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INTRODUCTION TO MICROFLUIDICS

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Hydrodynamics of Complex Fluids in Micro-Channels

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Hydrodynamics of complex fluids in micro-channels.

(limit of hydrodynamic description, statistical fluctuations, dynamic similarity, local dynamics versus global, hydrodynamics of polymer solutions, elastic instability and Elastic turbulence, etc)

Lecture 1

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Basic concepts and dynamic similarity in micro-flows.

Main characteristics of gas flow (hard sphere molecules):

- mean molecular diameter- $d \approx 4 \cdot 10^{-4}\ \mu m$
- mean molecular spacing- $\delta \cong n^{-1/3} \approx 3.5 \cdot 10^{-3}\ \mu m$
- mean free path- $\lambda \equiv (2^{1/2} \pi d^2 n)^{-1} \approx 6.5 \cdot 10^{-2}\ \mu m$

(for air at normal conditions)

$\delta / d >> 1$ -dilute gas limit

$\bar{c} = (3P/\rho)^{1/2} = (3RT)^{1/2} \approx 486 m/s$

Limitations of continuum-based description (e.g. hydrodynamic approach)

1. How small should a sample size be so that we can still assign it mean properties?

2. At what scales will the statistical fluctuations be significant?

Sampling a volume that contains 10 000 molecules results in 1% statistical fluctuations in averaged quantities, i.e. for air at standard conditions the smallest sample volume that will result in 1% statistical variations is about $3.7 \cdot 10^{-4}\ \mu m^3$

or about 65 nm on a side.

Rarefaction effects

A key non-dimensional parameter that is defined the importance of the rarefaction effects is

$$Kn \equiv \frac{\lambda}{L} \propto \frac{Ma}{\mathrm{Re}}; Ma \equiv \frac{V}{c_s} \approx \left(\frac{6}{5}\right)^{1/2} \frac{V}{\bar{c}}; \mathrm{Re} \equiv \frac{VL}{\nu}$$

In complex micro geometries where 3D spatial gradients are expected, definitions of instantaneous macroscopic values and their gradients become problematic for flows with ***Kn>1*** as the concept of macroscopic property distribution breaks down

Classification of different flow regimes:

- for ***Kn < 0.01*** the fluid can be considered as a continuum
- for ***Kn > 10*** it is considered a free-molecular flow.
- slip flow (0.01< ***Kn*** <0.1) and transition flow (0.1< ***Kn*** <10)

Concept of dynamical similarity

- Are the low-pressure rarefied gas flows *dynamically similar* to the gas micro flows?

The answer depends on the onset of statistical fluctuations and on wall surface effect.

Examples

Statistical fluctuations are significant for $L/\delta \le 20$

Below- the statistical fluctuations exceed 1%

1. At normal conditions for air ***Kn=1*** is obtained at 65 nm $(\lambda \cong 65nm)$ length scale; at this scale statistical fluctuations are below 1%
2. At P=100Pa 1% fluctuations limit sets at L=0.65 μm but ***Kn=1*** corresponds to L=65 μm , i.e. macroscopic properties can be defined without any significant fluctuations though description is kinetic
3. For dense gas flow regime N-S equations are valid but the fluctuations are large at ***Kn>0.1*** (or at scale below 0.1 μm)

Micro-flows of complex fluids

- Hydrodynamic description is broken at larger scales due to lower density of fluid constituents (both λ and δ are inversely proportional to particle density n in power “-1” and “-1/3”, respectively)
- Statistical fluctuations are more significant for the same reason
- In addition internal local dynamics of constituents that is specific for each complex fluid affect rheology (or relation between the stress and the rate of strain) in this fluid

Hydrodynamics of a polymer solution in micro-channels as a canonical example of a complex fluid micro-flow

- Non-Newtonian fluid mechanics
- Elastic instabilities at negligible ***Re***
- Random flows in micro-flows and role of local dynamics of a single polymer molecule

Two aspects of polymer hydrodynamics

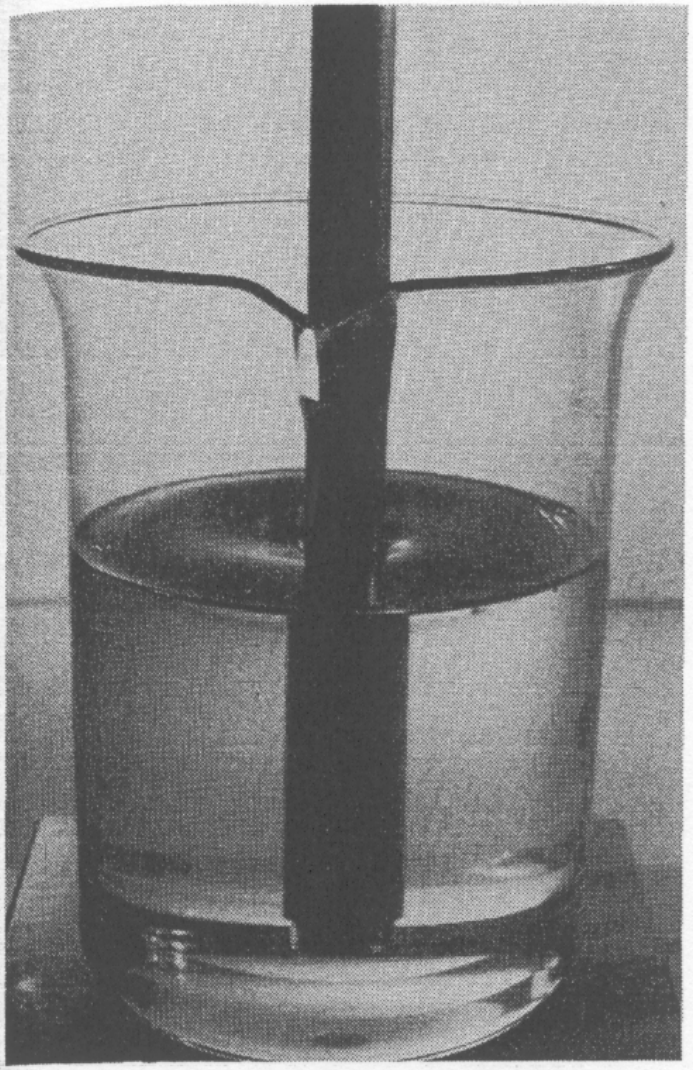
1. Influence of polymers on flow stability, flow structure and statistics

