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CONFERENCE ON STRONGLY INTERACTING SYSTEMS AT THE NANOSCALE
8 - 12 August 2005

Frustrating Mott physics of bosons on the triangular lattice

(Bosonic Mott Transitions on the Triangular Lattice)

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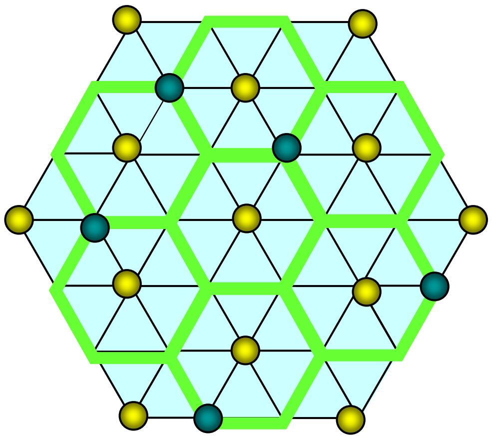
These are preliminary lecture notes, intended only for distribution to participants.

Bosonic Mott Transitions on the Triangular Lattice

- Leon Balents
- Anton Burkov

- Roger Melko
- Arun Paramekanti
- Ashvin Vishwanath
- Dong-ning Sheng



cond-mat/0505258

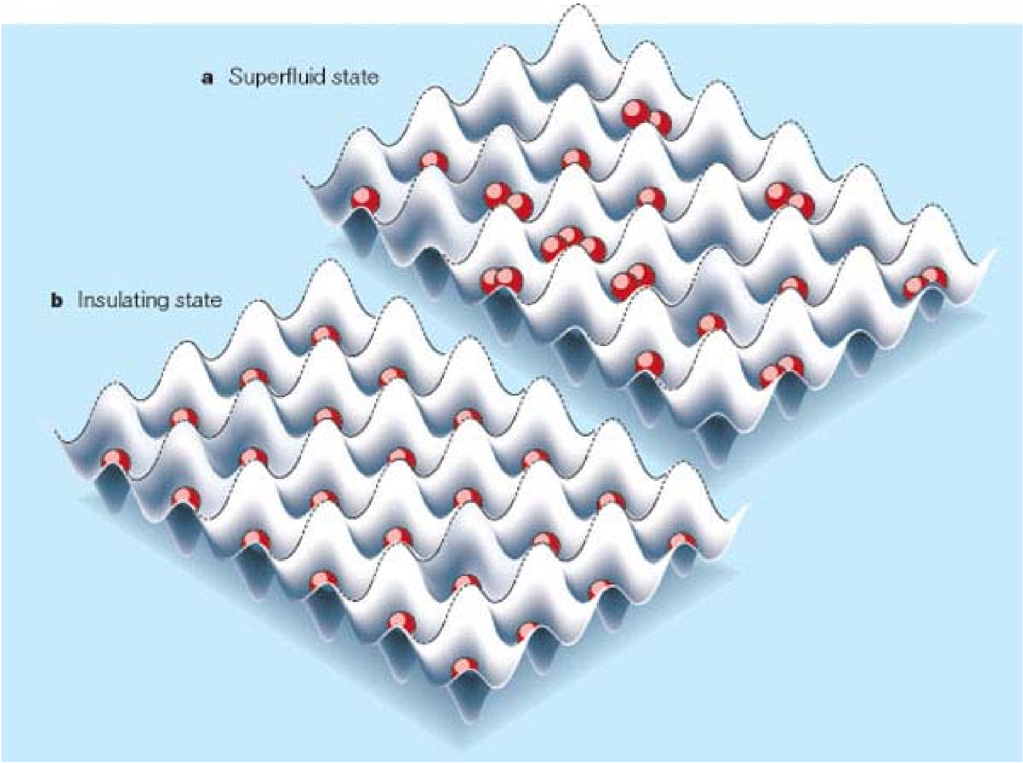
cond-mat/0506457

Outline

- XXZ Model
  - persistent superfluidity at strong interactions
  - supersolid
- Dual vortex theory of Mott transition
  - Field theory
  - Mott phases in (dual) mean field theory
  - Supersolids and deconfined Mott criticality

Bose Mott Transitions

- Superfluid-Insulator transition of bosons in a periodic lattice: now probed in atomic traps



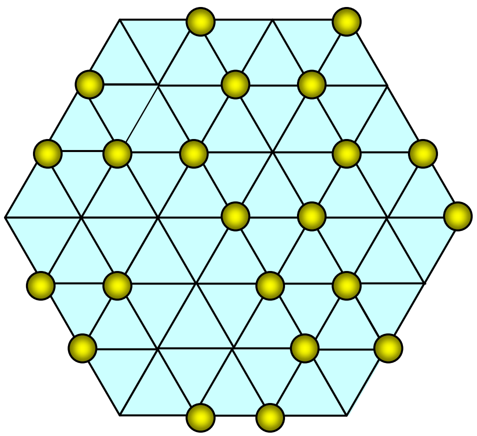
Filling f=1: Unique Mott state w/o order, and LGW works

$f \neq 1$: localized bosons must order

Interesting interplay between superfluidity and charge order!

