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Energy Agency

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Miramare-Trieste, Italy

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TBA

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Physics towards cell dynamics

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ESPCI

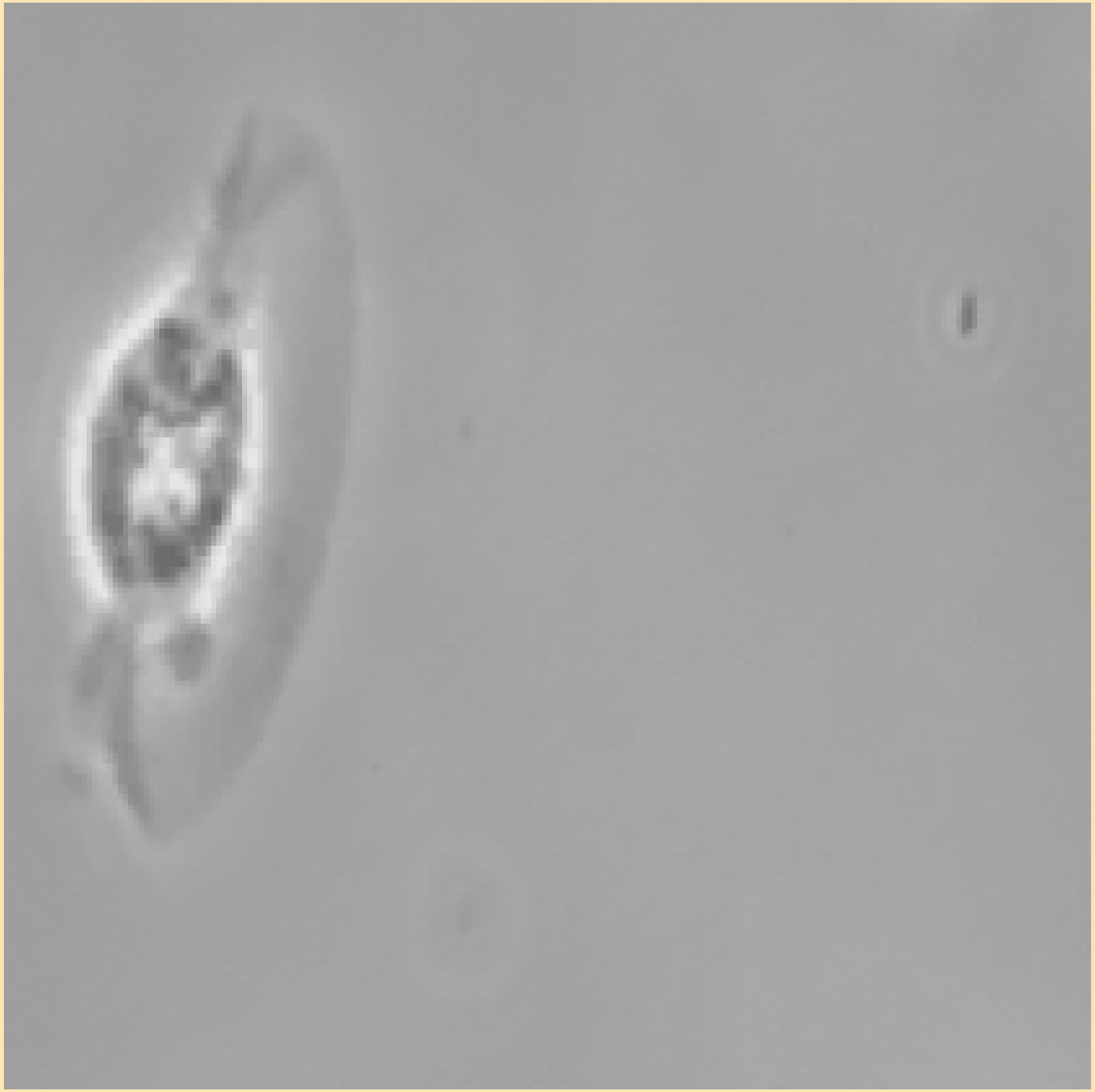
CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

Laboratoire PhysicoChimie Curie (Institut Curie), Paris

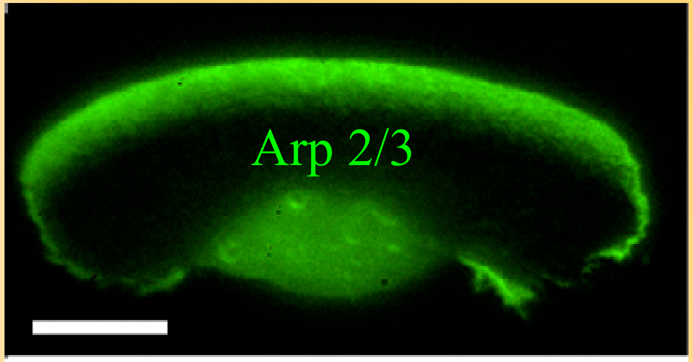
Institut Curie
La Science au Service de l'Homme
SECTION RECHERCHE

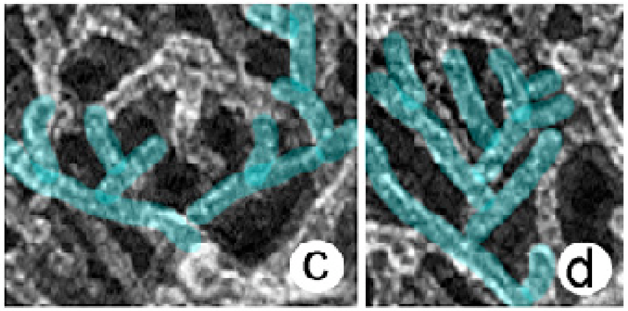
Physics “towards” cell dynamics

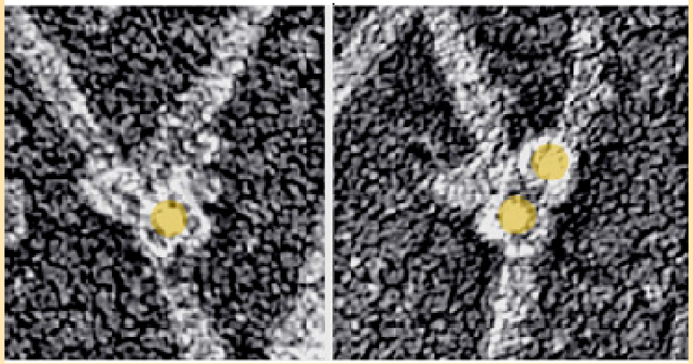
- K. Kruse
- J.F. Joanny
- F. Jülicher
- J.P.
- K. Sekimoto
- R. Voituriez
- K. Storm

