Wild branching surfaces and topological 4-manifolds

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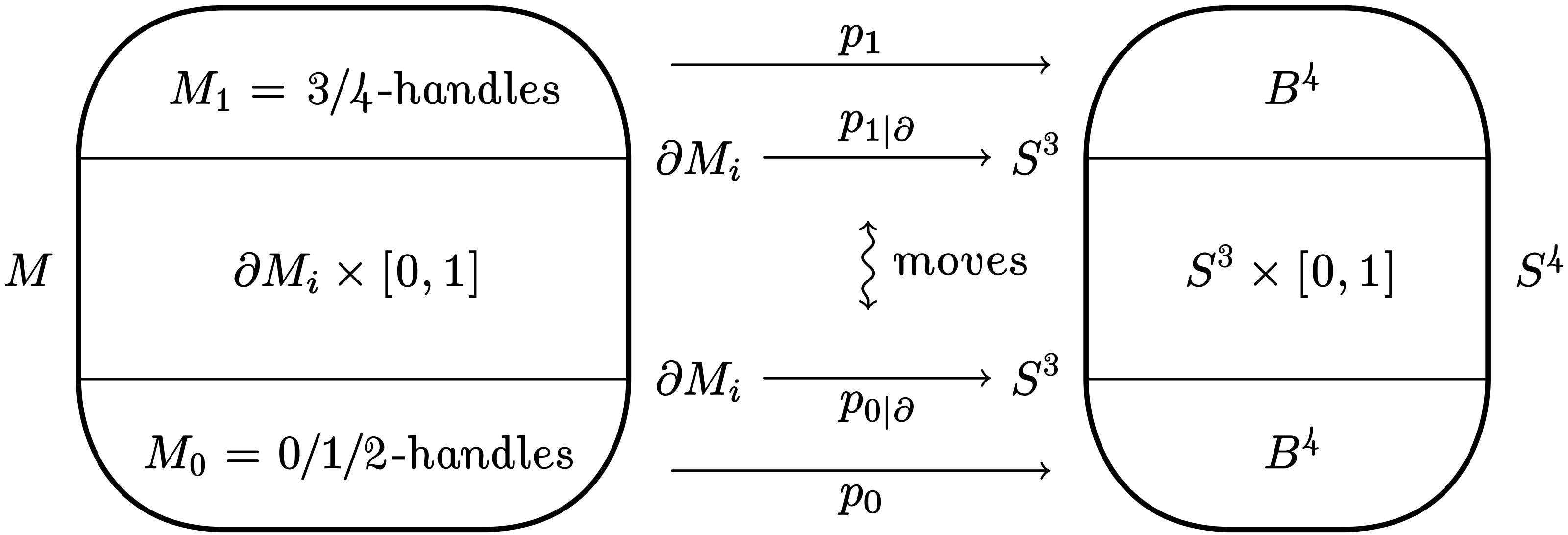
The starting point

For any closed orientable smooth 4-manifold M there is a simple branched covering $M \to S^4$

