



The Abdus Salam  
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Educational, Scientific  
and Cultural Organization

  
International Atomic  
Energy Agency



*Summer School on*  
**Design and Control of  
Self-Organization in Physical, Chemical, and  
Biological Systems**

**25 July to 5 August, 2005**

*Miramare-Trieste, Italy*

**1668/5**

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**Reproduction, Adaptation, and Evolution as a Universal  
Feature of Dynamical Systems with Growth**

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## Complex Systems Biology

cf. Life as Complicated System: (current trend)

Enumeration of molecules, processes  
detailed models mimicking the life process  
But understanding??

## Life as Complex System:

Understand General features at a System Level

Strategy:

1) Search for universal features in cellular processes :  
extension of Dynamical Systems & Statistical Physics

2) Constructive Approach: (Exp & Theory)

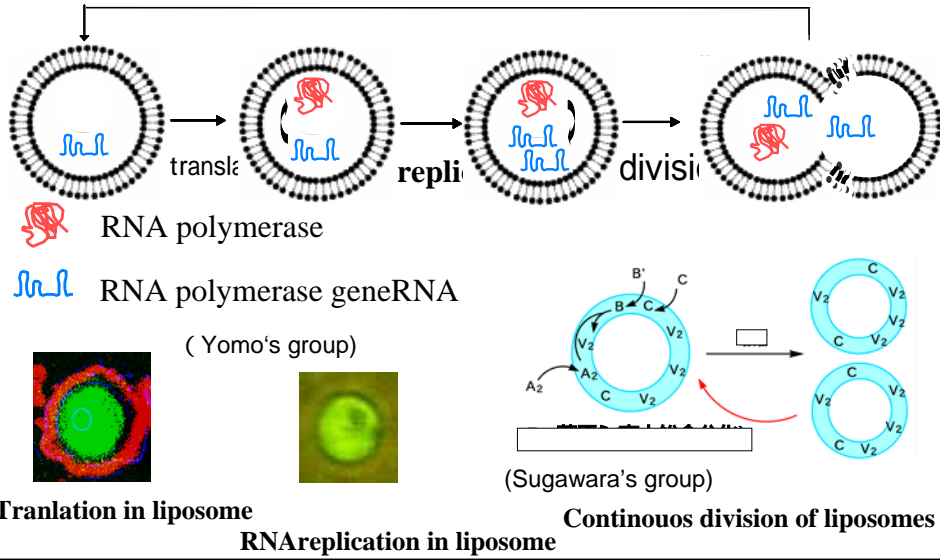
`construct simple system to catch universal features'  
as simple as possible

## Constructive Biology Project

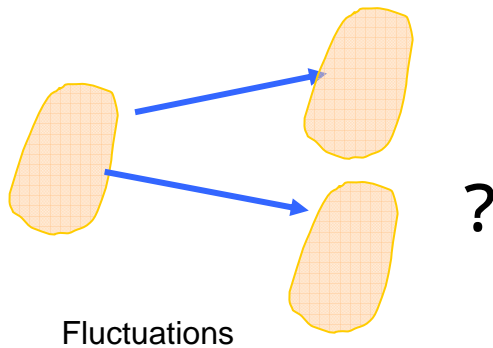
theme	experiment	theory	question
replicating system	in vitro replication with enzymatic reaction	minority control	origin of heredity; evolvability
cell system	replicating cell with internal reactions	universal statistics in reaction dynamics	condition for recursive growth
cell differentiation. development	differentiation of E. coli by interaction	emergence of differentiation rule from dynamics	irreversibility robustness
Spontaneous adaptation	Artificial gene network	Adaptive attractor selection by noise	Robust adaptation without signalling
evolution	Relevance of phenotypic fluctuation and dynamics	Genetic assimilation of phenotype fluct. and dynamics	geno-pheno type relationship

ERATO Project Complex Systems Biology (2004 -2010, with Tetsuya Yomo (experimentalist))

Replicating artificial cell (experiment)  
 (↔ theory; fluctuation, minority control)



How recursive production of a cell is sustained ?  
 each cell complex reaction network  
 with diversity of chemicals;  
 The number of molecules of each species  
 not so large



## Toy Cell Model with Catalytic Reaction Network Crude but whole cell model

C.Furusawa & KK, PRL2003

■ **k species of chemicals**,  $X_0 \dots X_{k-1}$   
number ---  $n_0, n_1 \dots n_{k-1}$

