



The Abdus Salam  
International Centre for Theoretical Physics



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**AUTUMN COLLEGE ON PLASMA PHYSICS**

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# Controlled Fusion by a Staged Z-pinch

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GTT International Inc.  
U.S.A.

# Controlled Fusion by a Staged Z-pinch

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**F. J. Wessel, and N. Rostoker**

University of California, Irvine, USA

Phys. Rev Lett. **74**, 714(1995)  
Phys. Of Plasmas, **58**, 367(1997)  
Phys. Of Plasmas, **11**, 5595(2004)

1. GTT International Inc.
2. San Jacinto College

6<sup>th</sup> Symposium on Current Trends in International Fusion Research: A Review,  
Washington DC 7-11 March 2005

# Outline

- Motivation from the experiments.
- 2D numerical simulation.
- Control and mitigation of RT-instability.
- Importance of high Z radiative liner.
- Possibility of breakeven in fusion energy.
- Experimental implementation.

# PINCH INSTABILITIES

## KINK m = 1

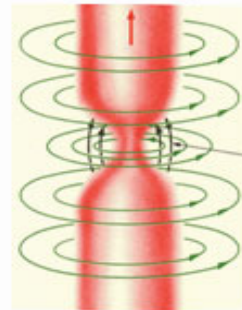


incompressible, sharp boundary:

$$\gamma = \frac{C_A(kr_0)}{r_0} \frac{I_m(kr_0)}{I'_m(kr_0)} \left[ 1 + \frac{m_2 K_m(kr_0)}{kr_0 K'_m(kr_0)} \right]$$

$$\sim (150 \text{ ns})^{-1}$$

## SAUSAGE m = 0



m = 0;

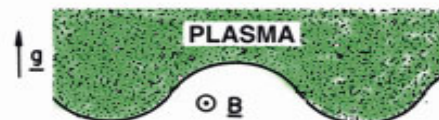
$$\gamma = \frac{C_A(kr_0)}{r_0} \frac{I_m(kr_0)}{I'_m(kr_0)} ;$$

compressible, k → 0;

$$\gamma = \frac{C_A}{r_0} (2 - \alpha^2)^{-1/2} kr_0$$

$$\sim (50 \text{ ns})^{-1}$$

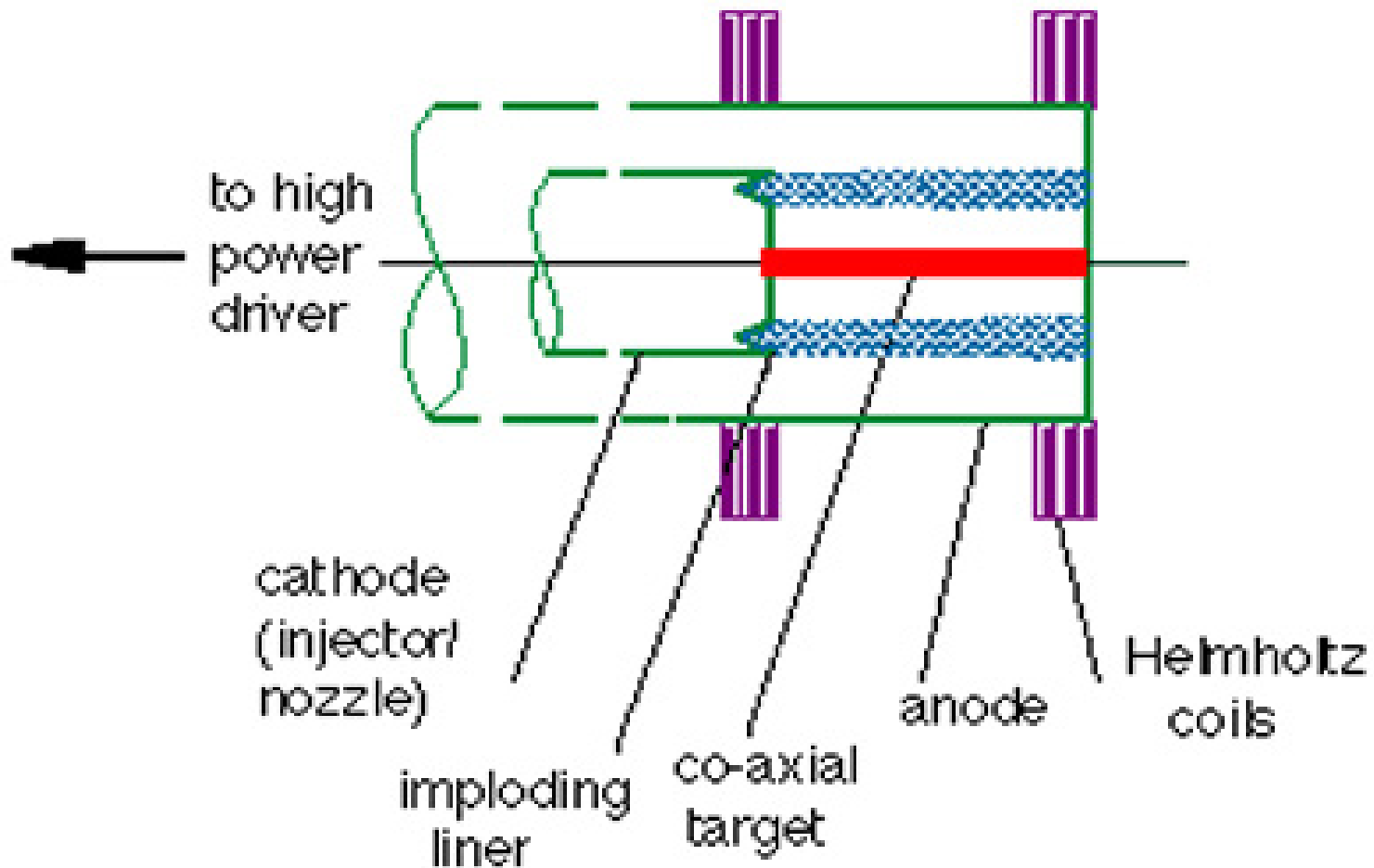
## RAYLEIGH TAYLOR



$$\gamma^2 = -kg + \frac{(\bar{k} \cdot \bar{B}_0)^2}{4\pi\rho_0}$$

$$\sim (10 \text{ ns})^{-1}$$

# Staged Z-pinch



# MAGNETO-INERTIAL FUSION

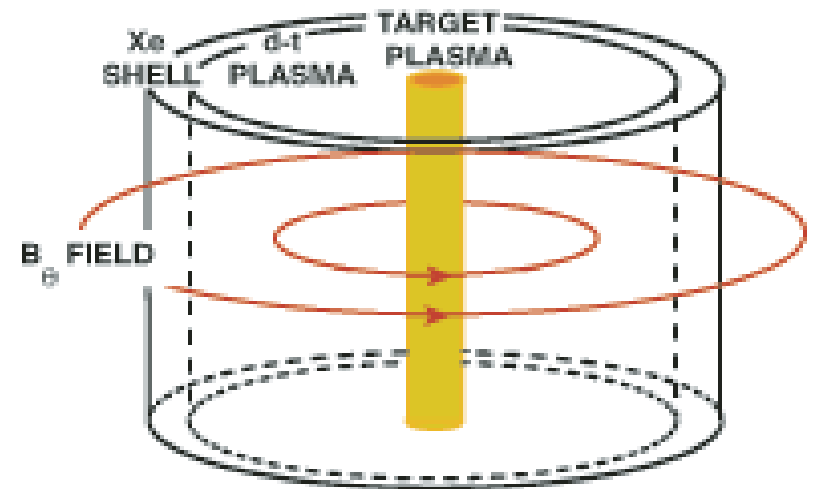
## PINCH DYNAMICS

CURRENT DIFFUSES THROUGH HIGH Z LINER  
INNER LAYER OF LINER PEALS OFF  
PEALED OFF LAYER COMPRESSES TARGET  
UNSTABLE PART OF LINER STAYS BEHIND  
AT PEAK COMPRESSION, CURRENT  
TRANSFERS TO INNER STABLE LAYER.

