The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR 1669 - 1

CONFERENCE ON STRONGLY INTERACTING SYSTEMS AT THE NANOSCALE
8 - 12 August 2005

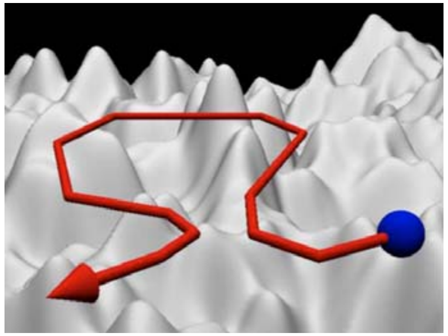
Localization of matter waves in 2D-disordered optical potentials

Cord MÜLLER
Universität Bayreuth - Physikalisches Institut
D-95440 Bayreuth, Germany

Strada Costiera 11, 34014 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

Localization of matter waves

in 2D-disorderd optical potentials

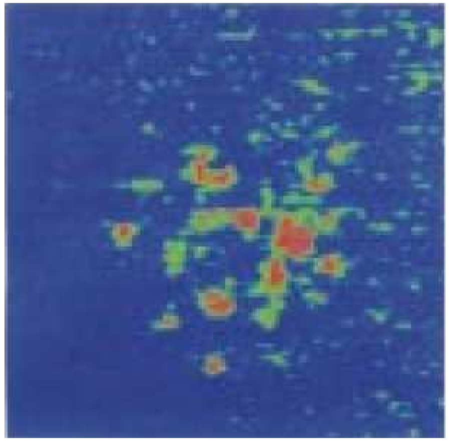


Cord Müller

UNIVERSITÄT BAYREUTH

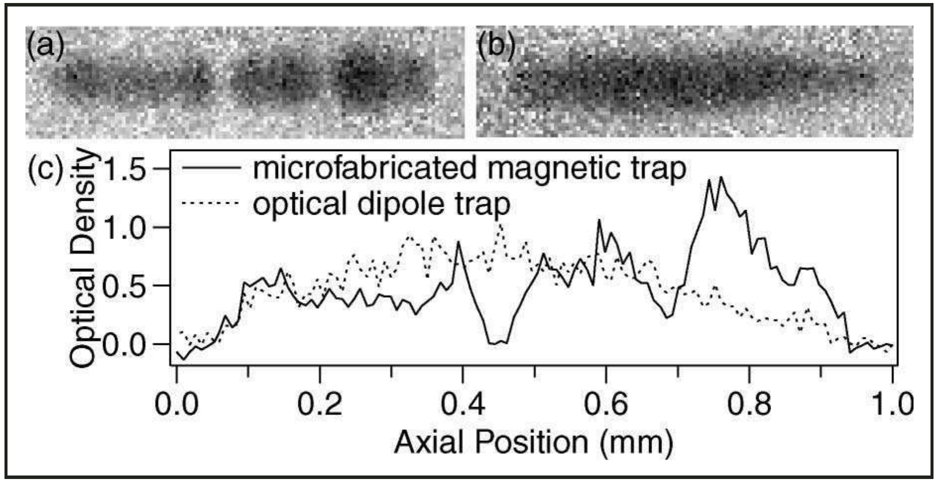
Cold atoms in disordered traps

Dissipative regime:



Cooling and trapping of cold atoms in speckle potentials
[Paris: Boiron et al, EPJ D (1999)] ▶
$\sim 10^3$ atoms per local field maximum

Coherent regime:

