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International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
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Summer School on

**Design and Control of
Self-Organization in Physical, Chemical, and
Biological Systems**

25 July to 5 August, 2005

Miramare-Trieste, Italy

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Onset of Collective Behaviour in Globally Coupled Oscillators

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Onset of collective behaviour in globally coupled oscillators

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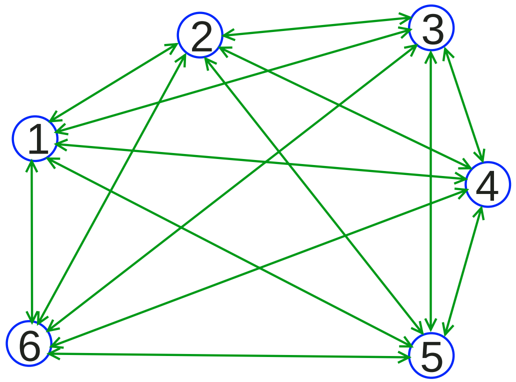
Trieste – July 2005

Collaborators: R. LIVI, P.K. MOHANTY, A. TORCINI, R. ZILLMER



- FROM BINARY UNITS TO OSCILLATORS
- FULL AND PARTIAL SYNCHRONIZATION
- TRANSIENT DYNAMICS AND LINEARLY STABLE CHAOS
- DISORDER AND NOISE

A PROTOTYPE SYSTEM: THE KURAMOTO MODEL

$$\dot{\theta}_i = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i) + \xi_i(t)$$

$$re^{i\psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

$$\dot{\theta}_i = \omega_i + Kr\sin(\psi - \theta_i) + \xi_i(t)$$

the order parameter

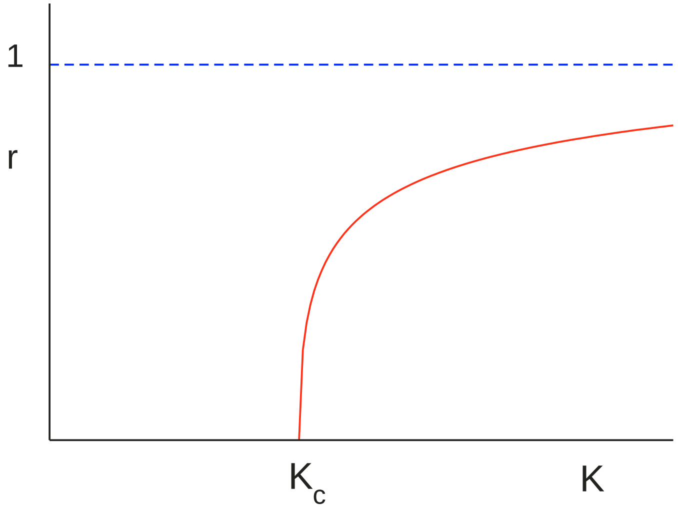
NONLINEAR FOKKER-PLANCK EQUATION

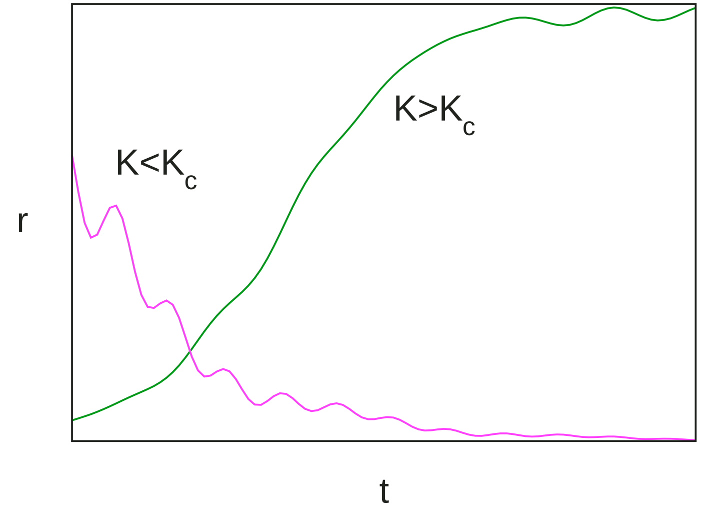
$$\frac{\partial \rho}{\partial t} = -\frac{\partial v\rho}{\partial \theta} + D\frac{\partial^2 \rho}{\partial \theta^2}$$

$$v = \omega + Kr\sin(\psi - \theta)$$

$$re^{i\psi} = \int_0^{2\pi} d\theta \int_{-\infty}^{+\infty} d\omega e^{i\theta}\rho(\theta, t, \omega)g(\omega)$$

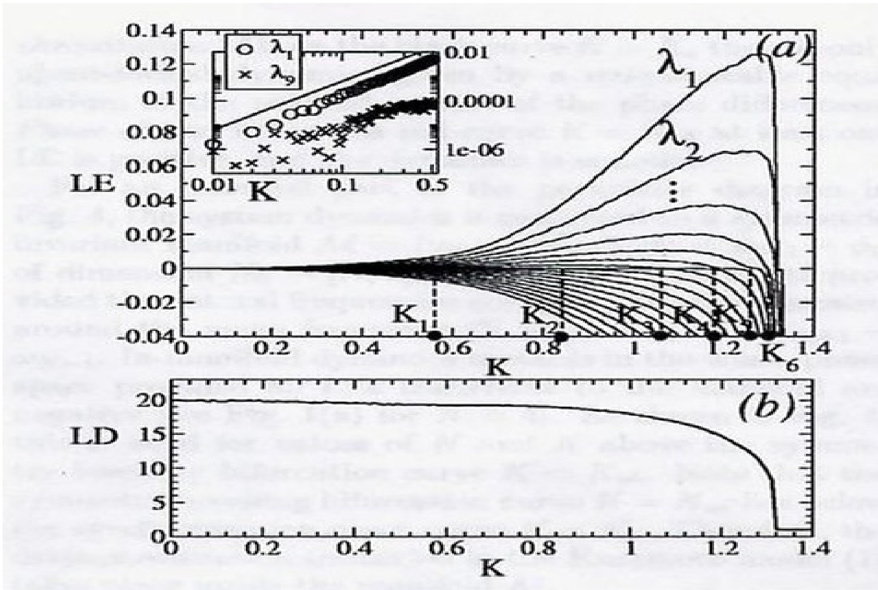
$$\int_0^{2\pi} d\theta\rho(\theta, t, \omega) = 1$$





