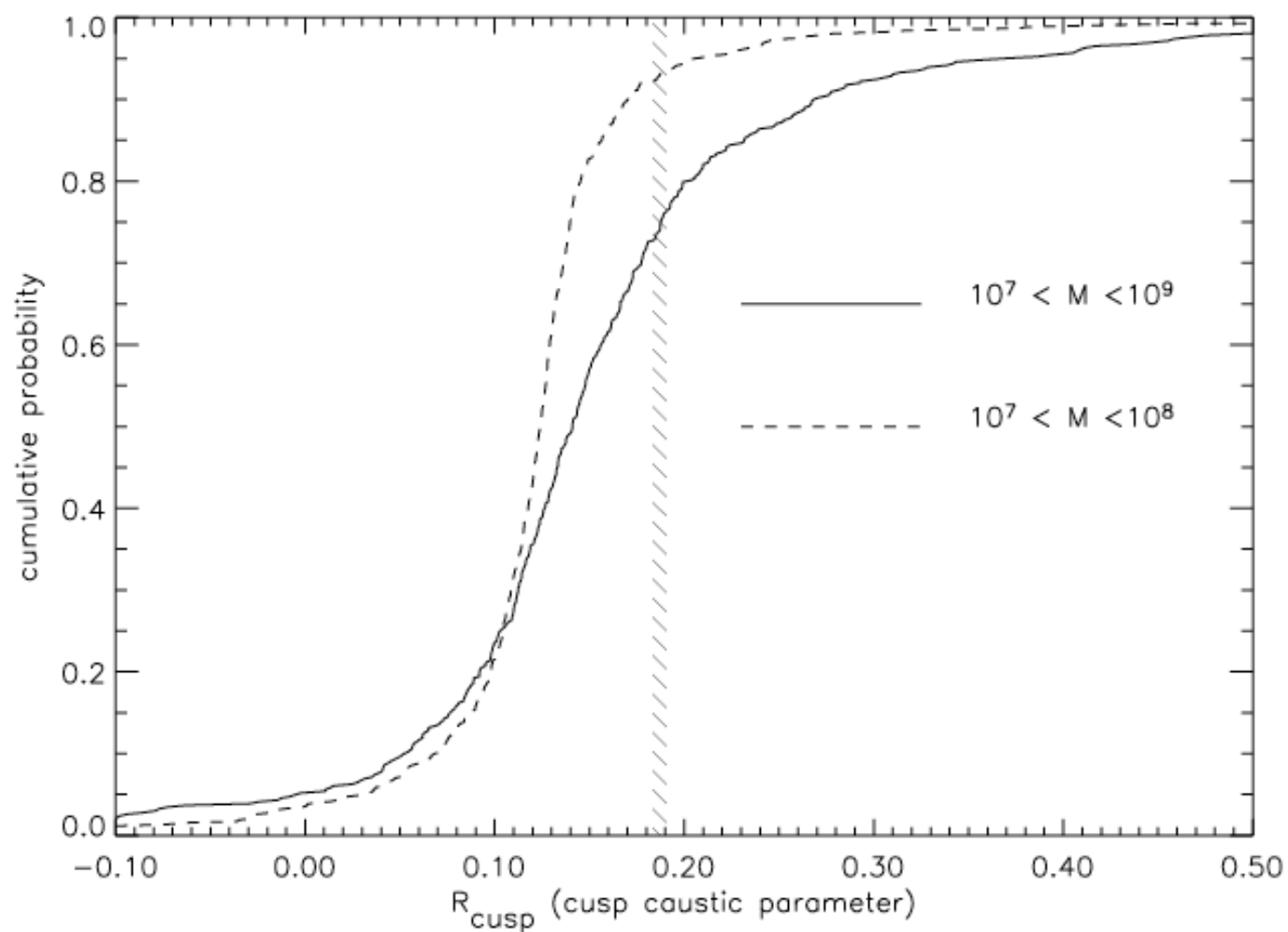
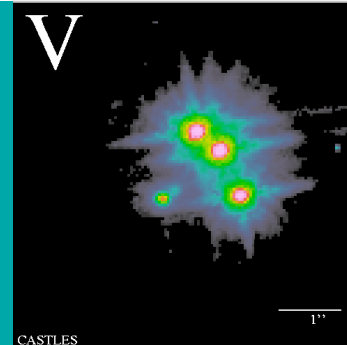


