Large-Scale Structure: Beyond the Power Spectrum

Galaxy Occupation Numbers from Bispectrum and Trispectrum

Sefusatti and Scoccimarro, PRD 71, 063001, astro-ph/0412626

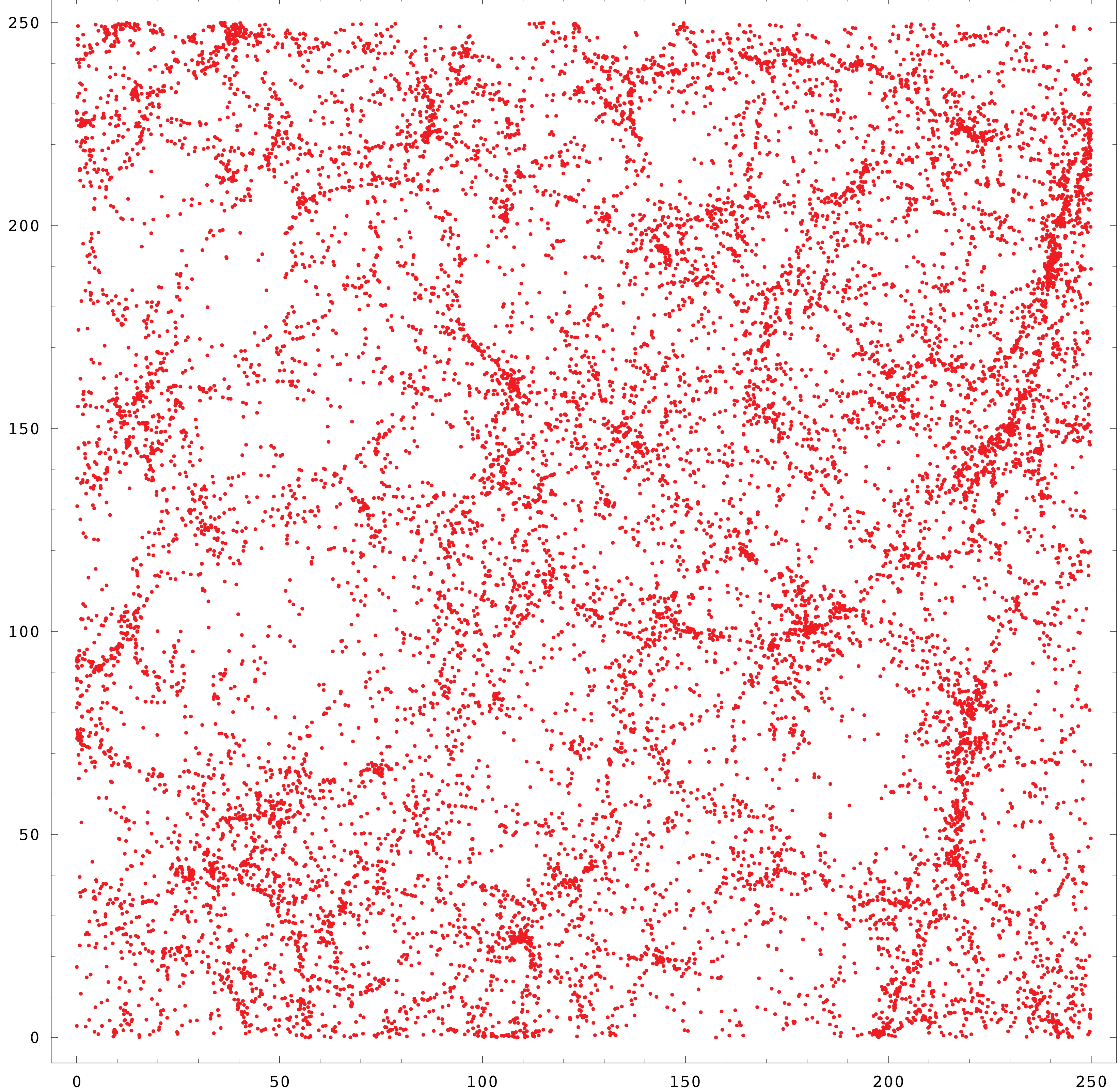
Cosmological Parameters from Joint Analysis of Power Spectrum and Bispectrum

Sefusatti, Crocce, Pueblas and Scoccimarro, in preparation (2005)

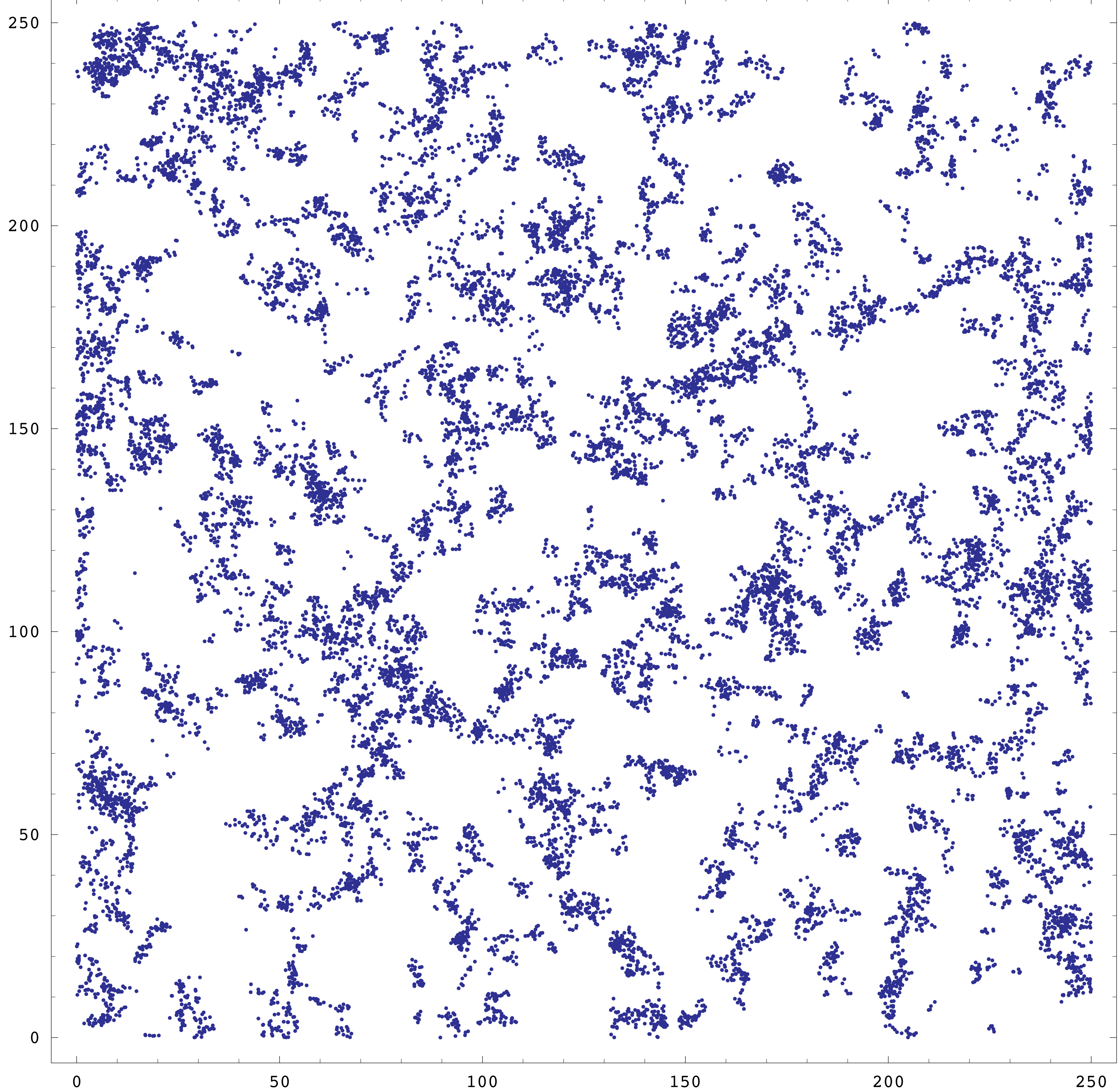
A New Approach to Gravitational Clustering