$\lambda \approx 1/N$

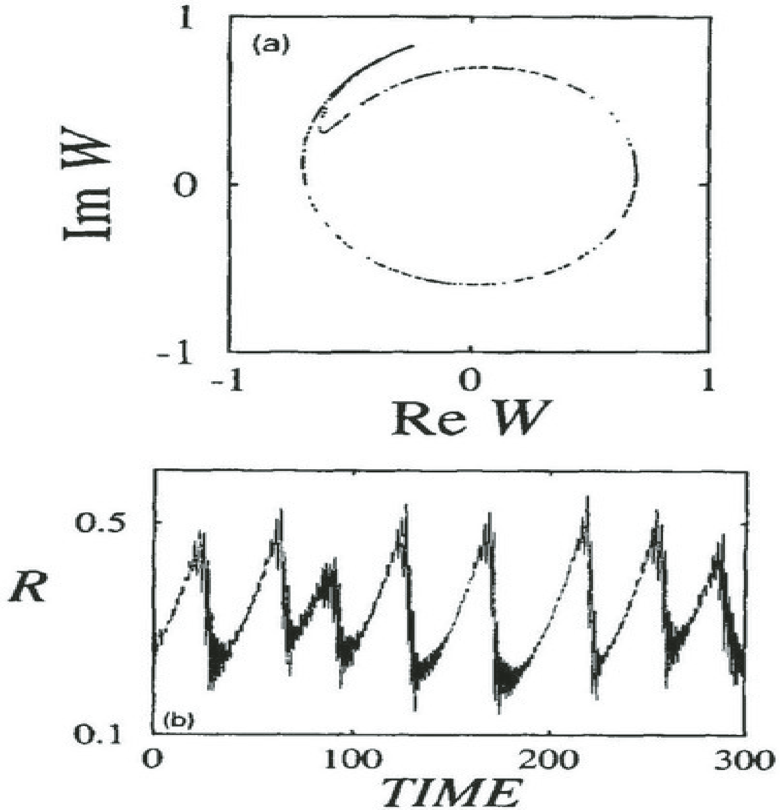
[O.V. Popovich, Y.L. Maistrenko, P. Tass, 2005]

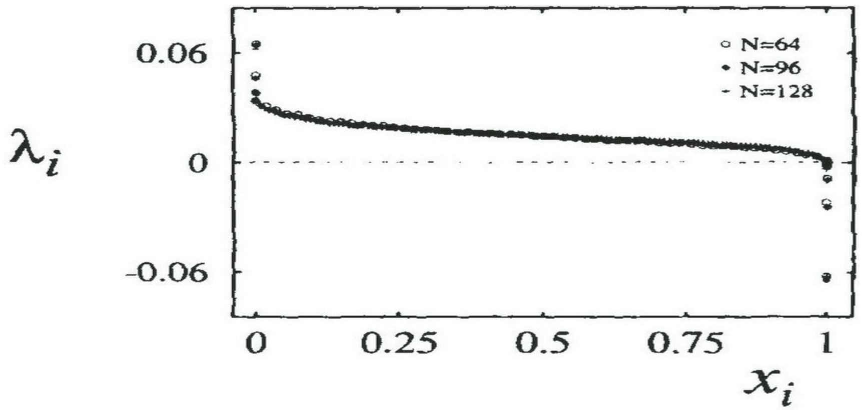


GLOBALLY COUPLED (Ginzburg)-LANDAU OSCILLATORS

$$\dot{W}_j = W_j - (1 + ic_2)|W_j|^2 W_j + K(1 + ic_1)(\overline{W} - W_j)$$

$$\overline{W} = \frac{1}{N} \sum_{j=1}^{N} |\overline{W}_j| \mathrm{e}^{i\theta_j}$$





(Nakagawa Kuramoto 1994)

PULSE-COUPLED OSCILLATORS

$\dot{\phi}_j = 1 \quad \phi_i((t_f+\tau))^+) = U^{-1}(U(\phi_i(t_f+\tau)+\varepsilon_{ij})$ At threshold ($\phi = 1$)

