The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR 1669 - 4

CONFERENCE ON STRONGLY INTERACTING SYSTEMS AT THE NANOSCALE
8 - 12 August 2005

Superfluid regimes in Fermi gases

Gora Shlyapnikov
LPTMS, Orsay, France
dan
University of Amsterdam, Netherlands

Superfluid regimes in Fermi gases

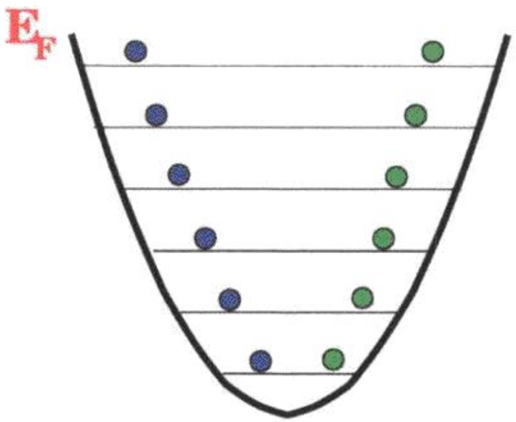
Gora Shlyapnikov
LPTMS, Orsay, France
University of Amsterdam

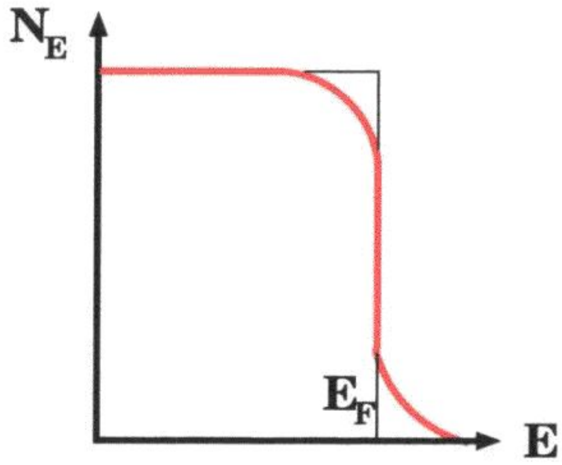
Outline

- Introduction. BCS-BEC crossover
- Strongly interacting regime
- Molecular BEC regime. Interaction between molecules
- Collisional relaxation
- Pauli principle and effect of the mass ratio
- Ideas for future and conclusions

Collaborations: D.S. Petrov (Harvard), C. Salomon (ENS)

Two-component trapped Fermi gas





$$E_F = \frac{\hbar^2 k_F^2}{2m}; \quad k_F = (3\pi^2 n)^{1/3}; \quad E_F \sim N^{1/3}\hbar\omega$$

Weakly interacting gas $n|a|^3 \ll 1;\ k_F|a| \ll 1$

$a < 0 \rightarrow$ Interspecies attraction $\rightarrow$ Cooper pairing at low T

