The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

International Atomic Energy Agency

Summer School on

Design and Control of Self-Organization in Physical, Chemical, and Biological Systems

25 July to 5 August, 2005

Miramare-Trieste, Italy

1668/27

Dynamics Properties of Cell-Cycle Network of Budding Yeast

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Spiral Instabilities and Their Control in Reaction-diffusion Systems

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Trieste, July. 27 2005

Reaction-diffusion system

$$\partial C / \partial t = D\nabla^2 C + F_R(C)$$

$C(r,t)$:	vector of chemical concentrations
D:	the diffusion coefficient matrix
$F_R(C)$:	the chemical kinetics

One of fundamental models

Chemical system (Belousov-Zhabotinsky reaction)

Biological system (morphogenesis)

Physiological system (heart fibrillation)

Physical system (Gas discharge)

Ecological system (Predator-Prey, semiarid vegetation)

Spirals

Exist every where

Reaction diffusion

Single cell

Bacteria colony

Heart system

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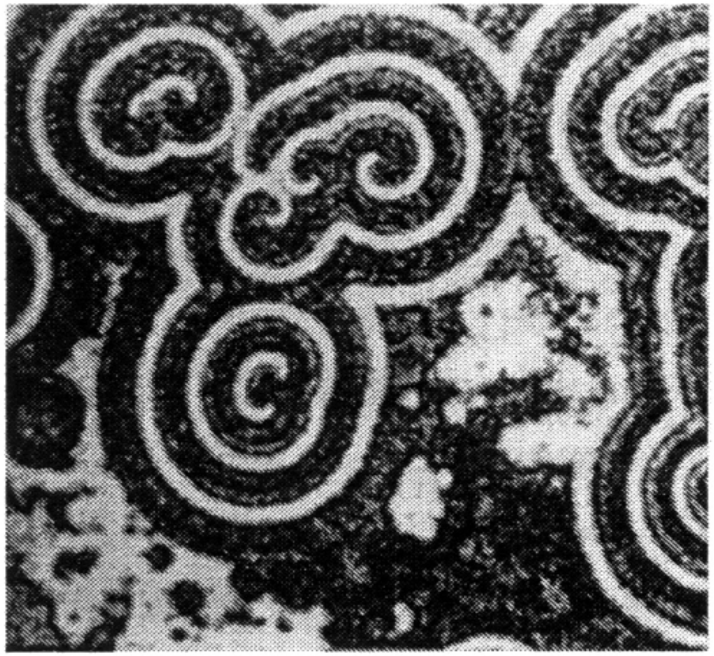
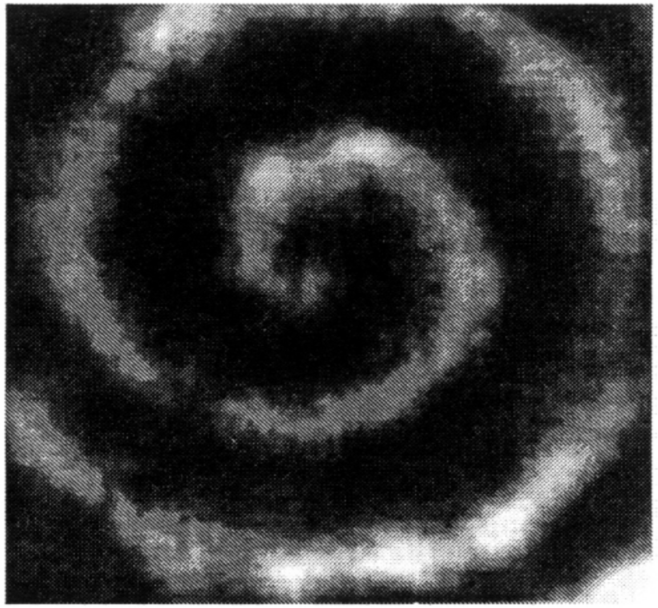
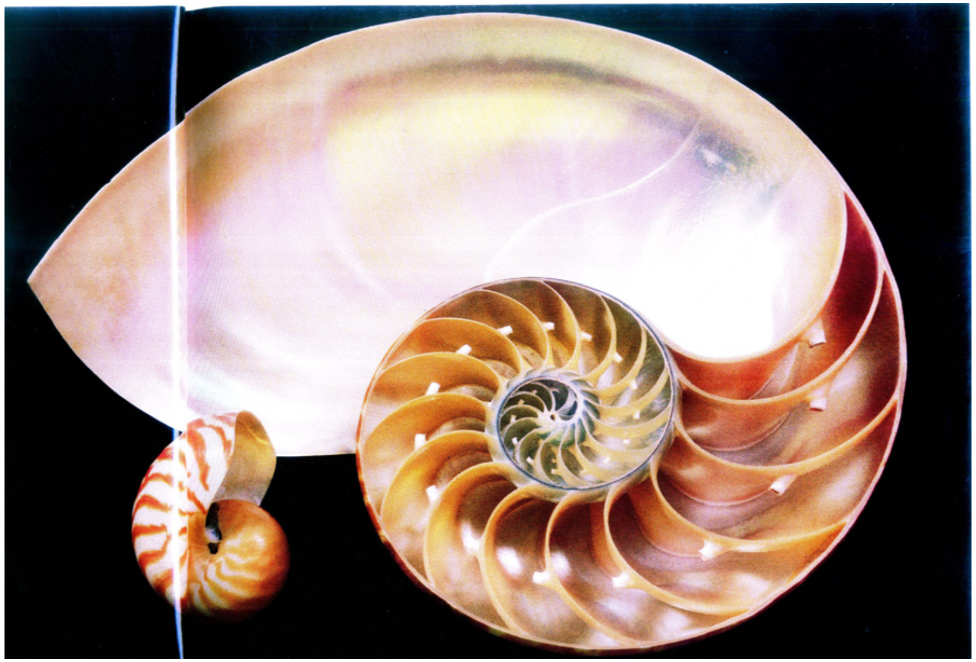
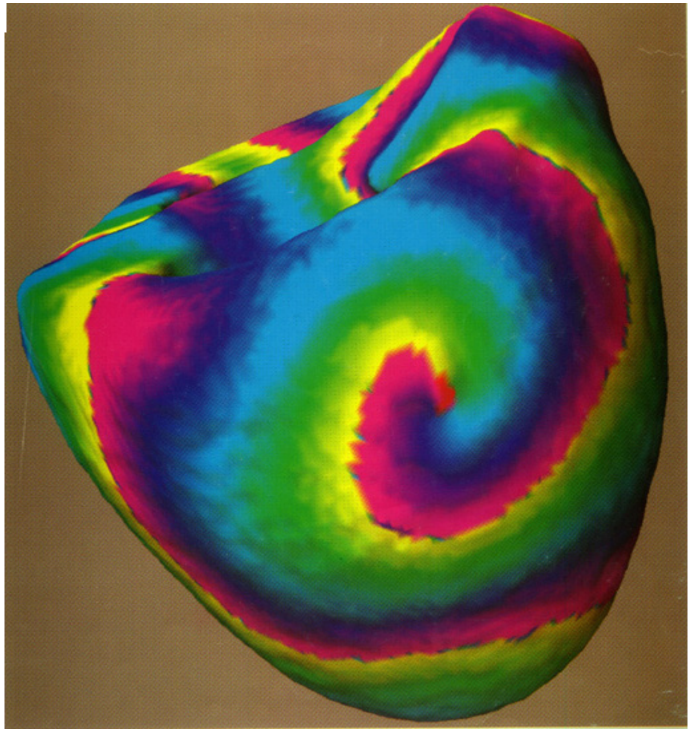








Excitable system

Excitable medium

$$\varepsilon \frac{\partial u}{\partial t} = f(u,v) + D_u \nabla^2 u$$

$$\frac{\partial v}{\partial t} = g(u,v) + D_v \nabla^2 v$$

$$\varepsilon \ll 1$$

In BZ reaction:

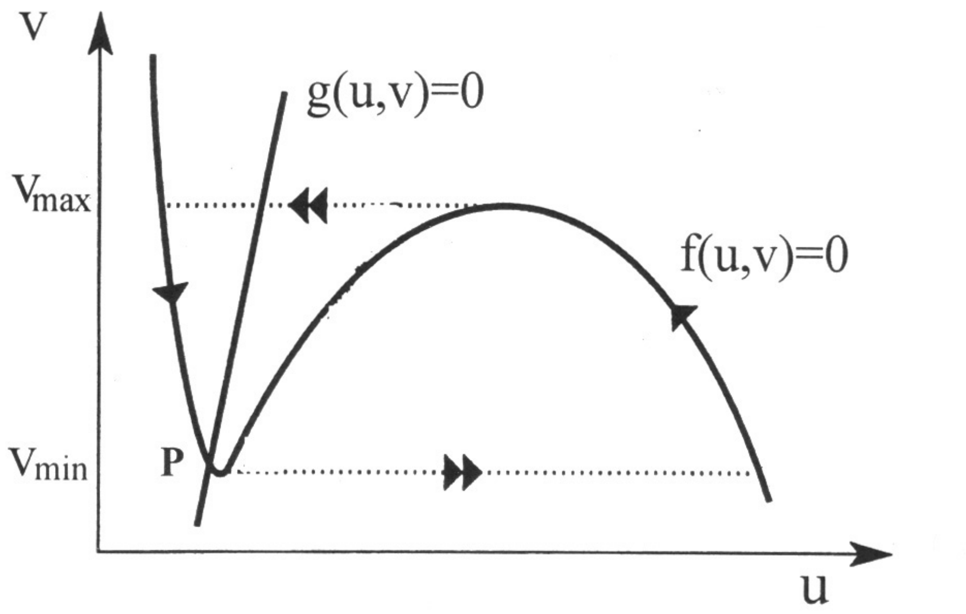
u: excitable variable:

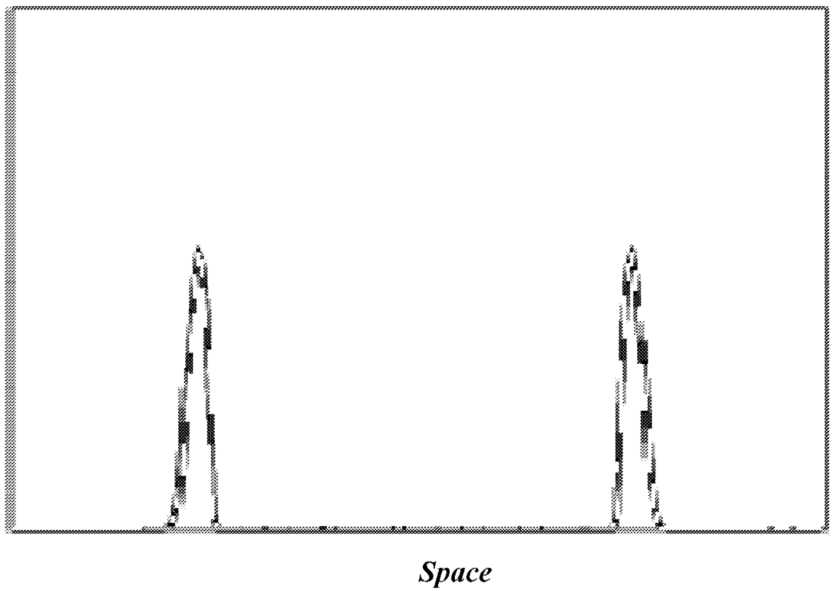
$[BrO_2]$

v: recovery variable:

[Ferroin]

Excitable medium

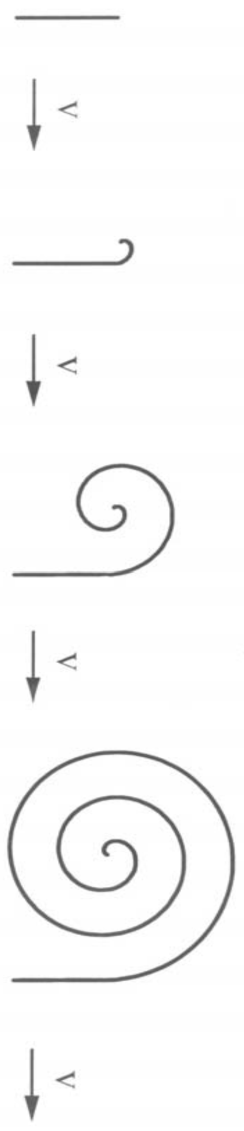


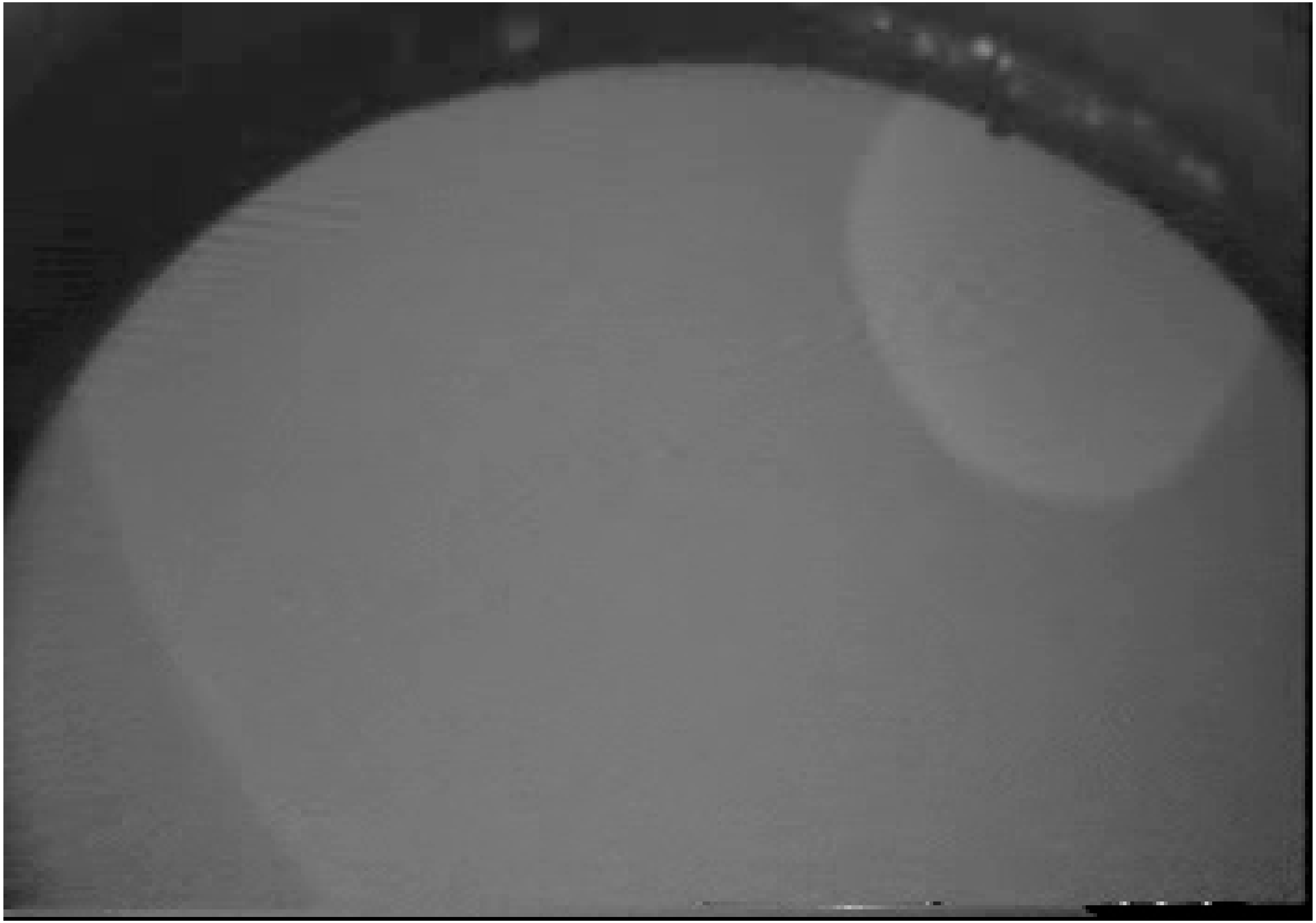


Excitable system

Mexican wave (La Ola) Created in 1986 World Cup

Spiral generation in an excitable medium





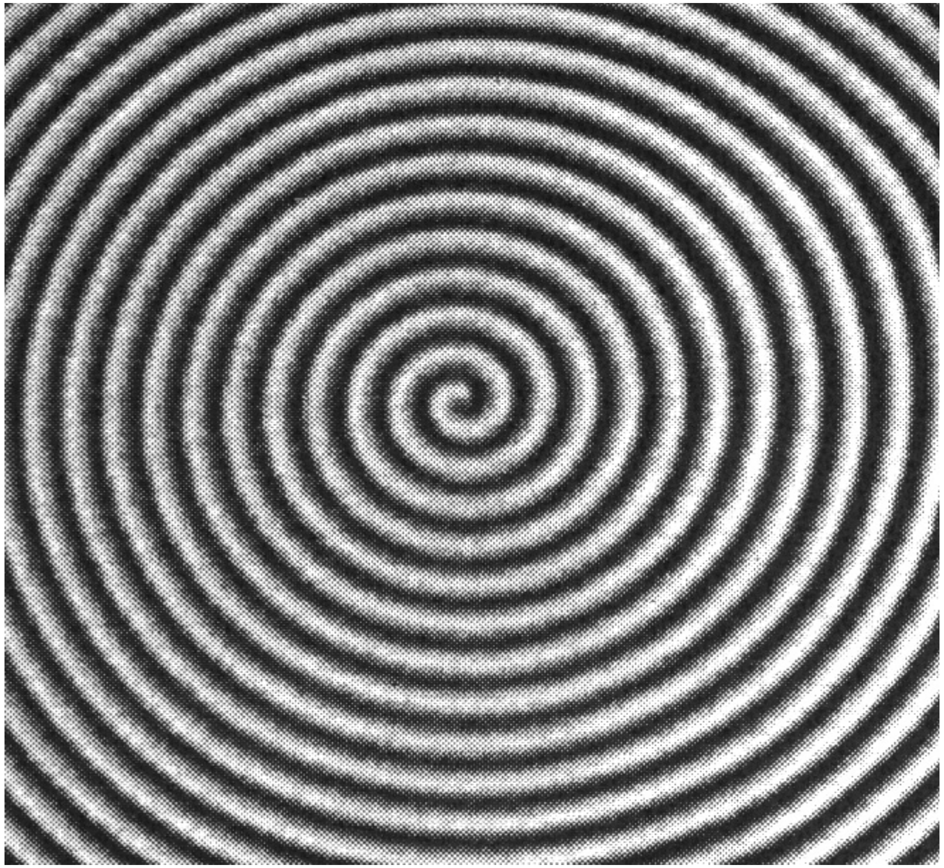
Phase Waves

Oscillation medium (near **Hopf**):

$$c = c_0 + Ae^{i(\omega t+\phi)} + c.c$$

Amplitude Equation (**Complex Ginzburg-Landau** Equation):

$$\frac{\partial A}{\partial t} = A + (1+i\alpha)\nabla^2 A - (1+i\beta)|A|^2 A$$



Traveling Wave Solution：

$$A = Fe^{i(\vec{q}\cdot\vec{r}+\omega_c t)}$$

$$F^2 = 1-q^2, \omega_c = \beta + (\alpha-\beta)q^2$$

Introduce Defect ——> Phase Wave Spiral

Eckhaus instability:

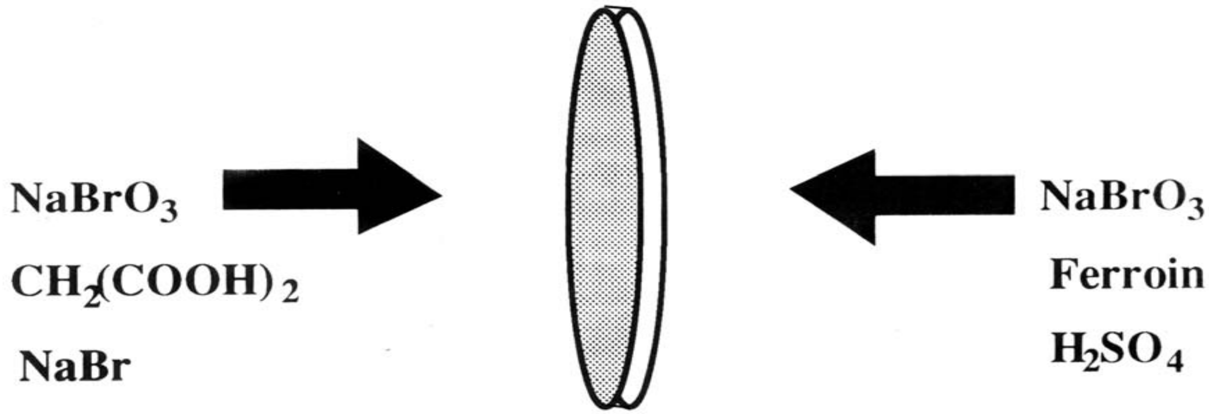
$$1+\alpha\beta - \frac{2(1+\beta^2)q^2}{1-q^2} < 0$$