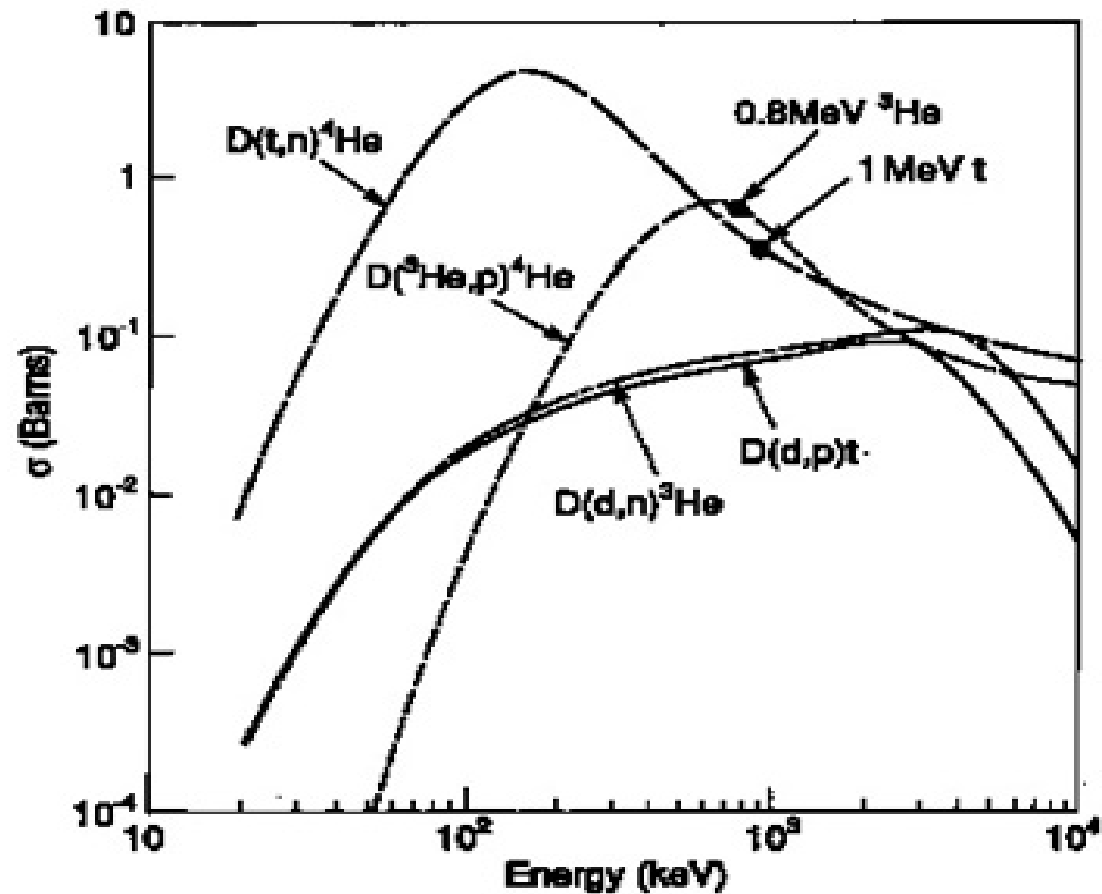
## BENEFITS

INERTIAL ENERGY TRANSFER TIMESCALES  
COMPRESSION IS RT STABLE  
BREAKEVEN FUSION IS PREDICTED

## STAGED Z-PINCH



# Fusion Cross Sections

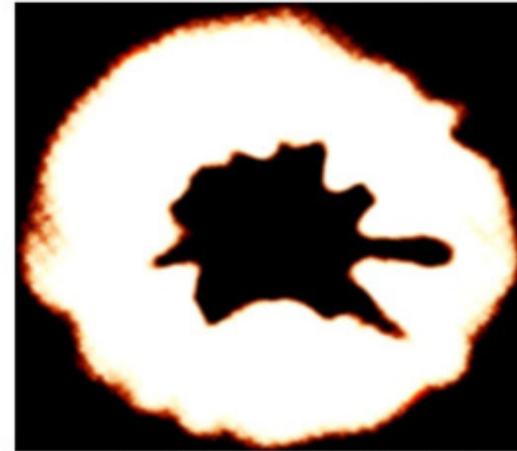


# STABILIZATION OF LINEAR PINCH

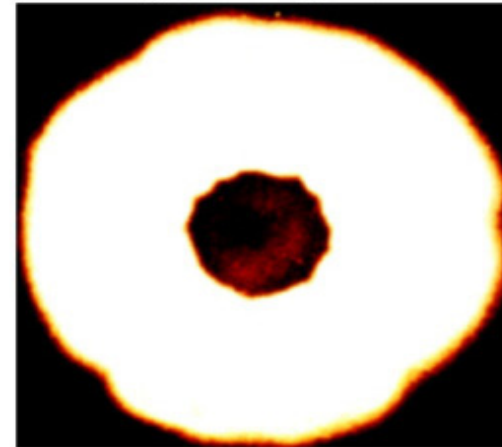
## Stability of RT-instability

End-on Kerr cell photos

Unstable pinch



Pinch stabilized with  
 $B_z$  and  $B_\theta$  fields

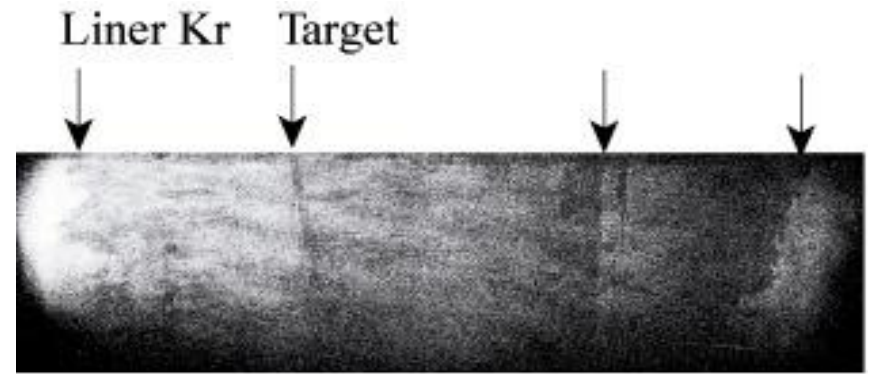


D.J.Albares, N. A. Krall and C. L. Oxley,  
"Rayleigh Taylor Instability in a  
Stabilized Linear Pinch Tube,"  
Phys. Fluids 4, 1031(1961).