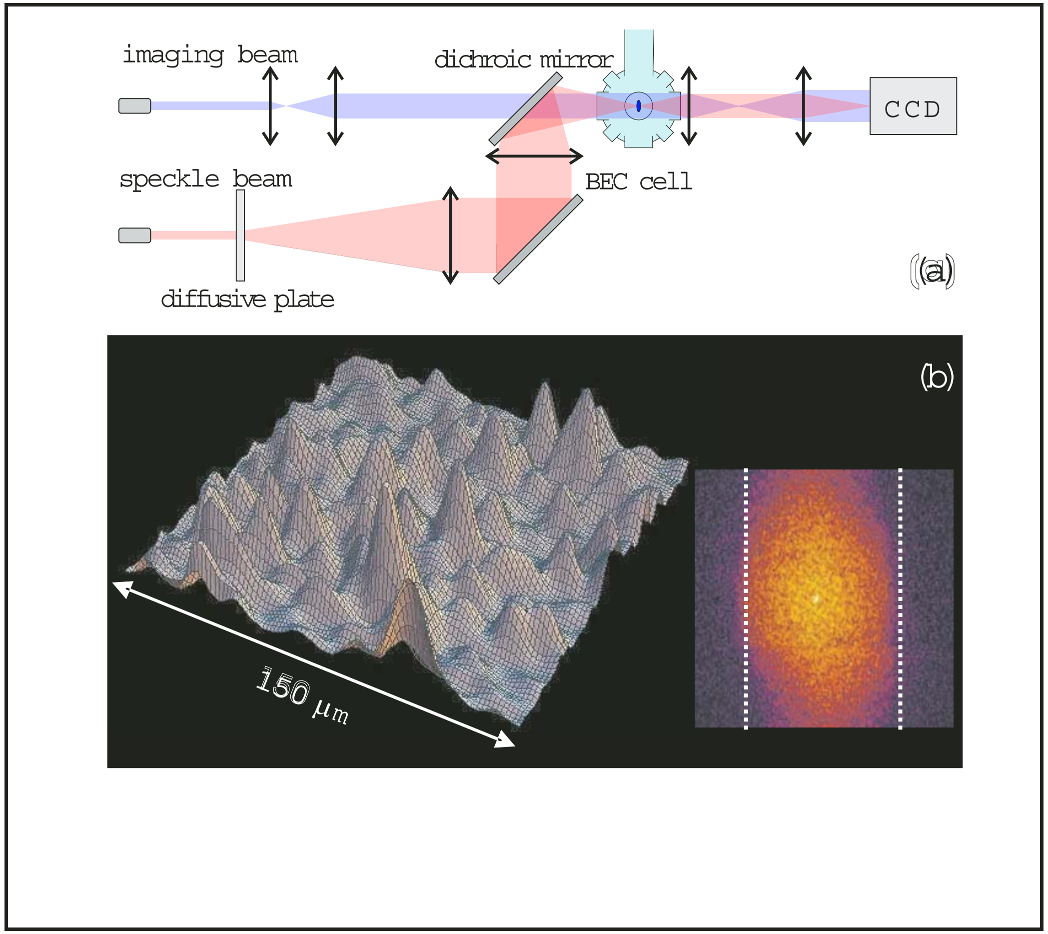


BECs trapped on atom chips
[MIT: Leanhardt et al., PRL (2003) ▶
Orsay: Estève et al., PRA (2004)]
Problem: fragmentation

Recent experiments

BEC in optical speckle potential:

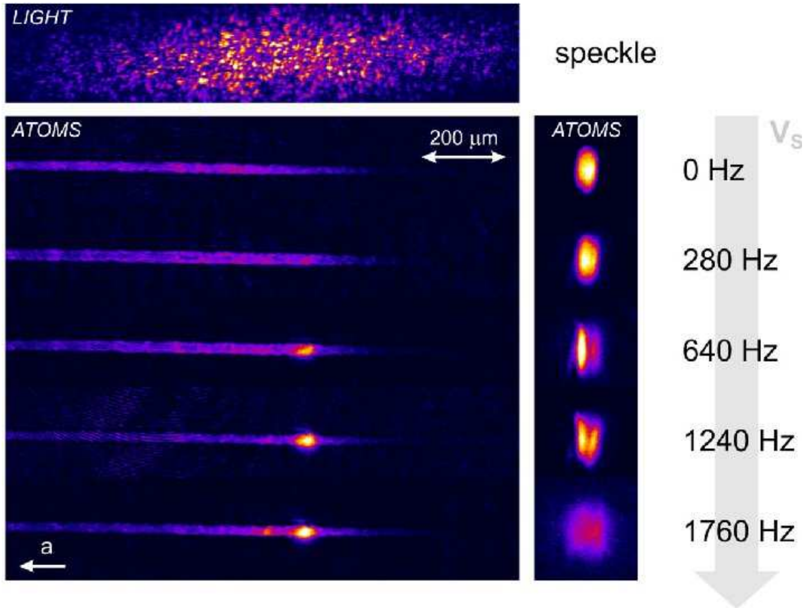
[Florence: Lye et al., cond-mat/0412167]



Recent experiments

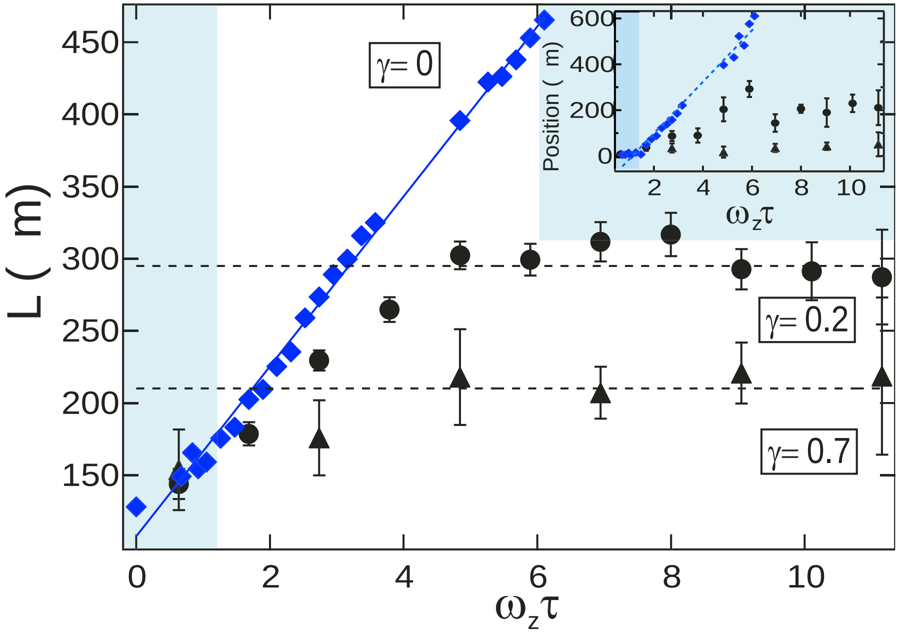
Florence:

[Fort et al., cond-mat/0507144]



Orsay:

[Clément et al., cond-mat/0506638]



Hannover: [Schulte et al., cond-mat/0507453]

► 1D-transport inhibited by disorder

Interacting *vs.* free regime

- Observation of "absence of [quantum] diffusion in certain random lattices" [Anderson, 1958]?
- Authors claim "No": simple trapping by large potential bumps

$$\delta V > E$$

in Thomas-Fermi regime of strong interaction

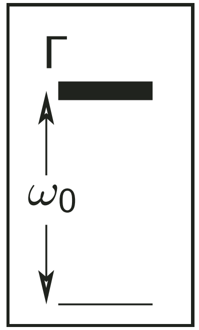
$$E = E_{\text{interaction}} \gg E_{\text{kin}}$$

Theory: numerics with Gross-Pitaevskii eq.