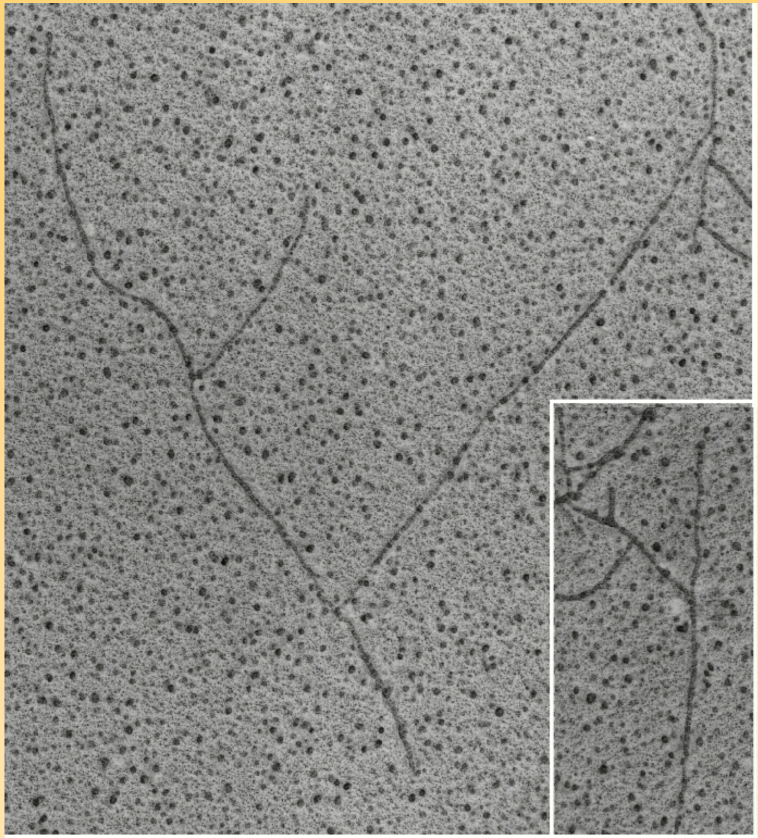


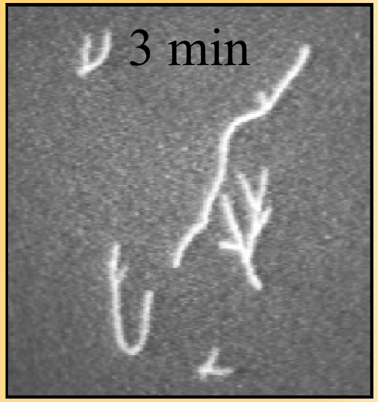
Filament branching array in lamellipodia

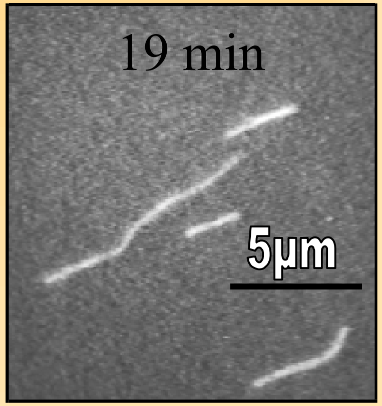






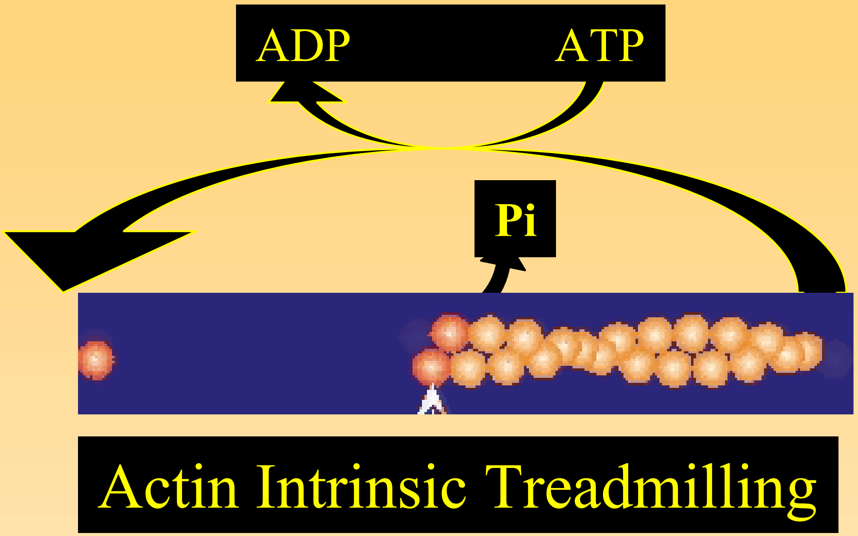




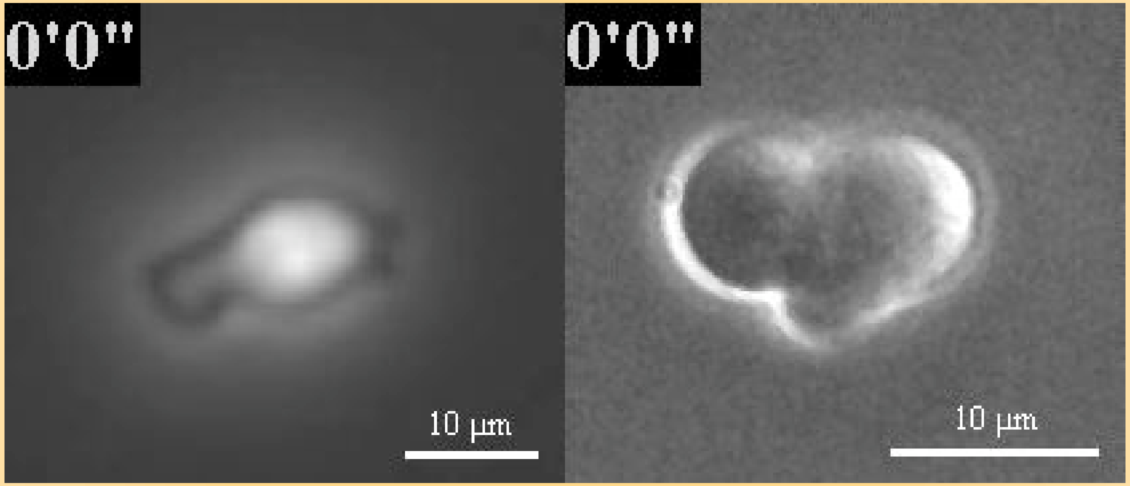


(T. Svitkina and G.G. Borisy, 1999; Blanchoin et al.; Pantaloni et al., 2000, Ishiwata et al 2000)