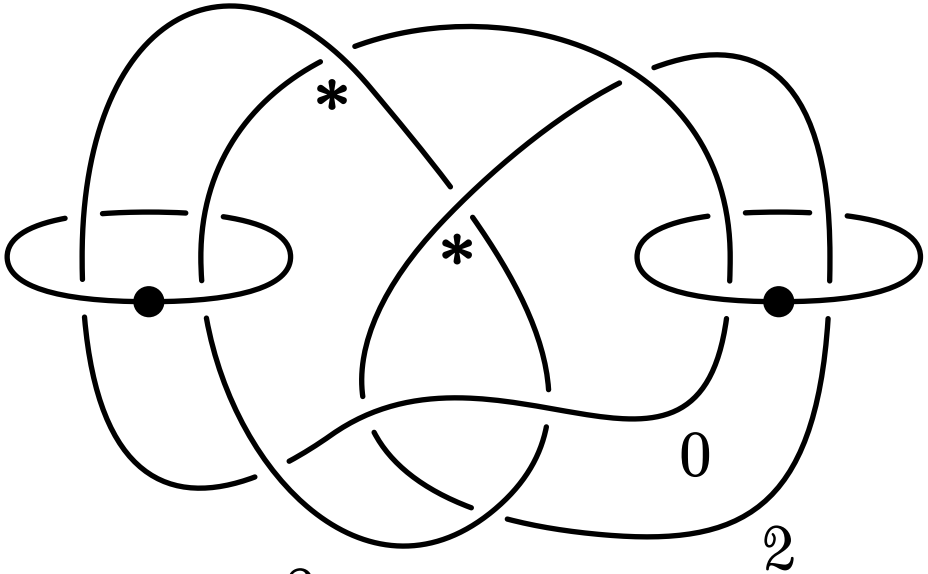
$M = M(F, \omega)$, with $F \subset S^4$ and $\omega : \pi_1(S^4 - F) \to \Sigma_d$



$$* \begin{cases} d = 4 & F = \text{smooth surface with nodes (P. 1995)} \\ d = 5 & F = \text{smooth surface (Iori-P. 2002)} \end{cases}$$

