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Conference on

VORTEX RINGS AND FILAMENTS IN CLASSICAL AND QUANTUM SYSTEMS

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Dynamics of Vortex Rings in a Rotating Fluid

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Dynamics of a classical vortex ring in a rotating environment

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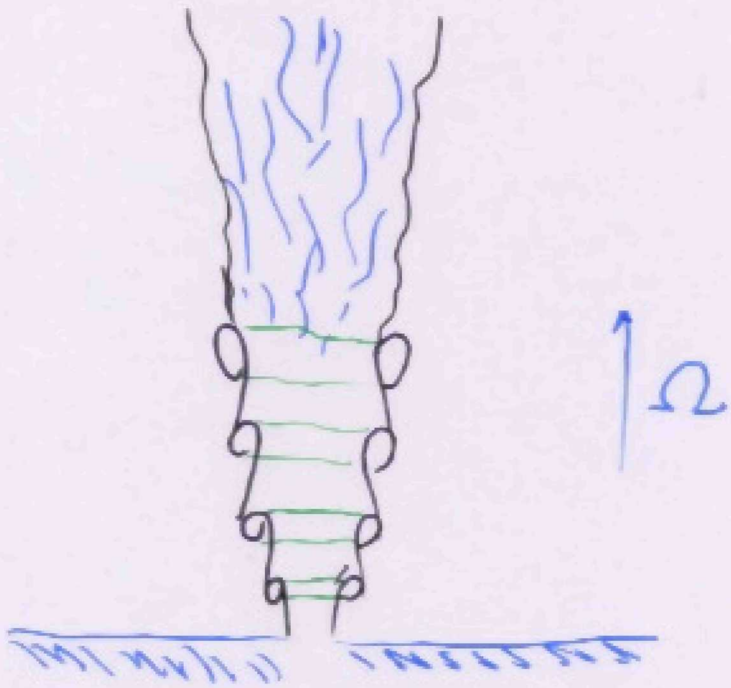
Motivation

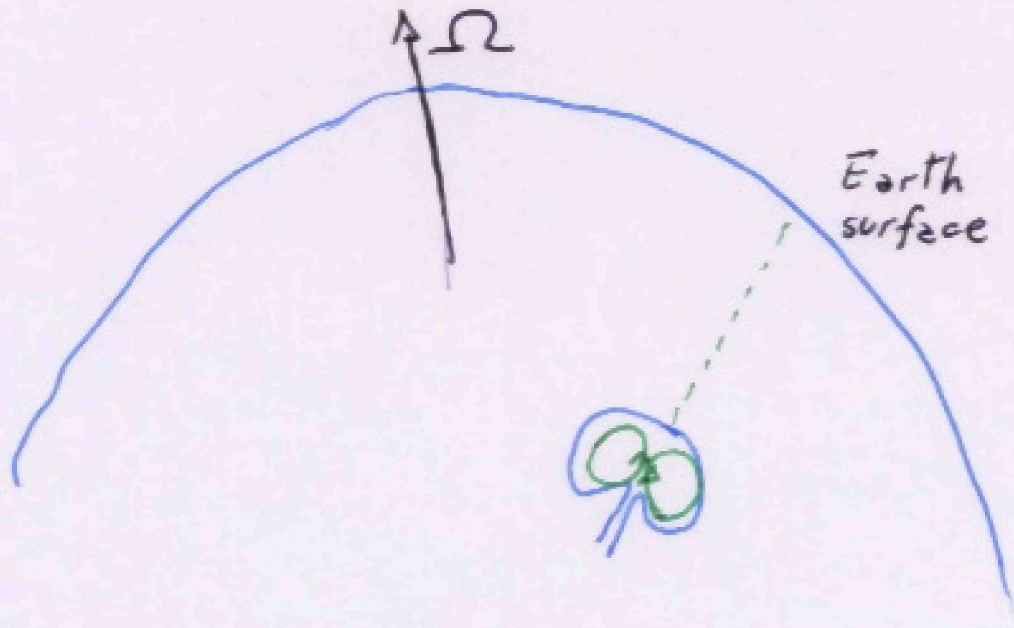
- Geophysical flows
- Flows in turbo-machineries
  (interaction between mean flow and imposed rotation)
- "Simple structure"
  complex vorticity dynamics
- One single structure → detailed investigation

  vortex ring // to background rotation

  vortex ring ⊥ to background rotation



The penetration of the plumes is reduced
by the ambient rotation (Ayotte & Fernando 1986)



Slow upward motion of "mushroom-like" structures

Depending on the latitude the ambient rotation
has different components // and ⊥ to the motion

Experimental set-up

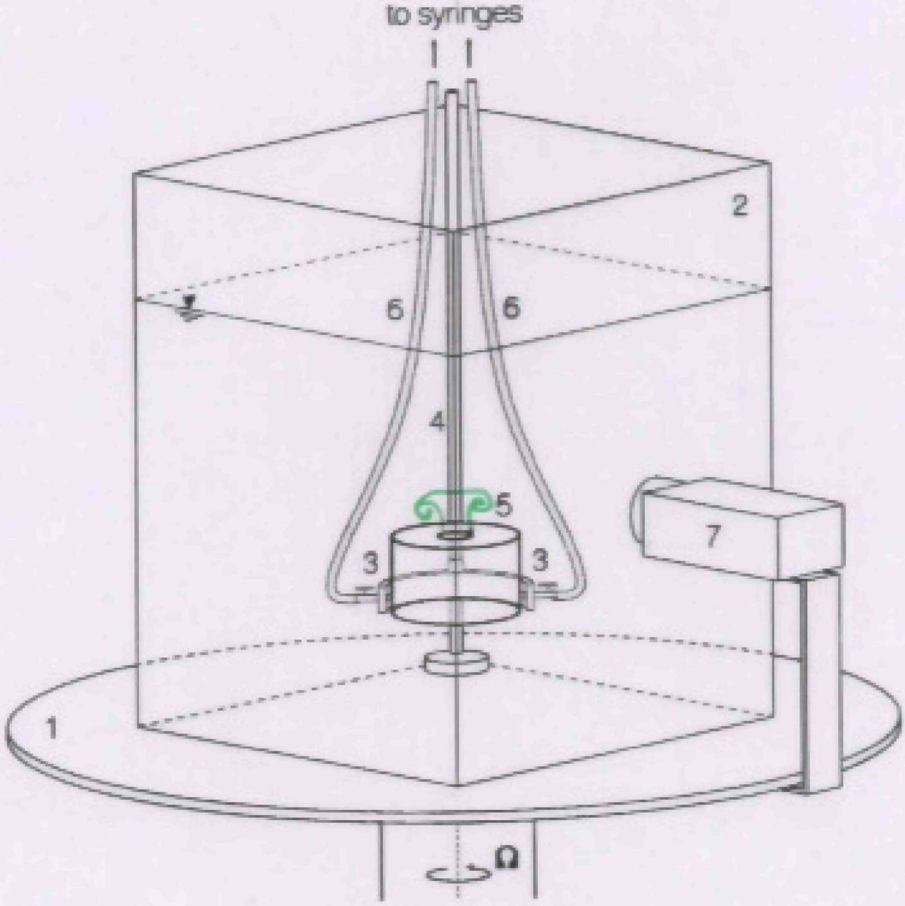


FIGURE 1. Sketch of the experimental apparatus: **1** rotating table, **2** glass tank, **3** fork, **4** stand of the vortex generator **5** cylindrical perspex box with the circular orifice on the top and taps in the lateral surface, **6** plastic tubes connected to the syringes driven by the step–motor system, **7** video-camera. The axis of rotation passes through the centre of the circular orifice.

- Flow visualization
- Particle tracking

Numerical Set-up

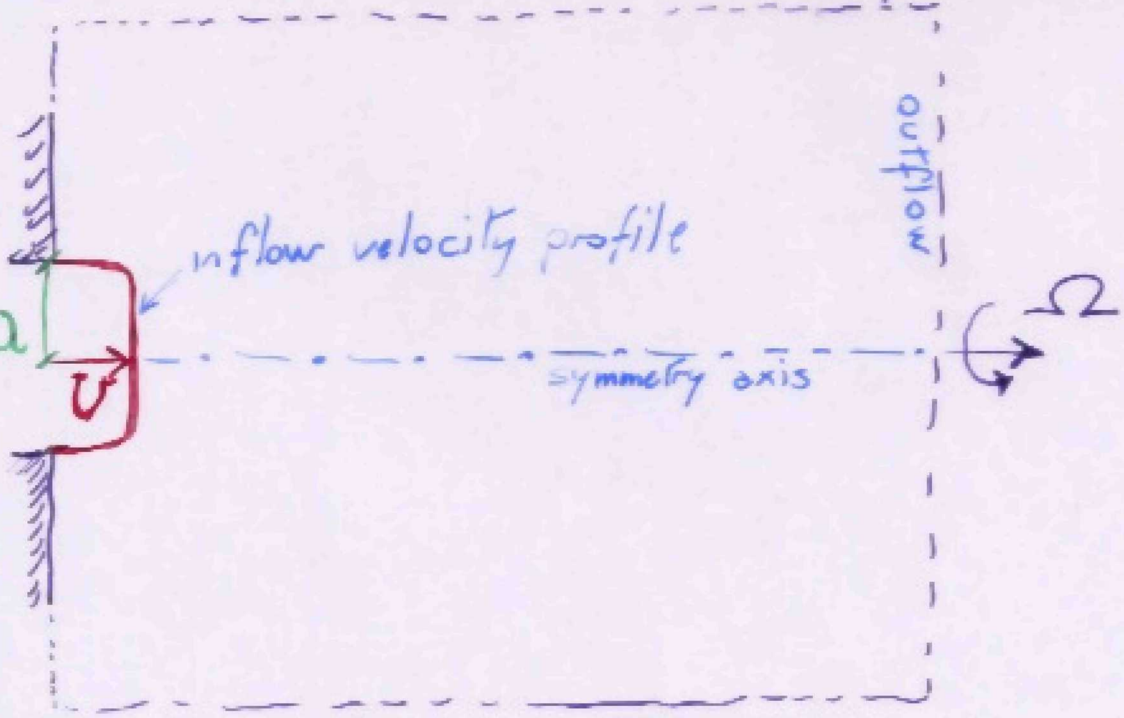
Direct simulation of 3D
Navier-Stokes equations in primitive variables

momentum $\frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \nabla \underline{u} = -\nabla p + \frac{1}{Re} \nabla^2 \underline{u} - \frac{1}{Ro} \hat{\Omega} \times \underline{u}$

continuity $\nabla \cdot \underline{u} = 0$

passive scalar $\frac{\partial c}{\partial t} + \underline{u} \cdot \nabla c = \frac{1}{Pe} \nabla^2 c$

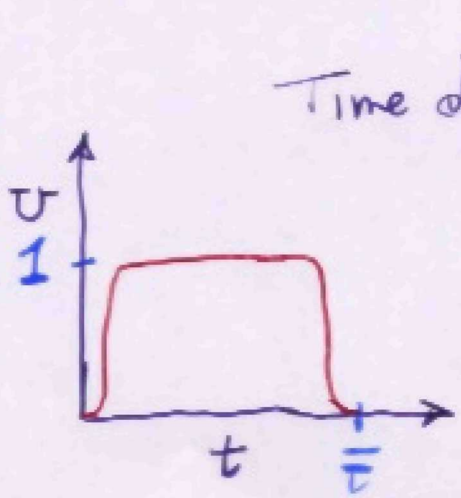
Computational domain



$Re = \frac{aU}{\nu}$

$Ro = \frac{U}{2 \Omega a}$

Time dependent injection



L injected slug length

$\frac{L}{R} = 1.2$

$\bar{T} \approx 1.3 \, R/U$

No rotation

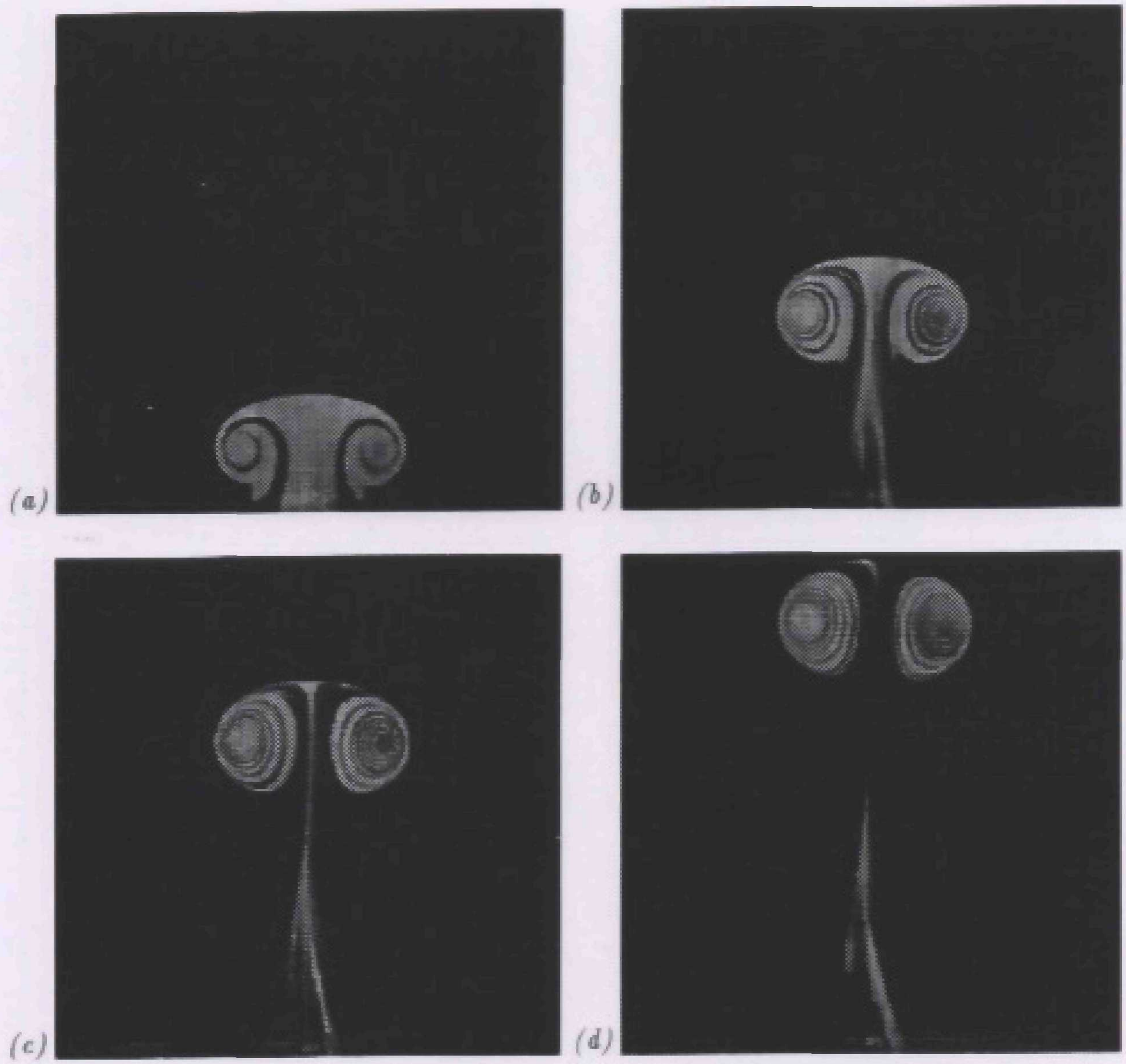
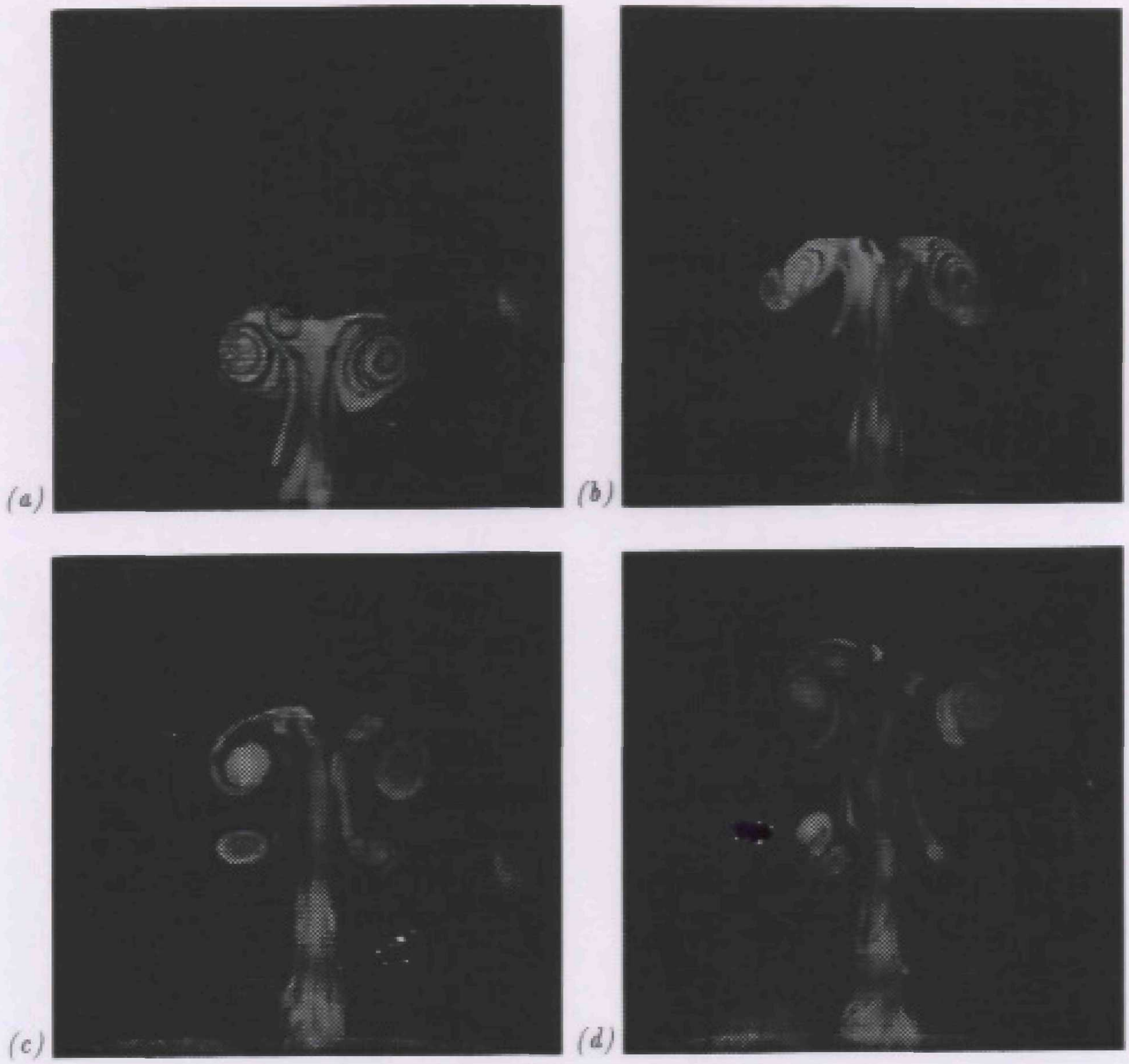


FIGURE 3. Dye–visualizations of a vortex ring evolution at Ro = ∞ and Re $\simeq$ 1000: *(a)* $t = 4$, *(b)* $t = 8$, *(c)* $t = 12$, *(d)* $t = 16$.