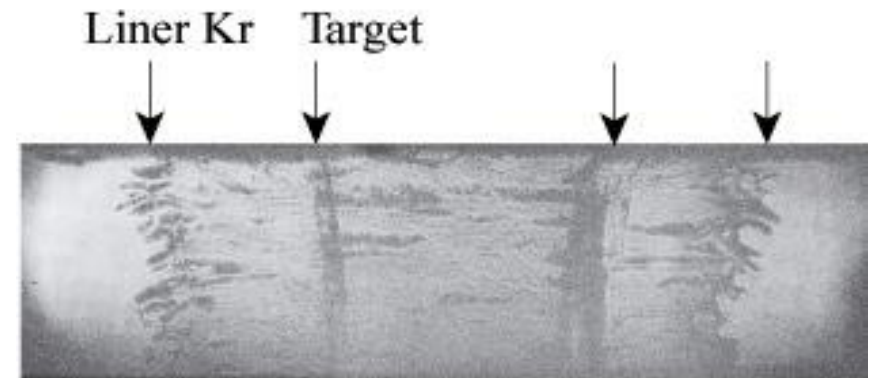


# Schlieren Images of Staged Z Pinch

Before peak implosion

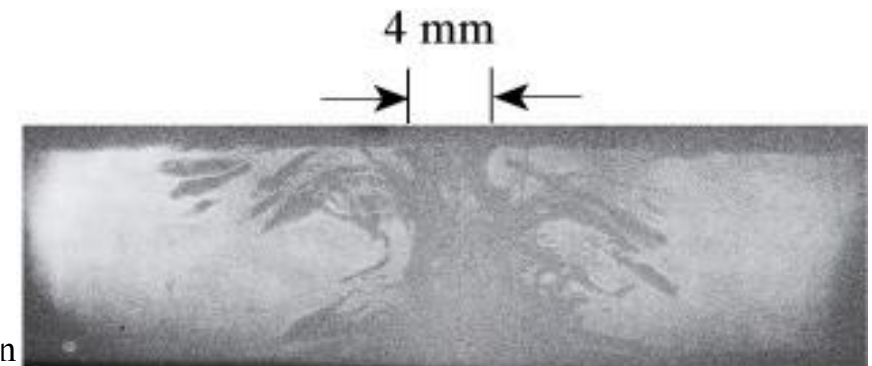


Shot 963 (a)



Shot 961 (b)

After peak implosion

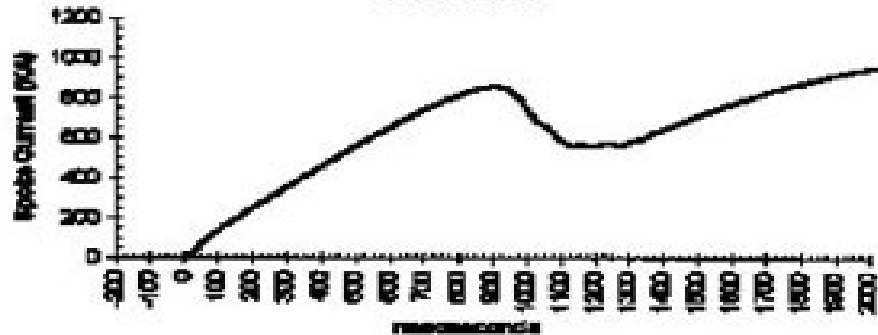
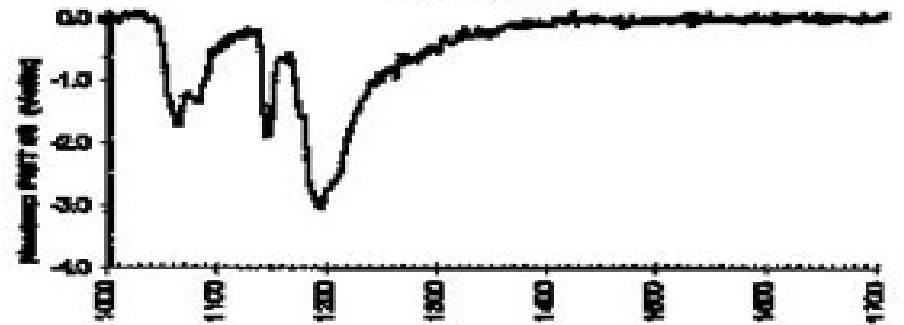
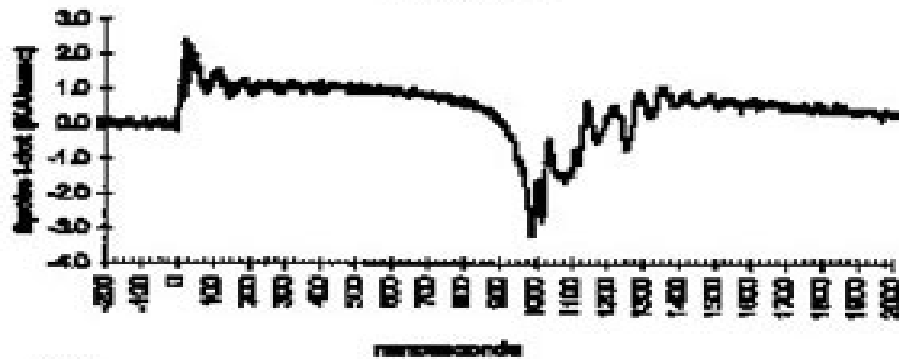
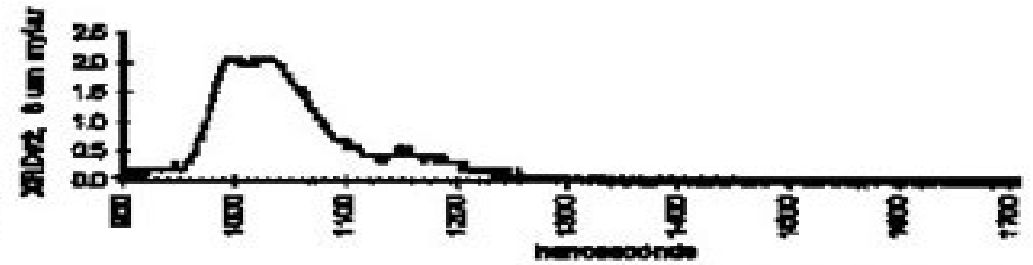
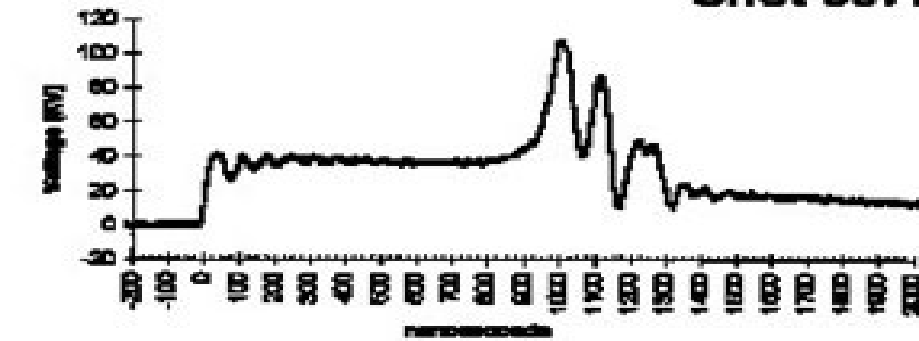


Shot 962 (c)

n  
al

# Electrical Signals

Shot 697: Kr + D2 + 100 G

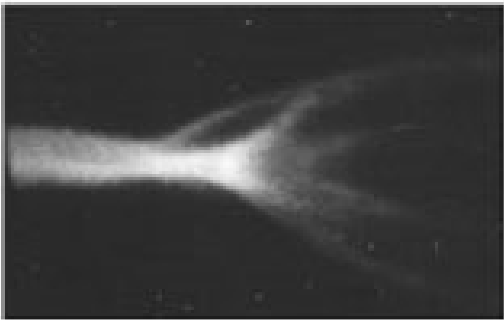


# Streak Images of Staged Z-pinch

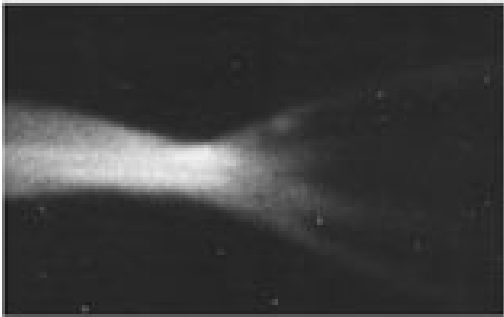
**Kr liner only**



**Kr + D2 gas**



**Kr + D2 + 100 G**



↑  
5 cm  
↓

← 200 ns →

# Numerical Simulation

- 2&1/2 dimensional, time-dependent, single fluid, MHD simulation code.
- Used in Eulerian mode.
- External capacitor bank circuit is modeled.
- Tabular (SESAME) equations of state.
- Implicit MHD with components of **B** and **U**.
- Multi-species plasma.
- Flux-limited, single group, implicit radiation diffusion.