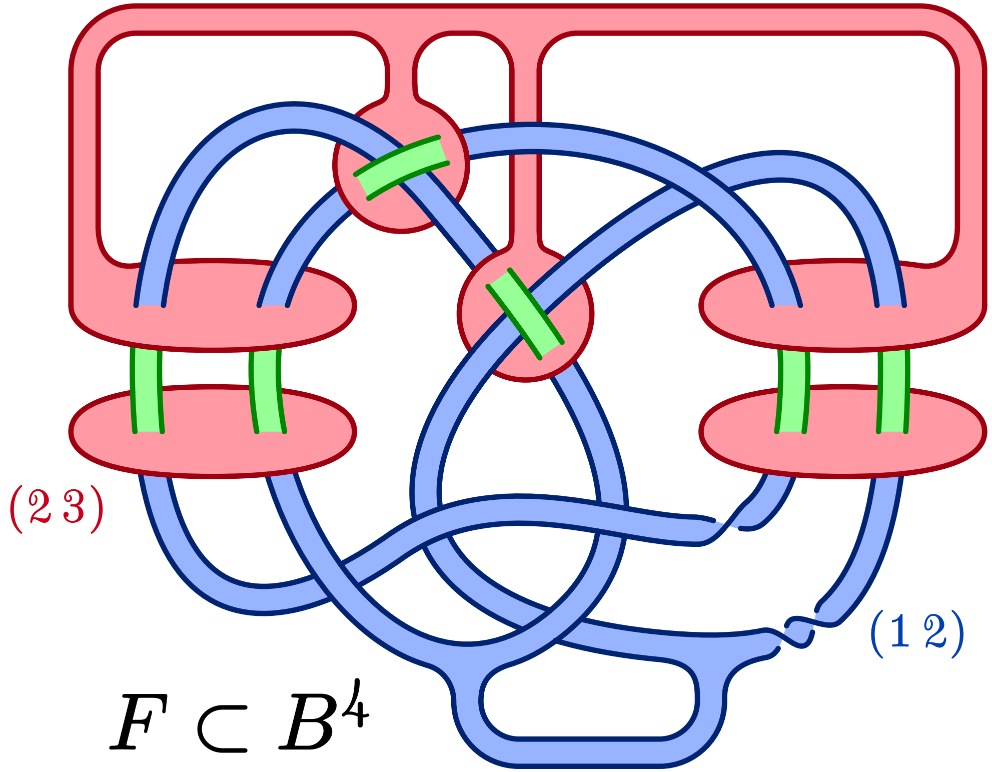
Handles and ribbons

Bobtcheva-P. 2004



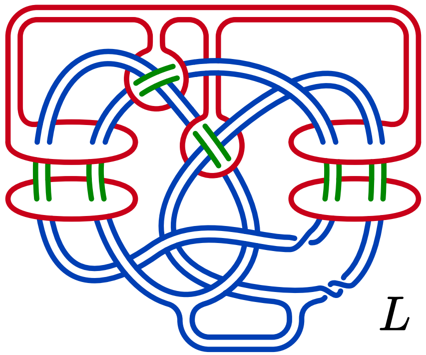
$K \subset S^3$

$\rightsquigarrow$



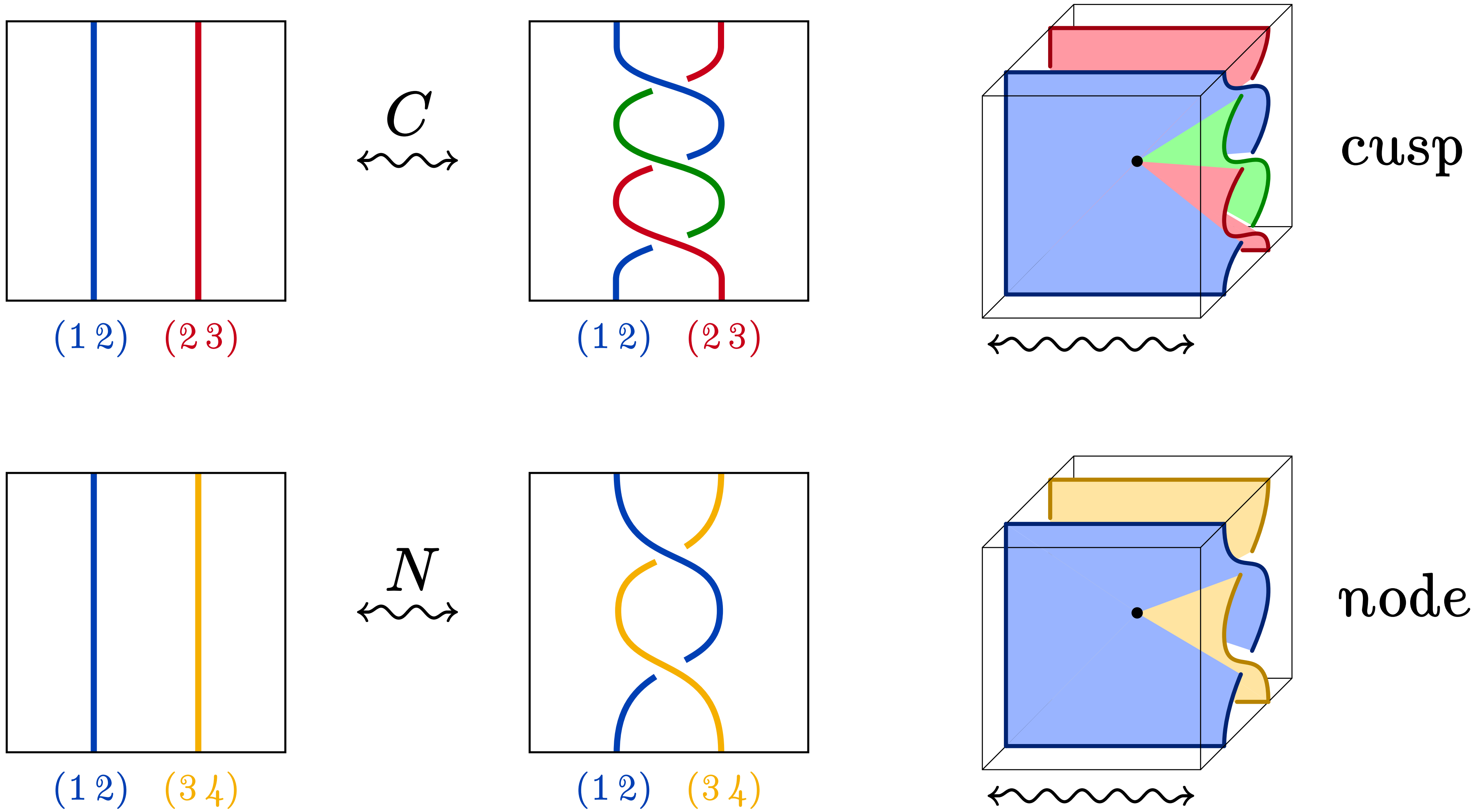