Renormalized Cosmological Perturbation Theory

Crocce and Scoccimarro, in preparation (2005)

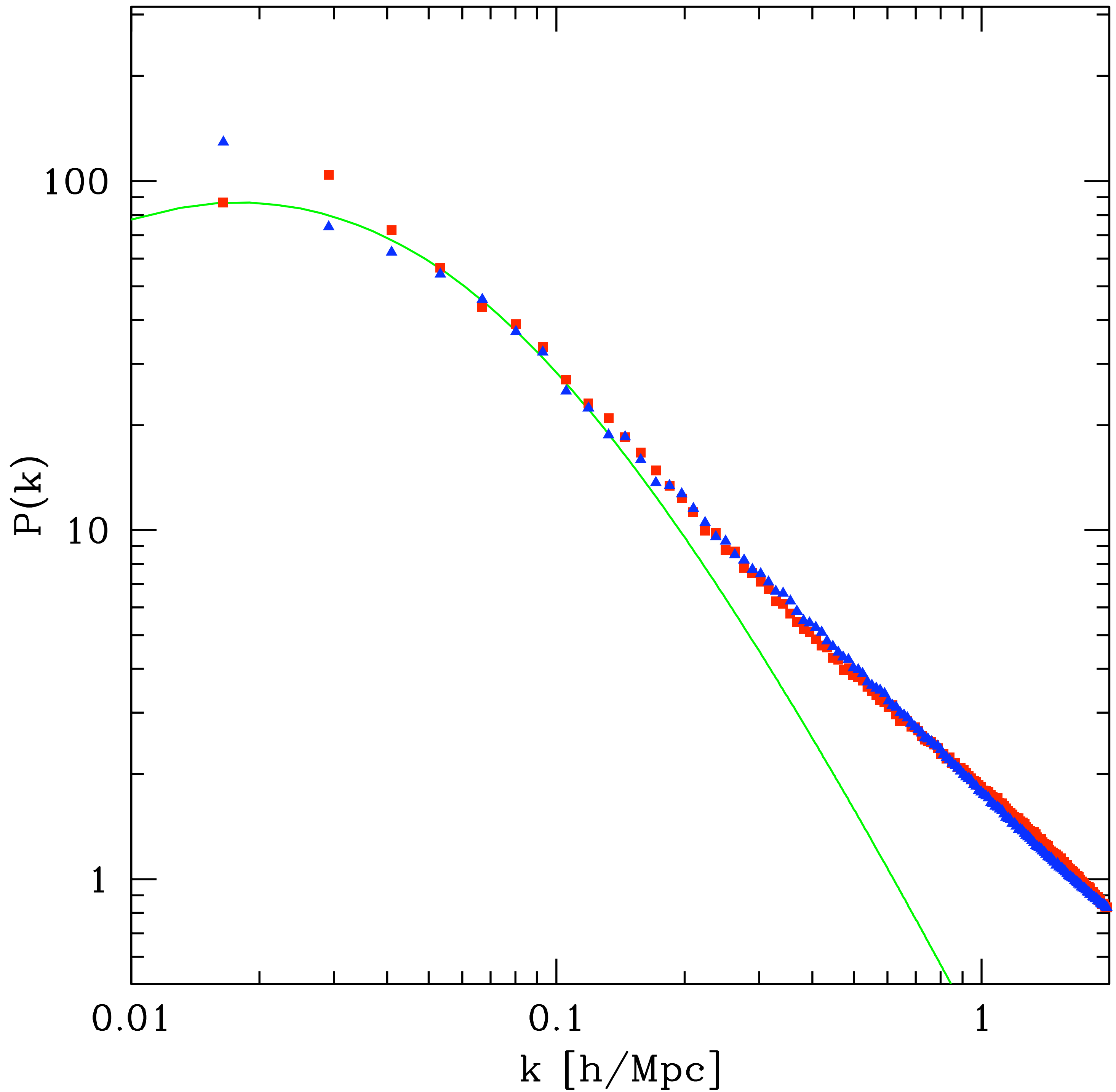


250
200
150
100
50
0
0
50
100
150
200
250



250
200
150
100
50
0
0
50
100
150
200
250

The two distributions have about the same Power Spectrum!



Galaxy Bias and HOD's from Large-Scale Correlations

At large scales the relationship between galaxies and dark matter can be approximated by,

$$\delta_g \approx b_1 \delta + \frac{b_2}{2!}\delta^2 + \frac{b_3}{3!}\delta^3$$