# Equation used in the simulation

MACH2

Continuity Equation:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{u})$$

Momentum Equation:

$$\rho \frac{\partial v^i}{\partial t} = -\rho v^j \nabla_j v^i + \nabla_j \left[ -(P + Q + \frac{1}{3} u_R) \delta^{ji} + \frac{1}{\mu_0} (B^j B^i - \frac{1}{2} B^2 \delta^{ji}) + \sigma_{ji}^d \right]$$

Electron Specific Energy Equation:

$$\rho \frac{\partial \epsilon_e}{\partial t} = -\rho \vec{v} \cdot \nabla \epsilon_e - P_e \delta^{ji} \nabla_i v_j + \eta J^2 - \vec{J} \cdot \left( \frac{\nabla P_e}{en_e} \right) + \nabla \cdot (\kappa_e \nabla T_e) - ac\rho\chi_{plank}(T_e^4 - T_R^4) - \rho c_{v_e} \frac{(T_e - T_i)}{\tau_{ei}}$$

Ion Specific Energy Equation:

$$\rho \frac{\partial \epsilon_i}{\partial t} = -\rho \vec{v} \cdot \nabla \epsilon_i + [-(P_i + Q) \delta^{ji} + \sigma_{ji}^d] \nabla_i v_j + \nabla \cdot (\kappa_i \nabla T_i) + \rho c_{v_e} \frac{(T_e - T_i)}{\tau_{ei}}$$

Radiation Energy Density:

$$\frac{\partial u_R}{\partial t} = -\rho \vec{v} \cdot \nabla u_R - \frac{4}{3} u_R \nabla \cdot \vec{v} + \nabla \cdot (\rho \chi_{ros} \nabla u_R) + ac\rho\chi_{plank}(T_e^4 - T_R^4)$$

Magnetic Induction:

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) - \nabla \times (\eta \vec{J}) - \nabla \times \left( \frac{\vec{J} \times \vec{B}}{en_e} \right) + \nabla \times \left( \frac{\nabla P_e}{en_e} \right)$$

Elastic Stress:

$$\frac{\partial \sigma_{ji}^d}{\partial t} = 2\mu d_{ji}^d - v^k \nabla_k \sigma_{ji}^d$$

Fusion Neutron Production Rate and Energy Gain:

$$P_{DT} = 5.6 \times 10^{-13} n_D n_T (\sigma \nu)_{DT}$$

$$P_{DD} = 3.3 \times 10^{-13} n_D n_D (\sigma \nu)_{DD}$$

$(\sigma \nu)_{DT}$  and  $(\sigma \nu)_{DD}$  are determined from a table look up.

**Elastic Stress:**

$$\frac{\partial \sigma_{ji}^d}{\partial t} = 2\mu d_{ji}^d - \nu^k \nabla_k \sigma_{ji}^d$$

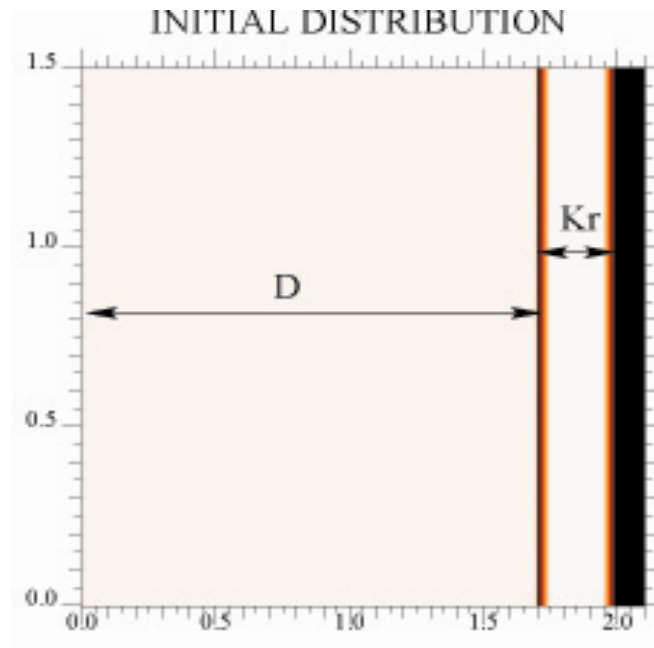
**Fusion Neutron Production Rate and Energy Gain:**

$$P_{DT} = 5.6 \times 10^{-13} n_D n_T (\bar{\sigma} \nu)_{DT}$$

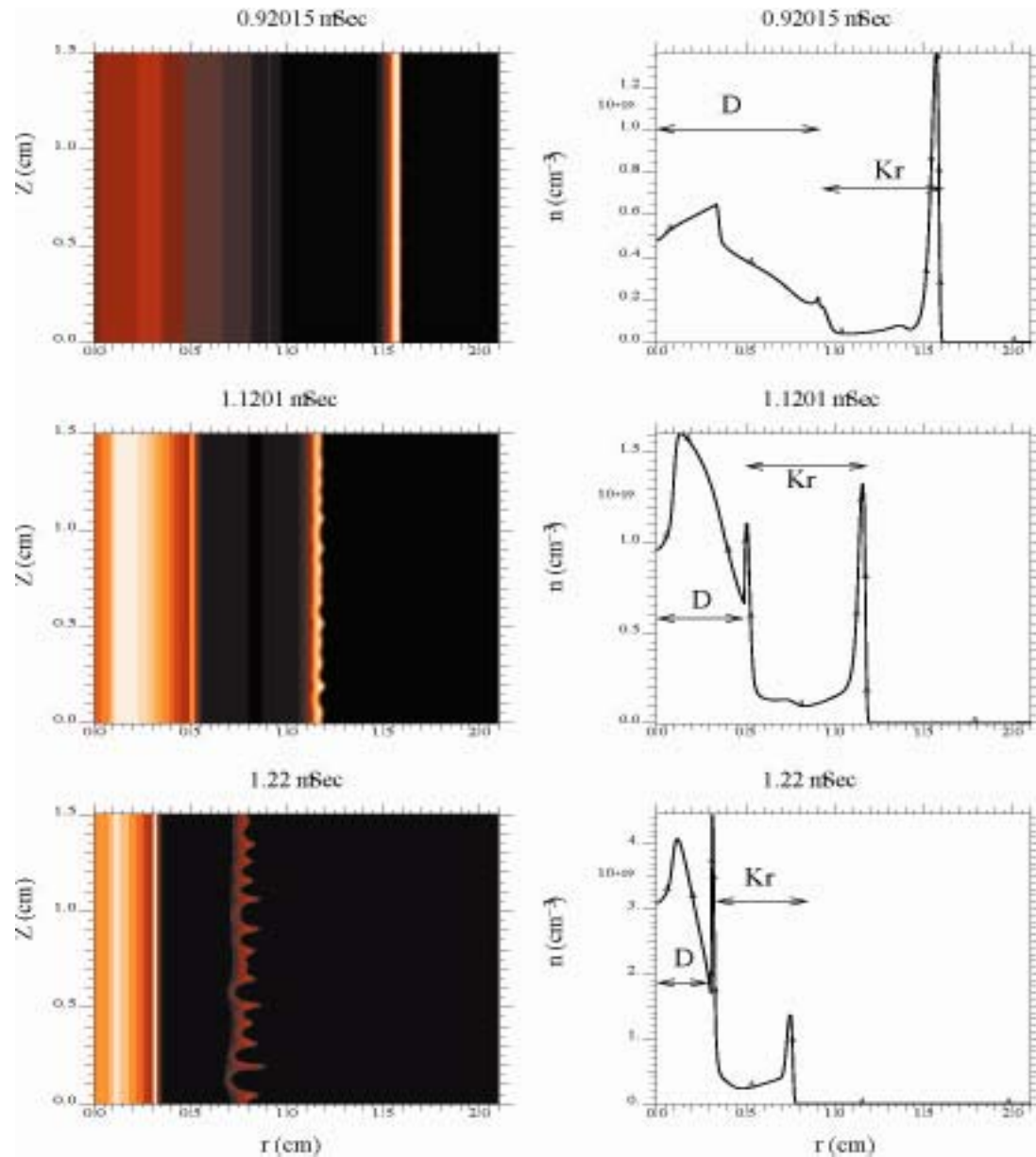
$$P_{DD} = 3.3 \times 10^{-13} n_D n_D (\bar{\sigma} \nu)_{DD}$$

$(\bar{\sigma} \nu)_{DT}$  and  $(\bar{\sigma} \nu)_{DD}$  are determined from a table look up.

