The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

International Atomic Energy Agency

SMR.1661 - 4

Conference on

VORTEX RINGS AND FILAMENTS IN CLASSICAL AND QUANTUM SYSTEMS

6 - 8 June 2005

Soliton Collisions and Hybrid Soliton Vortex-Ring Structures in Bose-Einstein Condensates

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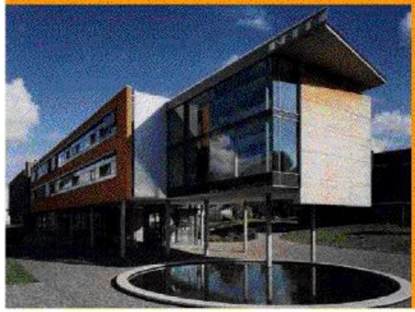
Soliton collisions and hybrid soliton vortex-ring structures in Bose-Einstein condensates

BEC solitons under the influence of transverse confinement

Joachim Brand

Max Planck Institute for the Physics of Complex Systems, Dresden

In collaboration with



Stavros Komineas, Cambridge (UK)
Naomi Ginsberg, Harvard
Lene V. Hau, Harvard
William P. Reinhardt, UW Seattle

Solitary waves in the BEC

repulsive interactions

quasi-1D: Dark solitons

Nonlinear Schrödinger equation

+ transverse dimensions

Waveguide geometry:

Solitons, vortex rings and solitonic vortices

THIS TALK

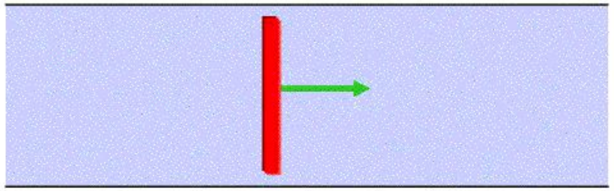
+ confinement

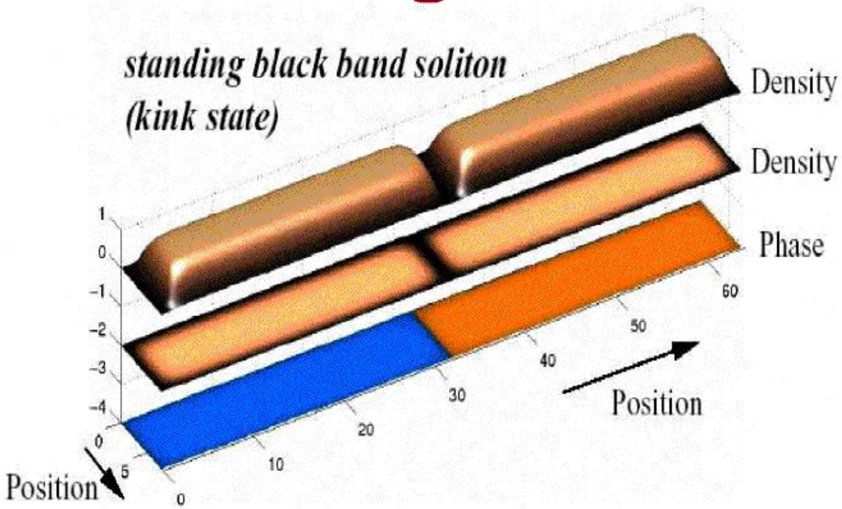
3D bulk: Vortex rings and rarefaction pulses

Jones and Roberts '82

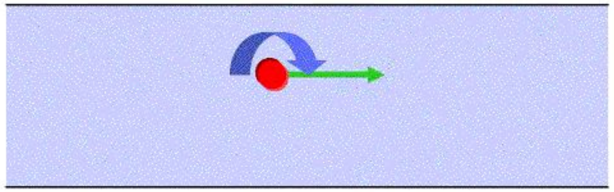
Solitary waves in 2D waveguides

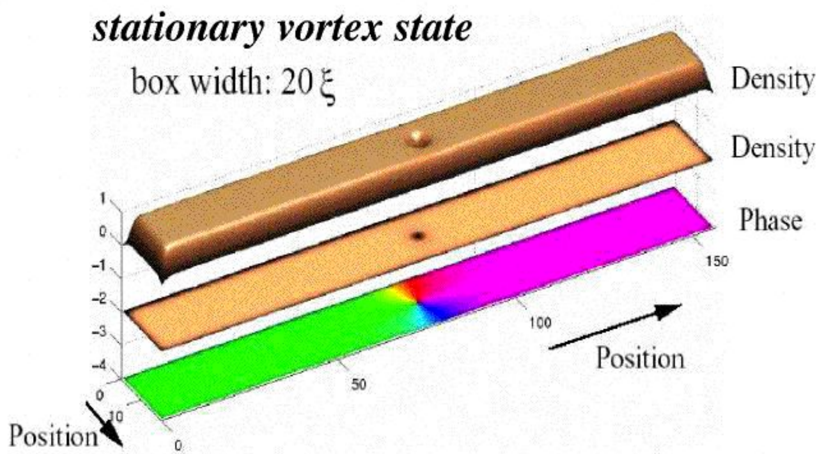
solitary wave front (soliton)



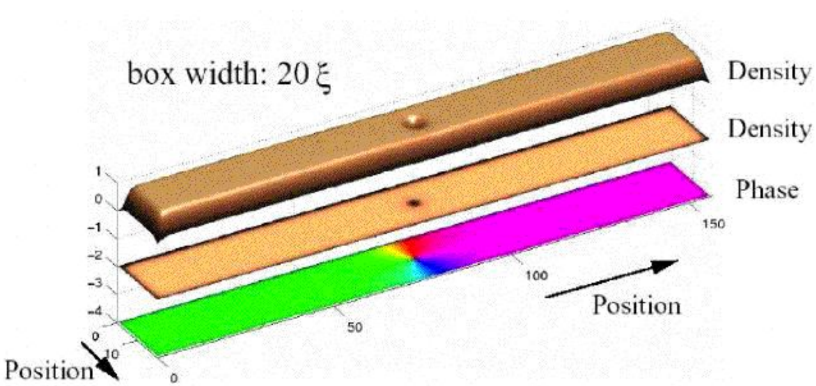


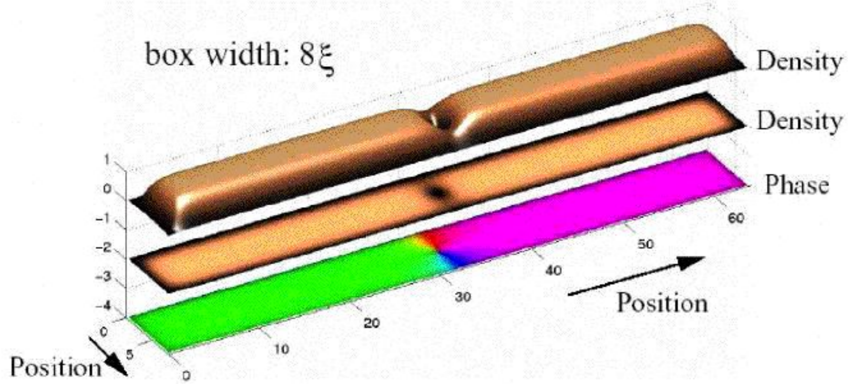
vortex (solitonic vortex)