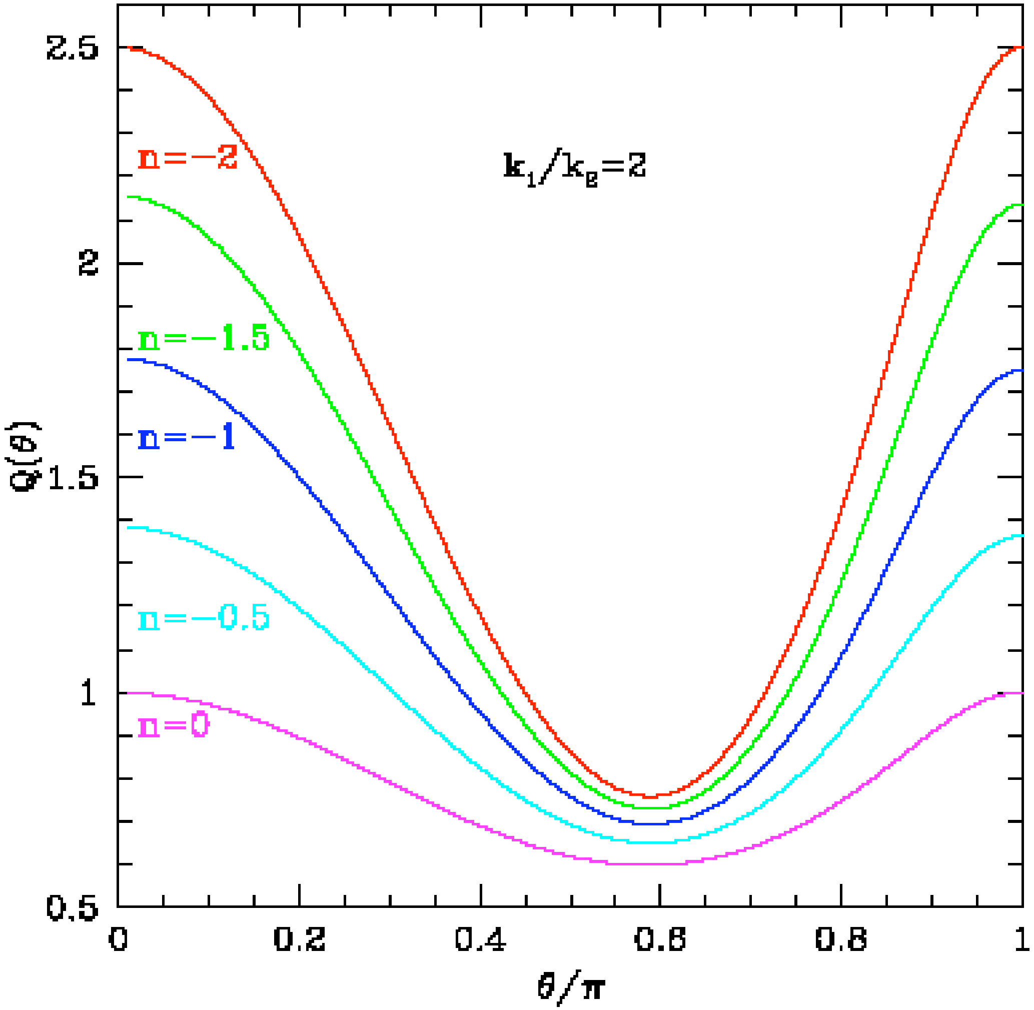
One can use higher-order correlation functions to determine the bias parameters. The three-point function (bispectrum) can be estimated by,

$$B_{123} \propto \int d^3x \; \delta_{k_1}(\mathbf{x})\delta_{k_2}(\mathbf{x})\delta_{k_3}(\mathbf{x})$$

When suitably normalized, it depends only on galaxy bias,

$$Q_B = \frac{B_{123}}{P_1 P_2 + P_2 P_3 + P_3 P_1}$$

and on spectral index,



$$Q_B^g = \frac{1}{b_1} Q_B + \frac{b_2}{b_1^2}$$

Similarly, for the four-point function (trispectrum), one can calculate an analogous quantity,

$$T_{1234} \propto \int d^3x \ \delta_{k_1}(\mathbf{x}) \delta_{k_2}(\mathbf{x}) \delta_{k_3}(\mathbf{x}) \delta_{k_4}(\mathbf{x})$$

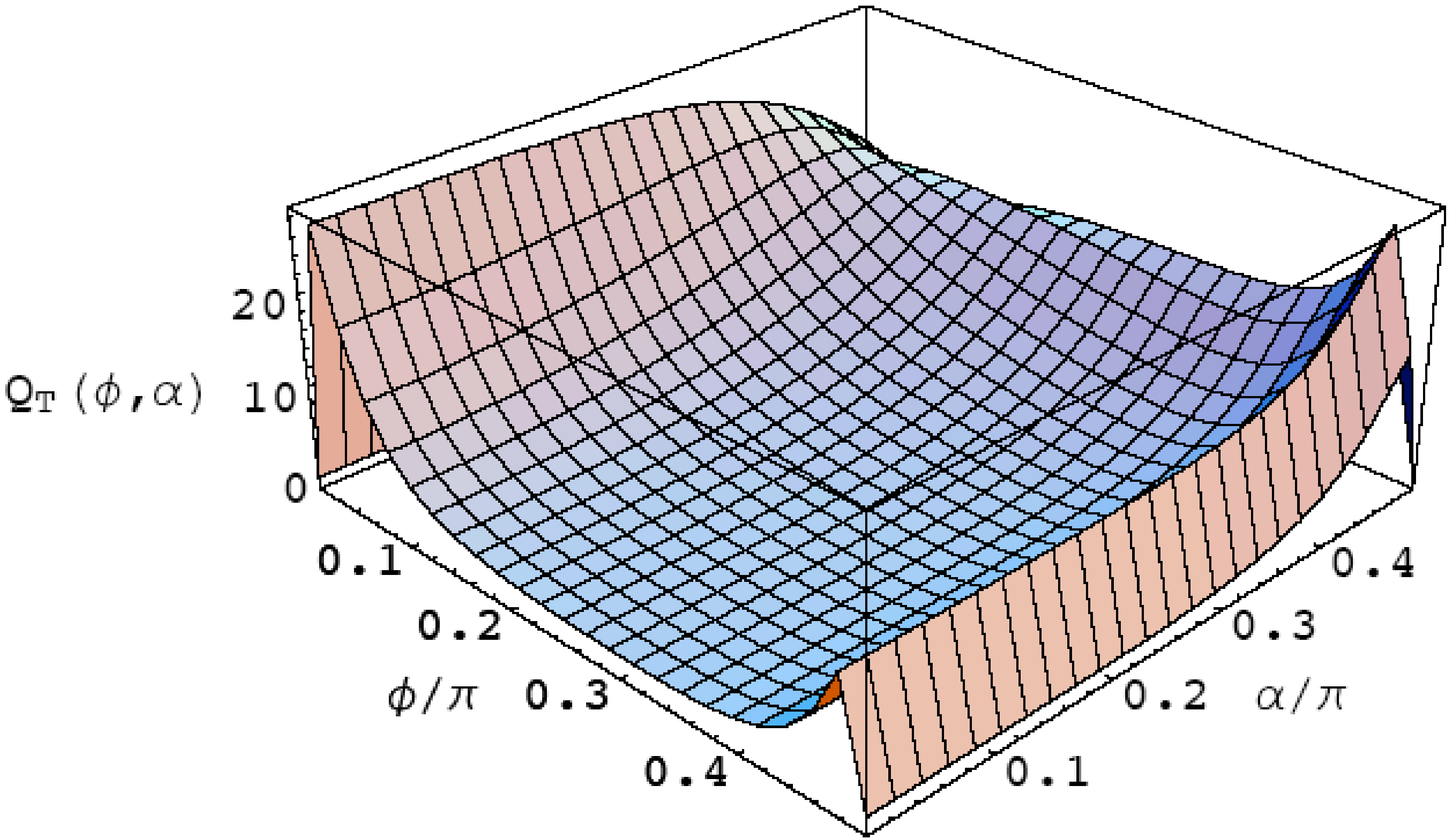
This is actually an average over the full trispectrum, which depends on 6 variables, but it is easy to calculate. The dependence on bias parameters is similar to the bispectrum case,