Vortex evolution

The ring translates "steadily" preserving its shape

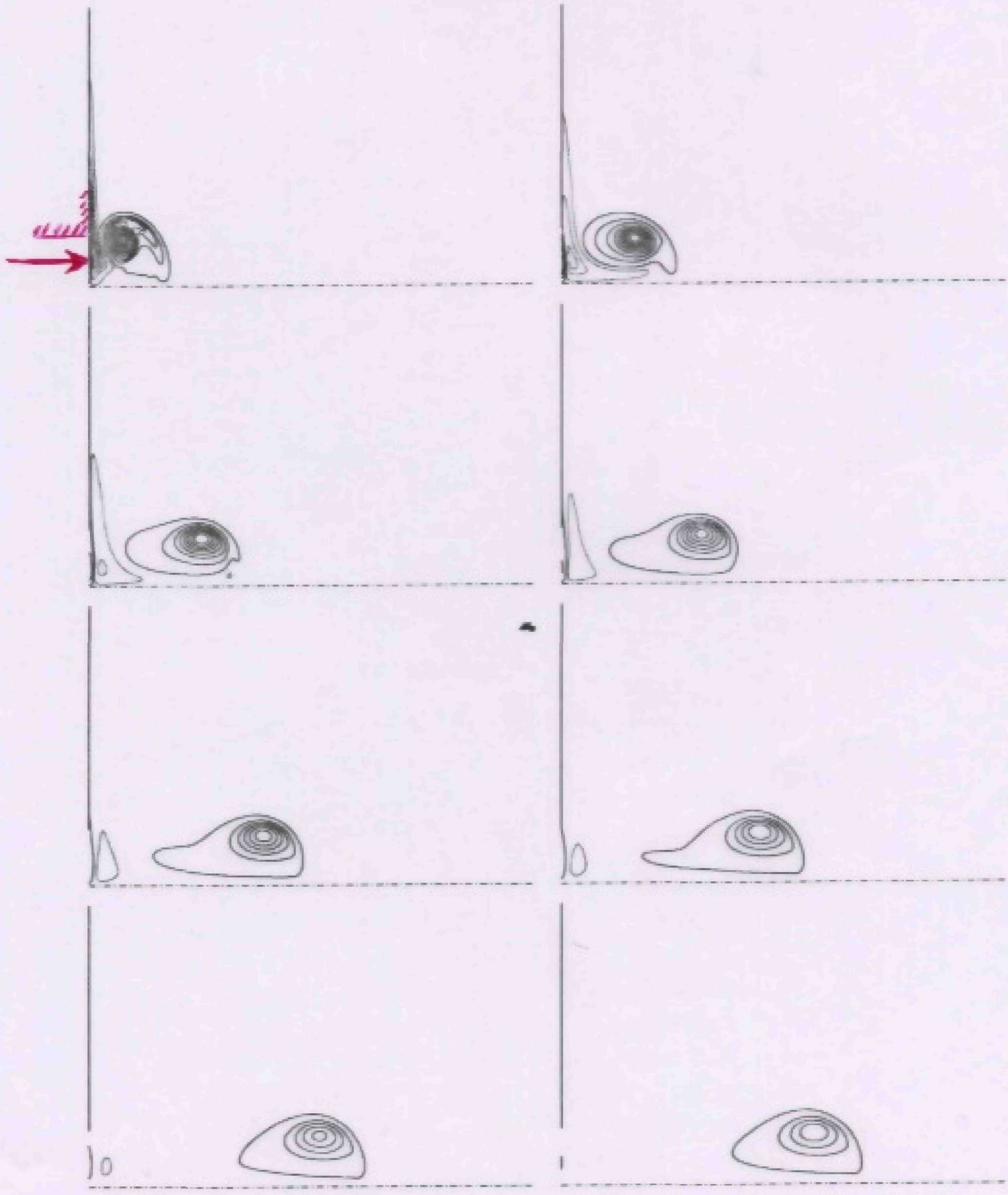
Rotation Ro = 4.8



$\uparrow \Omega$

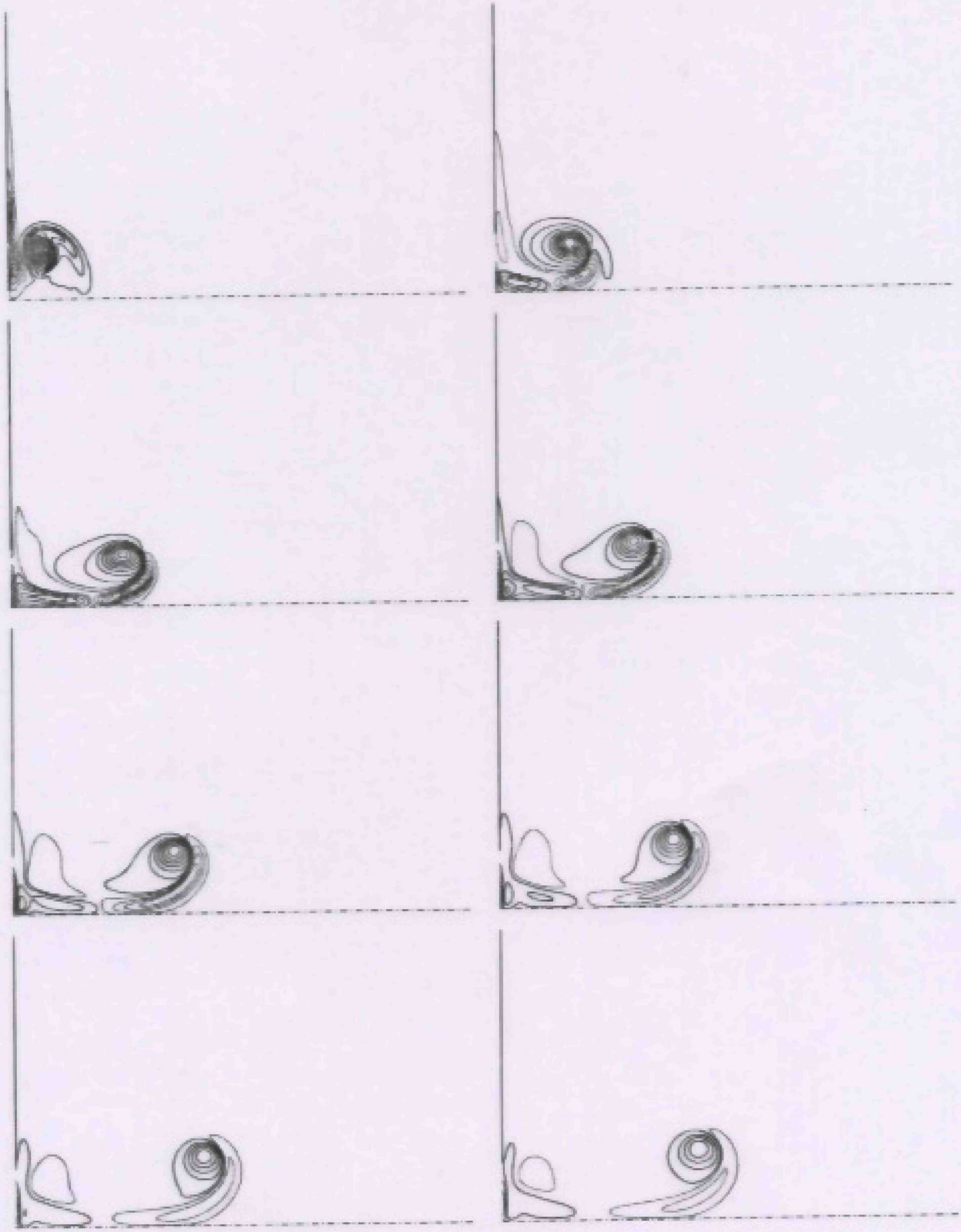
FIGURE 4. Dye–visualizations of a vortex ring evolution at Ro = 4.8 and Re $\simeq$ 1000: *(a)* $t = 7$, *(b)* $t = 10$, *(c)* $t = 13$, *(d)* $t = 16$.

The ring decelerates and secondary structures are shed



Contour plots of azimuthal vorticity at Re= 484 and Ro= ∞ vorticity increments $\Delta\omega =$ ± 0.40, —— for positive - - - - for negative values.

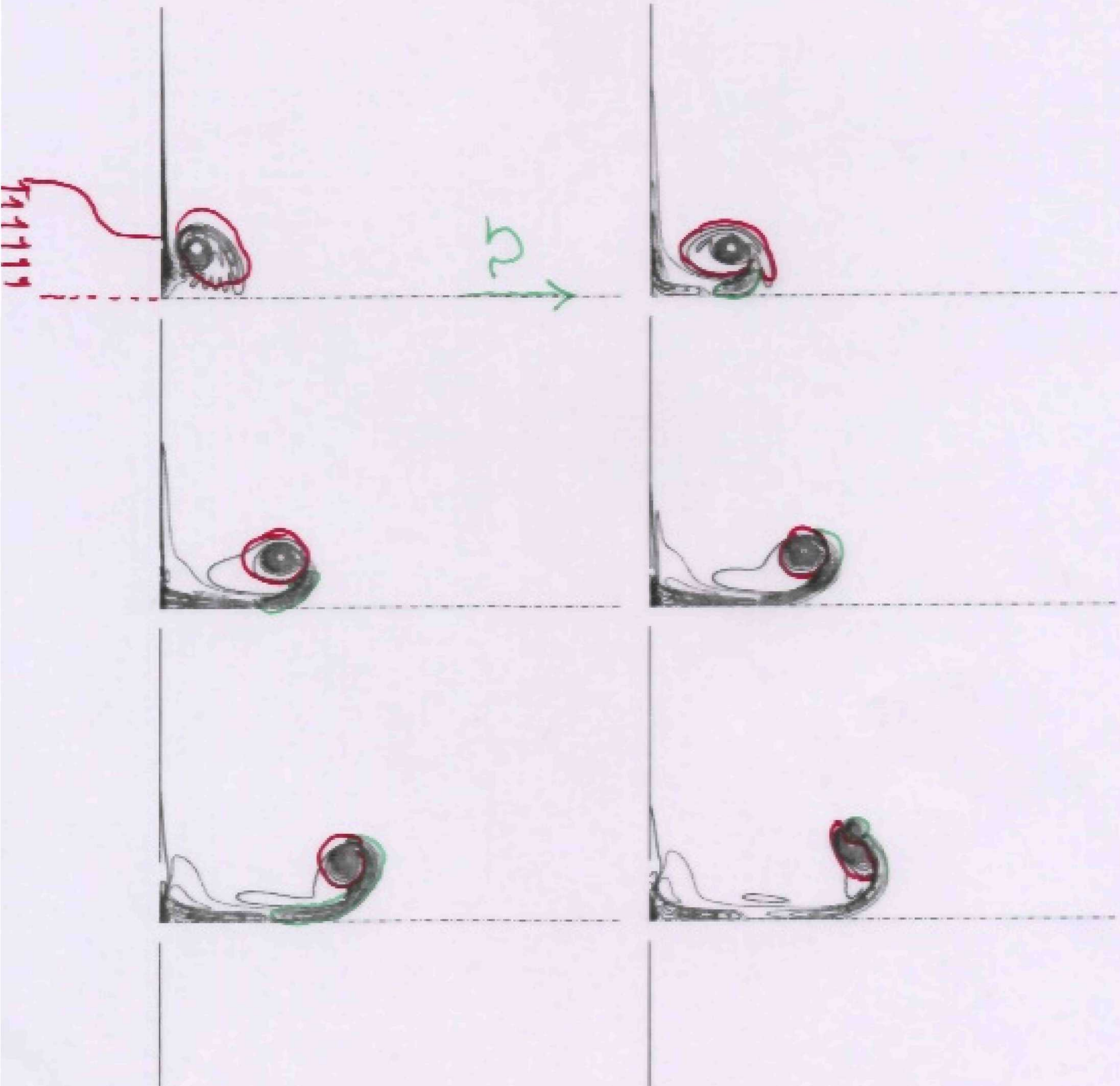
No rotation, $Ro = \infty$



Contour plots of azimuthal vorticity at Re= 484 and Ro= 5.5 vorticity increments $\Delta\omega = \pm 0.40$, —— for positive - - - - for negative values.

Ro = 5.5

High Reynolds (Re=1500)

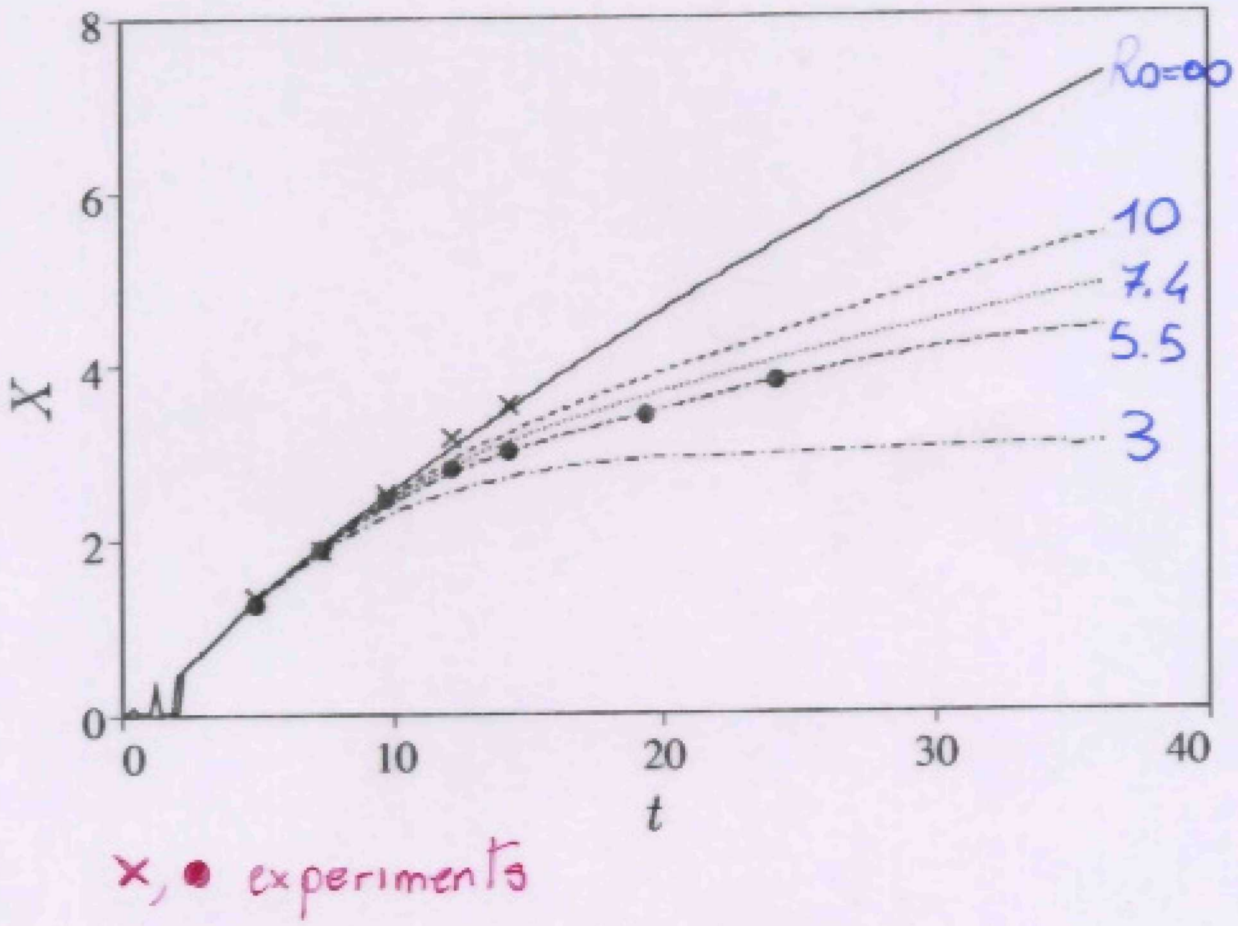


Contour plots of azimuthal vorticity at Re= 1500 and Ro= 10 vorticity increments $\Delta\omega = \pm 0.40$, —— for positive ---- for negative values.