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Triangular Lattice



- "Hard-core": no double occupancy

$\mathcal{P}$ = hard-core projector

$$H = -t \sum_{\langle ij \rangle} \mathcal{P} \left(b_i^\dagger b_j + \text{h.c.} \right) \mathcal{P} + V \sum_{\langle ij \rangle} n_i n_j$$

$$H = \sum_{\langle ij \rangle} -\frac{J_\perp}{2} \left(S_i^+ S_j^- + \text{h.c.} \right) + J_z \, S_i^z S_j^z$$

- S=1/2 XXZ model with FM XY and AF Ising exchange

Ising particle-hole symmetric

- *Frustration*: Cannot satisfy all J_z interactions
  - no *simple* "crystalline" states near half-filling

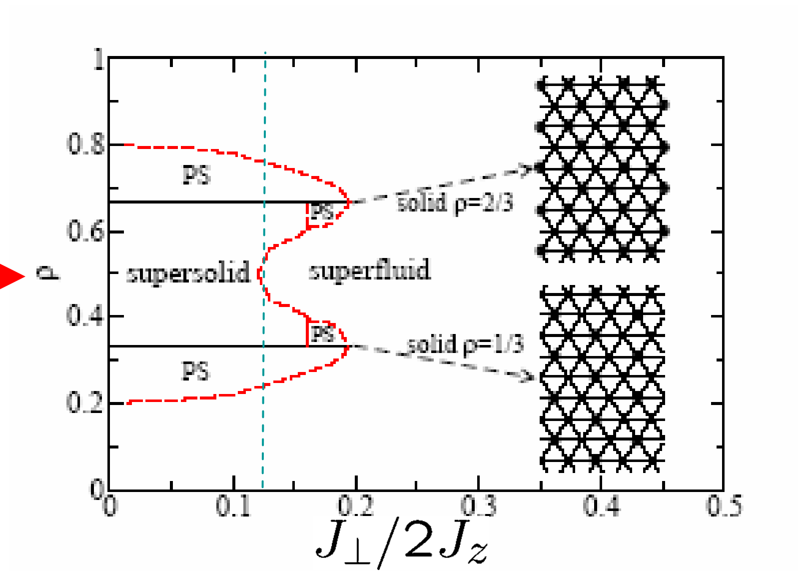
Supersolid Phase

- Recent papers on XXZ model find *supersolid* phase near ½-filling

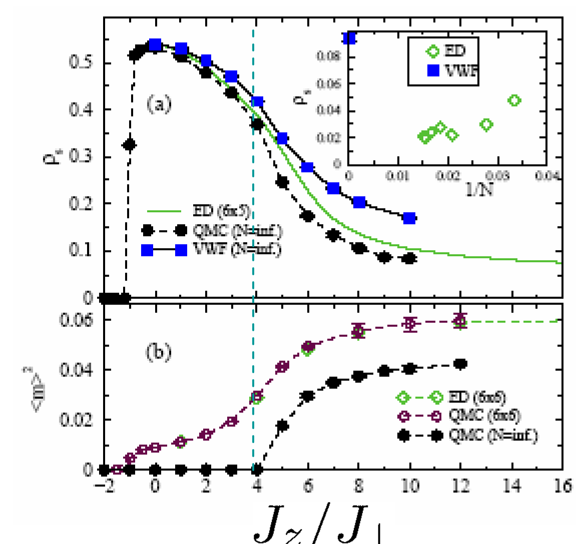
- D. Heidarian, K. Damle, cond-mat/0505257
- R. G. Melko *et al*, cond-mat/0505258
- M. Troyer and S. Wessel, cond-mat/0505298

T=0

½ filling



from M. Troyer and S. Wessel



from Melko *et al*

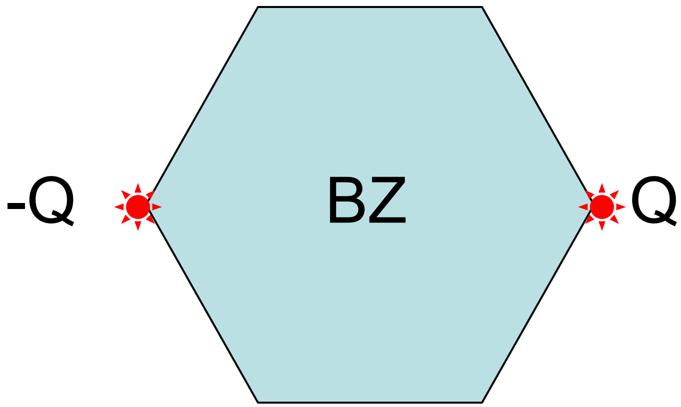
ODLRO

+

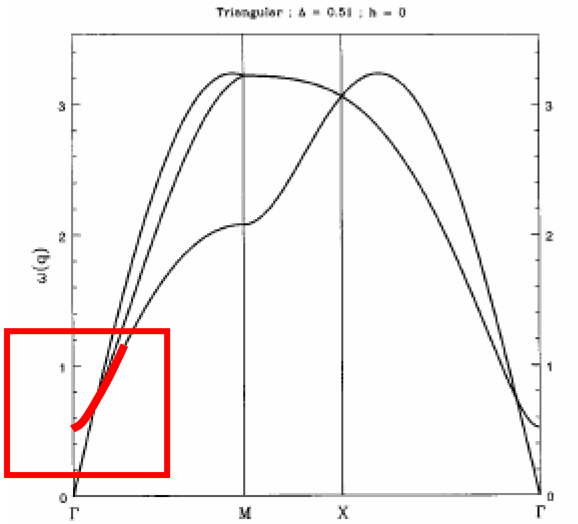
DLRO

Murthy *et al* (1997)
Melko *et al*

Spin Wave Theory



soft “roton” for $J_z/J_\perp > 1$



- Order parameter $S_i^z \sim M + \text{Re}\left[\psi e^{i\mathbf{Q}\cdot\mathbf{r}_i}\right]$

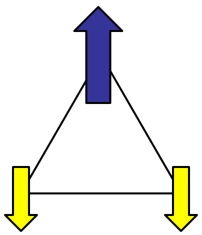
$$= M + \text{Re}\left(\psi, \psi e^{2\pi i/3}, \psi e^{4\pi i/3}\right)$$

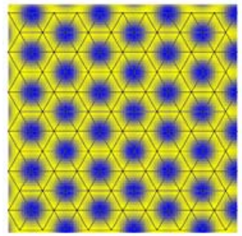
3 sublattice *diagonal* order

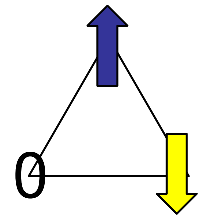
- Landau theory of superfluid-supersolid QPT:

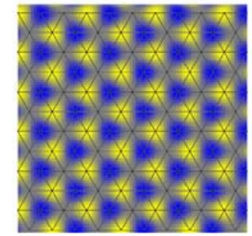
$$\mathcal{S} = \int d^2x \int_0^\beta d\tau \left[|\partial_\tau \psi|^2 + c^2 |\nabla \psi|^2 + r|\psi|^2 + u|\psi|^4 \right.$$
$$\left. + \; v|\psi|^6 + w\text{Re}\,(\psi^6) + M^2/(2\chi) - \lambda M\,\text{Re}\,(\psi^3)\right]$$

Supersolid Phases





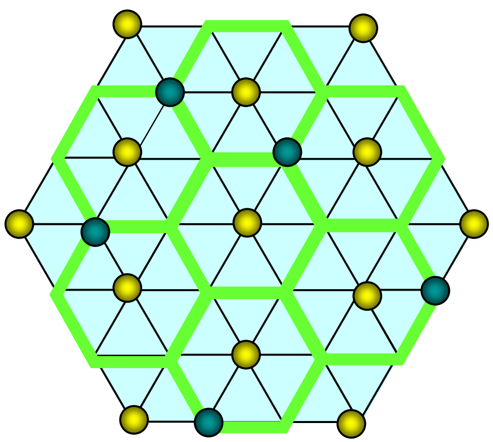


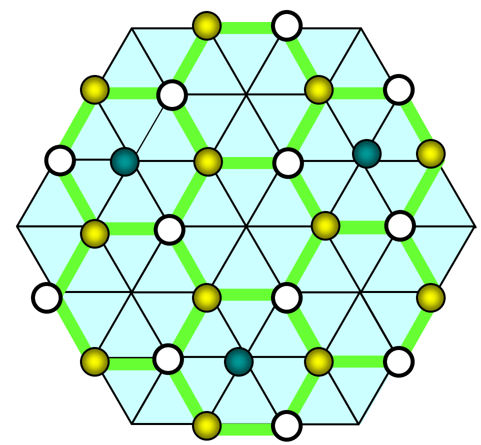


“ferrimagnetic”

“antiferromagnetic”

spontaneous magnetization=
phase separation





superfluid on ≈ ¼-filled honeycomb “interstitial lattice“ of 1/3-triangular solid

superfluid on ≈ 1/2 -filled triangular “interstitial lattice“ of honeycomb “antiferromagnetic” solid

particle-hole transform not identical

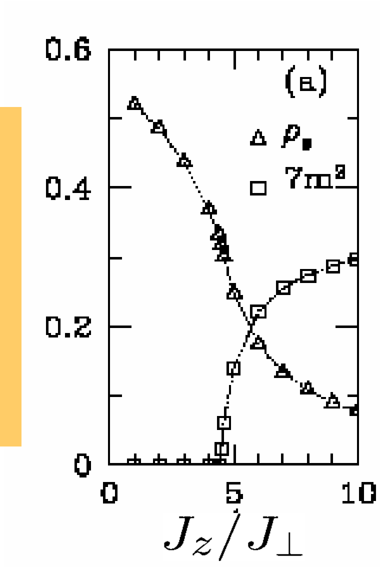
expect stabilized by 2nd neighbor hopping

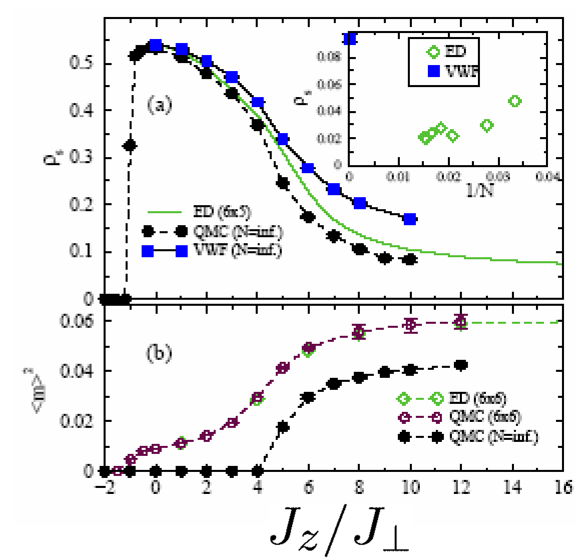
Surprises

- Superfluidity survives even when $V=J_z \to \infty$!

Symptomatic of frustration: superfluid exists within extensively degenerate classical antiferromagnetic ground state Hilbert space

topology of this space leads to “proof” of diagonal LRO at $J_z = \infty$





- Persistent superfluidity is exceedingly weak

$$\rho_s(J_z=\infty) \approx 0.02\rho_s(J_z=0)$$

close to Mott insulator

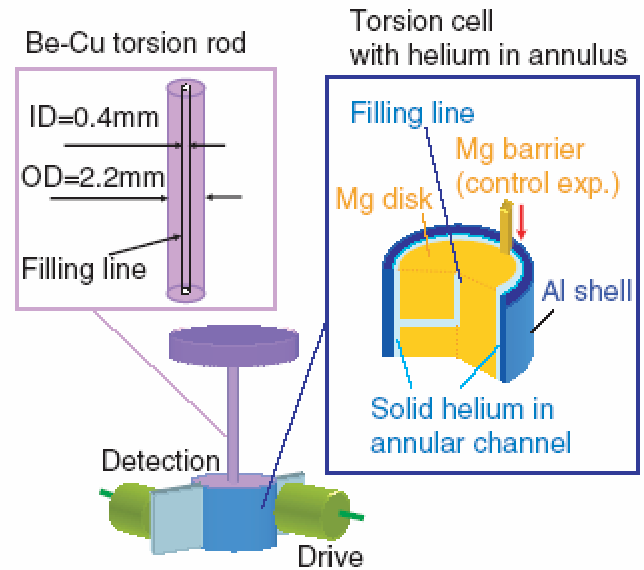
- Energy difference between 2 supersolid states is *nearly unobservable*