- Low density, fast atoms $\Rightarrow$ quasi-free regime
- Our approach: 1-particle-dynamics with disorder

$$H = \frac{p^2}{2m} + \overline{V} + \delta V(\mathbf{r})$$

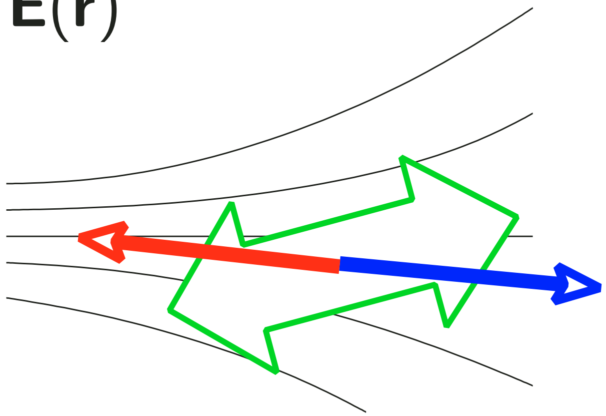
Optical dipole potential



- Atom: Energy $E = \hbar^2 k^2/2m$, momentum $\hbar\mathbf{k}$, optical resonance ω_0, Γ
- Light: monochromatic ω_L, laser power P, detuning $\delta = \omega_L - \omega_0$
- Electric dipole interaction $H_{int} = -\mathbf{D} \cdot \mathbf{E}(\mathbf{r})$
- Induced atomic dipole: potential

$$V(\mathbf{r}) \propto \frac{|\mathbf{E}(\mathbf{r})|^2}{\delta}$$

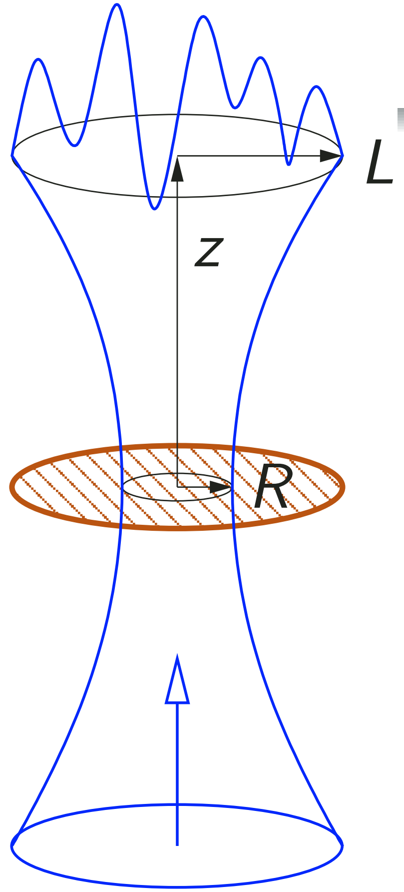
far off resonance $|\delta| \gg \Gamma$



- Inelastic scattering rate

$$\gamma_{inel} \propto \frac{|\mathbf{E}(\mathbf{r})|^2}{\delta^2} \propto \frac{V(\mathbf{r})}{\delta}$$

Speckle potential



- Scalar Gaussian random field $E(\mathbf{r})$

$$\gamma(\mathbf{r}) = \frac{\overline{E^*(\mathbf{r}' + \mathbf{r})E(\mathbf{r}')}}{\overline{|E|^2}} = 2\frac{J_1(\alpha k_L r)}{\alpha k_L r}$$

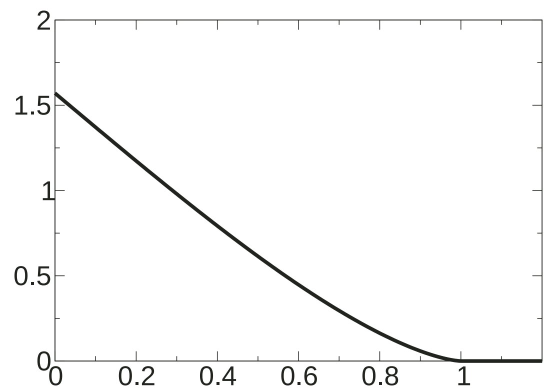
$\alpha = R/z \ll 1$: aperture

- Correlation scale $\xi = 1/\alpha k_L \approx 1\,\mu\text{m}$

- Intensity potential *not* Gaussian: $\overline{\delta V^3} \sim \overline{|E|^6} \neq 0$
- Correlation $\mathcal{P}(\mathbf{r}) = \overline{\delta V(\mathbf{r})\delta V(0)}/\overline{V}^2 = |\gamma(\mathbf{r})|^2$

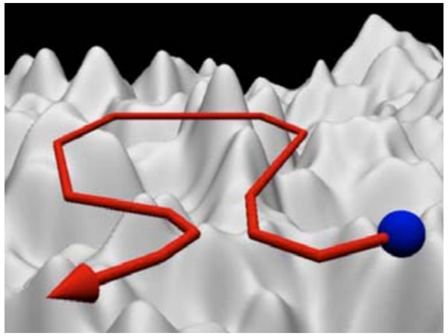
$$\mathcal{P}(\mathbf{k}) = \mathcal{F}(k\xi/2)\,\Theta(1 - k\xi/2)$$

Potential smooth on scale $< \xi$



Goal

- Predict 2D-dynamics (marginal dimension) with quenched disorder



$$H = \frac{p^2}{2m} + \overline{V} + \delta V(\mathbf{r})$$

- Local conservation law and linear response $\Rightarrow$ diffusion
- Calculate diffusion constant and localization corrections as function of relevant parameters:
  - atom momentum k
  - laser power P and detuning δ
  - correlation ξ

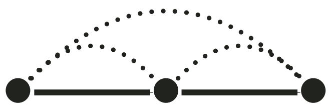
Quantum transport formalism

- Particle density $p(\mathbf{r}, t) = \langle \mathbf{r} | \overline{U(t)\rho_0 U^\dagger(t)} | \mathbf{r} \rangle$
- Resolvent $G(E) = (E - H)^{-1} = G_0 + G_0 \delta V G$
- Dyson:

$$\overline{G} = G_0 + G_0 \overline{\delta V G_0 \delta V} G_0 + \cdots = G_0 + G_0 \Sigma \overline{G}$$

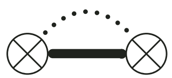
- Self-energy:

$$\Sigma = \overline{\delta V G_0 \delta V} + \cdots =$$

 +  + ...