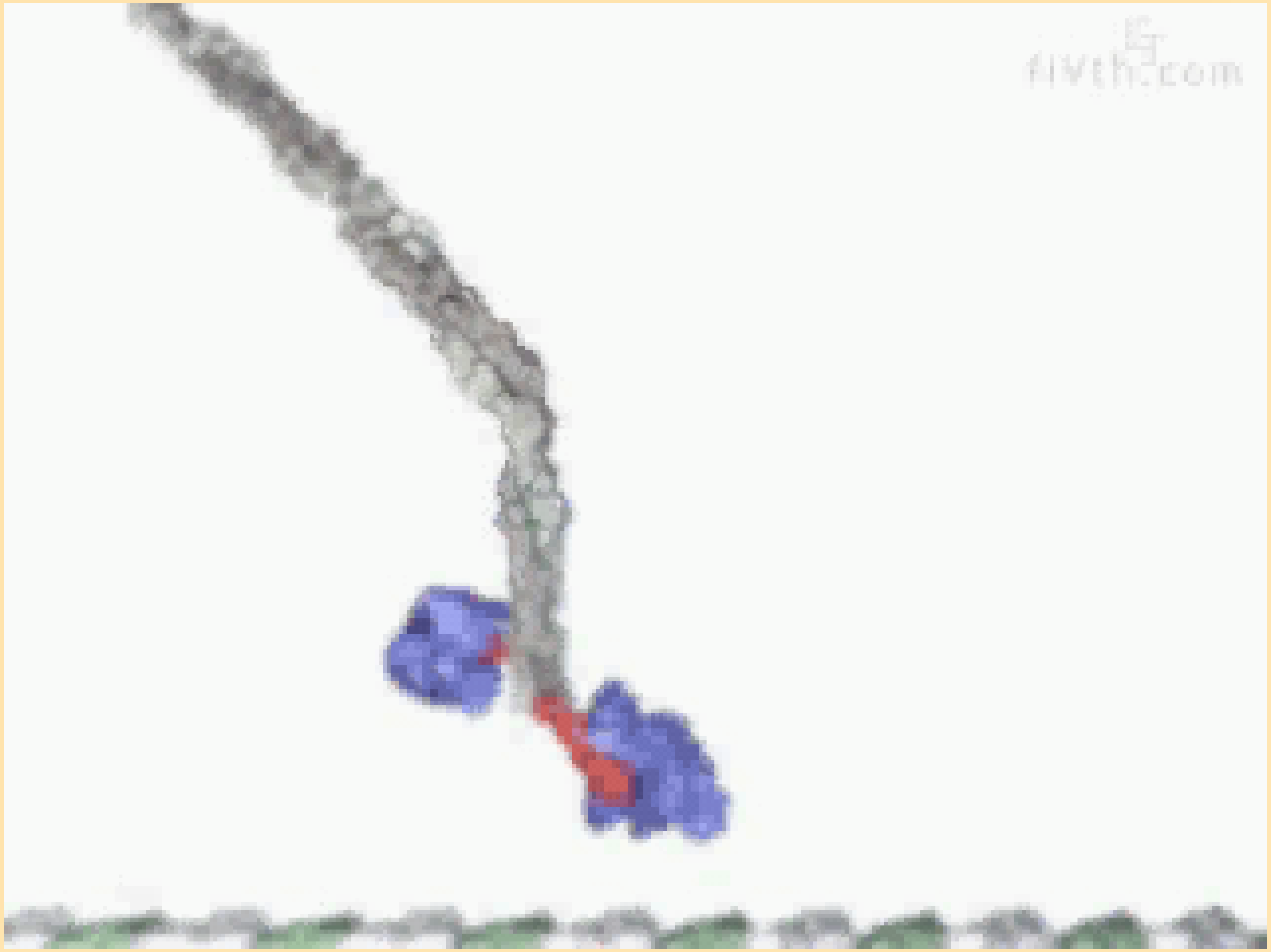
Actin-based motility



Fibroblast Fragments
Depolymerized Microtubules

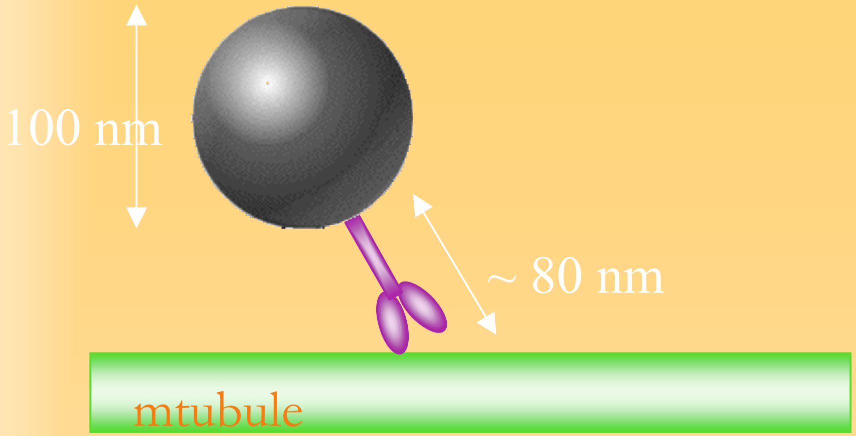


E. Paluch et al, E.480
Bornens et al, 1988

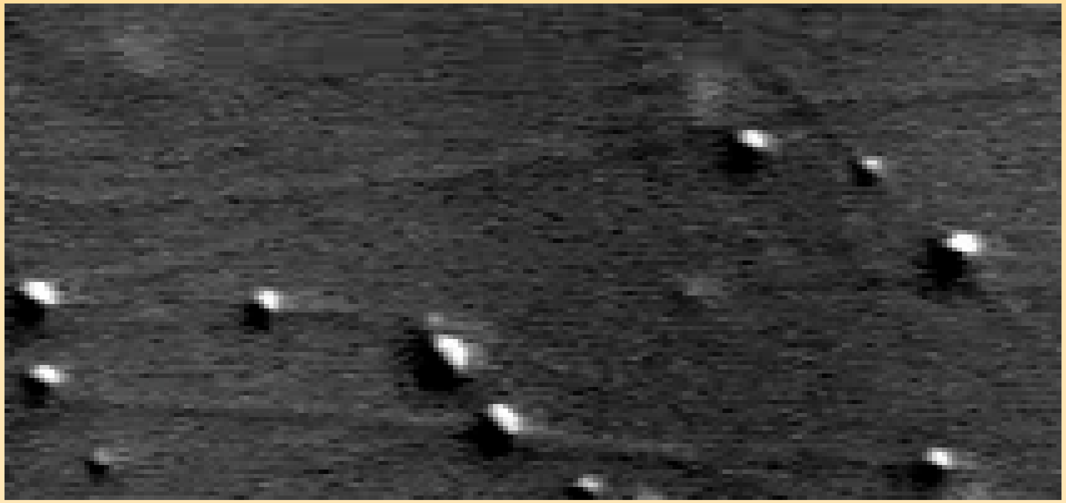


R. Vale

Kinesin in vitro



1) Speed: ~ 500 nm/sec
2) Force: few pN



MITOSIS

Cell division in the Chinese
Hamster Ovary (CHO) cell

EXPERIMENT

(François Nedelec et al)

Conserved quantities, Broken symmetries

- Actin (monomer+polymer)
- Myosin (bound, unbound)
- Momentum (force)
- Polarization

Conservation laws

$$\frac{\partial \rho}{\partial t} + \nabla(\mathbf{v}\rho) = -k_d\rho + k_p\delta_S$$

$$\frac{\partial \rho^{(a)}}{\partial t} + \nabla\mathbf{j}^{(a)} = k_d\rho - k_p\delta_S$$

$$\frac{\partial c^{(m)}}{\partial t} + \nabla\mathbf{j}^{(m)} = k_{\mathrm{off}}c^{(m)} - k_{\mathrm{on}}\rho(c^{(m)})^n$$

$$\frac{\partial c^{(b)}}{\partial t} + \nabla c^{(b)}\mathbf{v} + \nabla\mathbf{j}^{(b)} = -k_{\mathrm{off}}c^{(b)} + k_{\mathrm{on}}\rho(c^{(m)})^n$$

$$\partial_\alpha(\sigma_{\alpha\beta}^{\mathrm{tot}} - \Pi\delta_{\alpha\beta}) + f_\beta^{\mathrm{ext}} = 0$$

Dissipative

$$j_\alpha^{(a)} = -D^{(a)}\partial_\alpha \rho^{(a)}$$

$$j_\alpha^{(m)} = -D^{(m)}\partial_\alpha c^{(m)}$$

$$j_\alpha^{(b)} = -D^{(b)}\partial_\alpha c^{(b)} + \lambda p_\alpha \Delta\mu$$

$$J_\rho = \rho v$$

Reactive

flux	$\leftrightarrow$	force
$\sigma_{\alpha\beta}$	$\leftrightarrow$	$u_{\alpha\beta}$
P_α	$\leftrightarrow$	h_α
r	$\leftrightarrow$	$\Delta\mu$
$j_\alpha^{(i)}$	$\leftrightarrow$	$\partial_\alpha \mu^{(i)}$

$$h_\alpha = -\frac{\delta F}{\delta p_\alpha}$$

$$P_\alpha = \frac{D}{Dt} p_\alpha = \frac{\partial p_\alpha}{\partial t} + (v_\gamma \partial_\gamma) p_\alpha + \omega_{\alpha\beta} p_\beta$$