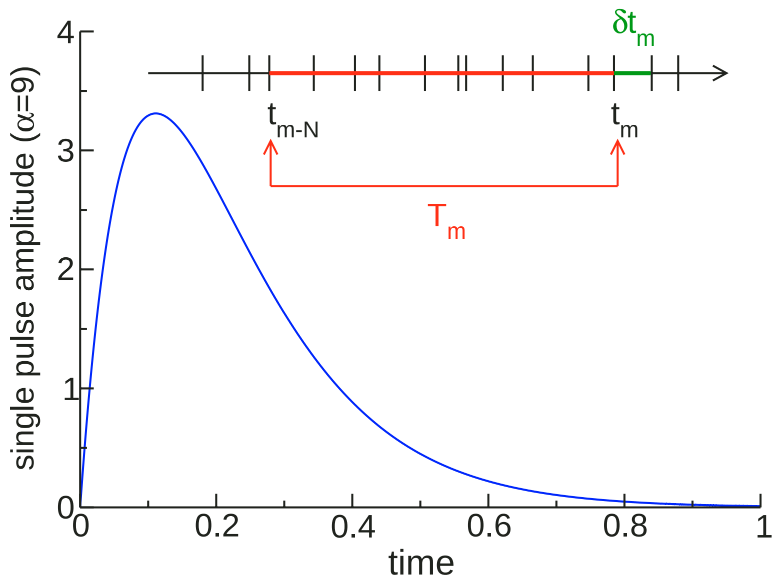
Leaky integrate-and-fire-neurons

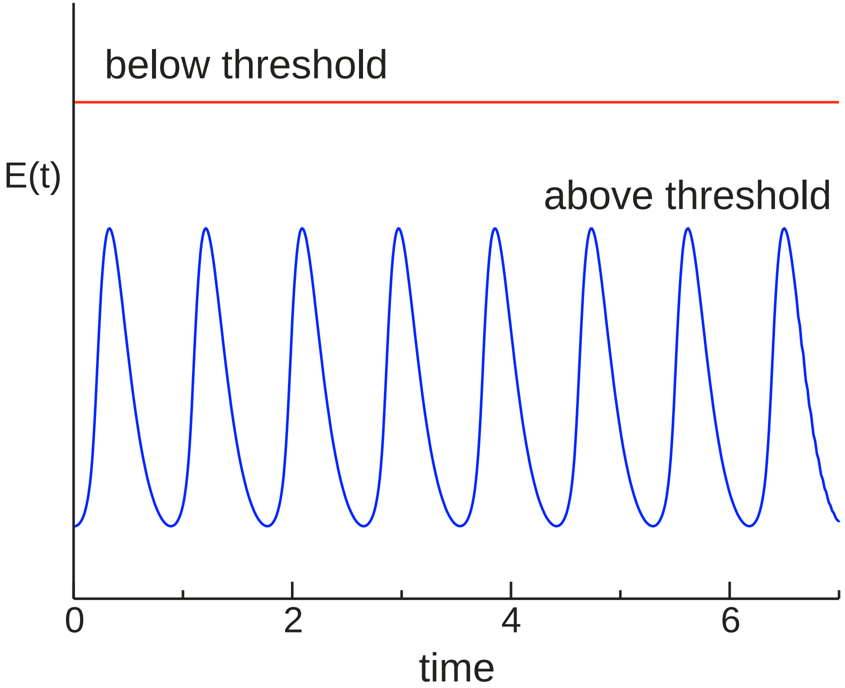
$\dot{x}_i = a - \lambda x_i + gE(t)$

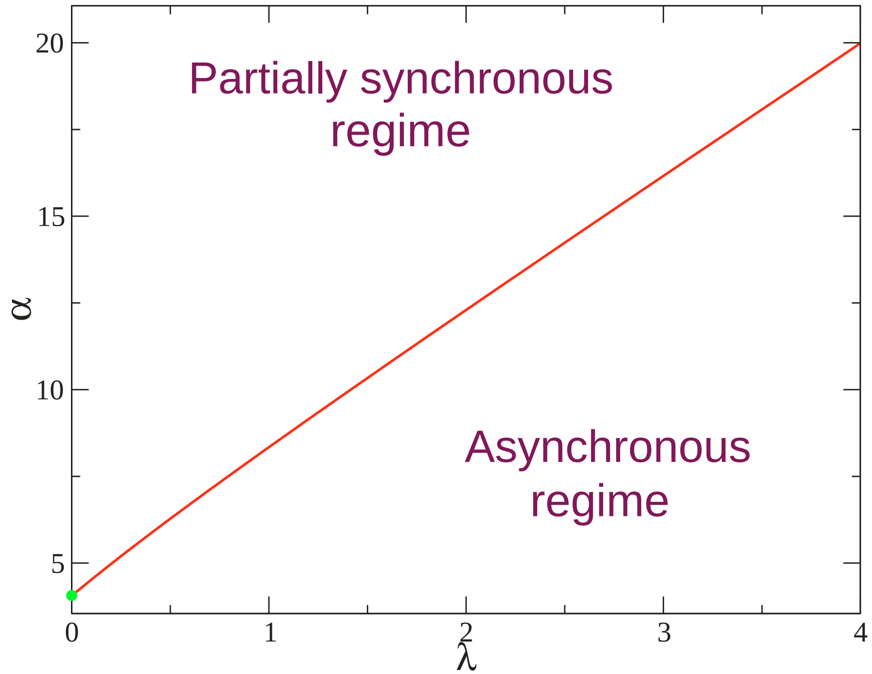
$E(t) = \alpha^2 \sum_m (t-t_m)\mathrm{e}^{\alpha(t-t_m)}$

[C. van Vreeswijk,

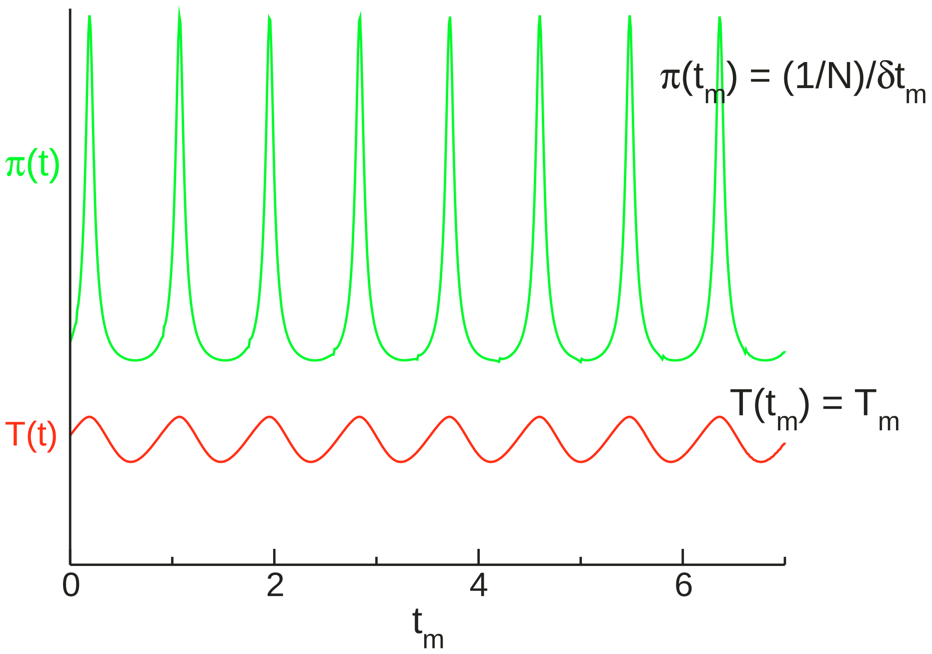
PRE, **54** 5522 (1996)]