# Initial configuration for UCI Pinch

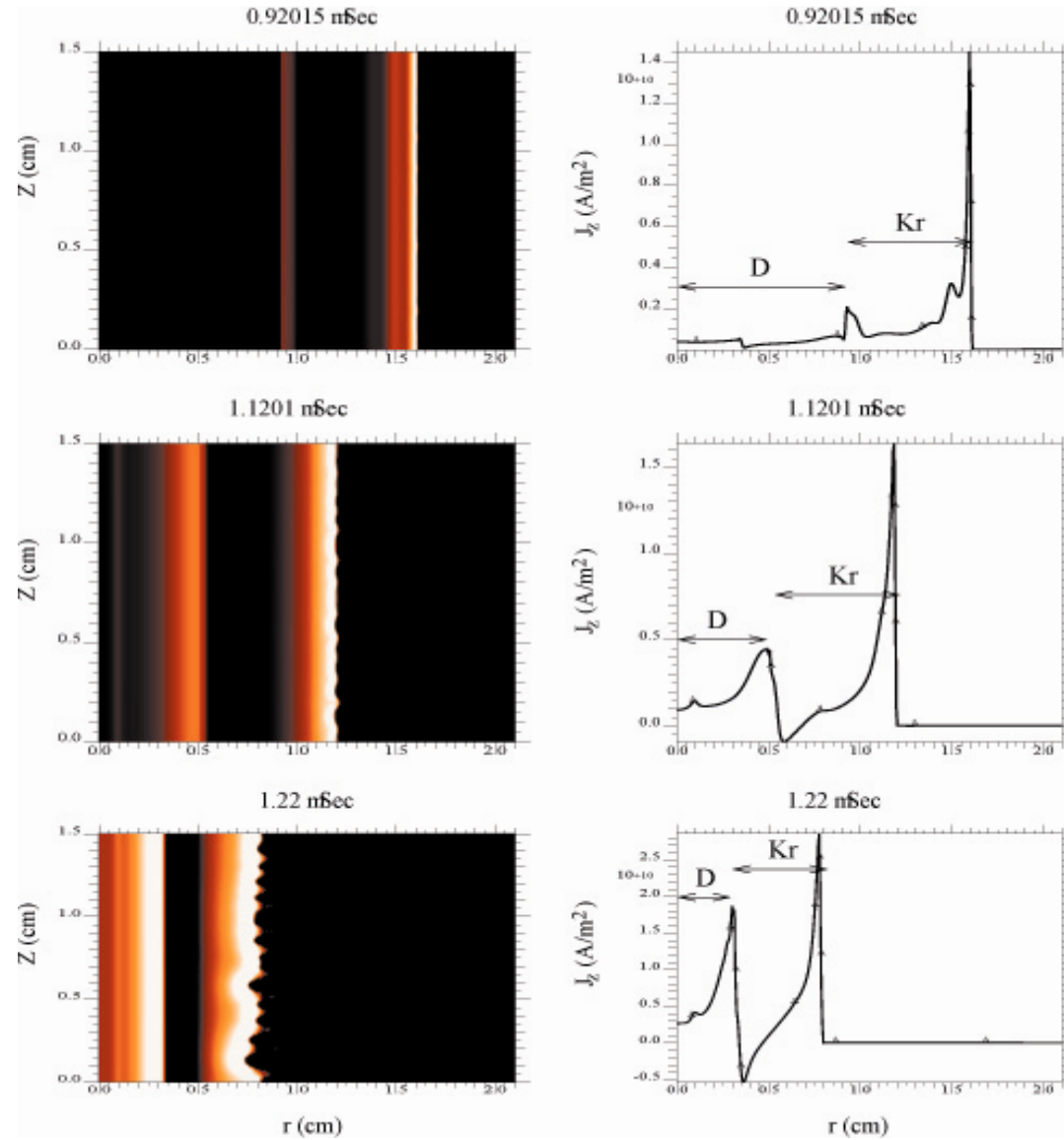


# Ion density During run-in phase

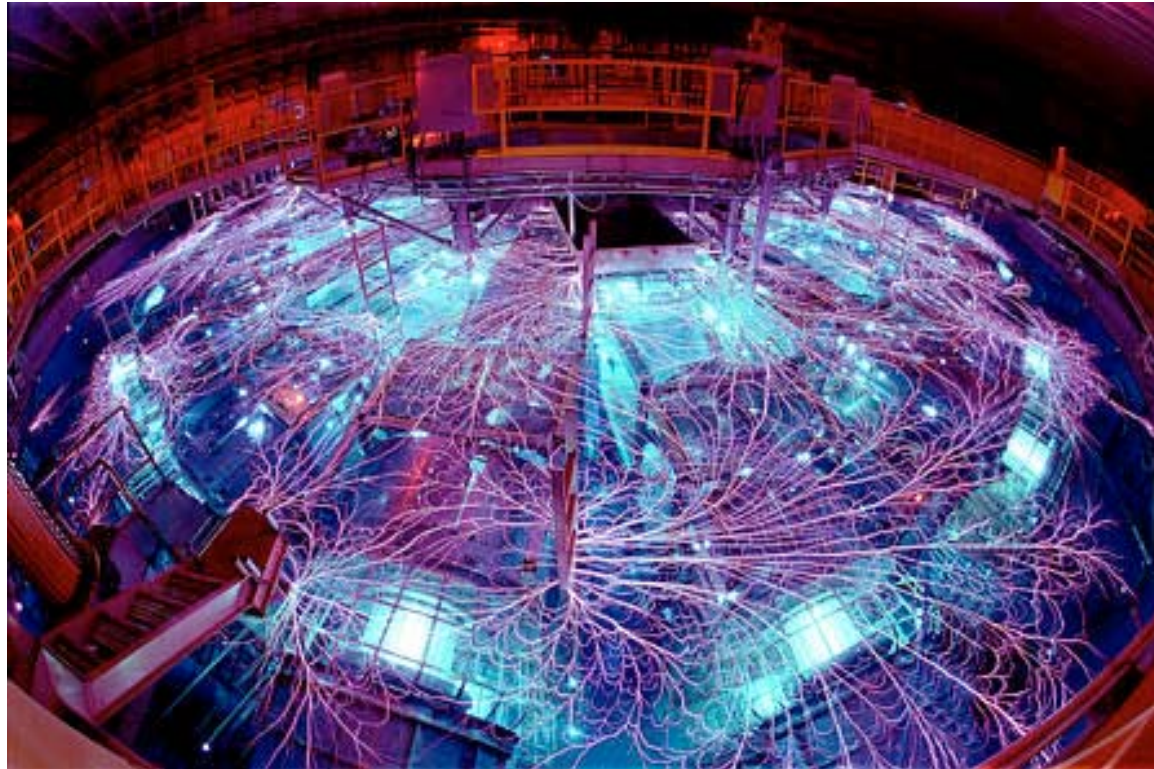




# Current Density During run in phase



# Z Machine



6<sup>th</sup> Symposium on Current Trends in International Fusion Research: A Review,  
Washington DC 7-11 March 2005