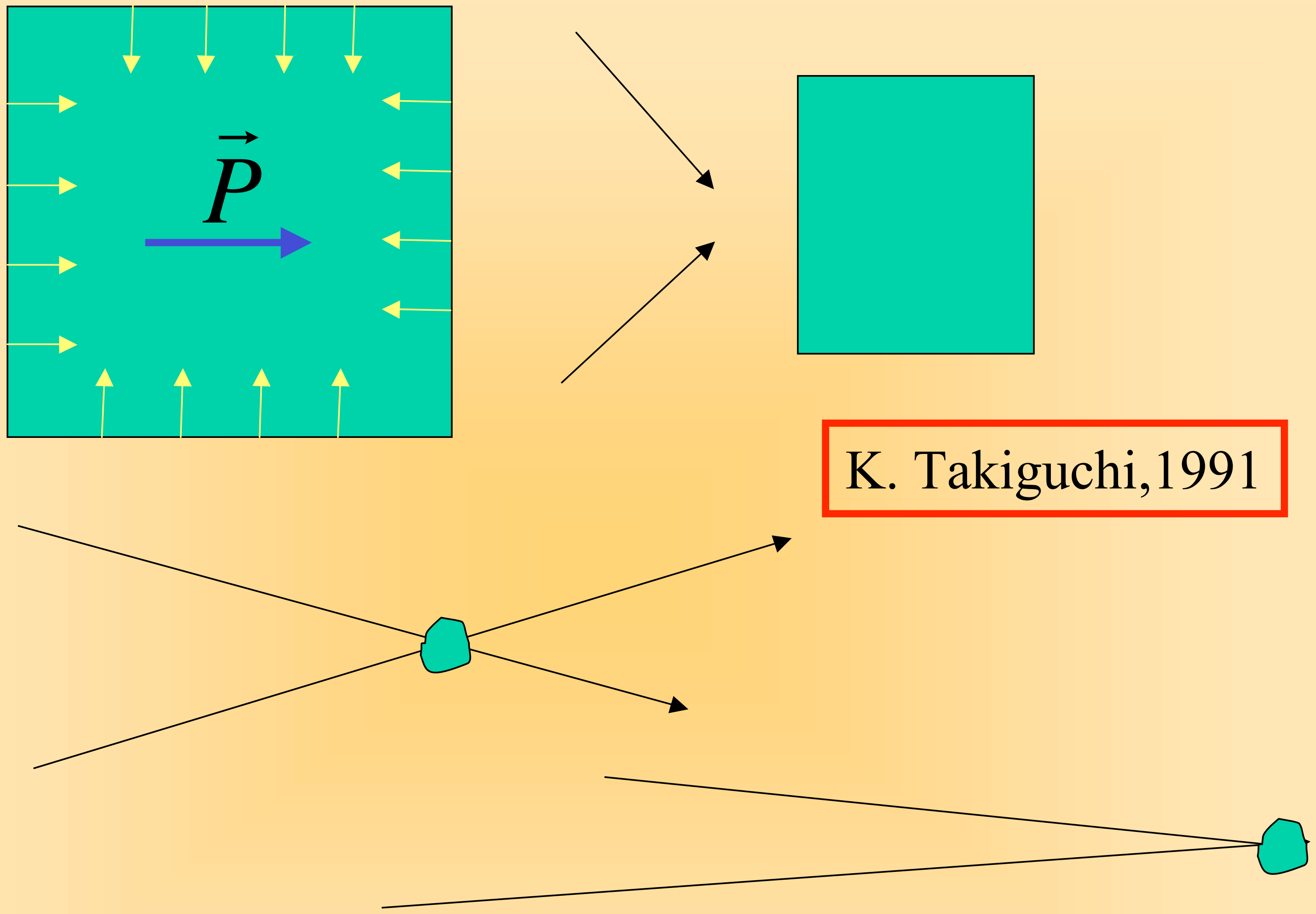
$$
\begin{aligned}
2\eta u_{\alpha\beta} &= \{\sigma_{\alpha\beta} + \zeta \Delta\mu p_\alpha p_\beta \\
&\quad + \zeta' \Delta\mu p_\gamma p_\gamma \delta_{\alpha\beta} + \bar{\zeta} \Delta\mu \delta_{\alpha\beta} + \quad \}
\end{aligned}
$$

$$
-
$$

$$
\frac{Dp_\alpha}{Dt} = \left(\frac{1}{\gamma_1} h_\alpha + \lambda_1 p_\alpha \Delta\mu\right)
$$

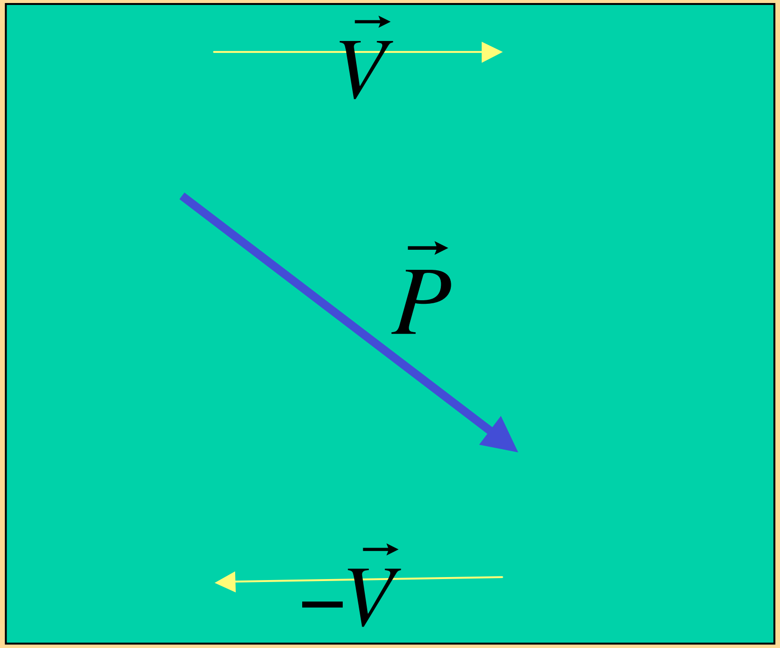
$$
-
$$

$$
\begin{aligned}
r &= \zeta p_\alpha p_\beta u_{\alpha\beta} + \bar{\zeta} u_{\alpha\alpha} + \zeta' p_\alpha p_\alpha u_{\beta\beta} \\
&+ \Lambda \Delta\mu + \lambda_1 p_\alpha h_\alpha \quad .
\end{aligned}
$$