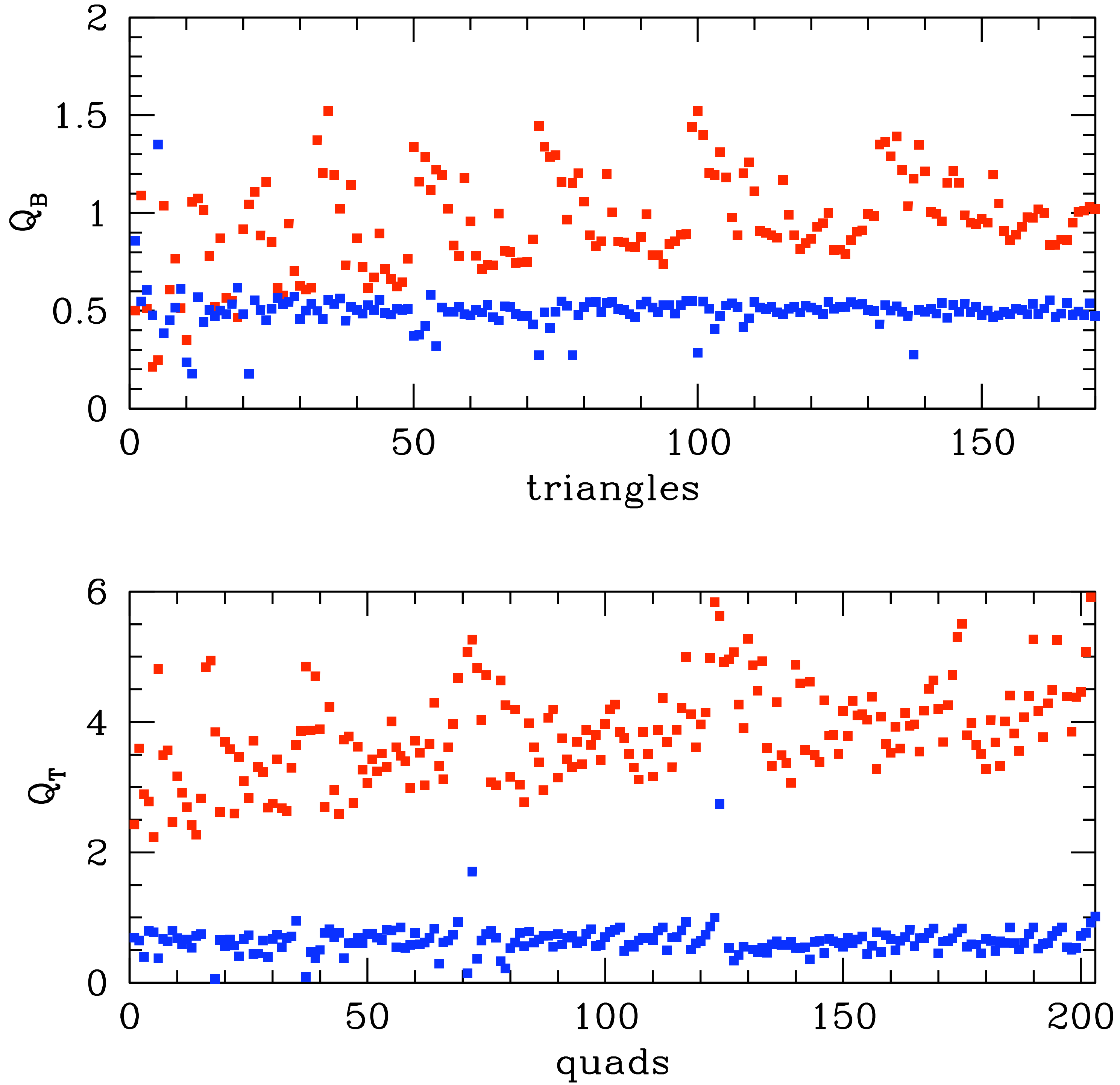
$$Q_T = \frac{T_{1234}}{P_1 P_2 P_3 + \mathrm{cyc}}$$

$$Q_T^g = \frac{1}{b_1^2} Q_T + \frac{b_2}{2b_1^3} Q_2 + \frac{b_2^2}{4b_1^4} Q_3 + \frac{b_3}{b_1^3}$$

$$k_1 = k_2 = k_3 = k_4 = 0.25\ h/\mathrm{Mpc}$$

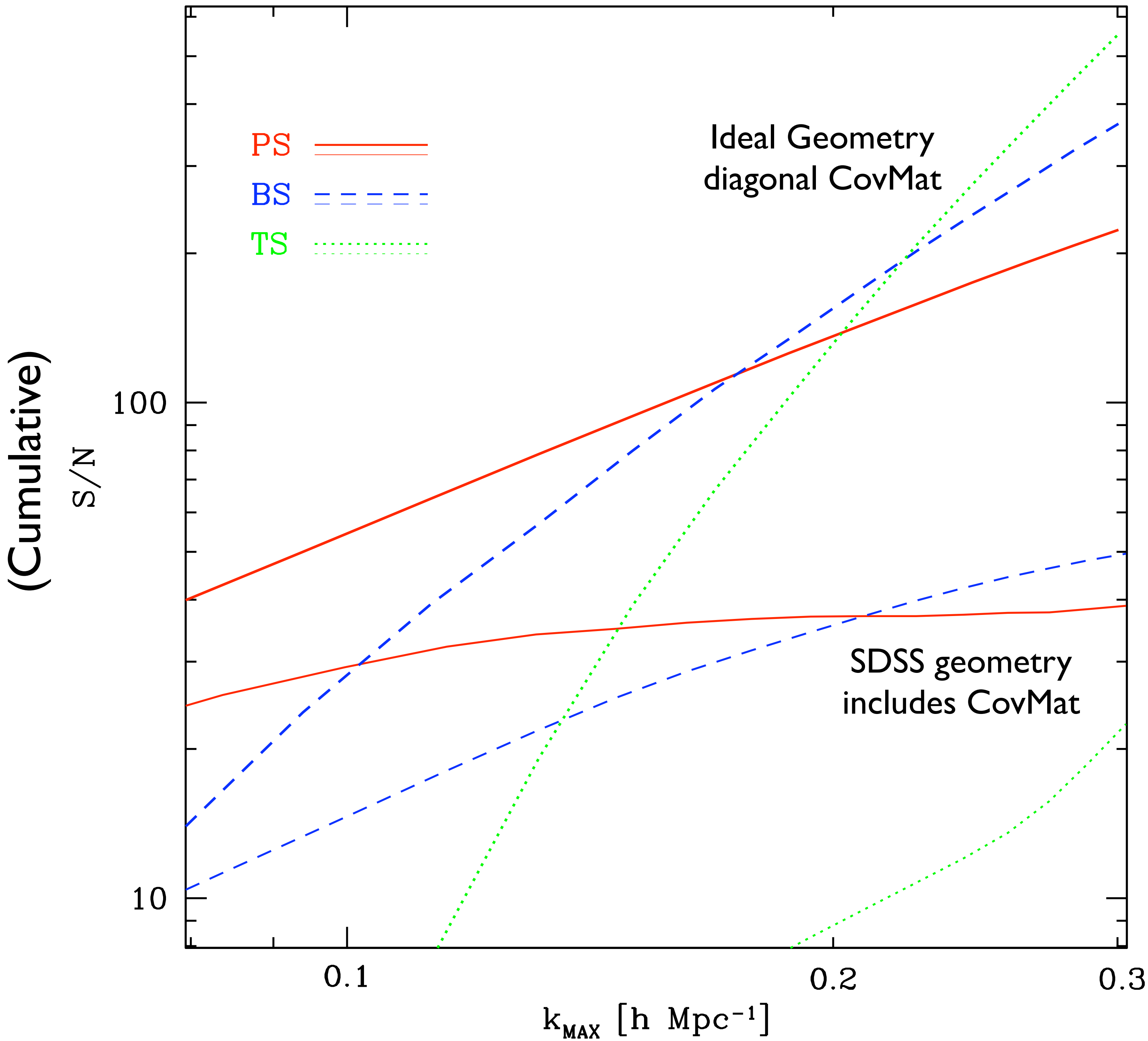
$$\mathbf{k}_1 \cdot \mathbf{k}_2 = \mathbf{k}_3 \cdot \mathbf{k}_4 = \cos(2\phi)$$