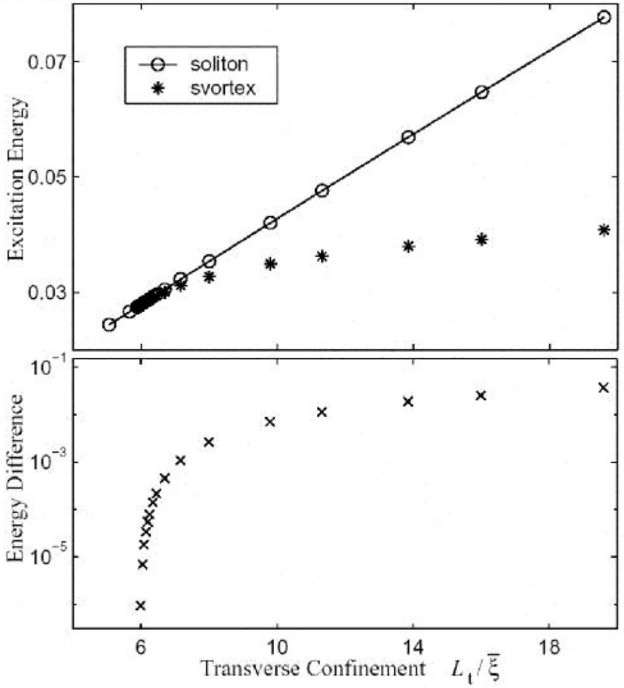


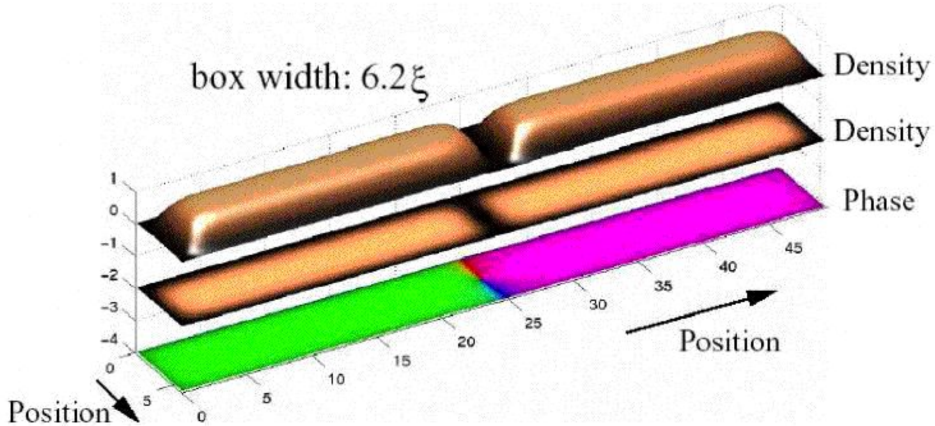


Transition from a vortex to a soliton







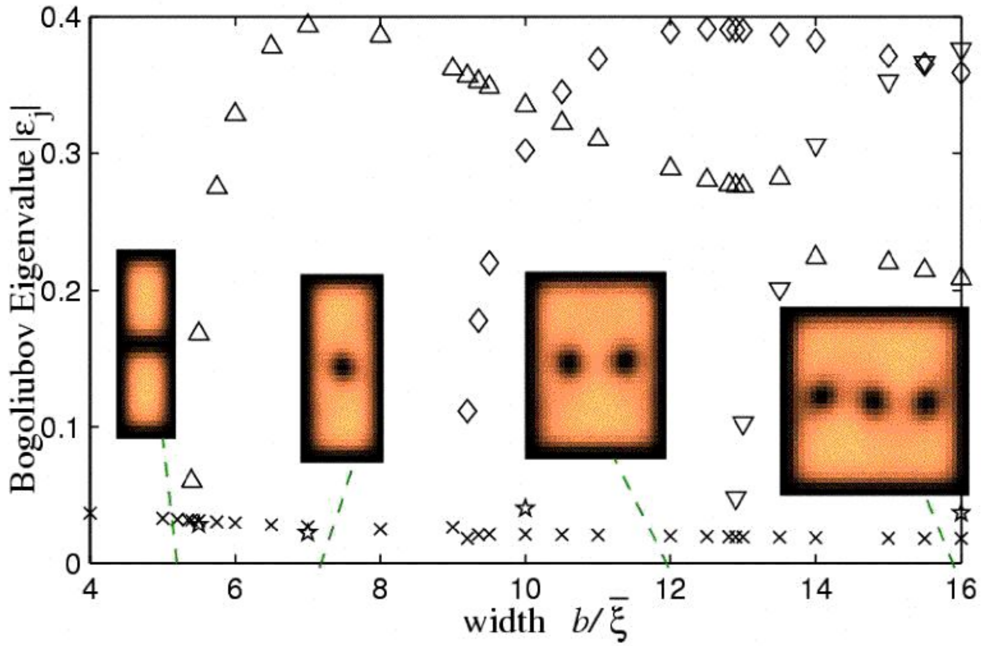


J. Brand, W.P. Reinhardt, PRA 65, 043612 (2002)

Is the soliton stable?

Linear stability analysis with Bogoliubov equations

„Snake instability" can promote the decay of a black soliton stripe into any number of vortices, in particular into a single solitonic vortex!



J. Brand, W.P. Reinhardt, PRA **65**, 043612 (2002)

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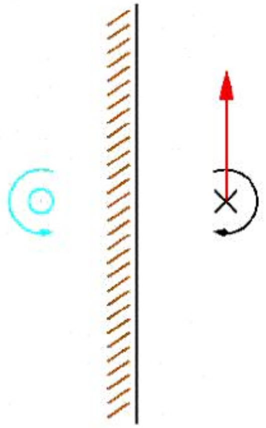
Does a vortex in a channel move?

Vortex dynamics induced by confining walls can be solved analytically with the image vortex method!

Solve for the dynamics and velocity field of a single vortex in front of a wall.
Approximation: assume incompressible fluid



Idea: arrange image vortex such that the boundary condition for the velocity field is matched.

Solution:
The vortex will move in the velocity field induced by the image vortex

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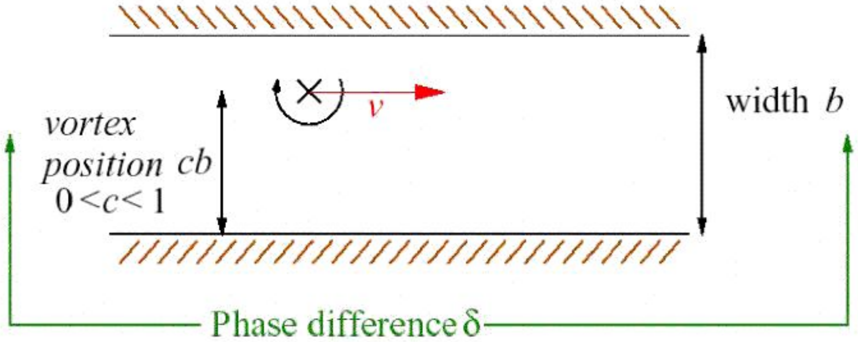
Image vortex method

The channel walls generate a doubly infinite array of image vortices. Using a conformal mapping, a simple solution for the vortex velocity can be found:

$$v = -\frac{\pi\xi}{\sqrt{2}b} c_{\max} \cot(c\pi)$$



The phase difference δ between the opposite far ends of the channel can easily be computed by integrating the velocity field.

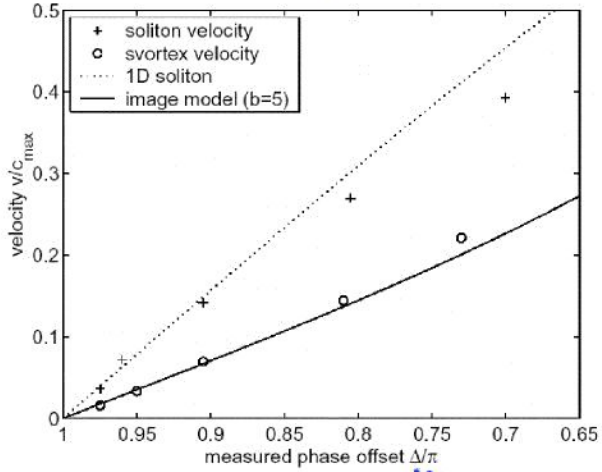
$$\delta = 2\pi(1 - c)$$

Together this yields a **phase-offset—velocity relation**:

$$v = -\frac{\pi\xi}{\sqrt{2}b} c_{\max} \cot(\pi - \delta/2)$$

Compare with the phase-offset—velocity relation for solitons in the 1D NLS:

$$v = c_{\max} \cos(\delta/2)$$



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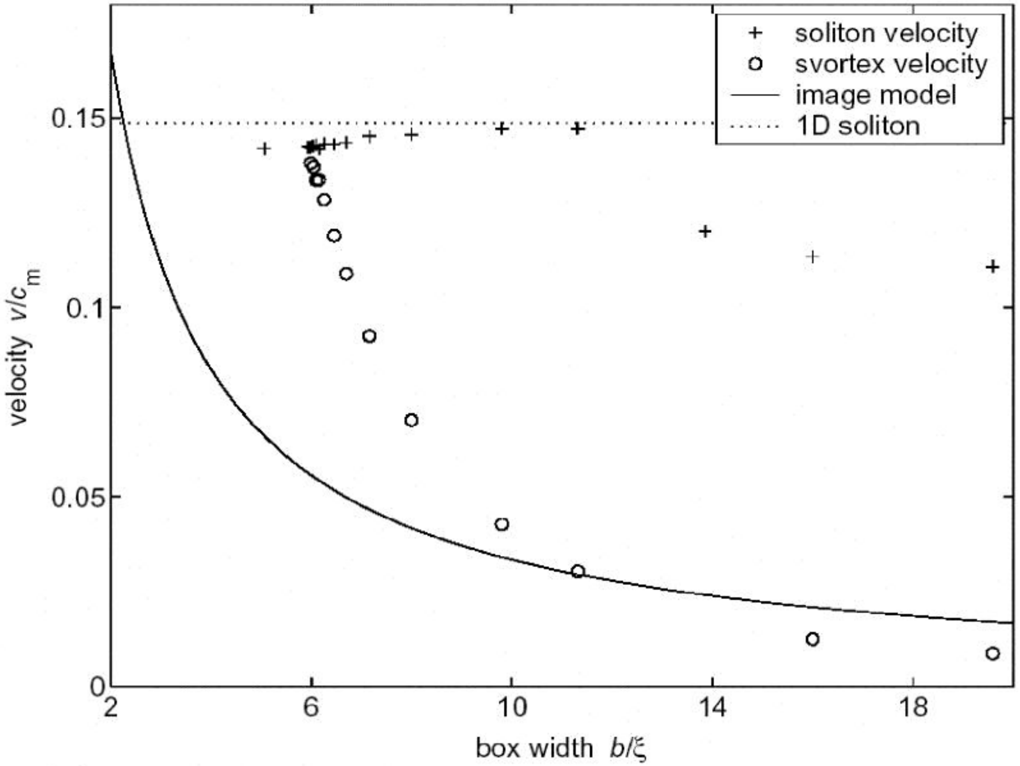
Solitonic vortex in a 2D channel

Influence of confinement on the velocity



$$v = -\frac{\pi\xi}{\sqrt{2}b} c_{\max} \cot(\pi - \delta/2)$$

imprinted phase offset: 0.2π
measured (actual) phase jump: 0.9π
box size: b x $8b$