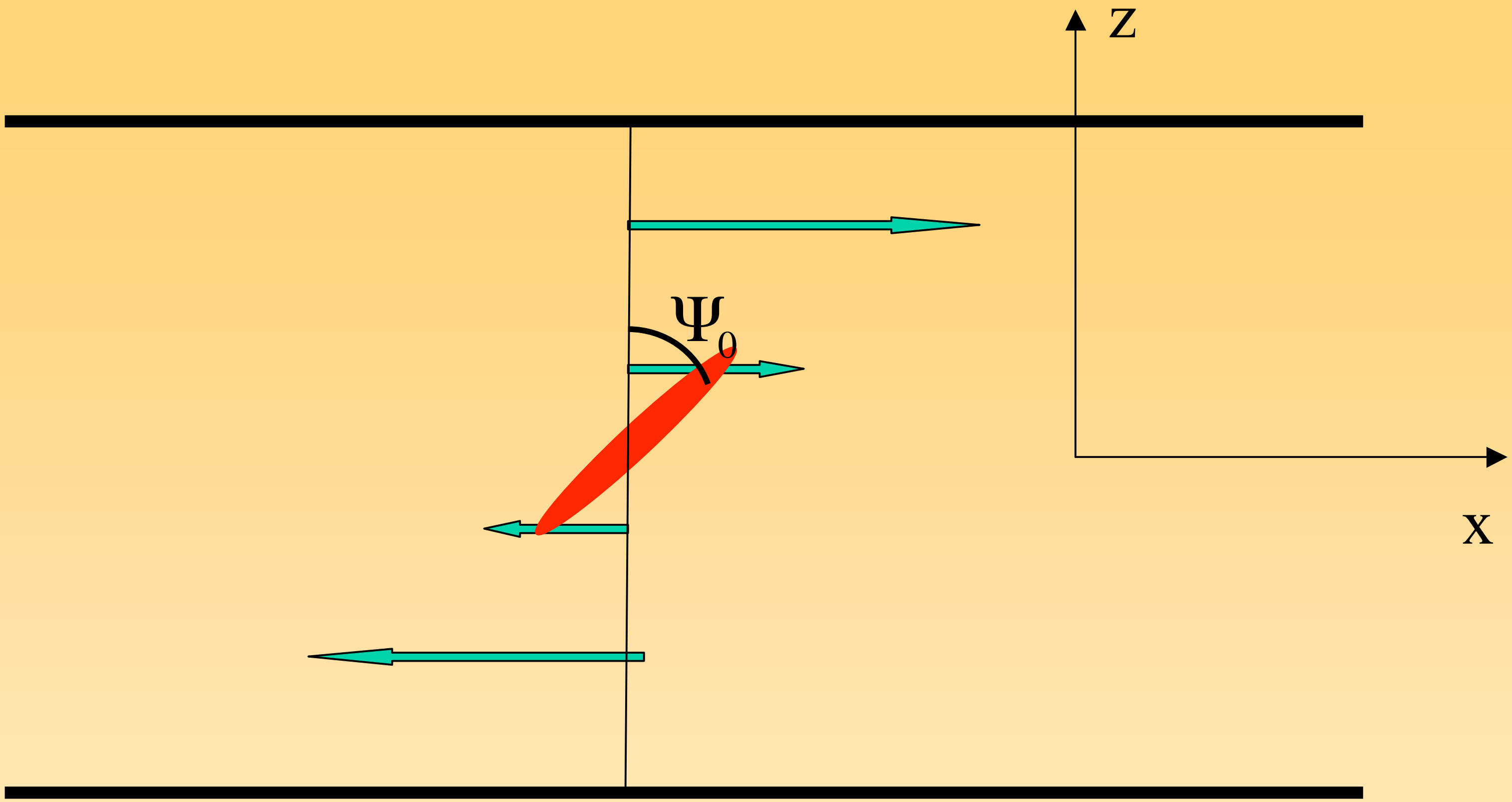
K. Takiguchi,1991

F. Nedelec et al 1997, K. Kruse, F. Jülicher, 2000, T. Liverpool, C. Marchetti 2002, I. Aranson2005, B. Mulder et al 2005



$$2\eta u_{\alpha\beta} = \ldots\ldots + \zeta\Delta\mu p_\alpha p_\beta + \ldots$$

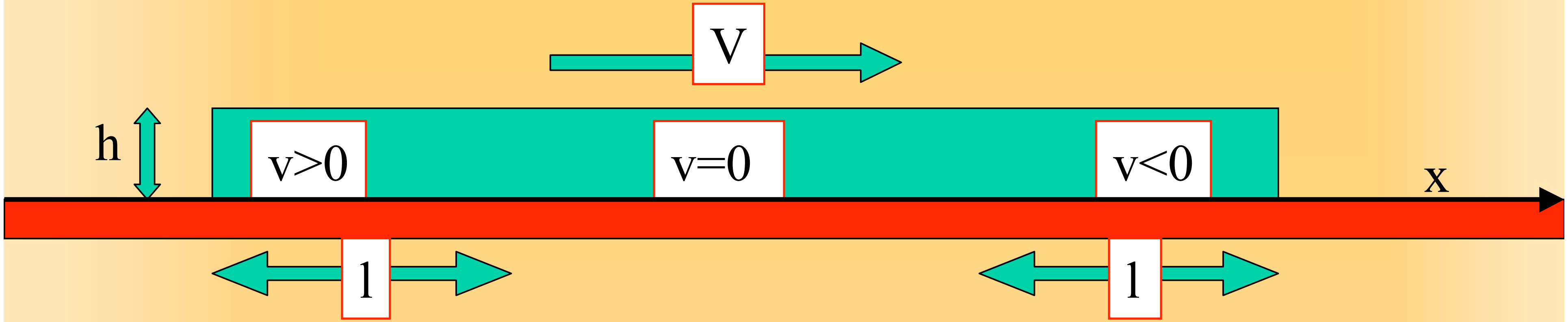
Nematic Hydrodynamics



$$\frac{Dp_\alpha}{Dt} = \ldots\ldots\ldots + \omega_{\alpha\beta} p_\beta + \ldots. = \ldots.. + \nu_1 u_{\alpha\beta} p_\beta + \ldots.$$

$$\cos(2\Psi_0) = 1/\nu_1$$

Thin slab



$$2\eta \frac{\partial v}{\partial x} = \sigma \qquad + \zeta \Delta \mu$$

$$\frac{\partial h \sigma}{\partial x} = \xi v$$

$$l^2 = 2\eta h / \xi$$

$$v_o = |\zeta \Delta \mu| l / 2\eta$$

$$\tau_o = 2\eta / |\zeta \Delta \mu|$$

$$V = (V_p + V_{dp})/2$$

Ramaswamy propagating mode

$$\frac{\partial c_b}{\partial t} + v_0 \frac{\partial c_b}{\partial x} = -k_{off} c_b + k_{on} c_{ub}$$

$$\frac{\partial c_{ub}}{\partial t} = +k_{off} c_b - k_{on} c_{ub}$$

$$\frac{\partial (c_b + c_{ub})}{\partial t} + v_0 \frac{\partial c_b}{\partial x} = 0$$

$$c_{ub} \approx c_b \frac{k_{off}}{k_{on}}$$

$$\frac{\partial c_b}{\partial t} + \bar{v}_0 \frac{\partial c_b}{\partial x} = 0$$

$$\bar{v}_0 = v_0 / (1 + k_{off} / k_{on})$$

M.G. Vicker 2002,
T. Bretscneider et al 2004,
G. Giannone et al 2004

Lamellipodium shape and motion?

$$\mathbf{v}^p = \mathbf{n} k_p \rho_{wa}(x)$$

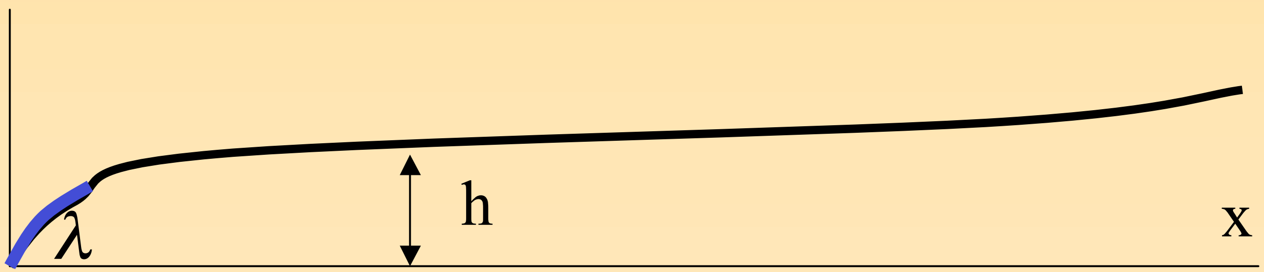
$$\rho_{wa}(x) = \rho^0_{wa} \exp(-x/\lambda)$$

$$\frac{dv}{dx} = \frac{1}{4\eta}\left(\frac{F}{h} + \zeta\Delta\mu\right)$$

$$\frac{dF}{dx} = \xi v$$

$$h(x) = \frac{k_p}{u + v(x)} \int_0^x dx' \rho_{wa}(x')$$

$$\lambda << d$$



$$u \simeq \left[v_d + |\zeta \Delta\mu| \left(\frac{h_0}{4\eta\xi} \right)^{1/2} \right] \Big/ \left[1 + \frac{\xi_{cb} L_{cb}}{(4\eta\xi h_0)^{1/2}} \right]$$

