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# INTRODUCTION TO MICROFLUIDICS

**8 - 26 August 2005**

## Mixing in a Random Flow in Curved Channels

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# Mixing in a random flow in curved channels

(elastic turbulence in curved channels, mixing in a random flow in macro- and micro-channels)

## Lecture 2

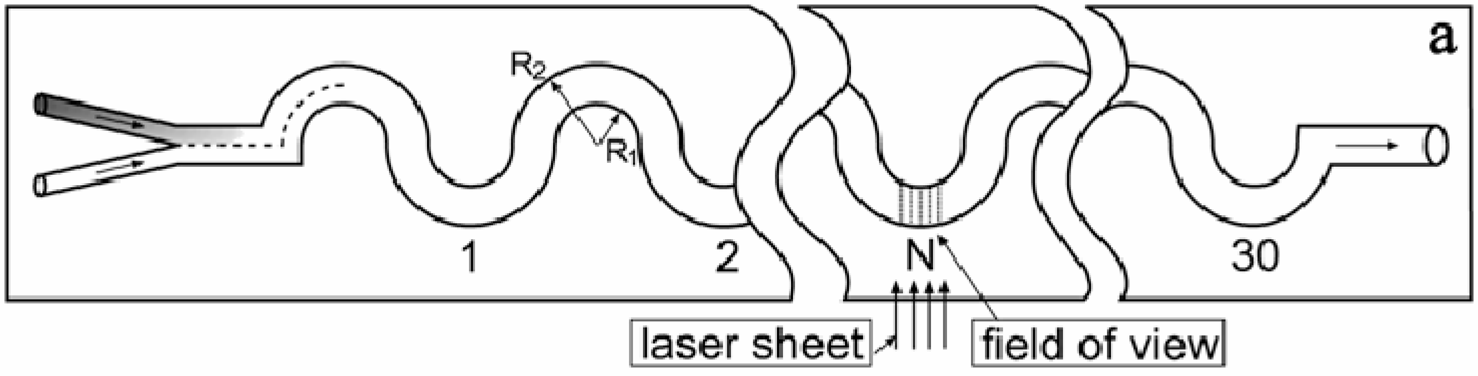
V. Steinberg

Summer School in Microfluidics,
August 8-26, 2005, ICTP, Trieste, Italy

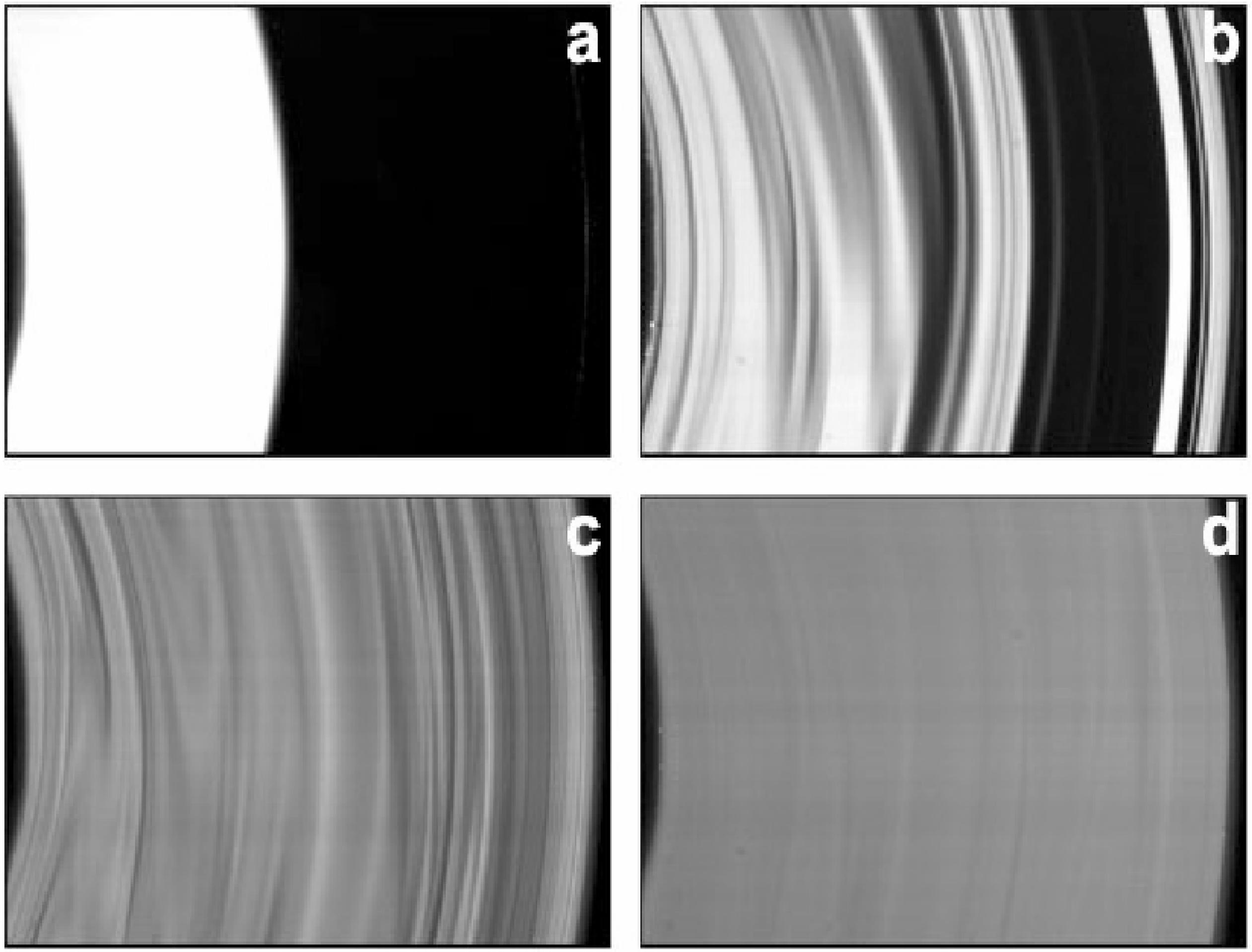
WEIZMANN INSTITUTE OF SCIENCE

# ***Elastic Turbulence in a curved channel: macro- and micro-flows***

A. Groisman and V. Steinberg, ***Nature* 410**, 905 (2001)
A. Groisman and V. Steinberg, ***New J. Phys*. 6**, 29 (2004)
T. Burghelea, E. Segre, V. Steinberg, ***PRL* 92**, 164501 (2004)
T. Burghelea, E. Segre, I. Bar-Joseph, A. Groisman,V. Steinberg, ***PRE* 69**, 066305 (2004)
V. Steinberg, *Elastic Turbulence in Viscoelastic Flows (review),* Ch C2.3 in "***Springer Handbook of Experimental Fluid Mechanics***", 2005
T. Burghelea, E. Segre, V. Steinberg, "*Elastic turbulence in macro- and microchannel flow*", to be published




***Macrochannel***
***d=3 mm, d/R1=1***
***N=40-64 units***

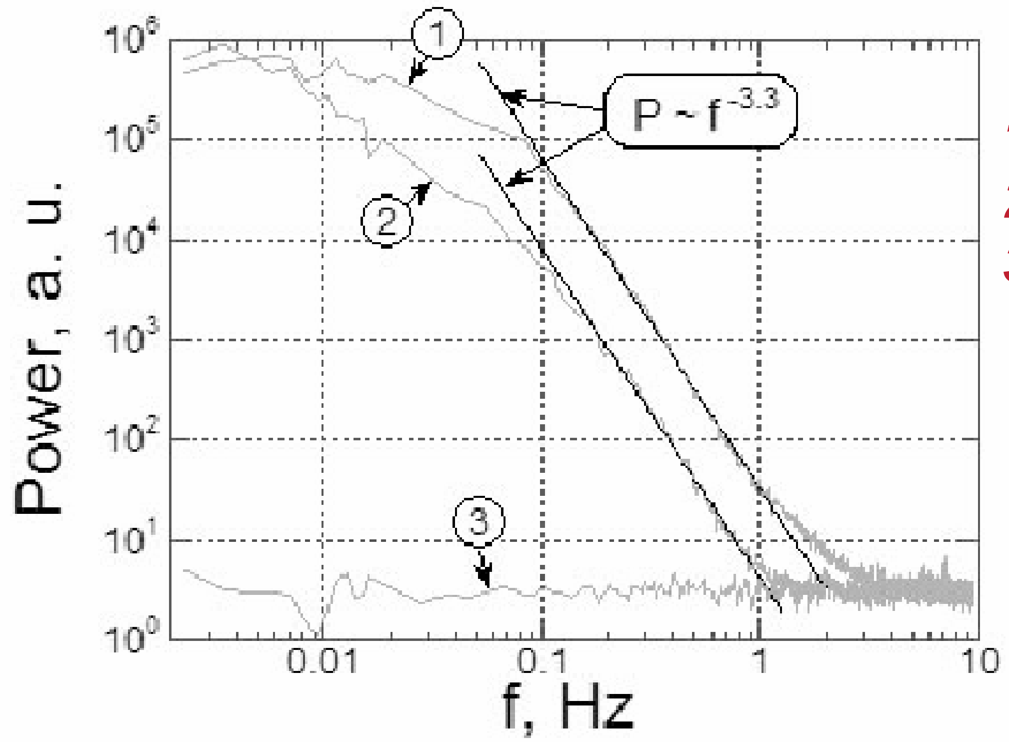



Photographs of the flow taken with laser sheet at different N. Bright regions correspond to high fluorescent dye concentration

***(a) N=29-pure solvent***
***(b) N=8***
***(c) N=29***
***(d) N=54 at Wi=6.7***
***(polymer solution)***

A.Groisman and V. Steinberg
*Nature* **410**, 905 (2001)

LDV measurements




*1-longitudinal velocity*
*2-transversal velocity*
*3-transversal velocity in a pure solvent*