2. Influence of flow on polymer dynamics, conformations and statistics

Main aspects of polymer hydrodynamics:

short course

- Polymer stretching and elastic stresses
- Role of elastic stresses and elastic non-linearity on flow stability
- Elastic turbulence as an example of strong influence of local dynamics on flow properties



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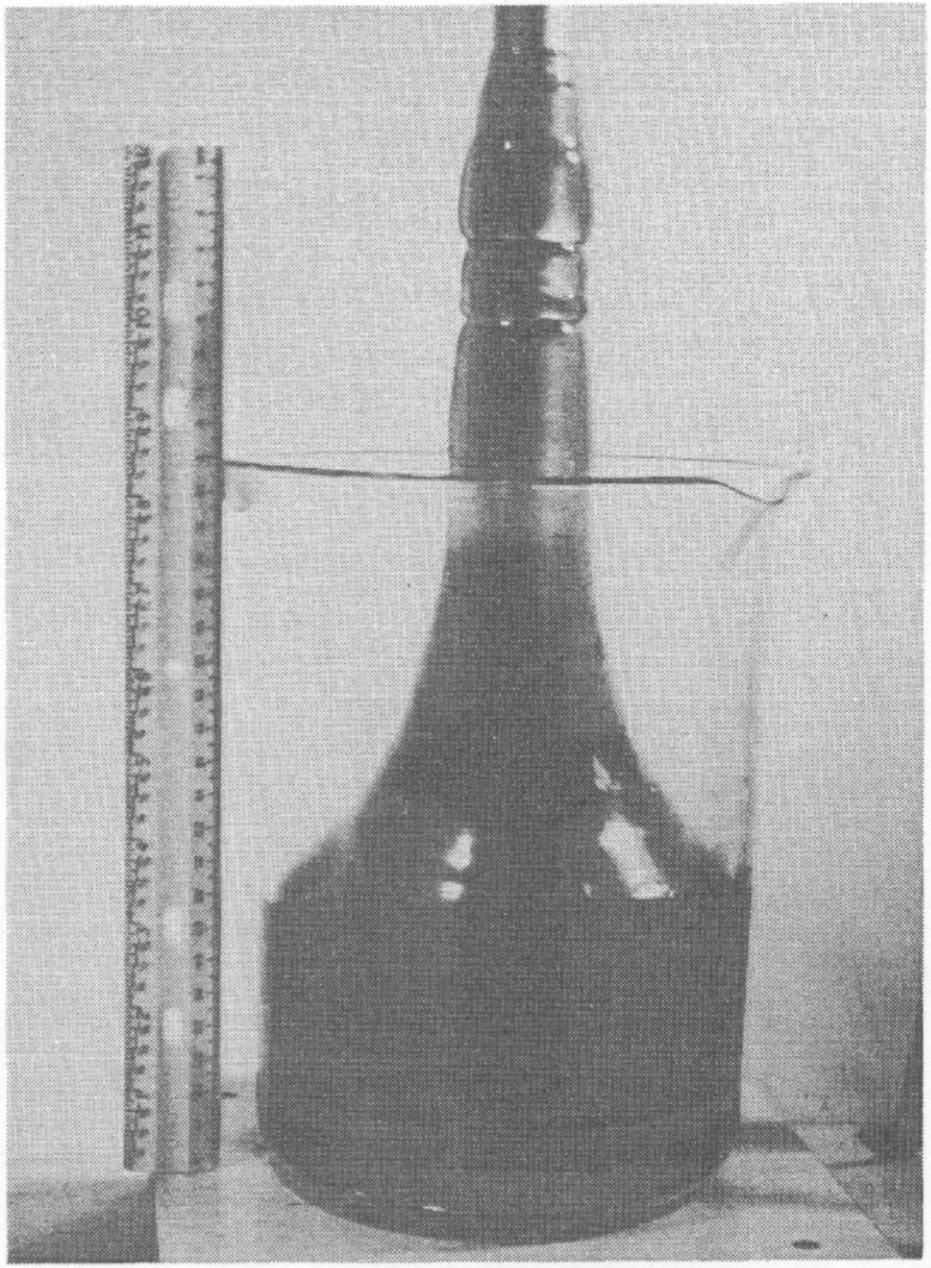

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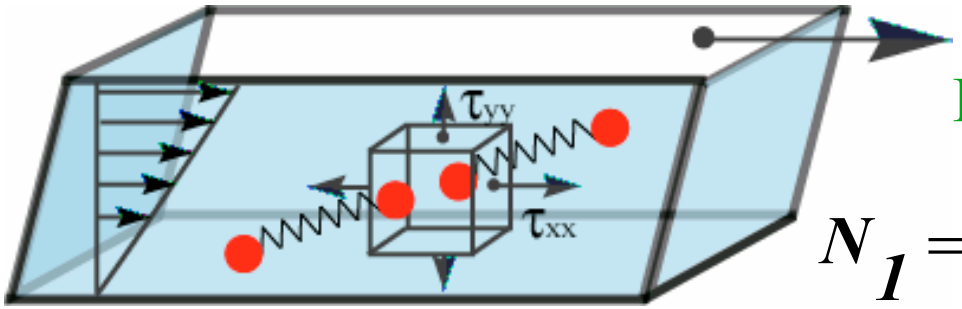
Polymer molecules in a shear flow.