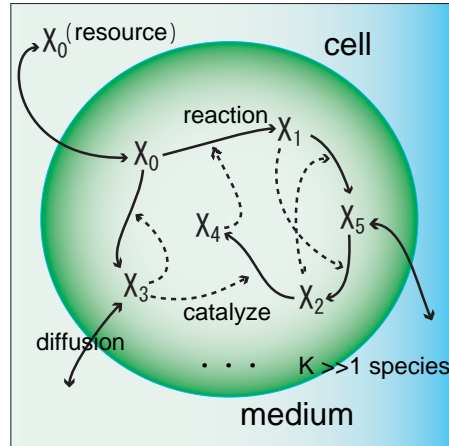
■ **random catalytic reaction network**  
with the path rate  $p$   
for the reaction  $X_i + X_j \rightarrow X_k + X_l$

■ some chemicals are **penetrable**  
**through the membrane with the**  
**diffusion coefficient D**

■ resource chemicals are thus  
transformed into impenetrable  
chemicals, leading to the growth in  
 $N = n_i$ , when it exceeds  $N_{\max}$   
**the cell divides into two**

model

(Cf. KK&Yomo 94,97)



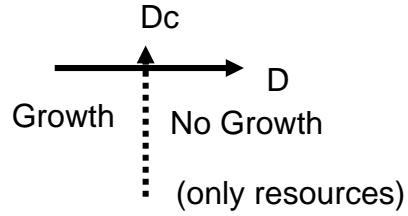
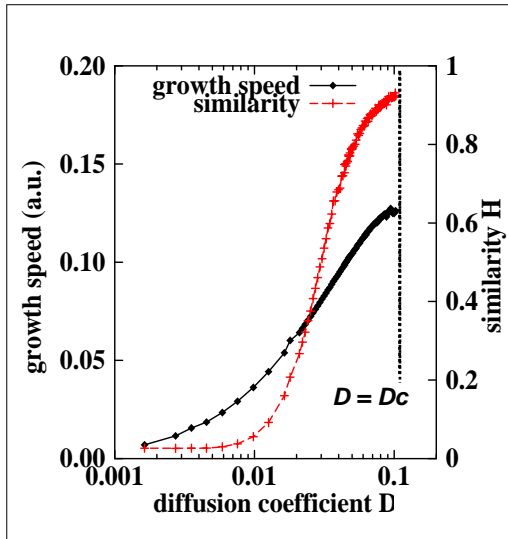
$dX/dt = \dots$ ; rate equation;  
Stochastic model here

In continuum description, the following rate eqn.,  
but we mostly use stochastic simulation

$$\begin{aligned} \frac{dn_i}{dt} = & \sum_{j,\ell} \text{Con}(j, i, \ell) \epsilon n_j n_\ell / N^2 \\ & - \sum_{j,\ell'} \text{Con}(i, j, \ell') \epsilon n_i n_{\ell'} / N^2 \\ & + D \sigma_i (\bar{n}_i / V - n_i / N), \end{aligned}$$

where  $\text{Con}(i, j, \ell)$  is 1 if there is a reaction  $i + \ell \rightarrow j + \ell$ ,  
and 0 otherwise, whereas  $\sigma_i$  takes 1 if the chemical  $i$  is  
penetrable, and 0 otherwise. The third term describes the  
transport of chemicals through the membrane, where  $\bar{n}_i$  is

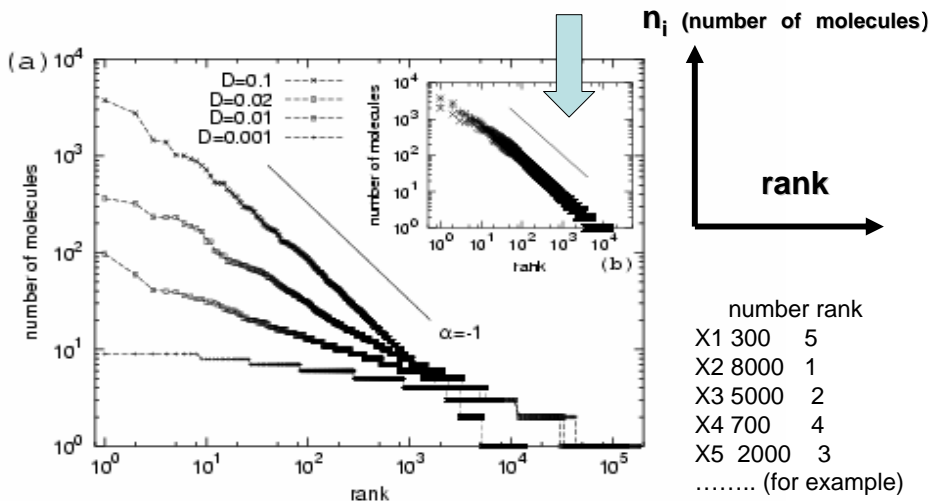
## Growth speed and fidelity in replication are maximum at $D_c$



similarity is defined from inner products of composition vectors between mother and daughter cells