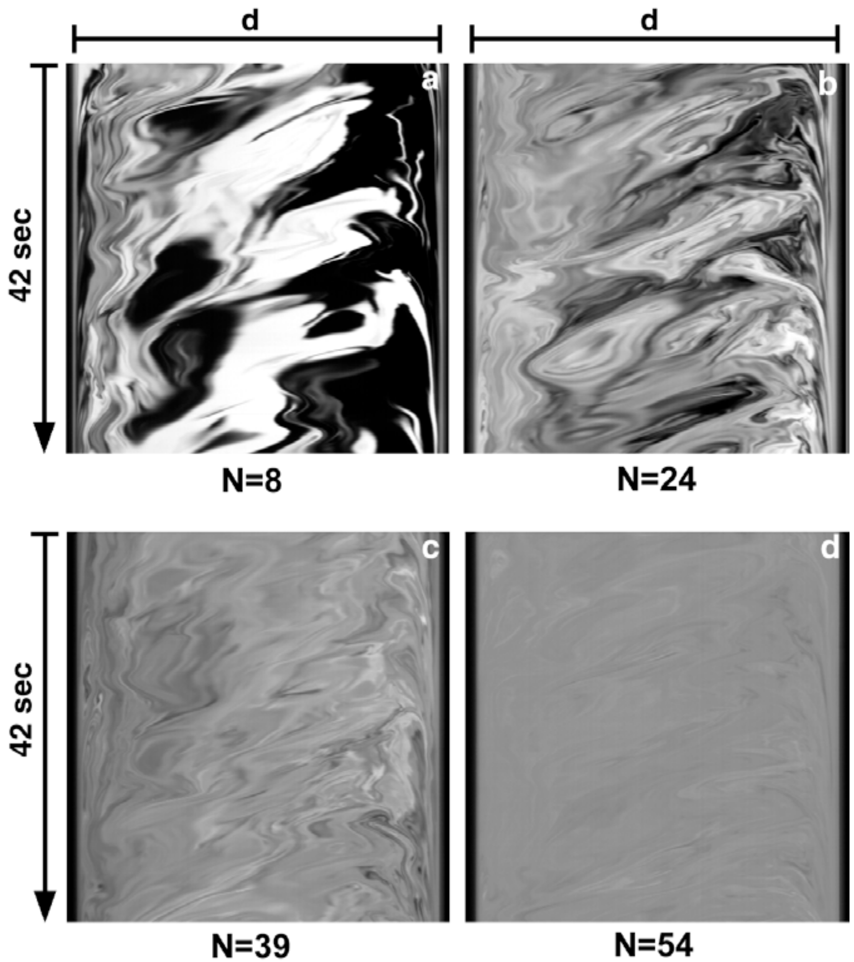
*Spectra of velocity fluctuations measured in the middle of the channel at N=12 with LDV.*

**Average velocity - V = 6.6 mm/s; Wi=6.7**
**fluctuations: longitudinal - Vrms = 0.09V, transversal - Vrms = 0.04V.**

# Mixing by the elastic turbulence in a macro-channel:

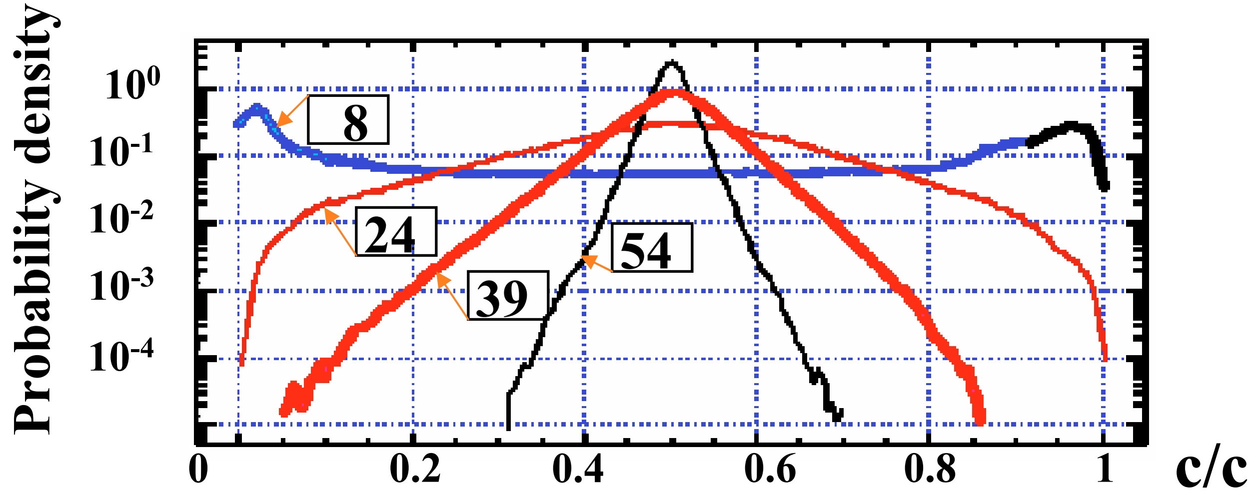
**tools for characterization of mixing effectiveness in a random flow**

# Space-time plots at different positions, N.

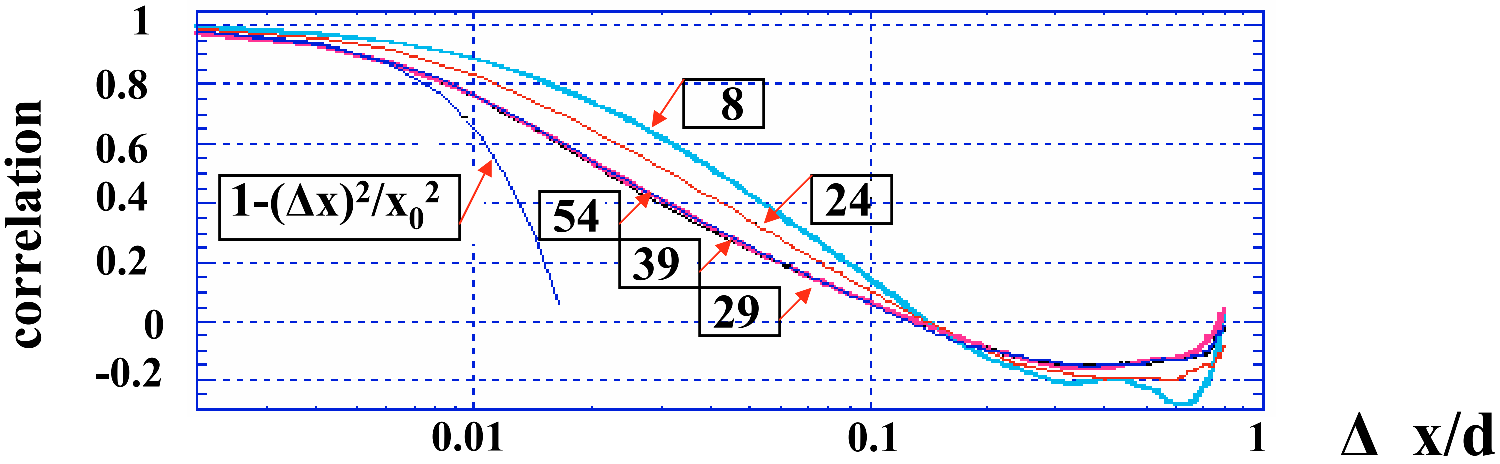



As the liquid advances downstream and gets mixed, the contrast decreases but the characteristic size of the structures does not seem to change very much.

## Probability density functions for the passive scalar in a point at different positions, N, along the channel.



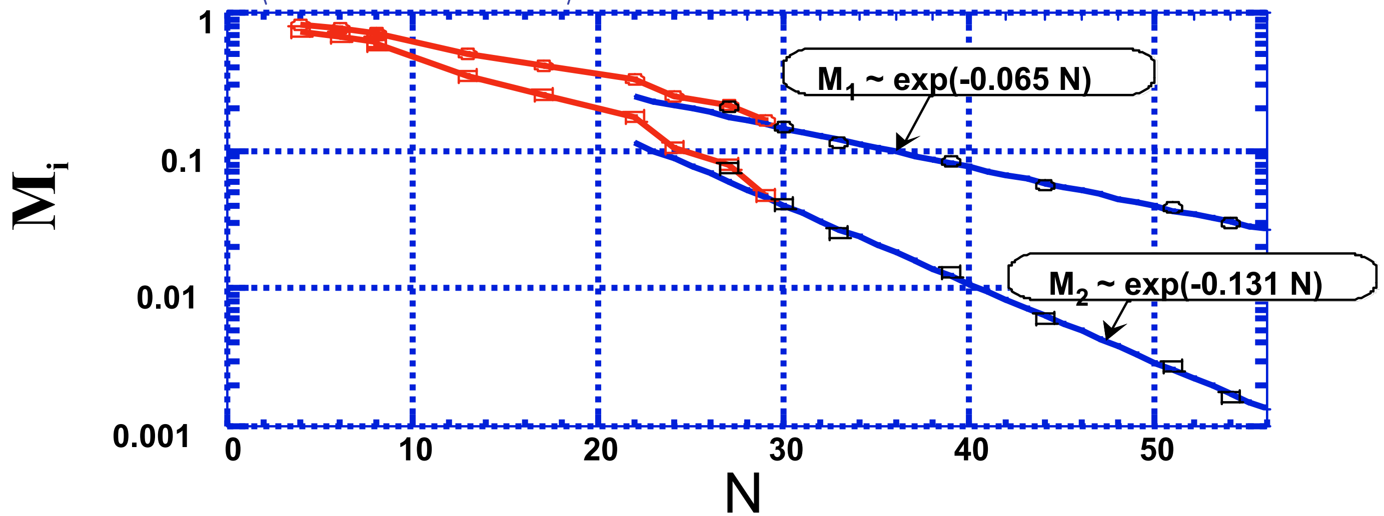

## Spatial correlation of the concentration across the channel at different N. Each line - statistics over about $10^7$ points, and over the total discharge of $2 \cdot 10^3\ d^3$.




$x_0 = 50\ \mu m$; molecular diffusion scale $(D/(V_{rms}/d))^{1/2} = 25\ \mu m$, illuminating light sheet - 40 μm thick.

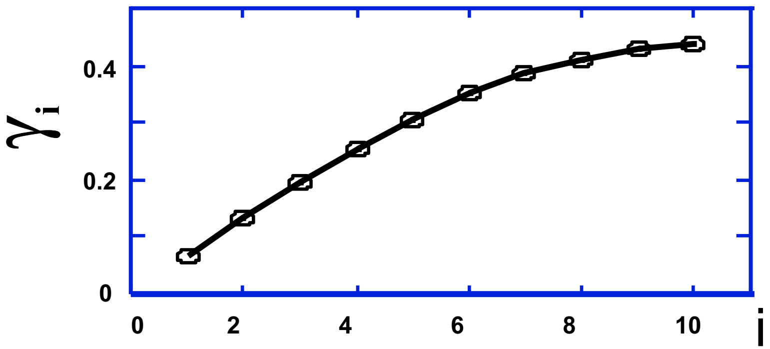
# Dependence of the moments, $M_i$, of the passive scalar PDF on the position, N, along the channel.

$$M_i \equiv \langle | \theta - \bar{\theta} |^i \rangle / \bar{\theta}^i$$




Exponential decay of the moments with the position, N, along the channel and with the mixing time, ***t =7.6 2π 4.5/V=21.5 sec.***

Mixing time in the flow – 21.5 sec; diffusion time $d^2/D \approx 10^5$ sec. $\mathbf{M_i \sim exp(-\gamma_I N)}$ was also found for higher order moments.