- A secondary vortex detachs

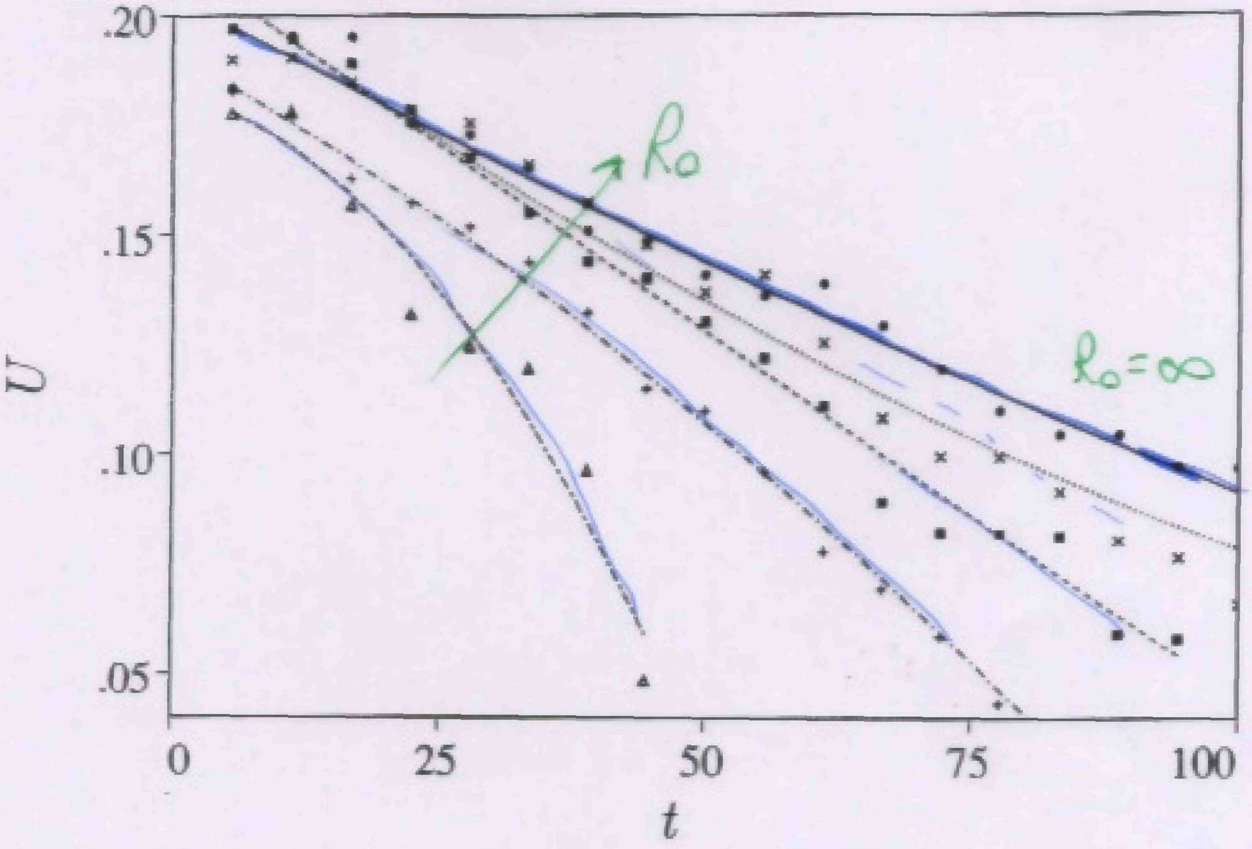
Trajectories

- for higher rotation rates the ring decelerates



Ring velocities

- Translation velocity decreases as *Ro* decreases



• and —— for $Ro = \infty$, × and ········ $Ro = 23$, ■ and - - - - $Ro = 15.6$, + and —·— $Ro = 12.7$, ∇ and —-— $Ro = 8.2$.

Particle Tracking

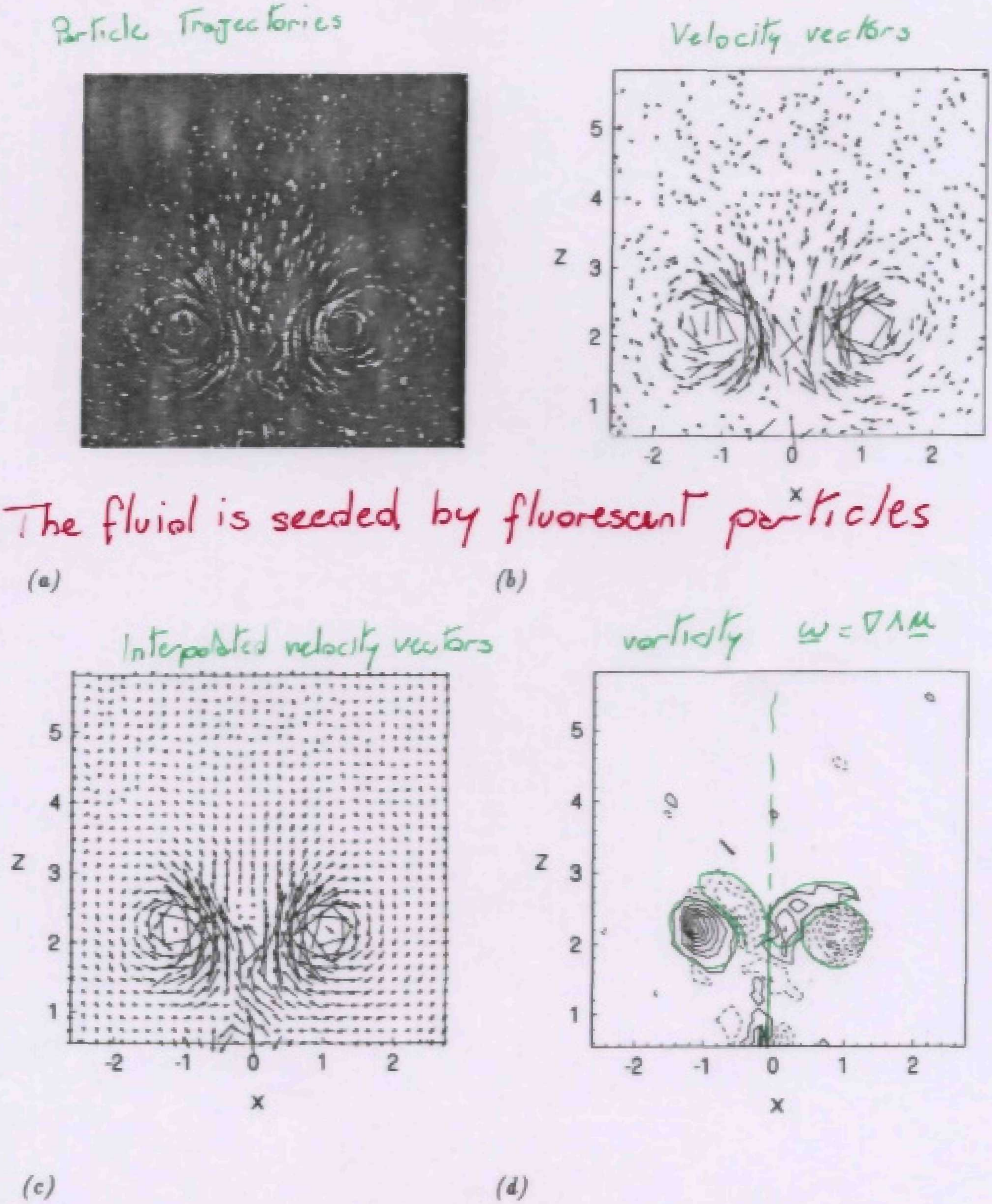
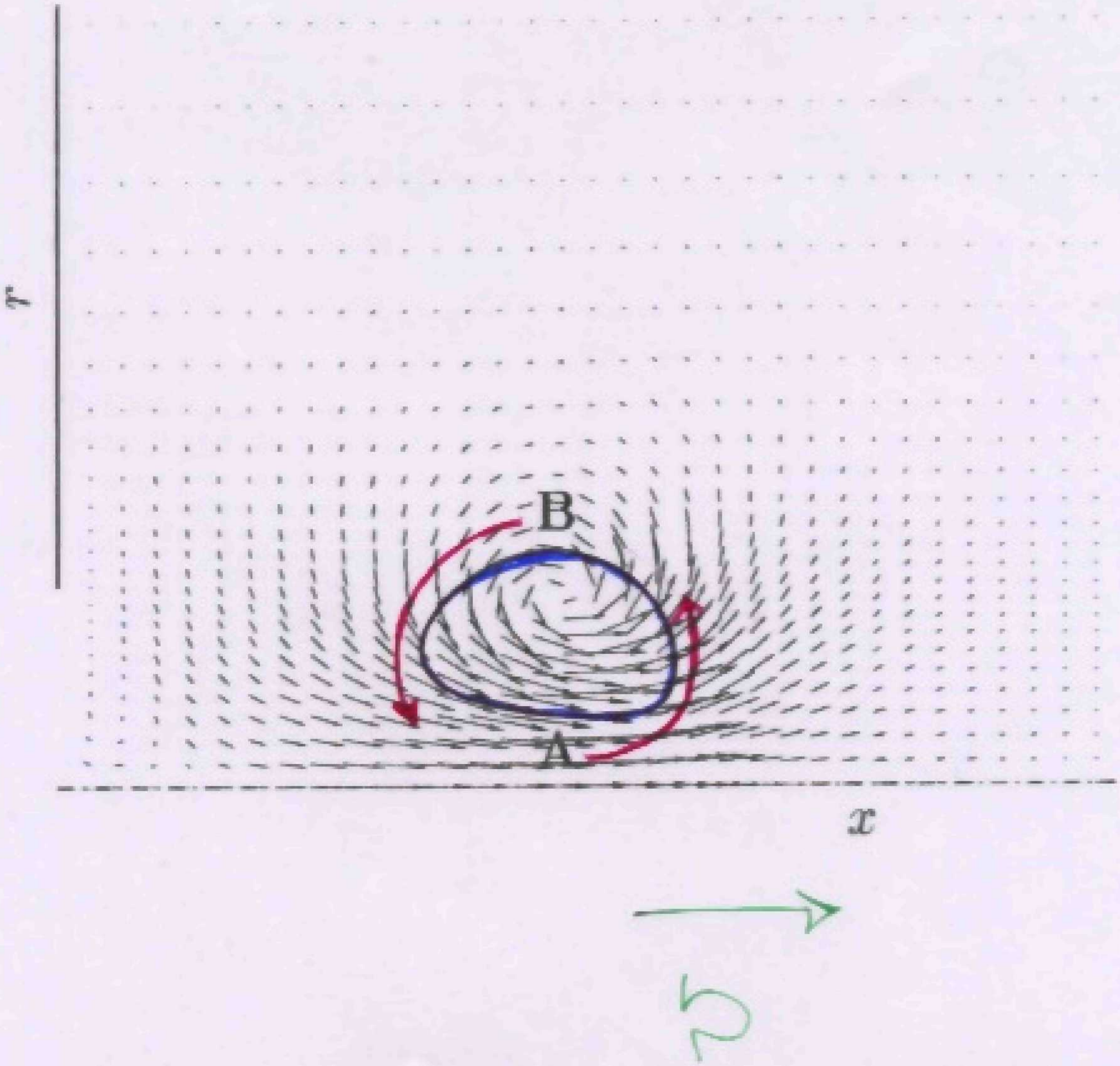


FIGURE 8. Particle trajectories *(a)*, measured velocity *(b)*, interpolated velocity *(c)* and azimuthal vorticity *(d)*, of a vortex ring at Ro = 3.5 and Re = 484 and $t = 6$. Vorticity increments $\Delta\omega = 0.5$, —— for positive and ········ for negative values. $X \equiv r$ for $X > 0$ and $X \equiv -r$ for $X < 0$.

Rotation effects

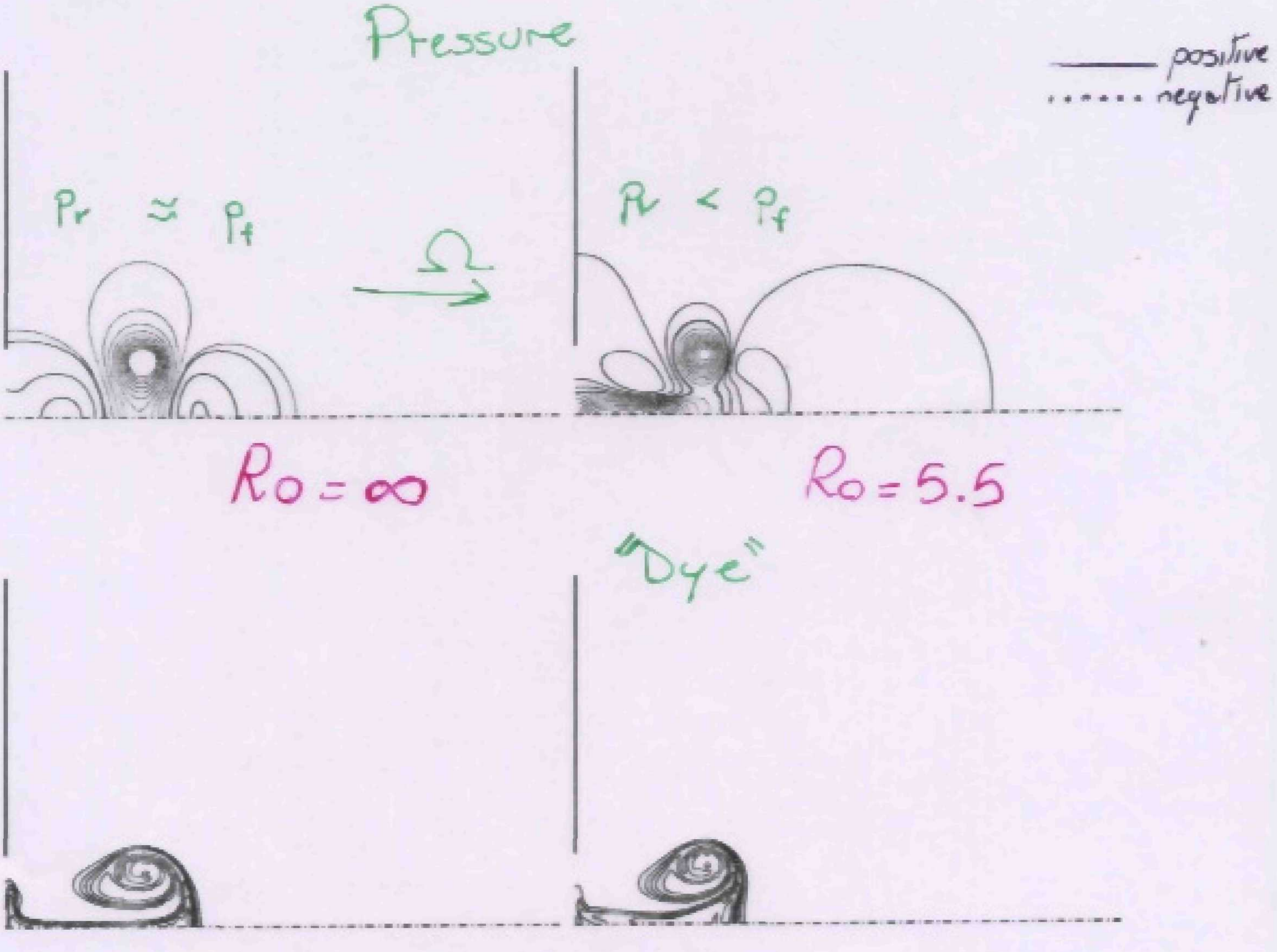
- the ring slows down and cross–diffusion weakens the structures.

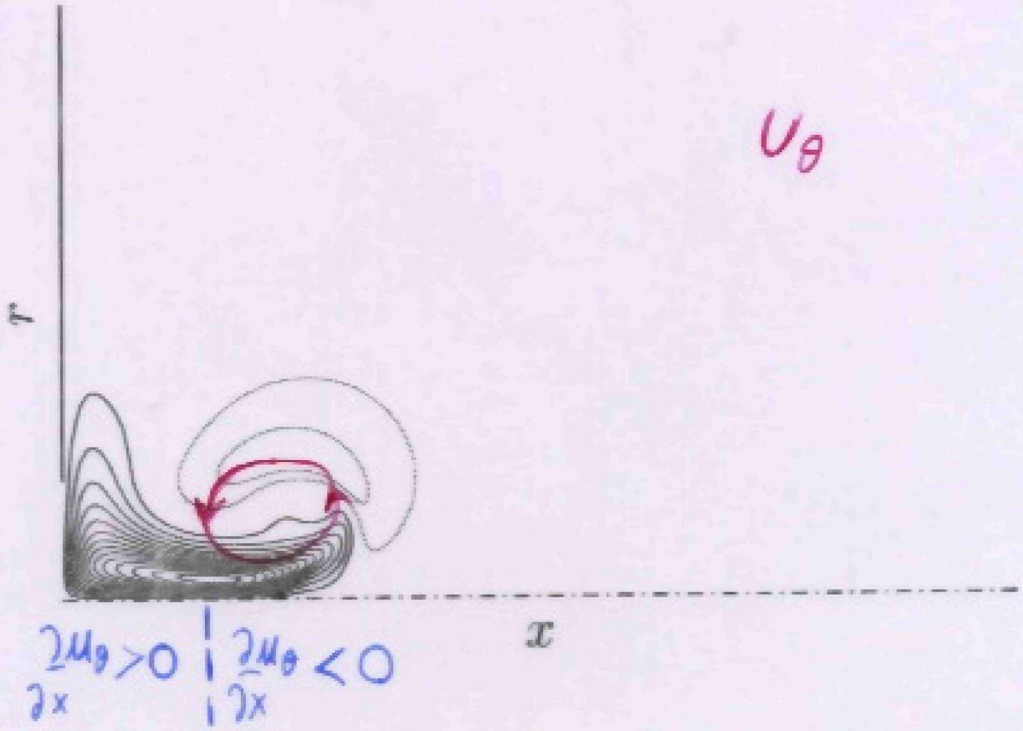
- an oppositely–signed vortex ring forms ahead of the primary ring.