$\vec{\mathbf{k}}$ ● ● $-\vec{\mathbf{k}}$

Superfluid BCS transition $\rightarrow T_c \sim E_F \exp\{-\pi/2k_F|a|\}$

$T_c \ll 0.1 E_F$ for ordinary a Very hard to reach

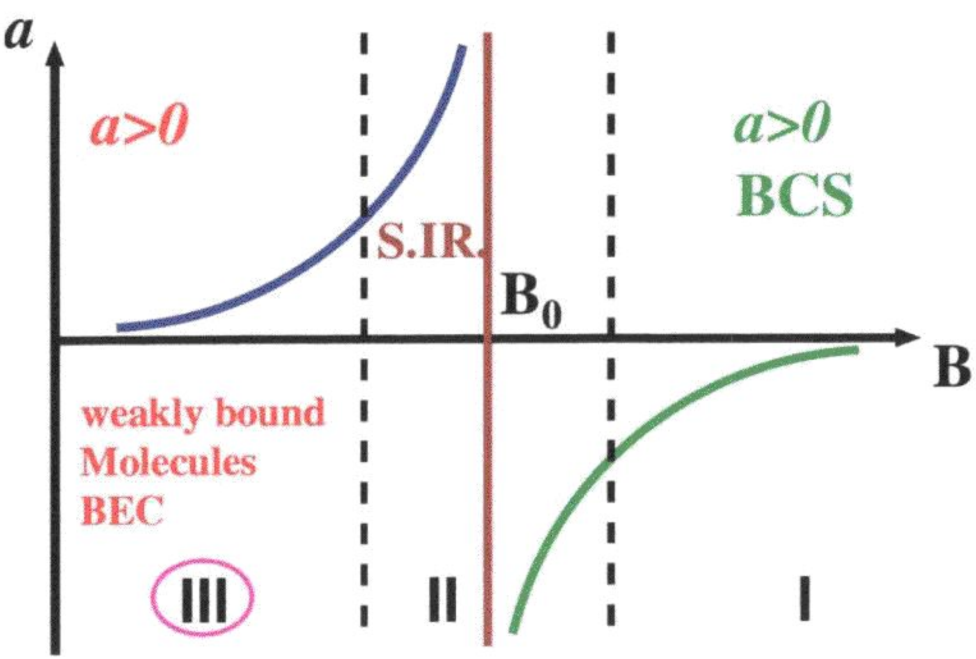
Experiments ^{40}K ^{6}Li

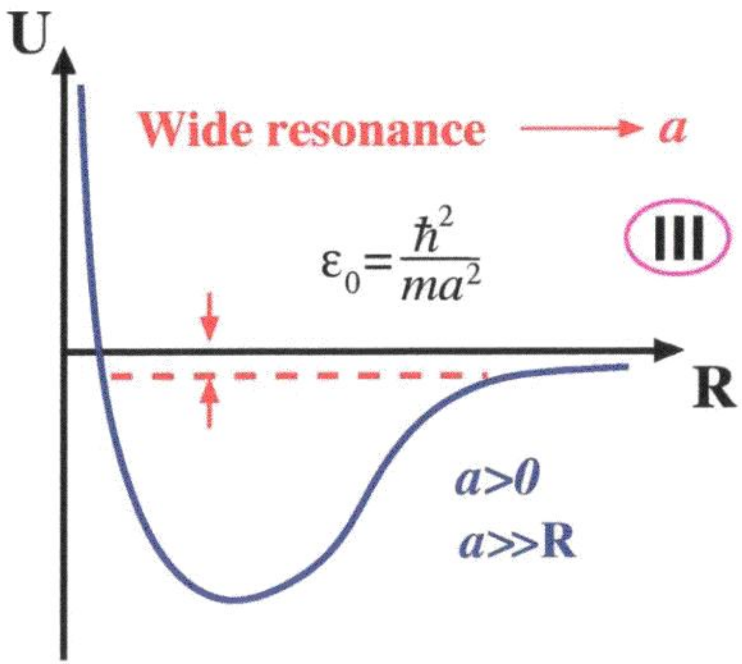
Dilute limit $nR_e^3 \ll 1$ Ultracold limit $\Lambda_T \gg R_e$

Quantum degeneracy $\rightarrow$ JILA 1998 ^{40}K

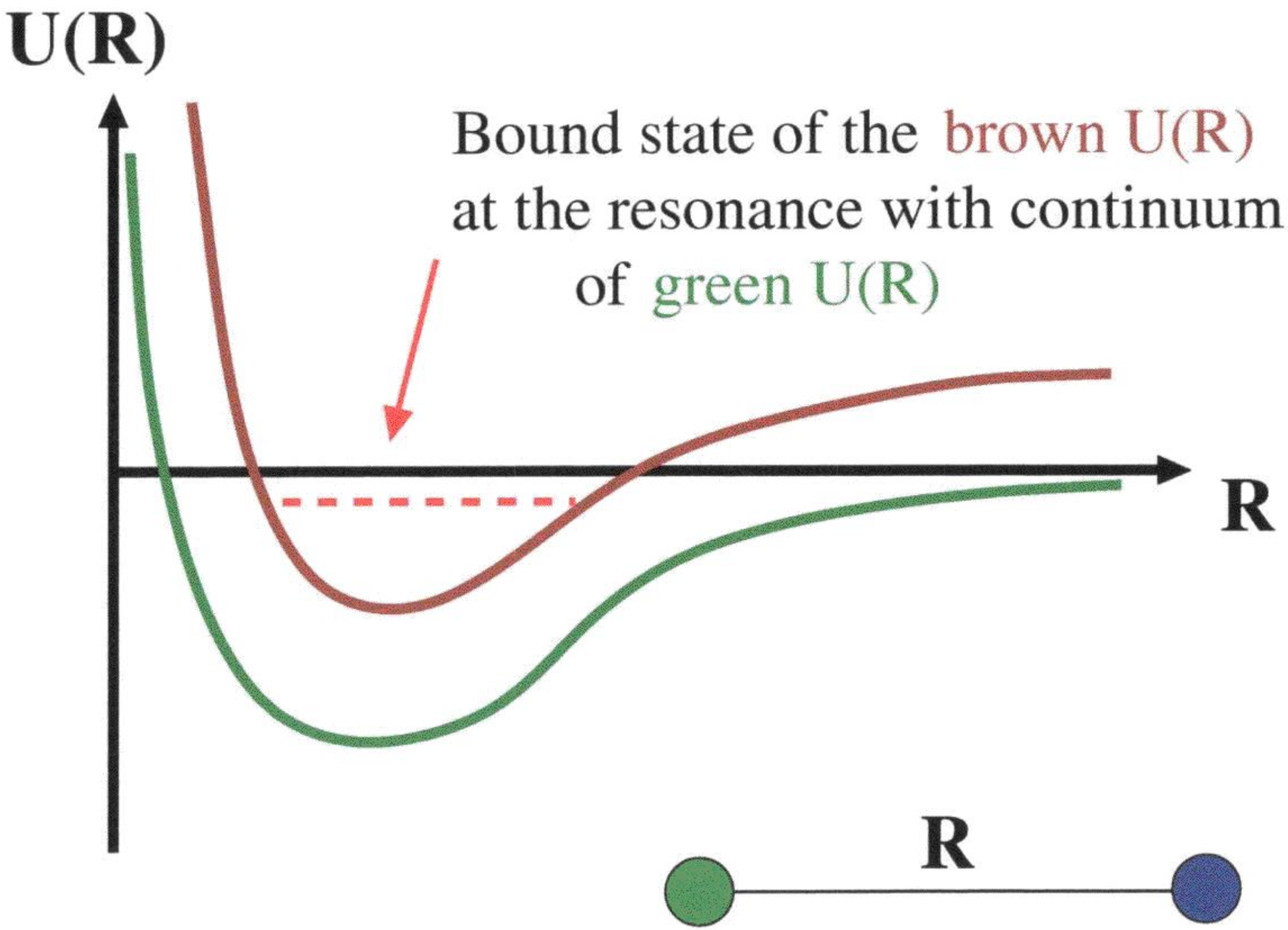
At present $n \sim 10^{13} - 10^{14} \mathrm{cm}^{-3}$; $T \sim 1\mu$K