Furusawa & KK, 2003, PRL

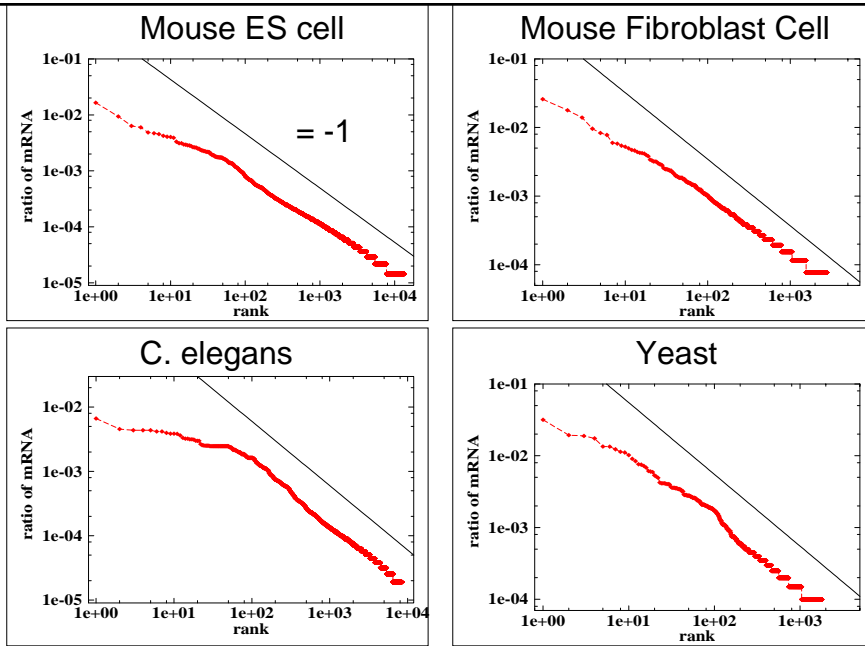
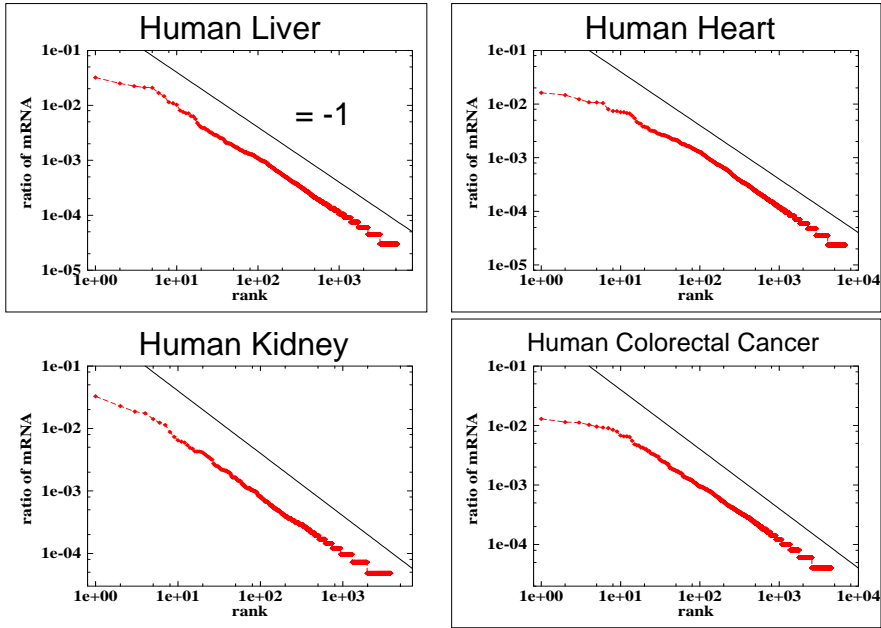
Zipf's Law is observed at  $D = D_c$



Average number of each chemical  $1/(\text{its rank})$

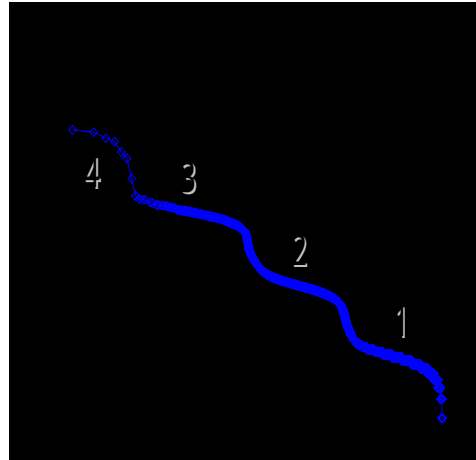
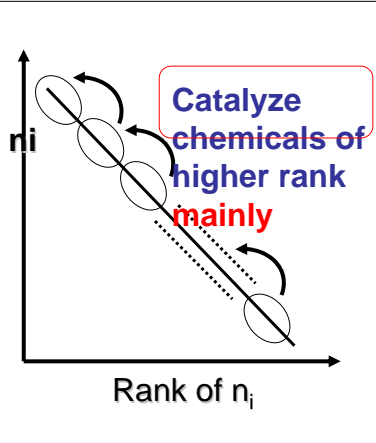
(distribution of  $x$ :  $(x) \propto x^{-2}$ )