Pressure

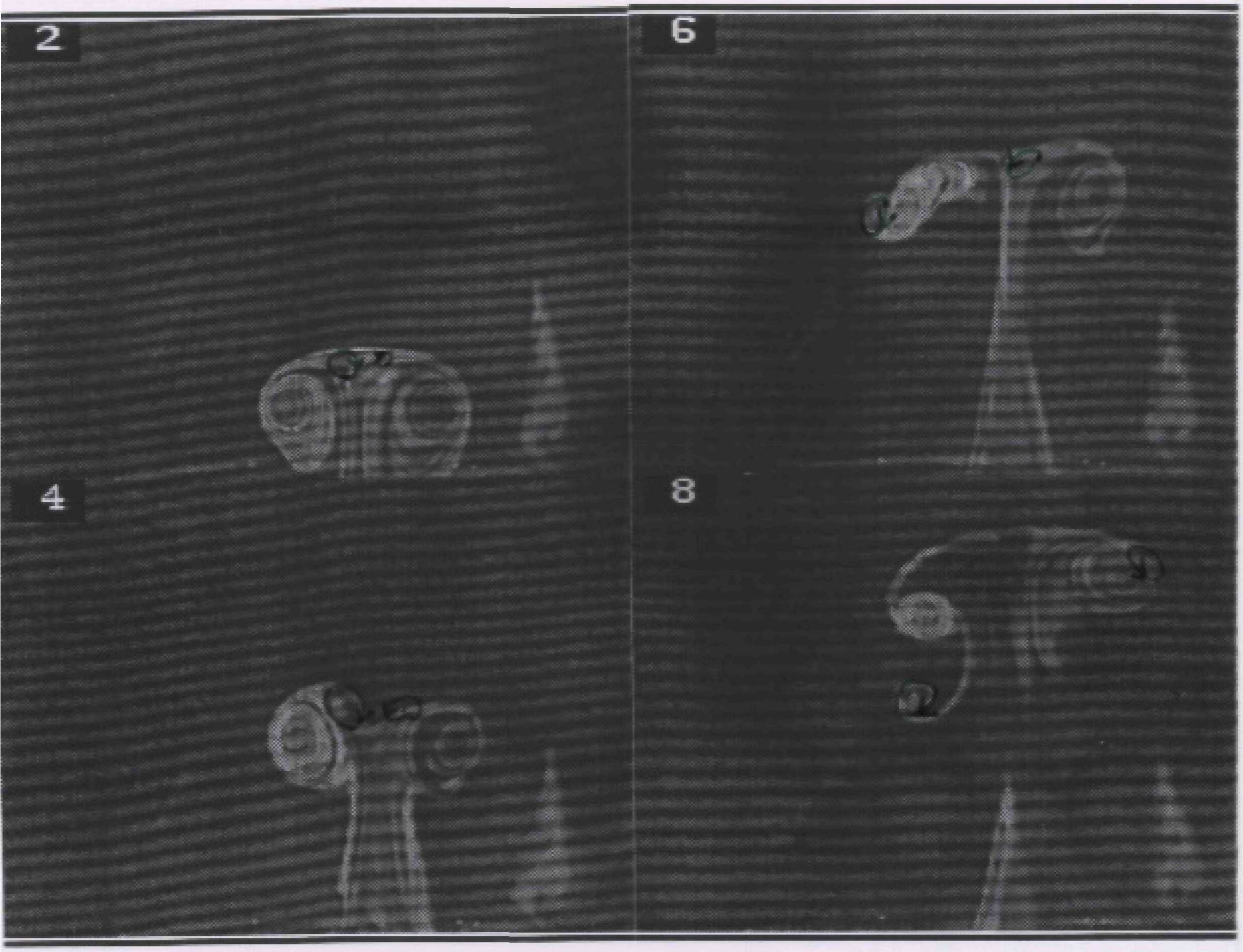
- Anticyclone ⇒ 'high pressure'
- Cyclone ⇒ 'low pressure'





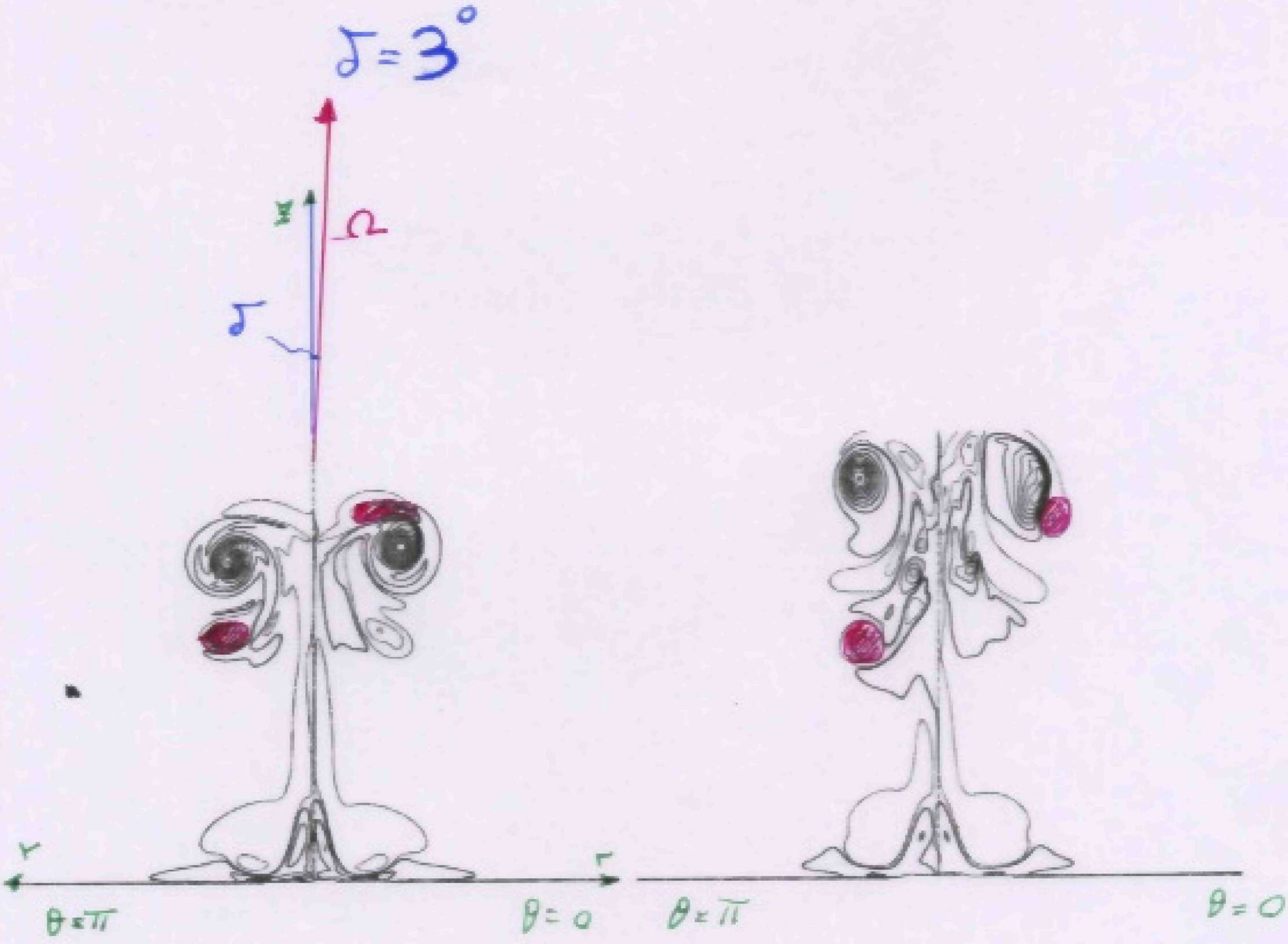
$$\frac{\partial \omega_\theta}{\partial t} + \mathbf{u} \cdot \nabla \boldsymbol{\omega} \mid_\theta = \boldsymbol{\omega} \cdot \nabla \mathbf{u} \mid_\theta + \frac{1}{\mathrm{Ro}} \frac{\partial u_\theta}{\partial x} + \frac{1}{\mathrm{Re}} \nabla^2 \boldsymbol{\omega} \mid_\theta$$

3D phenomena



- experimental evidence of asymmetry
- no Widnall instability

• small misalignment between axis of rotation and propagation of the vortex

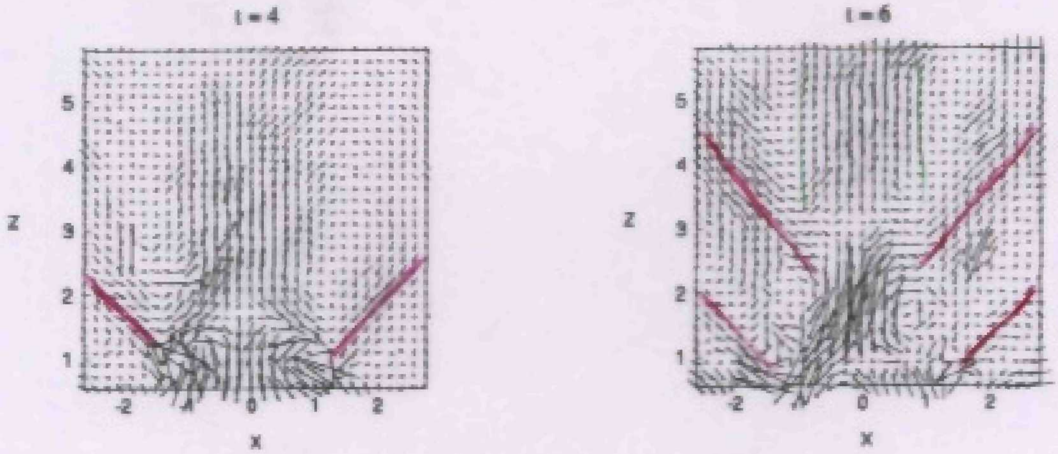


$$\Omega_z = \Omega \cos(\delta) \simeq \Omega$$

$$\Omega_r = \Omega \sin(\delta) \underline{\cos(\theta)} \simeq \delta \Omega \underline{\cos(\theta)}$$

High Rotations

Ro ≤ 1



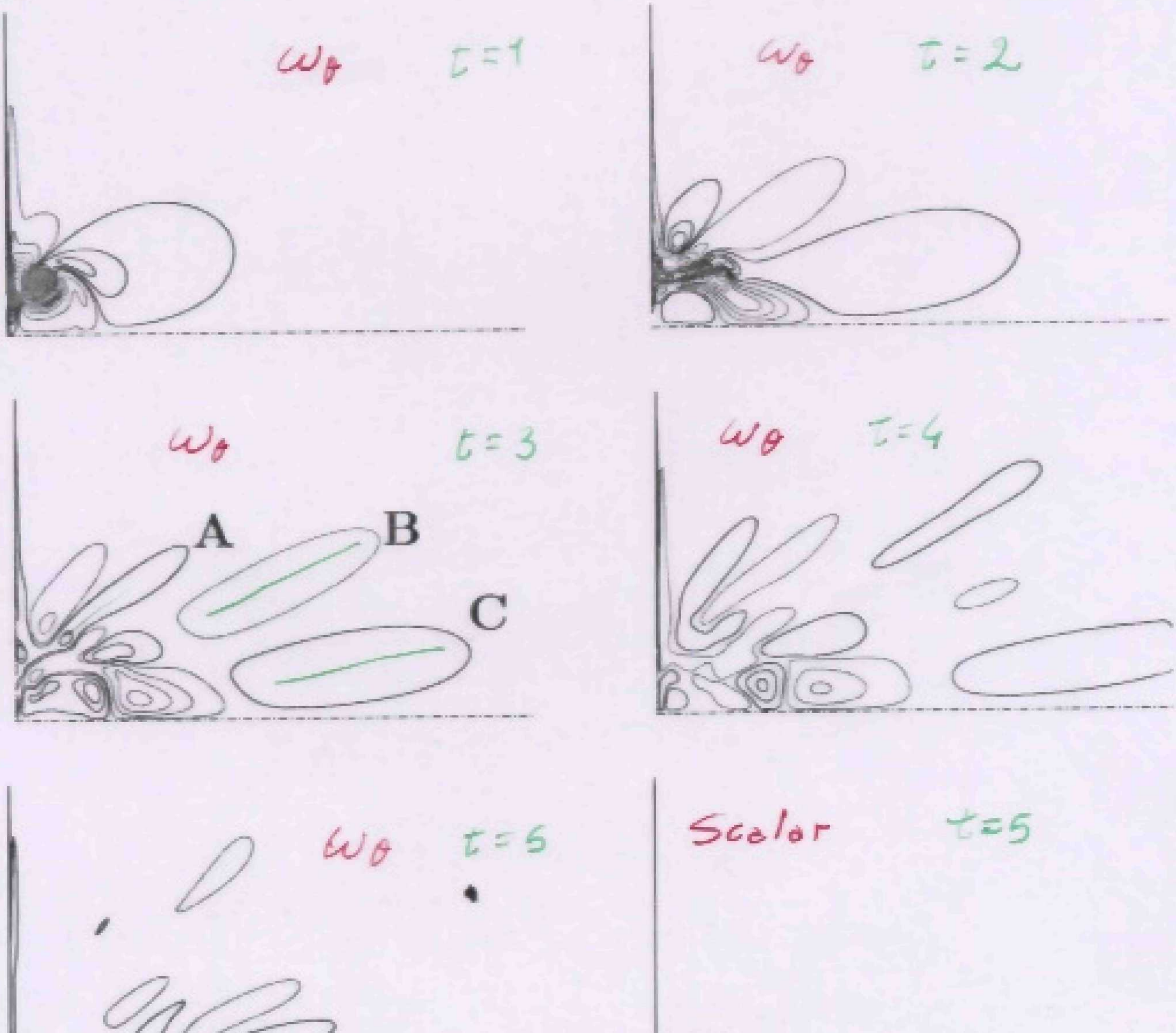
(a) (b)

FIGURE 14. Interpolated velocity field of a vortex ring at Ro = 0.605 and Re = 484: *(a)* $T = 4$, *(b)* $T = 6$.

- no ring formation
- inclined shear-layers

High rotations

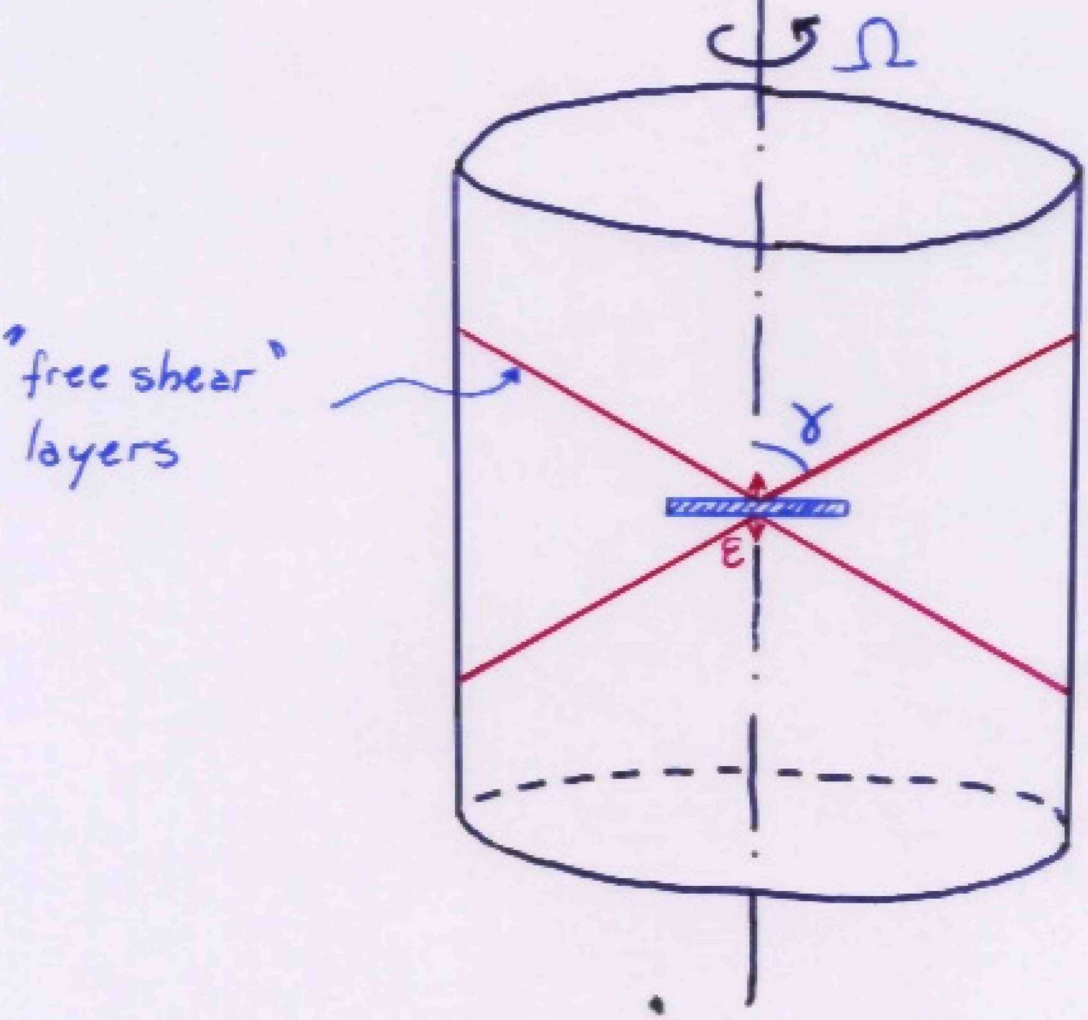
- the ring does not form



- inclined shear layers as for the problem of 'oscillating disk' (Greenspan, 1968).