- Plane waves $|k\rangle$ decay as $\overline{G(r)} \propto e^{-r/2\ell_s}$
  ℓ_s: scattering mean free path
- Weak disorder, Born approximation:

$$\Sigma = \overline{\delta V G_0 \delta V} =$$

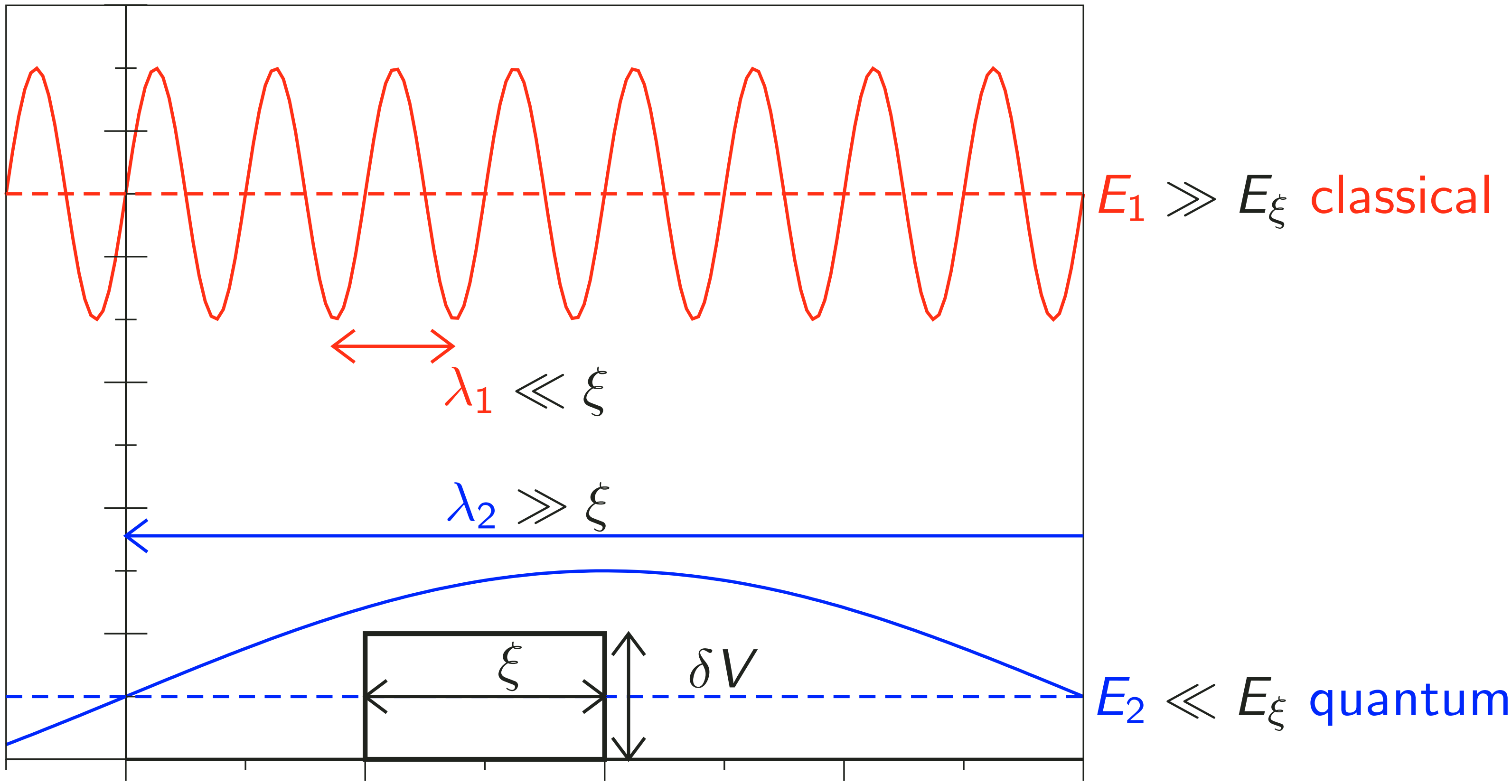
 =: 

Weak disorder criterion

Correlation energy $E_\xi = \hbar^2/m\xi^2 = 2\alpha^2 E_{\text{recoil}}$

$$\Sigma = \overline{\delta V G_0 \delta V} \ll E : \qquad E \gg \frac{\overline{V}^2}{E_\xi}$$