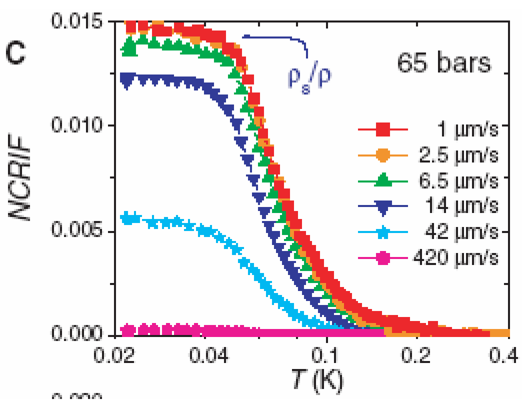
SCIENCE VOL 305 24 SEPTEMBER 2004

Observation of Superflow in Solid Helium

E. Kim and M. H. W. Chan*

We report on the observation of nonclassical rotational inertia in solid helium-4 confined to an annular channel in a sample cell under torsional motion, demonstrating superfluid behavior. The effect shows up as a drop in the resonant oscillation period as the sample cell is cooled below 230 millikelvin. Measurement of 17 solid samples allows us to map out the boundary of this superfluid-like solid or supersolid phase from the melting line up to 66 bars. This experiment indicates that superfluid behavior is found in all three phases of matter.





- Superfluid grain boundaries?

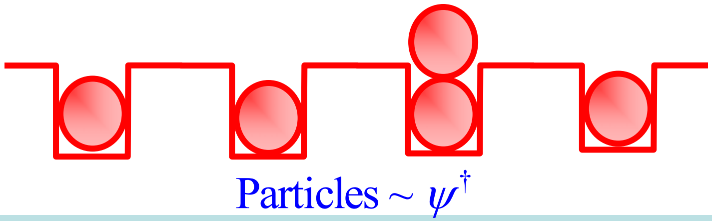
Burovski *et al, 2005*

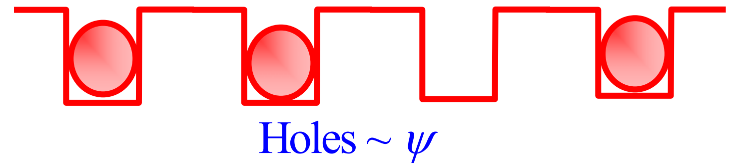
- He^4 atoms at boundaries frustrated by incommensurate quasiperiodic superposition of potentials from two crystallites?
- Persistent superfluidity stabilized by frustration despite strong interactions?

- Superflow? Bulk or defect related?

Mott Transition

- Goal: continuum quantum field theory
  - describes “particles” condensing at QCP





- Conventional approach: use extra/missing bosons
  - -Leads to LGW theory of bose condensation
  - -Built in diagonal order, the same in both Mott and SF state

- Dual approach: use vortices/antivortices of superfluid
  - non-LGW theory, since vortices are non-local objects
  - focuses on “Mottness”, diagonal order is secondary
  - theory *predicts* set of possible diagonal orders

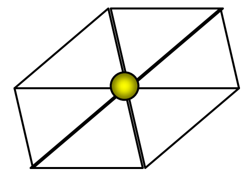
Duality

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett*. **47**, 1556 (1981); D.R. Nelson, *Phys. Rev. Lett*. **60**, 1973 (1988); M.P.A. Fisher and D.-H. Lee, *Phys. Rev*. B **39**, 2756 (1989);

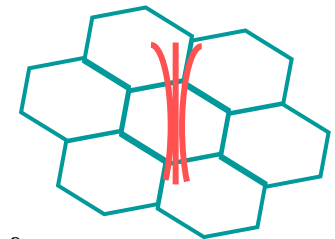
- Exact mapping from boson to vortex variables

- Dual magnetic field B = 2πn

$$n = \frac{1}{2\pi}\vec{\nabla} \times \vec{A}$$

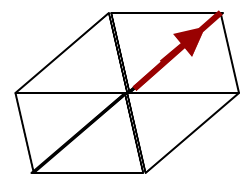


$n = 1$

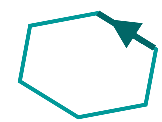


$$\int d^2x\, B = 2\pi$$

$$\vec{\nabla}\phi = 2\pi\hat{z} \times \vec{E},$$



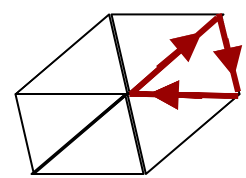
$v_{sf} \propto \vec{\nabla}\phi$



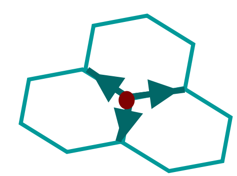
$\vec{E}$

- Vortex carries dual U(1) gauge charge

$$\vec{\nabla} \cdot \vec{E} = N$$



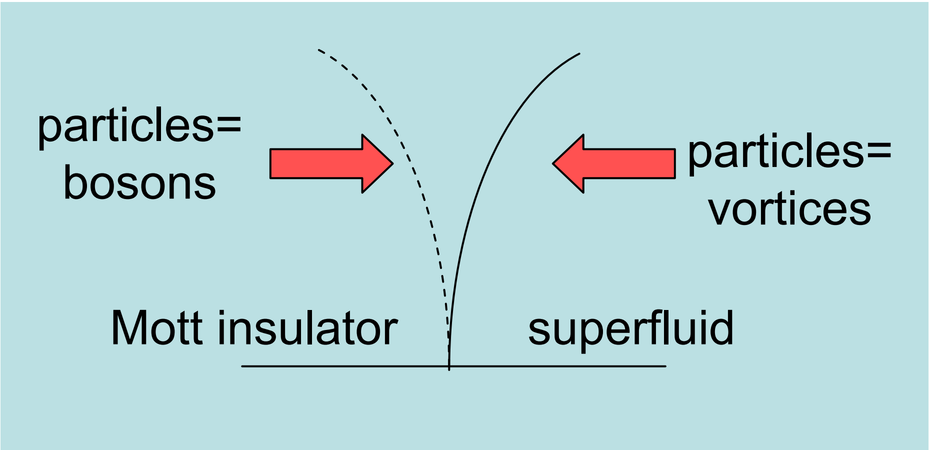
$$\oint \vec{\nabla}\phi \cdot d\vec{\ell} = 2\pi$$