$Re = 484 \quad Ro = 0.6$

Oscillating disk (Greenspan, 1968)



The disk performs 'infinitesimal' oscillations

$\varepsilon(t) = \varepsilon_0 \sin(f^* t)$

if $2\Omega \geqslant f^*$ $\qquad \gamma = \sin^{-1}(f^*/2\Omega)$

Steady, infinitesimal forcing

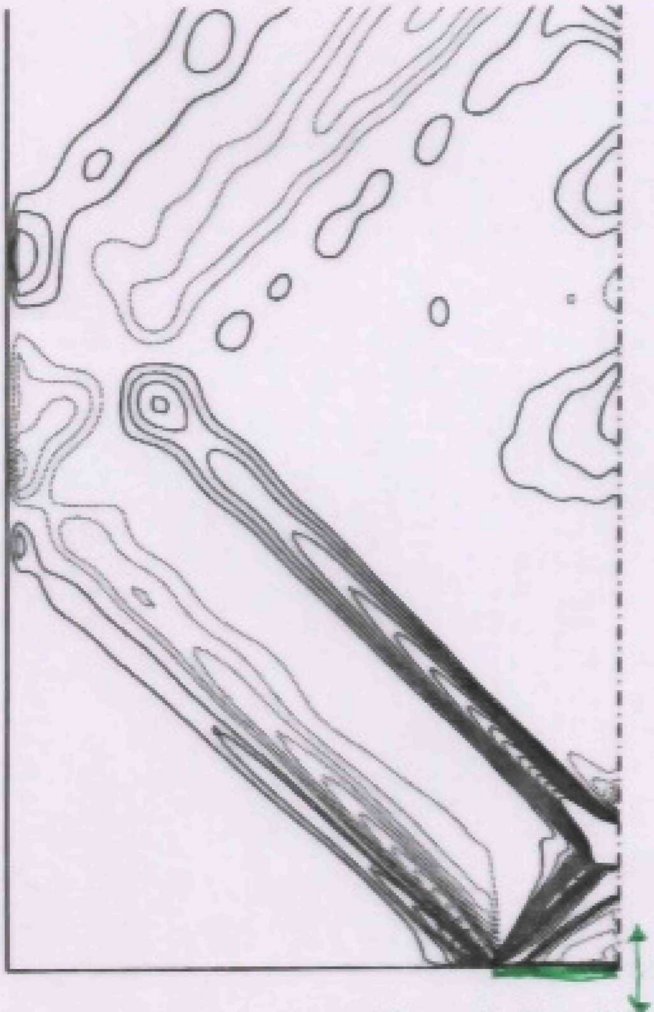


FIGURE 17. Contour plots of azimuthal vorticity at Re= 484, Ro= 0.707 and $t = 10\pi$. Vorticity increments $\Delta\omega = \pm 0.01$, —— for positive, - - - - for negative values.

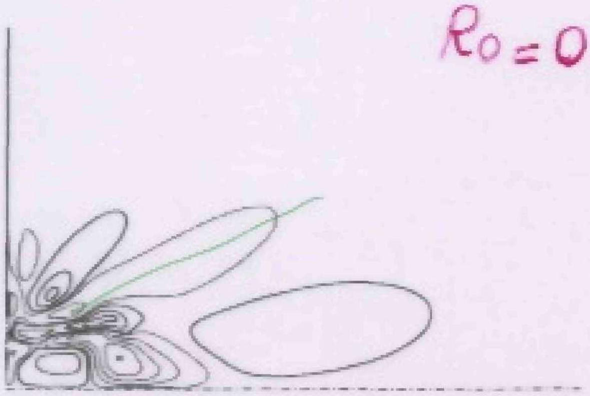
$f = 1 \quad Ro = \frac{\sqrt{2}}{2} \Longrightarrow \gamma = 45°$

t = const

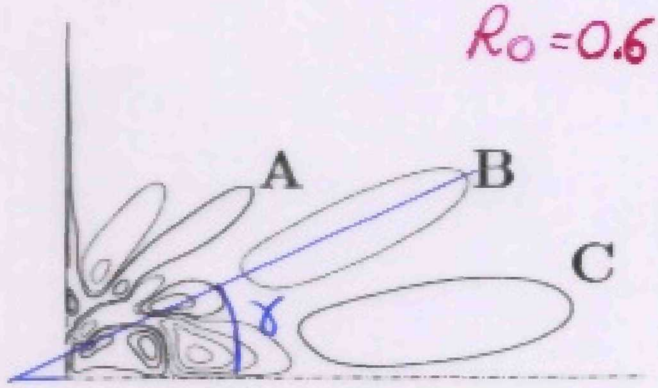
- $\gamma = \sin^{-1}(\text{Ro} f)$

f calculated from a simulation

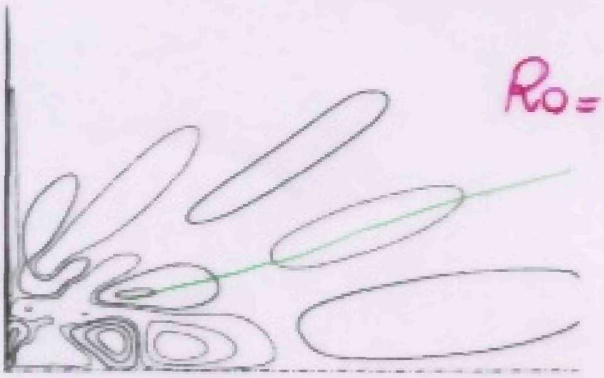
Ro = 0.8



Ro = 0.6



Ro = 0.48

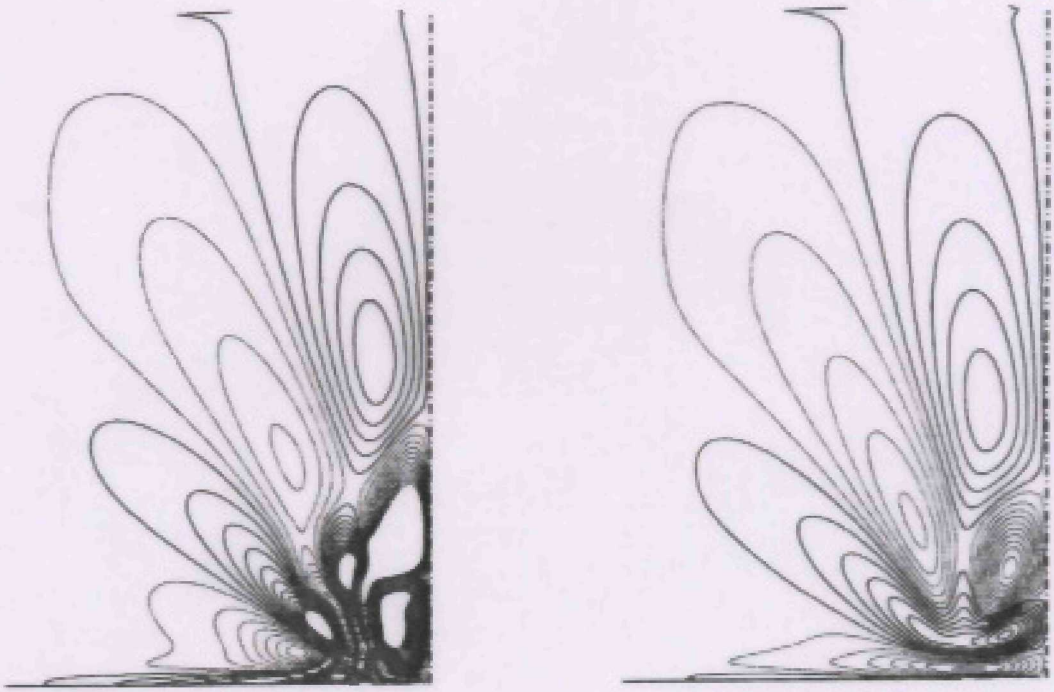


⇓

$f \simeq 0.685$

	γ (predicted)	γ (measured)
Ro = 0.8	33°	30°
Ro = 0.48	19.5°	19°

- The same agreement for 'A' and 'C' structures



(a) Non-linear (b) Linear

FIGURE 16. Contour plots of azimuthal vorticity at Re= 484, Ro= 0.707 and t = 7.2: *(a)* full Navier–Stokes run, *(b)* linear run. Vorticity increments $\Delta\omega = \pm 0.05$, —— for positive, ---- for negative values.

- Shear-layers due to linear dynamics

Transient ("impulsive") forcing ⟹

- many angles of the structures
- angles diminishes in time

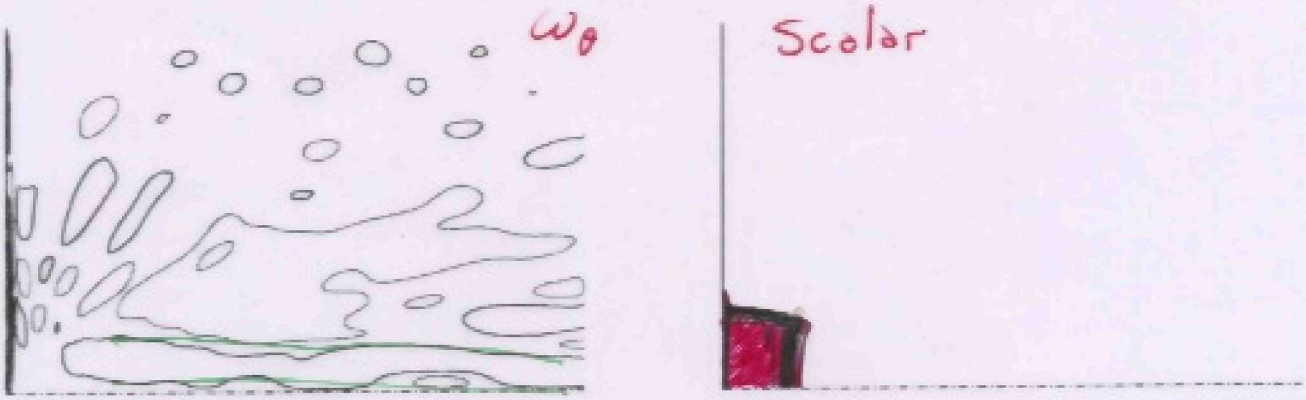
'Very high' rotations

- $\mathrm{Ro} \to 0$

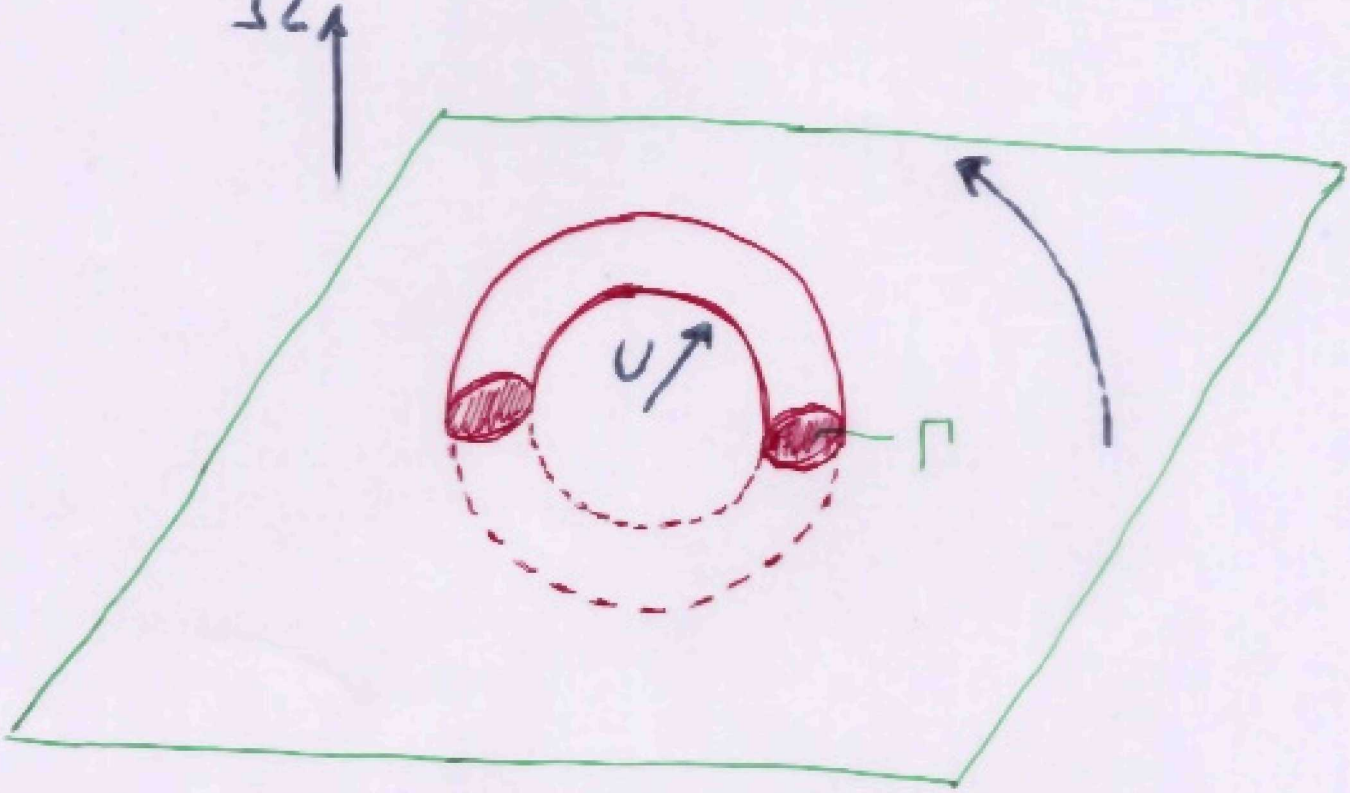
$$\frac{\partial \omega_\theta}{\partial t} + \mathbf{u} \cdot \nabla \boldsymbol{\omega} \mid_\theta = \boldsymbol{\omega} \cdot \nabla \mathbf{u} \mid_\theta + \frac{1}{\mathrm{Ro}} \frac{\partial u_\theta}{\partial x} + \frac{1}{\mathrm{Re}} \nabla^2 \boldsymbol{\omega} \mid_\theta,$$

- $\to \partial u_\theta / \partial x = 0$

'two dimensional' Taylor–Proudman column



$Ro \simeq 0.1$

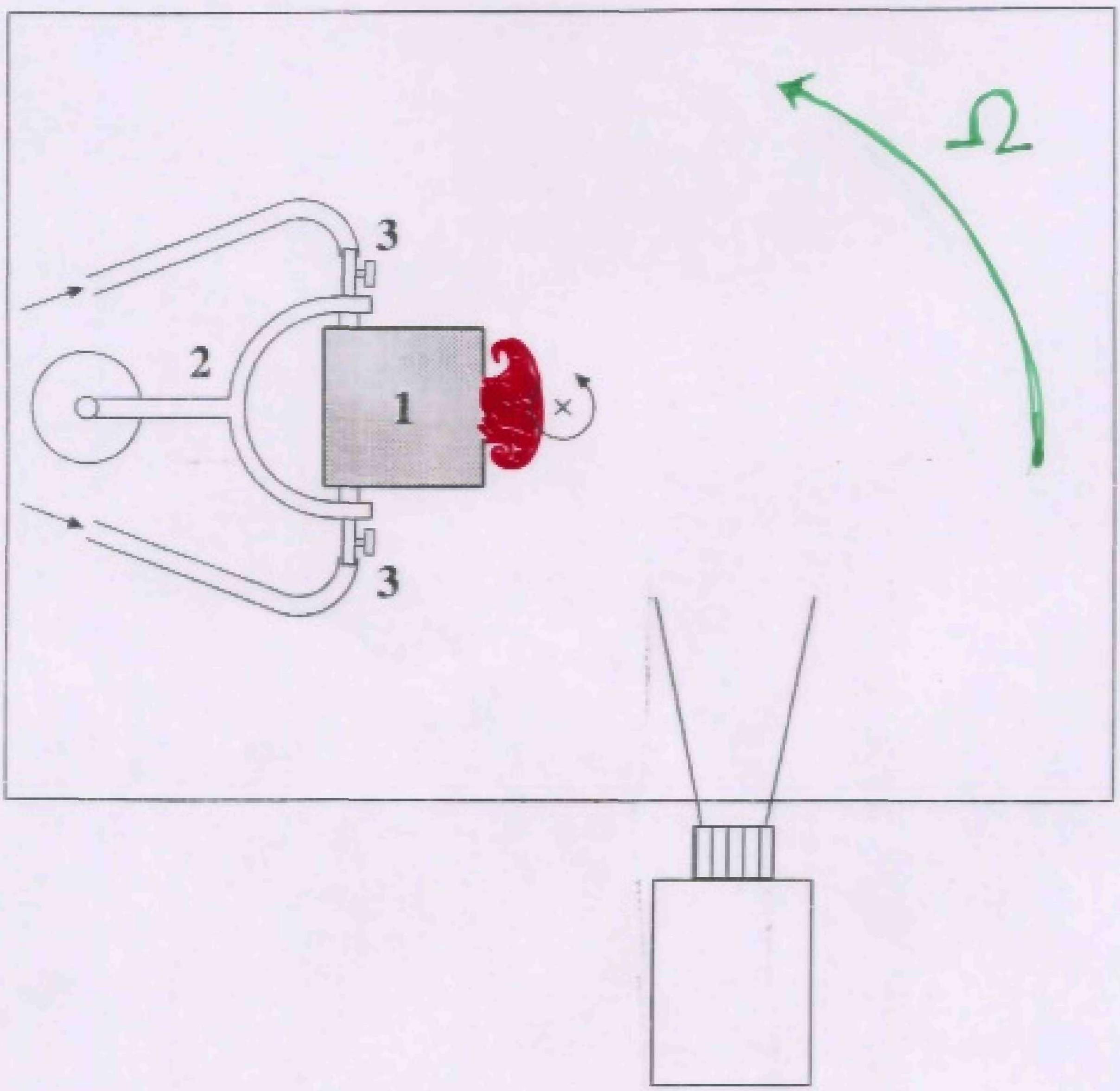


$$Re = \Gamma/\nu \qquad Ro = \Gamma/2\Omega R^2$$

Γ = ring circulation.