Benjemin-Feir instabilty:

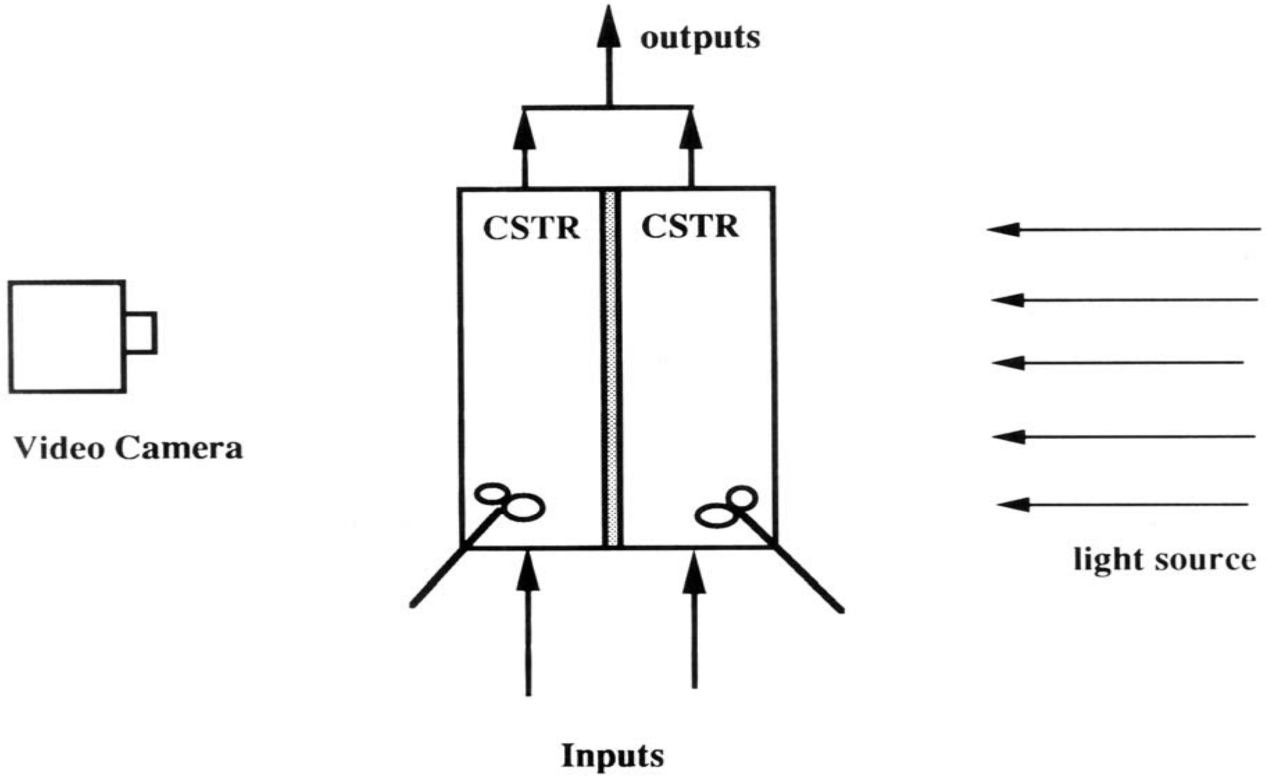
$$1+\alpha\beta < 0$$

Spatial Open Reactor

Reaction medium

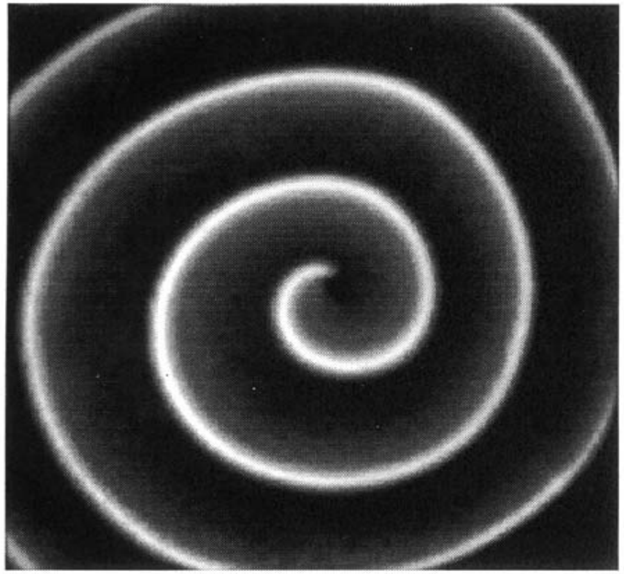


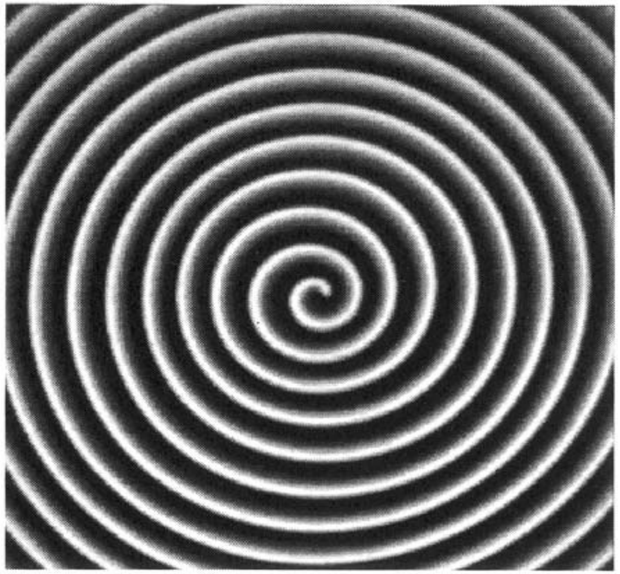
Spatial open Reactor



Spirals in BZ Reaction

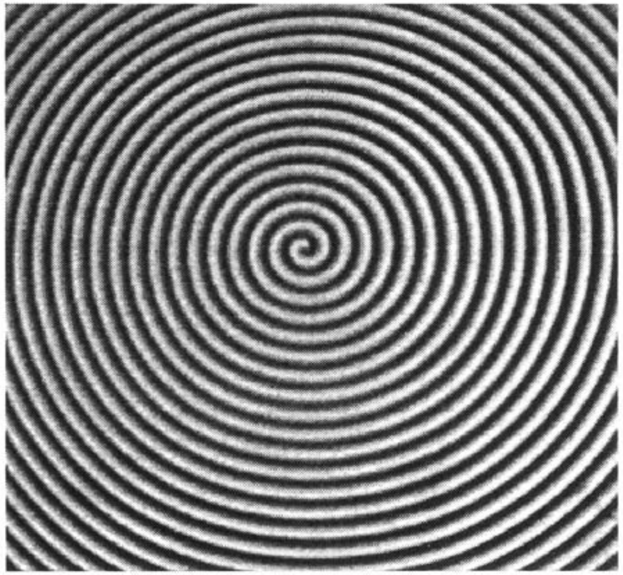
$[H^+]$=0.15 M
λ = 2.04 mm
T = 168 s
Excitable Spiral