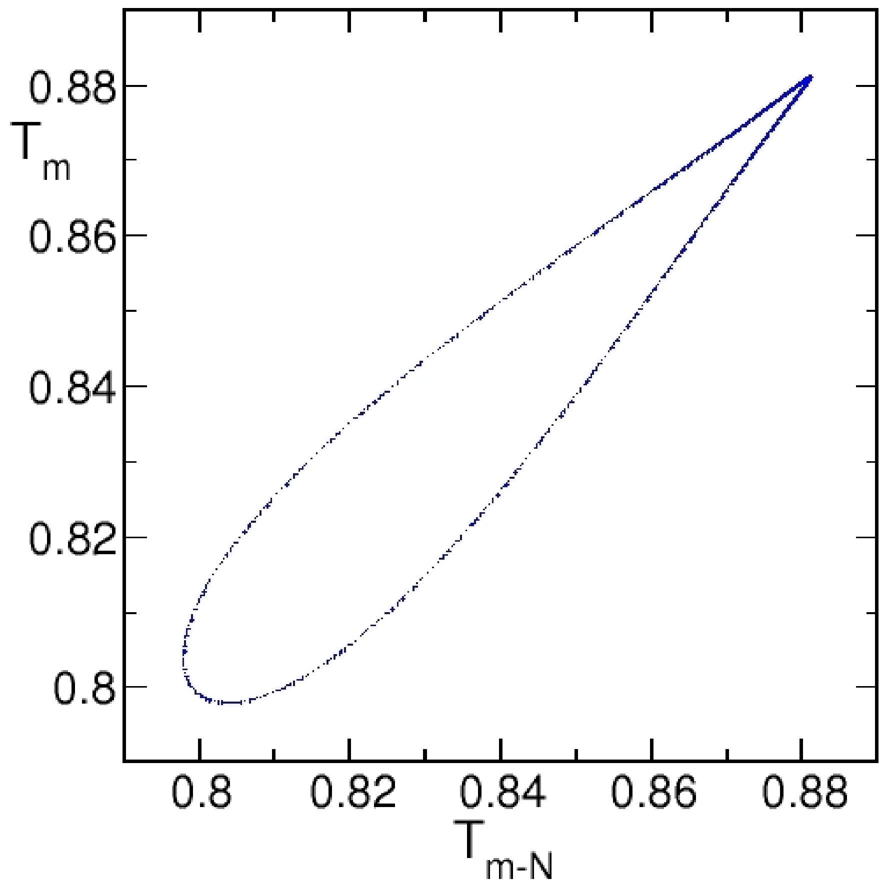


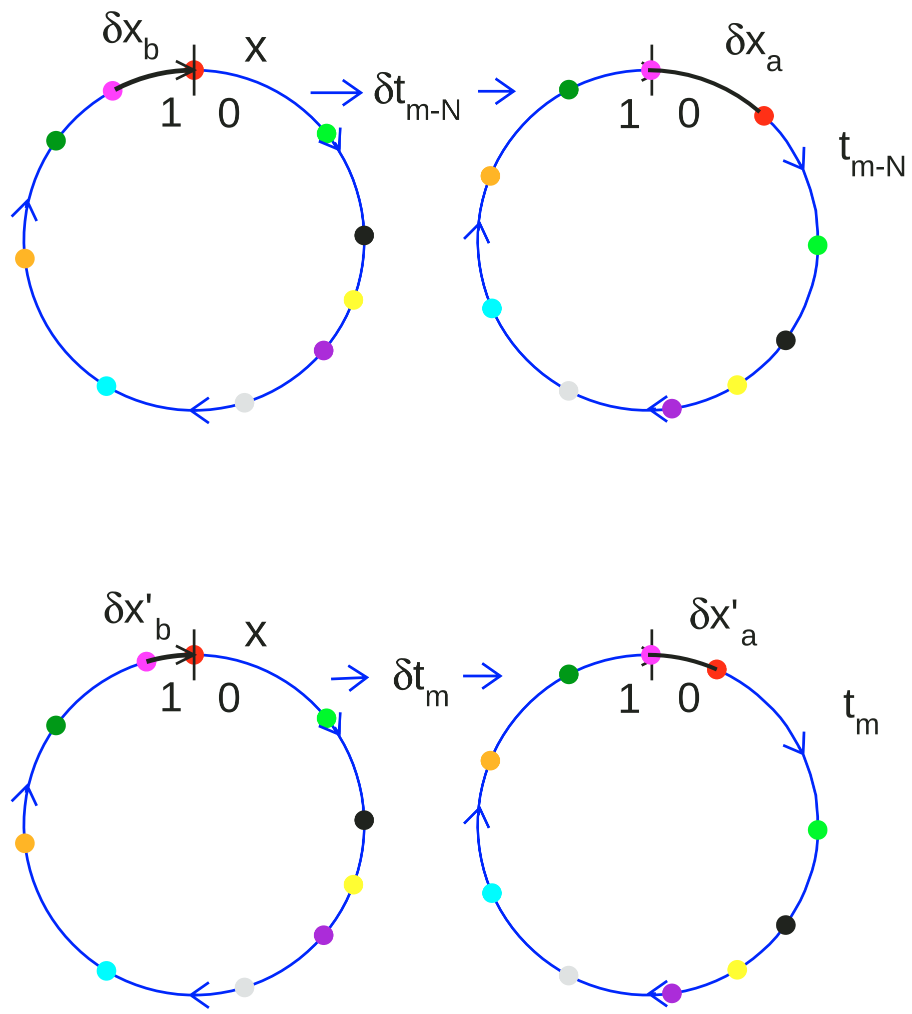

below threshold
E(t)
above threshold
0
2
4
6
time


20
15
10
5
α
Partially synchronous regime
Asynchronous regime
0
1
2
3
4
λ

A MICROSCOPICALLY QUASIPERIODIC DYNAMICS







$$\frac{\delta x_b}{v_b} = \delta t_{m-N} = \frac{\delta x_a}{v_a}$$

$$\delta x'_b = \delta x_a \mathrm{e}^{-\lambda T_m}$$

$$\frac{\delta x'_b}{v'_b} = \delta t_m = \frac{\delta x'_a}{v'_a}$$

A MICRO-MACRO APPROACH