Confirmed by gene expression data



Later confirmed by several other groups

## Formation of cascade catalytic reaction

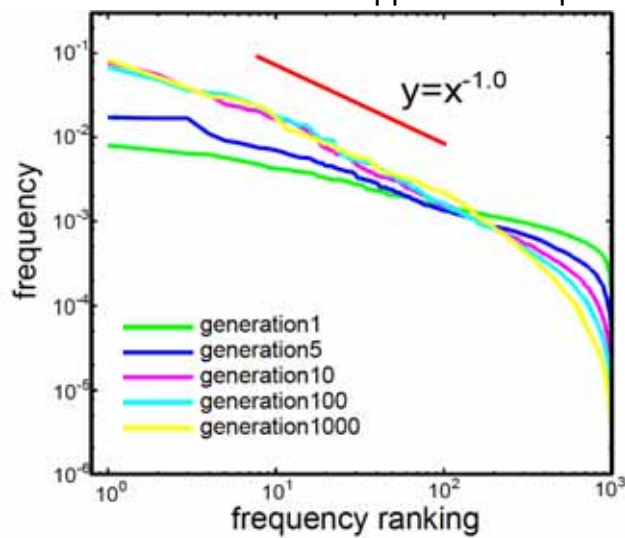


With conservation law,  
The exponent -1 is explained  
Mean-field type (self-consistent) calc.)

- 1 : minority molecules
- 2 : catalyzed by 1, synthesized by resource
- 3 : catalyzed by 2

## Evolution of Network to satisfy Zipf's law? Yes

Critical D value depends on connectivity in the network;  
mutation of network + selection → approaches Zipf's law

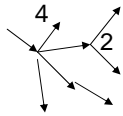


Furusawa

Fig1. rank distribution of chemical concentrations

Zipf's law holds, **irrespective of network structure**, but  
 Later, the connectivity in the network approaches  
 "scale-free" network through evolution.  
 statistical properties; embedded into network structure  
 Dynamics (abundance) first, structure (equation for dynamics) later

evolutionary embedding  
 of dynamics into network



probability for a path to  
 chemical with abundances  $x$   
 is selected;  $q(x) \rightarrow$   
 transformation of abundance  
 distrib. to connectivity distrib.

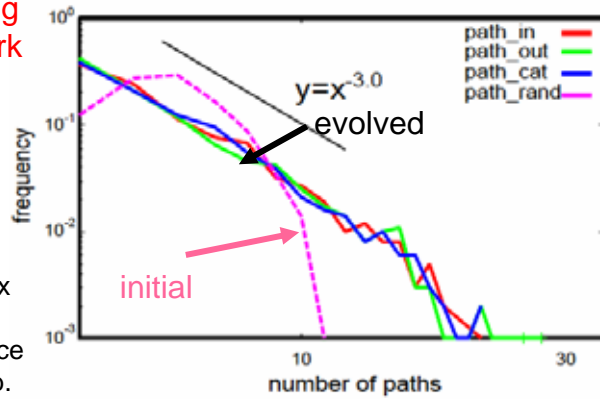


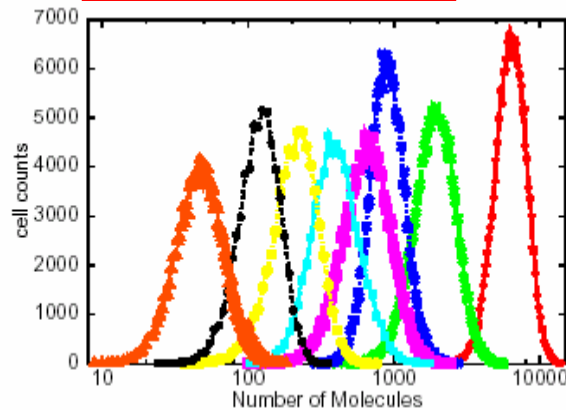
Fig2. connectivity distributions

Furusawa, KK, submitted

So far average quantity of all components;

Next question: fluctuation by cells:  
 distribution of each  $N_i$  by cells

Log normal distribution !