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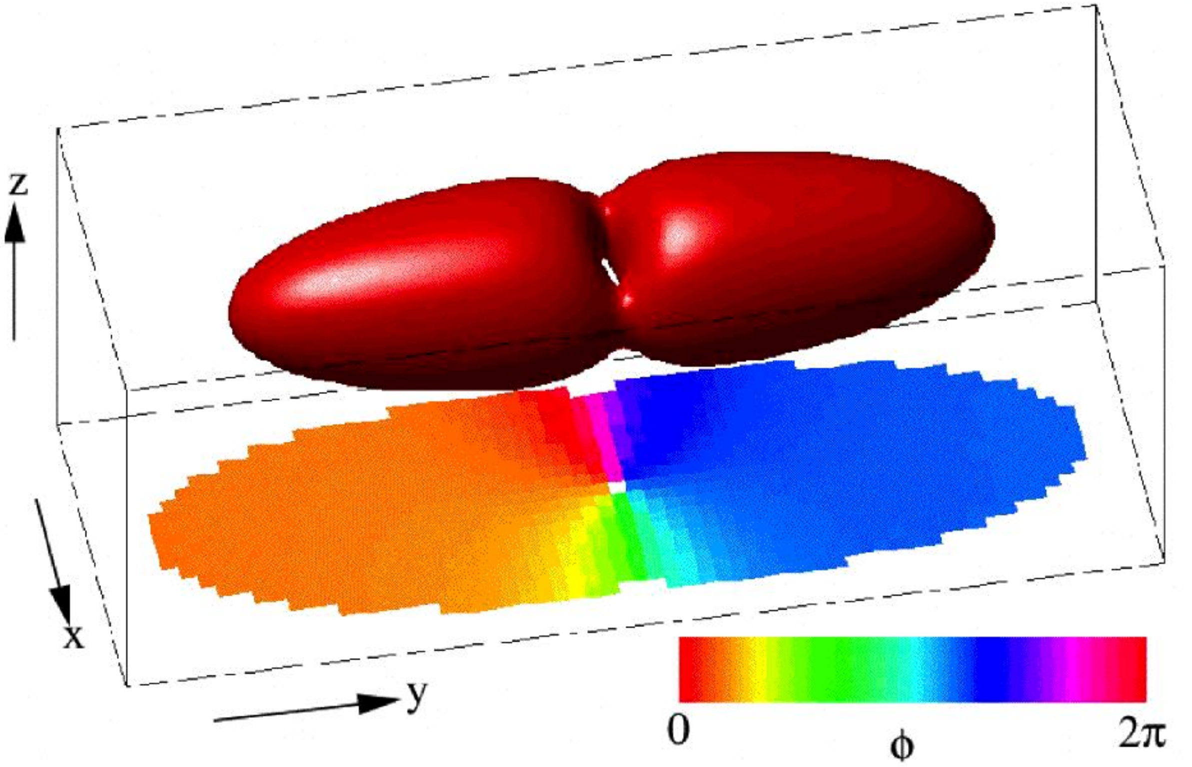
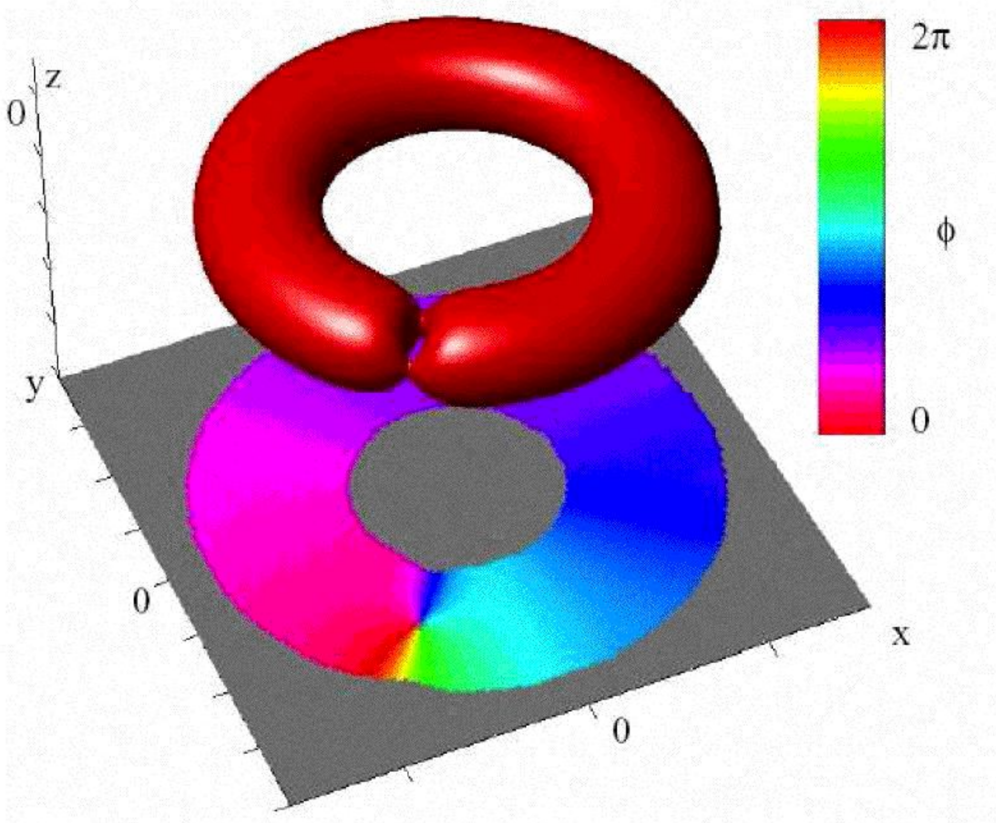
Solitonic Vortex generated by decay of a soliton sheet in a 3D harmonic trap



J. Brand, W.P. Reinhardt, PRA **65**, 043612 (2002)

Solitonic Vortex in a toroidal trap

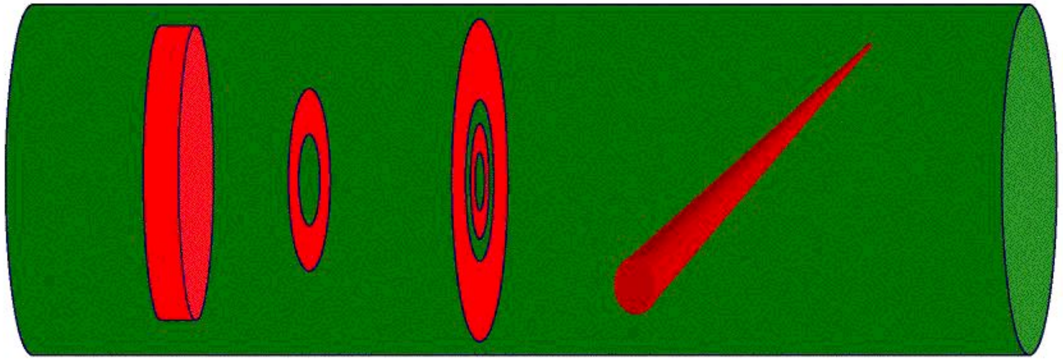
generated by stirring with a laser beam



J. Brand, W.P. Reinhardt, J. Phys. B (2001)

Solitary waves in 3D waveguides

axisymmetric non-axisymmetric



planar soliton solitonic vortex

vortex ring

double ring

more ...

Solitary waves in 3D waveguides

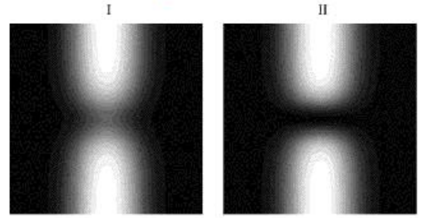
rescaled Gross-Pitaevskii equation

$$i\,\frac{\partial\Psi}{\partial t} = -\frac{1}{2}\,\Delta\Psi + \frac{1}{2}\,\rho^2\,\Psi + 4\pi\gamma\,|\Psi|^2\Psi,$$

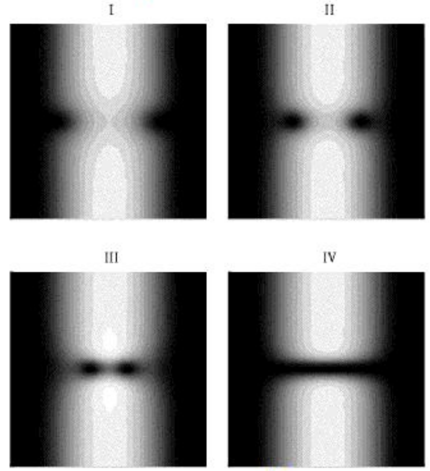
dimensionless parameter:

$$\gamma = n_1 a_s$$

γ=1



γ=10



Regimes:

$\gamma \ll 1$ quasi-1D -> NLS dark solitons

$\gamma > 1$ 3D Thomas-Fermi

S. Komineas, N. Papanicolao PRL **89**, 070402 (2002);
PRA **67** 023615 (2002); PRA **68**, 043617 (2002)

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Solitary waves in 3D waveguides

Energy

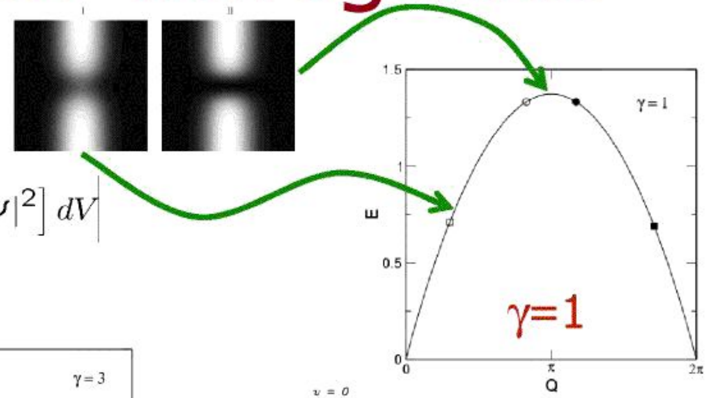
$$E = W - W_0$$

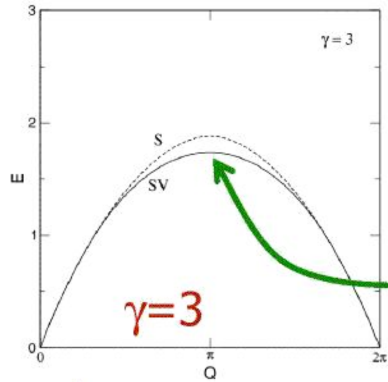
$$W = \frac{1}{2}\int \left[|\nabla\Psi|^2 + \rho^2|\Psi|^2 + 4\pi\gamma|\Psi|^4 - 2\mu|\Psi|^2 \right] dV$$

