Second Sound as a Tool to Study He II Flow

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New Experimental Techniques for the Study of Quantum Turbulence,

Trieste, June 9-10, 2005

Outlook

Introduction- second sound

Second sound attenuation technique and sensors

design

performance

sensitivity

examples of detected flows

Summary

Acknowledgements

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Cryogenic helium flow and turbulence

flows that can be studied by the second sound attenuation technique

Cryogenic He gas, He I

Classical turbulence:
High Ra convection (Eugene, Grenoble, Trieste…)
High Re pipe flow (Eugene…)
Taylor – Green flow (Paris..)
Turbulent jet (CERN) Grid turbulence (Yale)

^{4}He

He II T>1K

Coflow (Eugene, Paris, Bariloche, Tallahassee….)
Counterflow (Prague, Tallahassee …)
Pure superflow (no net normal flow) Prague

He II low T

Pure superflow (Lancaster, Regensburg, Paris…)

^{3}He

B phase T>0.3 Tc

Normal component (almost) at rest
100 times bigger vortex core (10 nm)
smooth (quartz) walls – no pinning
(Helsinki)

B phase low T

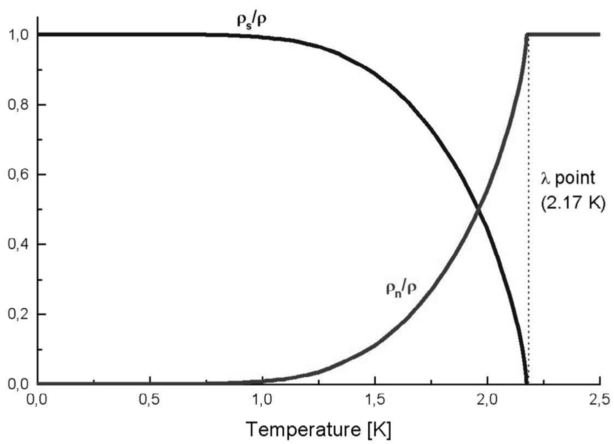
(Lancaster)

Turbulence in superfuid He II

There is no fundamental theory of superfluidity in He II

Two-fluid model (Landau)

He II flow is well described only in a limit of low velocities



Very low kinematic viscosity of the normal fluid (above appr. 1 K)

	T (p)	ν (cm^2/s)	$\alpha/\nu\kappa$
air	20 C	0,15	0,122
water	20 C	$1{,}004 \times 10^{-2}$	14,4
SF_6	50 C (50 bar)	$2{,}8 \times 10^{-4}$	$7{,}5 \times 10^{5}$
mercury	20 C (VP)	$1{,}14 \times 10^{-3}$	3,43
Helium I	2,25 K (VP)	$1{,}96 \times 10^{-4}$	$3{,}25 \times 10^{-5}$
Helium II	1,8 K (VP)	$9{,}01 \times 10^{-5}$	X
He-gas	5,5 K (2,8 bar)	$3{,}21 \times 10^{-4}$	$1{,}41 \times 10^{8}$

Sound propagation in He II

Two – fluid model (Landau)

I. Continuity eq. :

$$\frac{\partial \rho}{\partial t} = -\nabla \vec{j} = -\nabla \rho \vec{v} \qquad \frac{\partial \rho}{\partial t} = -\nabla\left(\rho_s \vec{v}_s + \rho_n \vec{v}_n\right)$$

II. If dissipation can be neglected, entropy is conserved :

$$\frac{\partial(\rho s)}{\partial t} = -\nabla(\rho s \vec{v}) \qquad \frac{\partial(\rho s)}{\partial t} = -\nabla(\rho s \vec{v}_n)$$

III. Navier-Stokes equation of motion :

$$\rho\left\{\frac{\partial \vec{v}}{\partial t} + (\vec{v}\nabla)\vec{v}\right\} = -\nabla p + \eta \Delta \vec{v} \qquad \rho_s\left\{\frac{\partial \vec{v}_s}{\partial t} + (\vec{v}_s\nabla)\vec{v}_s\right\} + \rho_n\left\{\frac{\partial \vec{v}_n}{\partial t} + (\vec{v}_n\nabla)\vec{v}_n\right\} = -\nabla p + \eta \Delta \vec{v}_n$$

$$[(\vec{v}\nabla)\vec{v}]_x = v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}$$

Combining I.+II.+III. , we get for entropy and density:

$$\frac{\partial^2 \rho}{dt^2} = \left(\frac{\partial p}{\partial \rho}\right)_s \Delta\rho + \left(\frac{\partial p}{\partial s}\right)_\rho \Delta s$$

$$\frac{\partial^2 s}{dt^2} = \frac{\rho_s}{\rho_n} s^2\left[\left(\frac{\partial T}{\partial \rho}\right)_s \Delta\rho + \left(\frac{\partial T}{\partial s}\right)_\rho \Delta s\right]$$

After linearization, we seek the plane wave solution, in a form

$$\rho = \rho_0 + \rho' \exp\left\{i\omega\left(t - \frac{z}{u}\right)\right\}$$

$$s = s_0 + s' \exp\left\{i\omega\left(t - \frac{z}{u}\right)\right\}$$

Two solutions:

$$u_1^2 = \left(\frac{\partial p}{\partial \rho}\right)_s \qquad u_2^2 = \frac{\rho_s}{\rho_n} s^2 \left(\frac{\partial T}{\partial s}\right)_\rho$$

„First sound“

$$u = u_1; s' = 0; \rho' \neq 0; \nabla T = 0; \vec{v}_n = \vec{v}_s$$

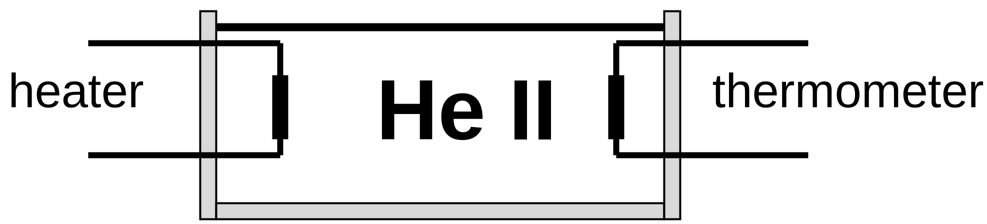
Normal sound, i.e., density waves, propagates both in He II and in He I

„Second sound“

$$u = u_2; s' \neq 0; \rho' = 0; \nabla p = 0; \rho_n \vec{v}_n = \rho_s \vec{v}_s$$

Entropy (temperature) wave at constant density; normal fluid and superfluid oscillating in antiphase. No analogy in classical liquids. A powerful tool to detect quantized vortices.

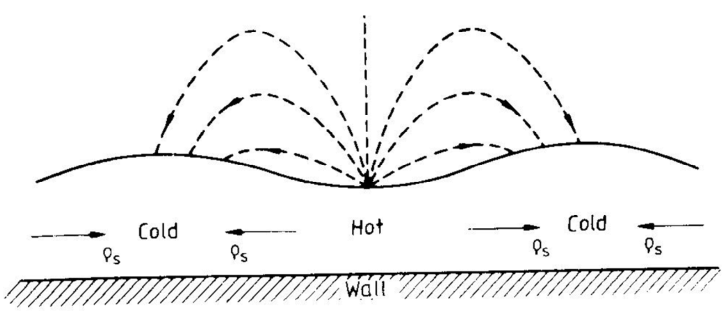
Experiment: 1944 Peshkov



Note: both second and first sound is generated

„Third sound“ –

kind of a surface wave on superfluid film

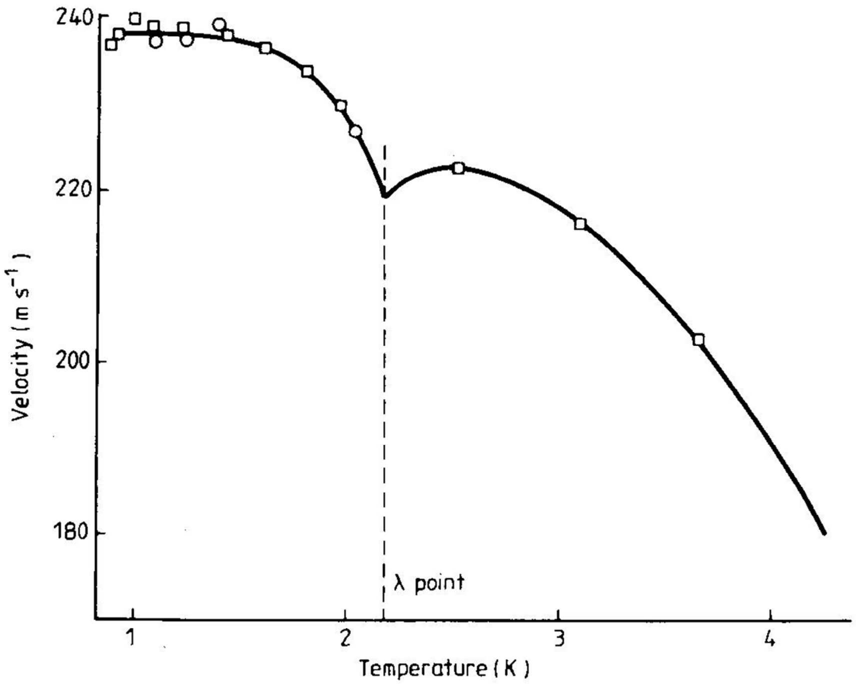


„Fourth sound“

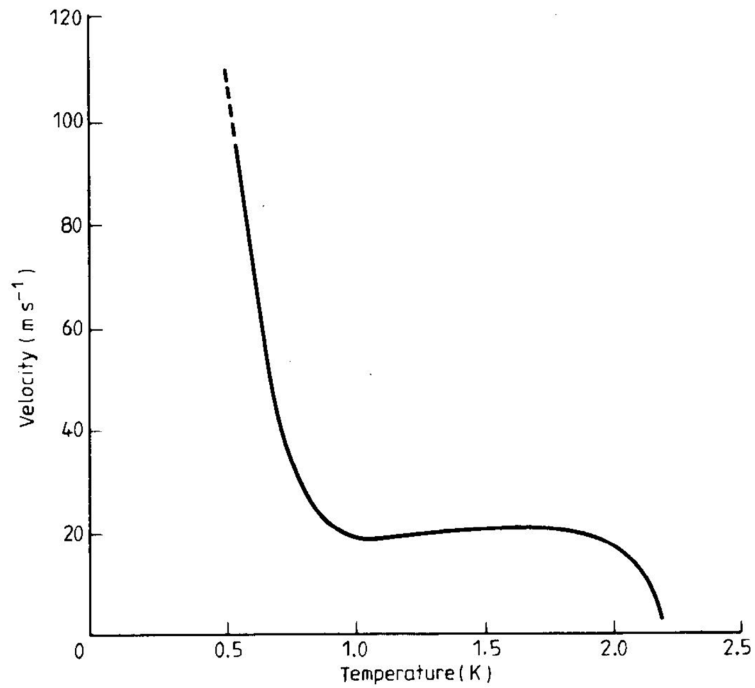
in porous media, where normal fluid motion is inhibited due to finite viscosity

The velocity of sound versus temperature in He II

First sound



Second sound



Experimental observation: quantized vortices attenuate second sound

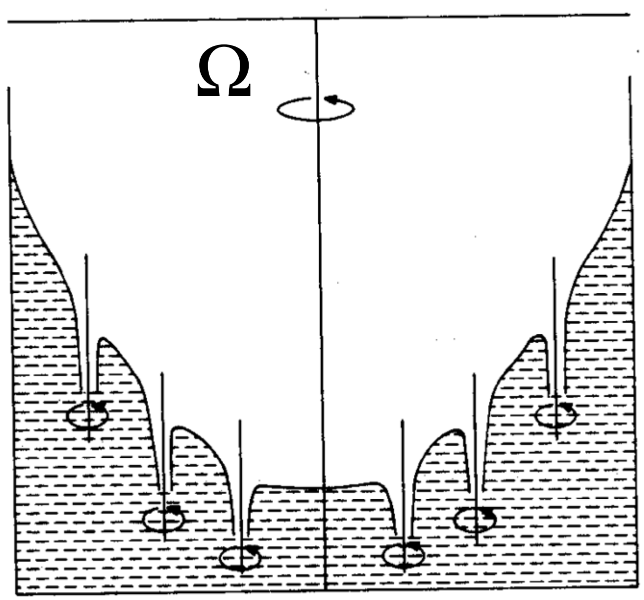
This results from the scattering of the elementary excitations, i.e.,