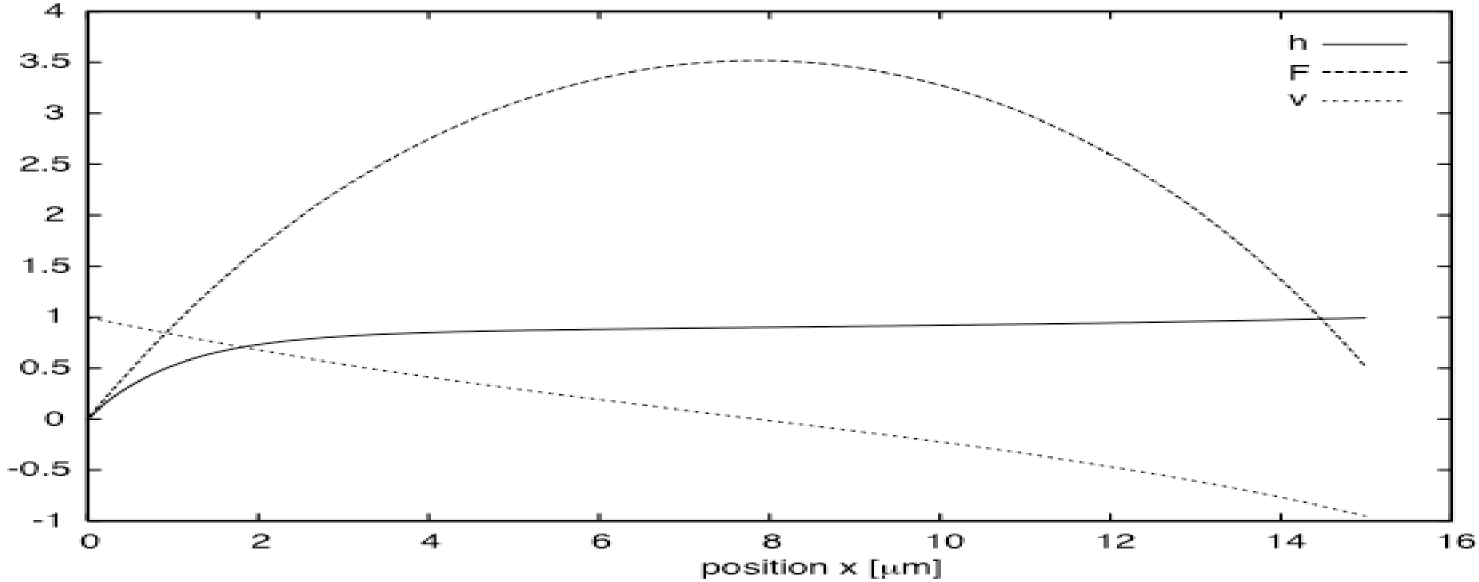
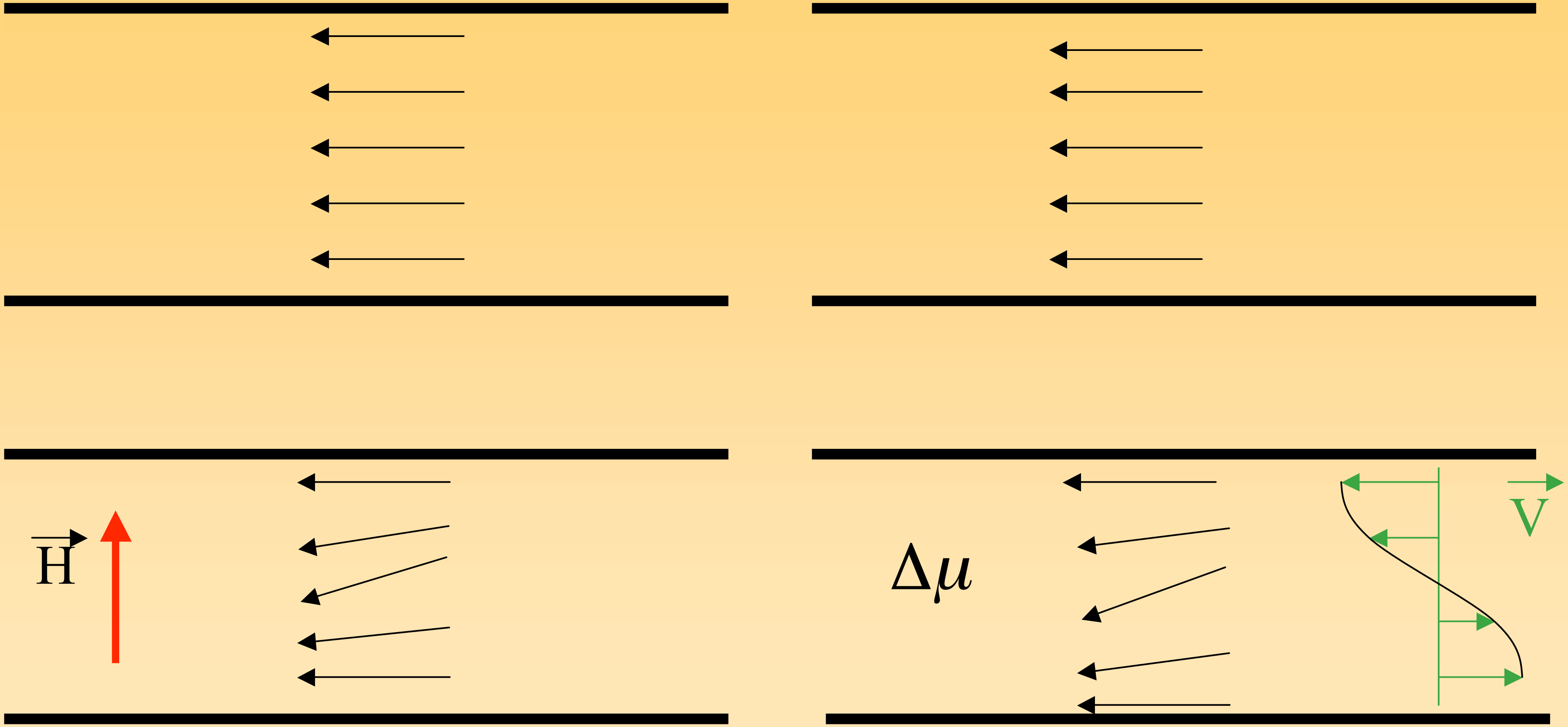


Fig. 1 (Kruse et al.)

Spontaneous Frederiks transition

R. Voituriez



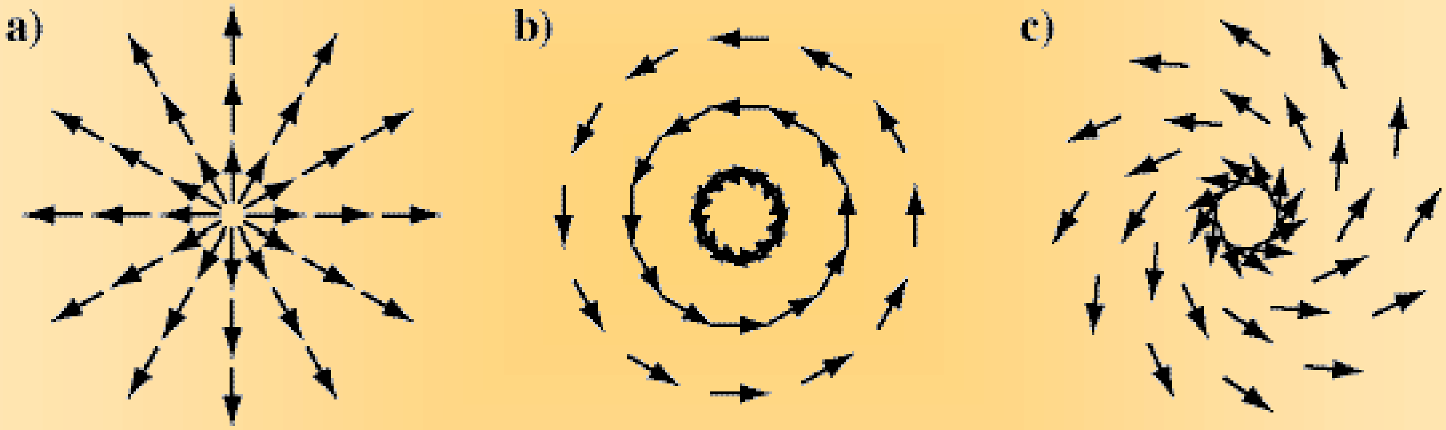
$$\frac{\partial \alpha}{\partial t} = \frac{K}{\gamma_1}\frac{\partial^2 \alpha}{\partial z^2} + (1+\nu_1)u_{zx}$$