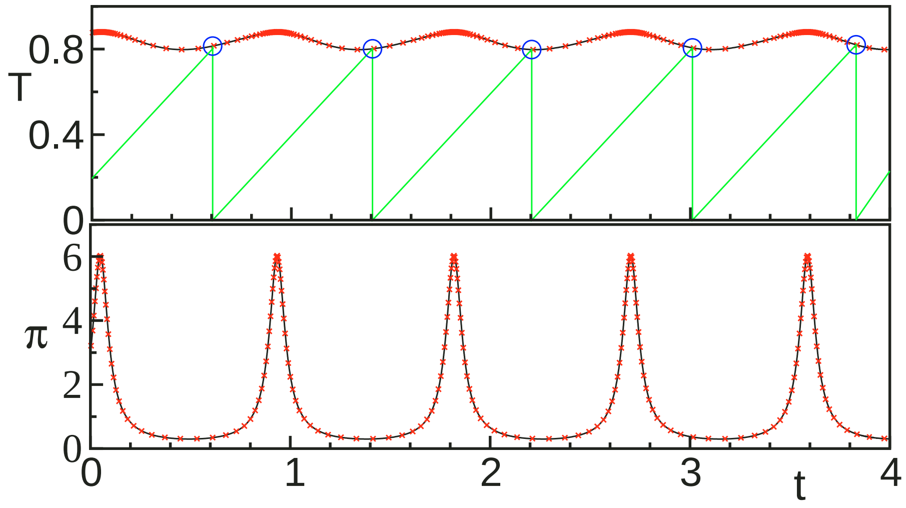
$\pi(t) = \frac{1/N}{\delta(t)}$

$\int_{t-T(t)}^{t} \pi(\tau)\, d\tau = 1$

$\pi(t) = \frac{a-\lambda+E(t)}{a+E(t-T)}\pi(t-T)\mathrm{e}^{-\lambda T}$

$\dot{T} = 1 - \frac{\pi(t)}{\pi(t-T)}$

$\ddot{E} = \alpha^2(g\pi - E) - 2\alpha\dot{E}$

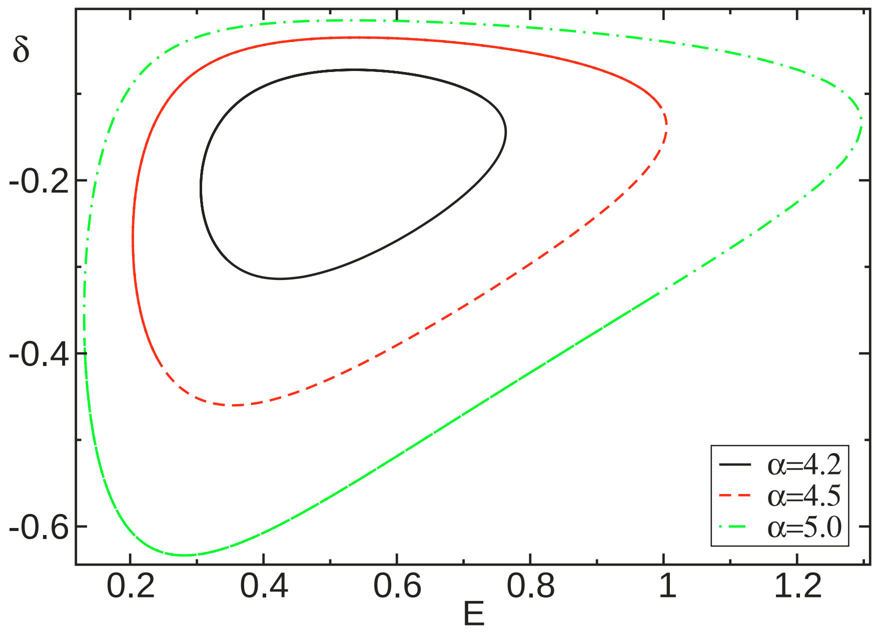


PERTURBATIVE ANALYSIS ($\lambda \ll 1$)

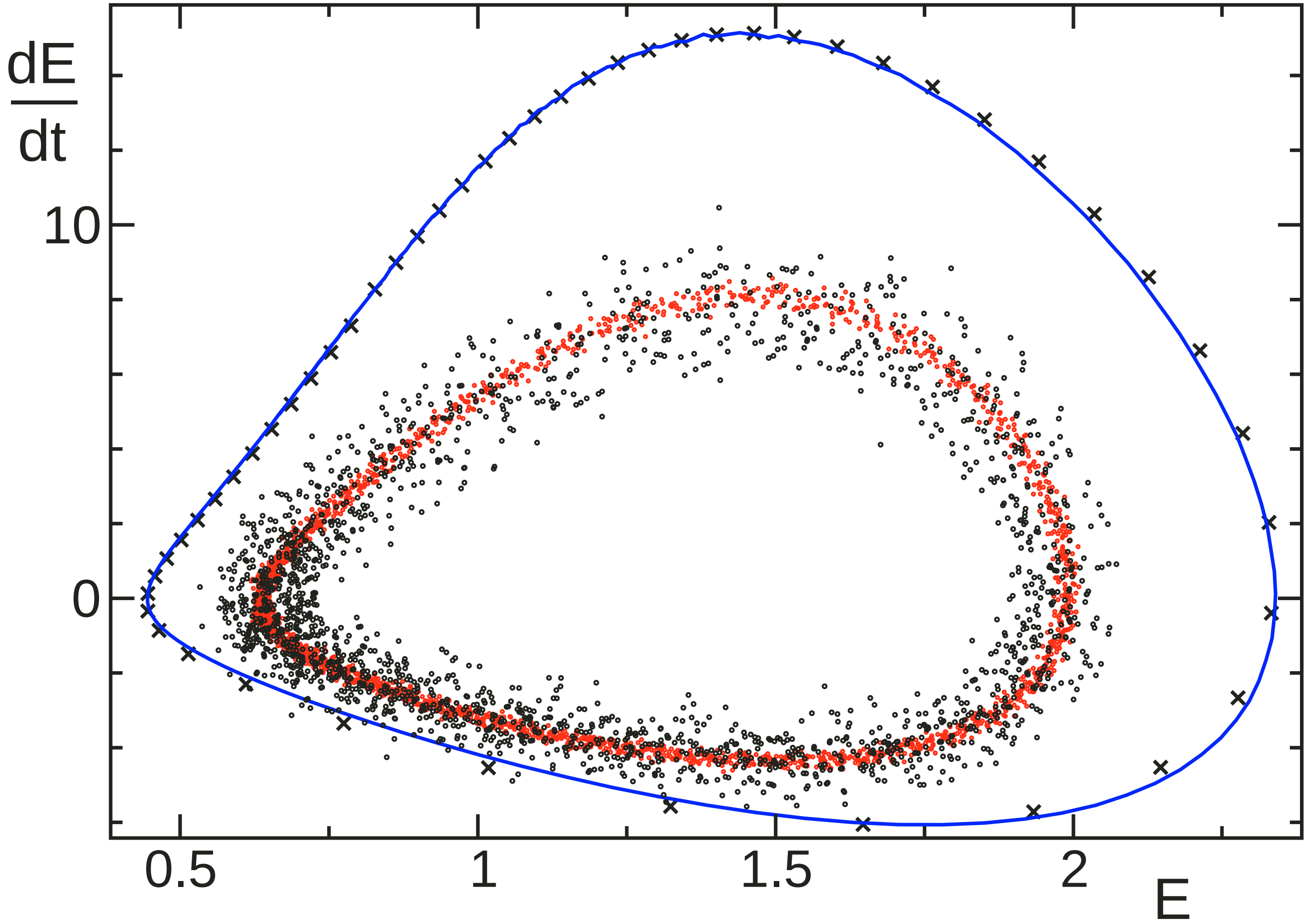
$$u(t-T) = u(t-P-\lambda\tau) = u(t-\lambda\tau) = u(t) - \lambda\tau\dot{u}(t) + O(\lambda^2)$$