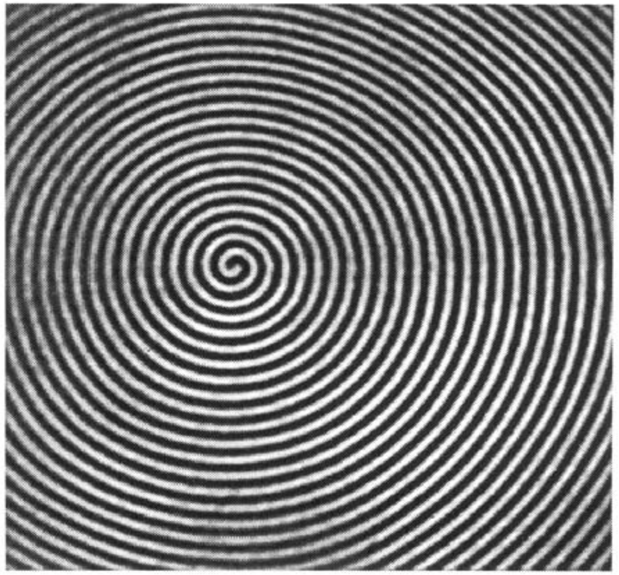




$[H^+]$=0.3 M
λ = 0.74 mm
T = 28.6 s
Excitable Spiral

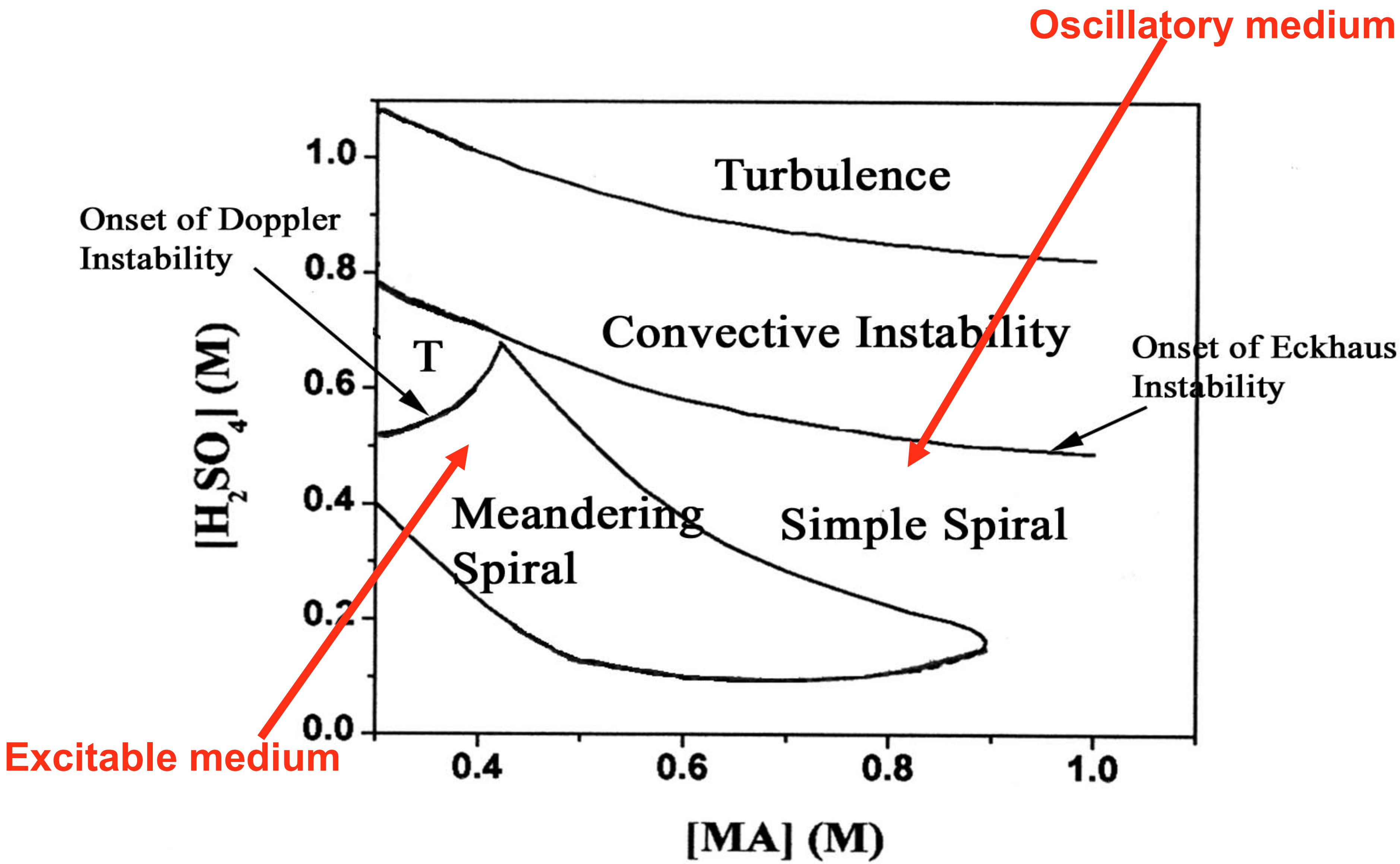
$[H^+]$=0.8 M
λ = 0.38 mm
T = 7.3 s
Phase Wave Spiral





$[H^+]$=1.2 M
λ = 0.35 mm
T = 5.4 s
Phase Wave Spiral

Phase diagram



Long wavelength Instability

L.Q.Zhou and Q. Ouyang PRL (2000)