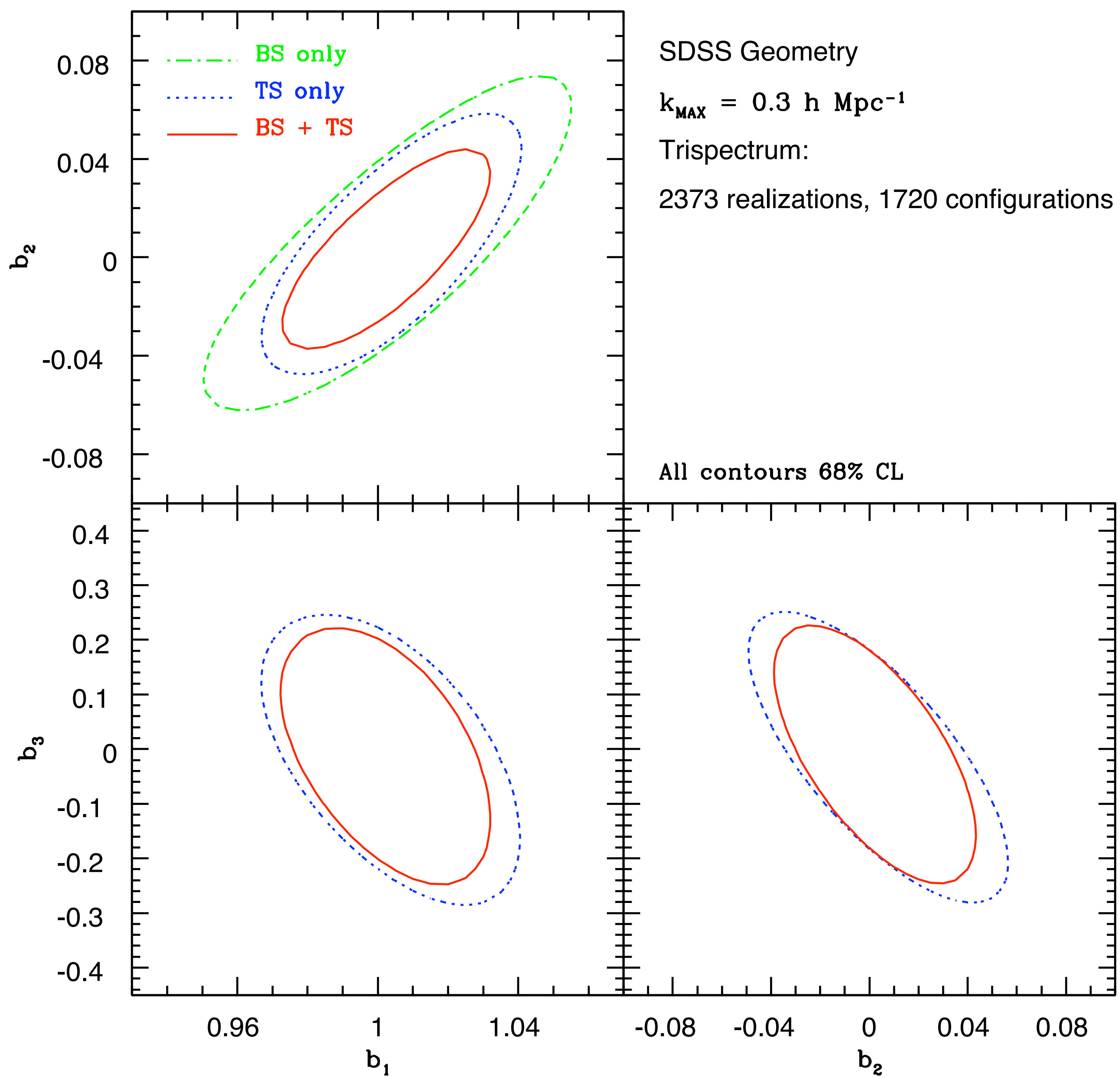


Q_B
triangles
0
0.5
1
1.5
2
0
50
100
150
Q_T
quads
0
2
4
6
0
50
100
150
200

Signal to Noise: Bispectrum and Trispectrum vs. Power Spectrum



How well can we measure the bias parameters?



Constraints expected from bispectrum alone,

$$b_1 = 1.000^{+0.038}_{-0.030} \qquad b_2 = 0.000^{+0.047}_{-0.042}$$

Constraints from bispectrum plus trispectrum,

$$b_1 = 1.000^{+0.023}_{-0.017} \qquad b_2 = 0.000^{+0.030}_{-0.025} \qquad b_3 = 0.000^{+0.14}_{-0.15}$$

In addition to significantly improving error bars, and constraining and additional parameter, one can do analysis of trispectrum alone and check for consistency with the bispectrum analysis.

The constraints on linear bias translate into constraints on sigma8 and Omega matter when combined with the measurement of the power spectrum. Also measurements of higher-order statistics improve constraints on spectral index.

What do we learn from the constraints on the nonlinear bias parameters?

In the halo model, the mean of the HOD determines the mean galaxy density, and the large-scale bias parameters,

$$\bar{n}_g = \int n(m) \, \langle \, N_{\rm gal}(m) \, \rangle \, {\rm d}m.$$

$$b_1(k) = \frac{1}{\bar{n}_g} \int n(m) \langle N_{\rm gal}(m) \rangle u_m(k) b_1(m) dm \approx \frac{1}{\bar{n}_g} \int n(m) dm \, b_1(m) \langle N_{\rm gal}(m) \rangle$$

in general, the large-scale bias parameters are

$$b_i \approx \frac{1}{\bar{n}_g} \int n(m) {\rm d}m \; b_i(m) \;\; \langle \, N_{\rm gal}(m) \, \rangle$$

From the point of view of large-scale clustering, the important thing is that the halo model gives a precise relation between bias parameters and mean occupation of galaxies as a function of halo mass. Since the other ingredients are well-known from theory, one can hope to constrain <Ngal(m)> by measuring large-scale clustering alone.