## Dependence of the exponents, $\gamma_i$ on the order, *i*, of the momentum.

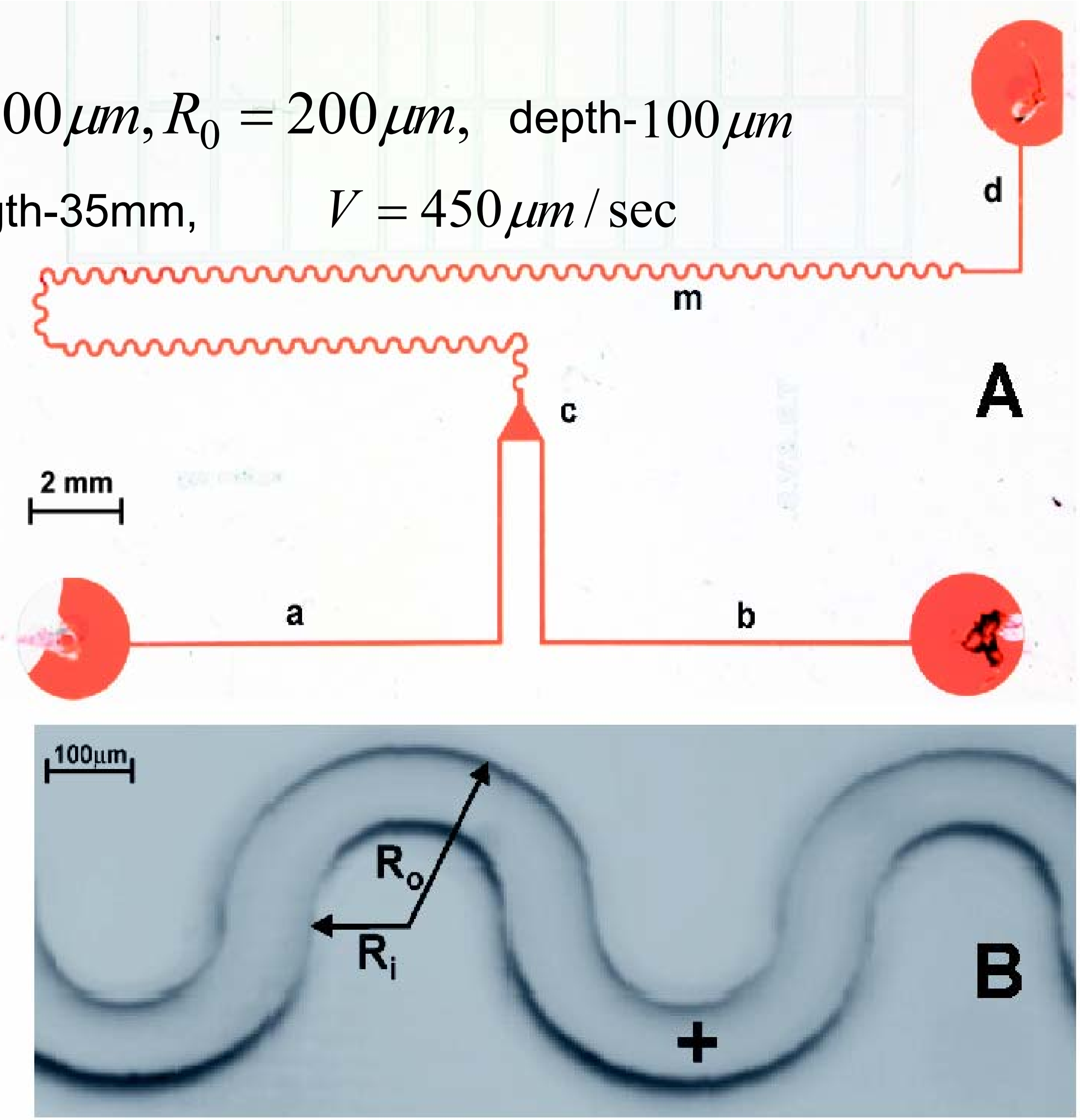
Linear growth of $\gamma_i$ at small *i* and saturation at larger *i*, in complete agreement with the theoretical predictions.

# ***Mixing by the elastic turbulence in micro-flows.***

Size: $R = 100\mu m, R_0 = 200\mu m,$ depth-$100\mu m$

length-35mm, $V = 450\mu m/\sec$

**Microchannel**




T. Burghelea, E. Segre, and V. Steinberg, *PRL* **92**, 164501 (2004).

# Parameters for our experiment.

$$Sc \approx 1.8\times 10^6,$$

$$Pe = \langle \dot{\gamma}_{fluct} \rangle d^2 / D \approx 10^4,$$

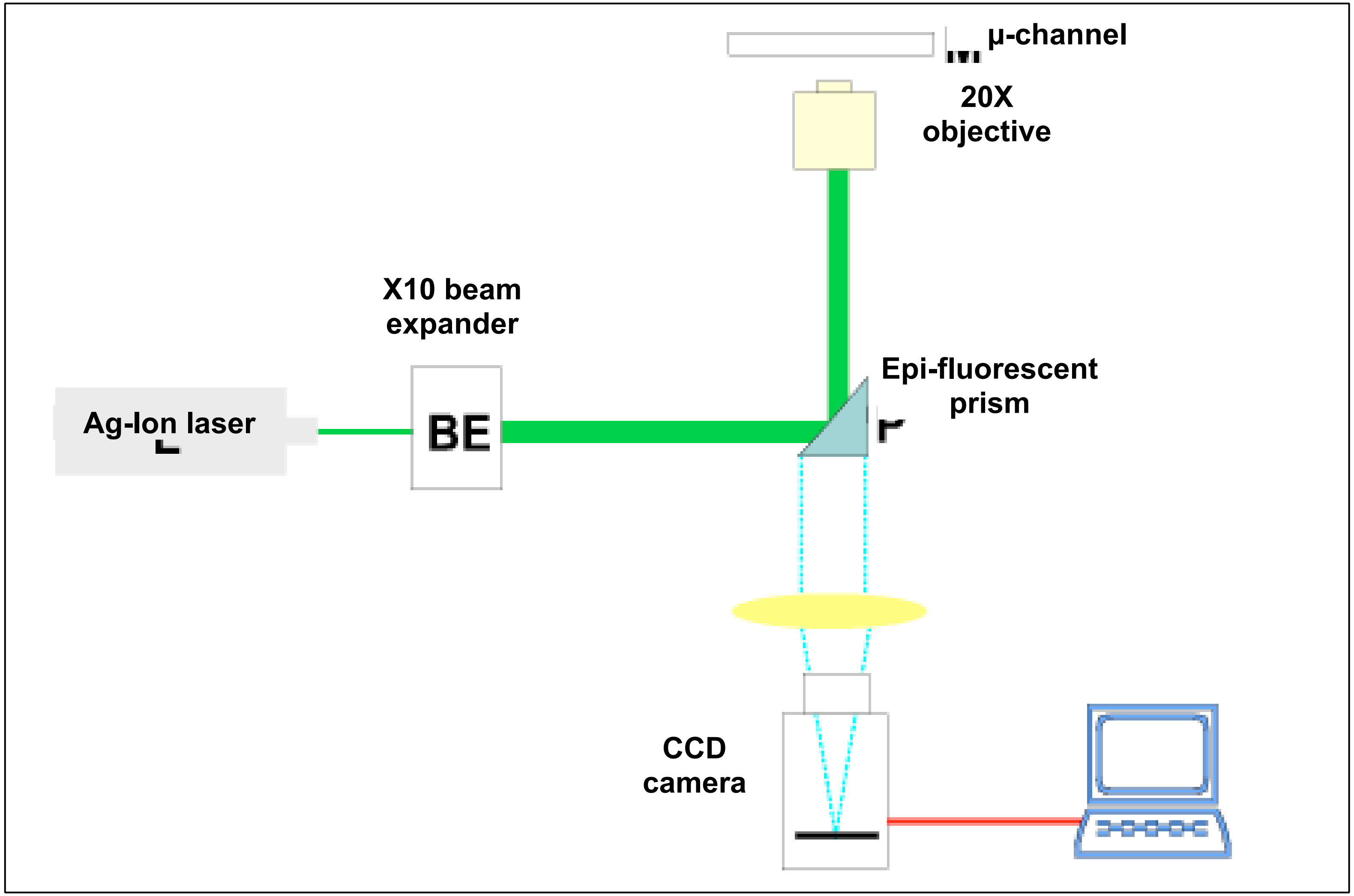
$$\eta_B \approx d \cdot Pe^{-1/2} = 3\times 10^{-3}\ \mathrm{cm}$$

$$\eta_K = (Sc)^{1/2}\eta_B \approx 1.3\times 10^3\ \eta_B$$

$$\eta_K \approx 4\ \mathrm{cm}$$

# Characterization of velocity field

# MicroPIV epi-fluorescent setup

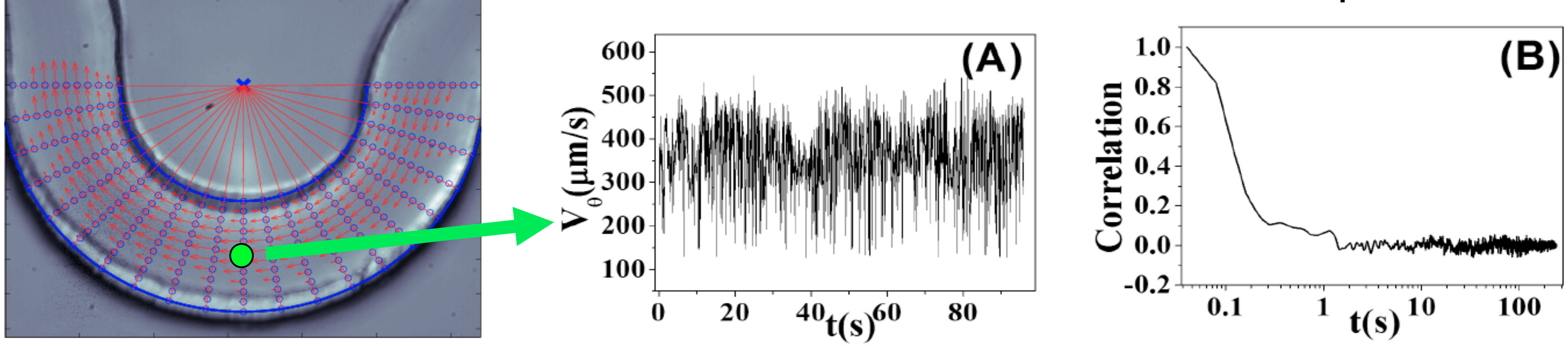



- 0.2μm yellow-green fluorescent beads

# PIV data

- Standard image processing for removing glares, background
- morphological filter removes out of focus beads
- Low spatial resolution, usable results only in the center of the channel, but time resolved measurement
- Flow becomes non-stationary above a critical *Wi*. We can get velocity data below and above transition!