LOG SCALE

Furusawa,..  
 KK,  
 Biophysics2005

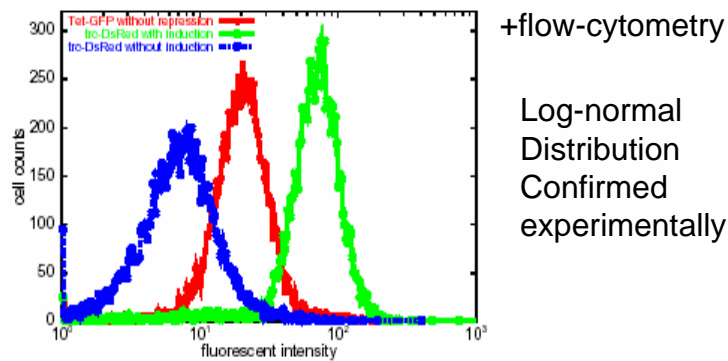
e.g.  
 cell1 X1 10000  
 cell2 8000  
 cell3 15000  
 cell4 20000

.....  
 histogram

Each color  
 gives  
 different  
 chemical  
 species



### Experiment; protein abundances measured by fluorescence



+flow-cytometry  
Log-normal  
Distribution  
Confirmed  
experimentally

Furusawa, Kashiwagi, Yomo, KK

Figure 3: The number distribution of the proteins measured by fluorescent intensity. Distributions are obtained from three *Escherichia coli* cell populations containing different reporter plasmids, i.e., EGFP (enhanced green fluorescent protein) under the control of the tetA promoter, DsRed (red fluorescent protein) under the control of the trc promoter with and without IPTG induction. Note that, although the IPTG induction changes the average fluorescent intensity, both the distributions (with and without the induction) can be fitted by log-normal distributions well.

Also studied in GFP synthesis in liposome

### Heuristic explanation of log-normal distribution

Consider the case that a component X is catalyzed by other component A, and replicate; the number  $N_X, N_A$

$$d N_X / dt = N_X N_A$$

then

$$d \log( N_X ) / dt = N_A$$

If,  $N_A$  fluctuates around its mean  $\langle N_A \rangle$ , with fluct. (t)

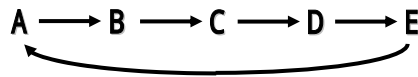
$$d \log( N_X ) / dt = \langle N_A \rangle + \text{fluct.}(t)$$

$\log( N_X )$  shows Brownian motion  $\rightarrow N_X$  log-normal distribution

too, simplified, since no direct self-replication exists here

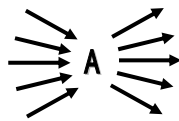
But with cascade catalytic reactions, fluctuations are successively multiplied, (cf addition in central limit theorem.); Hence after logarithm, central limit th. applied

Cascade leads to multiplicative propagation of noise (at critical region)

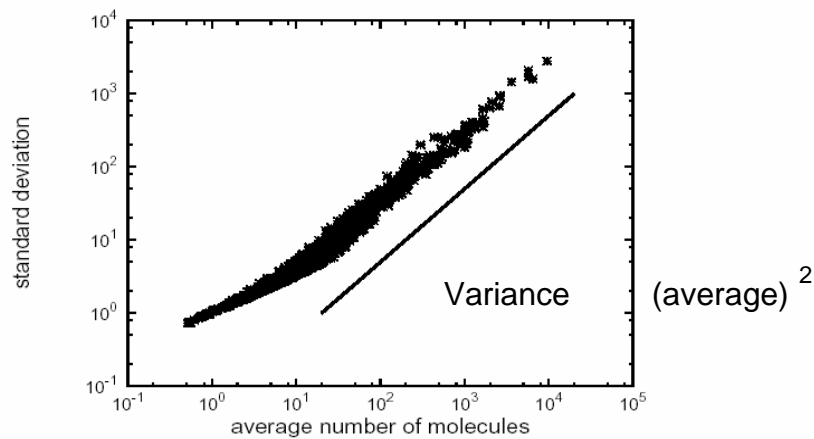


Propagation of fluctuation, feedback to itself, leading to log-normal distribution tail.

Cf. If parallel,



Fluctuations come in parallel:  
Usual central limit theorem is valid;  
normal distribution.



Also confirmed experimentally (indirectly)

2. Standard deviation versus average number of molecules. Using the same data set parameters as for Fig.1, the relationship between the average and standard deviation is plotted for all chemical species. The solid line is for reference.

Questions

(1) Large phenotypic fluctuation →  
relevance to biology ?