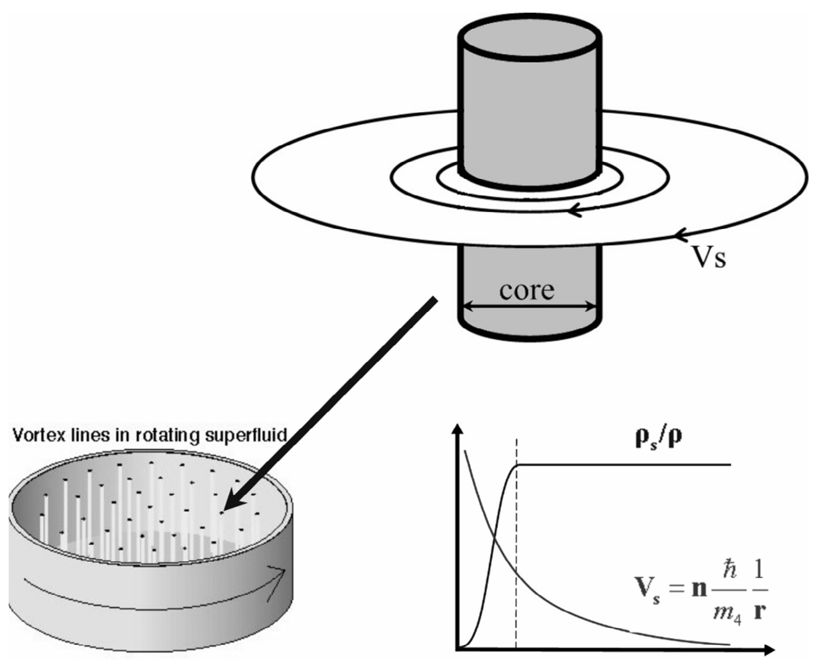
- phonons and rotons = normal fluid, by the vortex lines

"Calibration"

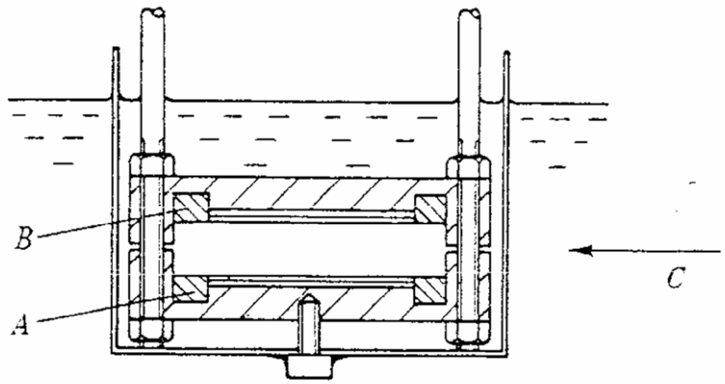
Simplest flow - rotating bucket of He II, with the superfluid mimicking the solid body rotation by an array of rectilinear quantized vortices so that on length scales exceeding the intervortex distance

$$\omega_N = 2\Omega \cong \langle \omega_S \rangle \cong \kappa L$$



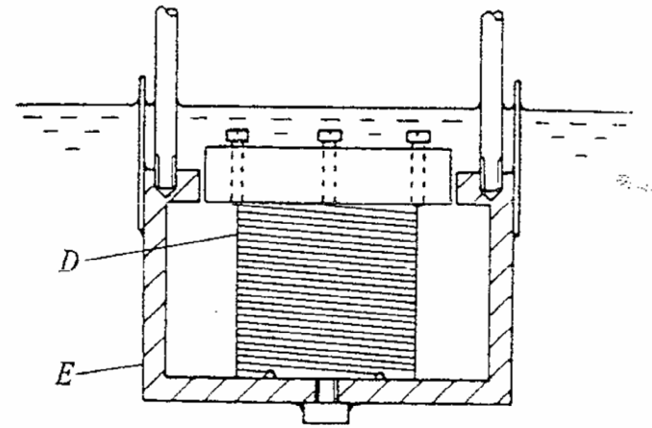


H.E. Hall and W.F. Vinen: **The rotation of helium II. Parts 1 and 2. Proc. Roy. Soc. London 238 (1957)**



a

Axial mode resonator



b

Radial mode resonator

$$\alpha = \alpha_0 + \alpha'(\Omega)$$

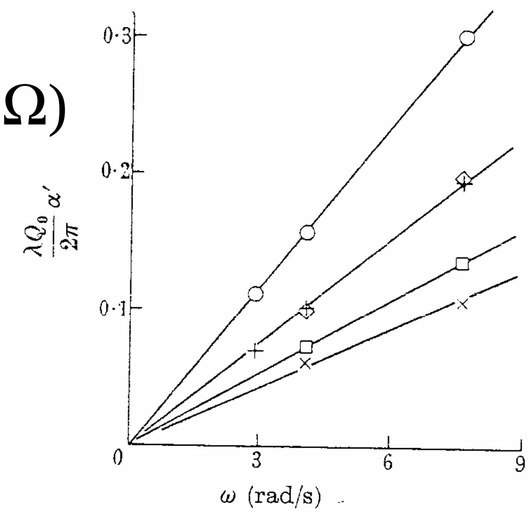


FIGURE 3. The steady attenuation in uniformly rotating helium as a function of angular velocity for the radial mode. ○, 1·289°K; +, 1·500°K; □, 1·635°K; ×, 1·834°K; ◇, 2·060°K (heat input 3 mW at 2·060°K).

$$\alpha' \propto B\Omega$$

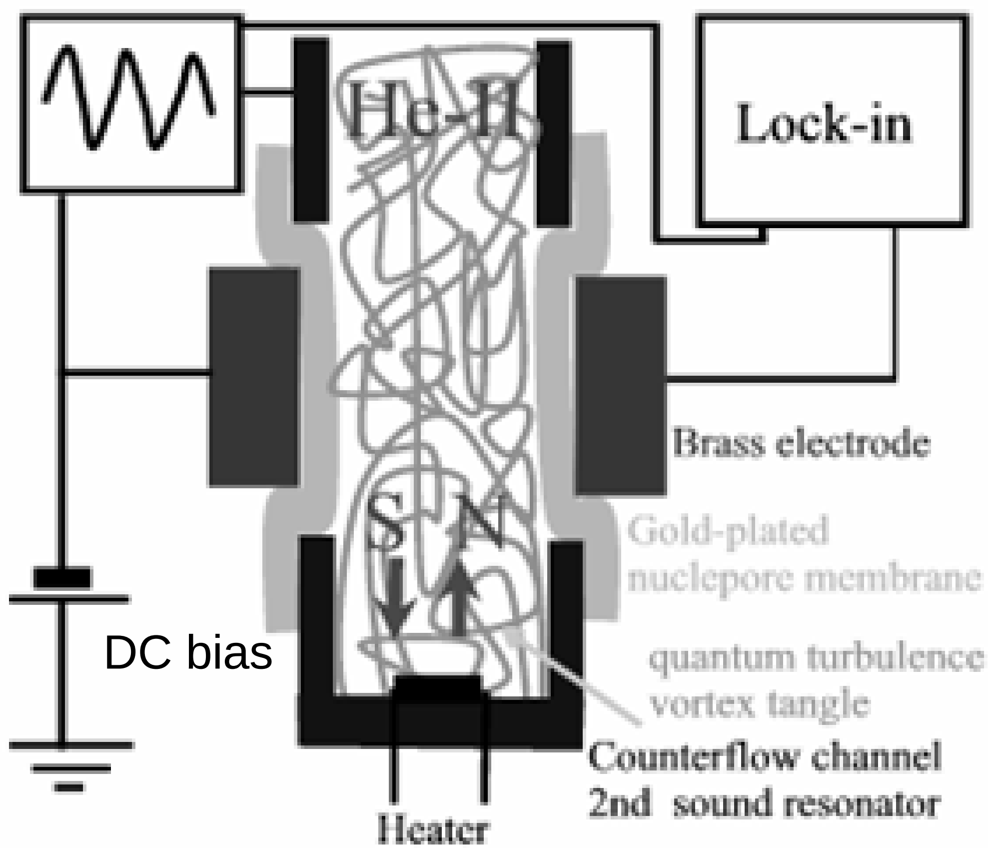
Experiment:

The velocity of the second sound did not change by more than 0.1% with rotating speed up to at least 1.2 rev/s

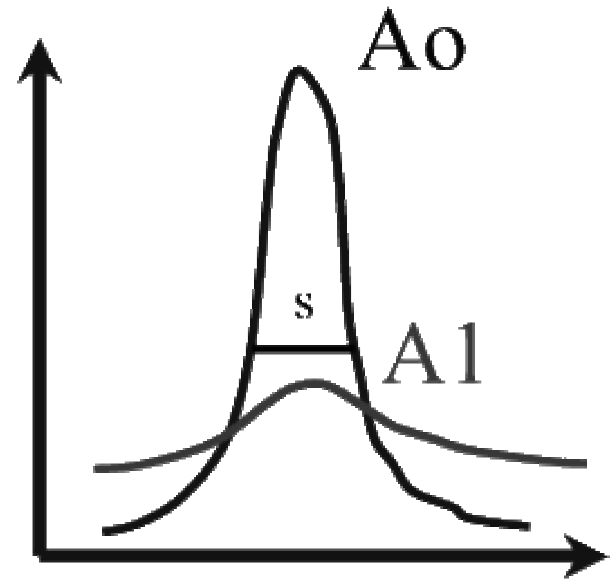
$$\alpha' = \frac{2\pi}{\lambda Q_0}\left(\frac{Q_0}{Q} - 1\right)$$

For solid body rotation we assume that $\omega = 2\Omega \cong \kappa L$

Second sound attenuation in a resonator, using semi-transparent gold-plated membranes



Excitation and detection of 2nd sound in He -II



Assuming that the tangle is random we get

$$L \cong \frac{16s}{B\kappa}\left(\frac{A_0}{A} - 1\right)$$

$$\boxed{\kappa L} = \frac{8u_2}{\pi B D} \ln \frac{1+\left(\frac{A_0}{A}\right)^2 a + \sqrt{2\left(\frac{A_0}{A}\right)^2 a + \left(\frac{A_0}{A}\right)^4 a^2}}{1+a+\sqrt{2a+a^2}}$$

where $$\boxed{a = 1 - \cos\left(\frac{2\pi D s}{u_2}\right)}$$

D size of the resonantor
u_2 second sound velocity
B mutual friction parameter
κ circulation quantum

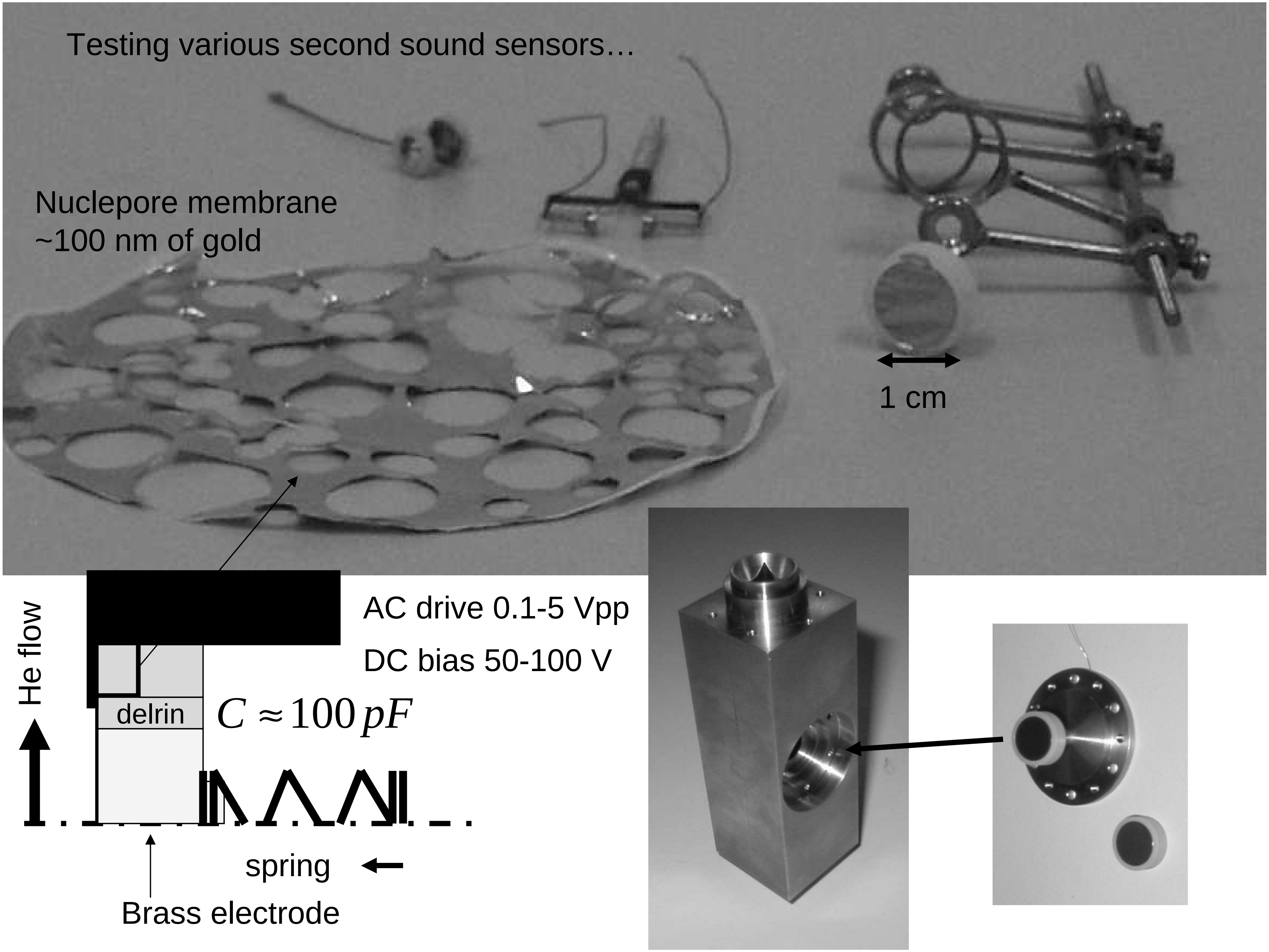
More accurate relationship must be used for high vortex line density, (practically of, order 10^4 cm $^{-2}$) in most cases it reduces to