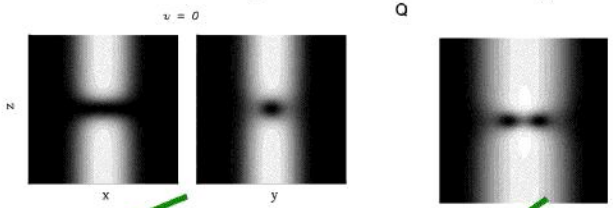
Impulse (canonical momentum)

$$Q = \int (n - n_0)\frac{\partial\phi}{\partial z} dV$$

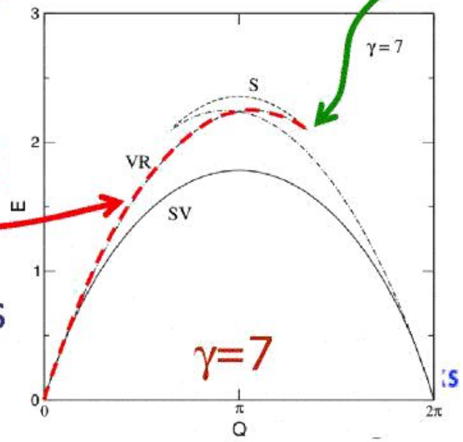
$$v = \frac{dE}{dQ}$$







THIS TALK



Regimes:

$\gamma<1.5$ solitons (S), single branch

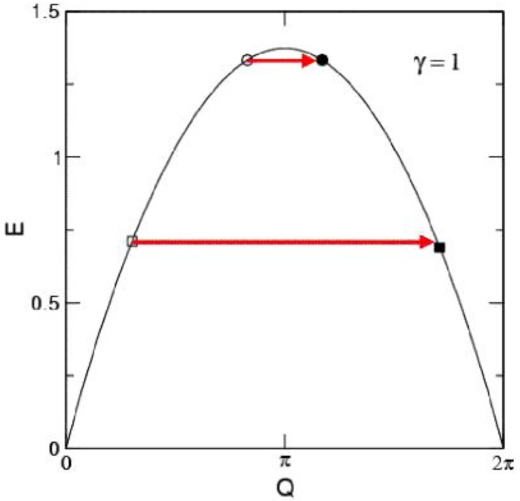
$\gamma>1.5$ solitons + solitonic vortex (SV)

$\gamma>4$ solitons + vortex rings (VR) + solitonic vortices

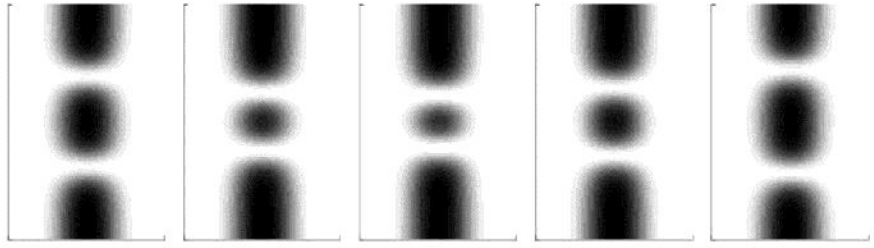
$$\gamma = n_1 a_s$$

Solitons: Elastic collisions

$\gamma = 1$

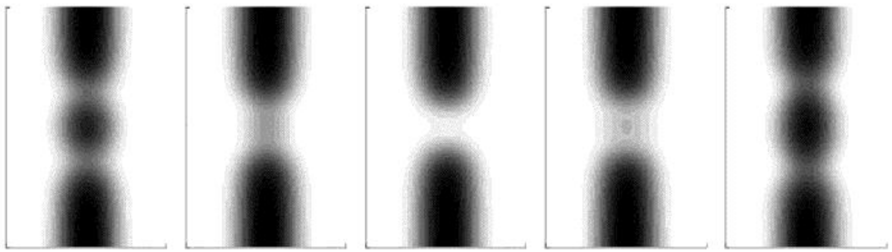


$v = 0.16\ v_s$



elastic bounce off

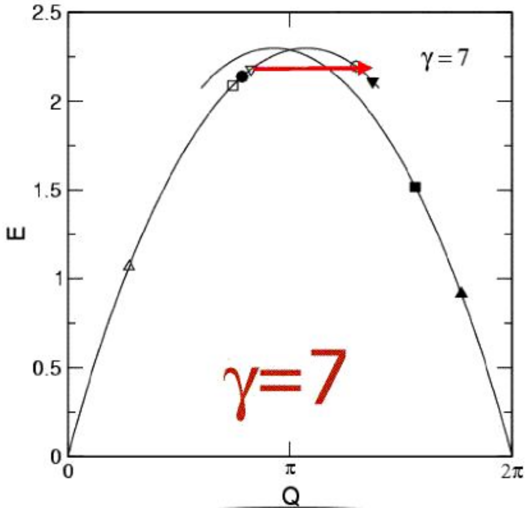
time



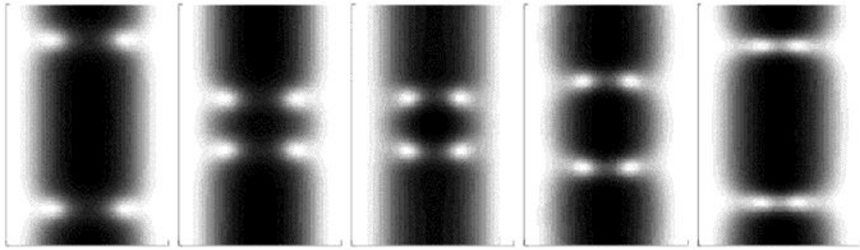
$v = 0.63\ v_s$

elastic pass through

Vortex rings: Collisions



v=0.23 v_s



elastic bounce off

time

Vortex Rings 05 Trieste

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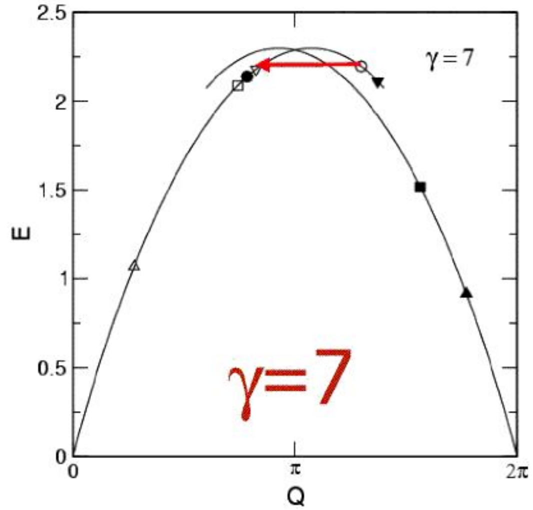
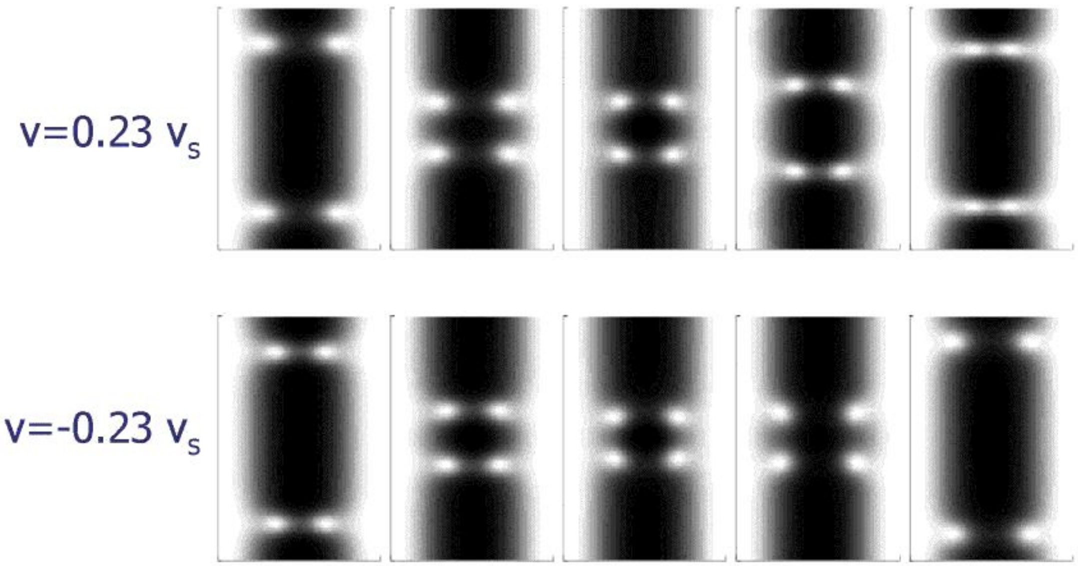
Vortex rings: Collisions