See Sefusatti's poster

Cosmological Parameters from
Joint Analysis of Power Spectrum and Bispectrum

- Measuring the galaxy bias allows us to break the degeneracy between bias and sigma8 present in the power spectrum

- The bispectrum measures mode-mode coupling and its strength is sensitive to the shape of the power spectrum, then we can hope to improve constraints on e.g. the primordial spectral index.

Let's consider a joint analysis of P+B and look at constraints on bias, sigma8 and ns, for fixed Omega_b, Omega_m and h in the case of the final SDSS geometry.

Marginalizing over the 3 remaining parameters in turn,

2σ error bars:

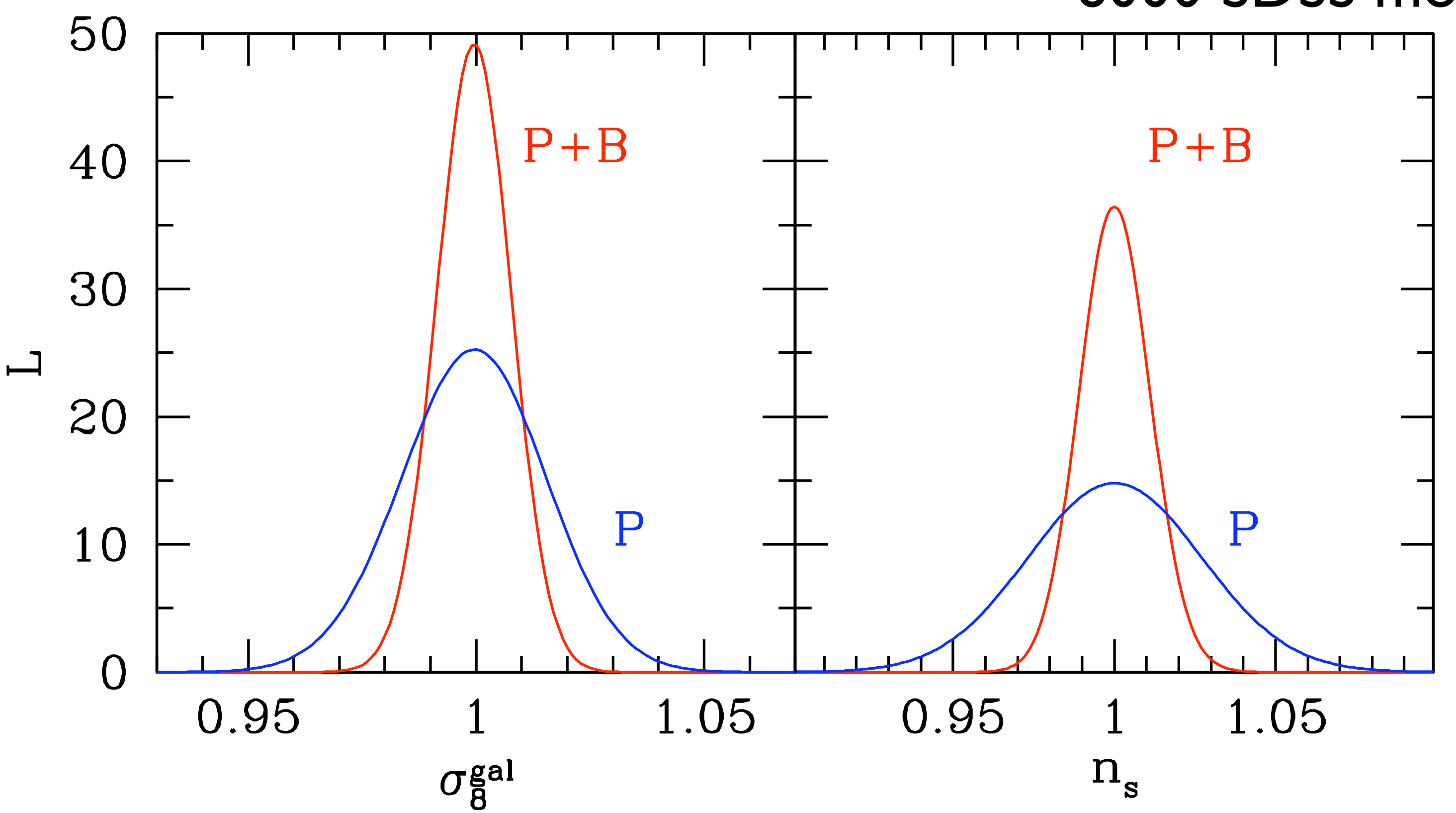
$\Delta\sigma_8^{gal} = 0.03$ (P), 0.015 (P+B)

$\Delta n_s = 0.05$ (P), 0.02 (P+B)

$\Delta b_1 = 0.06$ (P+B)

$\Delta b_2 = 0.09$ (P+B)

Includes Full P+B covariance matrix from 6000 SDSS mocks catalogs



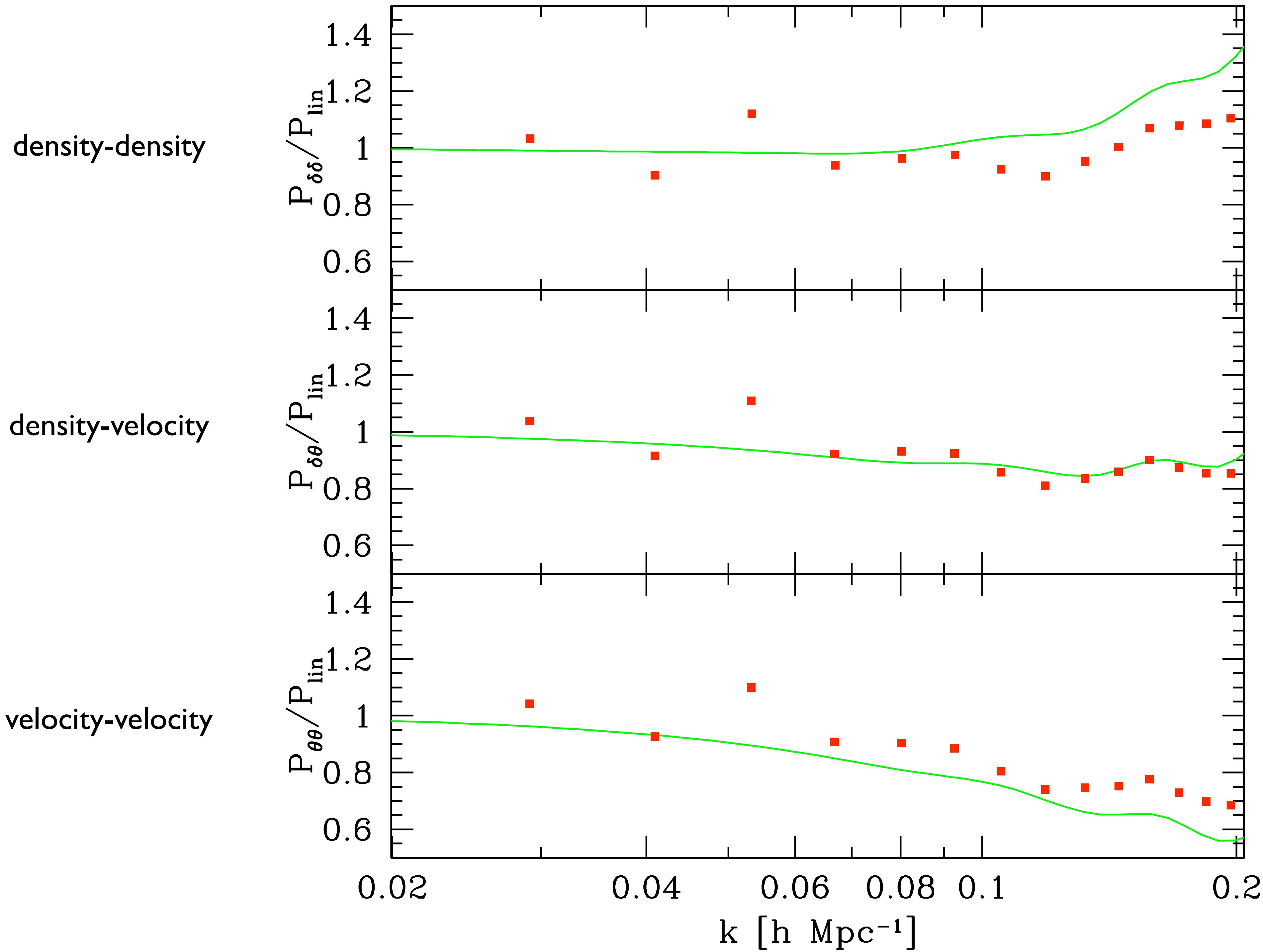
A New Approach to Gravitational Clustering