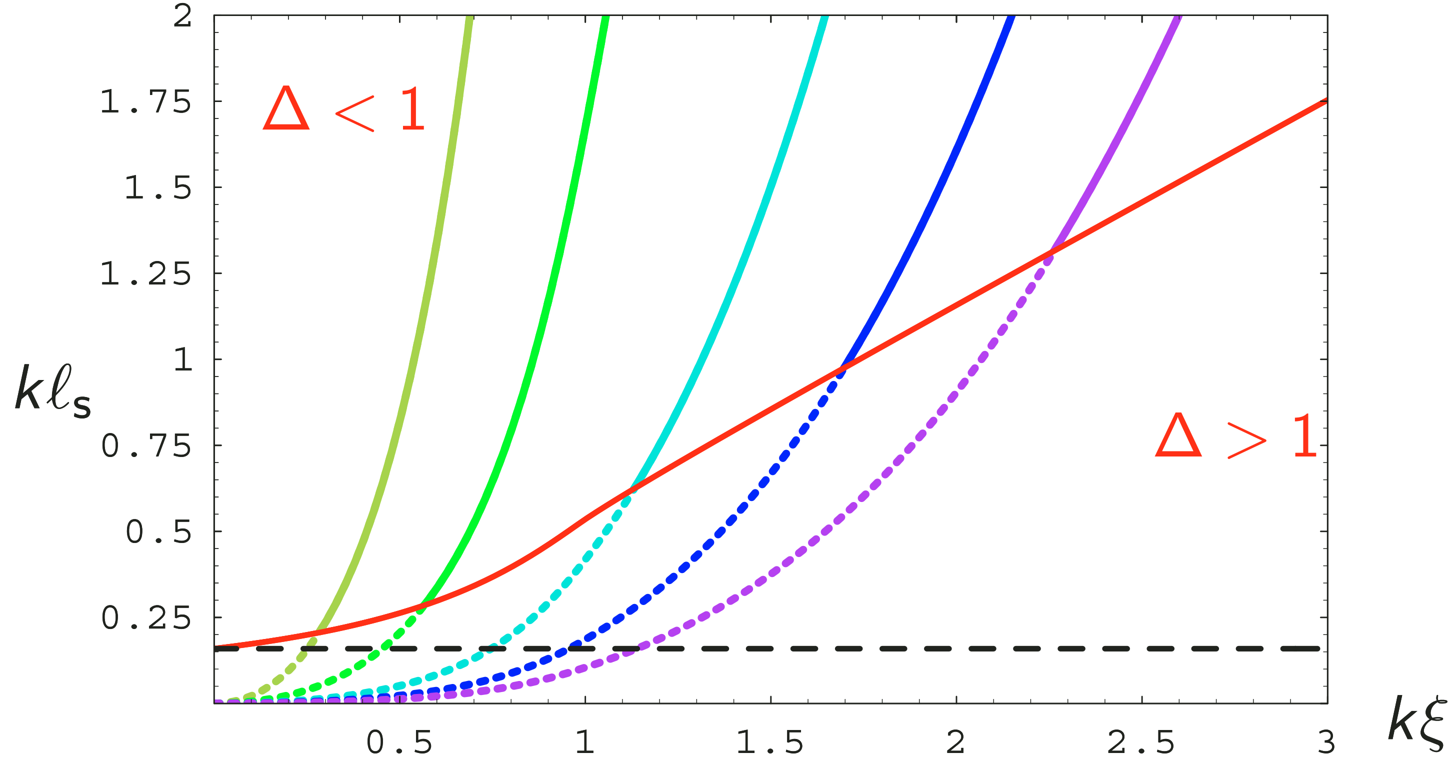
Small quantum reflection at square potential:



Elastic mean free path $k\ell_s$

$$\frac{1}{k\ell_s} = \Delta \int_0^{2\pi} \frac{\mathrm{d}\theta}{\pi}\, \mathcal{P}(k\xi \sin\theta/2), \qquad \Delta =: \frac{\overline{V}^2}{EE_\xi} \ll 1$$

speckle strength $\overline{V}/E_\xi = 0.2,\ 0.4,\ 0.8,\ 1.2,\ 1.6$

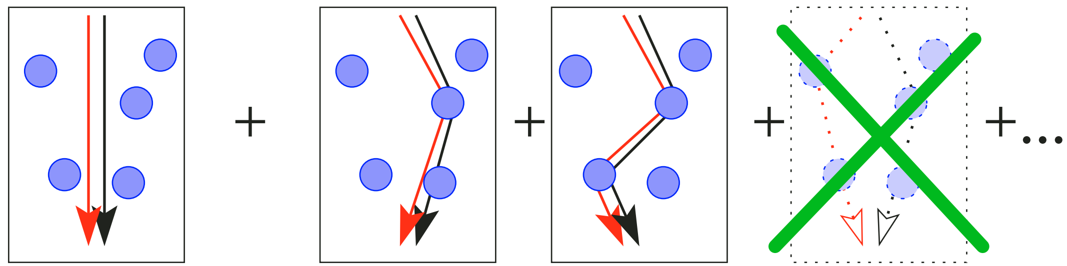


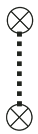
$k\ell_s \approx 1$: highly disordered medium $\qquad$ $k\ell_s \gg 1$: clean limit.

Intensity propagation

- Particle density $p(\mathbf{r},t) = \langle \mathbf{r} | \overline{U(t)\rho_0 U^\dagger(t)} | \mathbf{r} \rangle$
- Intensity propagation kernel (Bethe-Salpeter eq.)

$$\Phi = \overline{G^R G^A} = \overline{G^R}\ \overline{G^A} + \overline{G^R}\ \overline{G^A} U \Phi$$