JILA, LENS Innsbruck,MIT,ENS,Rice,Duke, ETH, Hamburg, Tuebingen





Feshbach resonance



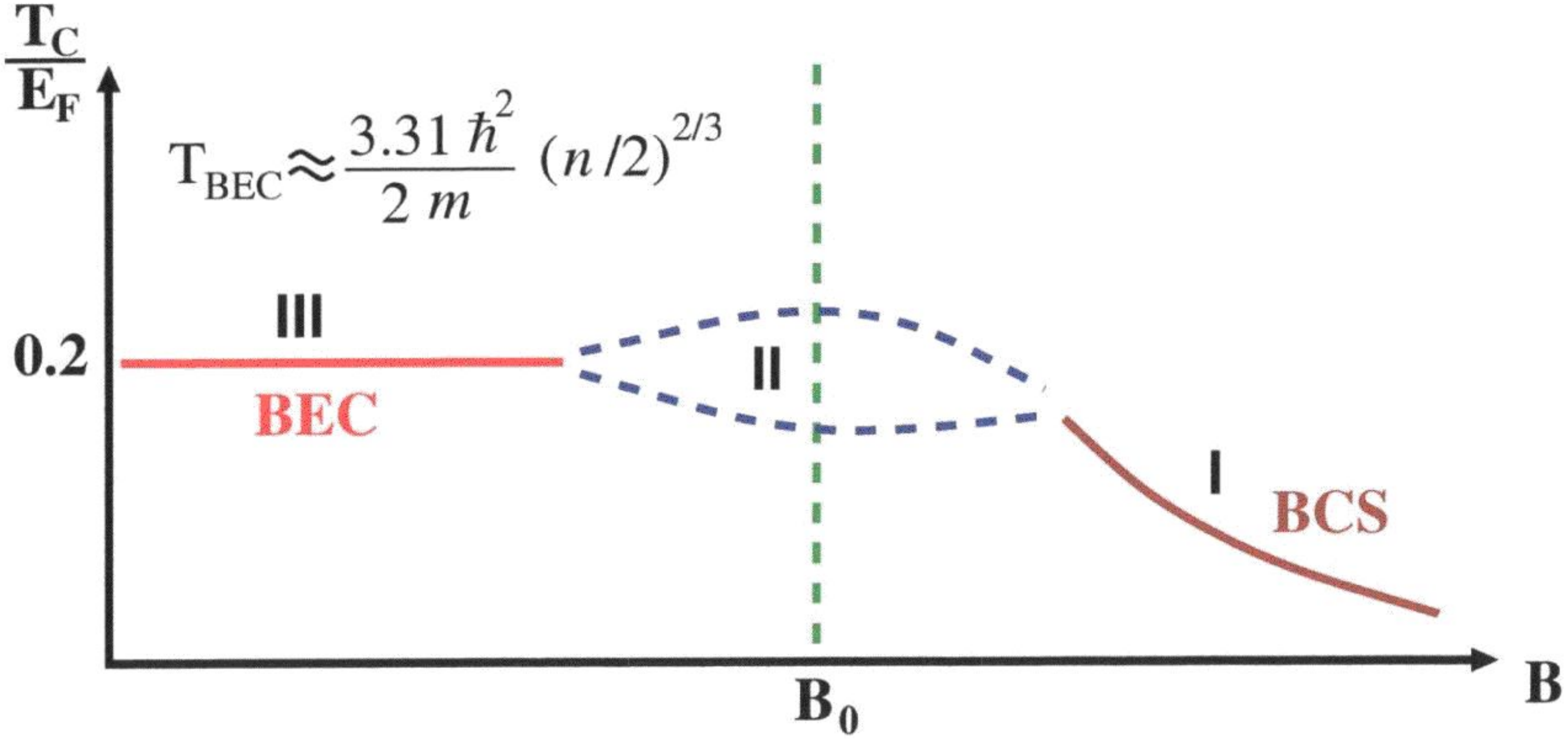
Superfluid regimes

I $k_F|a| \ll 1 \rightarrow$ **BCS**

II $k_F|a| > 1 \rightarrow$ **Strongly interacting regime**

III $na^3 \ll 1 \rightarrow$ **Gas of bosonic molecules**

$a \gg R_e \rightarrow$ BEC of weakly bound molecules



BCS-BEC crossover: Leggett, Nozieres-Schmitt-Rink

Strongly interacting regime

$T = 0 \quad k_F \gg 1 \quad \rightarrow$ Only one distance scale $n^{-1/3}$

Only one energy scale $E_F \sim \hbar^2 n^2 / m$

Universal thermodynamis (J. Ho)

Monte Carlo studies $\rightarrow \quad \mu = \beta E_F$

(Carlson et al, Giorgini/Astracharchik, etc.)

Nature of superfluid pairing, Transition temperature, Excitations

(Holland, Timmermans, Ho, Griffin, Stringari, Strinati, Tosi, Bulgac, Levine, Pethick, Bruun, Stoof, Cheng, etc.) $\rightarrow$ theory

Experiments

BEC-type behavior of fermionic atom pairs (JILA, MIT)

Excitation frequencies and damping rates (Innsbruck, Duke)

Pairing gap (Innsbruck), Heat capacity (Duke)

Correlations (JILA)

Vortices (MIT)

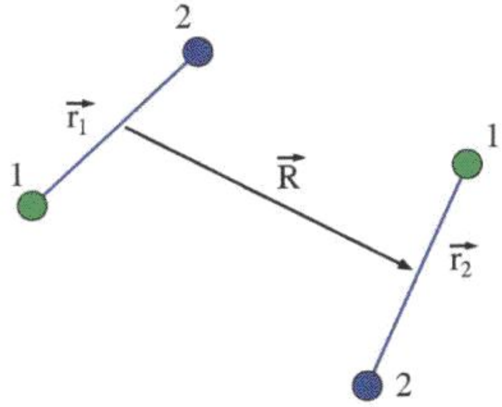
Weakly interacting gas of bosonic dimers