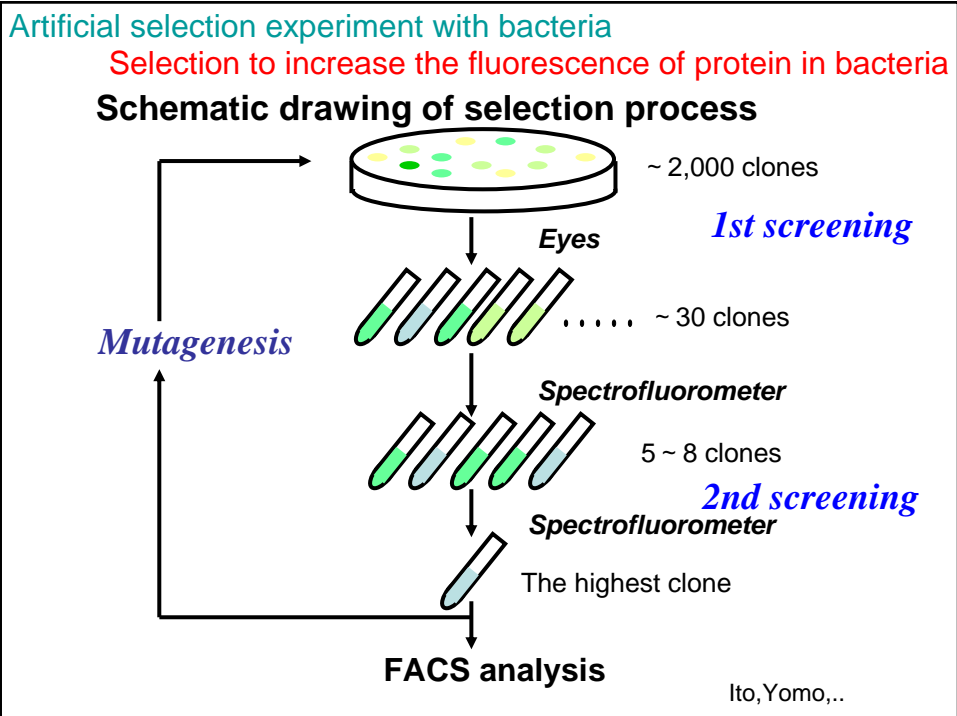
ans. **evolution** (Sato et al., PNAS, 2003) **adaptation**,.....  
--- recall in standard evolutionary genetics,  
only the distribution of gene is discussed, by assuming  
unique phenotype from a given genotype

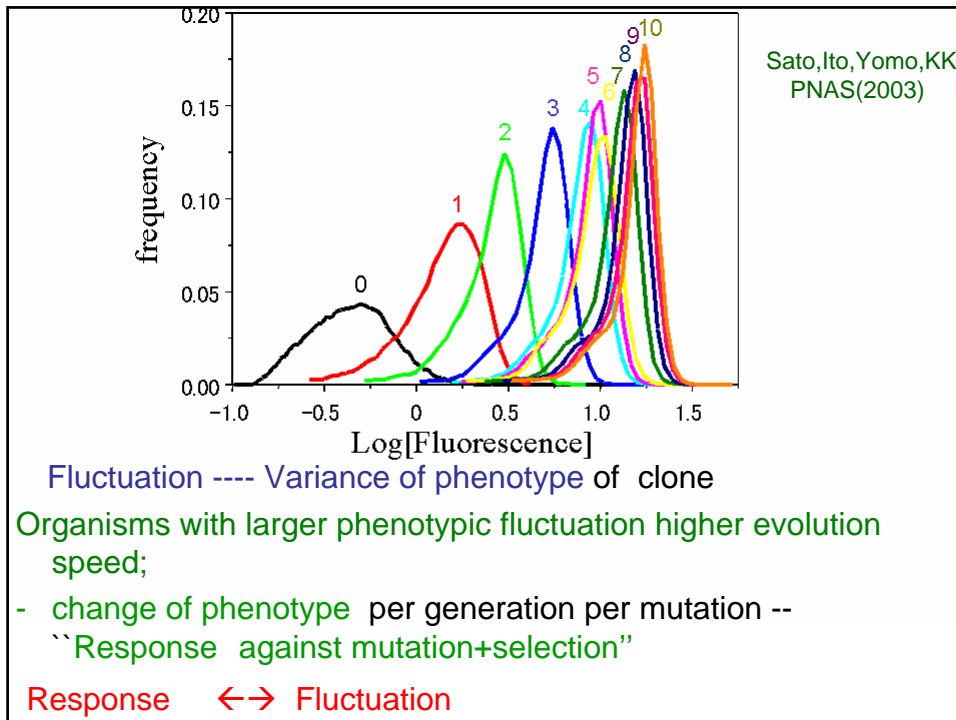
(2) All chemicals have such large fluctuations?  
Important ones are protected??

**Origin of heredity (genetic information)**  
**why is there genotype and phenotype**

--in terms of dynamical systems

gene: equation of dynamical systems, parameter,  
Phenotype: variable according to the dynamical  
systems (with fluctuations)





So-called fluctuation-dissipation theorem in physics:

Force to change a variable  $x$ ;

**response ratio** = (shift of  $x$ ) / force

**fluctuation of  $x$**  (without force)

**response ratio** proportional to **fluctuation**

originated by Einstein's paper a century ago...