R = ring radius

Experimental set–up



Vortex ring generator **1**, fork and stand**2**, watertaps and pipes **3**,

Initial Conditions

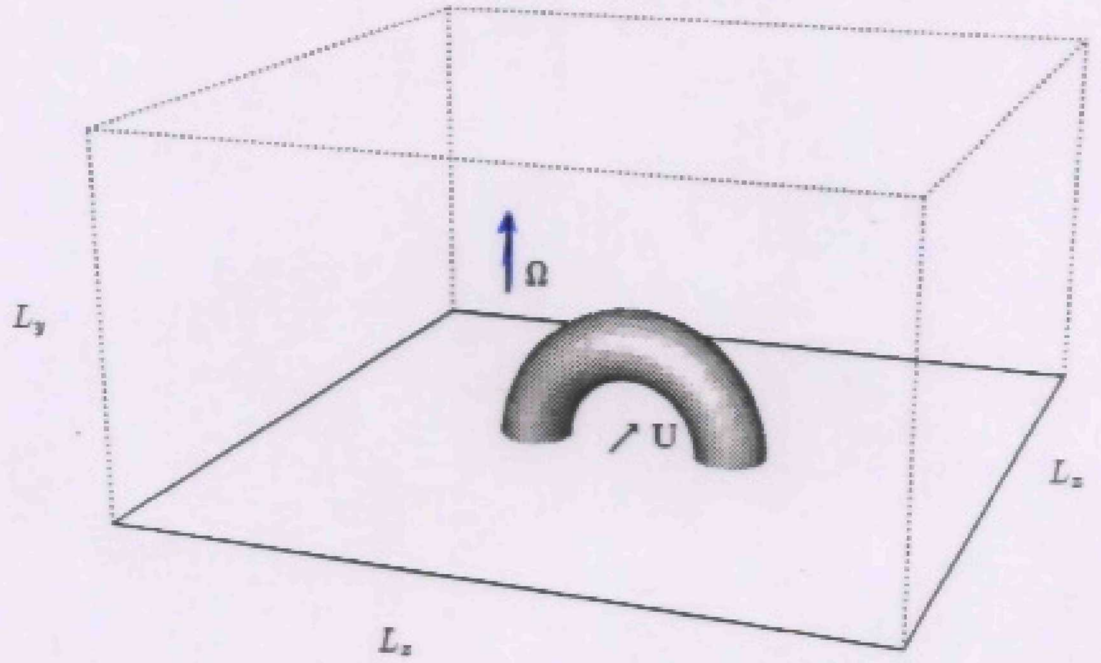
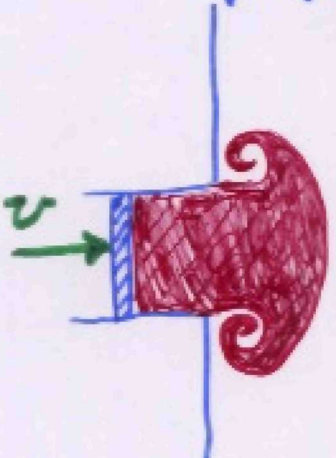


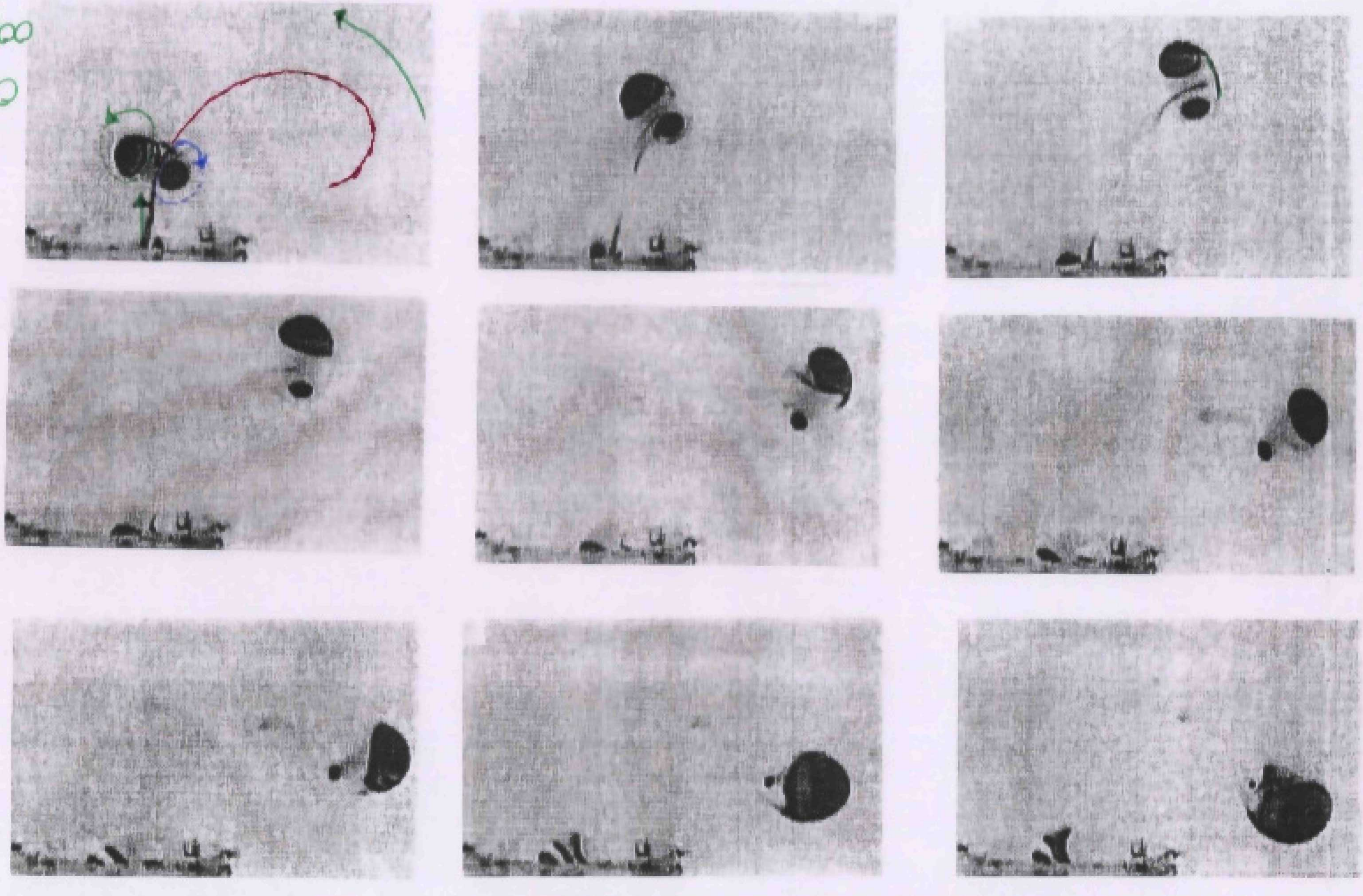
FIGURE 2. Schematic of the initial condition and computational box.

vortex ring such to match the experimental ring generation



only half ring was simulated

$Re \approx 900$
$Ro = 10$



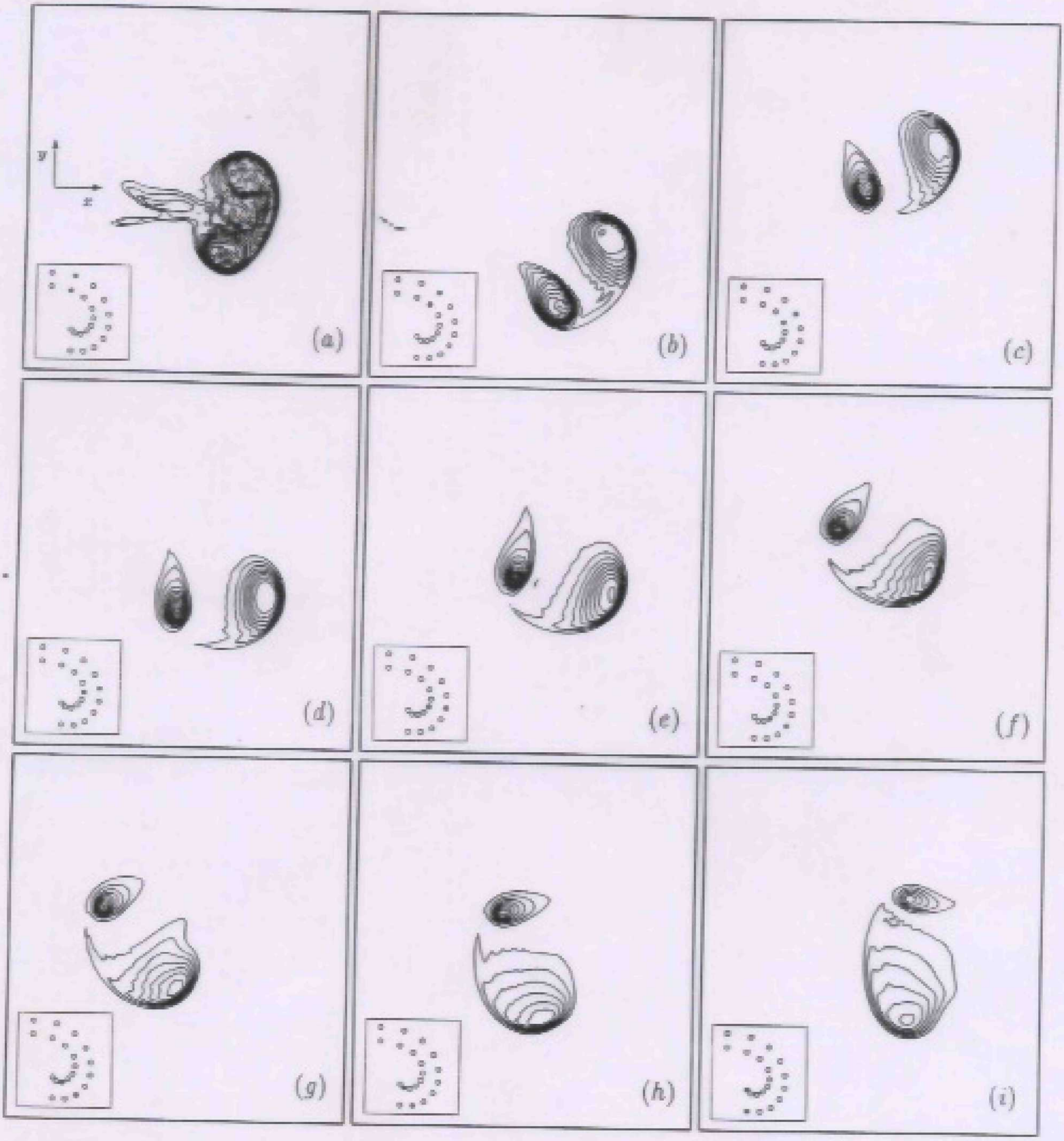


FIGURE 13: Contour plots of passive scalar concentration C in the horizontal symmetry plane $z = 0$ of the vortex ring, obtained from a numerical simulation with $Ro = 25$ and $Re = 500$. (a) $t = 17$, (b) $t = 33$, (c) $t = 50$, (d) $t = 67$, (e) $t = 83$, (f) $t = 100$, (g) $t = 117$, (h) $t = 133$, (i) $t = 150$. Contour increments $\Delta C = 0.1$; minimum contour levels: (a–c) $C = 0.25$, (d) $C = 0.2$ and (e–i) $C = 0.15$. The insets show the trajectories of the core centres, the instantaneous position of the vortex ring in each panel is denoted by black dots.

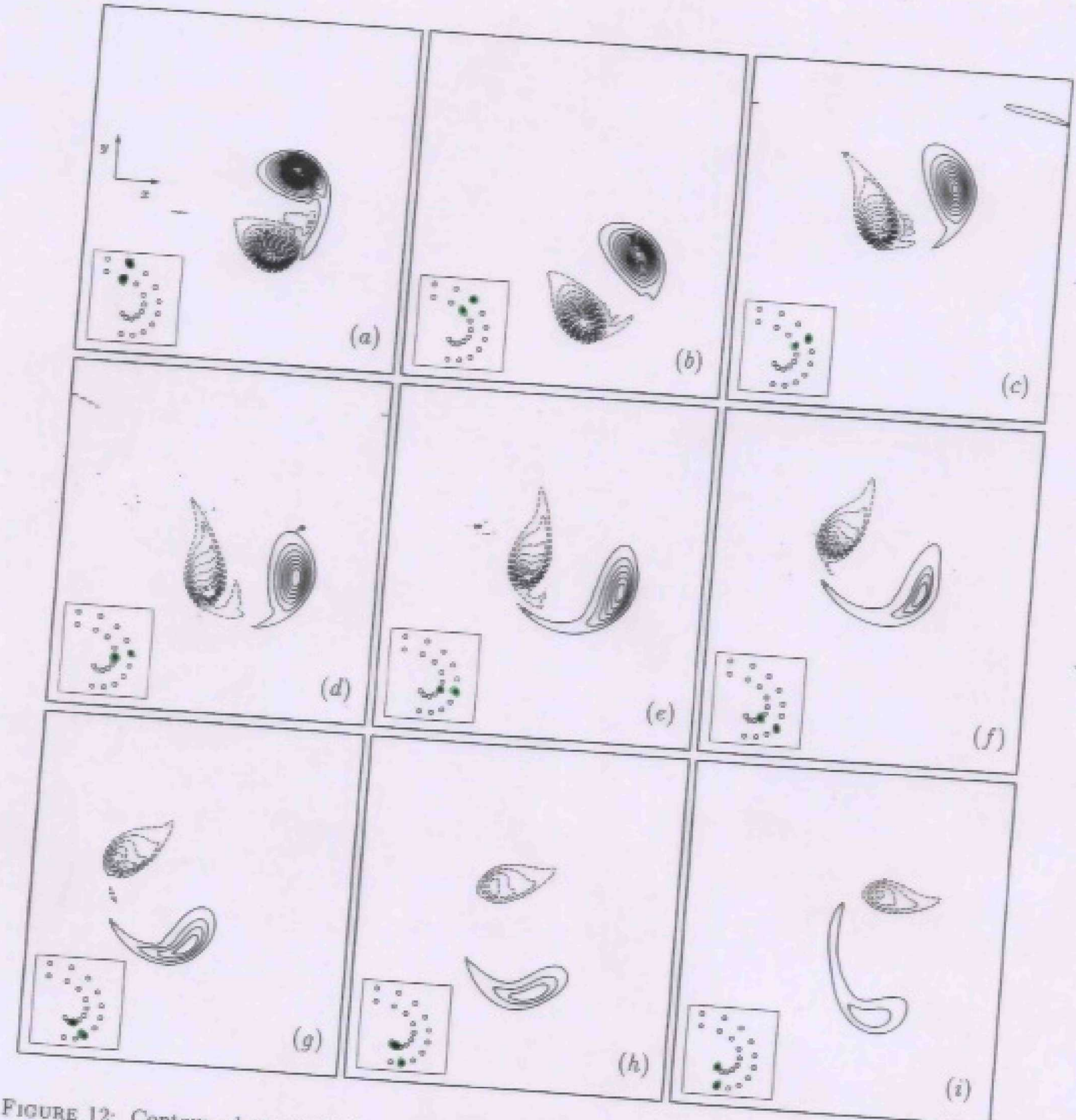
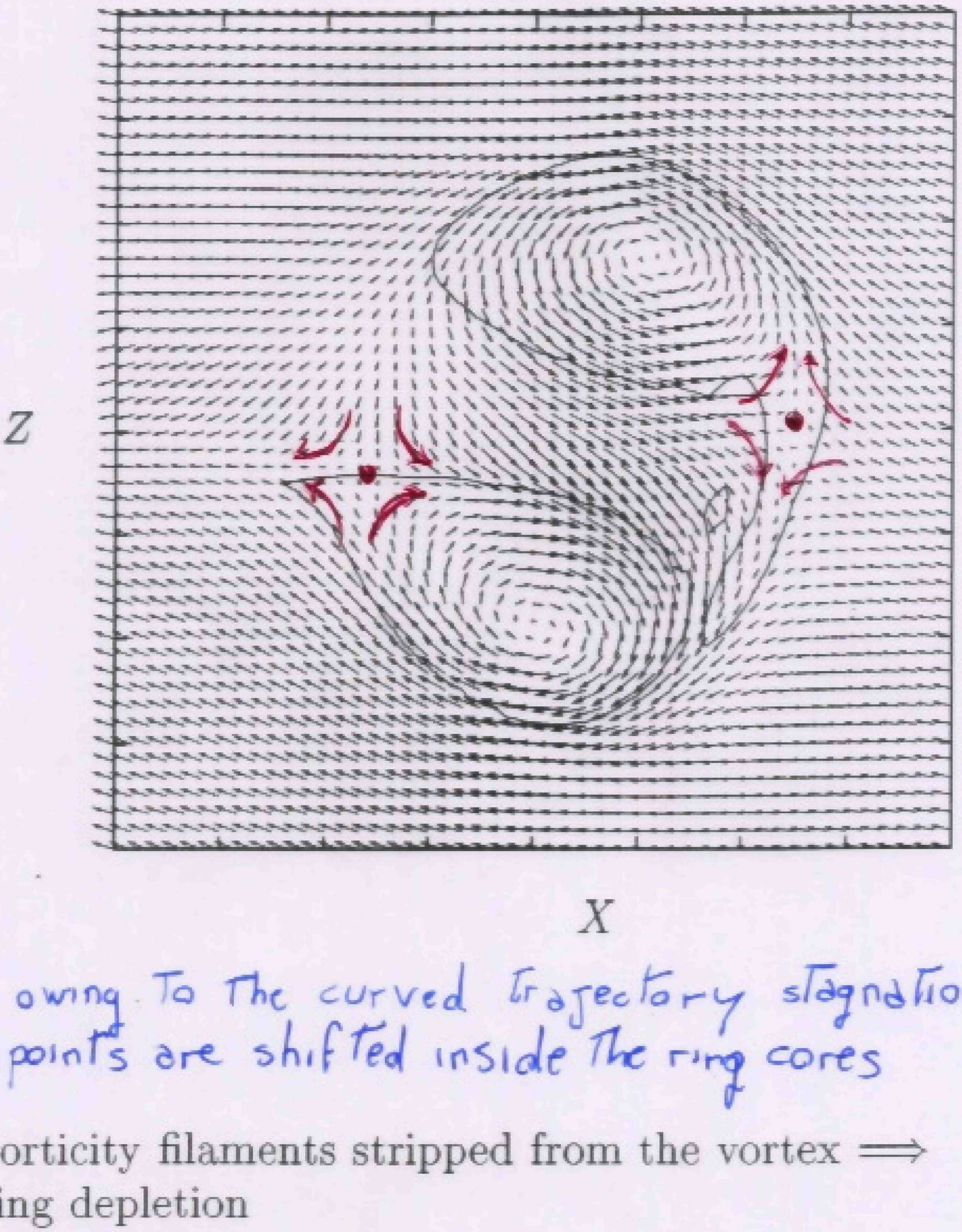


FIGURE 12: Contour plots of relative vorticity ω_z in the horizontal symmetry plane $z = 0$ of the vortex ring, obtained from a numerical simulation with $Ro = 23$ and $Re = 900$: (a) $t = 17$, (b) $t = 33$, (c) $t = 50$, (d) $t = 67$, (e) $t = 83$, (f) $t = 100$, (g) $t = 117$, (h) $t = 133$, (i) $t = 150$. Contour increments $\Delta\omega_z = \pm 0.1$, minimum contour level at $|\omega_z| = 0.1$; solid lines denote positive values and dashed lines negative values. The insets show the trajectories of the core centres, the instantaneous position of the vortex ring in each panel is denoted by black dots.