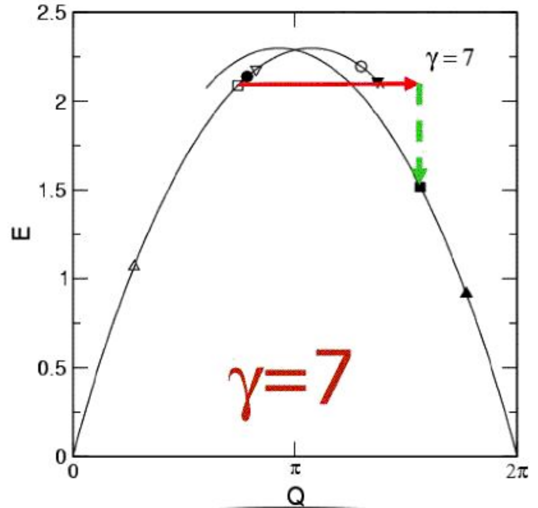




elastic bounce off

time →

Vortex rings: Collisions

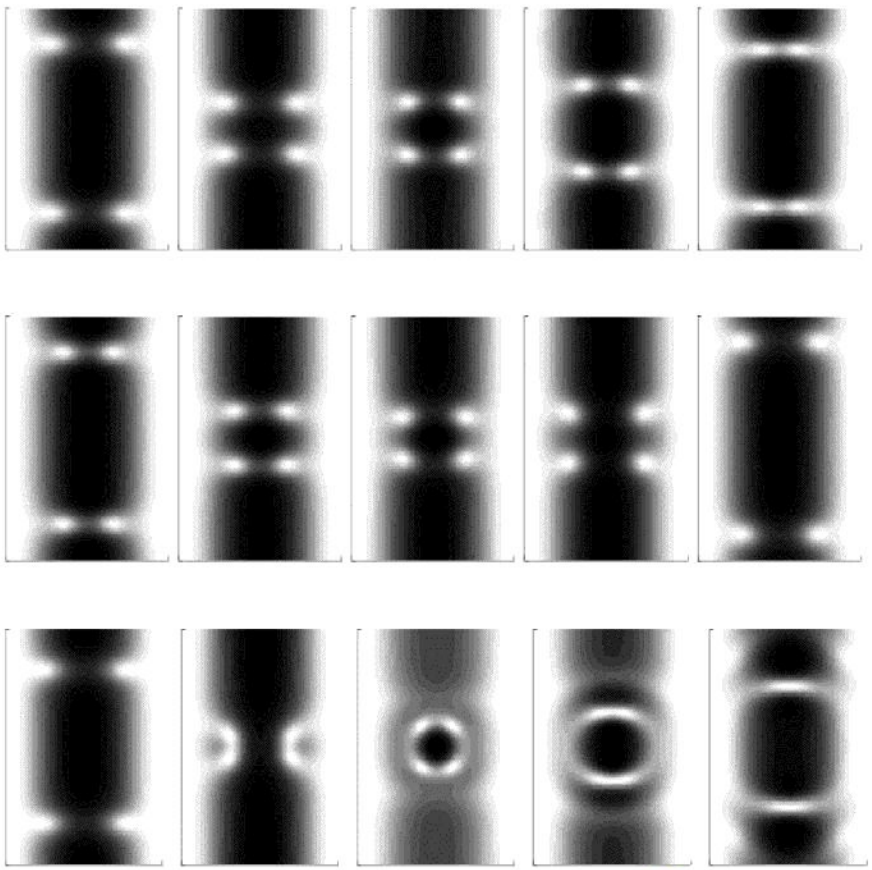


v=0.23 v_s

v=-0.23 v_s

v=0.42 v_s

inelastic!



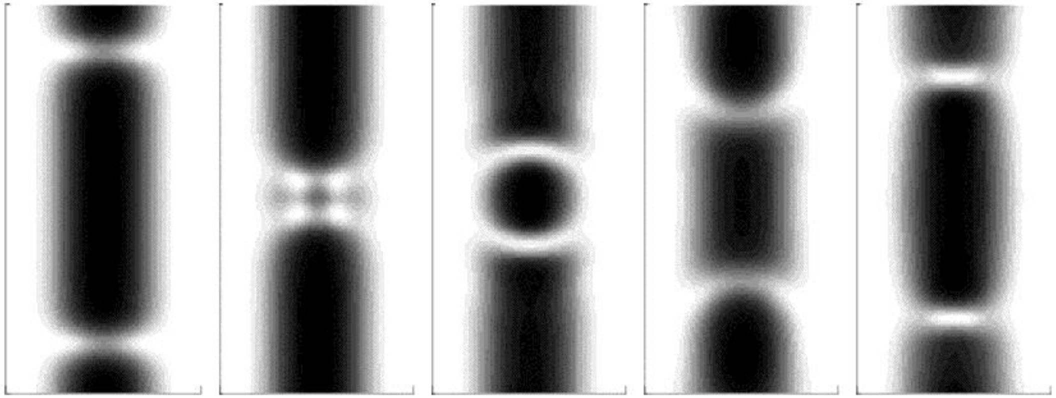
time

Vortex Rings 05 Trieste

„Spherical" Shells ?