- In standard perturbation theory (PT), one expands in the amplitude of density perturbations.

- This is well justified when looking for asymptotic behavior at large-scales, where fluctuations become small.

- How about nonlinear corrections to these results? Once these become important, one may need to sum up all orders in PT to obtain meaningful results.

For the power spectrum, taking into account the first corrections to the linear spectrum works well for steep spectra but not so well for CDM at z=0,



density-density
density-velocity
velocity-velocity
$P_{\delta\delta}/P_{lin}$
$P_{\delta\theta}/P_{lin}$
$P_{\theta\theta}/P_{lin}$
1.4
1.2
1
0.8
0.6
0.02
0.04
0.06
0.08
0.1
0.2
k [h Mpc⁻¹]

Renormalized Perturbation Theory (RPT)

- In RPT, one looks at the infinite series of diagrams in PT and sees how they organize themselves into a few characteristic physical quantities, the most important of which is the propagator

Final density / velocity div.

$$G_{ab}(k,\eta)\ \delta_{\rm D}(\mathbf{k}-\mathbf{k}') \equiv \left\langle \frac{\delta\Psi_a(\mathbf{k},\eta)}{\delta\phi_b(\mathbf{k}')} \right\rangle_c$$

Initial Conditions

where

$$\Psi_a(\mathbf{k},\eta) \equiv \Big(\delta(\mathbf{k},\eta),\ -\theta(\mathbf{k},\eta)/\mathcal{H}\Big), \qquad \eta \equiv \ln a(\tau).$$

The propagator is a measure of the memory of initial conditions, and reduces to the usual growth factors in linear theory,

$$g_{ab}(\eta) = \frac{\mathrm{e}^{\eta}}{5}\begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix} - \frac{\mathrm{e}^{-3\eta/2}}{5}\begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix},$$

The rest of the diagrams can be thought of as the effects of mode-coupling.

To illustrate these ideas, let's look at the Zel'dovich approximation nonlinear power spectrum, in this dynamics the exact (non-perturbative) result is known,

$$P(k) = \int \frac{d^3r}{(2\pi)^3}\ \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}}\ \Big[\mathrm{e}^{-[k^2\sigma_v^2 - I(\mathbf{k},\mathbf{r})]} - 1\Big],$$