$$\dot{\tau} = -P + \frac{1-g\tau E}{a+gE} \qquad \ddot{E} + 2\alpha\dot{E} + \alpha^2 E = \alpha^2\frac{C}{\tau}$$

$$\pi = C/\tau$$



NOISE ADDITION



"TRANSIENT" DYNAMICS

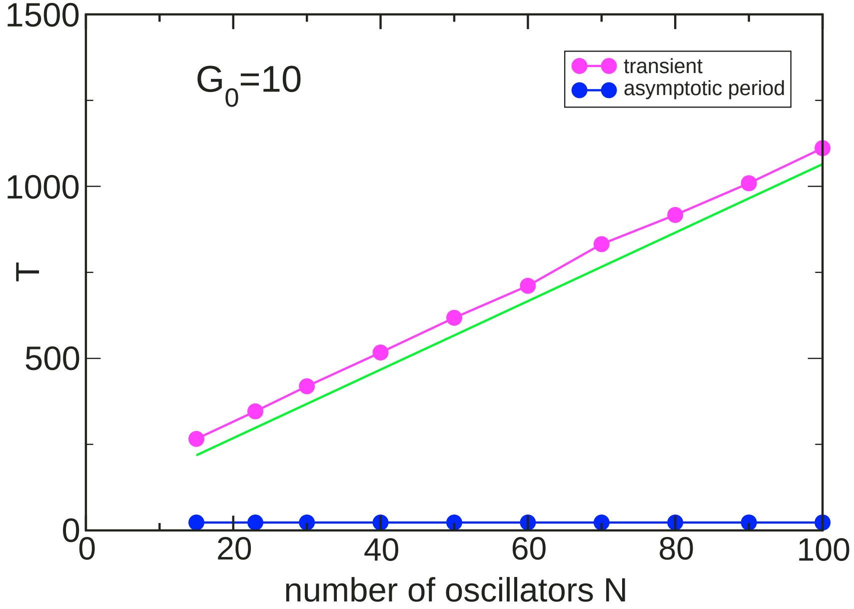
$$\tau \dot{v}_j = L_j - v_j - \frac{\tau}{N} \sum_m \left[G_{j,i(m)}(v_j + E) \right] \delta(t - t_m)$$

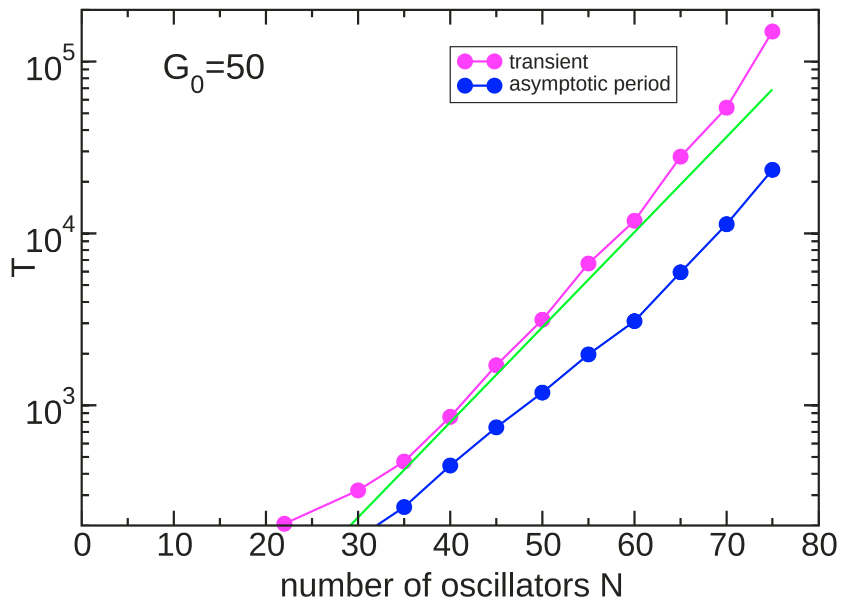
[D.Z. Jin, PRL **89**, 208102 (2002)]

v_j, L_j	membrane, resting potential
$G_{i,j}$	inhibitory synapse conductance
t_m	time of the mth spike emitted by the $i(m)$th neuron
Θ_j	threshold of jth neuron
R_j	resetting potential

$$\dot{x}_j = a - x_j - G_0 \sum_m F_{j,i(m)}(x_j + r)\delta(t - t'_m) \qquad a = 2,\ r = 4/7$$