$$\boxed{L \cong \frac{16s}{B\kappa}\left(\frac{A_0}{A} - 1\right)}$$

(for random tangle)

Various types of sensors-transducers/receivers have been used

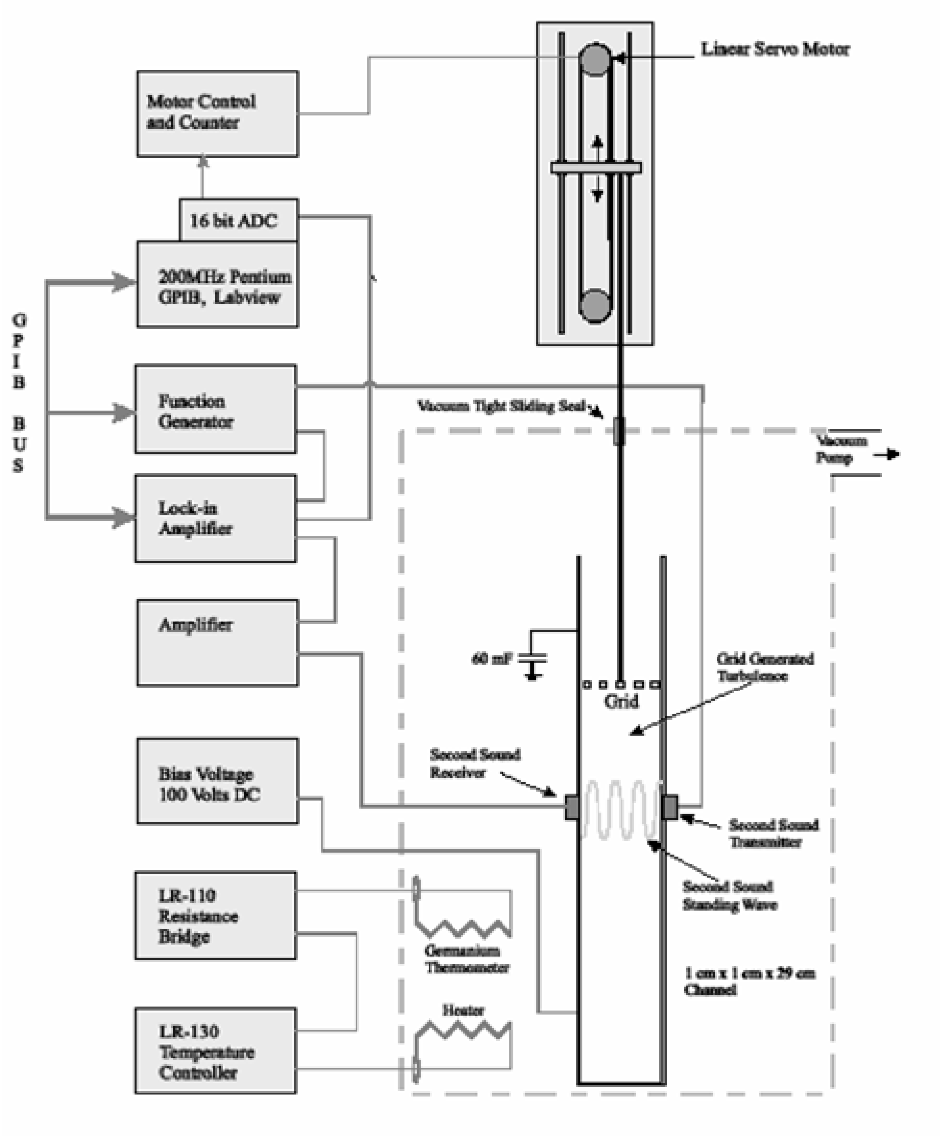
- Nichrome or “constantan” wire
- Brass or phosphor bronze wires (Hall, Vinen)
- Carbon resistors
- Thin film carbon layers
- $AuIn_2$ layers
- Superconducting bolometers
- Nuclepore membranes
- …


Testing various second sound sensors…
Nuclepore membrane
~100 nm of gold
1 cm
AC drive 0.1-5 Vpp
DC bias 50-100 V
He flow
delrin
$C \approx 100\,pF$
spring
Brass electrode

Examples of He II flows studied by the second sound attenuation technique

- Towed grid turbulence (Eugene), Eugene)
- Thermal counterflow (Vinen 1957
- Thermal counterflow and its decay (Prague)
- Flow between counterrotating discs and due to torsionally oscillating cylinder (Prague)
- Pure superflow through superleaks (Prague)

Grid turbulence in He II -
detection scheme used in the Oregon decay experiments



Detection method:
Second sound attenuation

Second sound is generated and detected by oscillating gold-plated porous membranes

Quantity detected:
quantized vortex line density, L
(its projection to a plane perpendicular to the direction of the second sound propagation, averaged over channel crossection)

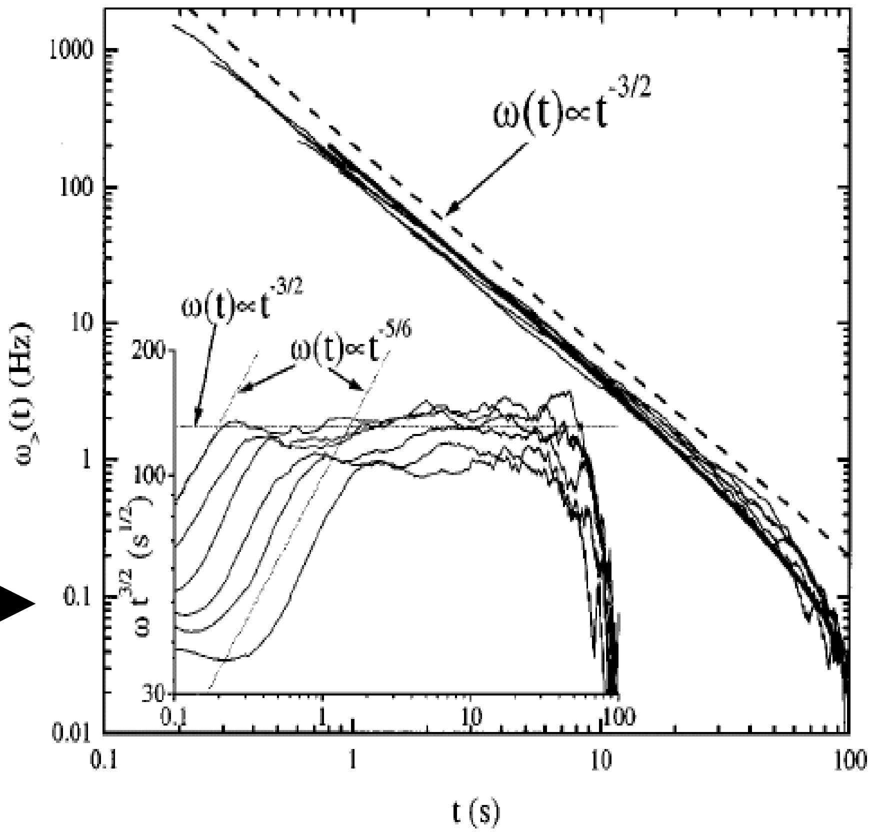


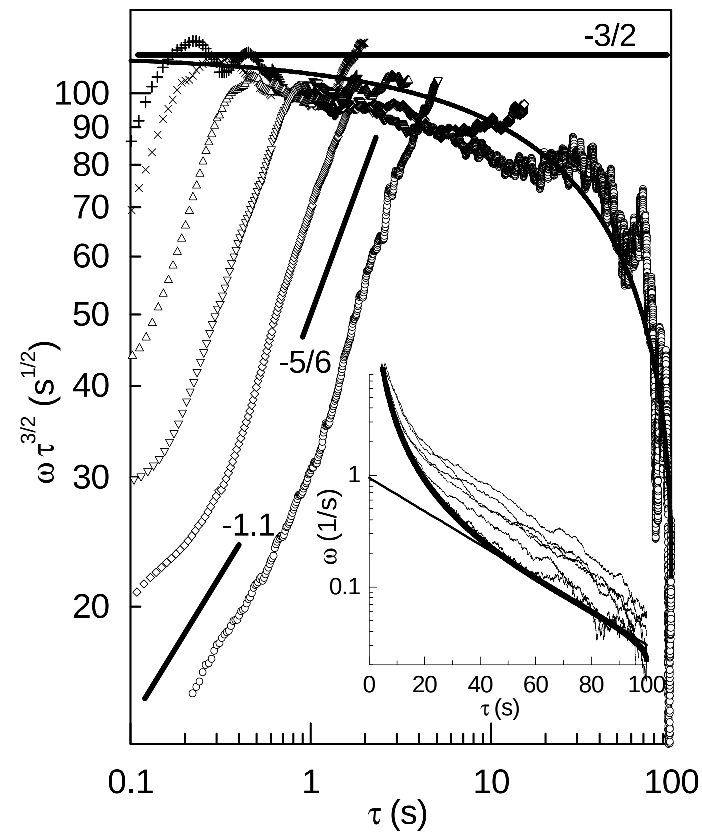
FIG. 11. Vorticity decay data obtained at $T = 1.75$ K and mesh Reynolds numbers of 2×10^5, 1.5×10^5, 1×10^5, 6×10^4, 4×10^4, and 2×10^4. Solid thick line is theoretically predicted decay as explained in text. Inset is same vorticity decay data multiplied by $t^{3/2}$ where t is the experimental time. Dotted lines represent pertinent vorticity decay power laws.