Elastic interaction BEC stability "Old answer" $\to 2a$

4-body problem **Exact solution for** $a \gg R_e$ (Petrov et al 2003)

$\Psi \to$ 9 variables

Zero-range approximation



$\Psi_{r_1 \to 0} \to f(\vec{r}_2, \vec{R})(1/4\pi r_1 - 1/4\pi a)$

Integral equation for f $k \to 0$ s-wave scattering; 3 variables

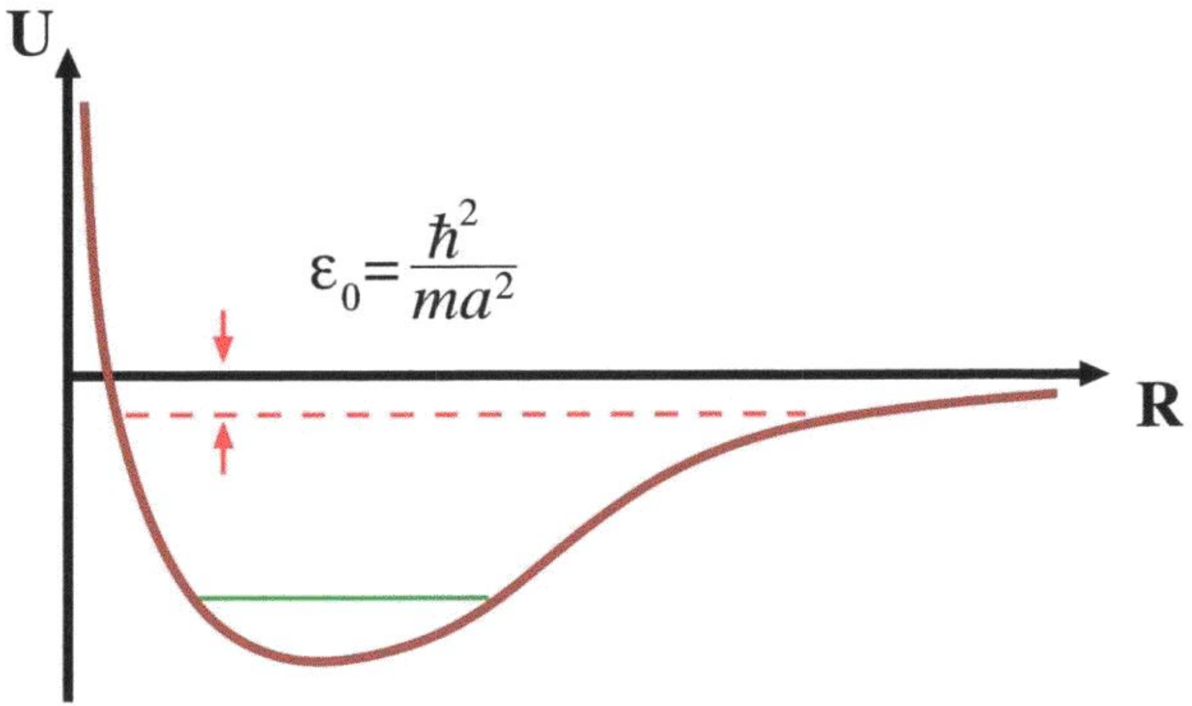
$$R \to \infty \quad \Psi = \phi_0(r_1)\phi_0(r_2)(1 - a_{dd}/R); \;\; \phi_0(r) = \frac{1}{\sqrt{2\pi a}} \exp(-r/a)$$

$$a_{dd} = 0.6a$$

Monte Carlo (Giorgini/Astracharchik, 2004)
Diagrammatic approach (M.Kagan et al, 2005)

Weakly bound dimers

Weakly bound dimers $\rightarrow$ The highest rovibrational state of the diatomic molecule



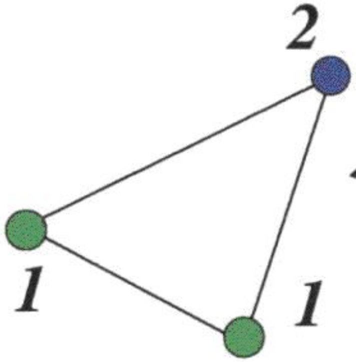
Collisional relaxation to deep bound states
(~ 1ms for Rb_2 at $n \sim 10^{13}$cm^{-3})