Dilution (a given fraction of links is cut)





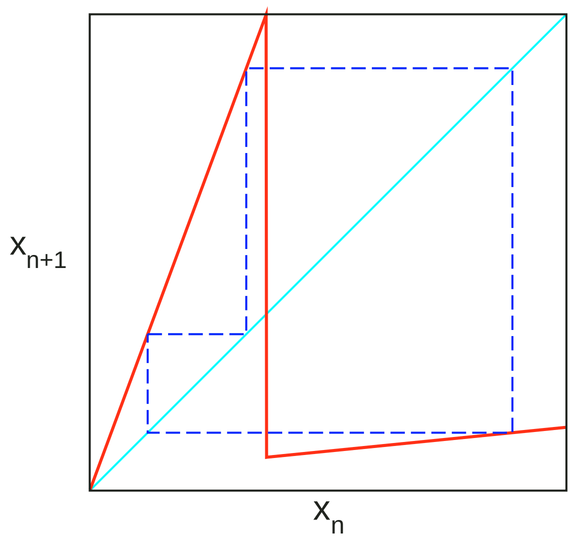
[A. Zumdieck, M. Timme, T. Geisel, F. Wolf, PRL **93** 244103 (2004)]

Main difference: destabilizing coupling (positive Lyapunov exponent) + delay

LYAPUNOV STABLE CHAOS

$$x^i_{n+1} = F(\bar{x}^i_n)$$

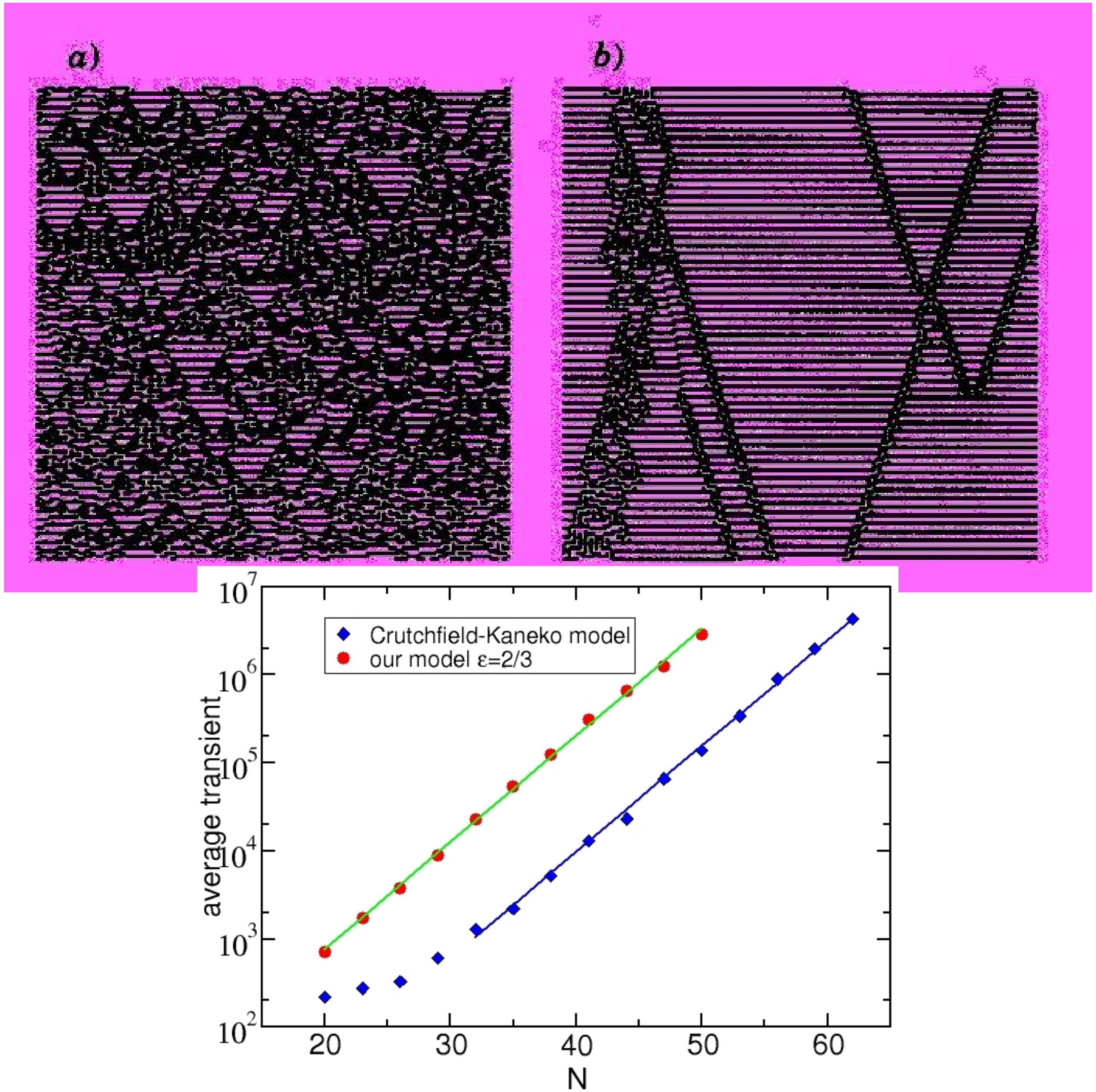
$$\bar{x}^i_n = \frac{\varepsilon}{2} x^{i-1}_n + (1-\varepsilon) x^i_n + \frac{\varepsilon}{2} x^{i+1}_n$$



[J.P.Crutchfield, K.Kaneko, PRL, **60** 2715 (1988)]

[A.P., R.Livi, G.-L.Oppo, R.Kapral, Europhys. Lett. **22** 571 (1993)]