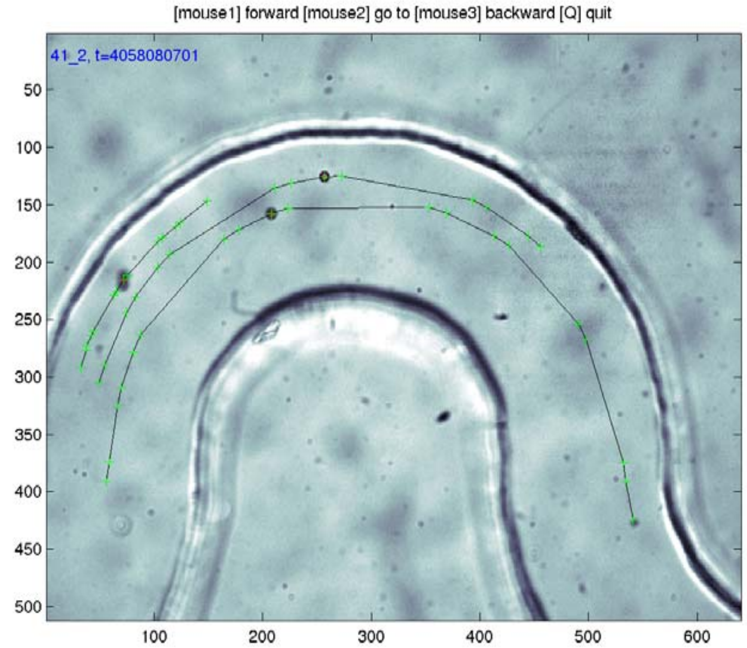
6000 points



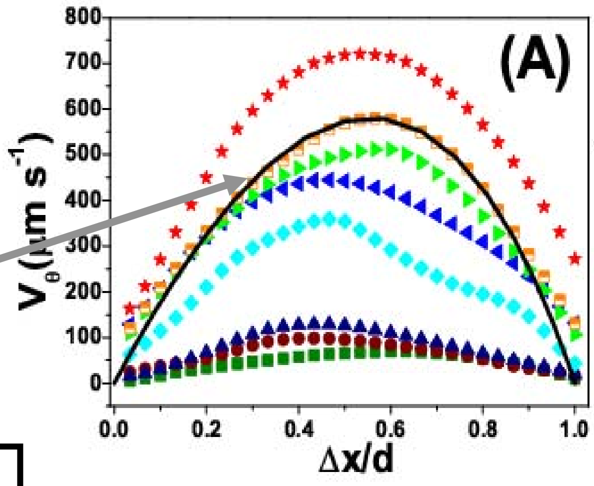

Longitudinal flow velocity at the center of micro-channel at Wi=21

# PTV data

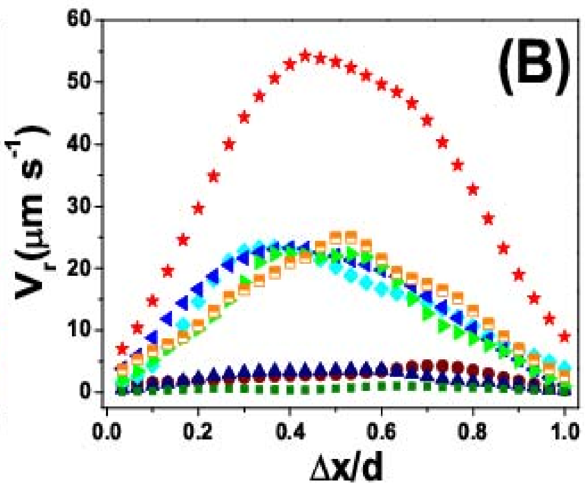
- Conventional microscopy, z selected by depth of field
- Tracking is easy at low seed concentration
- Interpolation and averaging of many velocity vectors gives the mean field as well
- μPIV validated