$M_{\max} = 10^9, 10^{10}, 10^{11}$

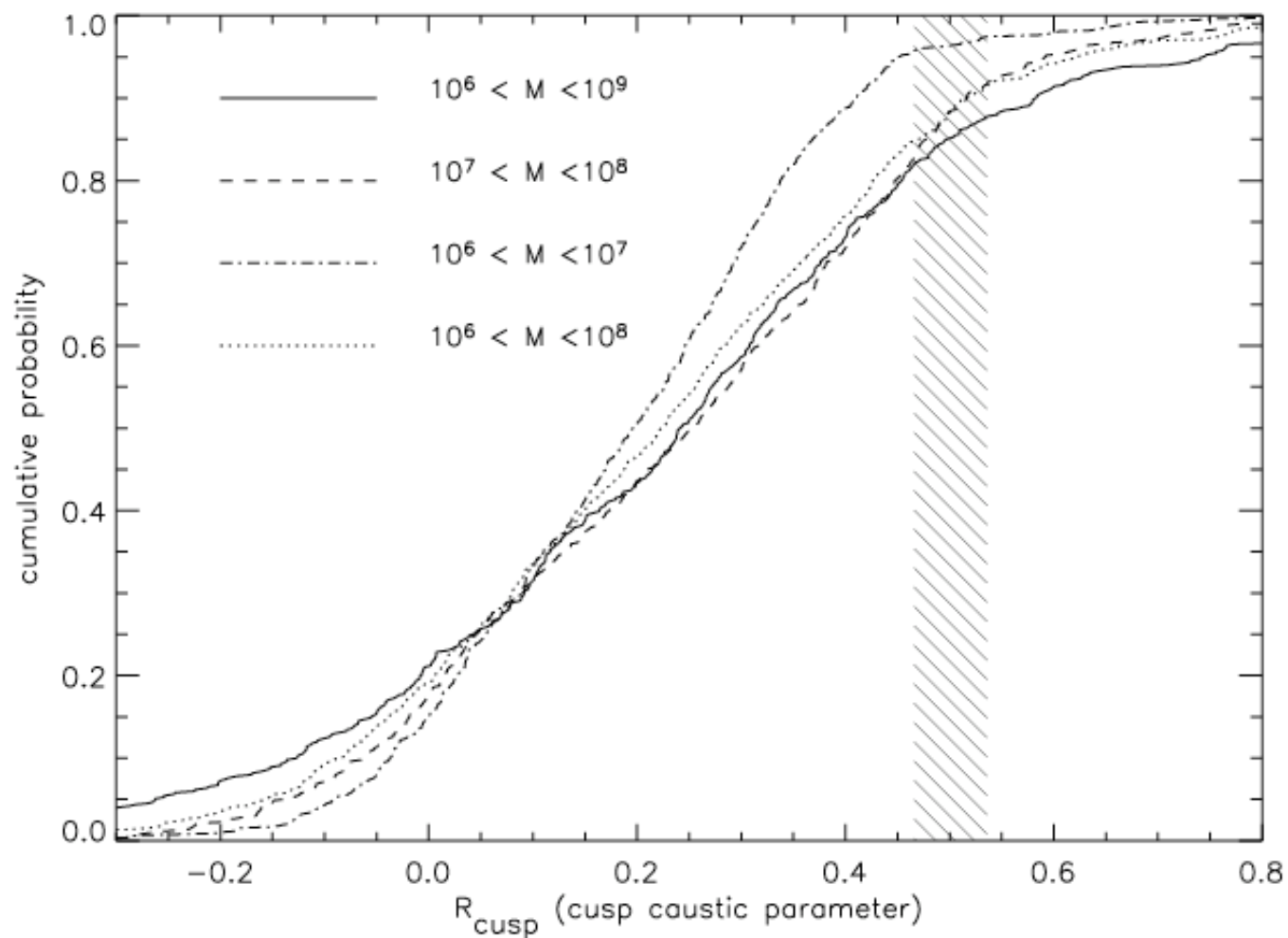
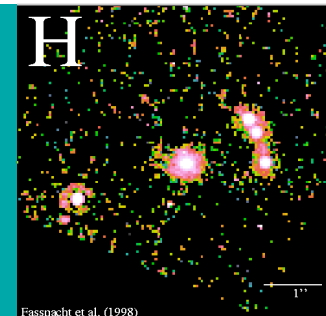
concentration  $\propto M^\beta / (1+z)$

NFW profile

# Cusp caustic lens Q1422+231 with only extragalactic "substructures"



# Cusp caustic lens Q2045+265 with only extragalactic "substructures"



## Conclusions:

The monochromatic flux ratio anomalies indicate that small-scales structure makes up between 0.25% and 1.5% of the surface density at projected radii of  $\sim 10$  kpc. This is believed to be in rough agreement with  $\Lambda$ CDM.

Spectroscopic gravitational lensing indicates that there is more substructure at a lower mass scale than expected, but we need more observations.

There is not enough substructure in the simulations at small projected radii to account for the observed anomalies.

- Could be because of resolution, could be because the subhalos are not concentrated enough or it could be that they really aren't there.

The indicators of flux ratio anomalies are robust in simulated galaxies.

Intergalactic Small-Scale Structure contributes significantly and inevitably to the flux anomalies and can be studied in this way.

- concentration as a function of halo mass
- slope of the primordial power spectrum as a function of scale