Atom-dimer collisions

Weakly bound dimer $\sim a$

Size $\rightarrow$

Deep bound state $\sim R_e$ (50 Å) $\ll a$

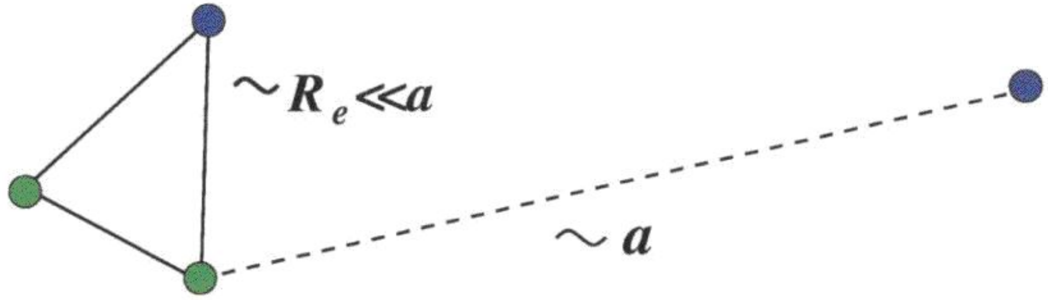


$\sim R_e$ 2 particles are identical fermions

Pauli principle

$$\alpha_{rel} \sim (k_{eff} R_e)^{2?} \sim (R_e/a)^{2?}$$

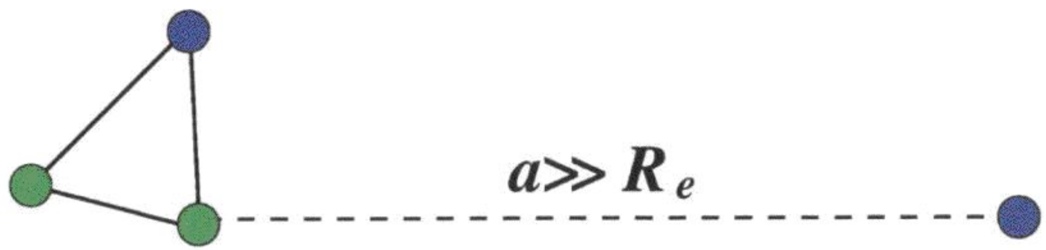
Molecule-molecule relaxation collisions



$$\alpha_{rel} = C\frac{\hbar R_e}{m}\left(\frac{R_e}{a}\right)^s; \quad s = 2.55$$

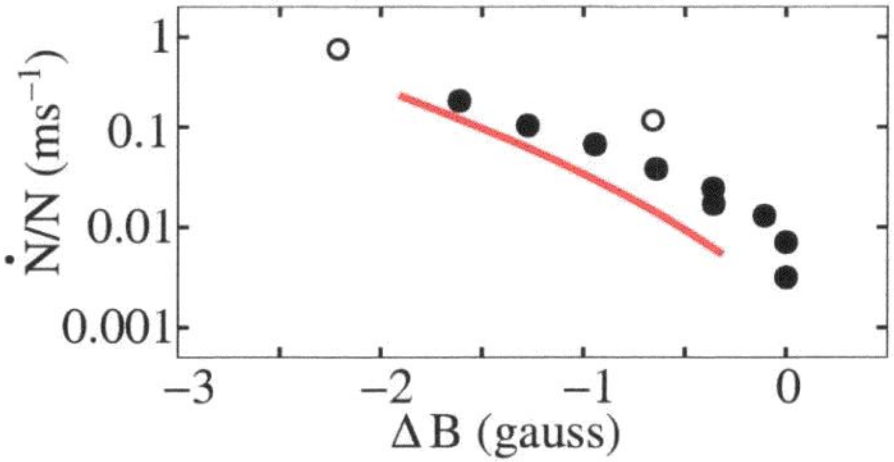
$\tau \sim (\alpha_{rel} n)^{-1} \sim$ seconds (Petrov et al 2003)