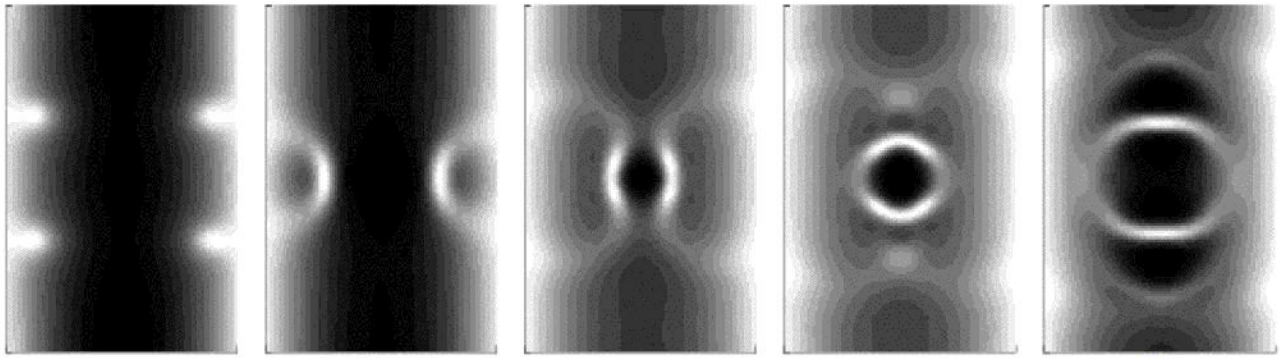
$\gamma = 3$

$v = 0.31\ v_s$

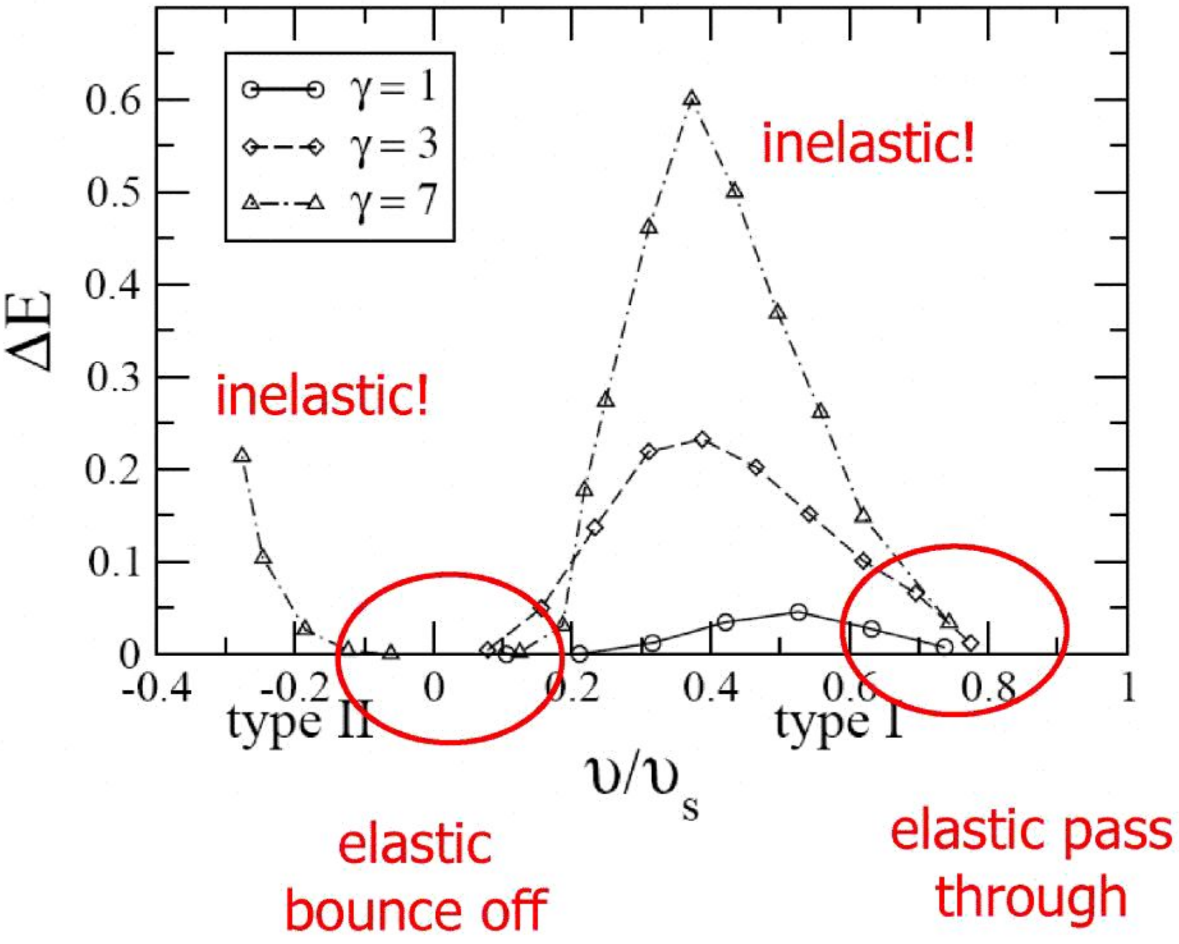


$\gamma = 20$

$v = 0.38\ v_s$



Energy loss during collision



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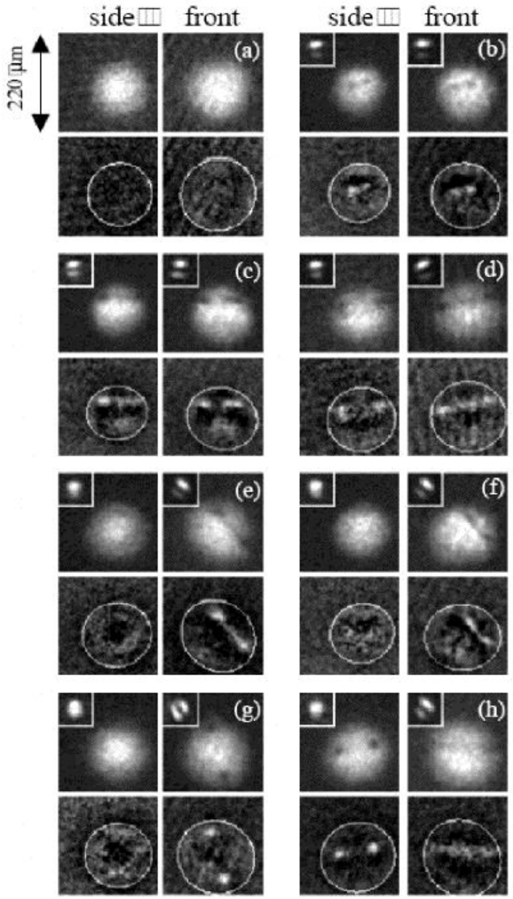
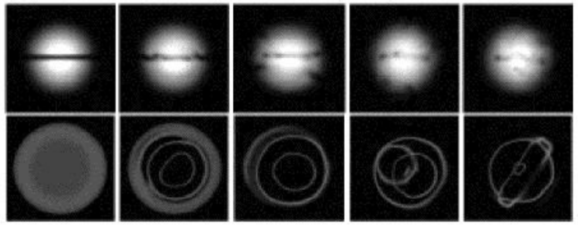
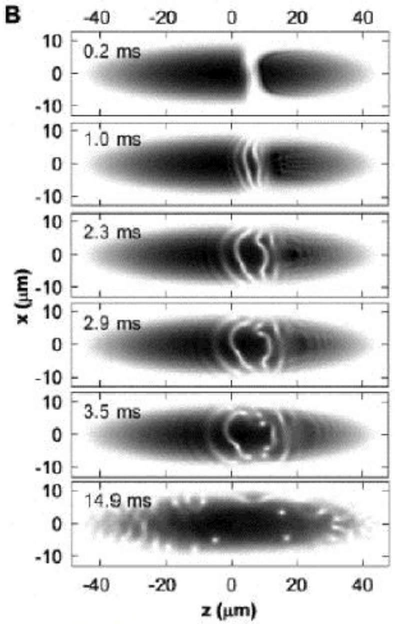
BEC Experiments on vortex rings

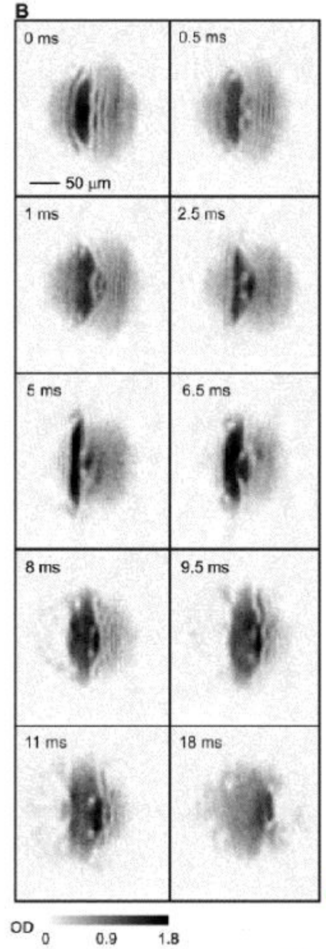
Cornell group JILA 2001

Hau group Harvard 2001

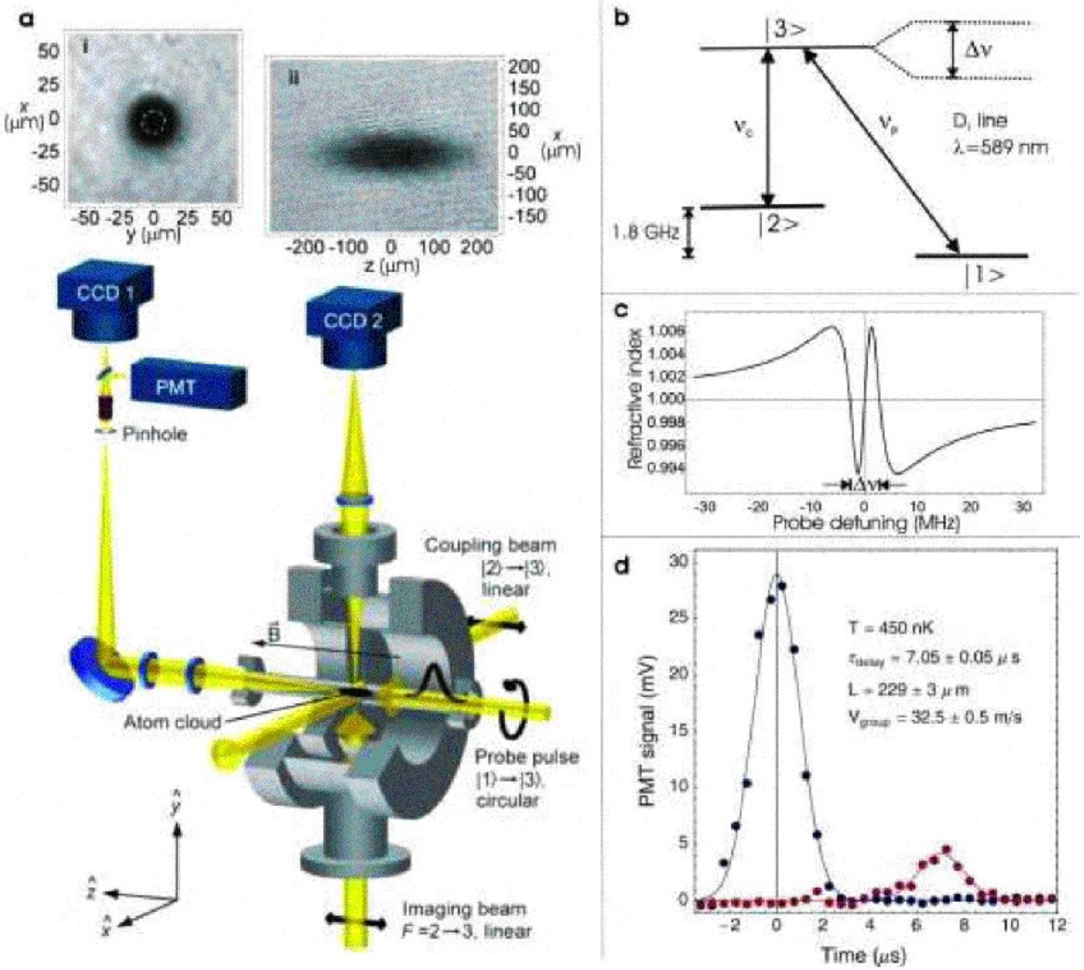








Experiments: BEC and slow light

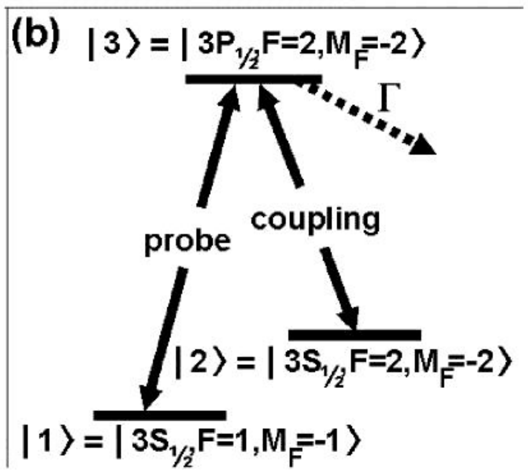


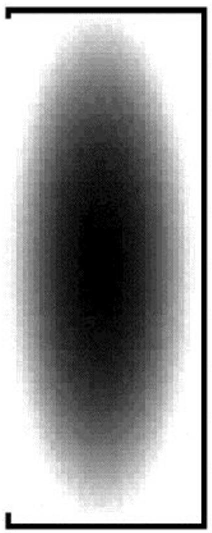
L. V. Hau *et al.* Nature **397**, 594(1999).

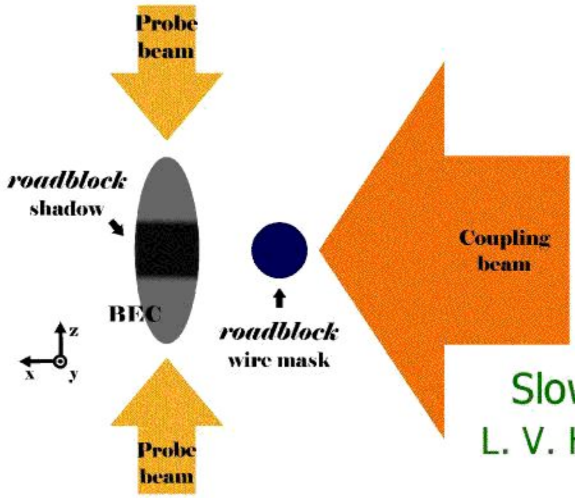
Dutton et al. Europhysics News (2004) Vol. 35 No. 2

New Experiment: Density voids

Electromagnetically induced transparency (EIT) is used to spatially compress the light pulses of two probe lasers at the shadow of a thin wire mask.