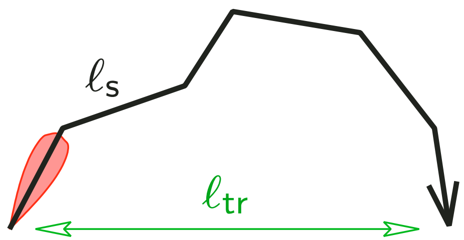


$U_B =$ 

- Diffuson: classical Boltzmann dynamics

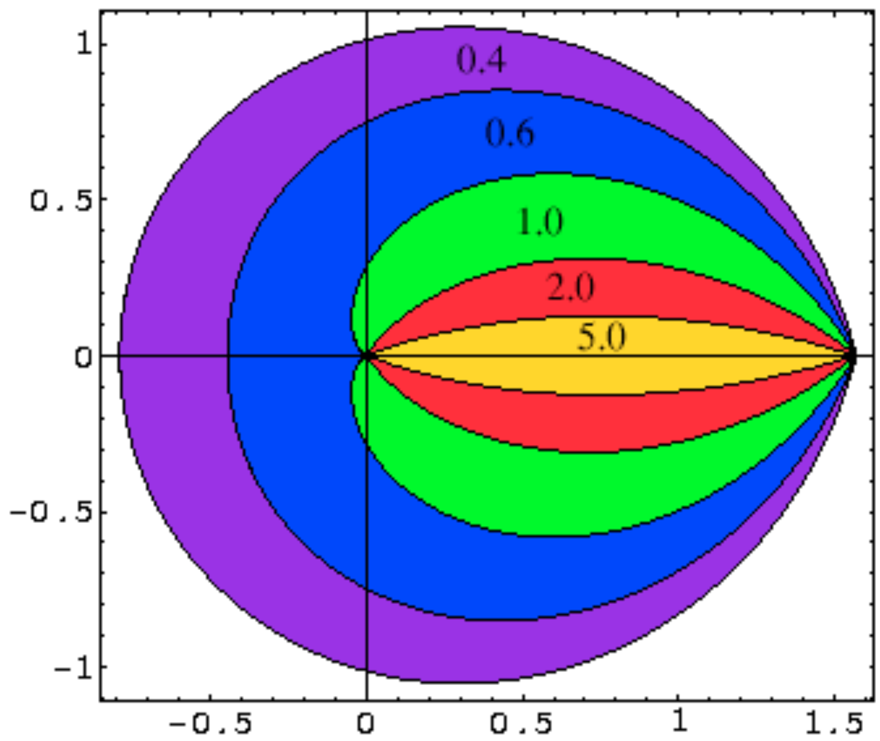
$$\Phi(k,q,\omega) = \frac{1}{\mathrm{i}\omega + D_B(k) q^2}, \qquad D_B(k) = \frac{\hbar k \ell_{tr}(k)}{2m}$$

- Transport mf path: $\ell_{tr} = \ell_s / (1 - \overline{\cos\theta})$



Scattering anisotropy

Phase function $f(\theta)$ peaked into forward direction:

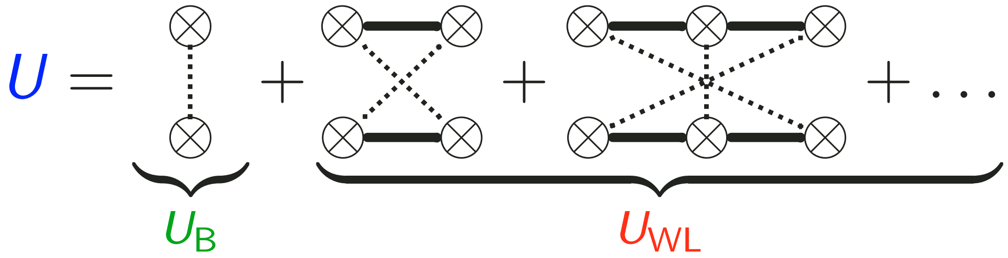


$$|\sin(\theta/2)| \leq 1/k\xi$$

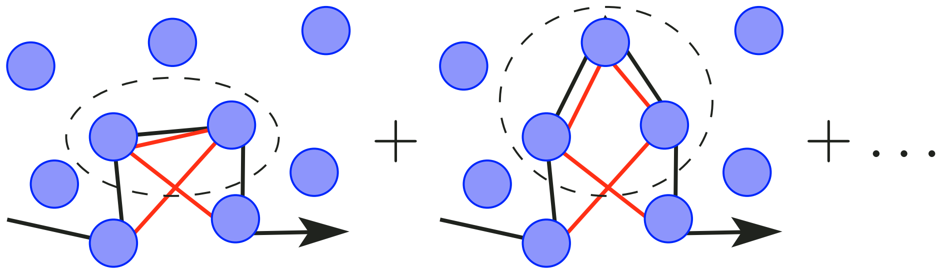
Only cold atoms far below recoil see an effective δ-correlated isotropic scattering potential.

Beyond Boltzmann: Weak localization

- Cooperon: maximally crossed diagrams



- Interference:



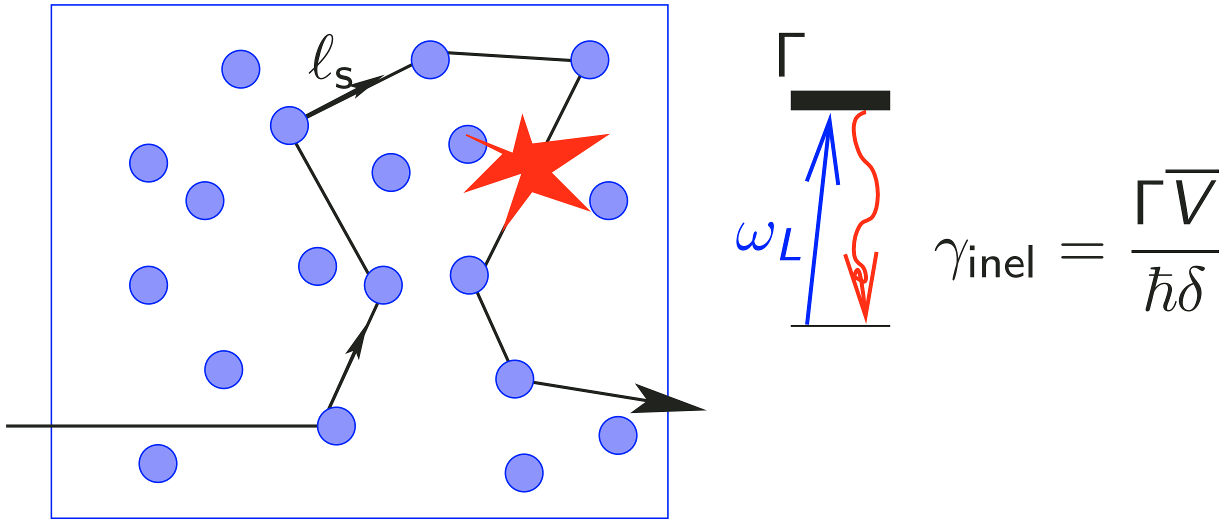
- Reduced diffusion constant $D = D_B - \delta D$. In 2D

$$\frac{\delta D}{D_B} = \frac{2}{\pi k \ell_{tr}} \int_{L_0^{-1}}^{\ell_s^{-1}} \frac{q^1 \mathrm{d}q}{q^2} = \frac{2}{\pi} \frac{\ln(L_0/\ell_s)}{k \ell_{tr}}$$