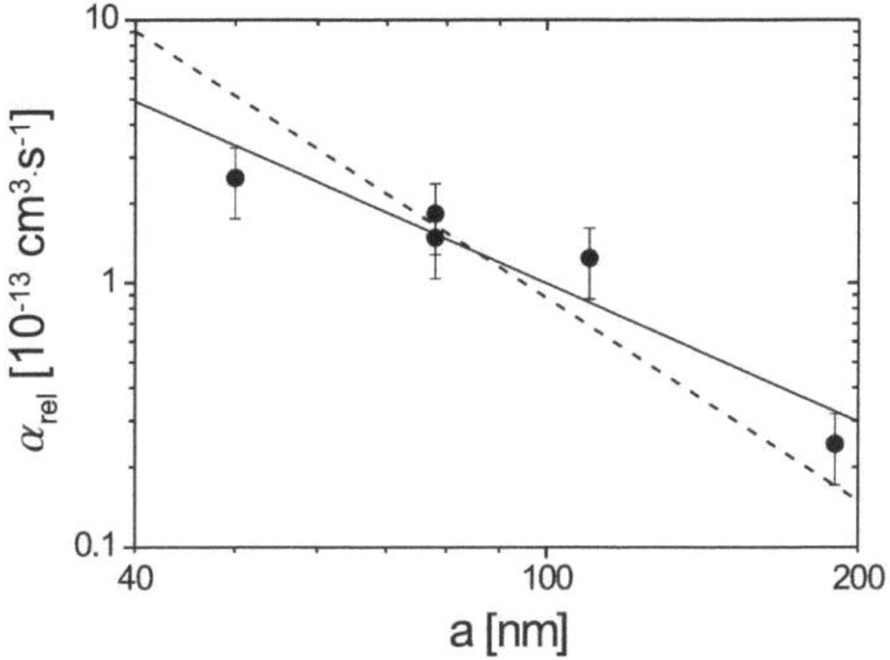
Molecules of bosonoc atoms



Resonant enhancement $\alpha_{rel} \sim \hbar a/m$ $\tau < 1$ms

Suppressed collisional relaxation





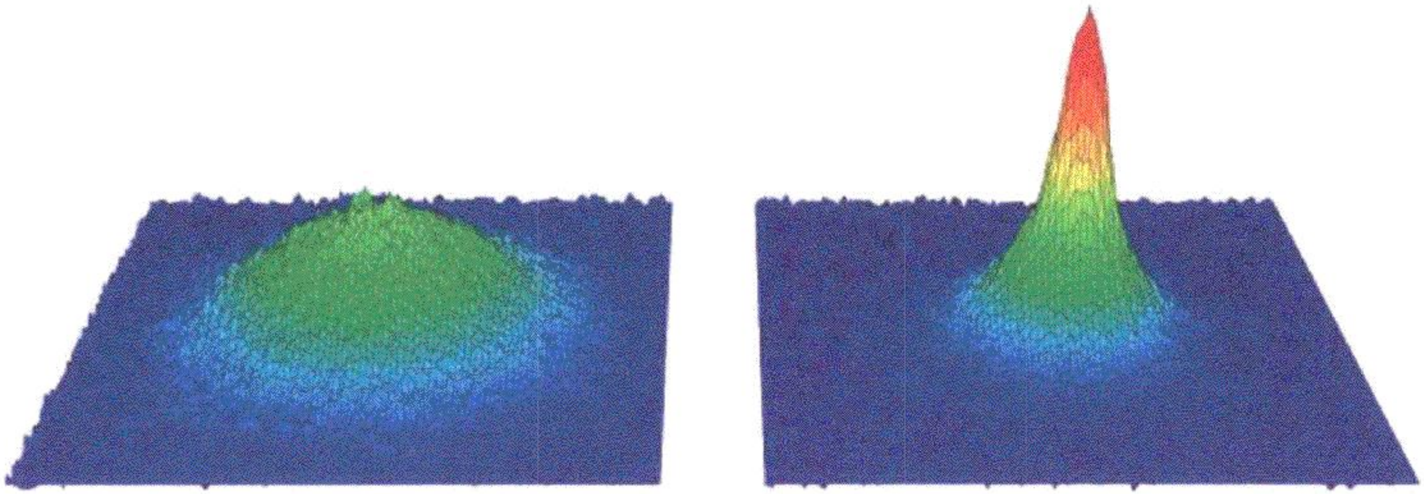
Bose-Einstein condensates of molecules

Suppressed relaxation **Fast elastic collisions** $a_{dd} = 0.6a$

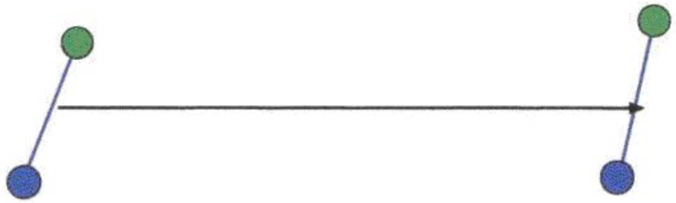
$$^6\mathrm{Li}_2 \rightarrow \frac{\alpha_{rel}}{\alpha_{el}} \leq 10^{-4}$$

Efficient evaporative cooling $\rightarrow$ BEC

JILA, Innsbruck, MIT, ENS, Rice

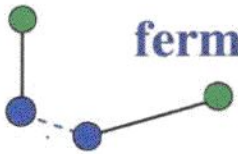


Composite bosons



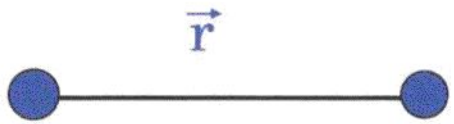
Bosonic behavior at large separations **BEC**

Small separations ?



fermionic correlations!

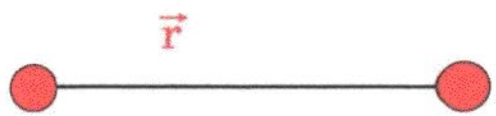
$$\Psi \propto (\exp\{i\vec{k}\vec{r}\} - \exp\{-i\vec{k}\vec{r}\})$$
$$\to 2\vec{k}\vec{r} \quad \text{for } kr \ll 1$$



$$|\Psi|^2 \propto k^2$$

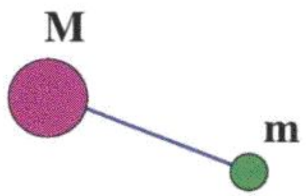
$$\Psi \propto (\exp\{i\vec{k}\vec{r}\} + \exp\{-i\vec{k}\vec{r}\})$$
$$\to \text{const for } kr \ll 1$$