Elastic dumbbell model:

$\lambda \propto \eta R/k$ **-polymer relaxation time**

η-viscosity, R-molecule size; k~1/R^2-spring constant.



First normal stress difference:

$N_1 = \tau_{xx} - \tau_{yy} \propto (\lambda\dot{\gamma})^2$, $\dot{\gamma}$ - shear rate.

Hydrodynamics of polymer solutions

Equation of motion:

$$\frac{\partial \vec{V}}{\partial t} + \underbrace{(\vec{V}\,\vec{\nabla})\vec{V}}_{\text{Inertial nonlinearity}} = -\vec{\nabla}p/\rho - \underbrace{\vec{\nabla}\tilde{\tau}/\rho}_{\text{linear}}$$

Constitutive equation for Oldroyd-B model

- Full stress tensor- $\tilde{\tau} = \tilde{\tau}_S + \tilde{\tau}_P$
- For solvent part- $\tilde{\tau}_S = -\eta_S\left[\vec{\nabla}\vec{V} + (\vec{\nabla}\vec{V})^T\right]$
- For polymer part $\tilde{\tau}_P + \lambda\frac{D\tilde{\tau}_P}{Dt} = -\eta_P\left[\vec{\nabla}\vec{V} + (\vec{\nabla}\vec{V})^T\right]$

Linear relax

and convective derivative:

$$\frac{D\tilde{\tau}_P}{Dt} \equiv \frac{\partial \tilde{\tau}_P}{\partial t} + \underbrace{(\vec{V}\,\vec{\nabla})\tilde{\tau}_P - \tilde{\tau}_P(\vec{\nabla}\vec{V}) - (\vec{\nabla}\vec{V})^T\tilde{\tau}_P}_{\text{elastic nonlinearity}}$$

$$\mathrm{Wi} = V\lambda/L = \frac{\text{nonlinearity}}{\text{relaxation}} \quad \text{or} \quad \frac{N_1(\dot{\gamma})}{\tau(\dot{\gamma})}$$

Weissenberg number

$$\mathrm{Re} = VL/\nu = \frac{\text{nonlinearity}}{\text{dissipation}}$$

Reynolds number

For Oldroyd-B model

$$N_1 \equiv \tau_{11} - \tau_{22} = -2\eta\lambda_d\dot{\gamma}^2$$

$$\tau \equiv \tau_{21} = -\eta\dot{\gamma}$$

$$Wi \equiv \frac{N_1}{\tau} = 2\lambda_d\dot{\gamma}$$

λ_d is the dumbbell (or Maxwell) relaxation time that can be considerably smaller than the longest polymer relaxation time λ

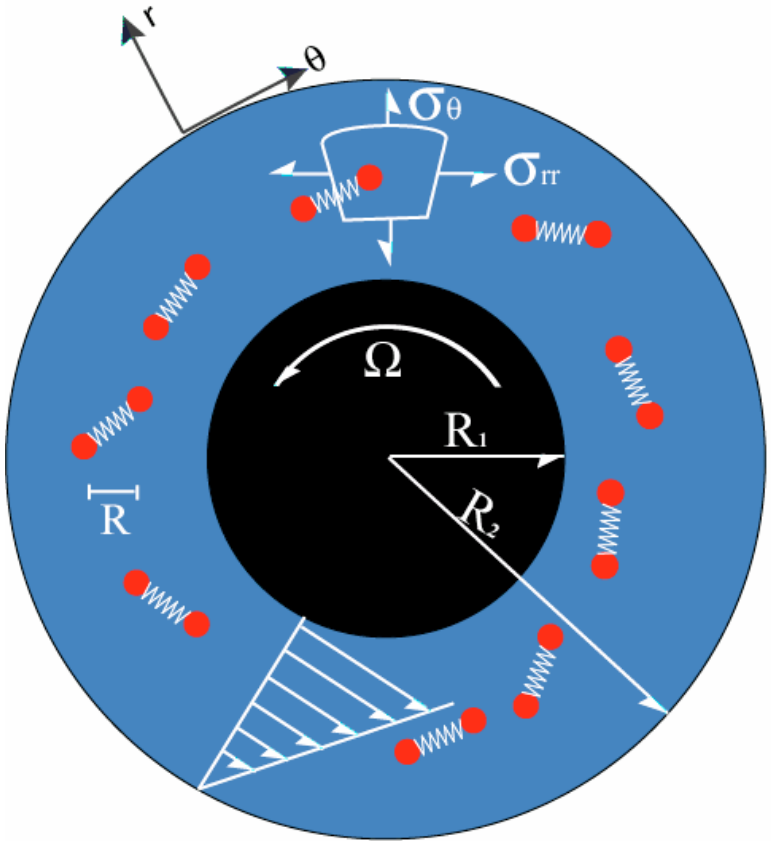
Three limiting cases:

1) $\mathrm{Re} >> 1; Wi \Rightarrow 0$ **Hydrodynamic turbulence**

2) $Wi >> 1; \mathrm{Re} \Rightarrow 0$ **Elastic turbulence**

3) $\mathrm{Re} >> 1; Wi >> 1$ **Turbulent drag reduction**

Rod Climbing (Weissenberg) Effect.