$N = 1$

- All non-locality is accounted for by dual U(1) gauge force

Dual Theory of QCP for f=1



• Two completely equivalent descriptions
- really one critical theory (fixed point) with 2 descriptions

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett*. **47**, 1556 (1981);

$$\mathcal{S} = \int d^3x \left[|\partial_\mu \psi|^2 + s|\psi|^2 + u|\psi|^4 \right]$$

$$\tilde{\mathcal{S}} = \int d^3x [|(\partial_\mu - iA_\mu)\varphi|^2 + \tilde{s}|\varphi|^2 + u|\varphi|^4 + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2]$$

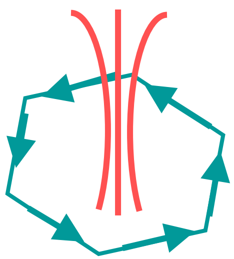
• N.B.: vortex field φ is not gauge invariant
- not an order parameter in Landau sense

• Real significance: "Higgs" mass $|\langle\varphi\rangle|^2 A^2$ indicates insulating dielectric constant $\epsilon_d \sim 1/|\langle\varphi\rangle|^2$

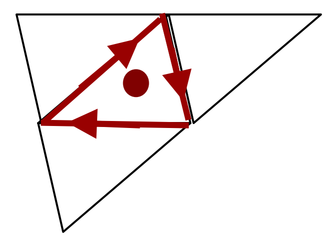
Non-integer filling f ≠ 1

- Vortex approach now superior to Landau one
  -need not postulate unphysical disordered phase

- Vortices experience average dual magnetic field
  - physics: phase winding



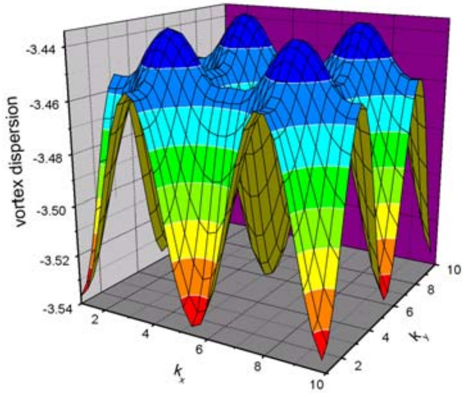
Aharonov-Bohm phase in vortex wavefunction encircling dual flux



2π winding of boson wavefunction on encircling vortex

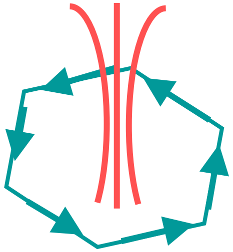
- Vortex field operator transforms under a *projective* representation of lattice space group

Vortex Degeneracy



- Non-interacting spectrum = honeycomb Hofstadter problem

- Physics: magnetic space group



$$T_1 T_2^{-1} T_3 T_1^{-1} T_2 T_3^{-1} = e^{2\pi i f}$$

(and other PSG operations)

- For f=p/q (relatively prime) and q even (odd), all representations are at least 2q (q)-dimensional

- This degeneracy of vortex states is a robust property of a superfluid (a “quantum order”)

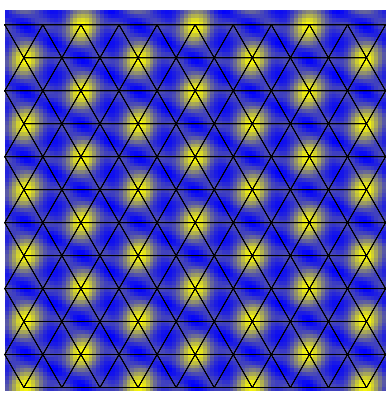
1/3 Filling

- There are 3 vortex "flavors" ξ_1, ξ_2, ξ_3 with the Lagrangian

$$\mathcal{L} = \sum_{\ell} \left[|(\partial_\mu - iA_\mu)\xi_\ell|^2 + s|\xi_\ell|^2 \right] + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2$$
$$+ u(\sum_{\ell} |\xi_\ell|^2)^2 + \sum_{\ell \neq \ell'} \{ v\, |\xi_\ell|^2 |\xi_{\ell'}|^2 + w\, \mathrm{Re}\, [(\xi_\ell^* \xi_{\ell'})^3] \}$$

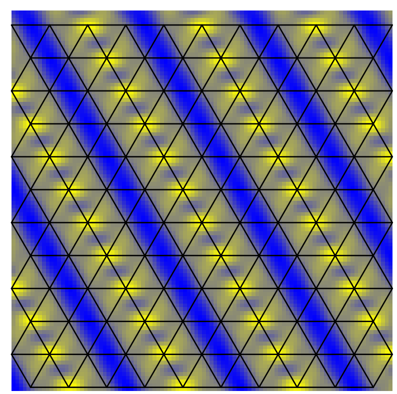
- Dual mean-field analysis predicts 3 possible Mott phases

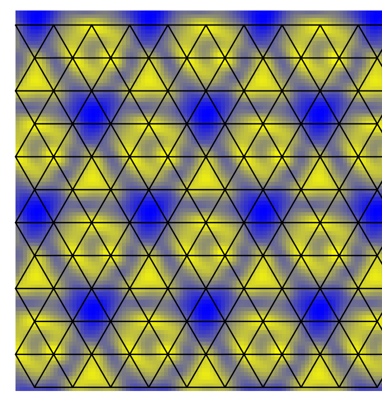
v>0:



1/3 solid of XXZ model

v<0:

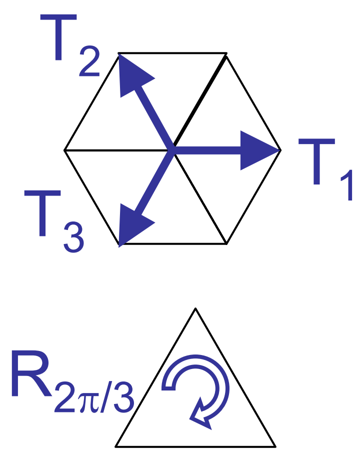


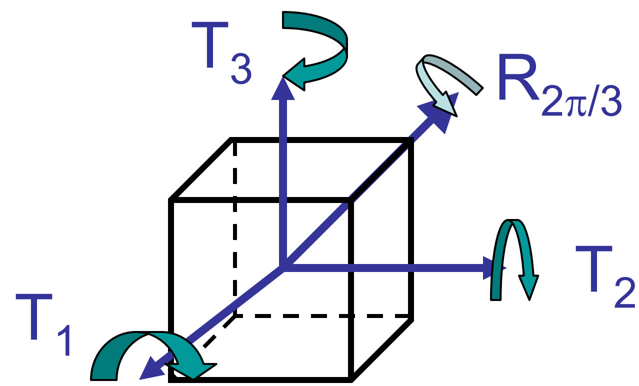


Expect "deconfined" Mott QCP with fluctuations included

½-Filling

- $2 \times 2 = 4$ vortex flavors with pseudo-spinor structure $z_{\pm\sigma}$
  - Space group operations appear as “rotations”





$\vec{S}$

- Order parameters

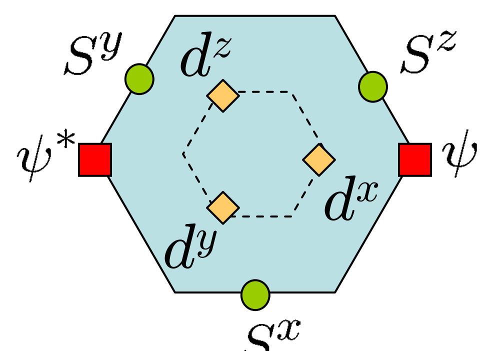
$$\vec{S}_\alpha = z_\alpha^* \vec{\tau} z_\alpha$$

$$\psi = e^{i\pi/4} z_{+\sigma}^* z_{-\sigma}$$

$$\vec{d} = z_+^* \vec{\tau} z_-$$

XXZ supersolid diagonal order parameter

ordering wavevectors



Dual ½-Filling Lagrangian

$$\mathcal{L} = \mathcal{L}_0 + u\left(|S_+| + |S_-|\right)^2 + v\,|S_+||S_-| + w_1\,\vec{S}_+\cdot\vec{S}_- \quad \} \text{ quartic}$$

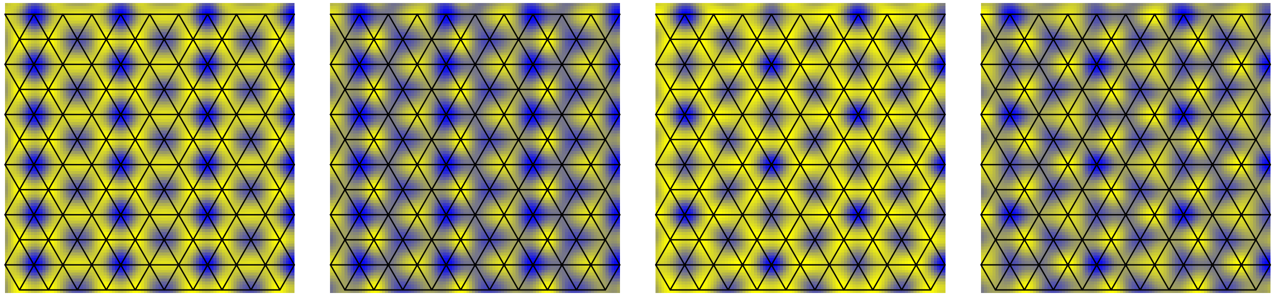
$$+ w_2 \sum_\alpha \left[(S_\alpha^x)^4 + (S_\alpha^y)^4 + (S_\alpha^z)^4\right] - w_3\,\mathrm{Re}\left(\psi^6\right) \quad \} \text{ 8th and 12th order}$$

- Emergent symmetry:
  - -Quartic Lagrangian has $SU(2)\times U(1)\times U(1)_g$ invariance
  - -$SU(2)\times U(1)$ symmetry is approximate near Mott transition
  - -Leads to "skyrmion" and "vortex" excitations of SU(2) and U(1) order parameters