Numerical simulation by Kumar



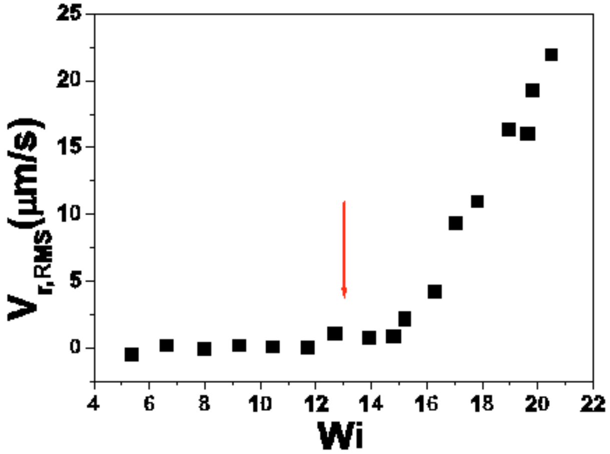

tangential

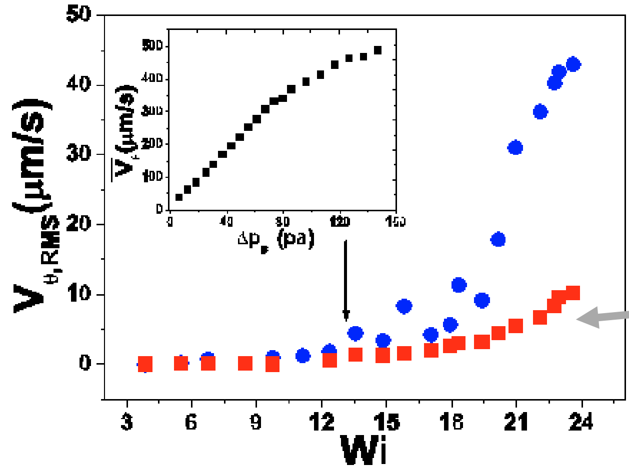


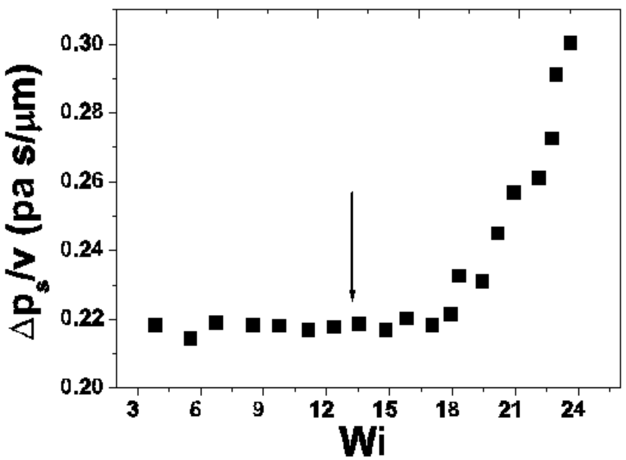

radial

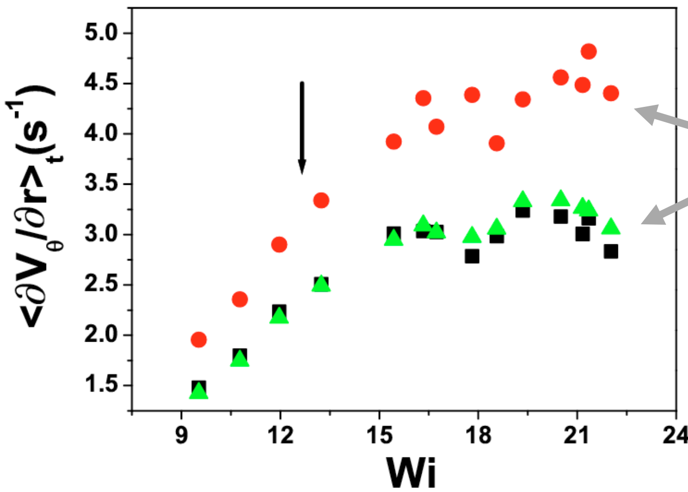
Wi from 4 till 22

# Transition to turbulence is evidenced by increase of velocity fluctuations, as well as by increase in drag



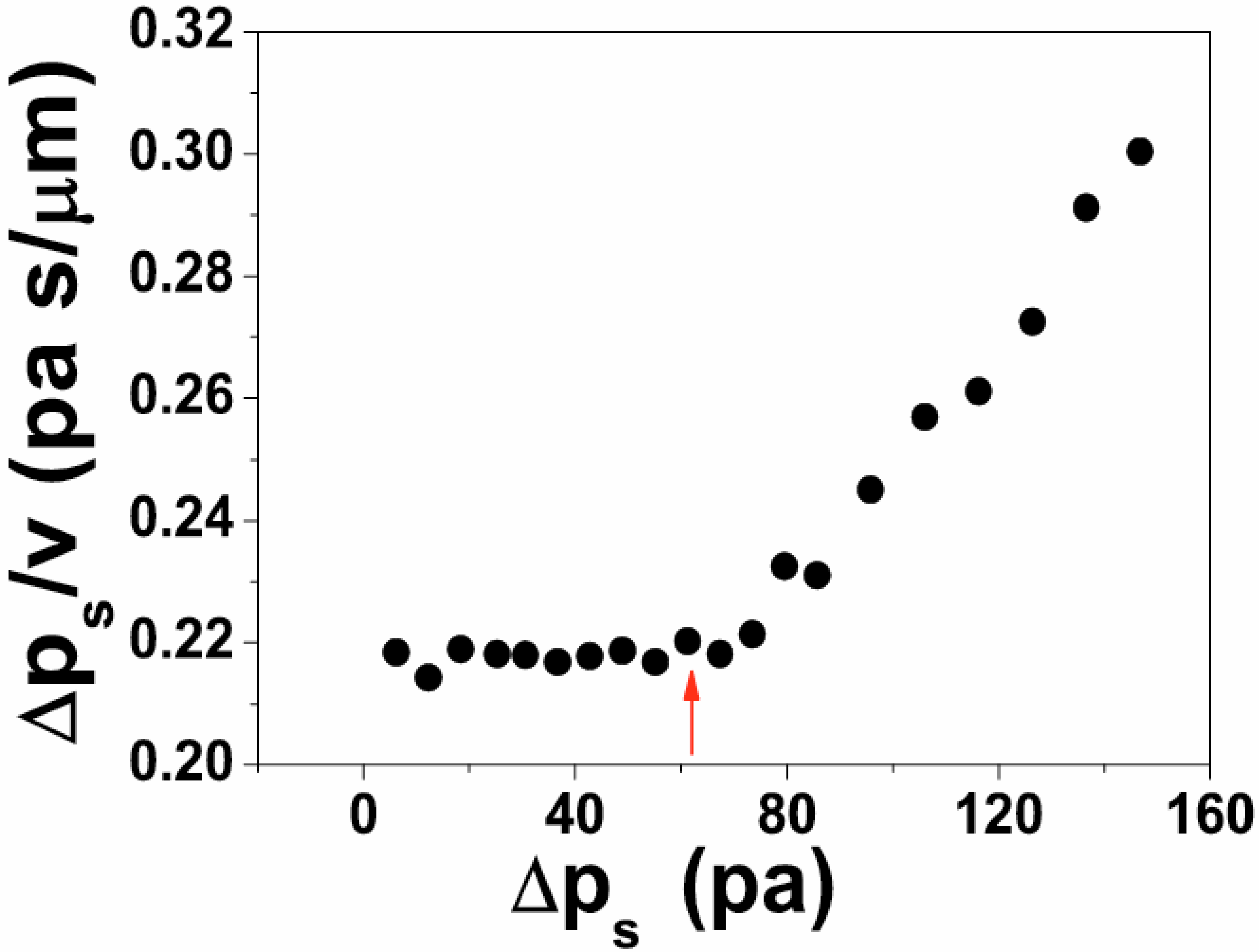



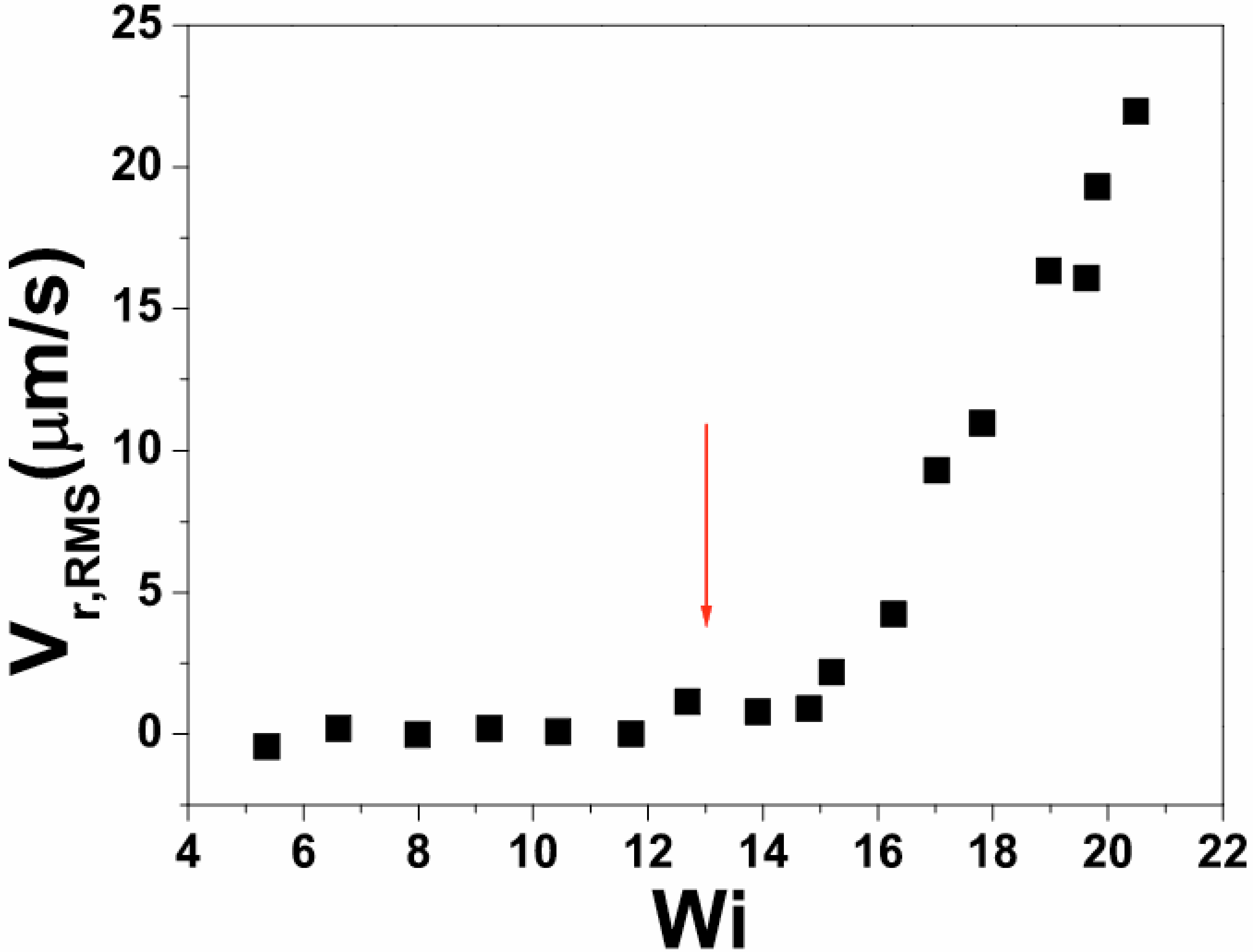
PTV measurements

# (1) Flow resistance



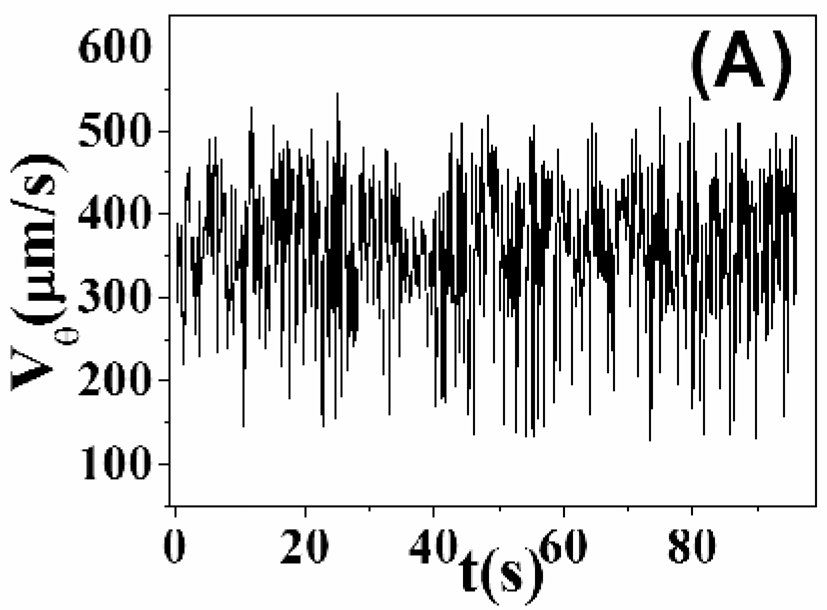

*Flow resistance in micro-channel.*
*The arrow indicates the onset of the elastic instability.*
$\Delta p_s$ *-pressure drop per segment*

## *PTV in micro-channel*

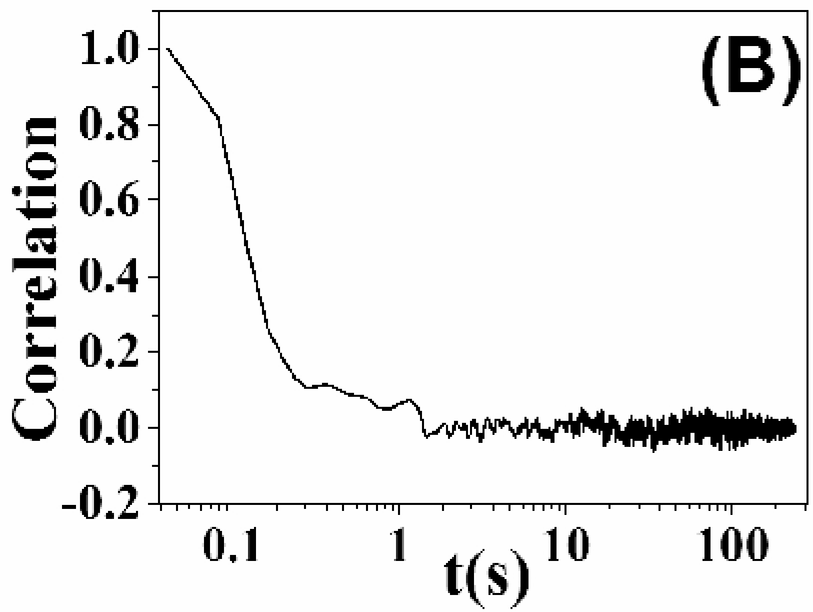



*RMS of radial velocity fluctuations as a function of Wi.*
*The arrow indicates the onset of the elastic instability.*