Velocity field

(In a frame moving with the ring)

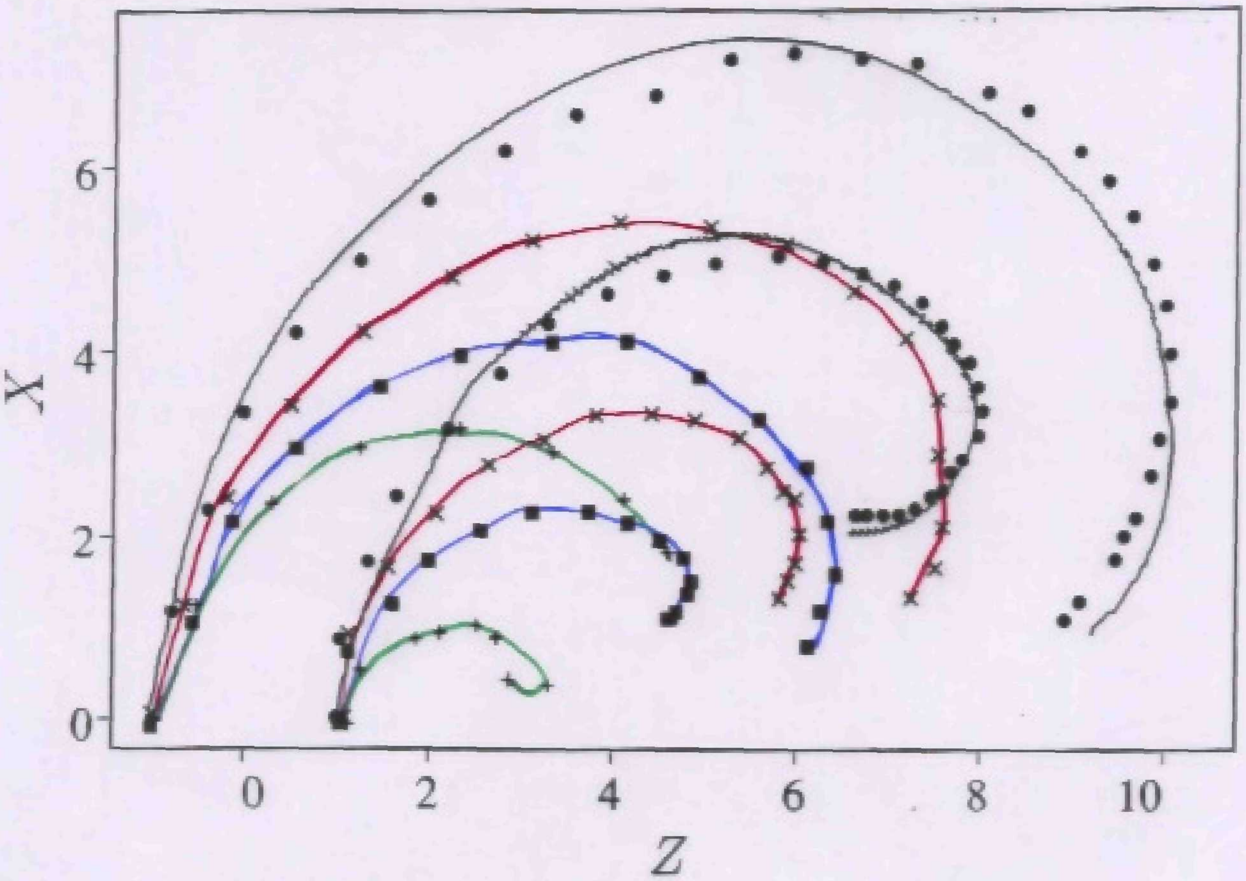


owing to the curved trajectory stagnation points are shifted inside the ring cores

vorticity filaments stripped from the vortex $\Longrightarrow$ ring depletion

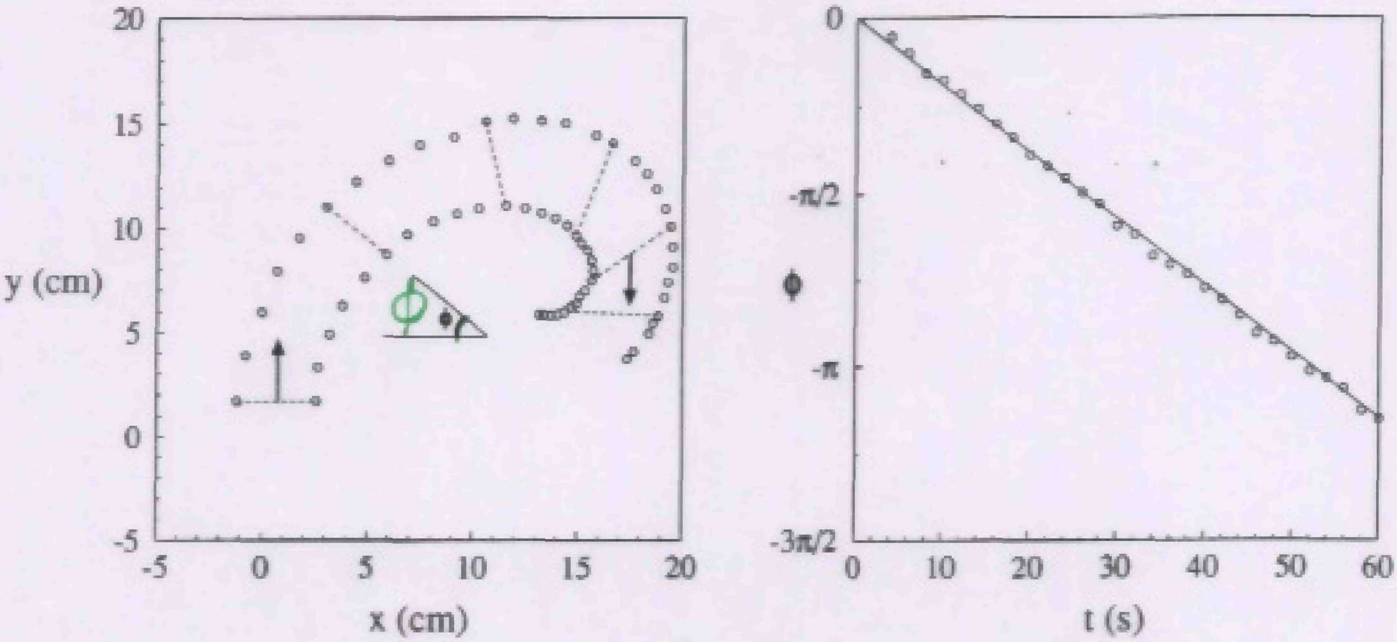
Ring trajectories

- Curvature enhanced as *Ro* decreases



• $Ro = 23$, —×— $Ro = 15.6$, —■— $Ro = 12.7$, —+— $Ro = 8.2$,
—— numerical simulation at $Ro = 23$.

Ring trajectory

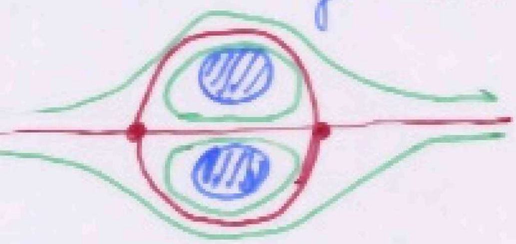


$\Omega_r = \frac{d\phi}{dt} = -\Omega$

- straight path in an inertial frame

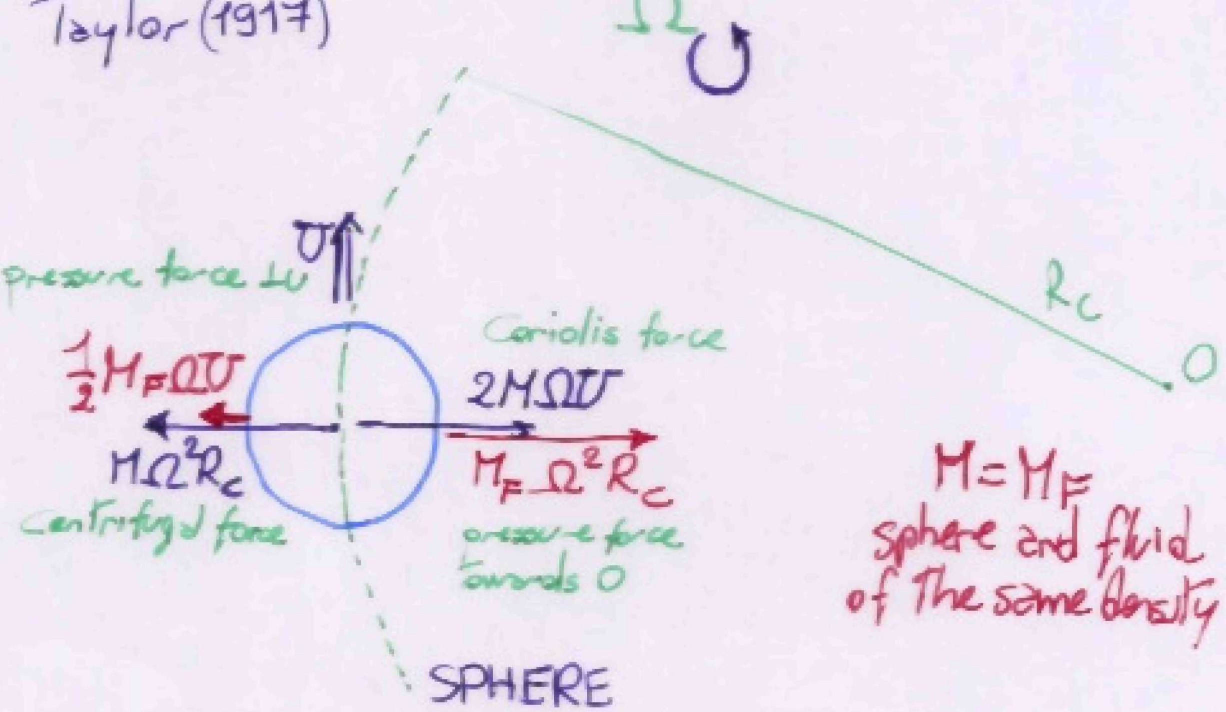
Exp.	Ω (s^{-1})	Ω_r (s^{-1})
1	0.06	-0.06 ± 0.002
2	0.09	-0.101 ± 0.002
3	0.11	-0.12 ± 0.002
4	0.17	-0.17 ± 0.002

"Thick" rings carry a spherical blob of fluid



(not thin rings) (Batchelor 1967)

From Taylor (1917)



net force $-\frac{3}{2}M\Omega U$ (Proudman 1916) (Taylor 1917)

sphere mass $M + \frac{M}{2} = \frac{3}{2}M$ (Batchelor 1967) in a non rotating fluid

added mass

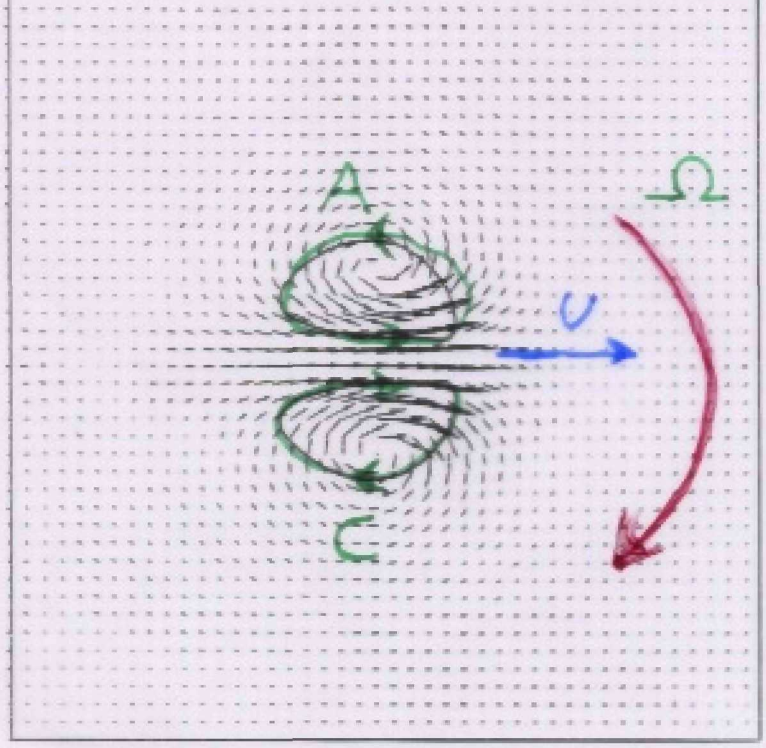
acceleration $a_\perp = -\Omega U$

centr. acc. $a_c \doteq U^2/R_c$

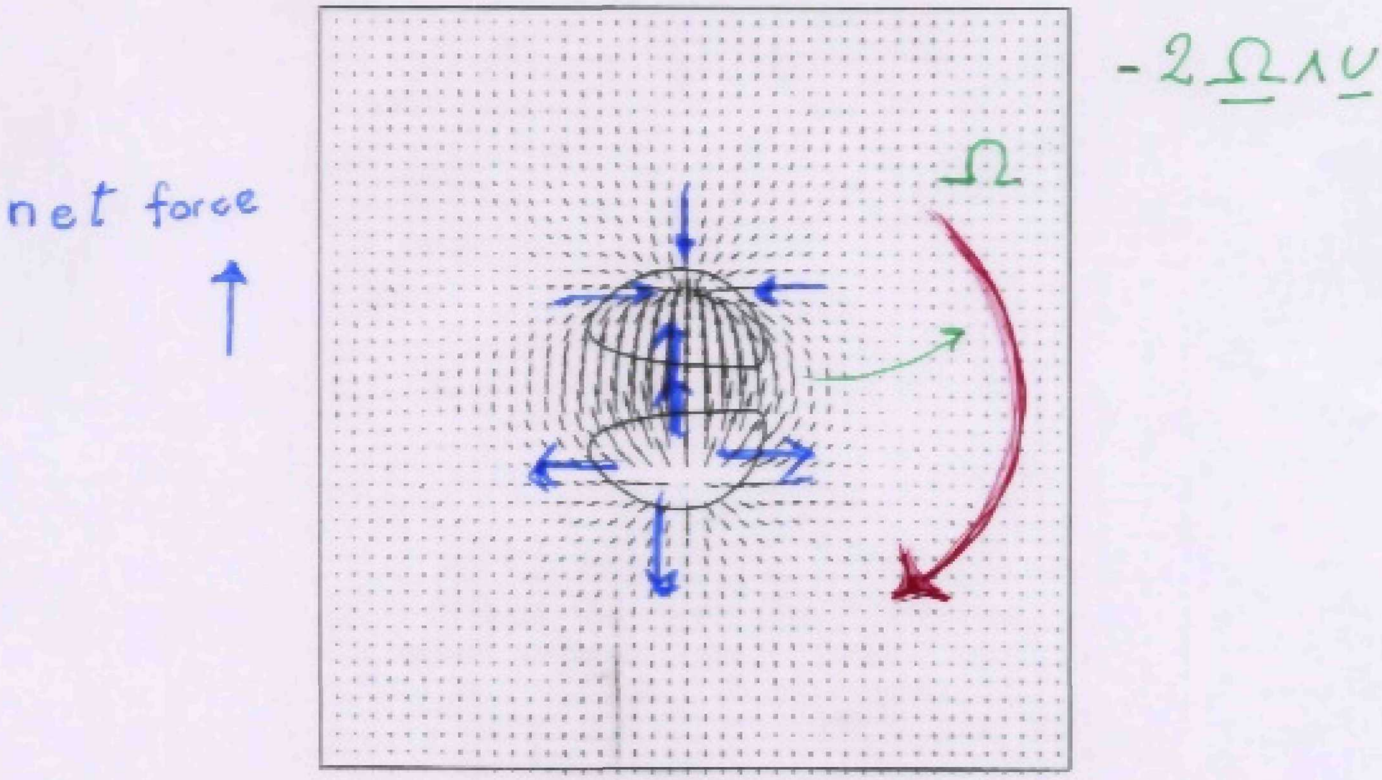
$\Big\}\; U = -\Omega R_c$

The sphere moves along a straight path in the inertial frame

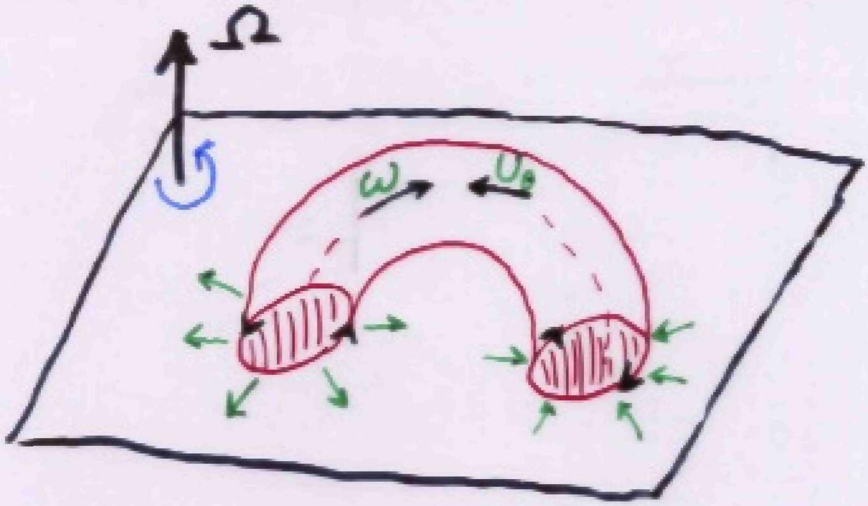
anti-cyclonic side shrinks
cyclonic side widens