Volume force: $\frac{N_1}{r}$

(hoop stress).

$$N_1 = \sigma_{\theta\theta} - \sigma_{rr} \propto \langle R^2 \rangle Wi^2$$

$$Wi = \lambda \dot{\gamma}, d = R_2 - R_1$$

$$\dot{\gamma} = \Omega \frac{R_1}{d}$$ **-shear rate.**

Criterion of elastic instability in the framework of Oldroyd-B model

(Larson, Shaqfeh, Muller,1990).

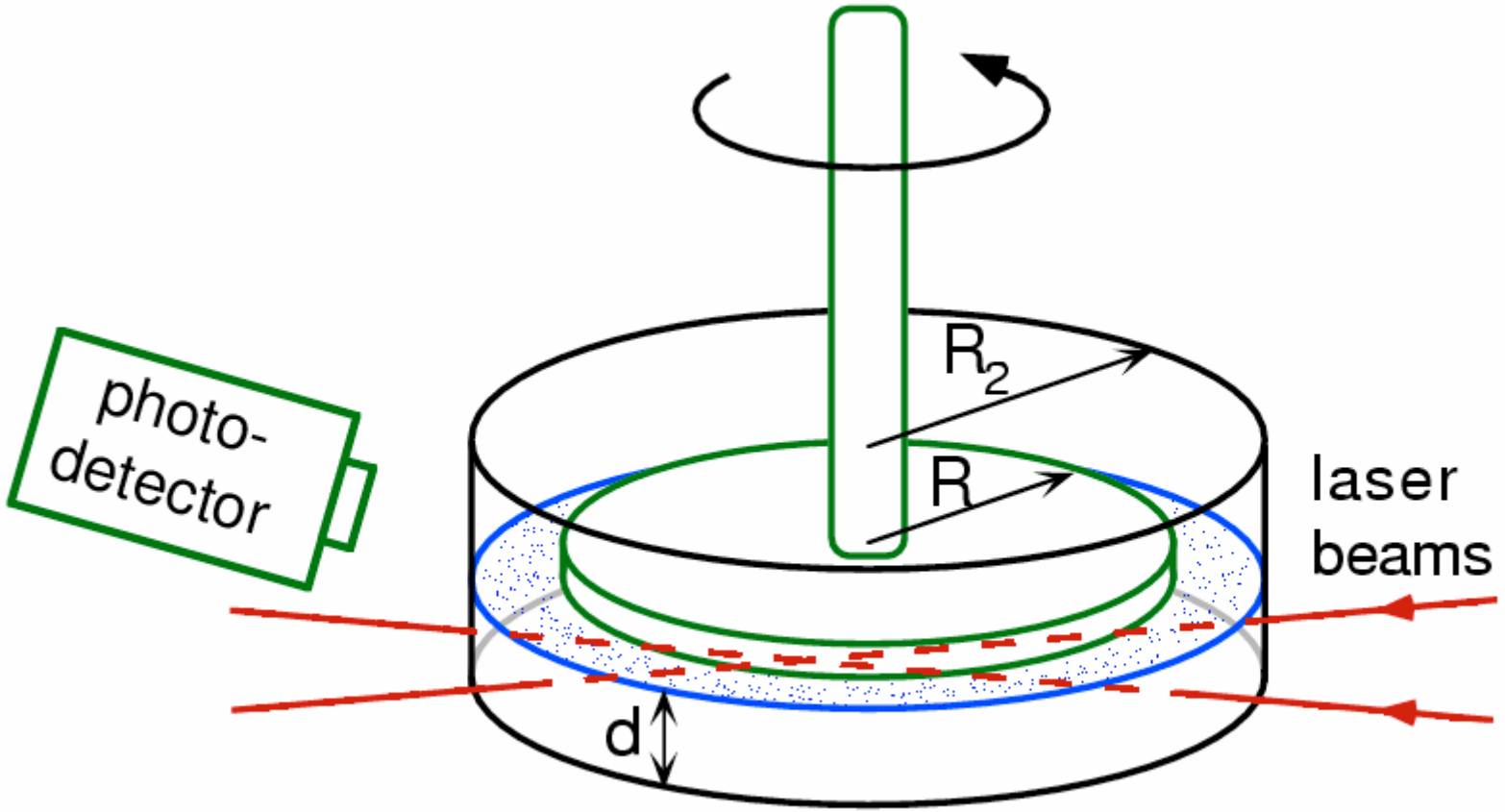
$$K \equiv \frac{\eta_P}{\eta}\frac{d}{R}Wi^2 = const$$

Strong shear primary flow → Polymer stretching

Weak radial elongation coupled to strong primary shear flow leads to energy release into secondary flow due to increase in hoop stress

Elastic Turbulence in a flow between two disks (von Karman swirling flow)

A. Groisman & V.Steinberg, ***Nature* 405**, 53 (2000)
A. Groisman and V. Steinberg, ***New J. Phys***. **6**, 29 (2004)
T. Burghelea, E. Segre, V. Steinberg, ***Europhys. Lett***. **68**, 529 (2004)
T. Burghelea, E. Segre, V. Steinberg, ***Phys. Fluids,*** accepted **17,** (2005).
V. Steinberg, *Elastic Turbulence in Viscoelastic Flows (review),* Ch C2.3 in "***Springer Handbook of Experimental Fluid Mechanics***", 2005
T. Burghelea, E. Segre, V. Steinberg, "*Elastic turbulence in swirling flow between two plates*", to be published (2005)