Simple spiral

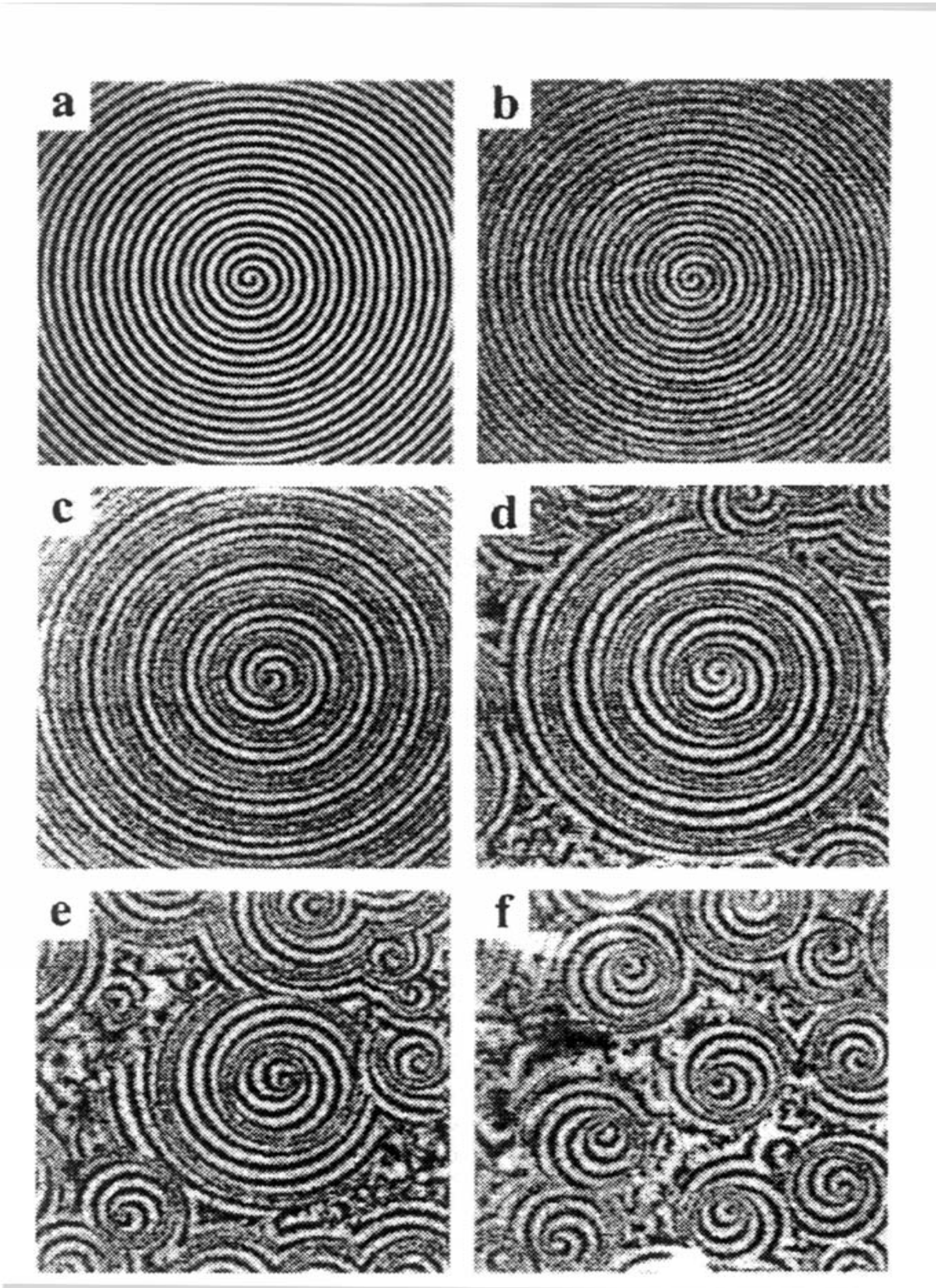
On set of Eckhaus instability

Full development of modulation waves

Spiral break up

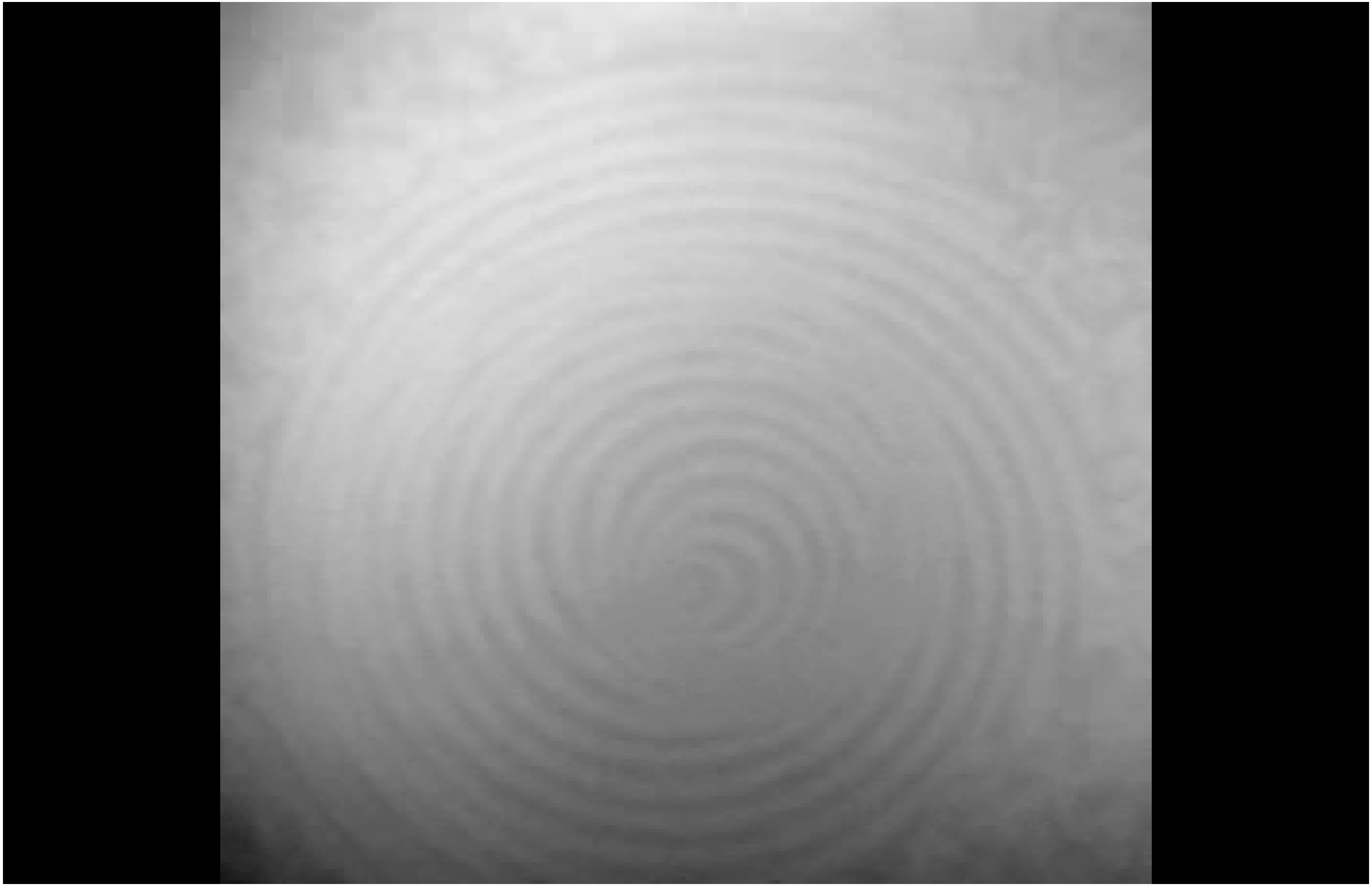
Development of turbulence

Ensemble of small spirals

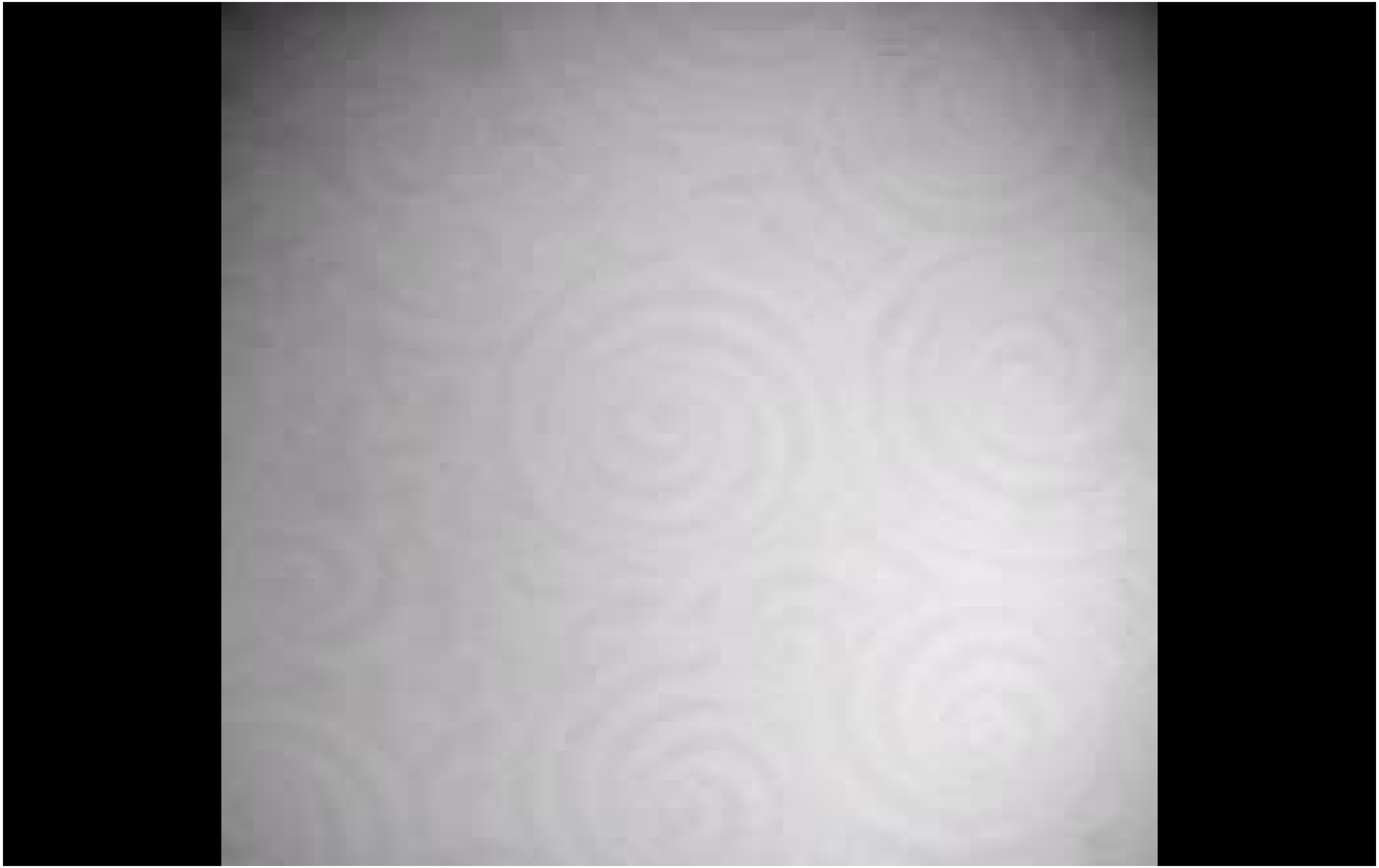


Modulated spiral. After the long wavelength instability

Spiral breakup



Ensemble of Small spirals



Spiral turbulence

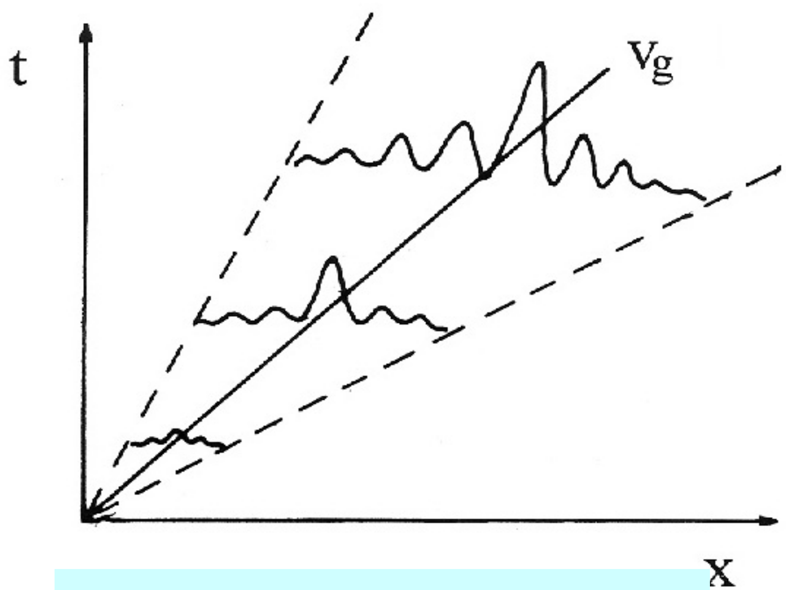
Theoretical explanation

CGLE: $$\frac{\partial A}{\partial t} = A + (1+i\alpha)\nabla^2 A - (1+i\beta)|A|^2 A$$

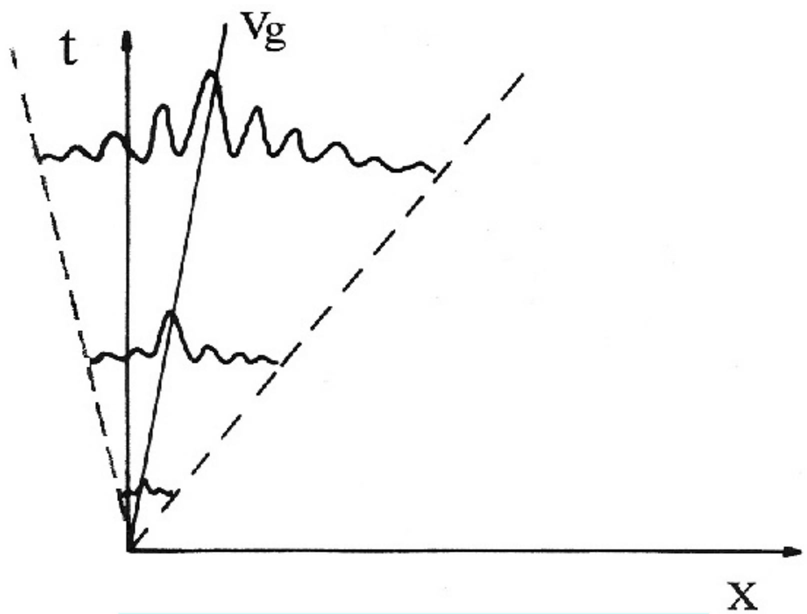
Traveling wave: $$A_0 = Fe^{i(\vec{q}\cdot\vec{r}-\omega t)}, F^2 = 1-q^2, \omega = \beta + (\alpha-\beta)q^2$$

Linear stability analysis ⟶ perturbation equation

$$\partial_t u = \vec{v}_g \cdot \nabla u + D_{\|}\nabla^2 u \qquad v_g = 2(\beta-\alpha)q \qquad D_{\|} = 1+\alpha\beta - \frac{2(1+\beta^2)q^2}{1-q^2}$$



Convective Instability



Absolute Instability

Control of long wavelength instability

Introducing **a target wave source** by locally (5x5points) **decreasing the control parameter β** from β to β_i.