- Mean field analysis predicts 10 Mott phases

- e.g. $v, w_1 < 0$

note similarity to XXZ supersolids



Hard-Spin Limit: Beyond MF analysis

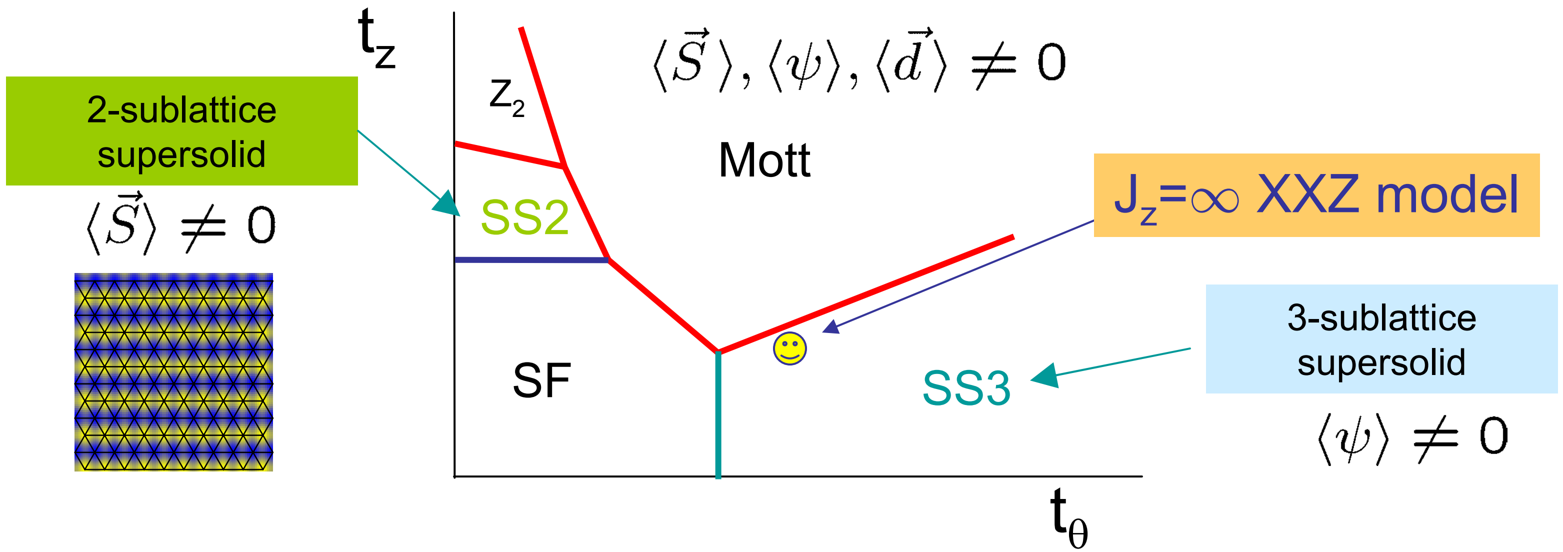
- Example: v,w_1<0: $\vec{S}_+ = \vec{S}_-$ $\quad |S_+| = |S_-| = \text{const.}$
  - Solution: $z_{\pm\sigma} = z_\sigma e^{\pm i\theta/2}$ $\qquad z^*_\sigma z_\sigma = 1$
  - Z_2 gauge redundancy: $\begin{cases} z_\sigma \longrightarrow -z_\sigma \\ \theta \longrightarrow \theta + 2\pi \end{cases}$

- Hard-spin (space-time) lattice model:

$$\mathcal{L}_{\text{eff}} = -t_z \sigma_{i\mu} e^{-iA_{i\mu}} z^*_{i\sigma} z_{i+\mu\sigma} - t_\theta \sigma_{i\mu} \cos(\Delta_\mu \theta_i/2) + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\Delta_\nu A_{i\lambda})^2$$

- Z_2 gauge field
- CP^1 field
- XY field
- U(1) gauge field

Phase Diagram



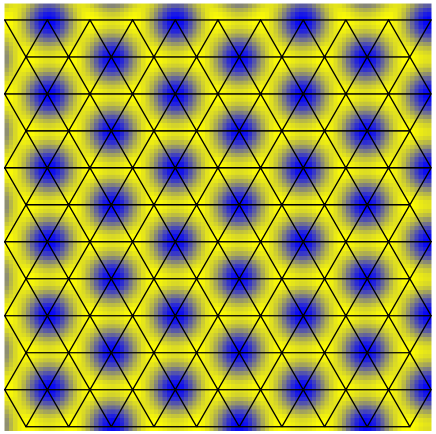
- Blue lines: LGW "roton condensation" transitions

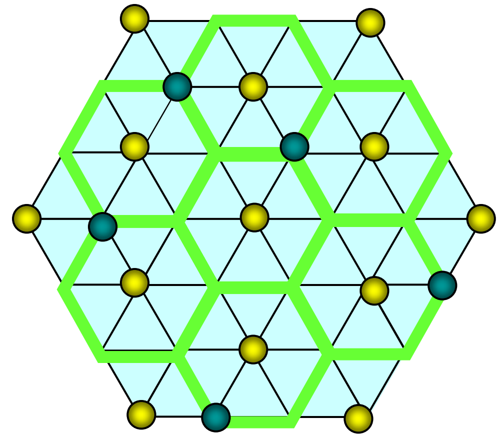
- Red lines: non-LGW transitions
  - Diagonal order parameters change *simultaneously* with the superfluid-insulator transition