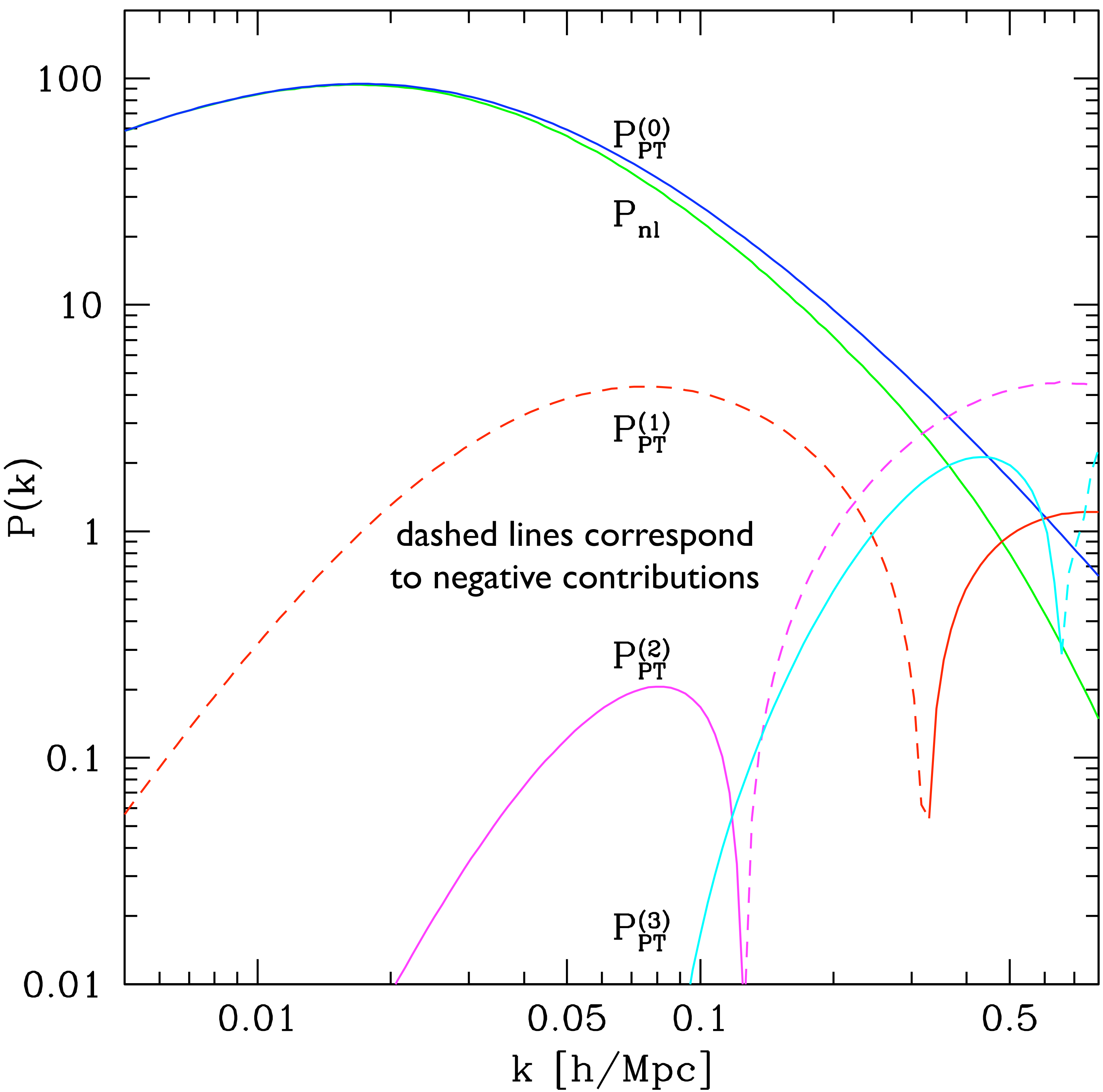
where,

$$I(\mathbf{k},\mathbf{r}) \equiv \int d^3q\, \frac{(\mathbf{k}\cdot\mathbf{q})^2}{q^2}\ \cos(\mathbf{q}\cdot\mathbf{r}) P_L(q), \qquad \sigma_v^2 = \frac{I(k,0)}{k^2}$$

Let's now expand this result in powers of the linear spectrum, to recover the standard PT expansion,

$$P(k) = \int \frac{d^3r}{(2\pi)^3}\ \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}}\ \sum_{n=1}^{\infty} \frac{(-1)^n}{n!}[k^2\sigma_v^2 - I(\mathbf{k},\mathbf{r})]^n \equiv \sum_{\ell=0}^{\infty}\ P_{\rm PT}^{(\ell)}(k),$$

ZA Nonlinear Power Spectrum in standard PT expansion



Different orders become comparable in the nonlinear regime

Now, it can be shown that in RPT the resummation of the propagator leads to the first exponential factor in the exact result

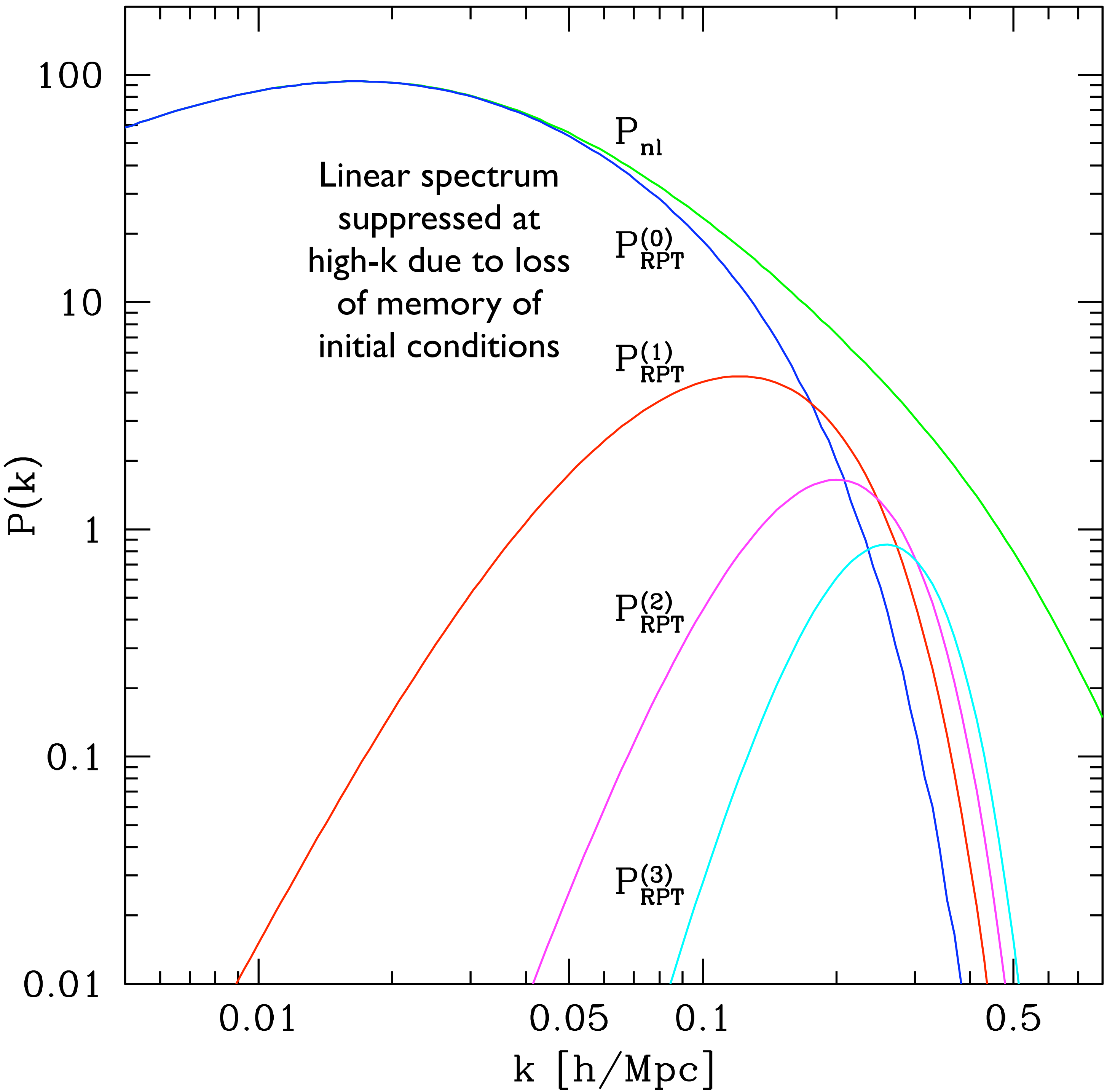
$$P(k) = \int \frac{d^3r}{(2\pi)^3}\ \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}}\ \left[\mathrm{e}^{-[k^2\sigma_v^2 - I(\mathbf{k},\mathbf{r})]} - 1\right],$$

And the RPT expansion correspond to expanding only the mode coupling terms,

$$P(k) = \int \frac{d^3r}{(2\pi)^3}\ \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}}\ \mathrm{e}^{-k^2\sigma_v^2} \sum_{n=1}^{\infty} \frac{[I(\mathbf{k},\mathbf{r})]^n}{n!} \equiv \sum_{\ell=0}^{\infty} P_{\mathrm{RPT}}^{(\ell)}(k),$$

Here the n=1 term gives the linear spectrum times the propagator resummation factor (no mode-coupling, unlike the rest of the terms)

ZA Nonlinear Power Spectrum in RPT expansion



Different orders dominate only in a narrow range of scales and are always positive due to being mode-coupling terms

In the exact dynamics the resummation of the propagator can be calculated (much more difficult than in ZA but can be done), see Crocce's poster

The real test of this method will be the calculation of the nonlinear power spectrum, which we are doing now. Stay tuned!