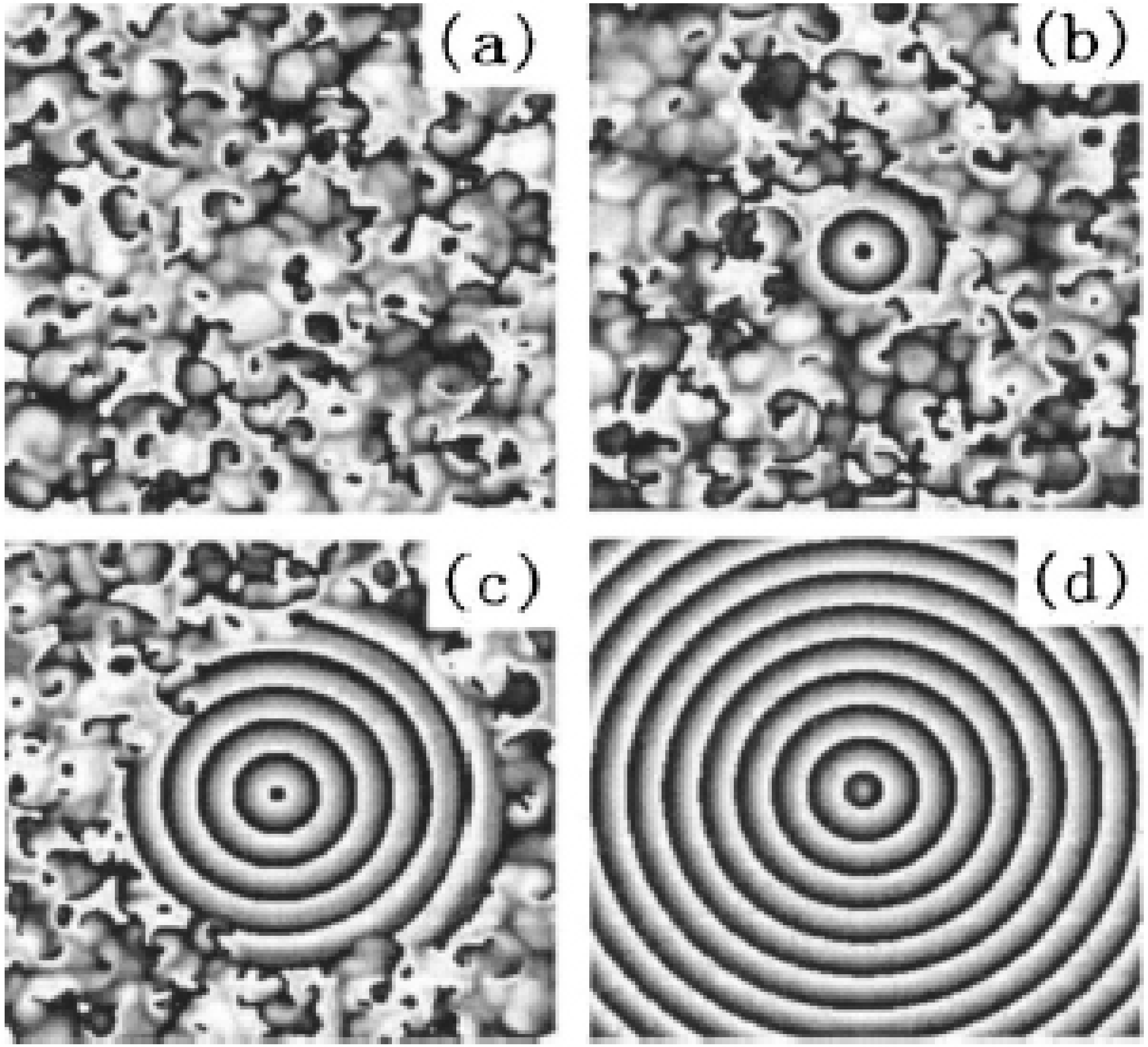
Dispersion relation:

$$\omega_c = \beta - (\alpha - \beta)k^2$$

$$\omega_{target} = \beta_i$$

Condition:

$$\beta - \beta_i \geq 0.3$$

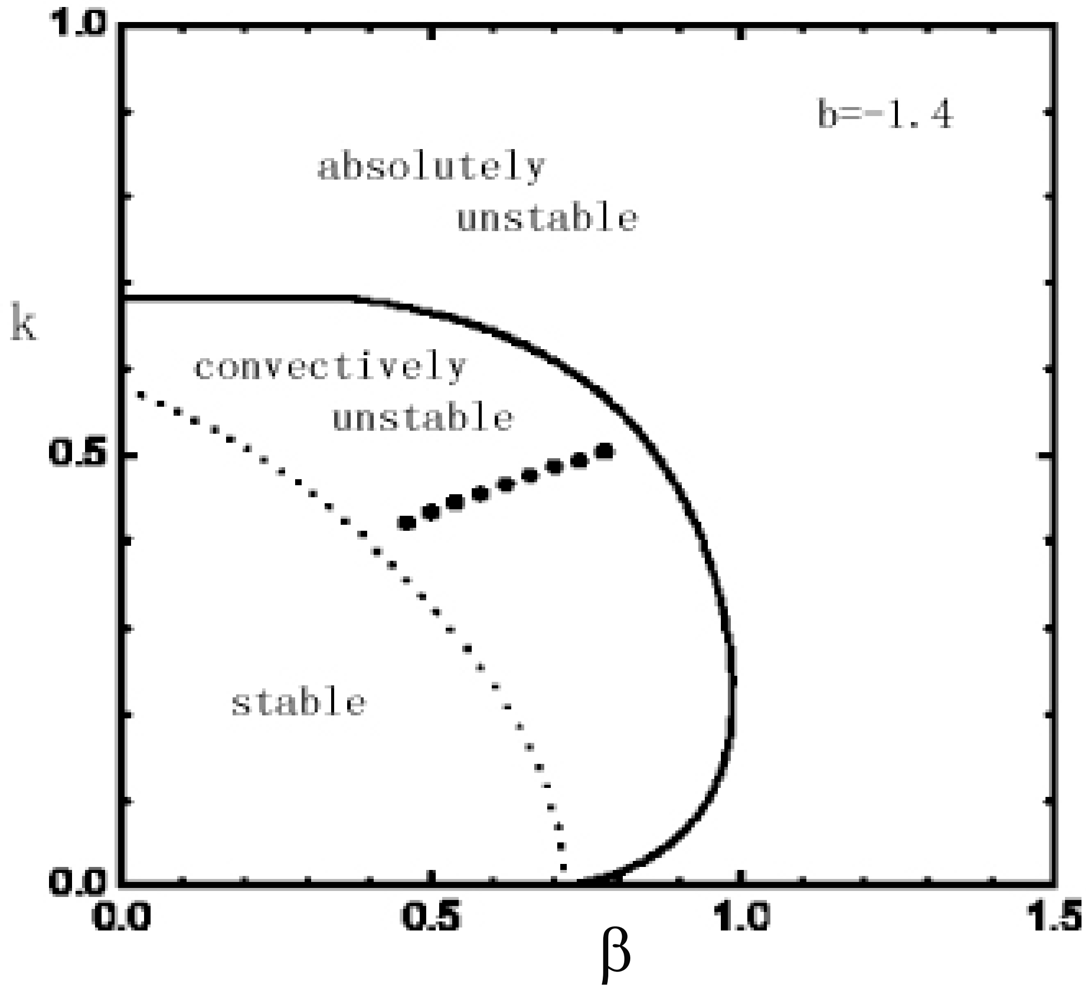


$\alpha=-1.4$, $\beta=0.9$, $\beta_i=0.6$

Explanation

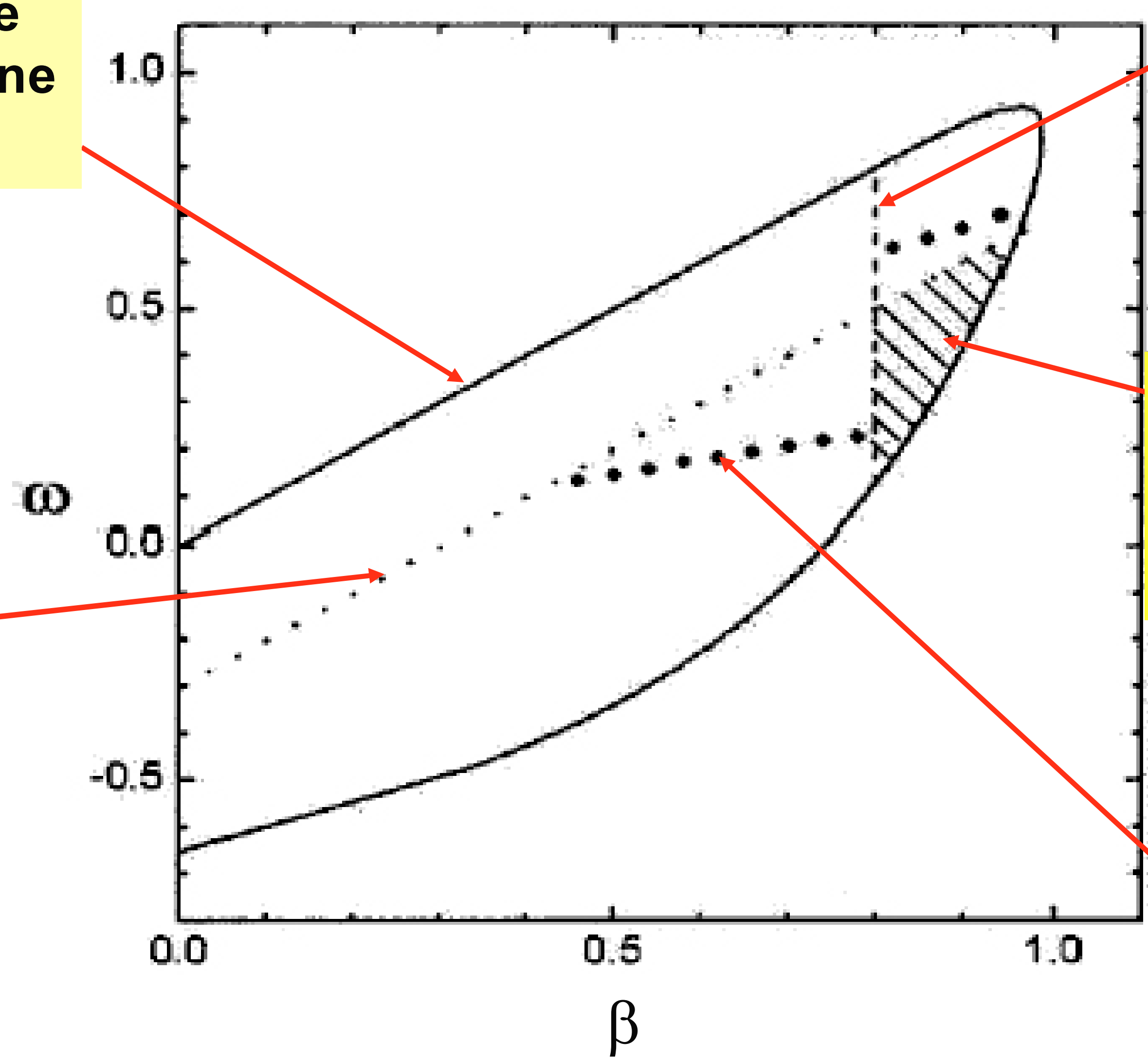
$$k = \sqrt{\frac{\omega_{target} - \omega_{spiral}}{\alpha - \beta}}$$

The increase of the frequency of target waves will decrease k, driving the system from the region of absolute instability to the region of convective instability.