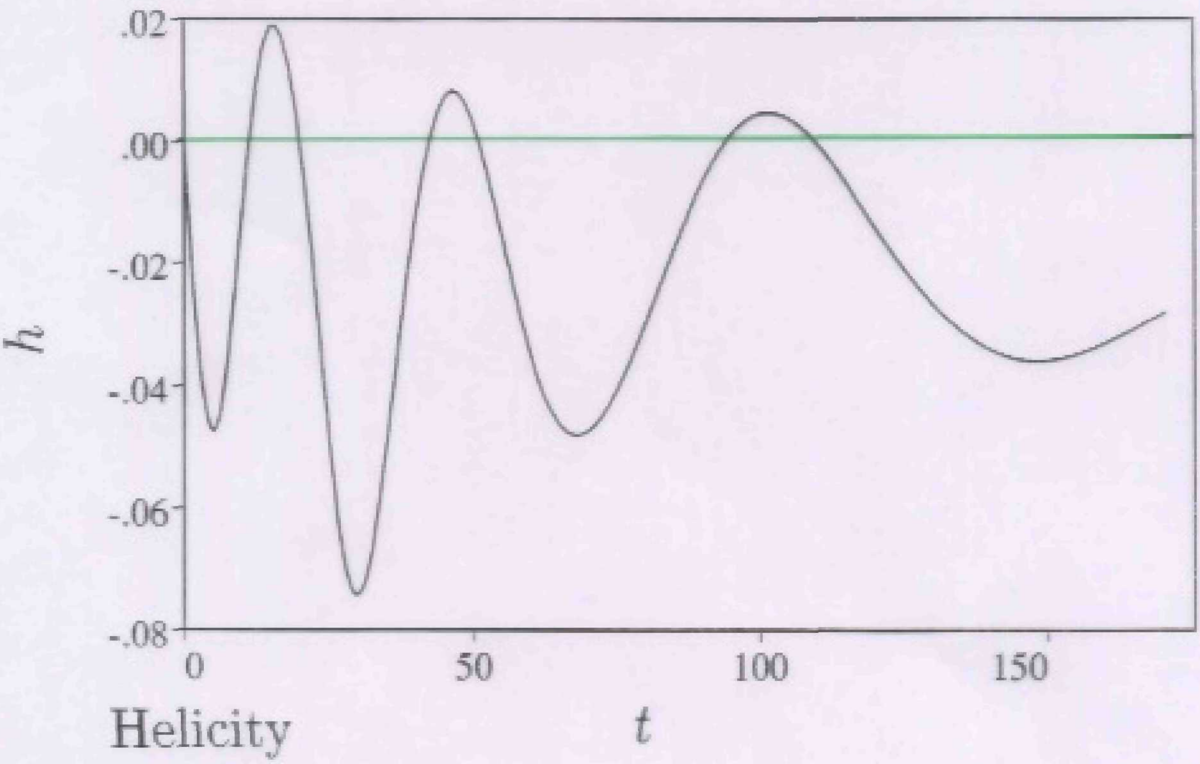
Velocity



Coriolis force

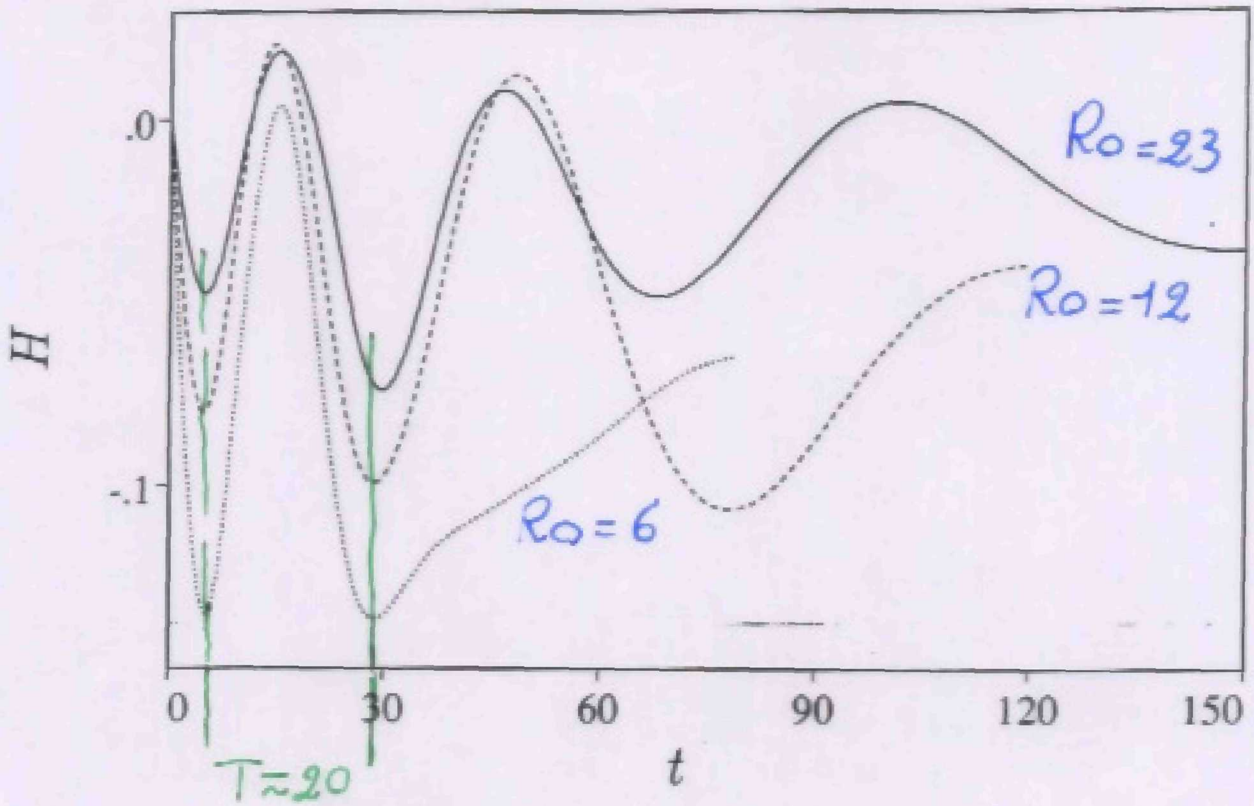


$$h = \int_V \underline{\omega} \cdot \underline{u} \, dV < 0$$



vorticity differences can not be steadily maintained along a vortex tube ⇒ Kelvin Waves

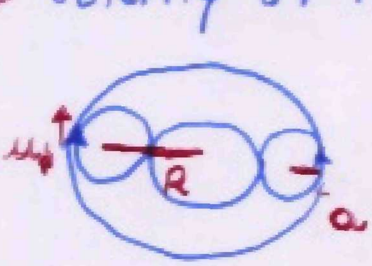
Waves in the ring



Oscillation period independent of $Ro \Longrightarrow$ Kelvin waves

oscillation period $T = R/c$

c celerity of the wave $c \simeq u_\phi$ (Kelvin 1880)

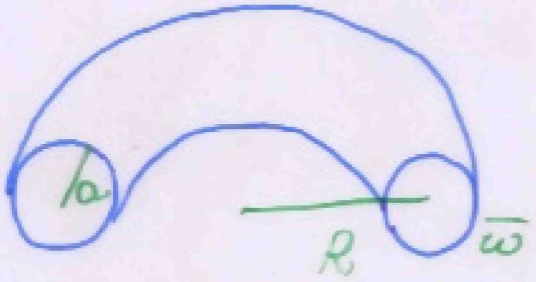


$u_\phi \sim \frac{\Gamma}{a}$

$T \sim aR/\Gamma$

non dimensional time

$T_{ad} = T\frac{\Gamma}{R^2} \sim a/R$



from Saffman (1982) $c \simeq .417\,\bar{\omega} a$

from in. con. $\bar{\omega} \simeq 2$ $a \simeq .4$

$$\Downarrow$$

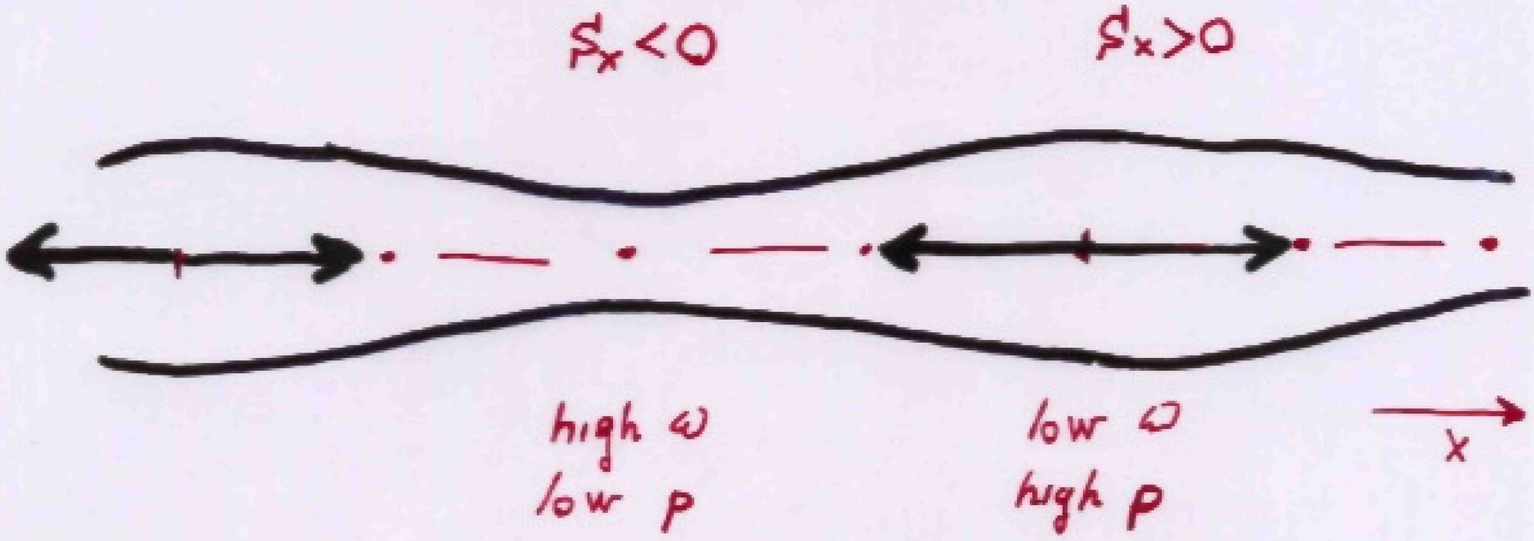
$$c \simeq .33$$

Wave period

$$T = 2\pi R/c \simeq 18$$

Kelvin waves

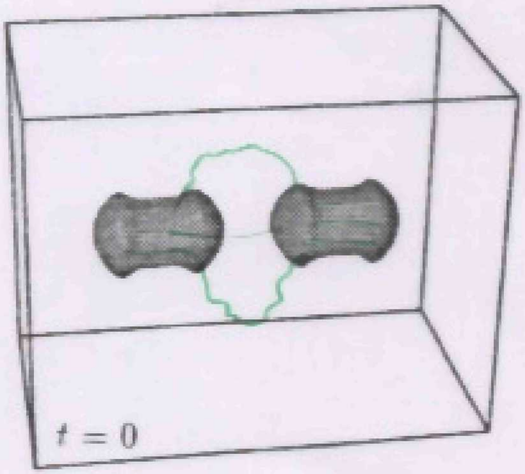
Non uniform vortex with constant circulation Γ

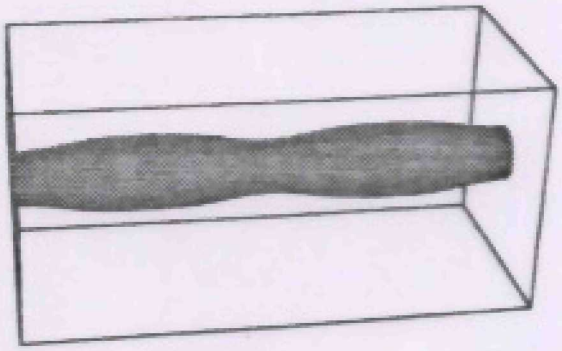


- axial pressure gradient
  - extensional strain in fat sections
  - compressive strain in narrow sections

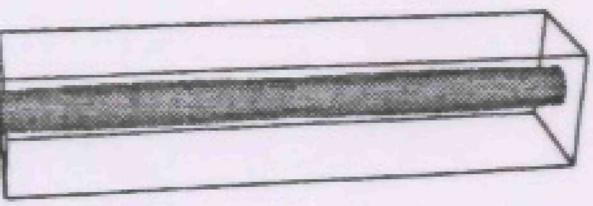
Aligned strained sticks

- $Re_\Gamma = 200$, $Re_L / Re_\Gamma = 0.06$

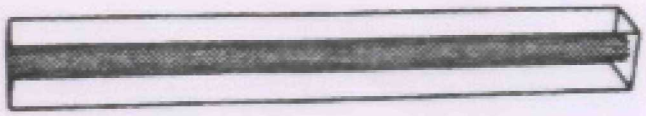




$t = 8$, $L_x = 6.61$, $L_y = L_z = 3.11$.



$t = 16$, $L_x = 10.93$, $L_y = L_z = 2.42$.



$t = 24$, $L_x = 18.07$, $L_y = L_z = 1.88$.

- Burgers vortex $\Rightarrow$ waves are compatible with stretching
- Uniform stretching $\Rightarrow$ high velocities incompatible with turbulence statistics

Wave mechanism works for structures not aligned

Angled short vortices ('sticks')

- $Re_\Gamma = 200$, $S_0 = 0$. (no strain)

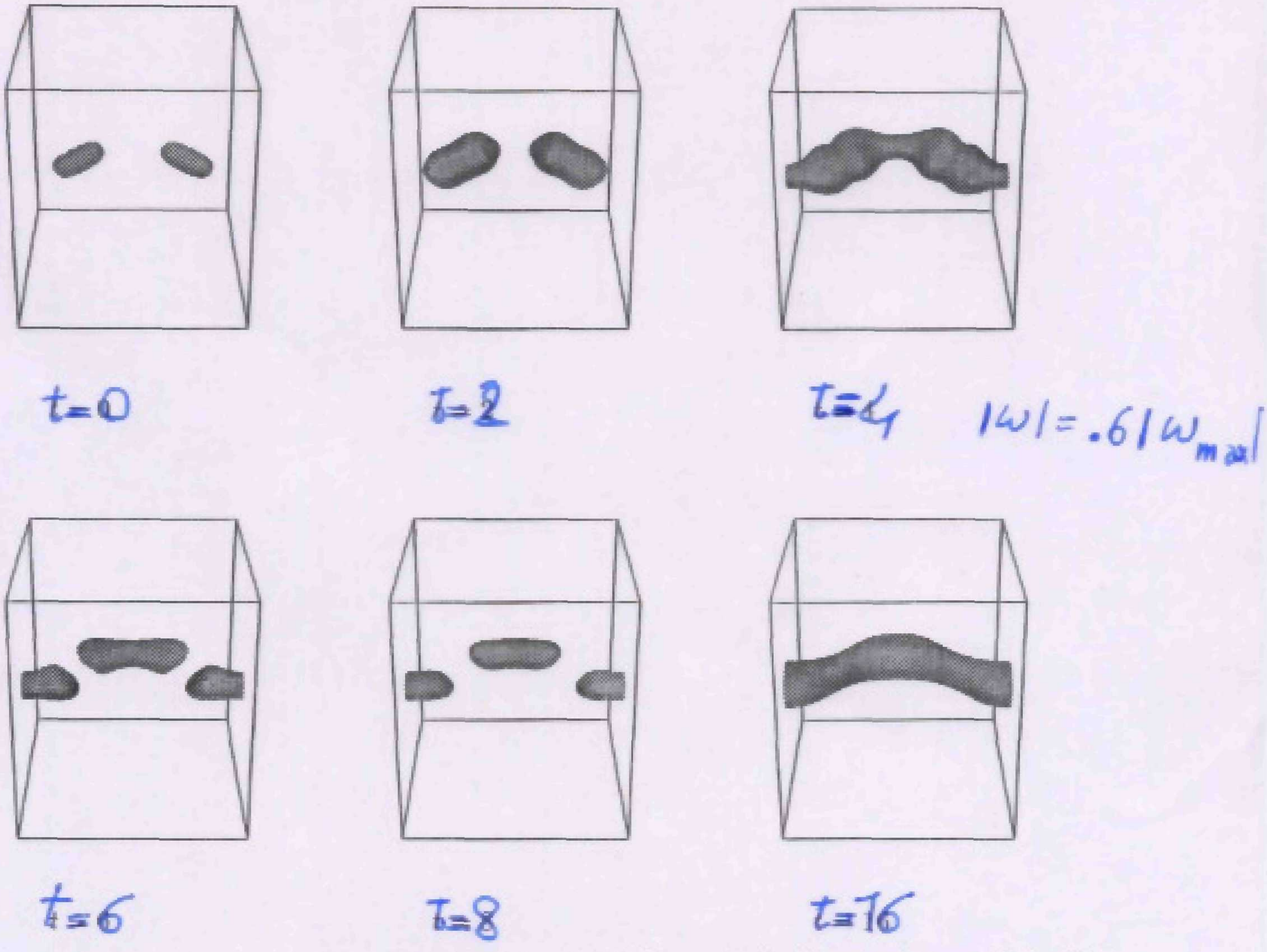


FIGURE 4. Iso-surfaces of vorticity magnitude for angled sticks at $Re_\Gamma = 200$. Iso-surface is always drawn at $|\omega| = 0.6 \mid \omega_{max}(t) \mid$. The initial angle between stick axes is 120^o. ($64 \times 64 \times 64$ grid).

- Uniform diffusing vortex

Conclusions

(ring motion // Ω)

- 'Low rotations': the ring slows down and weakens

- 'High rotations': formation of oblique shear layers

- 'Very high rotations': 2D Taylor column

- 3D: small misalignment very important.

Related papers

Vortex rings { Verzicco et al. JFM, 317, 1996
Eisenga et al. JFM 354, 1998

Vortex filaments in "model" turbulence { Verzicco Jiménez & Orlandi JFM, 299, 1995
Verzicco & Jiménez JFM, 394, 1999

Conclusions

(Ring motion $\perp \Omega$)

- The ring follows a curved trajectory

- Straight path in an inertial frame

- The ring weakens in time owing to a peeling process

- The ring remains 'toroidal' up to the end of the evolution

- Future work: higher rotation rates