Cutoff $L_0 = \min(\text{system size } L, \text{ phase coherence } L_\phi)$

Controlled decoherence

- Electronic negative magnetoresistance: $\dfrac{\mathrm{d}\rho}{\mathrm{d}B} < 0$
- Atoms: spontaneous emission of photons



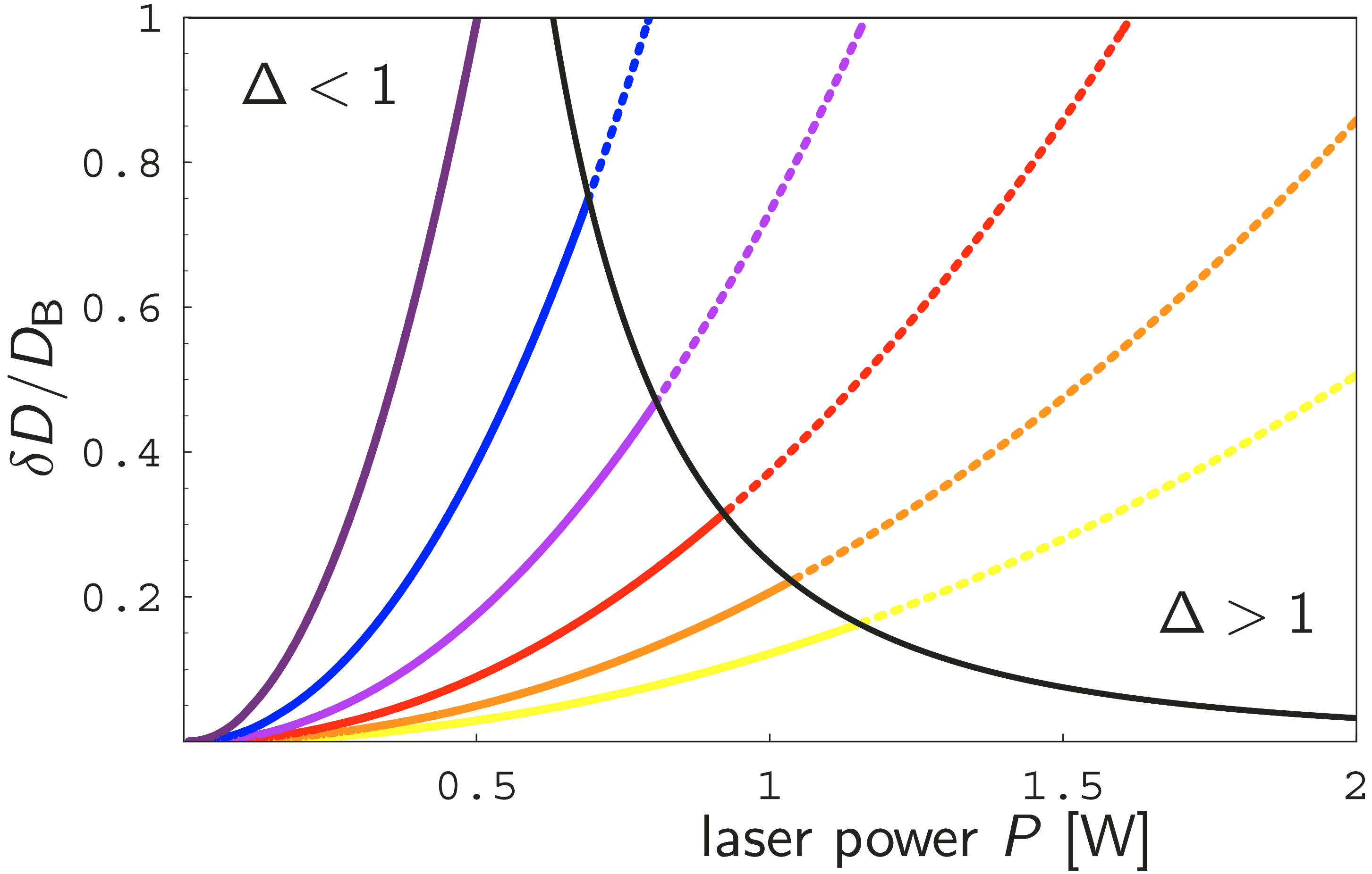
$$L_\phi = \sqrt{D_B/\gamma_{\text{inel}}}$$

- Transport quantities $\overline{V}, \ell_s, \ell_{tr}, D_B = f(P/\delta)$.

$\Rightarrow$ Change of intensity and detuning at fixed P/δ: controlled dephasing $L_\phi \propto \delta^2/P^{3/2}$ and $\dfrac{\mathrm{d}D}{\mathrm{d}P} > 0$

Weak localisation in speckle potentials

- Strong scattering ($k\ell_s$ small) in large sample ($L \gg \ell_{tr}$) under phase coherence ($L_\phi \geq L$) with limited (€) laser power P ?
- Size $L = 2\,$cm, detuning $\delta = 10^6\Gamma$, aperture $\alpha = 0.1$, atom wave numbers $k\xi =$ **1.25**, **1.5**, **1.75**, **2.0**, **2.25**, **2.5**