# PIV in micro-channel

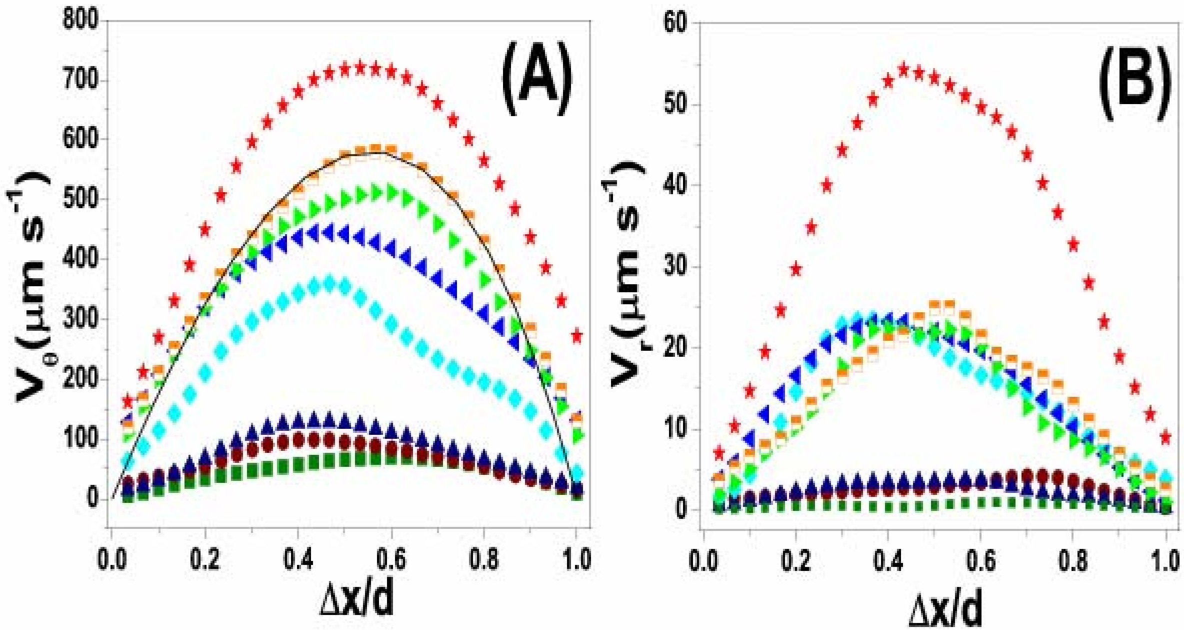



Time series of longitudinal flow velocity $V_\theta$ at the center of micro-channel at Wi=21



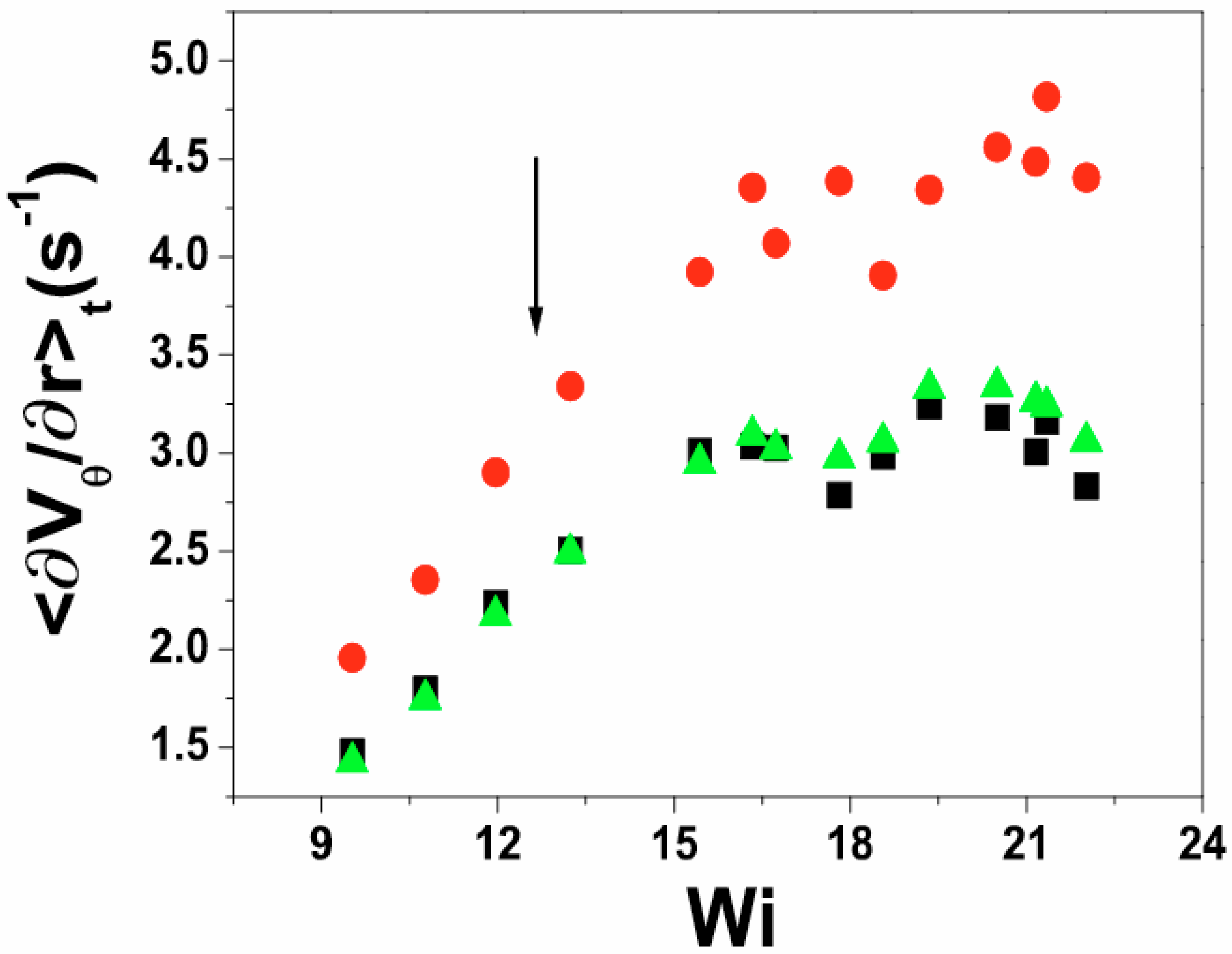

Autocorrelation function of $V_\theta$ based on about 6000 velocity measurements

**PTV measurements**




Green squares- Wi=3.85
Red circles- Wi=6.75
Violet triangles- Wi=8.5
Electric rhombus-Wi=13.6
Blue triangles- Wi=14.8
Green triangles- Wi=19.4
Yellow squares- Wi=20.1
Red stars- Wi=22
Full line-numerical simulations

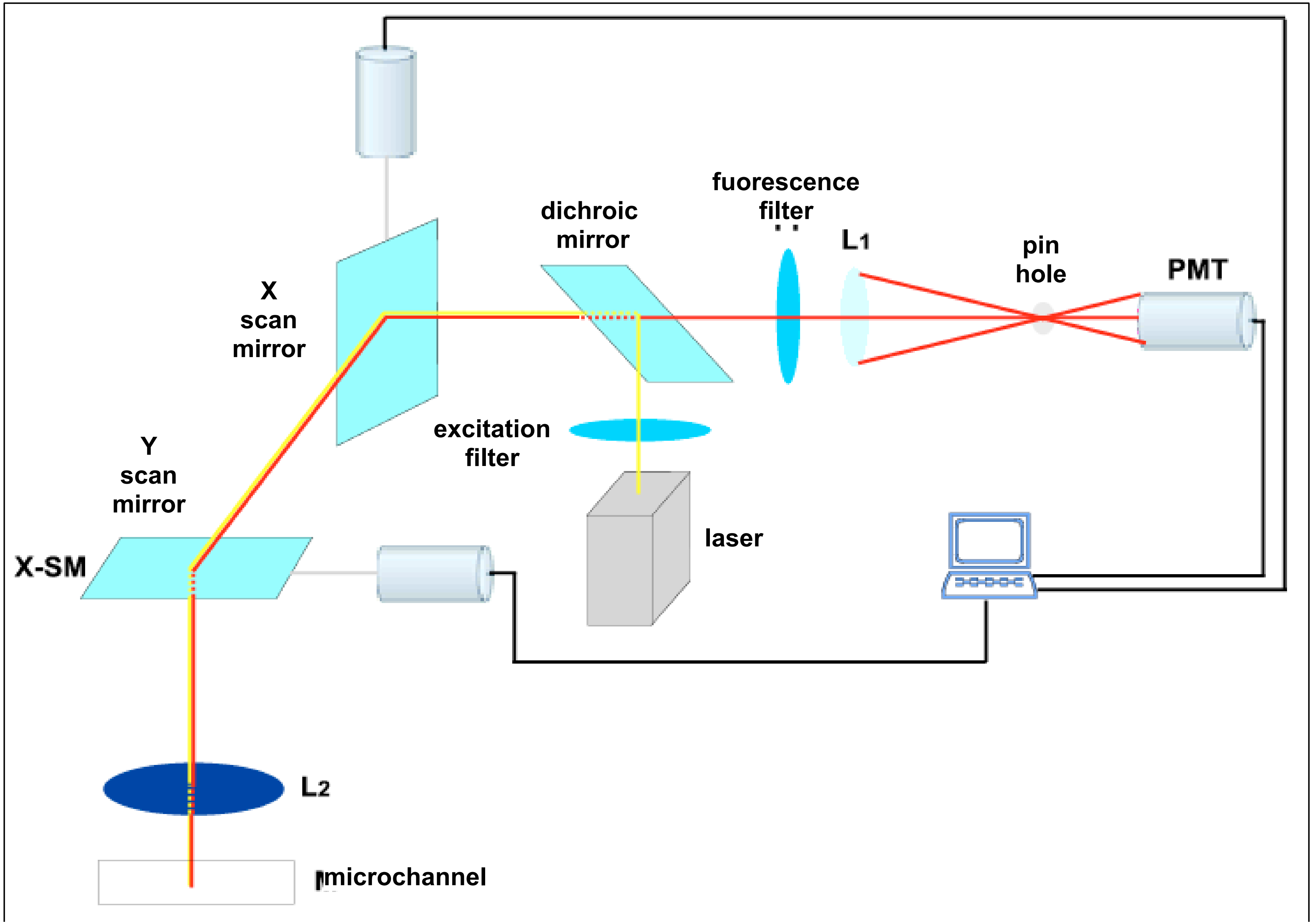
***Profiles of longitudinal (A) and transverse (B) velocity components in micro-channel flow***