- Should be able to understand supersolids as "partially melted" Mott insulators

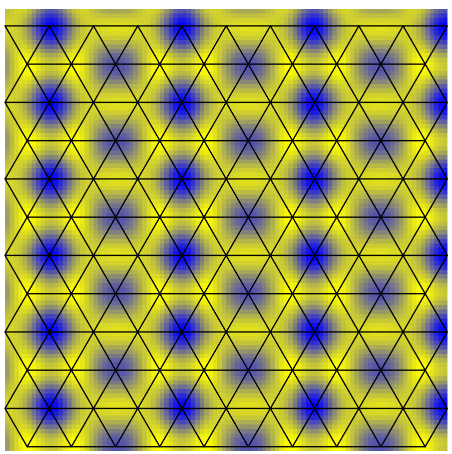
Physical Picture

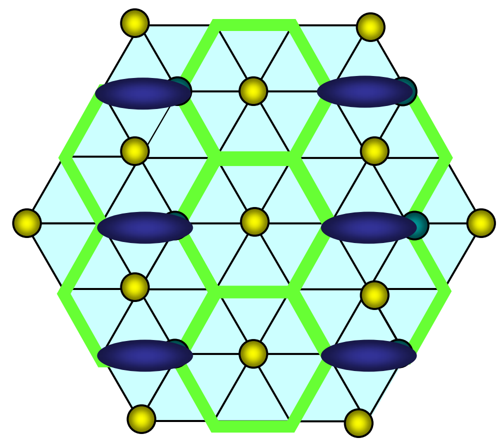
SS3
ferrimagnetic
supersolid





ferrimagnetic
columnar
solid

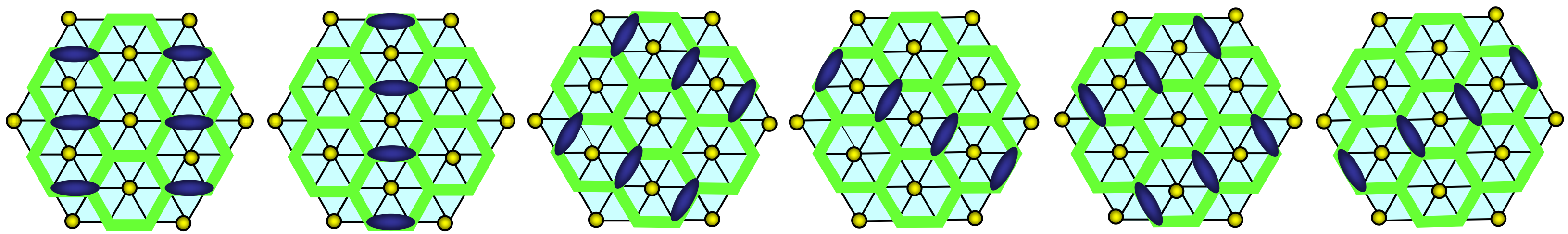




- Superfluid to columnar VBS transition of ¼-filled honeycomb lattice!

Skyrmion

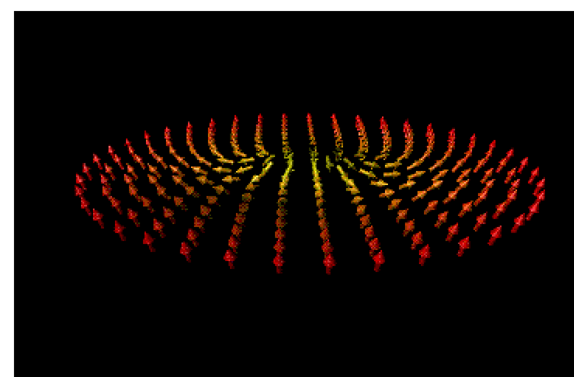
- VBS Order parameter: pseudo-spin vector $\langle \vec{S} \rangle = S_0 \hat{n}$



$\hat{n} =$ (100) (-100) (010) (0-10) (001) (00-1)

- Skyrmion:
  - -integer topological index
  - -finite size set by irrelevant "cubic anisotropy"

$$Q = \frac{1}{4\pi} \int d^2 r \, \hat{n} \cdot \partial_x \hat{n} \times \partial_y \hat{n}$$



- Boson charge is bound to skyrmion!

$N_b = Q$

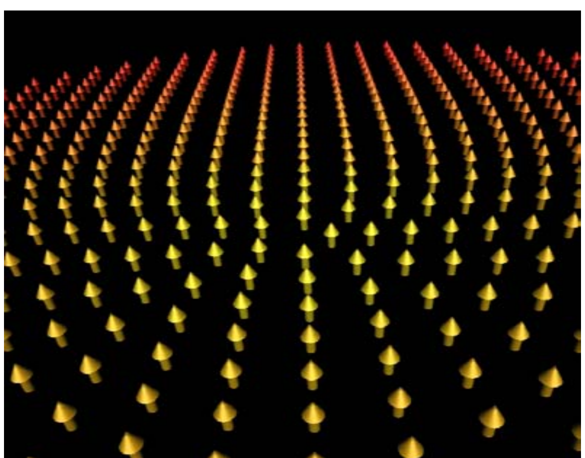
Mott-SS3 Criticality

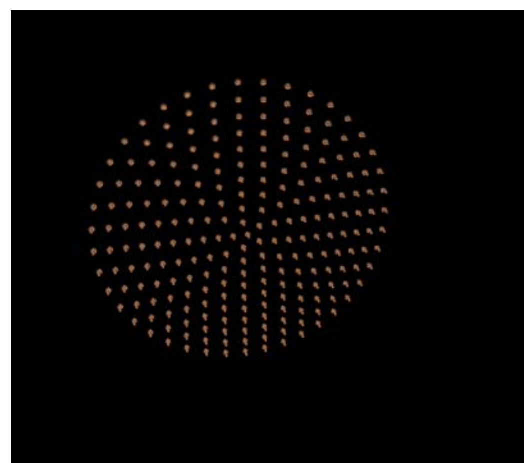
- SS3-Mott transition is *deconfined quantum critical point*
  - Non-compact CP^1 universality class
  - Equivalent to hedgehog-free O(3) transition

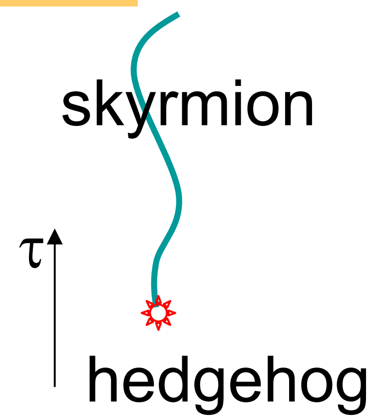
Motrunich+Vishwanath

- Disordering of pseudospin → skyrmions condense: superfluid

- Hedgehogs = skyrmion number changing events







Conclusions

- Frustration in strongly interacting bose systems seems to open up a window through to observe a variety of exotic phenomena
- The simplest XXZ model exhibits a robust supersolid, and seems already quite close to non-trivial Mott state
- It will be interesting to try to observe Mott states and deconfined transitions by perturbing the XXZ model slightly
  - Cartoon pictures of the supersolid and Mott phases may be useful in suggesting how this should be done

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