$$2\tilde{\eta}u_{zx} \approx \sigma_{zx} - \tilde{\xi}\Delta\mu \sin(2\alpha) + \nu_1 K\frac{\partial^2 \alpha}{\partial z^2}$$

$$\frac{\partial \alpha}{\partial t} = \frac{K}{\tilde{\gamma}_1}\frac{\partial^2 \alpha}{\partial z^2} - \Delta\mu\tilde{\tilde{\xi}}\alpha$$

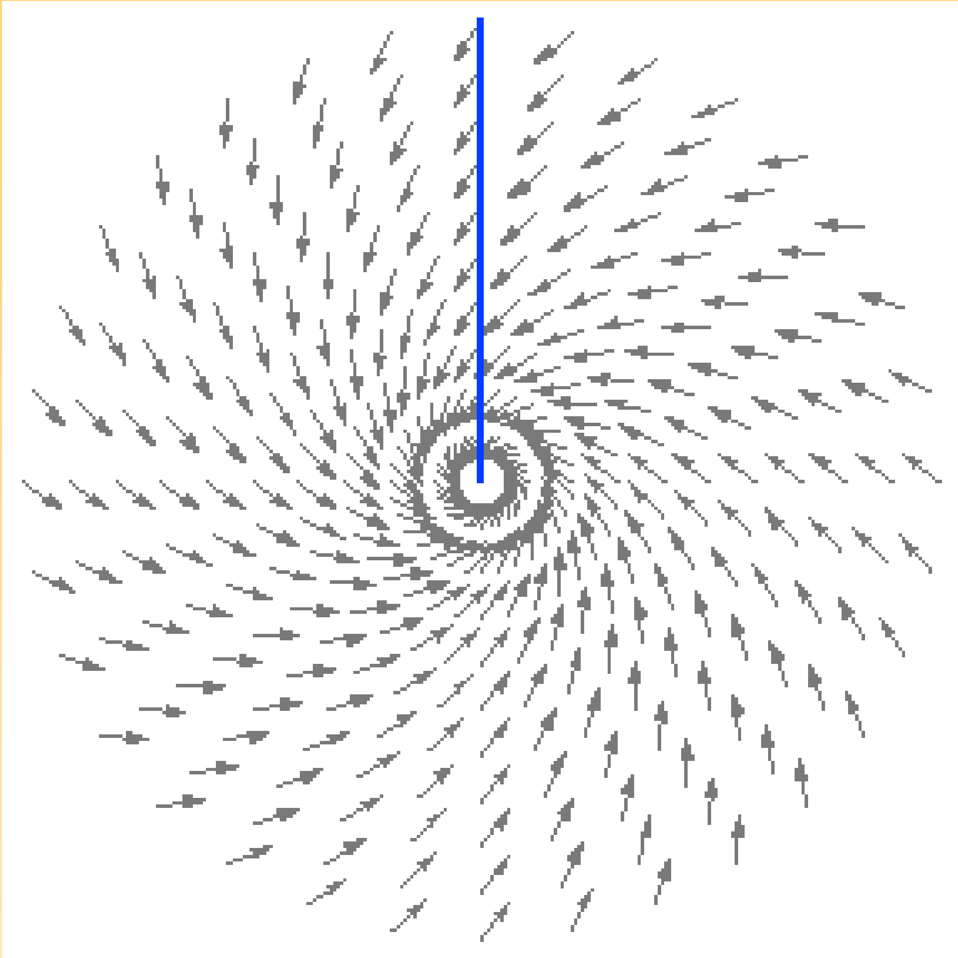
$$(\Delta\mu\tilde{\tilde{\xi}})_c = K\left(\frac{\pi}{D}\right)^2$$

$$F = \int d^2x \left[\frac{K}{2}(\nabla \cdot \mathbf{p})^2 + \frac{K+\delta K}{2}(\mathbf{p}\cdot\nabla\mathbf{p})^2 + k\nabla\cdot\mathbf{p} - h_{||}\mathbf{p}^2 \right]$$



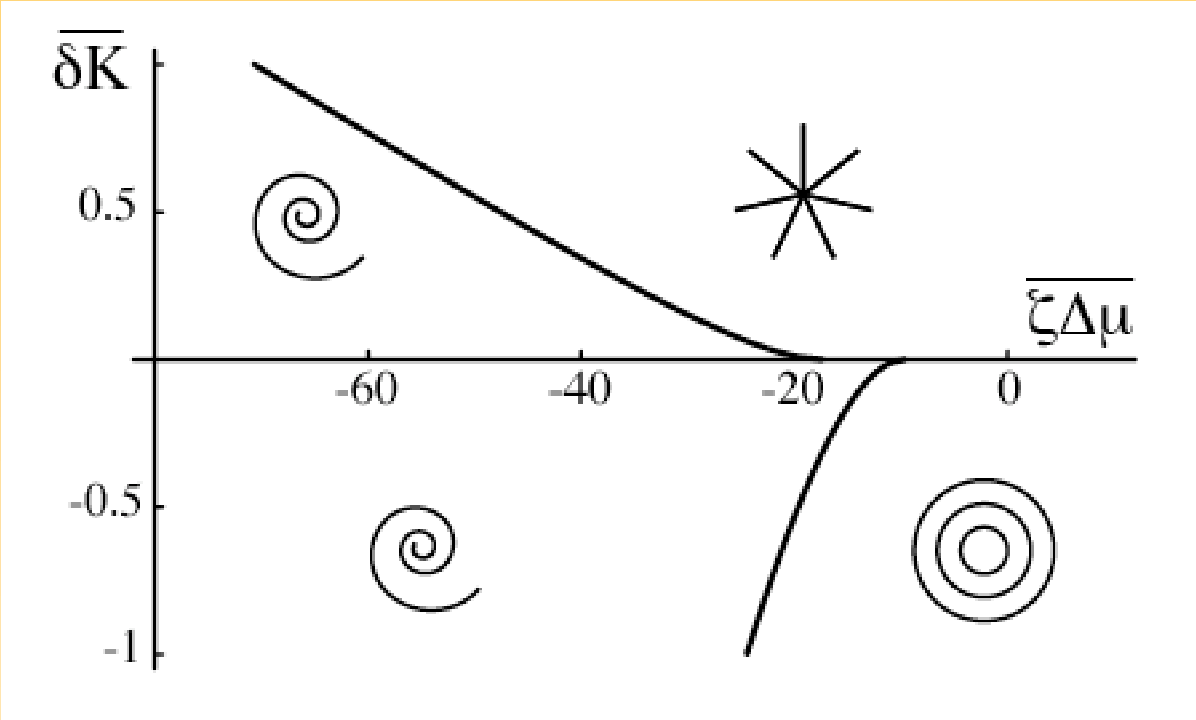
$\delta K > 0$ $\delta K < 0$ $\delta K = 0$