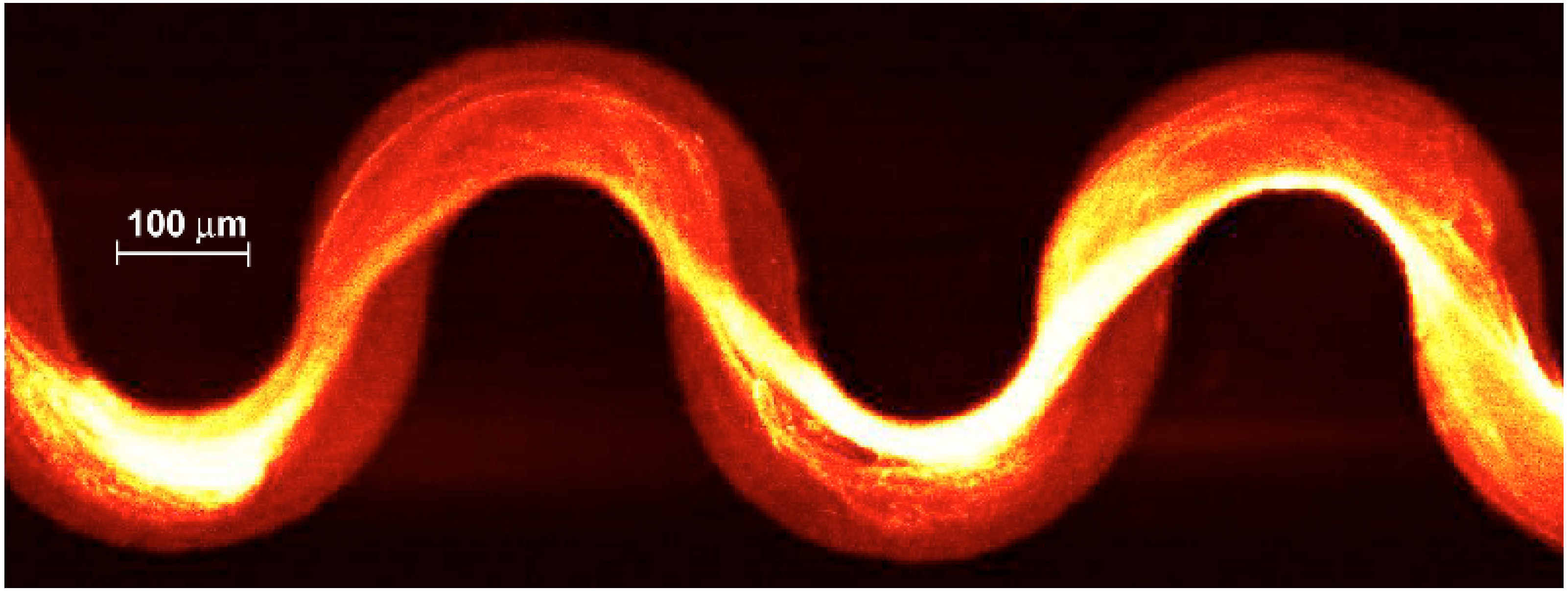



*Average gradients of longitudinal velocity as a function of Wi at three polar angles and at the same radial position, r=d/2. The arrow indicates the onset of the elastic instability. Velocity gradient saturates in the elastic turbulence regime.*

# Characterization of mixing in a micro-channel

# Passive scalar: scanning confocal setup

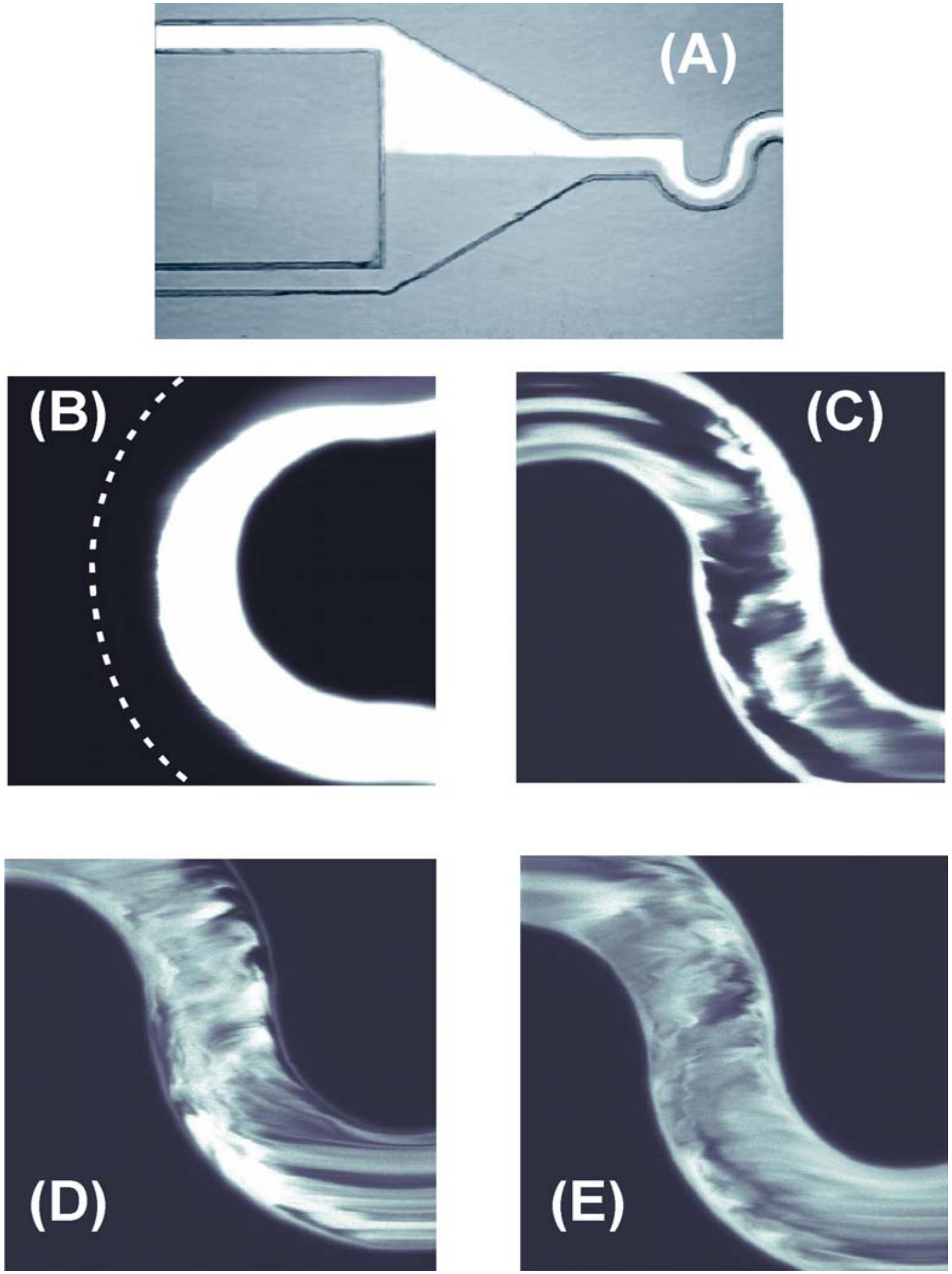





***Middle plane horizontal confocal microscope snapshot at Wi=20. The flow is seeded with 0.2 $\mu$m green fluorescent beads.***