Phase diagram

M. Jiang, et al. PRE (2004)



Onset of absolute instability for plane waves

existence condition for target waves

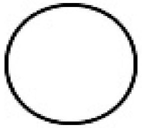
Onset of absolute instability for spiral waves

area where spiral turbulence can be controlled

frequency of spiral waves as a function of β

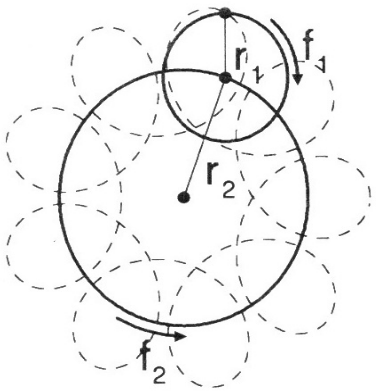
Meandering spiral

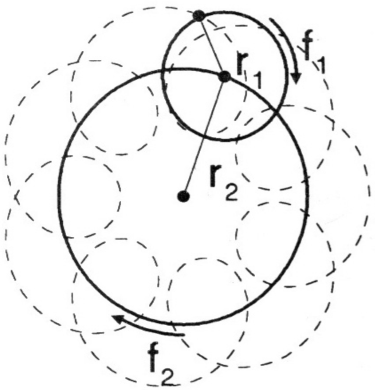
Simple Spiral

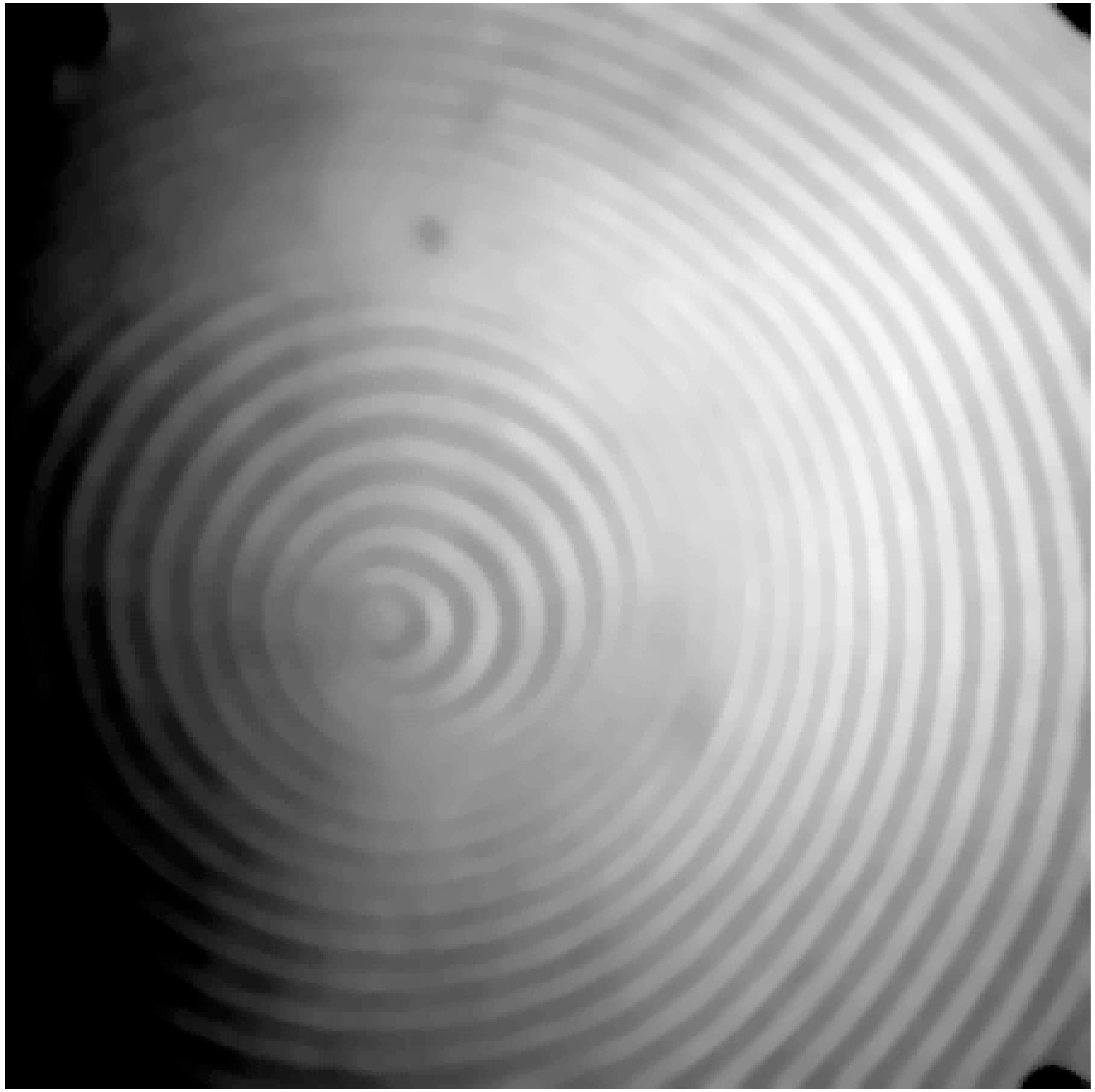


Meandering Spiral

Out-pedal Inner-pedal





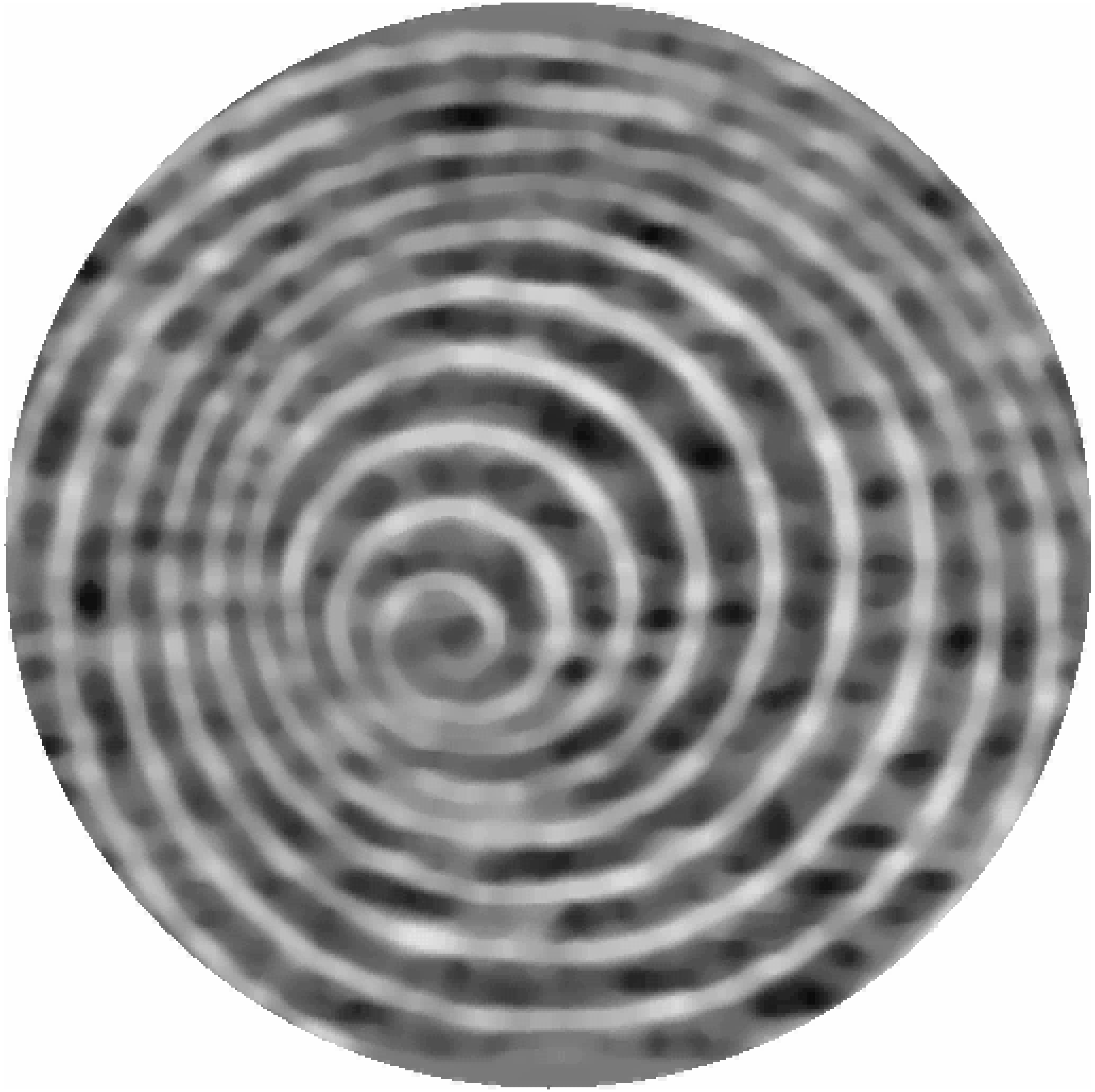


Doppler Instability

Q. Ouyang et al. PRL (2000)

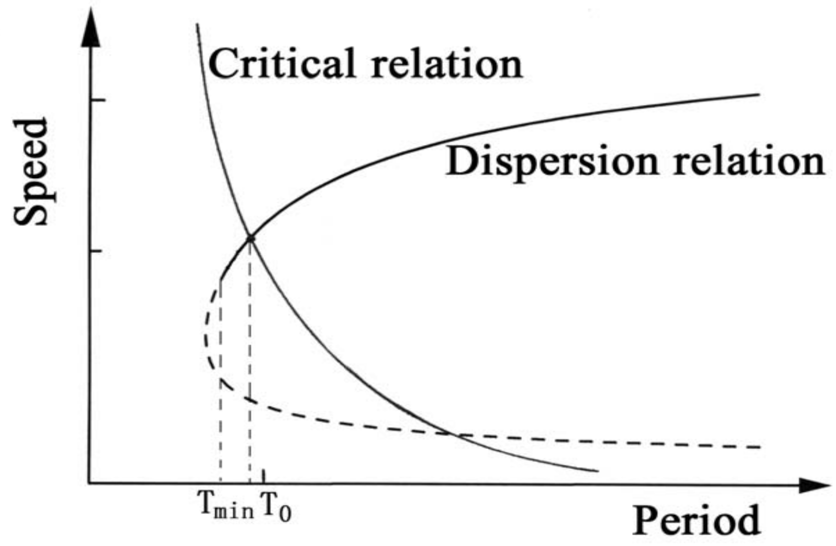
- Defects are generated when the diameter of primary circle is larger than a critical value;

- The number of defect increase monotonically after the instability.