Sensitivity 1m of vortex line in 1 cm³

Huge dynamic range – up to 5 orders of magnitude of vortex line density

Decay of the grid generated turbulence in He II – main features

Character of the decay does not change with temperature
(1,1 K < T < 2,1 K),
although the normal to superfluid density ratio does by more than order of magnitude



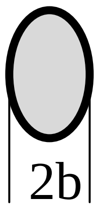
Stalp, Skrbek, Donnelly: *Decay of Grid Turbulence in a Finite Channel*, **Phys. Rev. Lett. 82** (1999) 4831
Skrbek, Niemela, Donnelly: *Four Regimes of Decaying Grid Turbulence in a Finite Channel*, **Phys. Rev. Lett. 85** (2000) 2973

There are four different regimes of the decay of vortex line density (vorticity – if defined as $\omega = \kappa L$) in the finite channel, characterized by:

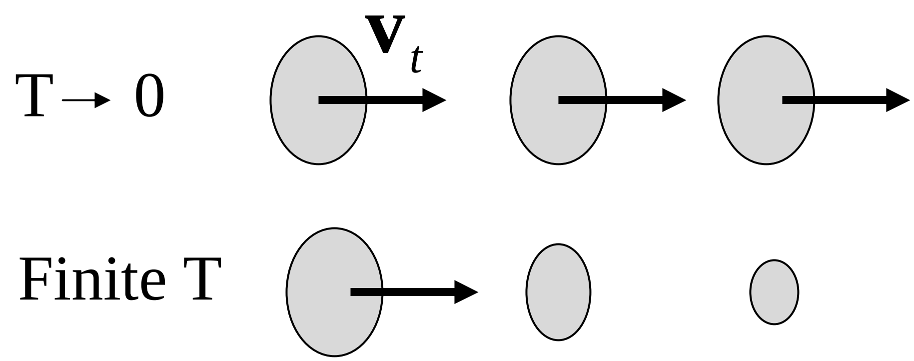
$t^{-11/10}$ $t^{-5/6}$ $t^{-3/2}$ $\exp\{-t/\tau\}$

Counterflow turbulence phenomenology (Vinen 1957)

Vortex ring



$$\mathbf{v}_t = \frac{\kappa}{4\pi b}\left(\ln\frac{8b}{a} - \frac{1}{4}\right) \cong \frac{\kappa}{b}$$



In counterflow, though, if $\left|\mathbf{v}_t\right| < \left|\mathbf{v}_n - \mathbf{v}_s\right| = V_{CF}$ rings with $b > b_c$ expand

Dimensional analysis and analogy with classical fluid dynamics leads to the Vinen equation:

$$\frac{dL}{dt} = \chi_1 \frac{B}{2}\frac{\rho_n}{\rho} V_{CF} L^{3/2} - \chi_2 \frac{\hbar}{m_4} L^2 + (g(V_{CF}))$$

production **decay**

L – voxtex line density

Reproduced by Schwarz (1988) – computer simulations

For steady V_{CF} there is a steady value of L

Early results reviwed by J.T. Tough **Turbulent states I, II, III**

Decay of counterflow turbulence:

$$\frac{dL}{dt} = \chi_2 \frac{\hbar}{m_4} L^2 \longrightarrow L(t) \propto \frac{1}{t + t_{VO}}$$

Numerous experiments

(Vinen, Schwarz, Milliken…)

Some yes, but…

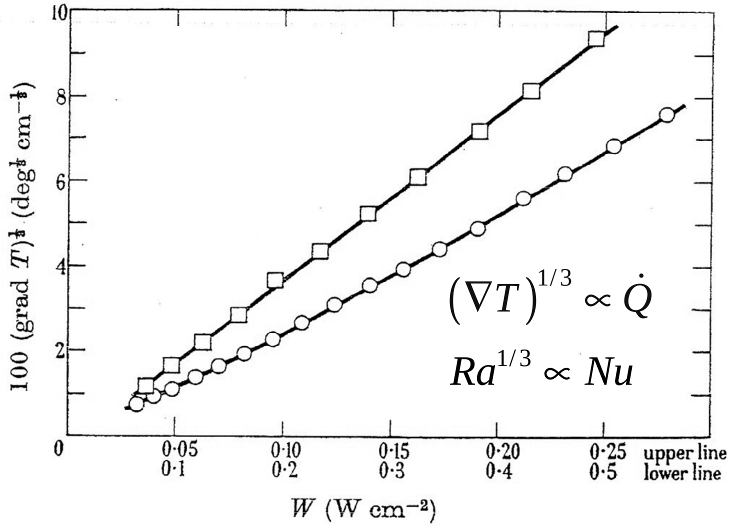


FIGURE 2. Thermal conduction in helium II; (grad T)$^{\frac{1}{3}}$ plotted as a function of W (apparatus no. 1): □, 1·312° K; ○, 2·111° K.

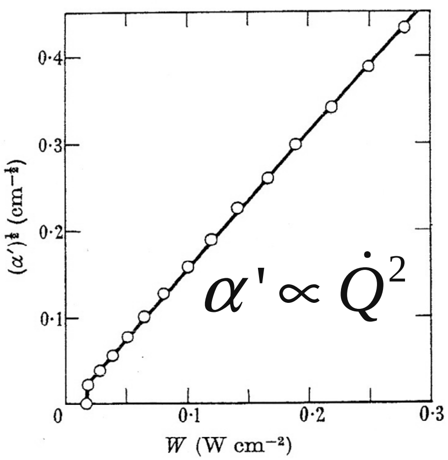


FIGURE 4. The attenuation of second sound in a heat current; (α')$^{\frac{1}{2}}$ plotted as a function of W (apparatus no. 1; 1·414° K).

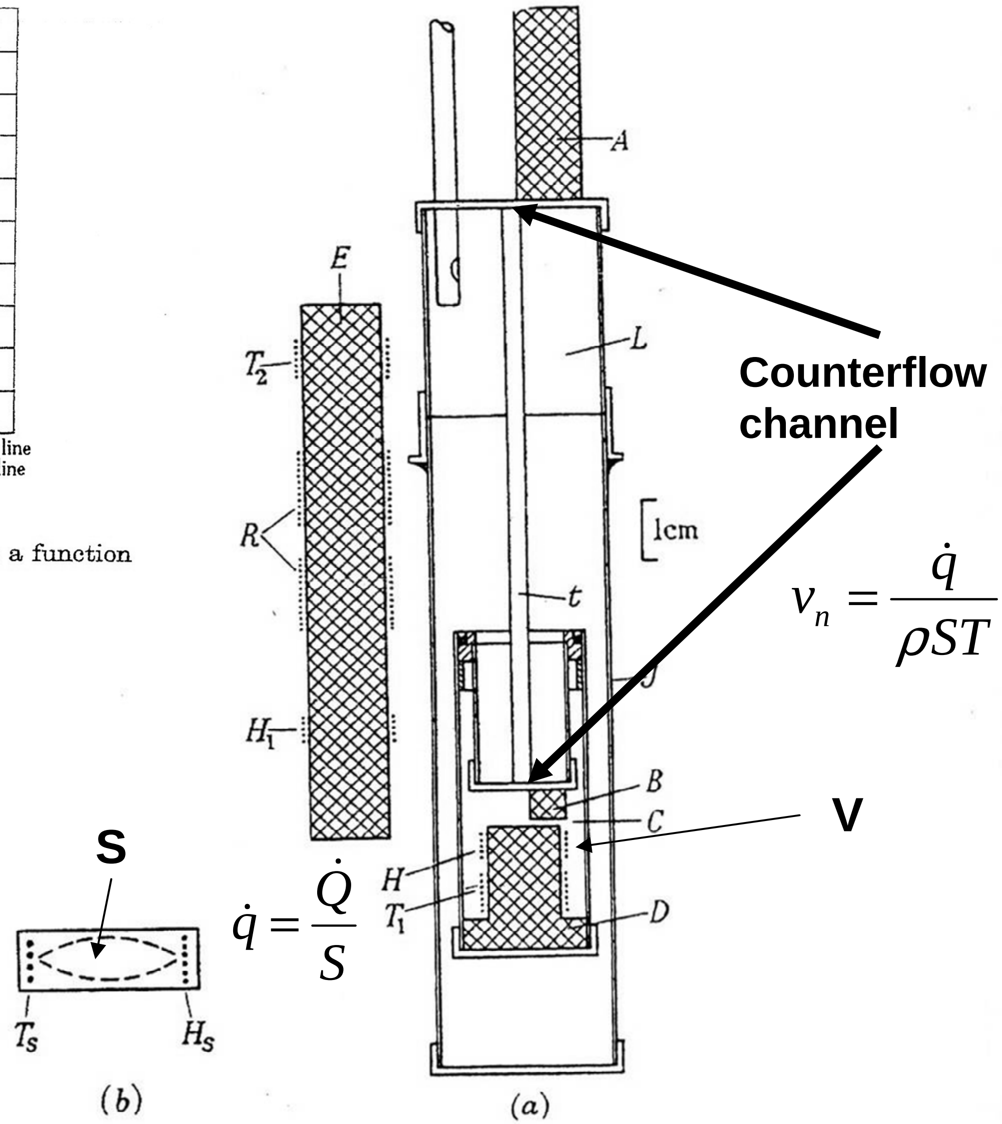


FIGURE 1. The apparatus. (a) Complete assembly within the helium Dewar vessel; (b) cross-section of tube t (enlarged).

W.F. Vinen, Proc. Roy. Soc. A240 114, (1957)
W.F. Vinen, Proc. Roy. Soc. A240 128, (1957)
W.F. Vinen, Proc. Roy. Soc. A242 493, (1957)
W.F. Vinen, Proc. Roy. Soc. A243 400, (1958)

Steady state data $\dot{q} = A(\nabla T)^{1/3}$

When heater is switched off, the following amount of heat must flow away through the channel (neglecting the channel volume)

$$Q = V\varrho c\ell \nabla T \longrightarrow \dot{q} = \frac{\dot{Q}}{S} = \frac{V\varrho c\ell}{S}\frac{d\nabla T}{dt}, \qquad \frac{V\varrho c\ell}{SA}\int_{\nabla T_0}^{\nabla T(t)} \frac{dx}{x^{1/3}} = -\int_0^t dt$$

Assuming quasi-equilibrium, for decaying temperature gradient we get

$$\nabla T(t) = \left[\nabla T_0^{2/3} - \frac{2SA}{3V\varrho c\ell}t\right]^{3/2}$$

Note: without knowing anything of quantized vortices, this simple model predicts that the extra attenuation decays linearly with time!

$$\sqrt{\alpha'} \propto \dot{q} \propto (\nabla T)^{1/3} \rightarrow \alpha' \propto t$$

We can use this simple model to estimate the time over which the temperature difference disappears

$$t_{dis} \cong \frac{3V\varrho c\ell}{2SA}\nabla T_0^{2/3} + \frac{3\varrho c\ell^2}{4A}\nabla T_0^{2/3}$$

-It is easy to take into account
-the volume of the channel itself, assuming linear T gradient along it (i.e., add half of its volume)

During this time, although the heater is switched off, the turbulence is NOT isothermal, but is driven by the (decaying) temperature gradient

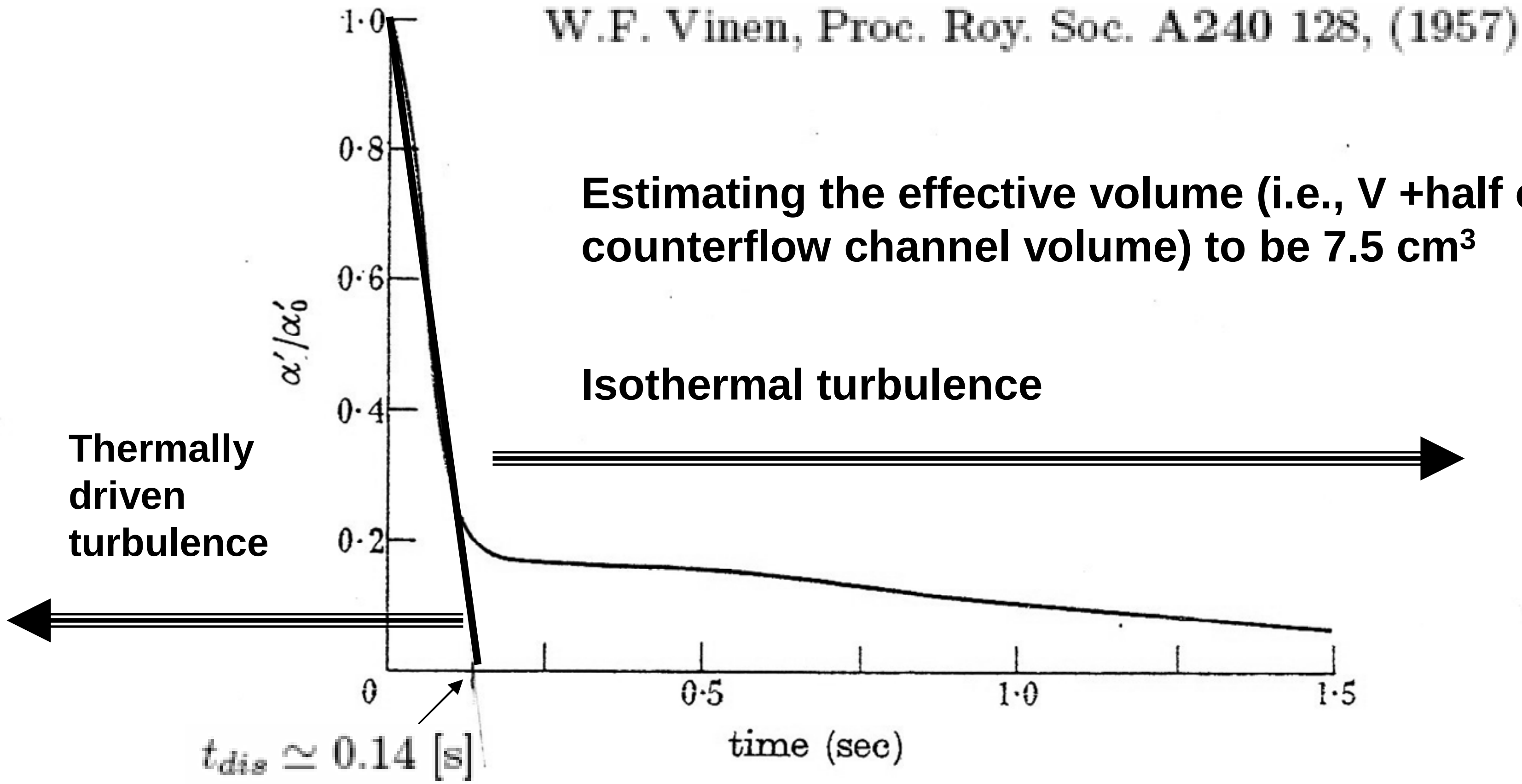


FIGURE 7. Variation with time of the excess attenuation α' in the conductivity tube after a moderately large heat current (0·140 W cm^{-2}) has been switched off (apparatus no. 1; 1·41° K). α'_0 is the excess attenuation in the steady heat current.