**Generalization::(mathematical formulation)**

**response ratio of some variable  $x$  against the change of parameter  $a$  versus fluctuation of  $x$**

$P(x;a)$   $x$  variable,  $a$ : control parameter  
change of the parameter  $a \rightarrow$   
peak of  $P(x;a)$  ( i.e.,  $\langle x \rangle$  average ) shifts

$$\frac{\langle x \rangle_{a+\Delta a} - \langle x \rangle_a}{\Delta a} \propto \langle (\delta x)^2 \rangle_a = \langle (x - \langle x \rangle)^2 \rangle$$

**Gaussian Distribution**

$$P(x; a) = N e^{-\frac{(x-\beta(a))^2}{2\alpha(a)}}$$

**Change of distribution by the change of parameter  $a \rightarrow a + \Delta a$**

$$\begin{aligned} P(x; a + \Delta a) &= N' e^{-\frac{(x-\beta(a+\Delta a))^2}{2\alpha(a+\Delta a)}} \\ &= N' e^{-\frac{(x-\beta(a))^2}{2\alpha(a)} + \varepsilon^{(1)}(a, \Delta a)(x-\beta(a)) + \frac{1}{2}\varepsilon^{(2)}(a, \Delta a)(x-\beta(a))^2} \end{aligned}$$

**where**

$$\varepsilon^{(1)}(a, \Delta a) = \frac{\beta(a + \Delta a) - \beta(a)}{\alpha(a)}$$

$$\varepsilon^{(2)}(a, \Delta a) = \frac{\alpha(a + \Delta a) - \alpha(a)}{\alpha(a)^2}$$

then

$$\langle x \rangle_{a+\Delta a} - \langle x \rangle_a = \varepsilon^{(1)}(a, \Delta a) \sigma_a^2$$

$$\sigma_{a+\Delta a}^2 - \sigma_a^2 = \varepsilon^{(2)}(a, \Delta a) (\sigma_a^2)^2$$

**If change of  $a$  is not large**

$$\varepsilon^{(1)}(a, \Delta a) = b \Delta a$$

**$b$ ; constant**

$$\boxed{\langle x \rangle_{a+\Delta a} - \langle x \rangle_a = b \Delta a \sigma_a^2}$$

**Response ratio is proportional to fluctuation**

Artificial selection experiment with bacteria  
 for enzyme with higher catalytic activity  
 for some protein with higher function

Change in gene (parameter;  $a$ )

“Response” ----- change of phenotype  $\langle x \rangle$   
 (e.g., fluorescence intensity)

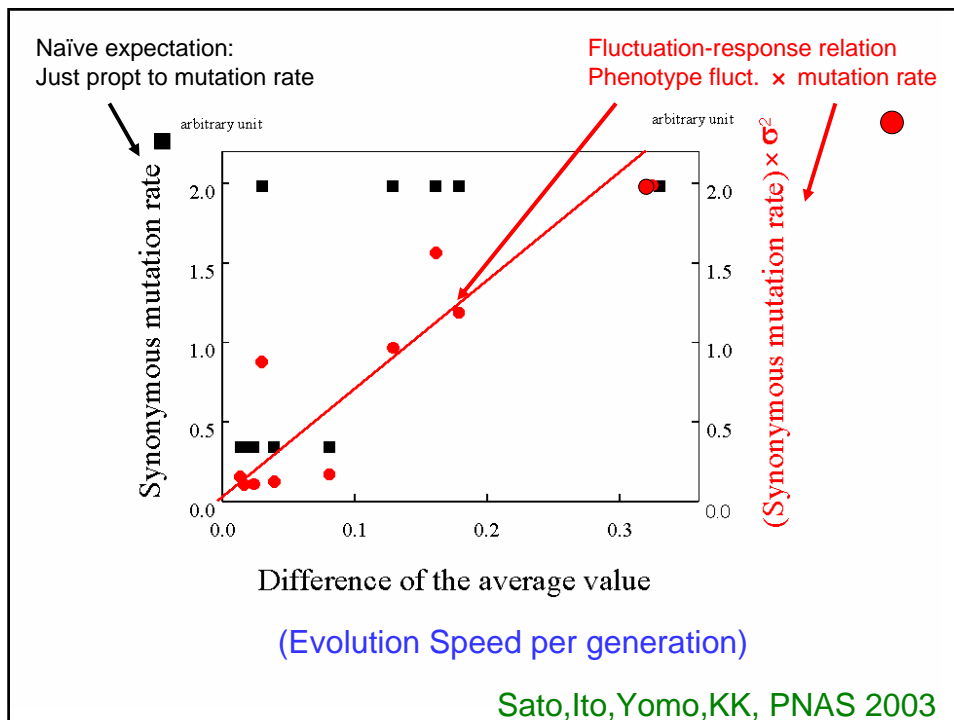
per generation per (synonymous) mutation rate

Fluctuation ---- Variance of phenotype  $x$  of clone

Fluctuation in the phenotype  $x$  of clone

speed of evolution to increase  $\langle x \rangle$

(proportional or correlated)