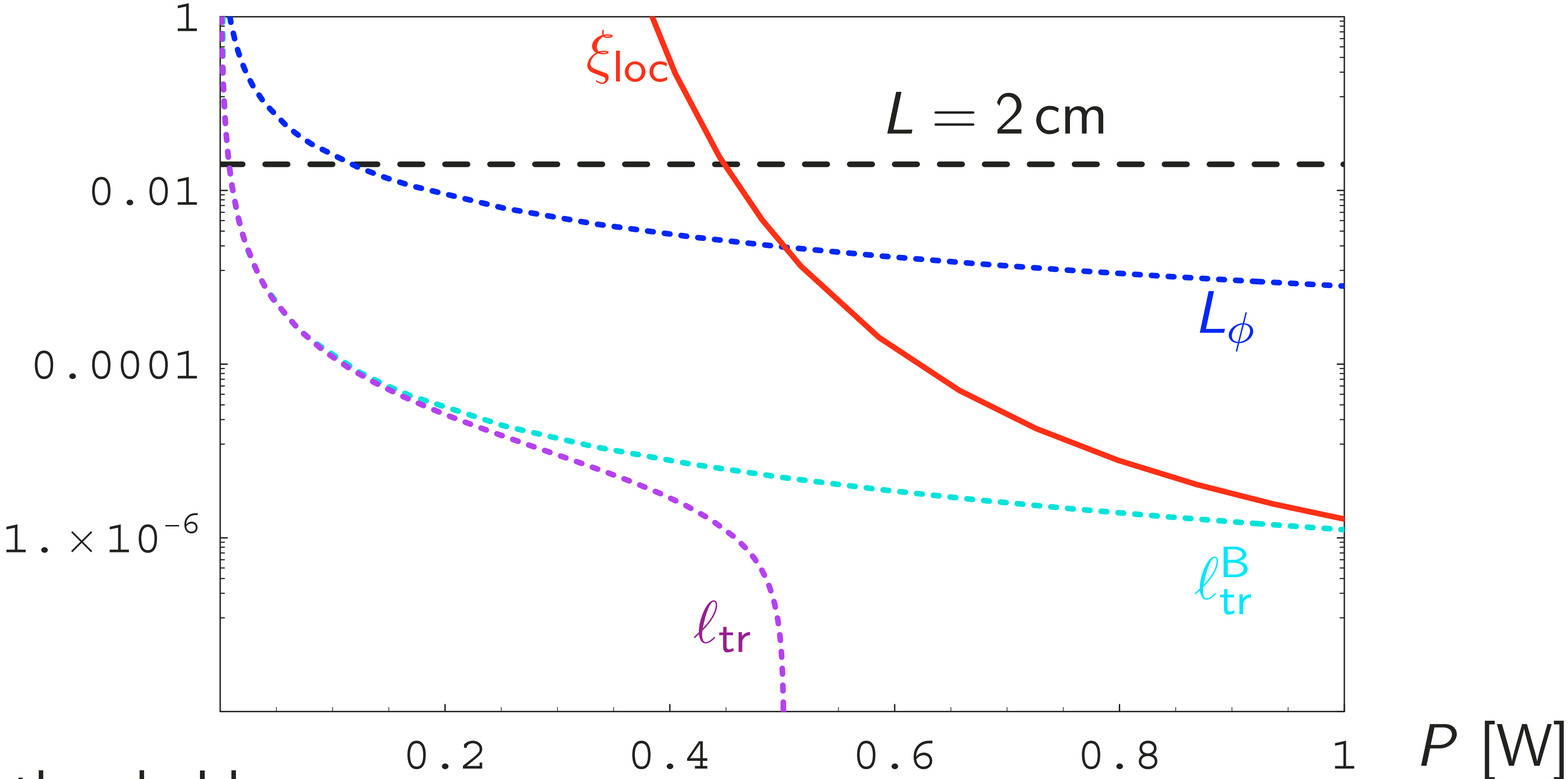


Towards strong localisation

- L_0 at the threshold $\delta D/D_B = 1$ is the localization length

$$\xi_{loc} = \ell_s \exp(\pi k \ell_{tr}/2)$$

For Rb^{87}, $k\xi = 1.25$, $\alpha = 0.1$, $\delta = 10^6\Gamma$:

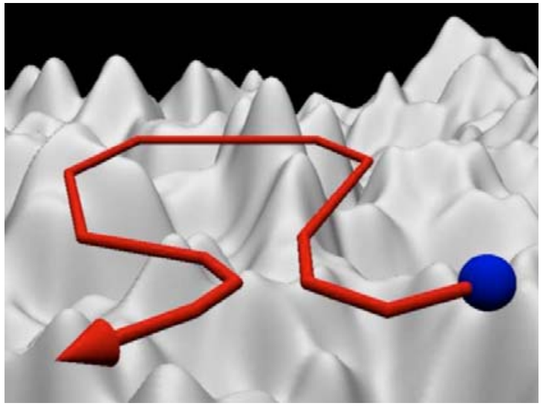


- At threshold:

$\xi_{loc} = 2\,\text{mm}$ $\quad \ell_{tr}^B = 5\,\mu\text{m}$ $\quad \ell_s = 1\,\mu\text{m}$

$k\ell_s = 1$ $\quad \overline{V}/E_\xi = 0.8 = \Delta$ $\quad E = \overline{V}$

Summary



- Multiple coherent scattering of cold matter waves in optical speckle potentials
- Sizable weak localization corrections. Strong localization reachable.
- Open issues:
  * Realistic thermal wavepackets or condensed clouds (no Fermi surface!)
  * Self-consistency in anisotropic case
  * Only perturbative predictions. What happens in the critical regime? Numerics, field theory, . . .
  * Add interactions !

Acknowledgements

This work: cond-mat/0506371

Robert Kuhn (Bayreuth/Nice)
Christian Miniatura (Nice)
Dominique Delande (Paris)
Olivier Sigwarth (Bayreuth)
Cord Müller (Bayreuth)

$\overleftrightarrow{h}_{\lambda\mu}$ *Quantum Transport of Light and Matter*

€ DFG: SPP 1116 “Ultrakalte Gase”

€ BFHZ-CCUFB, DAAD

UNIVERSITÄT BAYREUTH