Experiments on counterflow turbulence in superfluid He II

Joint Low Temperature Laboratory, Institute of Physics and Charles University, Prague

Detection method:

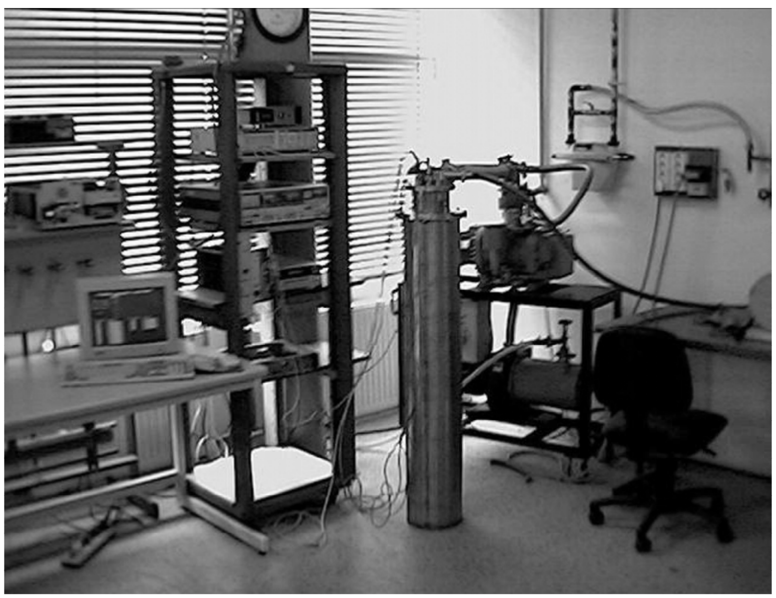
Second sound attenuation

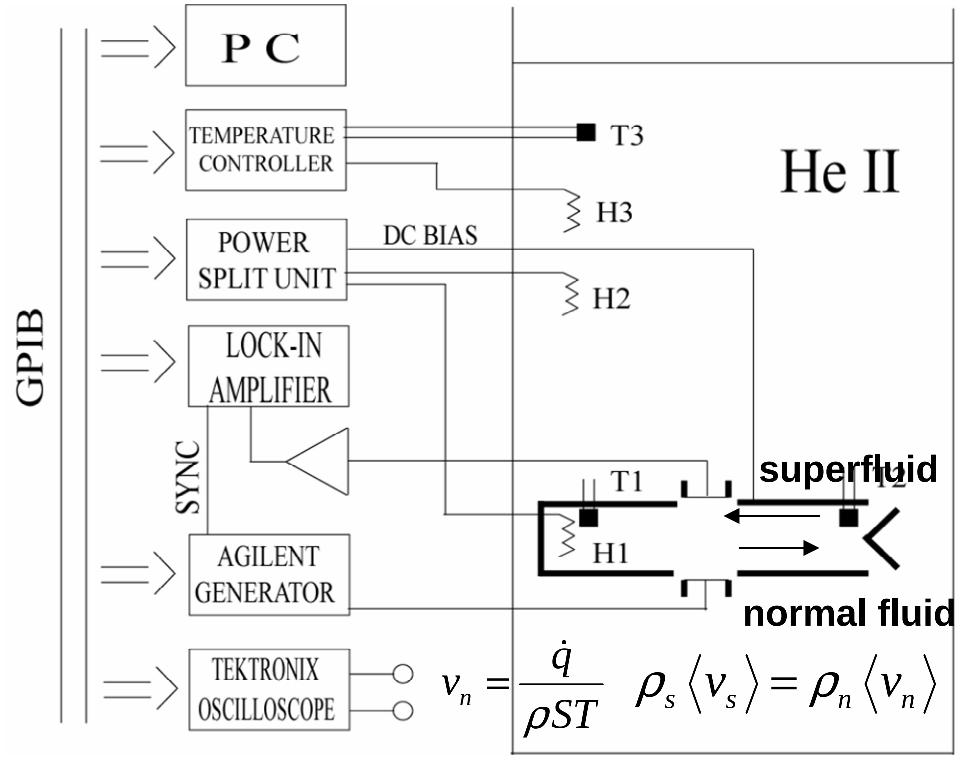
Second sound is generated and detected by oscillating gold-plated porous nuclepore membranes

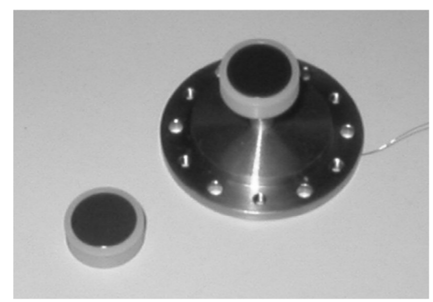
Three different CF channels:

C9

S6 **S10**



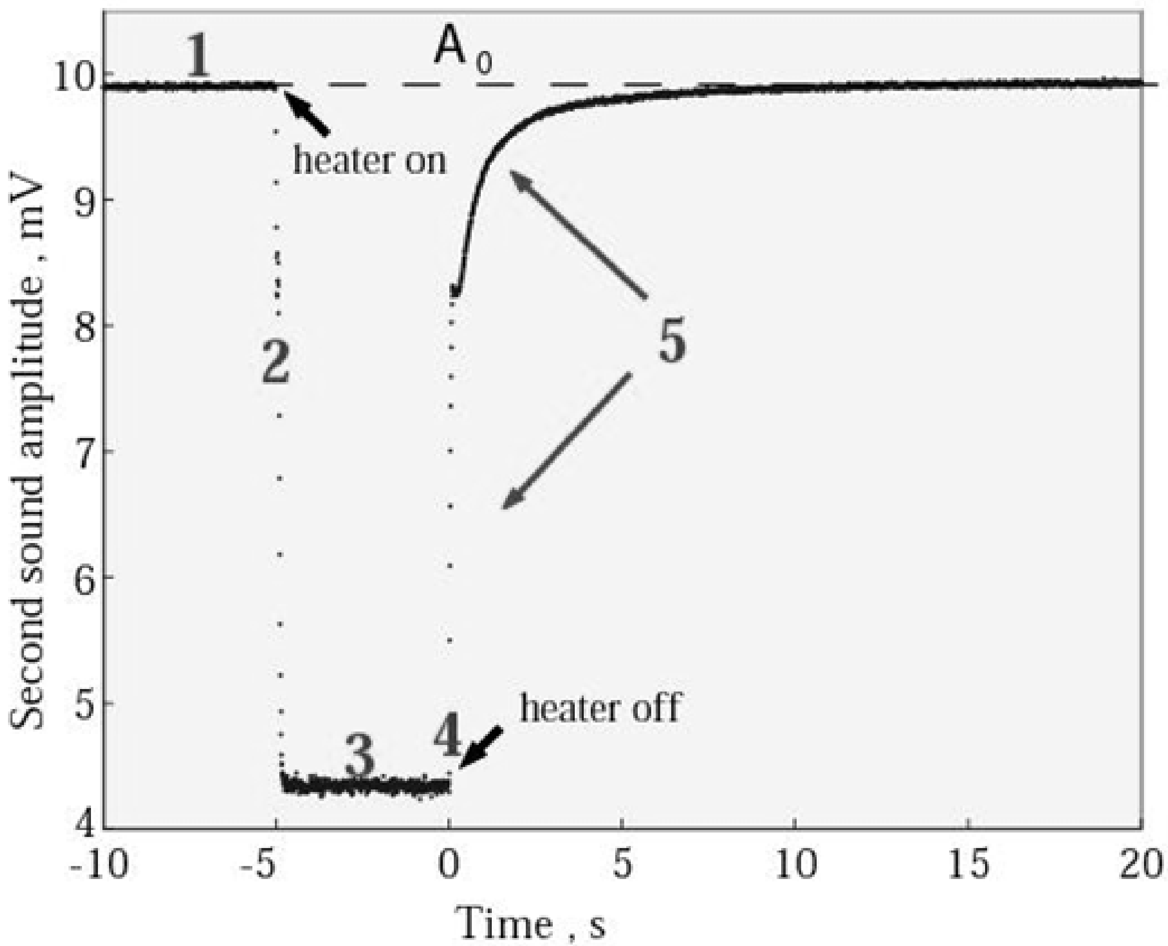




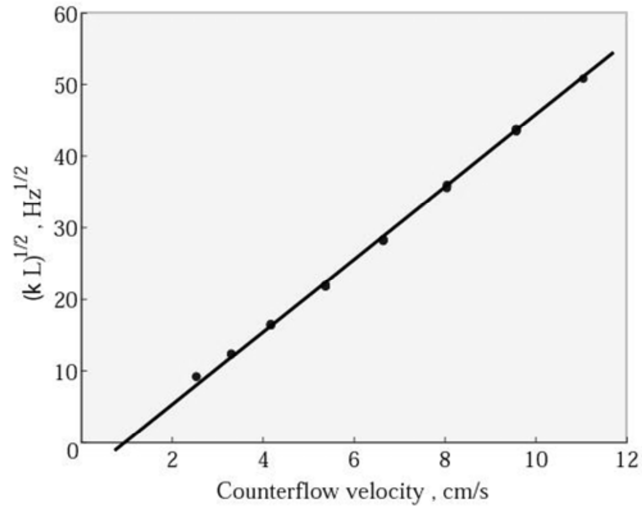
Stationary state data

agree well with previous experiments

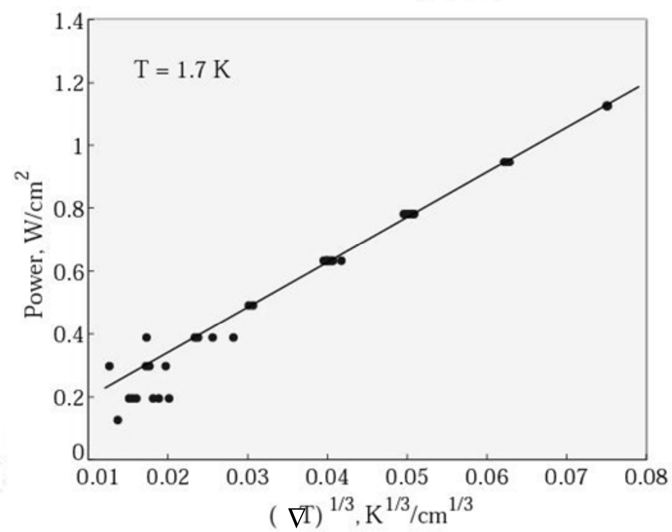
A typical set of row experimental data



Typical behaviour of the second sound amplitude at the resonant frequency f :



Vortex line density obtained using (eq.2) versus calculated counterflow velocity (eq.1).