# Initial Configuration For Z-Facility

## Load parameters

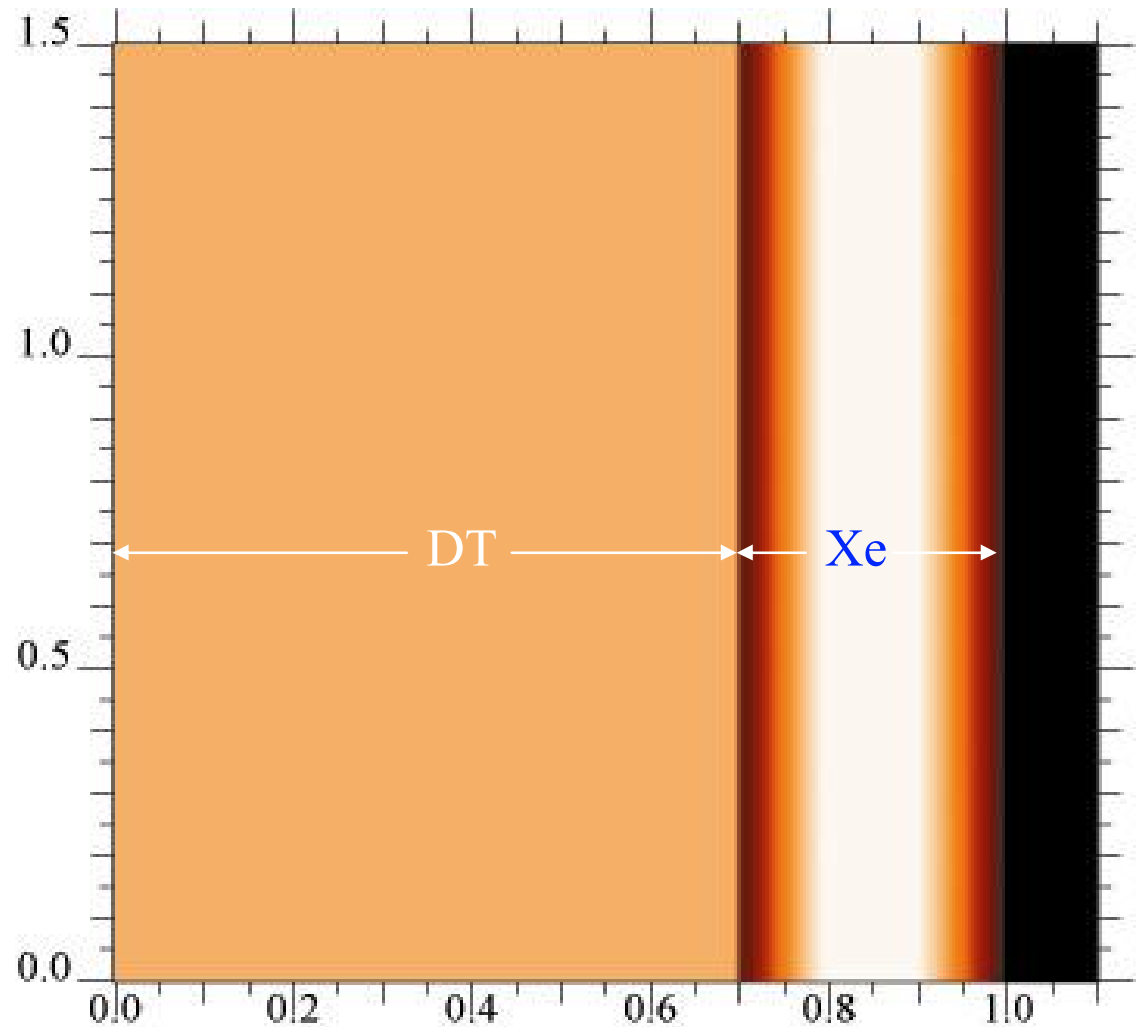
Density of Xe = 43 kg/m<sup>3</sup>  
Density of DT = 0.2 kg/m<sup>3</sup>  
Perturbation = 0.1%