# Mixing in the microchannel

inlets

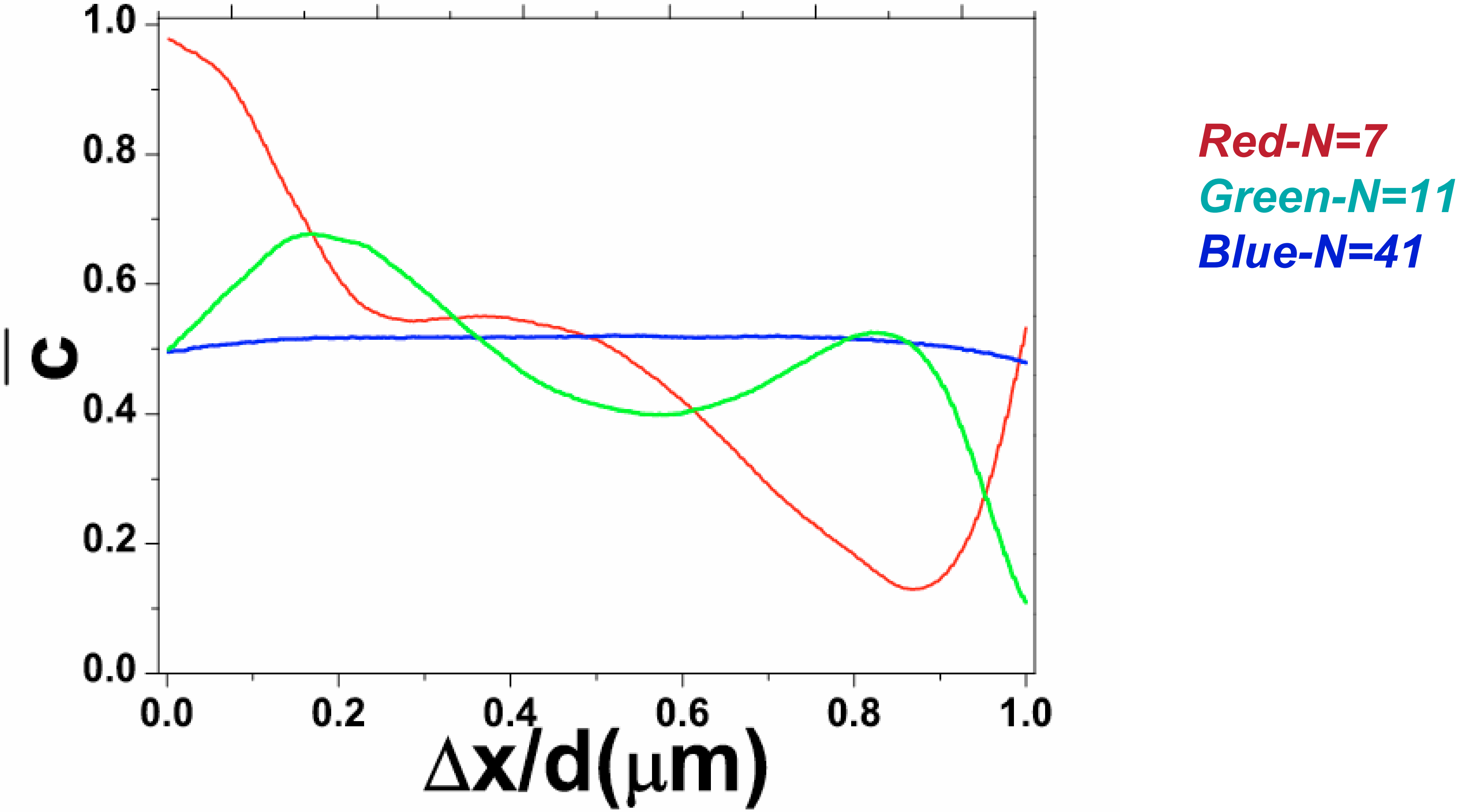



N=30, laminar

N=5, polymer

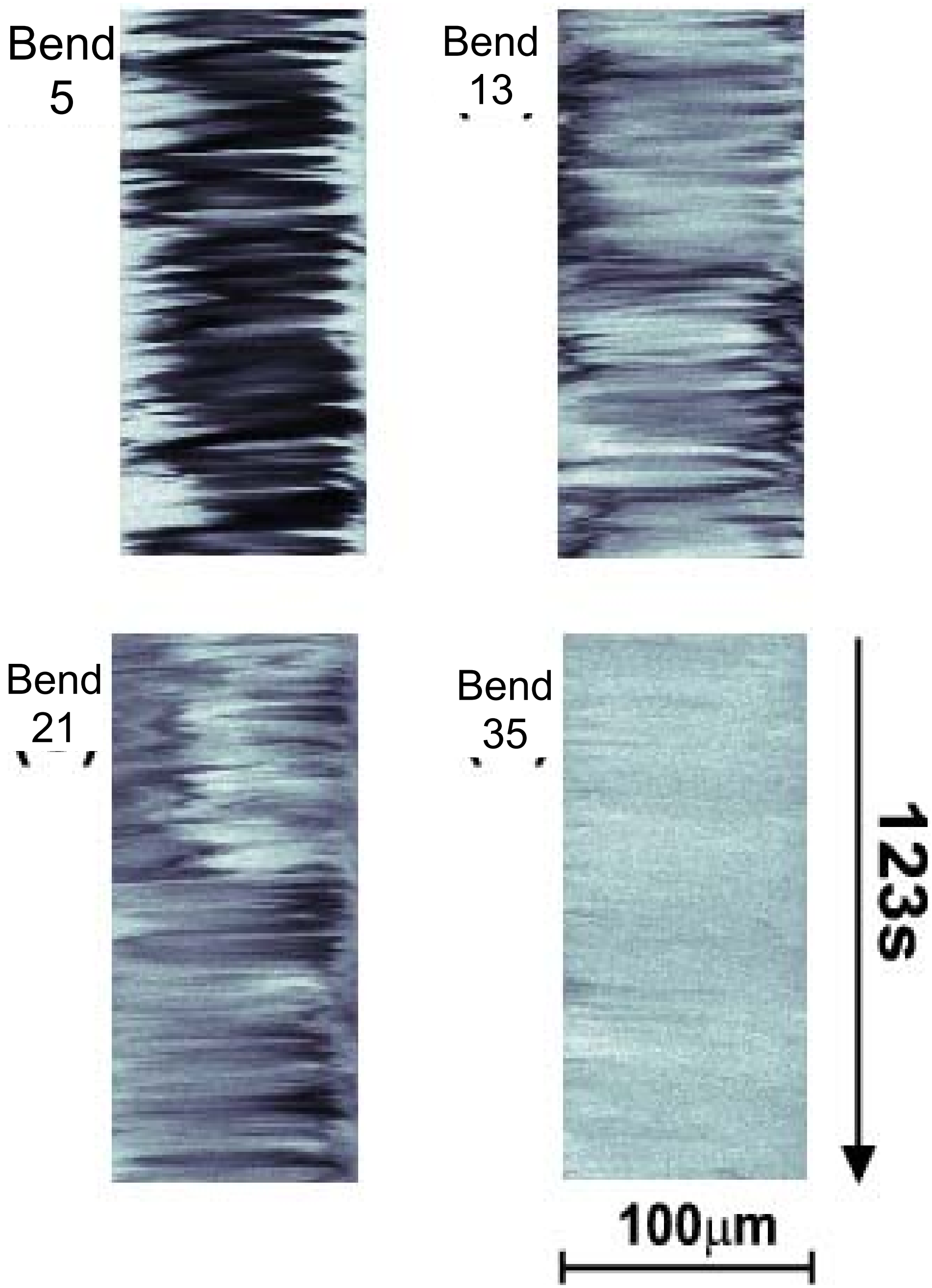
N=11, polymer

N=17, polymer



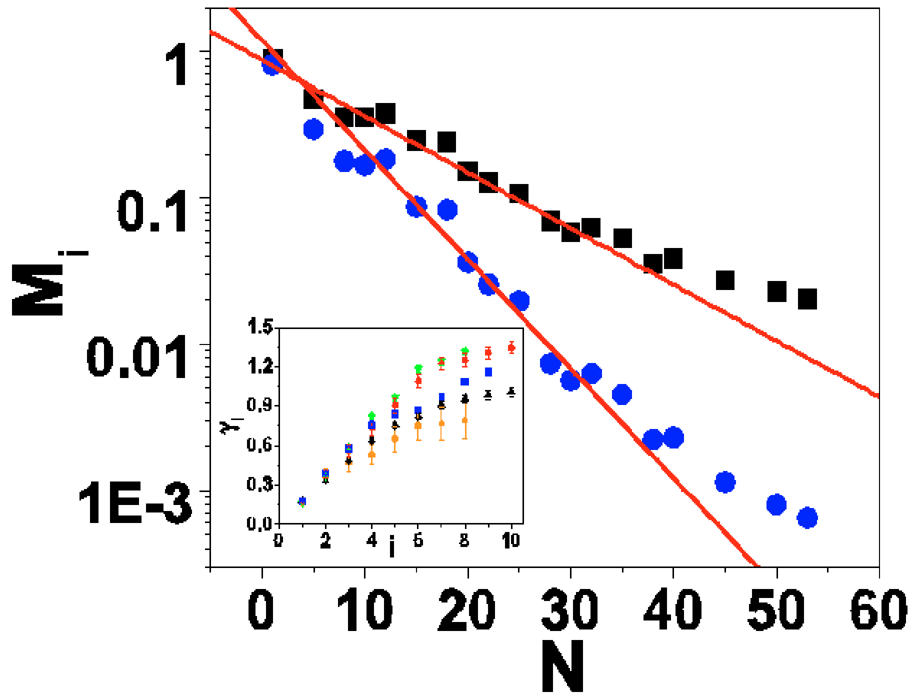

***Profiles of time average concentrations of fluorescent-conjugated dextran across the microchannel at different locations downstream at Wi=20***

# x-t patterns




Line profiles, as time advances, show mixing and persistence in boundary layers

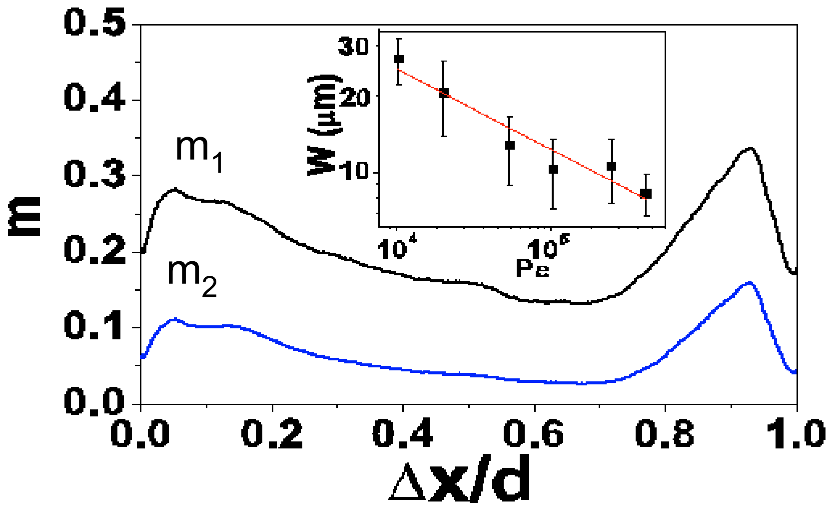
Moments of the scalar pdf decay exponentially downstream ✸ mixing length

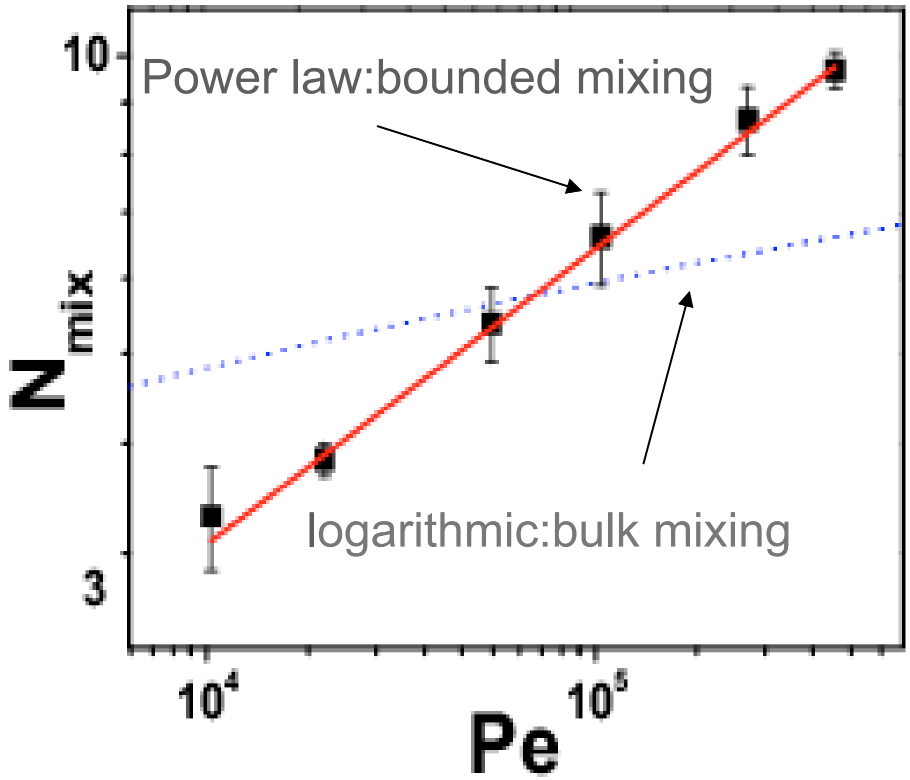


# Mixing as function of Peclet number

*Pe=Ud/D* is tuned up by using fluoresceine labelled dextranes (10kDa-2MDa)

Batchelor regime in a bounded system: boundary layers are a sink for the tracer.




This affects scaling:
theory: Chertkov and Lebedev (2003), $L_{mix} \sim Pe^{1/4}$
we get $N_{mix} \sim Pe^{0.26 \pm 0.01}$