bosons



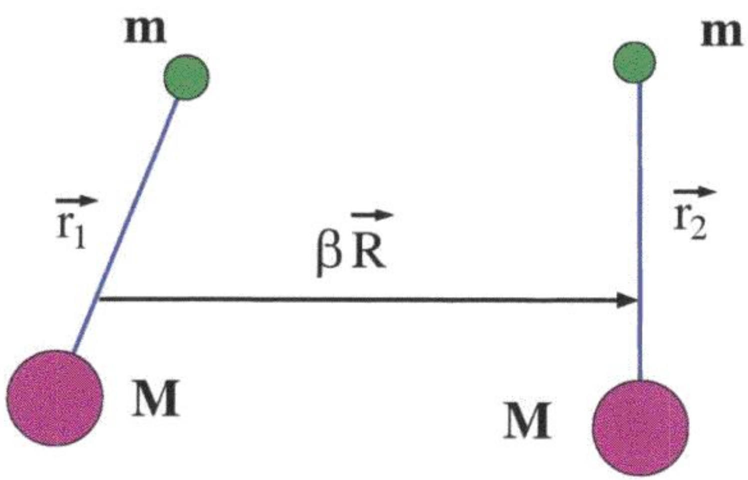
Mixtures of Fermi gases

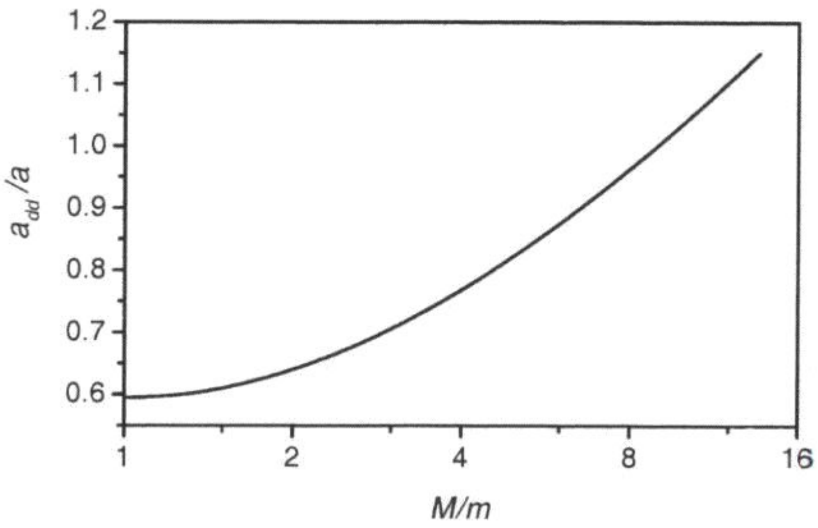
Molecules of different fermionic atoms $^{6}\mathrm{Li}^{40}\mathrm{K}$ $^{6}\mathrm{Li}^{87}\mathrm{Sr}$



What happens with collisional stability and molecular BEC?

Interaction between the molecules (a_{dd}) Petrov et al 2005



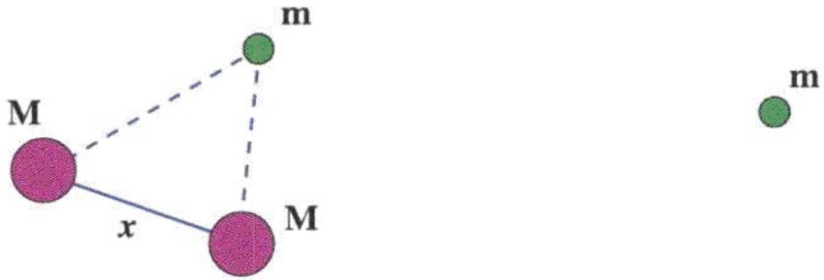


$$\beta = \sqrt{Mm/(M+m)}$$

Collisional relaxation

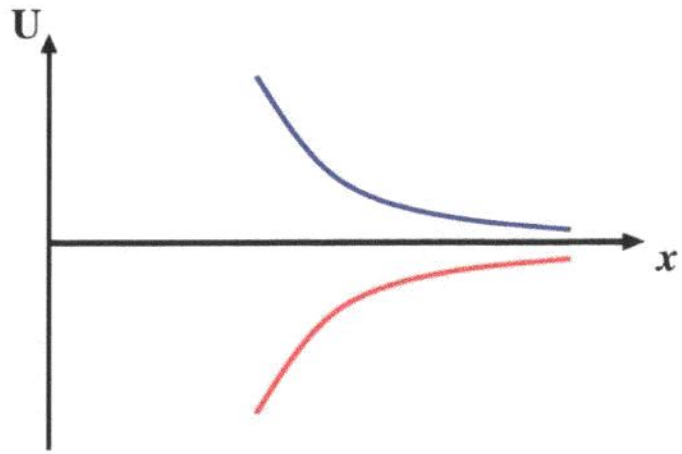
Exact solution for the dependence on a and M/m

$M \gg m \quad \rightarrow \quad$ Born-Oppenheimer picture



$x \ll a \rightarrow U_{eff} \approx -0.16\hbar^2/mx^2$

Centrifugal potential $U_c = 2\hbar^2/Mx^2$

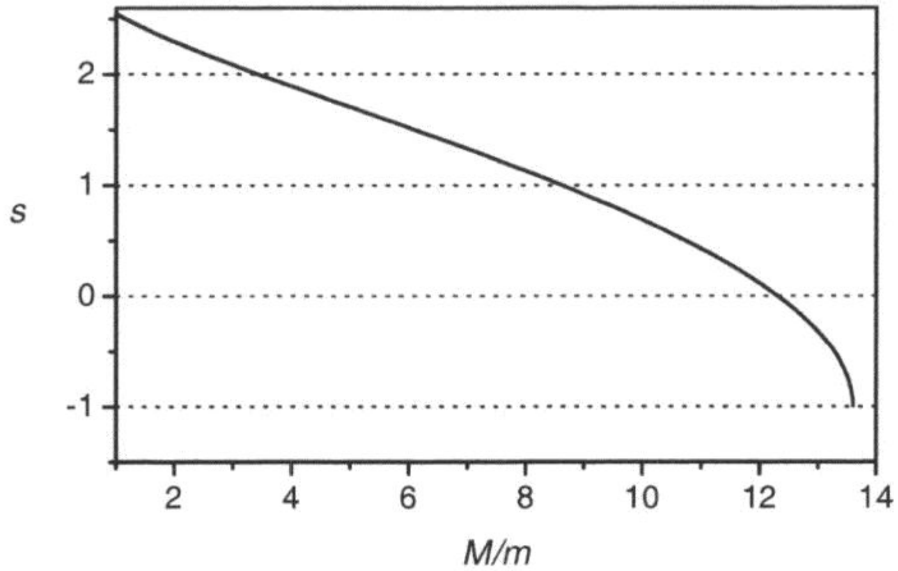


Mediated attraction competes
with Pauli principle

$$\alpha_{rel} \sim a^{-s}$$

s decreases with increasing M/m

Relaxation rate



$$\alpha_{rel} \sim \frac{1}{a^s}$$

$M/m > 12.33 \;\rightarrow\; s < 0 \;\rightarrow\; \alpha_{rel}$ increases with a