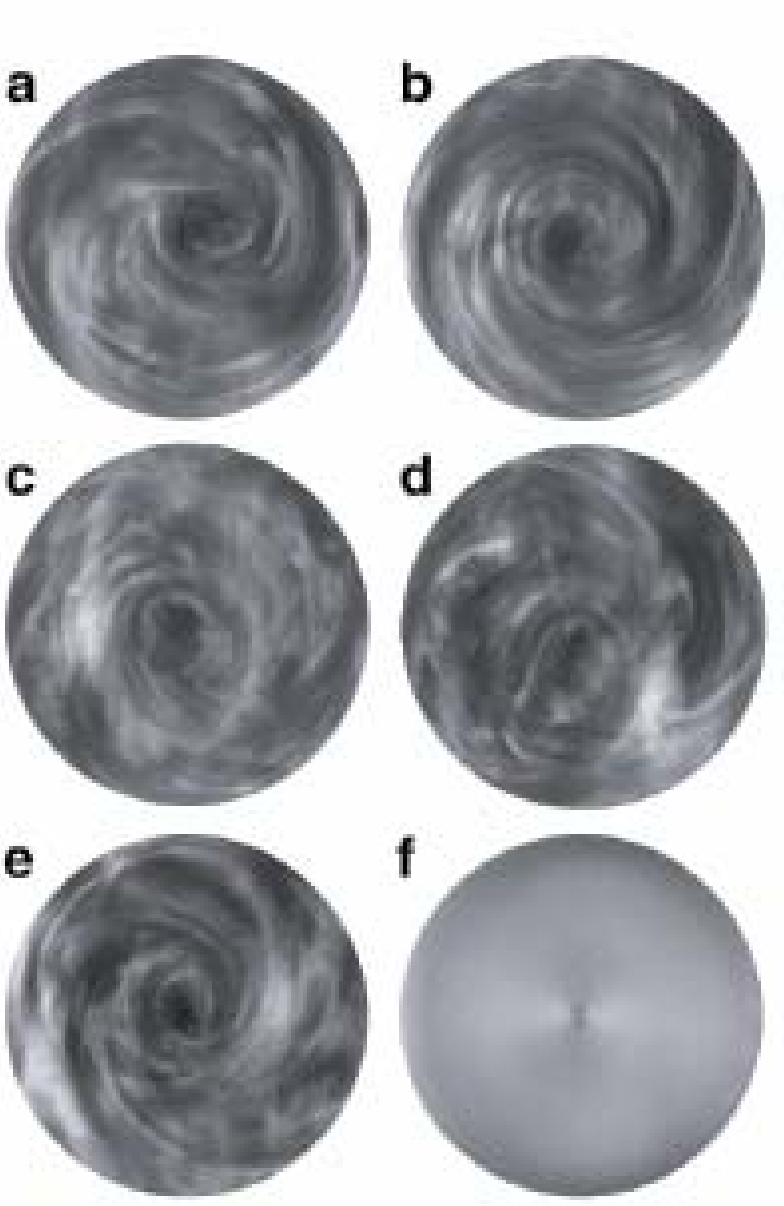
•**Solution:** 80 ppm PAAm, 65% saccharose and 1% NaCl,

$$\eta_s = 0.324 Pas, M = 18 \cdot 10^6 da; \eta = 0.424 Pas \quad \text{at} \quad \dot{\gamma} = 1 s^{-1}$$

relaxation time λ=3.4 sec at 12C (temperature at which the experiment run)

A. Groisman & V.Steinberg, ***Nature*** **405**, 53 (2000)

Visualization of Elastic Turbulence.



Wi=6.5; Re=0.35 (a,b)

Wi=13, Re=0.7 (c-e)

Re=1, pure solvent

Characterization and properties of the random flow

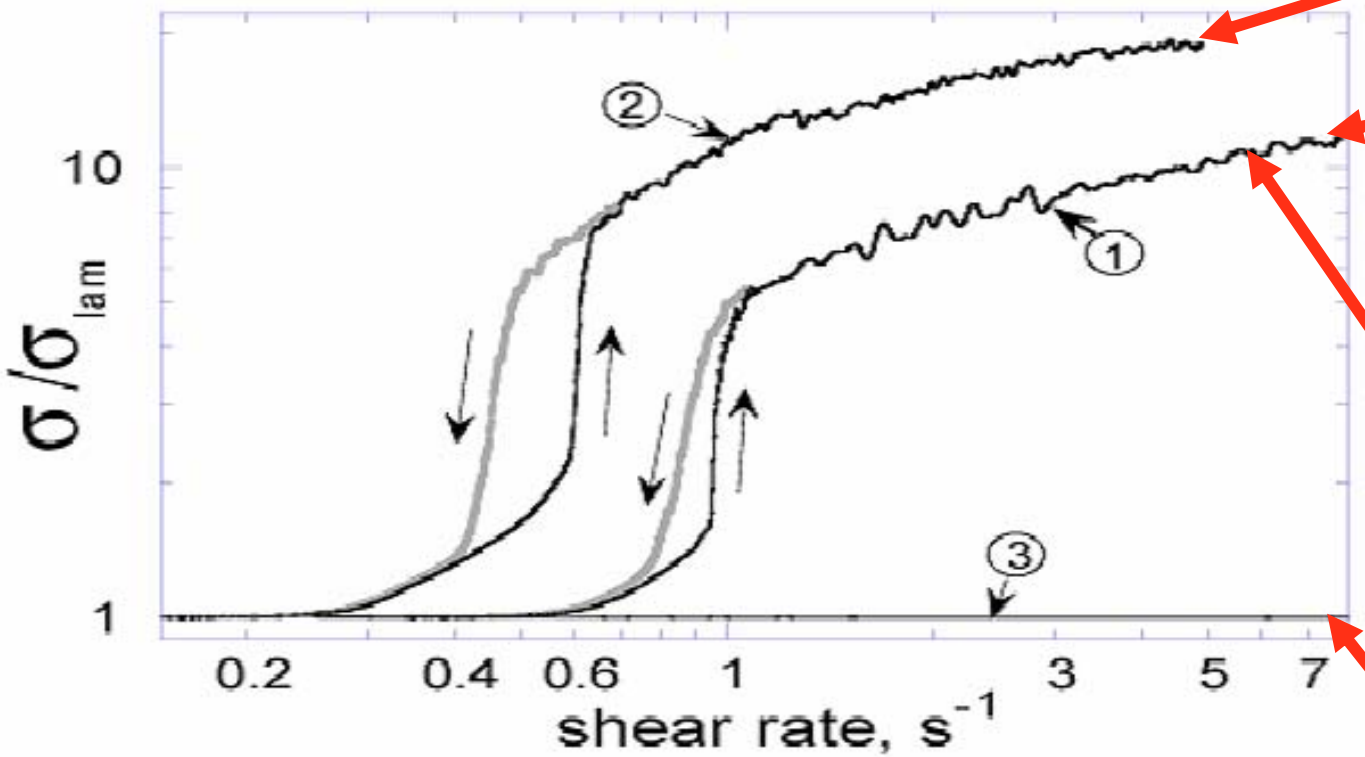
Hydrodynamic turbulence is also defined only by its properties:

- Drastic increase in flow resistance (turbulent drag)
- Energy spectra-existence of inertial range (algebraic decay in energy spectra and a wide range of excited modes in spatial and temporal domains)
- Turbulent mixing-drastic enhancement in mass diffusion

(1) Flow Resistance

σ-stress measured at the upper plate

σ_{lam} -stress in laminar shear flow



d=20 mm

d=10 mm

Polymer solution

solvent

Shear rate 1 1/s--Wi=3.4, Re=0.3

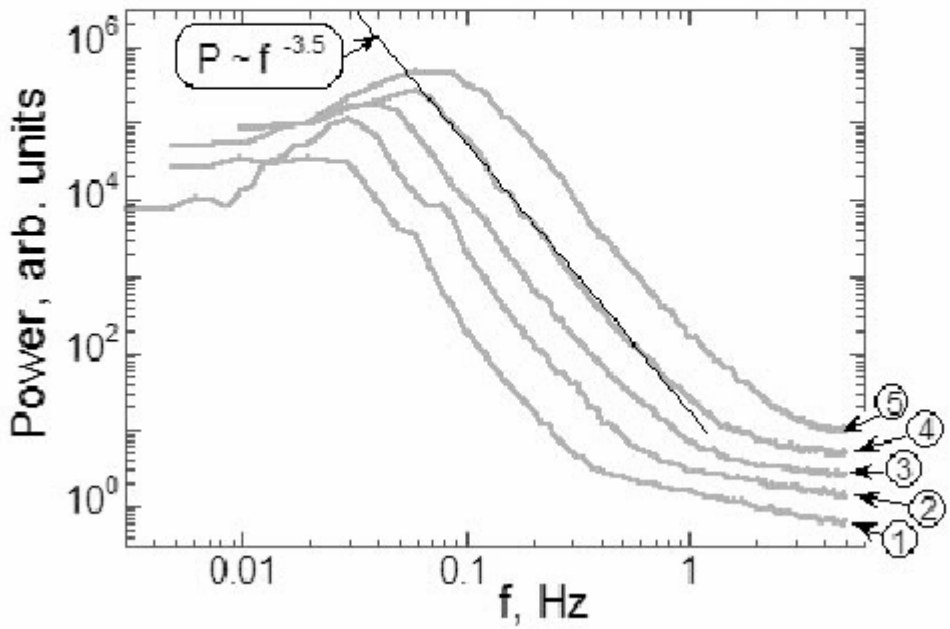
Flow resistance increases by factors of:

11.5 for d=10 mm and 19 for d=20 mm

as in a pipe at

$\mathrm{Re} = 10^5$

(2) Energy spectra

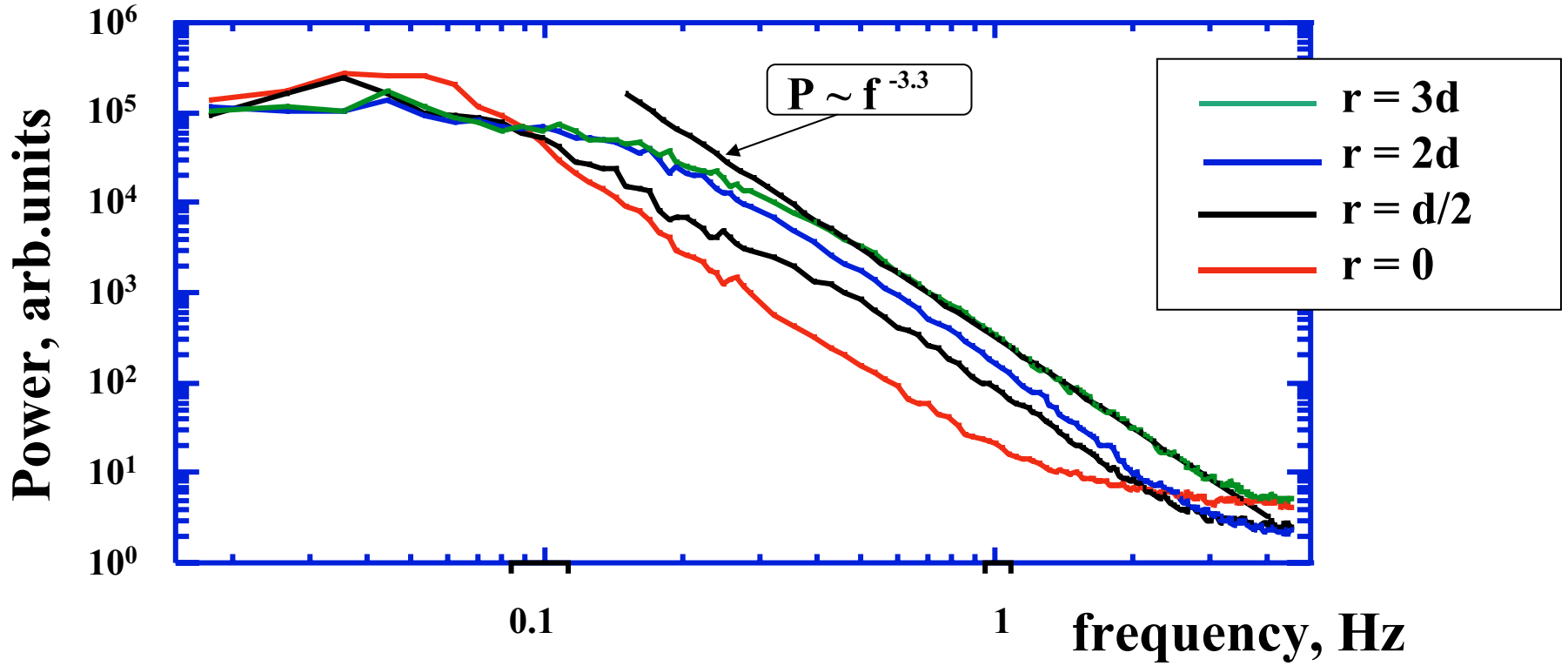


Power spectra of azimuthal velocity fluctuations at
(1) Wi=4.3; (2) Wi=6.3; (3) Wi=9.2; (4) Wi=13.6; (5) Wi=20.1

(LDV measurements)

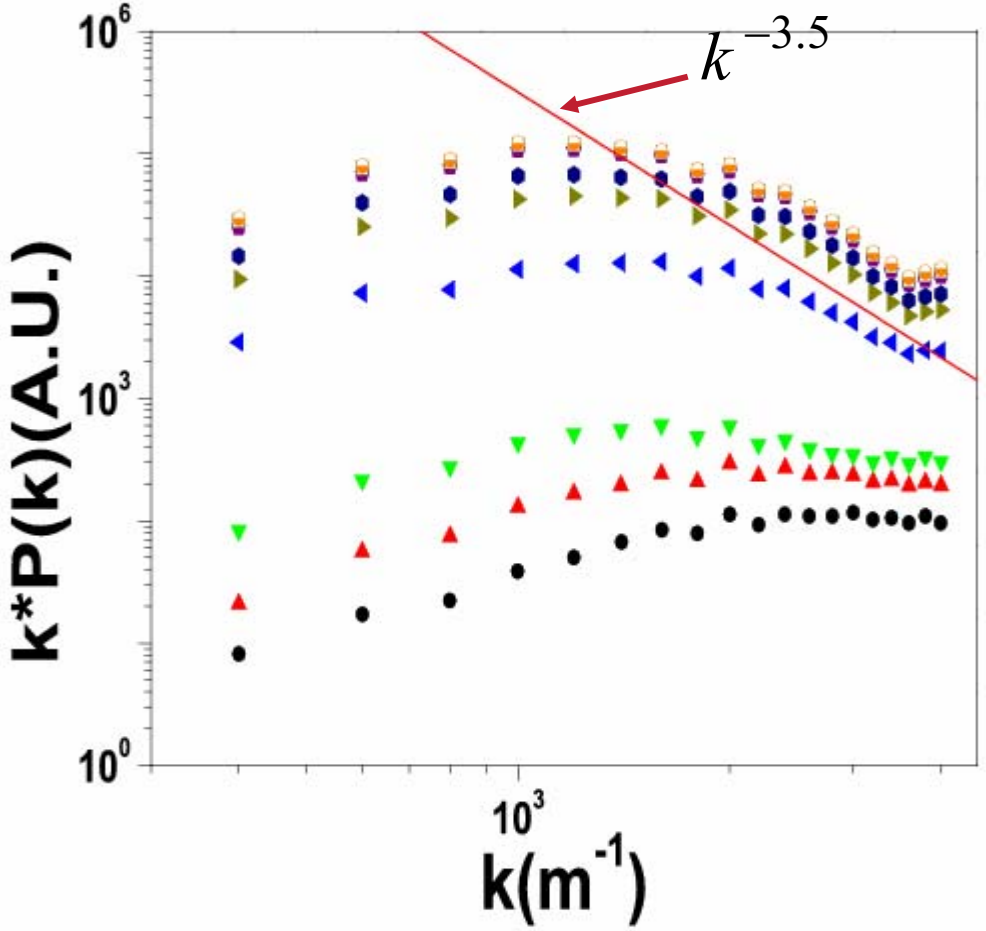
Fluid motion is excited in a wide range of temporal scales, due to fast decay only low frequencies contribute to power

Spectra of the radial velocity fluctuations, measured at different points.



Radial velocity, V_r, was measured at different radii at z = d/2 (midplane), at a shear rate 4 s^{-1}. The average azimuthal (mean flow) velocities in those points were 0, 0.13 mm/s, 3.8 mm/s and 7.0 mm/s at r = 0, d/2, 2d and 3d, respectively. The amplitudes of fluctuations were rather close, varying from 0.88 to 0.99 mm/s.

Temporal spectra at r = 2d and r = 3d should be mainly due to velocity fluctuations in space advected by the mean flow. No peaks - another evidence for many spatial scales excited.