$F \subset B^4$

Any closed orientable 3-manifold
is a boundary simple cover of S^3

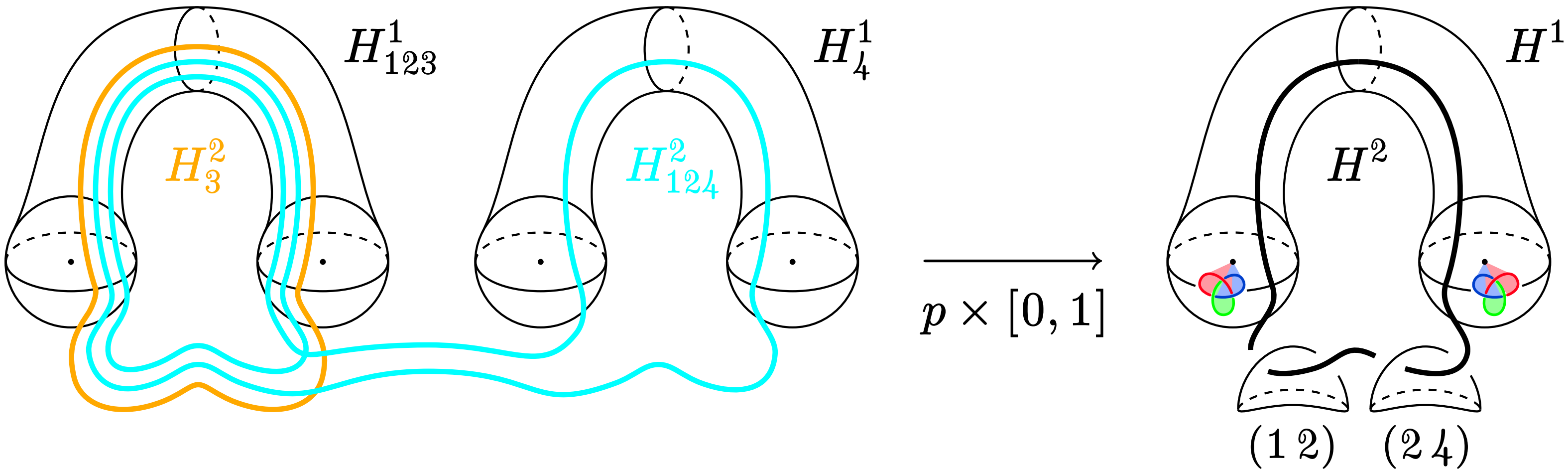


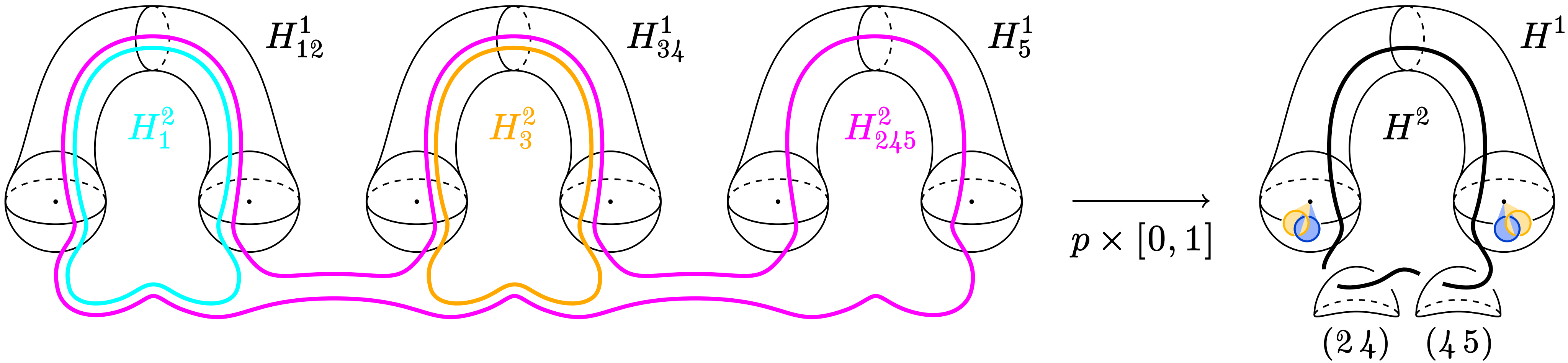