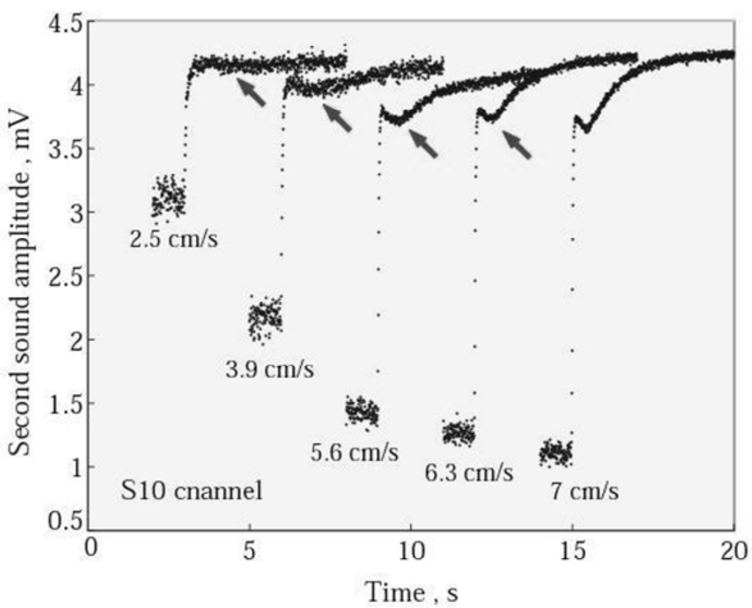
Heat transport efficiency versus measured temperature difference across the counterflow channel. In agreement with the early measurements of Chase [10], for high enough heat inputs.

Can we treat our data on decaying counterflow turbulence as isothermal ?

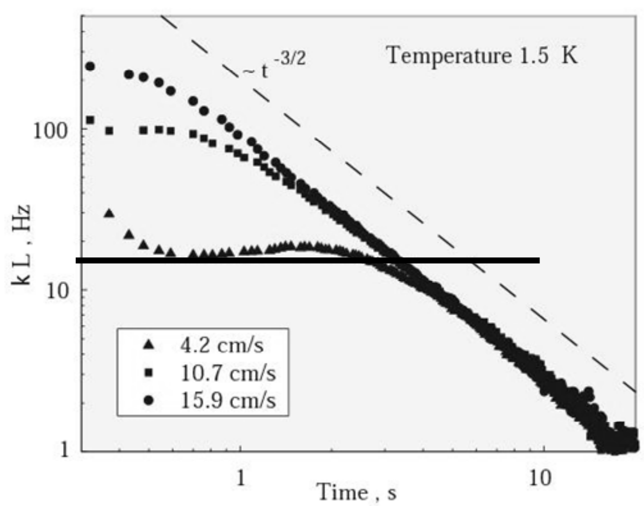
Yes (all our second sound data within experimentally accessible time domain)

- **Direct measurement of the temperature difference across the channel**
- **Simple model for decaying temperature gradient when the heater is switched off**

(Gordeev, Chagovets, Soukup, Skrbek, JLTP **138** (2005) 554)

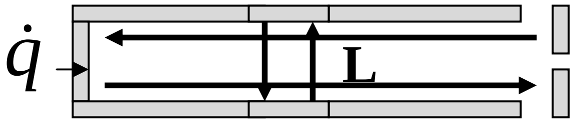


A net increase of the vortex line density during the decay ?



Depolarization of the vortex tangle?

Wang, Swanson, Donnelly, PRB 36 (1987) 5236
vortex tangle generated in the steady state counterflow turbulence is polarized

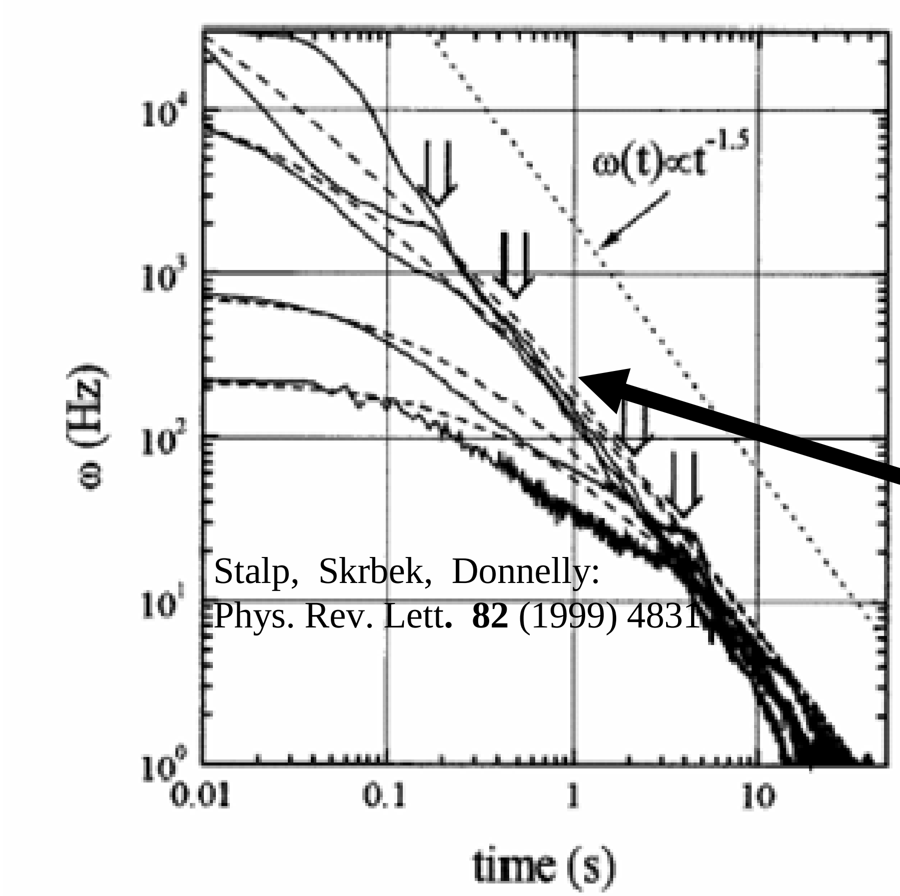


(i) The tangle is fully polarized
vortices lying randomly in planes perpendicular to counterflow velocity. Second sound is effectively attenuated by their projection to the plane perpendicular to the second sound wave, i.e., by $L_{2D} = 2L/\pi$

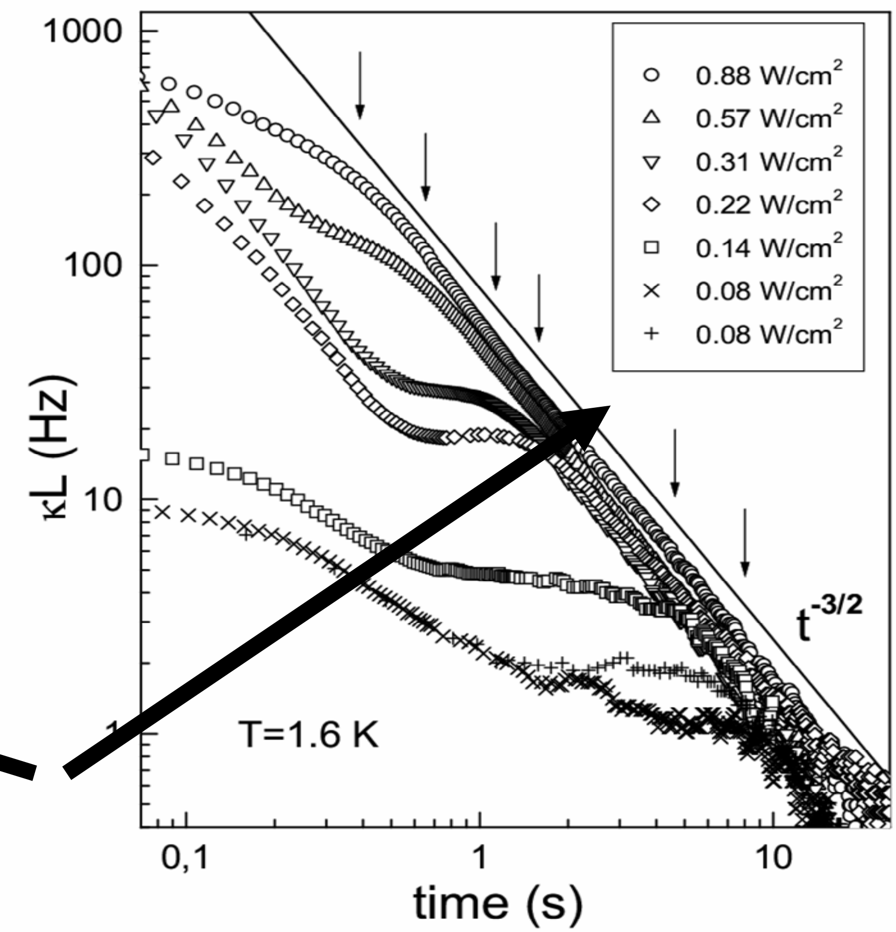
(ii) The tangle is random in 3D
Again, the second sound is effectively attenuated by the projection of these vortices to the plane perpendicular to the second sound wave, this time by $L_{3D} = \pi L/4$

"randomization" - transverse second sound would indicate a net growth by a factor up to $\pi^2/8$ **i.e., up to about 23 %**

Decay of counterflow turbulence in He II above 1K



Stalp, Skrbek, Donnelly: Phys. Rev. Lett**.** **82** (1999) 4831

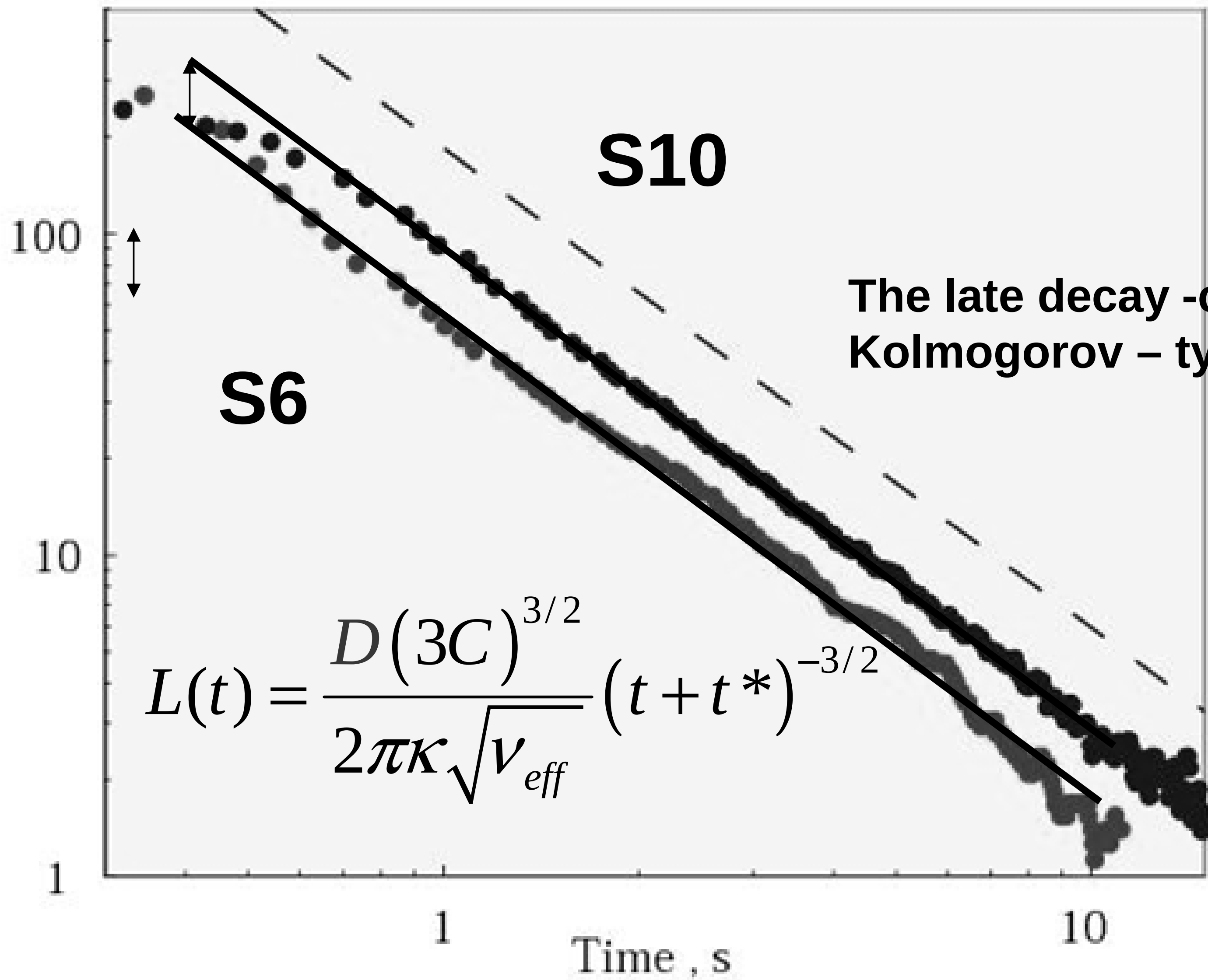


Skrbek, Gordeev, Soukup: **Phys. Rev. E (2003)**

for t > t_{sat} the decay of vortex line density (vorticity) both in grid-generated and counterflow-generated He II turbulence displays the classical exponent of -3/2