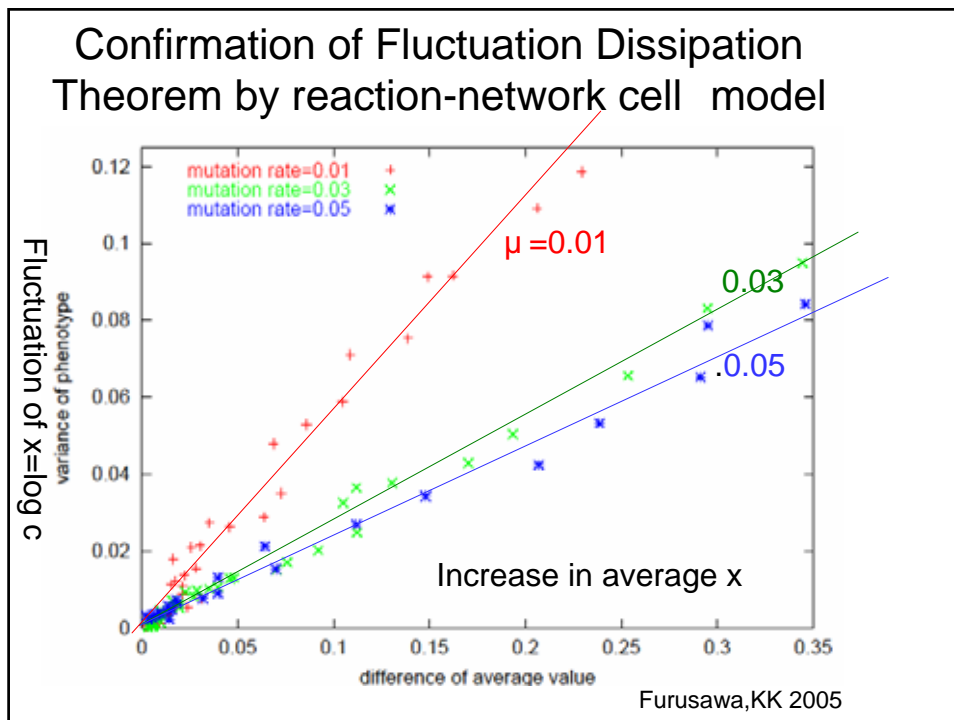


- Confirmation by numerical evolution experiment by the reaction-net cell model


Mutate the network ('gene') with mutation rate  $\mu$ ,  
(rewire the path of the network with the rate)  
and select such network

having highest concentration  $c$  of  
a given specific chemical

Measure the phenotypic fluctuations of clone  
concentration distribution is log-normal, so we  
choose  $\log c$  as a variable



(1) the use of log(fluorescence), because  
 log x is close to Gaussian distribution in experiments

(2): relationship between  
 phenotype fluctuation of clone vs evolution speed  
 in contrast to  
 phenotype fluctuation by gene variation (mutation) vs.  
 evolution speed (standard population genetics)  
 relationship?? 

what phenotype can vary  $\leftrightarrow$  what gene can change  
 fluctuation of variable (micro) vs  
 variation of equation (genetic evolution)

Indeed, by assuming also gene a variable, and  $P(x,a)$   
 as 2 variable distrib., a general relation between  
 geno- and pheno-type fluctuations is obtained  
 (KK, Furusawa submitted)  $\rightarrow$  theory of genetic assimilation  
 self-consistency between micro & genetic levels

- In the talk I derived general relationship between the phenotypic fluctuation of clone vs phenotypic fluctuation by genetic change.
- This leads to a theory of geno-pheno relationship at fluctuation level
- Since the contents are not yet published (under review), they are omitted here. Please ask me if you are interested in



In the talk, we have proposed  
novel Mechanism of Spontaneous Adaptation  
(without the use of signal transduction)  
based on stochastic fluctuation.

Here I omit this part since the results are not yet  
published ( I can tell you privately if you are  
interested)

### Summary

- How is recursive production of cells possible in the midst of diversity and fluctuations?  
→ Universal Statistics: amplification and regulation of fluctuations. (Zipf's law and log-normal distribution)
- Biological relevance of such large fluctuations?  
→ Phenotypic Fluctuation Evolution Speed
- Spontaneous adaptation with noise to select a state with higher growth in advance

All are 'universal features' in steady growth system!

- Collaborators:
  - Chikara Furusawa** (Reaction network of cell(Zipf's law,Log-normal), Cell-differentiation)
  - Katsuhiko Sato** (fluctuation-response relationship)
  - experiments:
    - Tetsuya Yomo**
    - Akiko Kashiwagi, Takao Suzuki, Yoichiro Ito**  
( Yomo's group)
- Most papers mentioned here are available at <http://chaos.c.u-tokyo.ac.jp>  
(PNAS,2003;PRL2003;...)