$L \subset S^3$

Moves and singularities



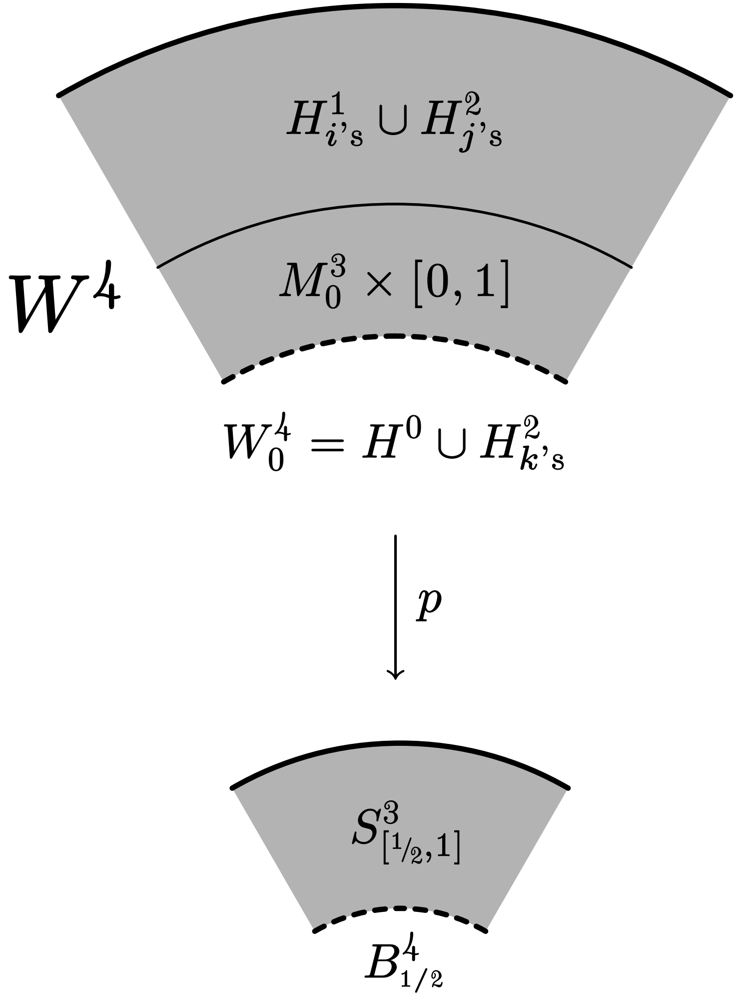
Removing the singularities

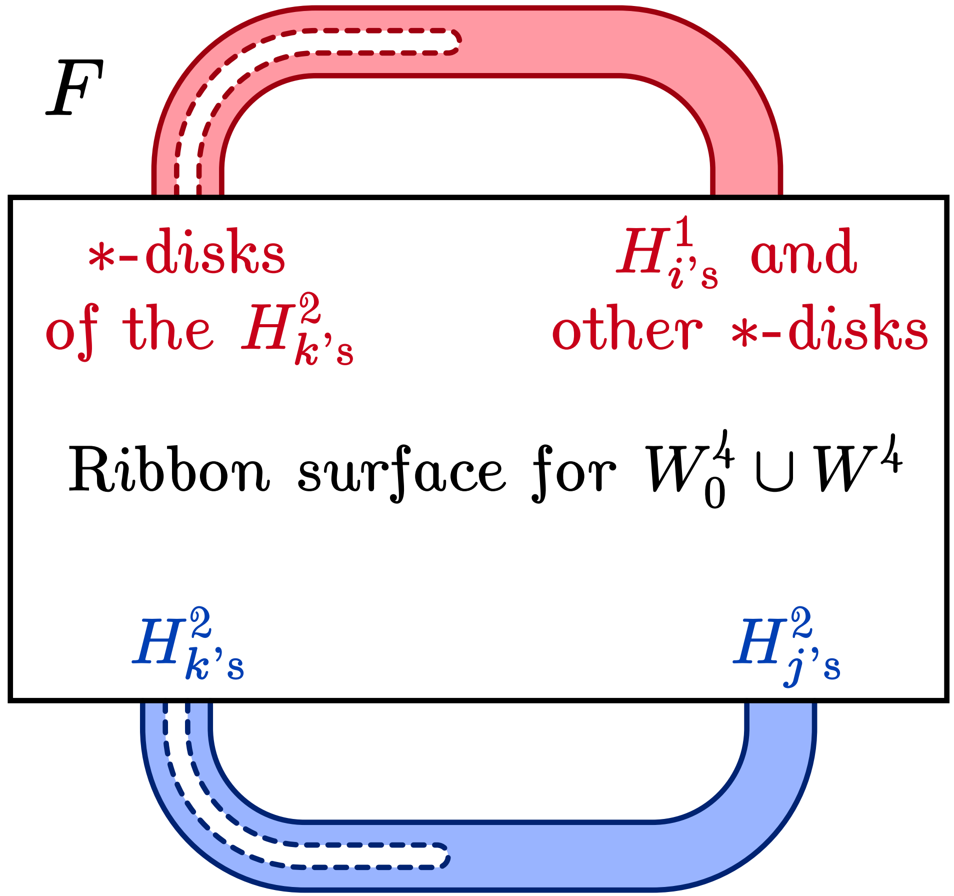




4-dimensional 2-cobordisms

Representing $W^4 = M_0^3 \times [0,1] \cup H^1_{i\text{'s}} \cup H^2_{j\text{'s}}$ (M_0 connected) as simple covering of $S^3_{[1/2,1]} = B^4 - \operatorname{Int} B^4_{1/2}$ branched over F