After saturation of the energy-containing length scale – universal decay law

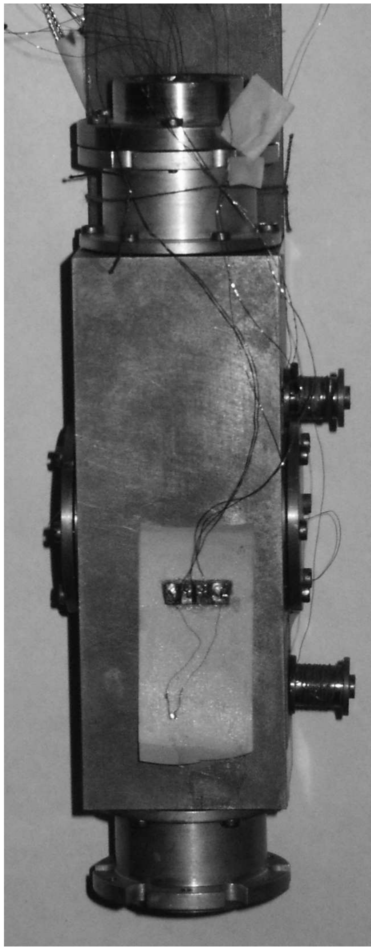
Dependence on the channel size experimentally confirmed for the first time (even for classical turbulence)

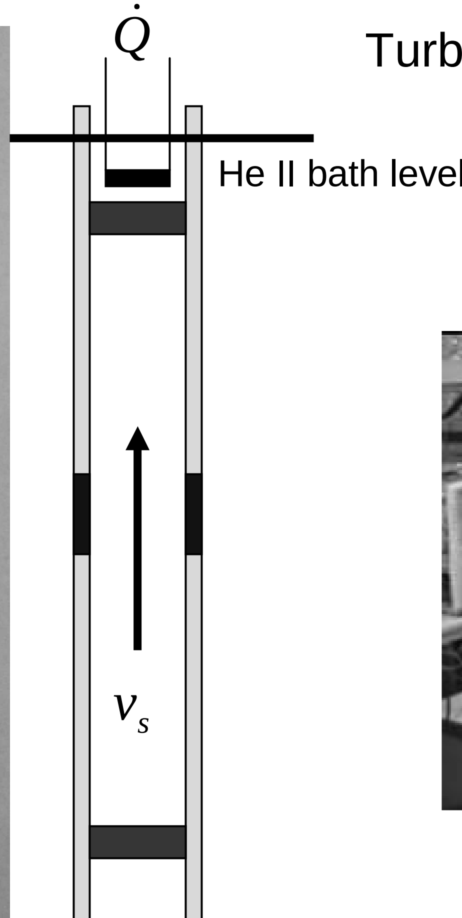


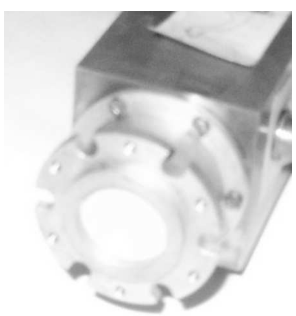
$$L(t) = \frac{D(3C)^{3/2}}{2\pi\kappa\sqrt{\nu_{eff}}}(t+t^*)^{-3/2}$$

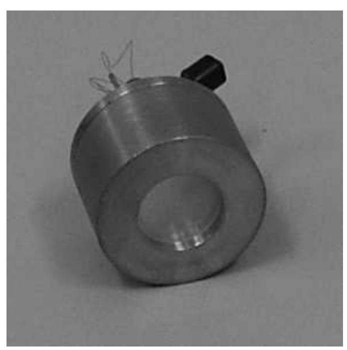
The late decay -consistent with Kolmogorov – type turbulence

Turbulence generated by pure superflow

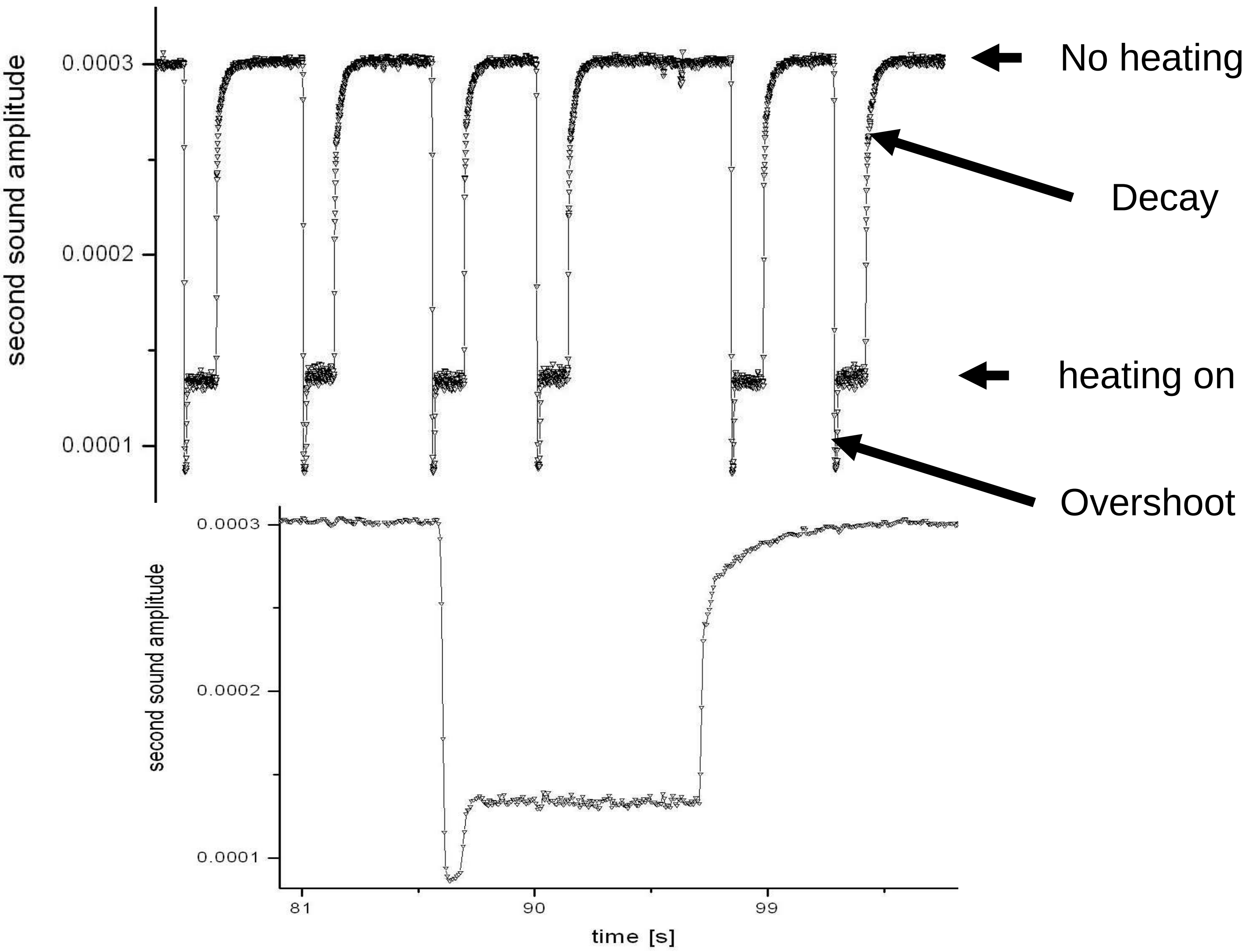






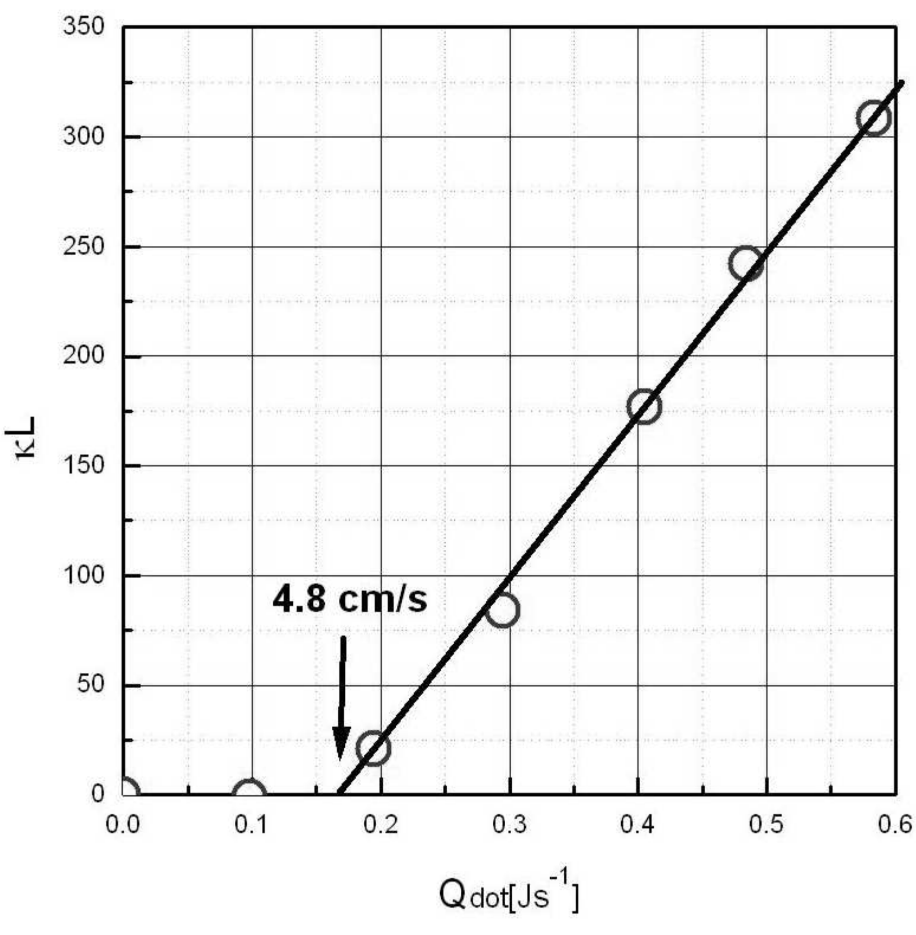


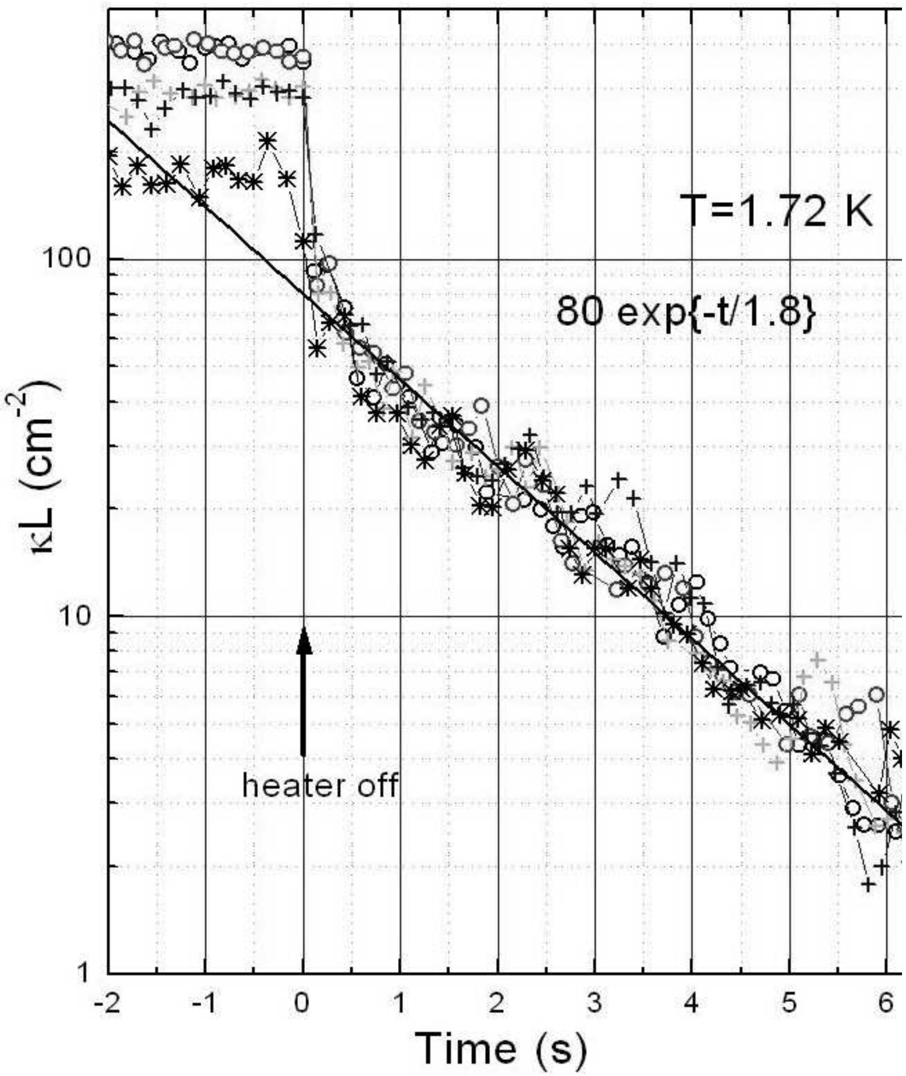
Typical decay data – pure superflow, T=1.72 K, 6x6 mm channel