Spontaneously moving topological singularity



$$v_\theta(r) = \omega_0 r \log(r/r_0)\,,\ \omega_0 = \frac{2\sin 2\psi_0}{4\eta + \gamma_1 \nu_1^2 \sin^2 2\psi_0}\zeta\Delta\mu$$

State Diagram



$\delta\overline{K} = \delta K / K$

$\overline{\zeta}\overline{\Delta}\overline{\mu} = R^2 \tilde{\zeta} \Delta\mu / K$

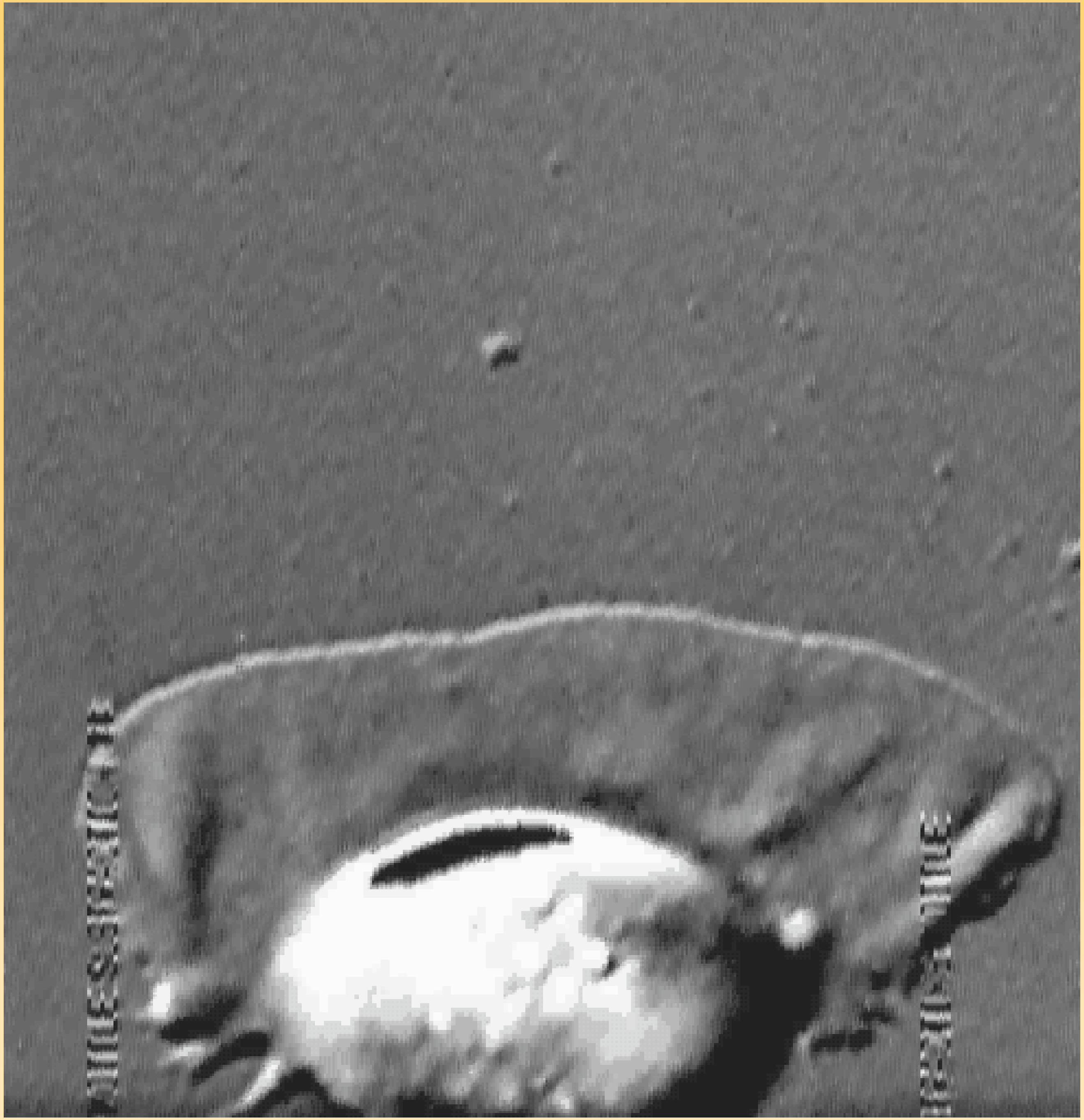
Substrate Friction

$$\partial_\alpha(\sigma_{\alpha\beta} - P\delta_{\alpha\beta}) = \xi v_\beta$$

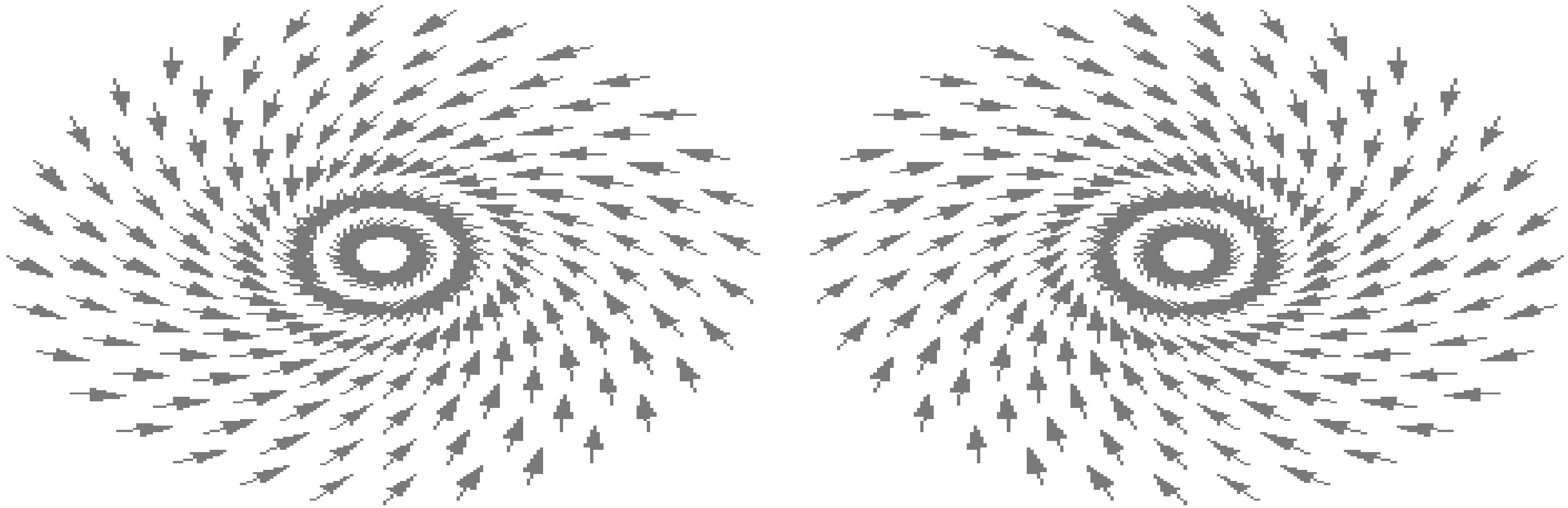
$$\lambda_f = (4\eta + \gamma_1 \nu_1 \sin^2 2\psi_o)^{1/2} / 2\xi^{1/2}$$

$$v_\theta(r) = 2\omega_o \lambda_f \left\{ K_1(r/\lambda_f) - \frac{\lambda_f}{r} \right\}$$

EXPERIMENT
(François Nedelec et al)



Keratocyte?



Merits?

- General Framework, measurable coefficients
- Retrograde motion
- Ramaswamy wave
- “Frederiks” Transition
- Rotating Asters, motility
- Gel unstable in spherical geometry
- Connection to other branches of science