Spatial power spectra of azimuthal velocity fluctuations at left-angles-Wi=11; right-angles-Wi=12.7; hexagons-Wi=13.8; diamonds-Wi=16, half-filled squares-Wi=18; empty circles-Wi=19 **(PIV measurements)**

Fluid motion is excited in a wide range of spatial scales; due to fast decay only large scales contribute to spectra

Elastic turbulence is "counterintuitive"

- Dependence on fluid viscosity, η

Inertial turbulence: higher η → higher V,

Elastic turbulence: higher η → lower V,

that is needed to excite turbulence

- Dependence on system size, d

Inertial turbulence: smaller d → higher V,

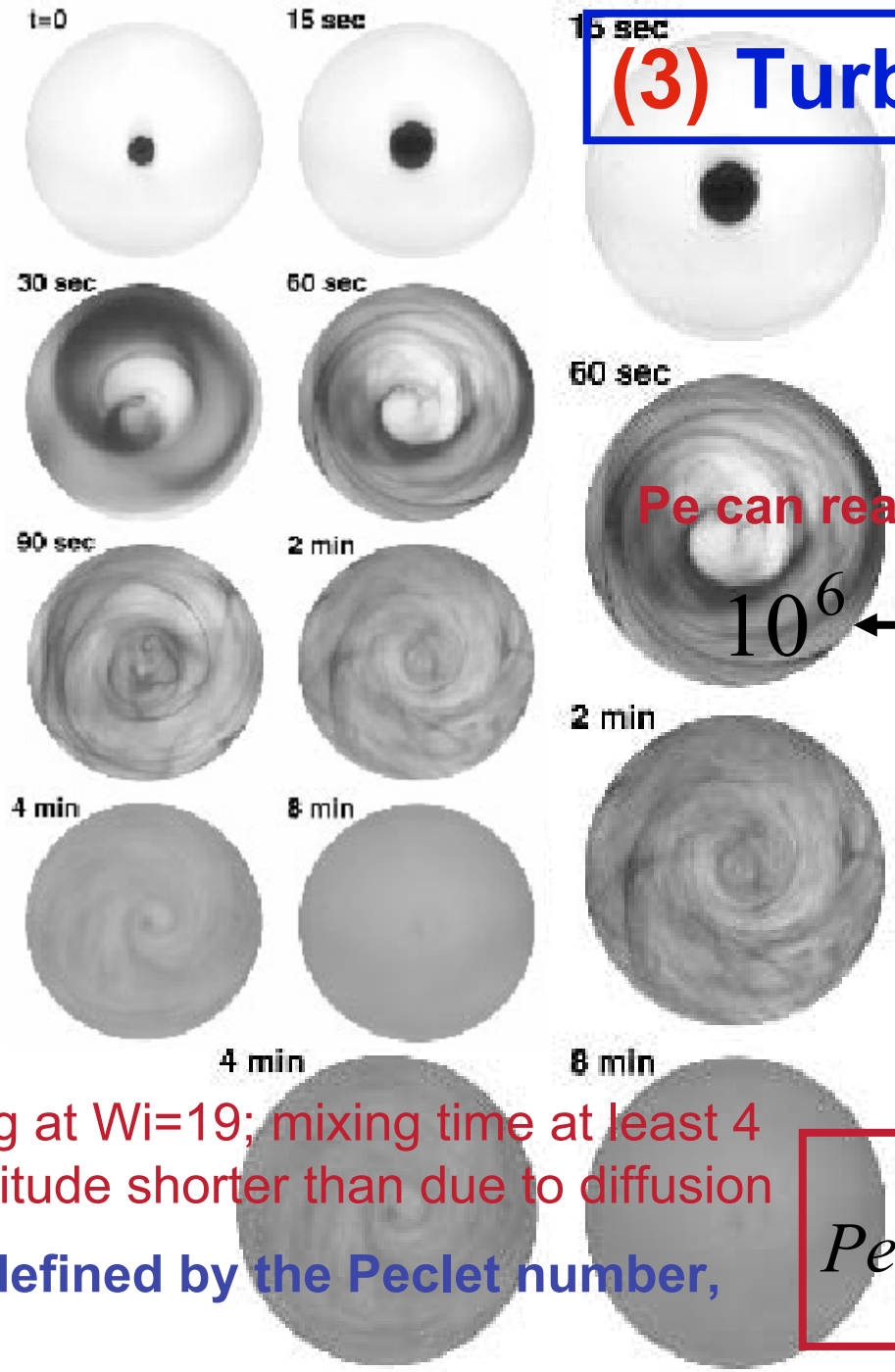
Elastic turbulence: smaller d → lower V,

that is needed to excite turbulence

- No apparent spatial scale other than d

only one time scale, λ , exists

(3) Turbulent mixing



Pe can reach values as high as

10^6 ← at very low D

Turbulent mixing at Wi=19; mixing time at least 4 orders of magnitude shorter than due to diffusion

It is defined by the Peclet number,

$$Pe = \frac{V}{R} \cdot \frac{R^2}{D} = \frac{VR}{D}$$

What is the source of high flow resistance in the elastic turbulence?

A. Groisman & V.Steinberg, PRL 86, 934 (2001)

Momentum balance equation:

$$\sigma_w = -\eta_s \dot{\gamma} + \rho(v_r' v_\theta' + v_\theta' v_r') + \tau$$

wall stress

solvent shear stress

Reynolds stress

Polymer stress (mostly elastic stress)

Solvent shear stress: the same as in a laminar shear flow, if averaged across the layer.

Reynolds' stress: low fluctuating velocities, low contribution into R. stress⇒less than 0.5% of σ_w in the standard set-up, and 16 times less in the 1:4 set-up.

Thus, the whole increase in flow drag is due to polymer stress τ.