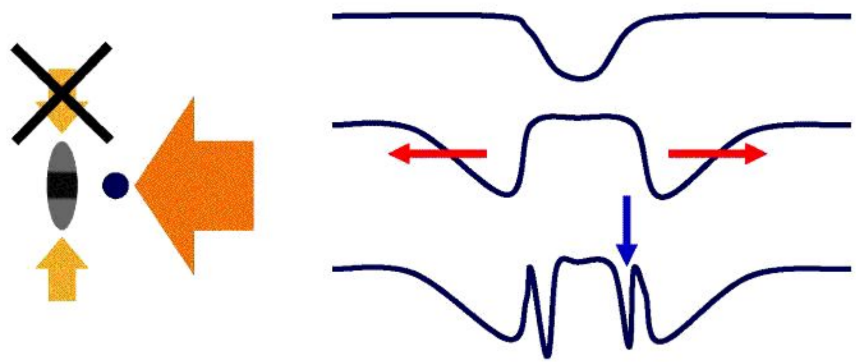




Slow light in BECs by EIT:
L. V. Hau *et al.* Nature **397**, 594 (1999).

Density voids are created by transfering atoms into the untrapped |2> state on a μ s time scale.

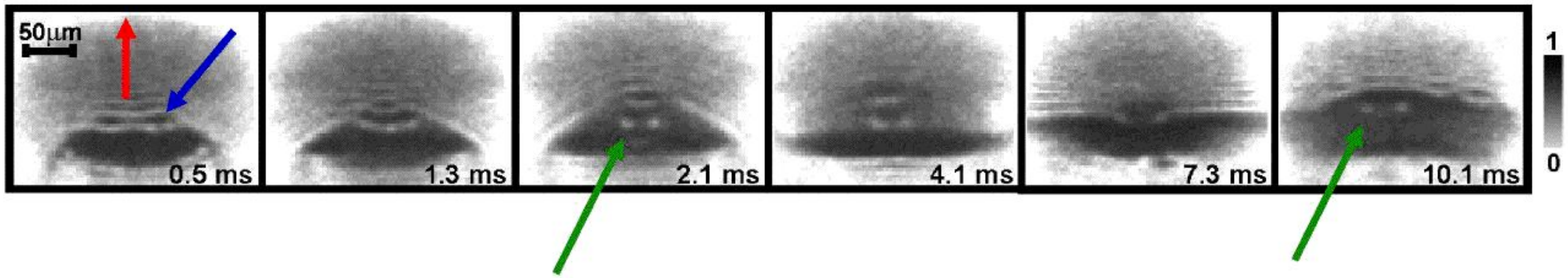
Control experiment: single defect



A defect is created

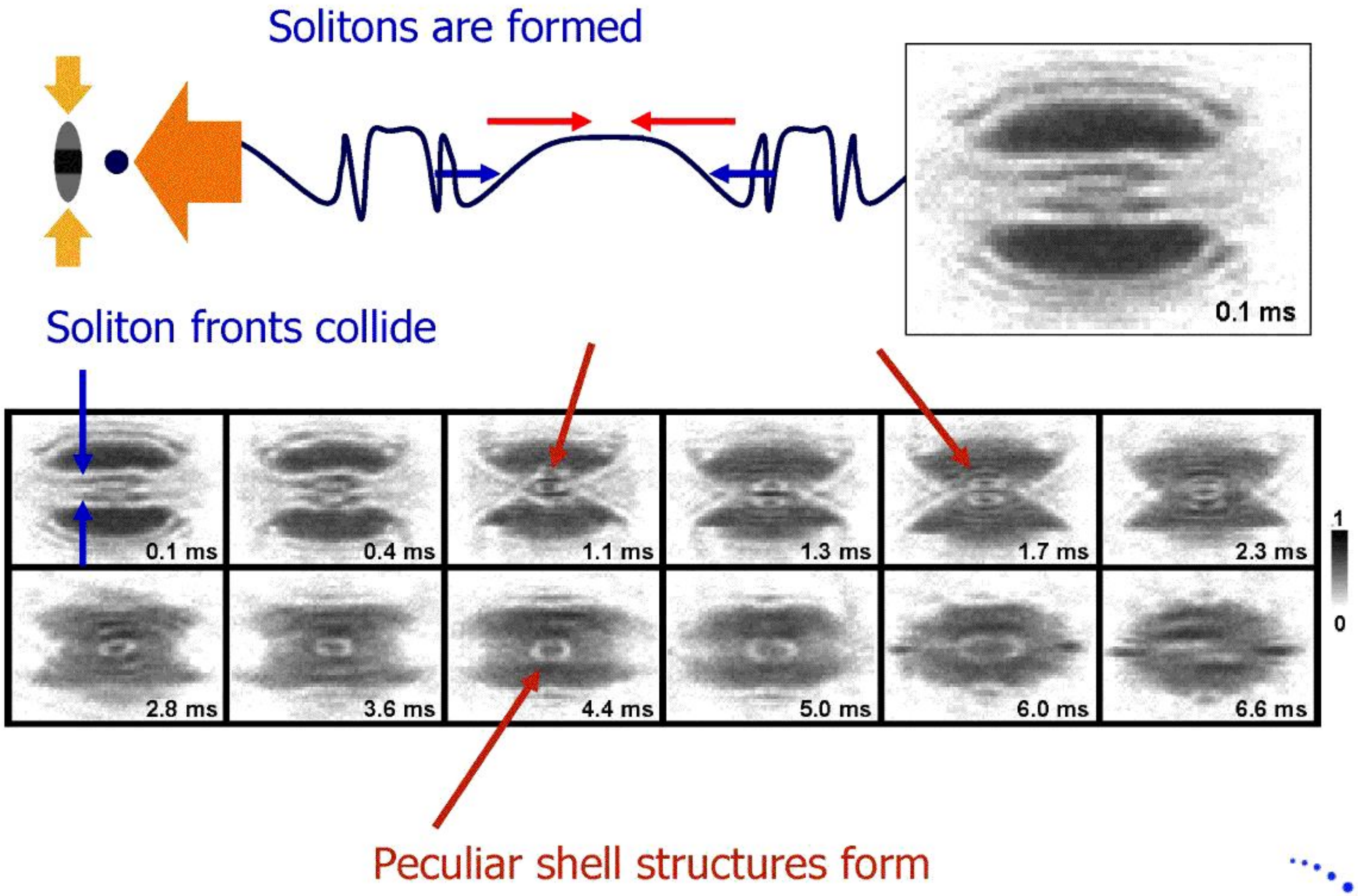
Shock fronts develop

Solitons are formed



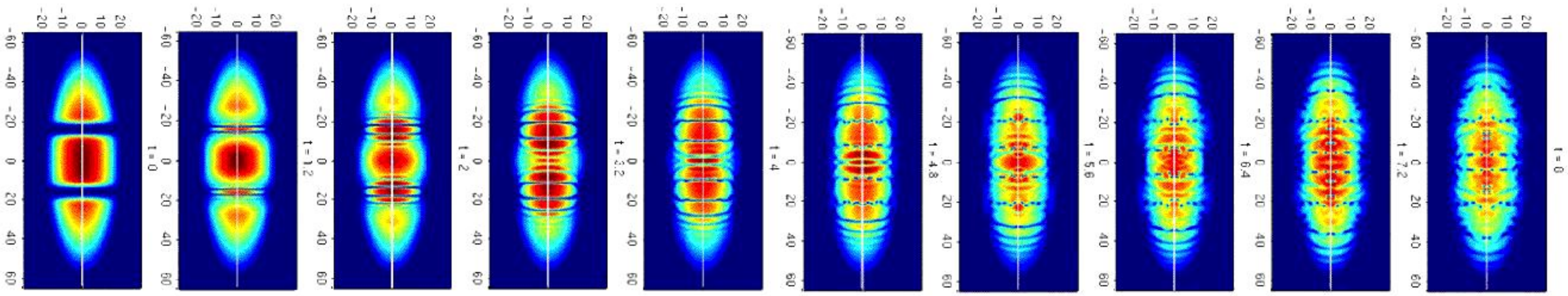
Vortex rings form as decay product of snaking instability

Collisions: double defect



Simulations of the experiment

in-trap evolution

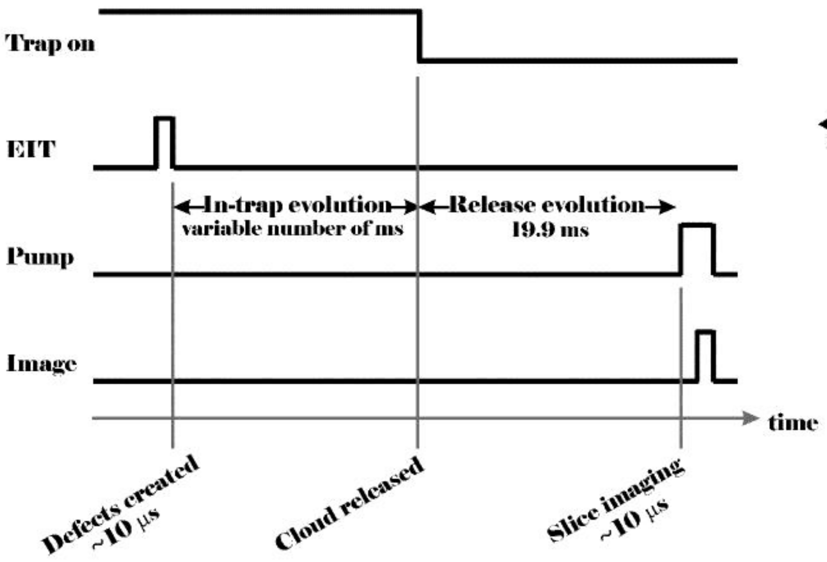


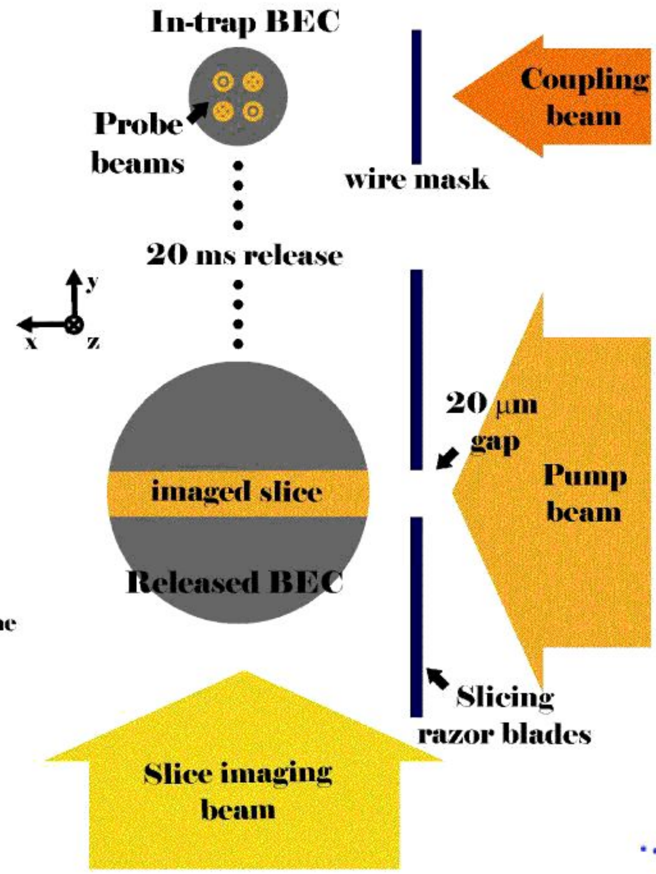
Where are the spherical structures?