## Z-parameters

Maximum current = 18 MA  
Current rise time = 90 ns  
Stored Energy = 2.1 MJ

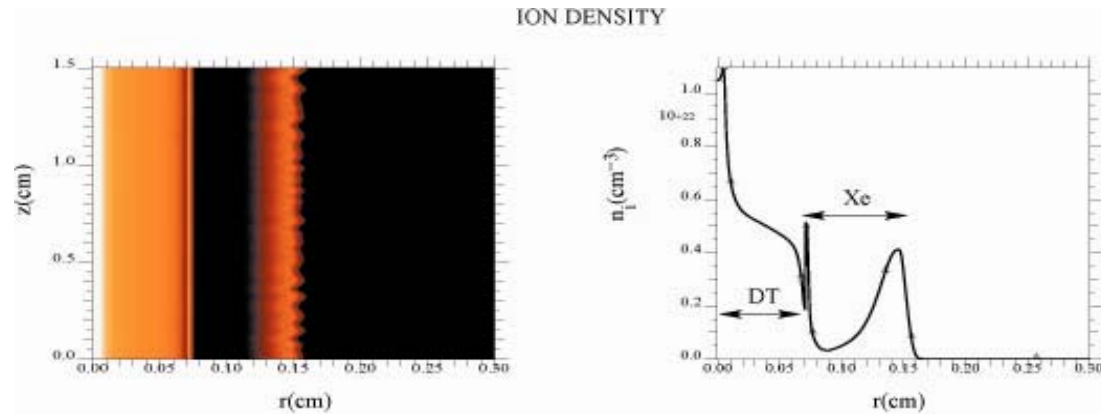
## GRID

352x128

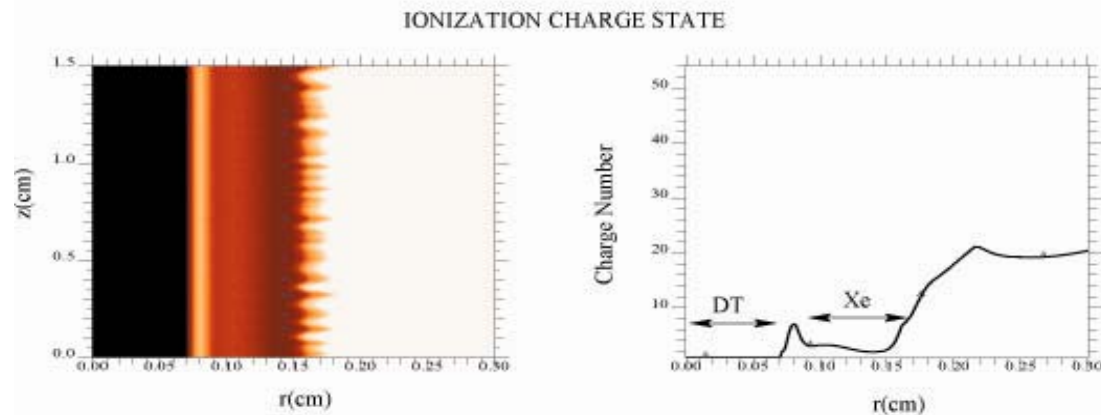


# Dynamics of pinch During run-in Phase

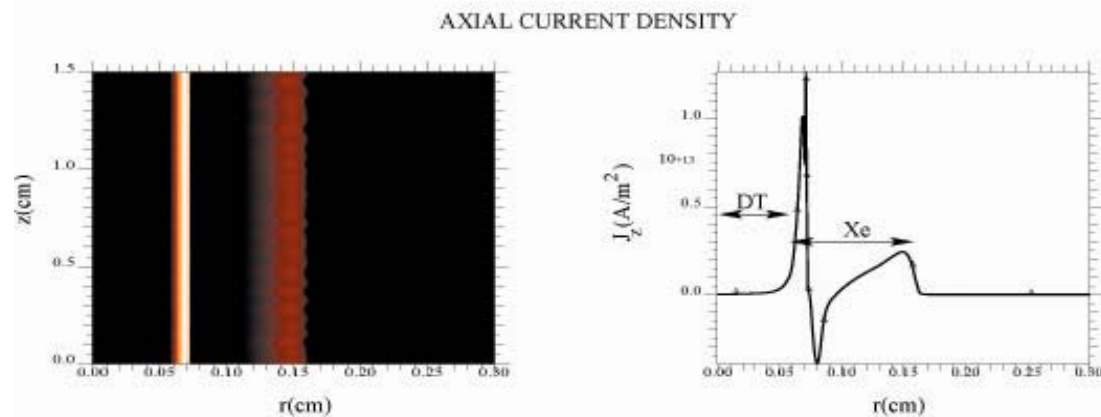
Density



Ionization Charge state



Axial current density



# Rayleigh-Taylor Instability

$$\xi = \xi_0 e^{\gamma t}$$

 $\xi_0$ 

Initial perturbations

$$\gamma = \sqrt{gk}$$

 $R$ 

Pinch radius

$$R = \frac{1}{2} g t^2$$

 $k$ 

Wave number

$$\xi = \xi_0 e^{\sqrt{2Rk} t}$$

 $g$ 

Pinch Acceleration

*Growth of perturbations depend upon the radius of the pinch*

# Energy coupling

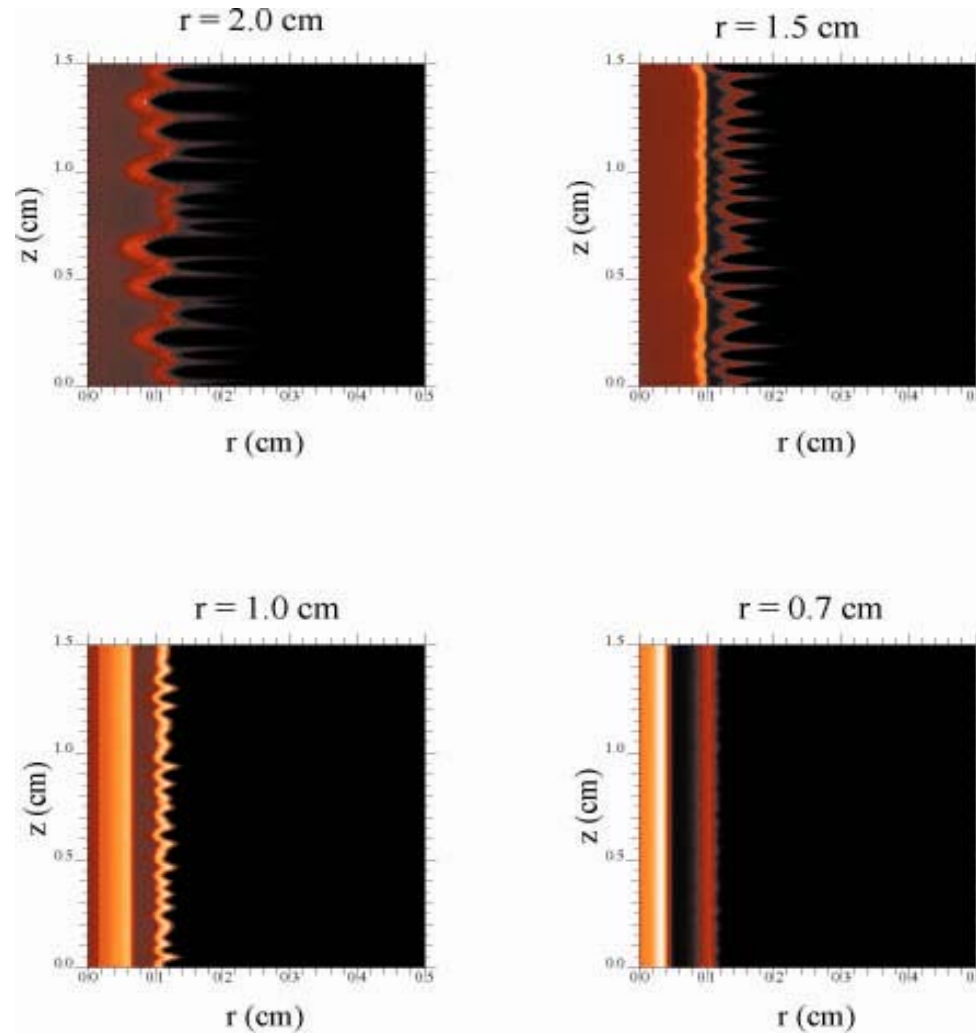
$$W = \int_{R_0}^R \frac{B_\theta^2}{8\pi} \cdot 2\pi R \cdot h dR$$

$$B_\theta = \frac{2I}{cR}$$

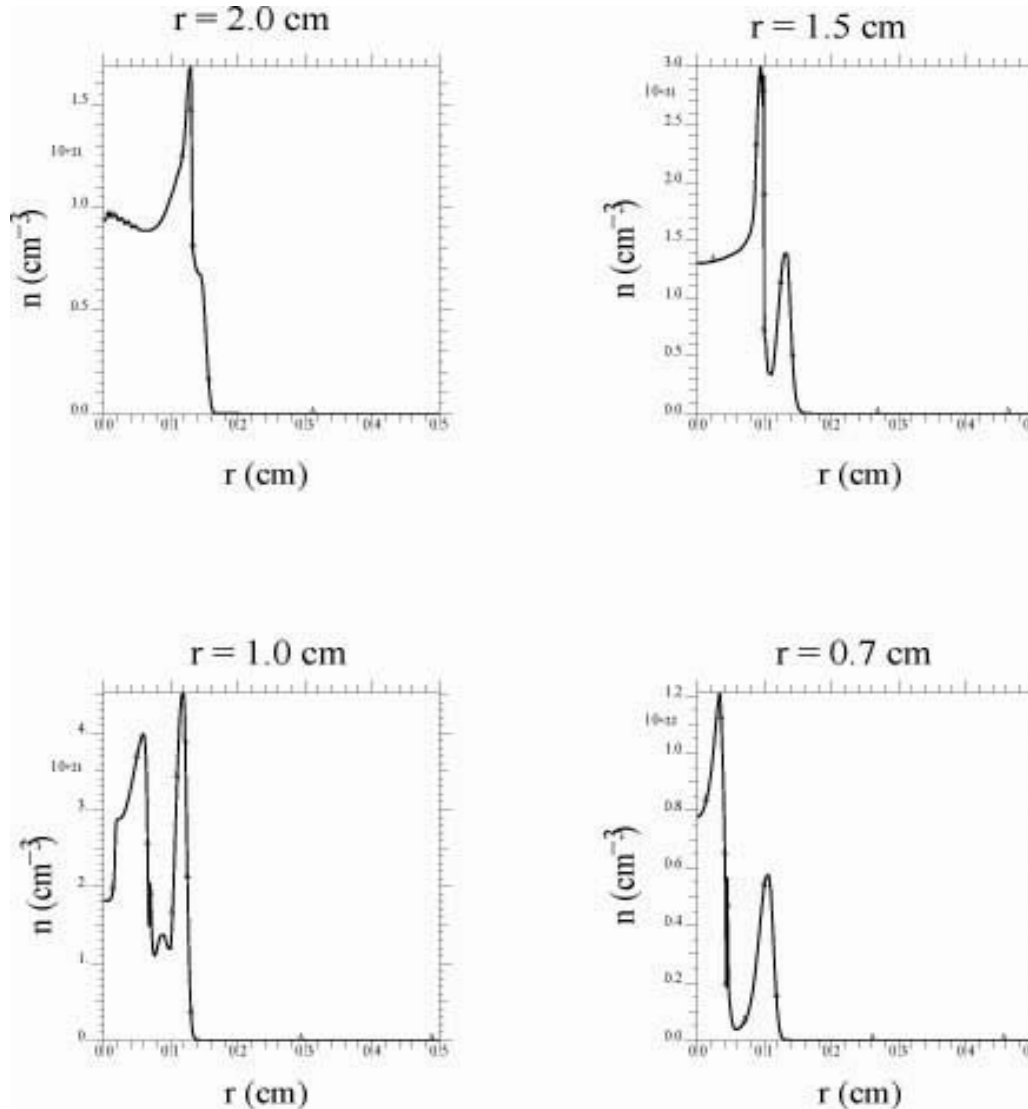
$$= h \left( \frac{I}{c} \right)^2 \ln \left( \frac{R_0}{R} \right)$$