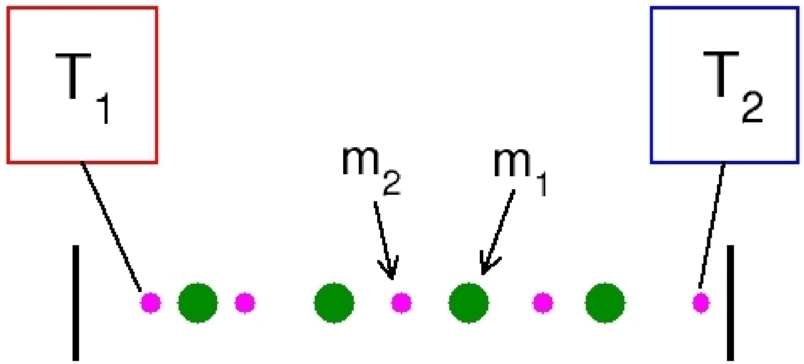
$\varepsilon = 2/3$



$\varepsilon = 0.76$

ANOTHER PHYSICAL SETUP: THE 1D HARD-POINT GAS

(with P. Cipriani)



$$v_i' = v_j \pm \frac{1-r}{1+r}(v_i - v_j)$$

$$r = \frac{m_1}{m_2} \quad \text{mass ratio}$$

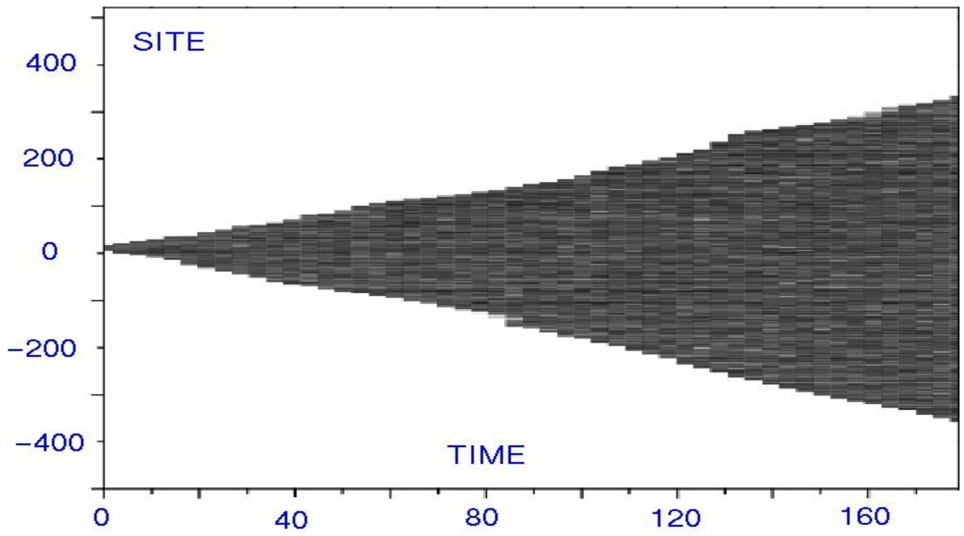
Evolution in tangent space equal to that in real space

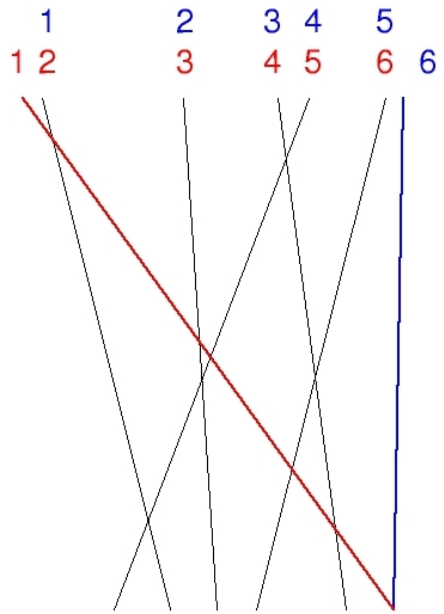
$\Downarrow$

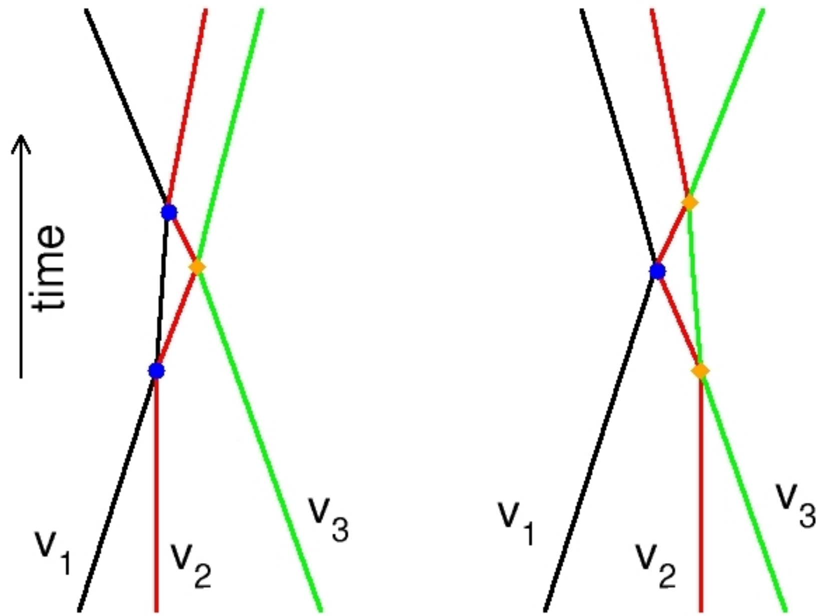
Conservation of the Euclidean norm of infin. perturbations

$\Downarrow$

NO STANDARD CHAOS


SITE
400
200
0
−200
−400
TIME
0
40
80
120
160


1
2
3 4
5
1 2
3
4 5
6 6


time
v_1
v_2
v_3
v_1
v_2
v_3

FUTURE PERSPECTIVES

- THE ROLE OF DELAY
- SYNAMPTIC PLASTICITY
- TOPOLOGY OF THE CONNECTIONS
- DISORDER