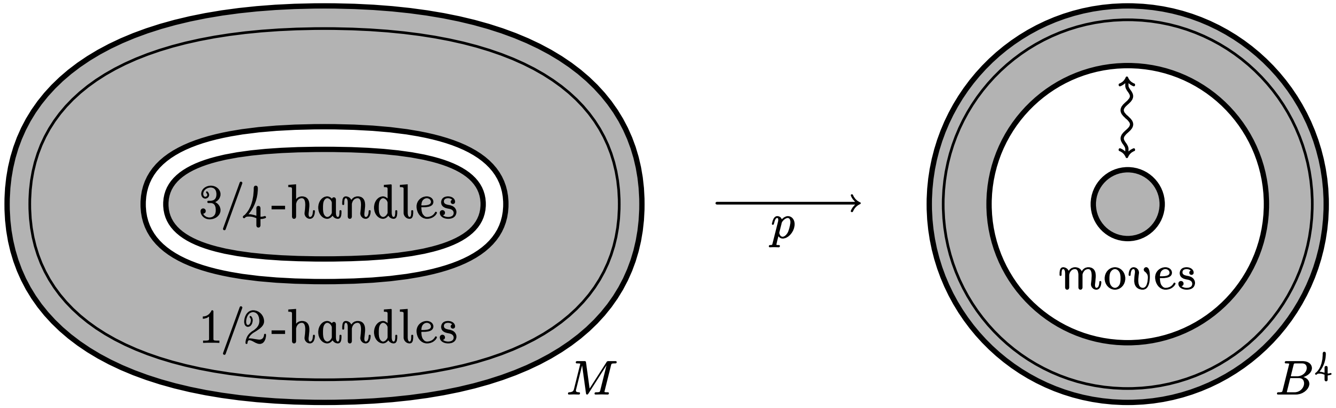




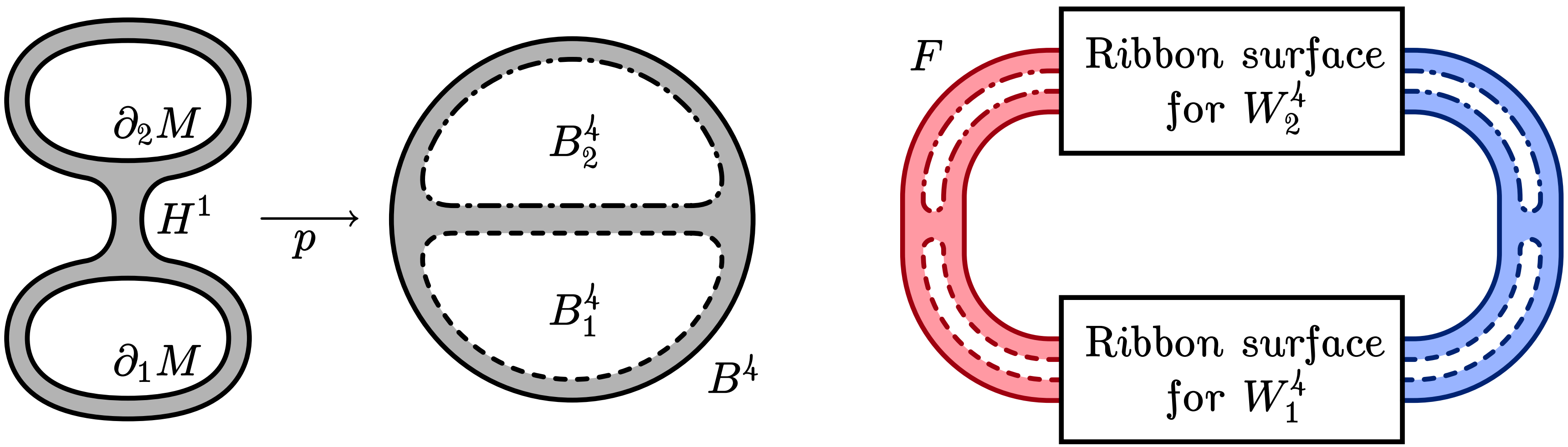
Bounded 4-manifolds

For any compact connected orientable smooth 4-manifold M with k boundary components there is a simple branched covering $M \to S^4 - \cup_{i=1}^{k} \operatorname{Int} B_i^4$ satisfying the property $*$