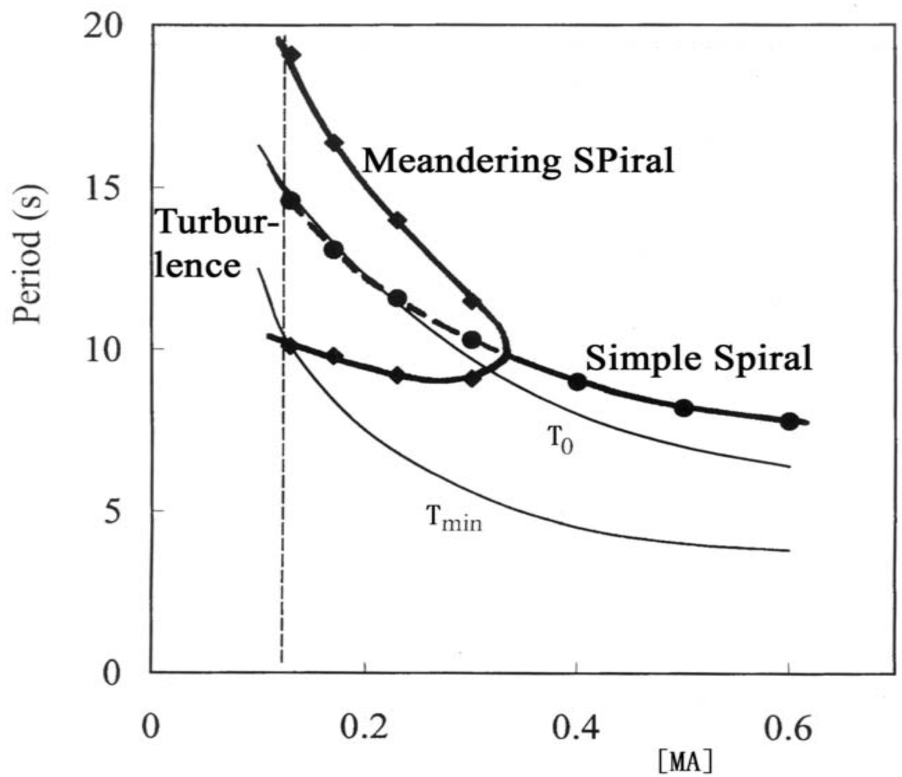


Explanation

Qi Ouyang, H. L. Swinney, G. Li, Phys. Rev. Lett., 84, 1047 (2000).



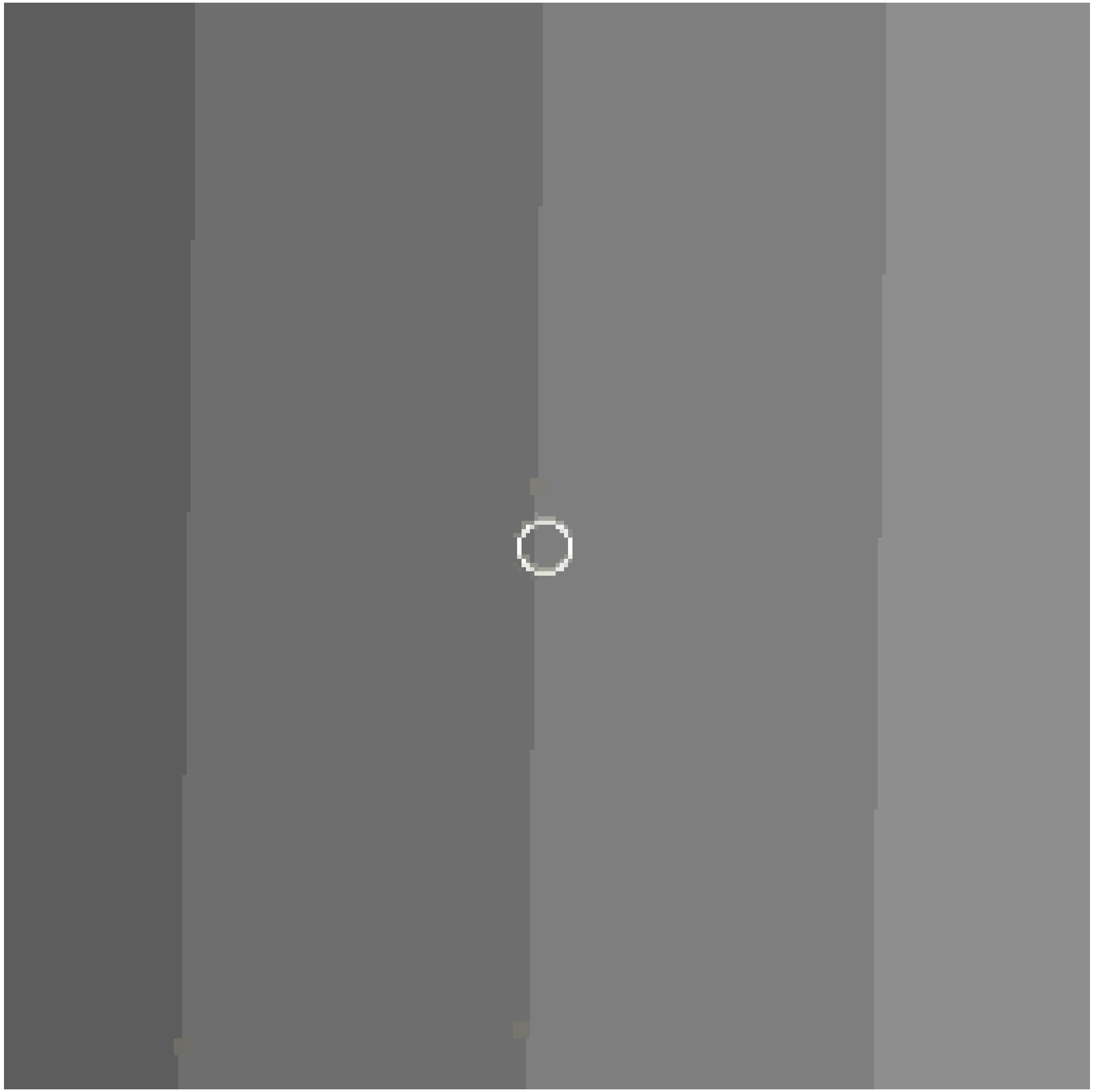
Critical and dispersion relations determine the behavior of a spiral waves



Doppler Effect makes local system passes critical point

Control of Doppler instability

X. Wang, et al. PRE (2004)



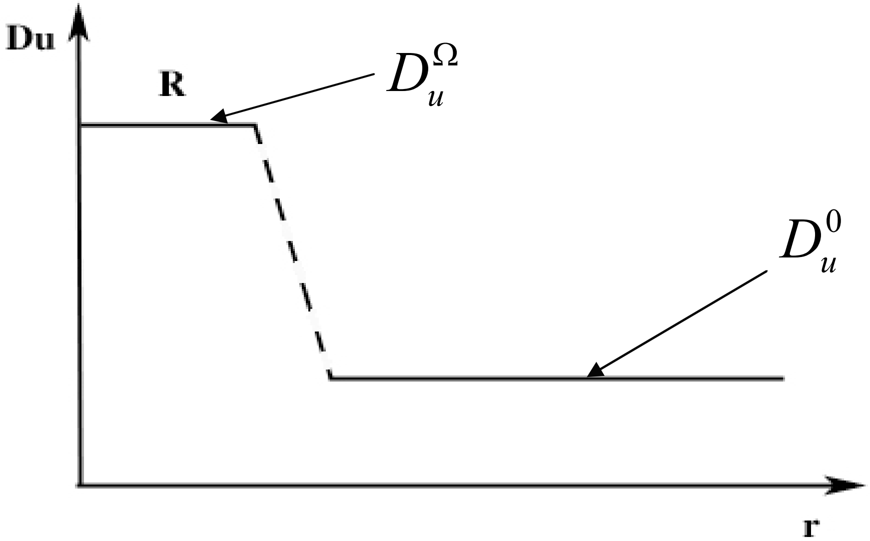
Strategy 1:

Locally increase the diffusion coefficient in the reaction medium

Control parameters:

- the size of the local area;
- the difference in diffusion coefficients.

Mechanism



Stability analysis:

$$N = C - D_u \kappa \qquad C = \alpha \sqrt{D_u}$$

$$\frac{\partial N}{\partial R}\Big|_{tip} = -\frac{1}{2}\kappa_{tip}\frac{\partial D_u}{\partial R} > 0$$

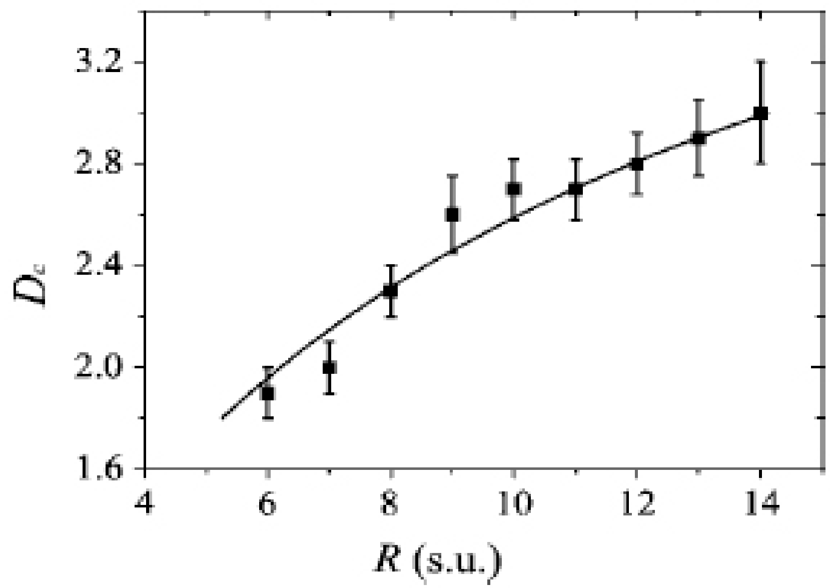
$$R = \frac{D_u}{c_0}\left(\frac{bK}{B_c - 2D_u/W}\right)^{3/2}$$

Critical condition: $D_u(R) = D_u^\Omega = D_c D_u^0$

V. Hakim and A. Karma, PRE (1999)

$D_u < D_u^\Omega$ **spiral tip can be trapped**

$D_u > D_u^\Omega$ **spiral tip will escape**



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