Expansion evolution is important!

Evolving and imaging the BEC

Timing





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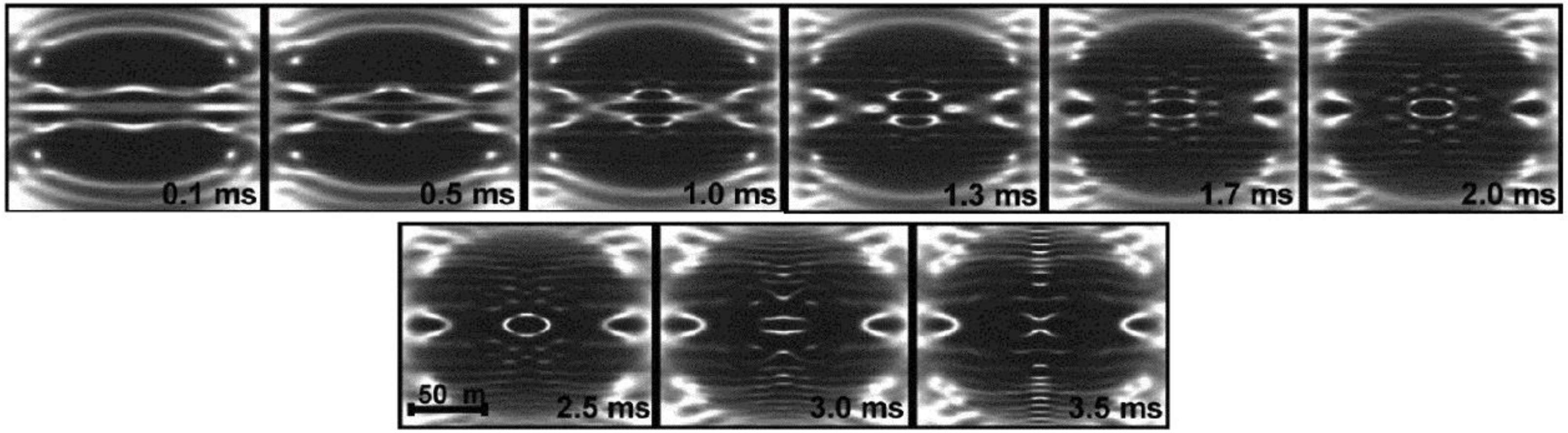
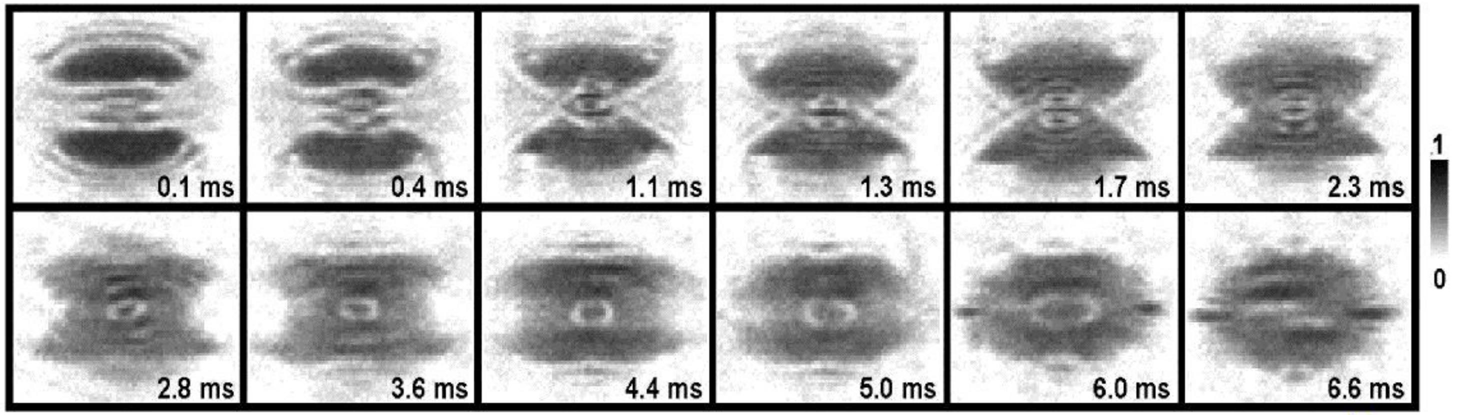
Simulations of the experiment

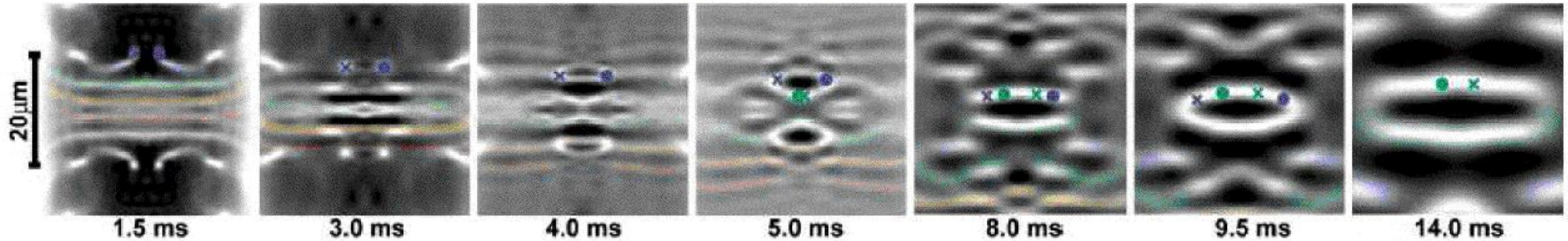




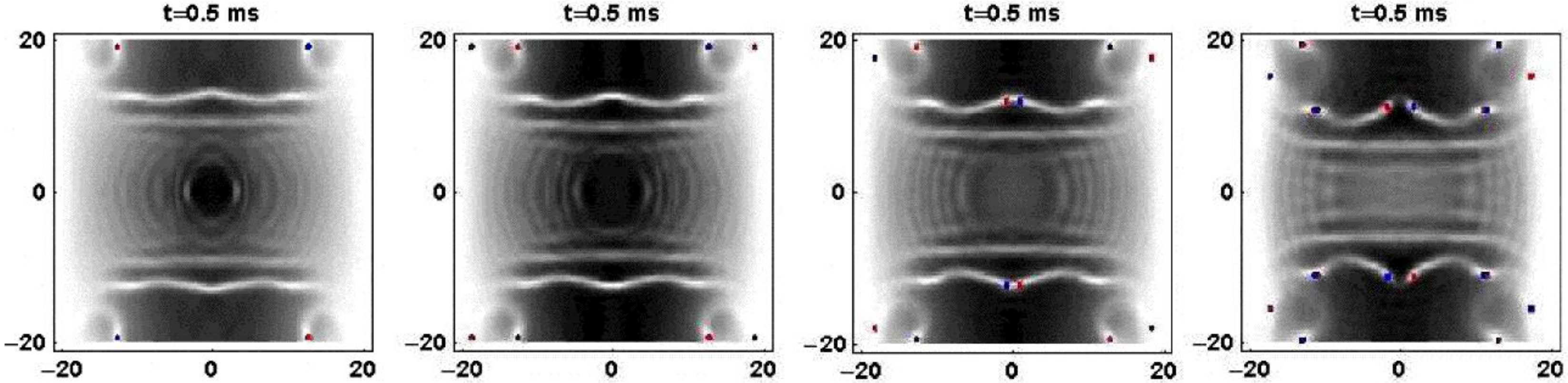
N.S. Ginsberg, J. Brand, L.V. Hau, Phys. Rev. Lett **94**, 040403 (2005)

Simulations of the experiment

Time evolution during free expansion phase (trap is off)



hold times **2.5 ms** **2.7 ms** **3.0 ms** **3.5 ms**



N.S. Ginsberg, J. Brand, L.V. Hau, Phys. Rev. Lett **94**, 040403 (2005)

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Conclusions

Solitonic vortices

- are fundamental non-axisymmetric solitary waves
- are the more stable decay products of solitons
- can be generated experimentally by the snake instability [Phys. Rev. A 65, 043612 (2002)] and by stirring in a toroidal trap [J. Phys. B L113 (2001)]

Vortex rings

- collide elastically at small and large velocities
- generate peculiar low-density shells in inelastic collisions
  S. Komineas and J. Brand, cond-mat/0504072 (2005)

Experimental evidence

- was given for complex hybrid structures consisting of low-density shells with vortex-core structures
  N.S. Ginsberg, J. Brand, L.V. Hau, Phys. Rev. Lett **94**, 040403 (2005)