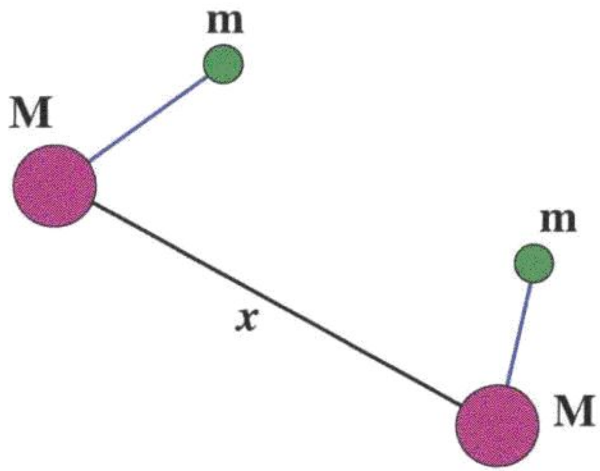
$M/m = 13.6 \;\rightarrow\; s = -1 \;\rightarrow\; \alpha_{rel} \sim a$

$M/m > 13.6 \;\rightarrow$ fall into center short-range physics

Suppression of relaxation ?

There is a suppression of relaxation for a large M/m

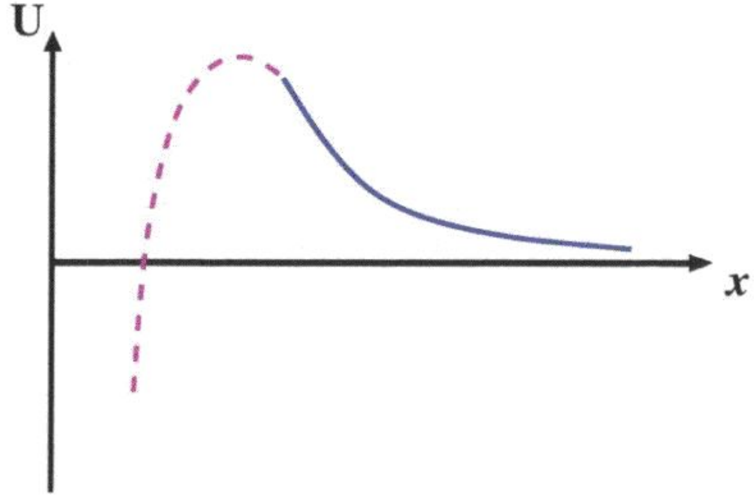
Born-Oppenheimer picture



$x \gg a$

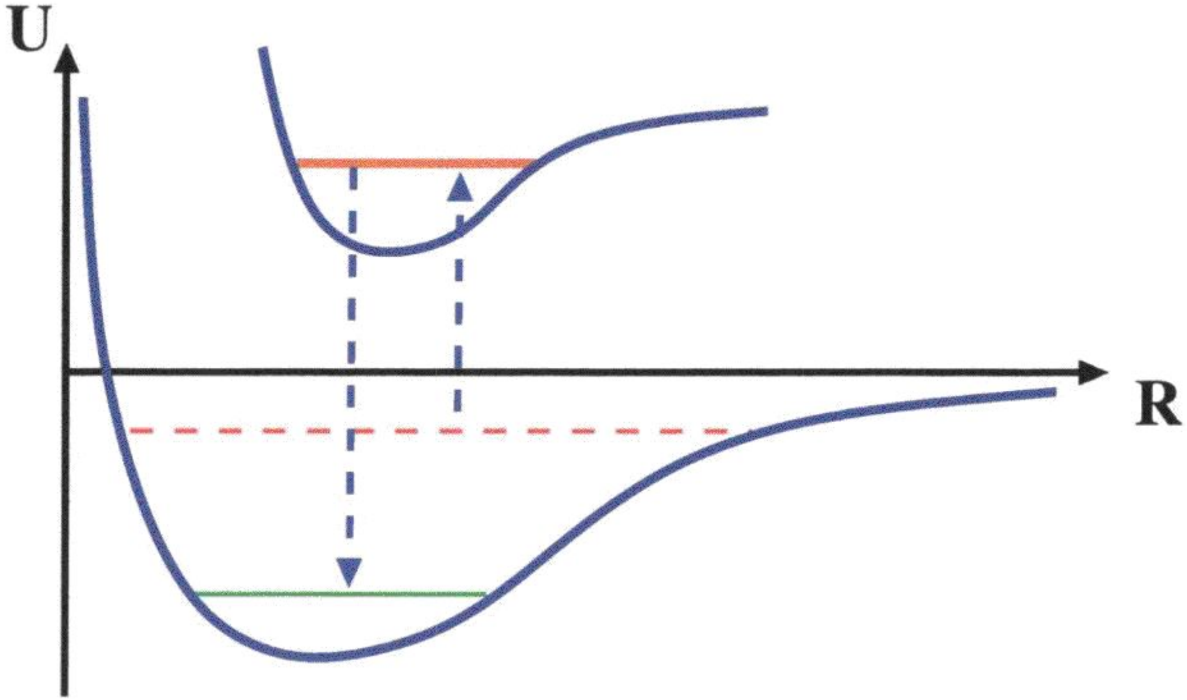
$U_{eff}(x) \approx (\hbar^2/2m)\exp(-2x/a)$

$P \sim \exp(-0.6\sqrt{M/m})$



Ideas for future

Idea from Yalle studies of molecules of bosonic atoms



Replace bosons by fermions

Large n Large τ

$\rightarrow$ Dipolar gas

Conclusions

- Remarkable physics of weakly bound molecules in cold Fermi gases
- Novel physics of molecular collisional stability in mixtures of Fermi gases
- Possibilities to create new macroscopic quantum systems