$k = 1$

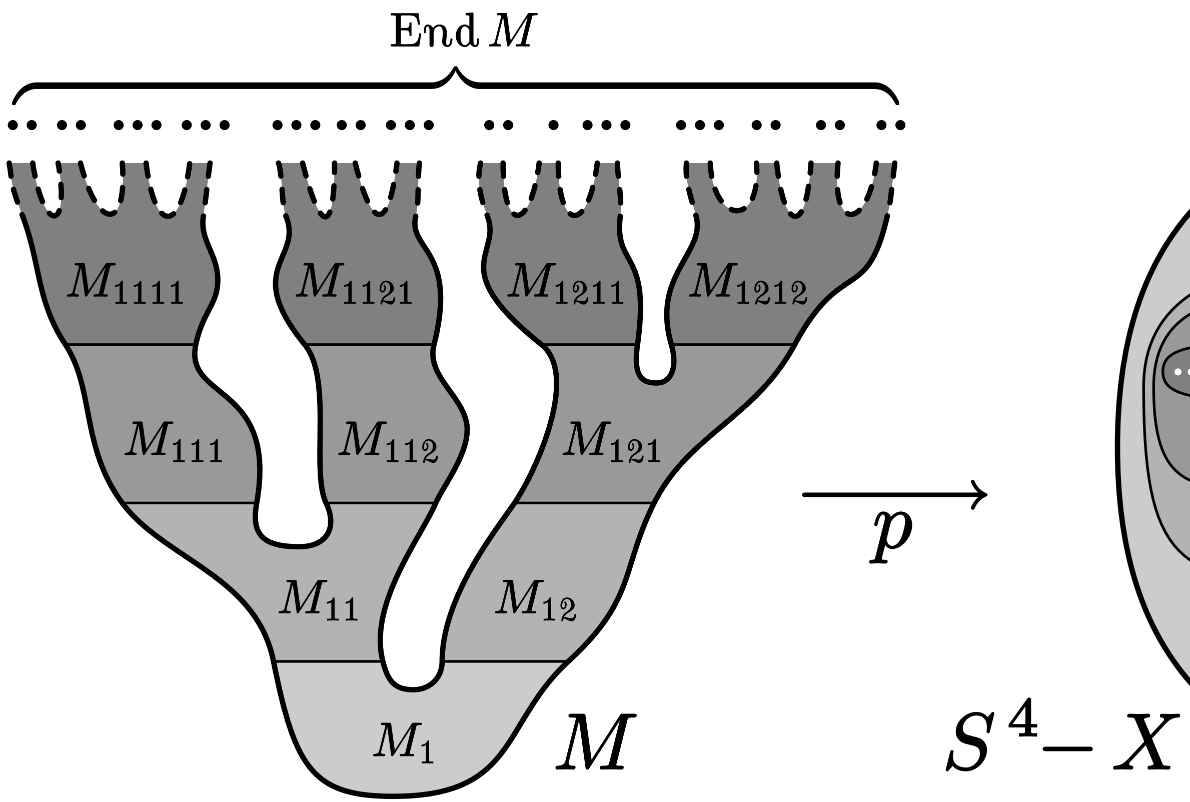


$k > 1$



Open 4-manifolds

For any open connected orientable 4-manifold M there is a simple branched covering $M \to S^4 - X$ with the property $*$, where $X \subset S^4$ is homeomorphic $\operatorname{End} M$



Any R^4_{exotic} is a simple branched cover of R^4_{standard}

Topological 4-manifolds

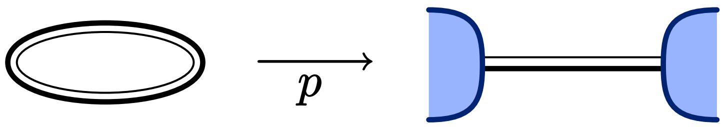
For any closed orientable topological 4-manifold M there is a simple branched covering $M \to S^4$ satisfying the property $*$ with a possibly wild branching surface F

$$\begin{array}{ccc} p : M - \{\mathrm{pt}\} \to R^4 & & \widehat{p} : M \to S^4 \\ \text{simple covering} & \rightsquigarrow & \text{simple covering} \\ \text{branched over } F \subset R^4 & & \text{branched over } \widehat{F} \subset S^4 \end{array}$$

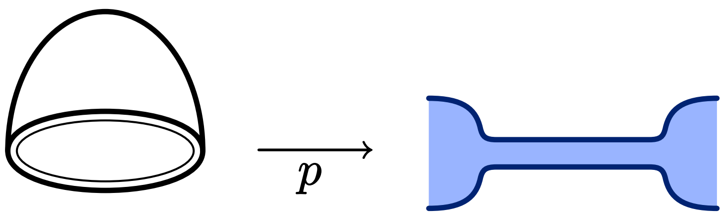
$\widehat{F}$ has one wild point (if M is not smooth)

Can we choose $\widehat{F}$ to be a topological surface and limit the wildness to the inclusion $\widehat{F} \subset S^4$?

Casson handle



2-handle



kinky handle