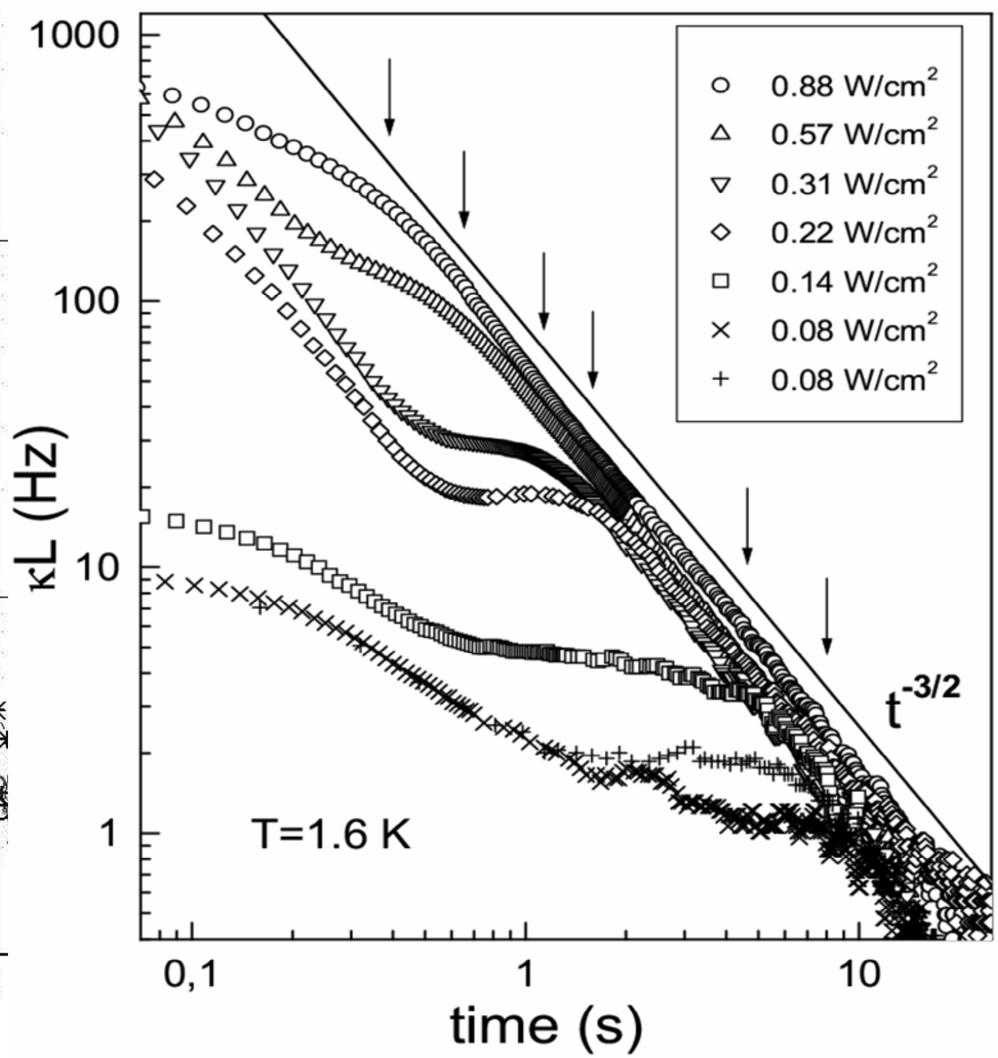
Stationary data – pure superflow of He II through superleaks

T=1.72 K

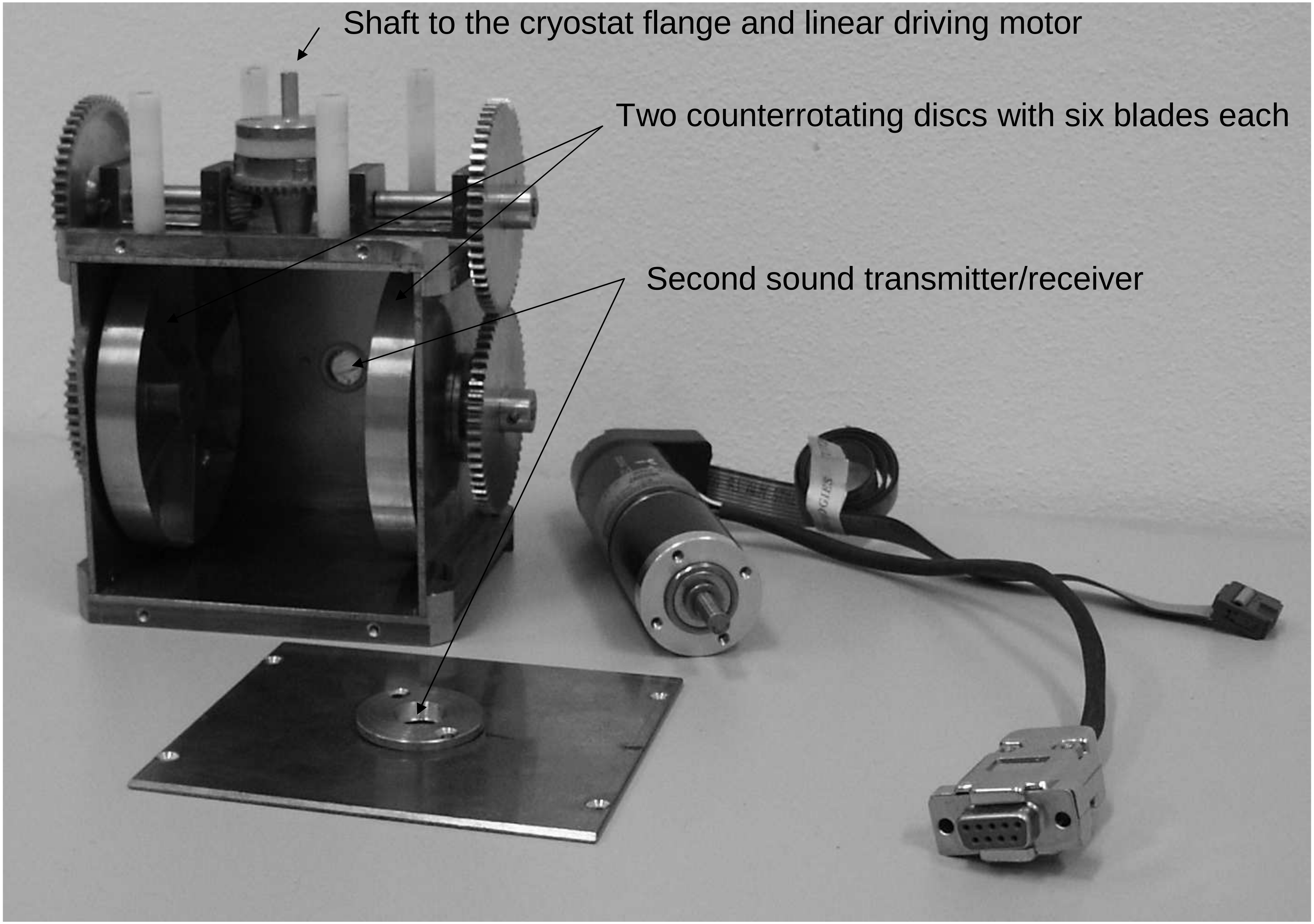




κL (cm⁻²)
T=1.72 K
80 exp{-t/1.8}
heater off
Time (s)



κL (Hz)
0.88 W/cm²
0.57 W/cm²
0.31 W/cm²
0.22 W/cm²
0.14 W/cm²
0.08 W/cm²
0.08 W/cm²
t^-3/2
T=1.6 K
time (s)


Shaft to the cryostat flange and linear driving motor
Two counterrotating discs with six blades each
Second sound transmitter/receiver

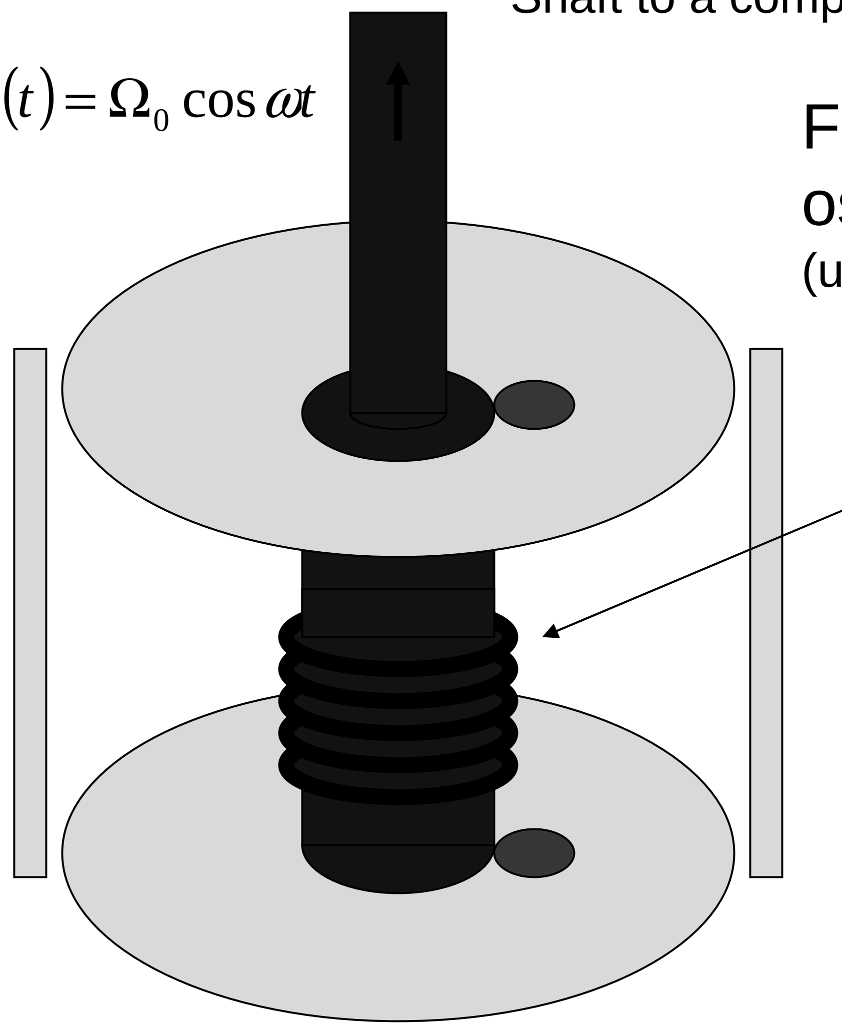
There is urgent need to develop small – local – second sound sensors,
In order to probe the flow of the superfluid component locally
These are under development….

Shaft to a computer controlled linear motor

$$\Omega(t) = \Omega_0 \cos \omega t$$

Flow of He II due to a torsionally oscillating cylinder

(under progress)



Pairs of Taylor vortices ought to appear above some critical Taylor number

Conclusions

second sound attenuation

- powerful technique for probing He II flowsby detecting (the projection of) the vortex line density
- in use over half a century
- temperature range - above 1 K to (close) the lambda point
- dynamic range – over 5 orders of magnitude
- typical sensitivity 1m of vortex line in 1 ccm
- typical time resolution 0.01-0.1 sec
- typical space resolution 0.1 -1 ccm

- there is still a lot of room for improvement