**Final energy of the pinch depends weakly on the compression ratio!**

# ION DENSITY (Four initial radii)

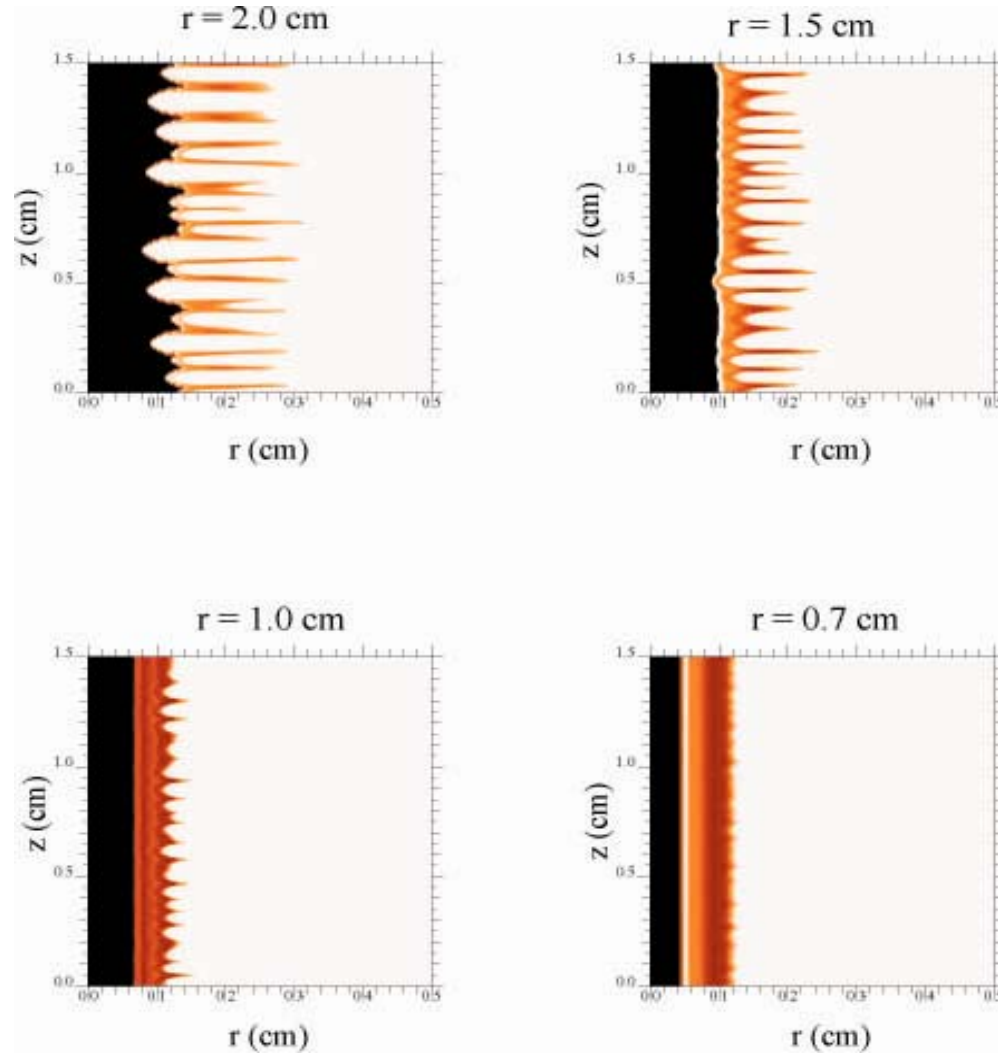


# Ion density profiles

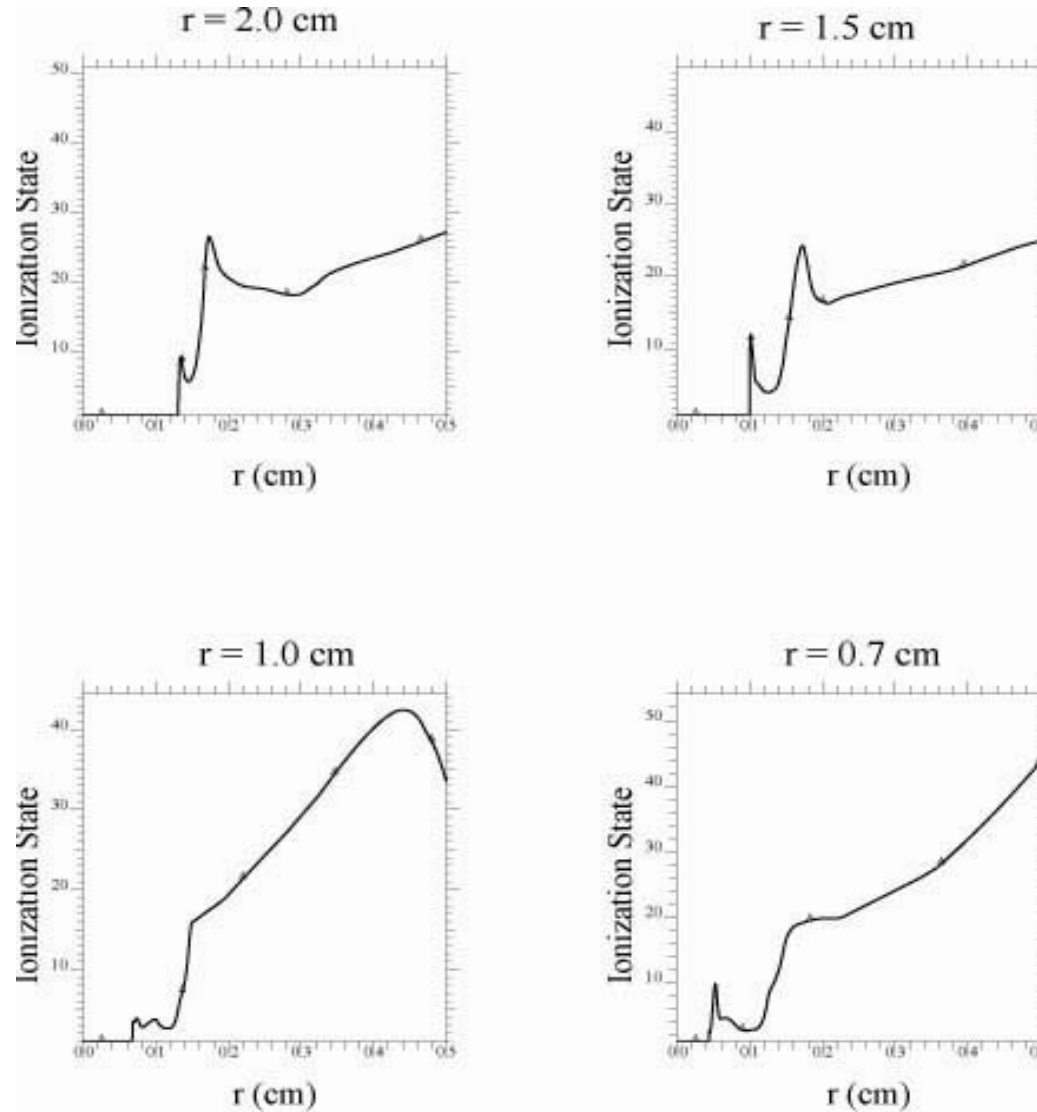




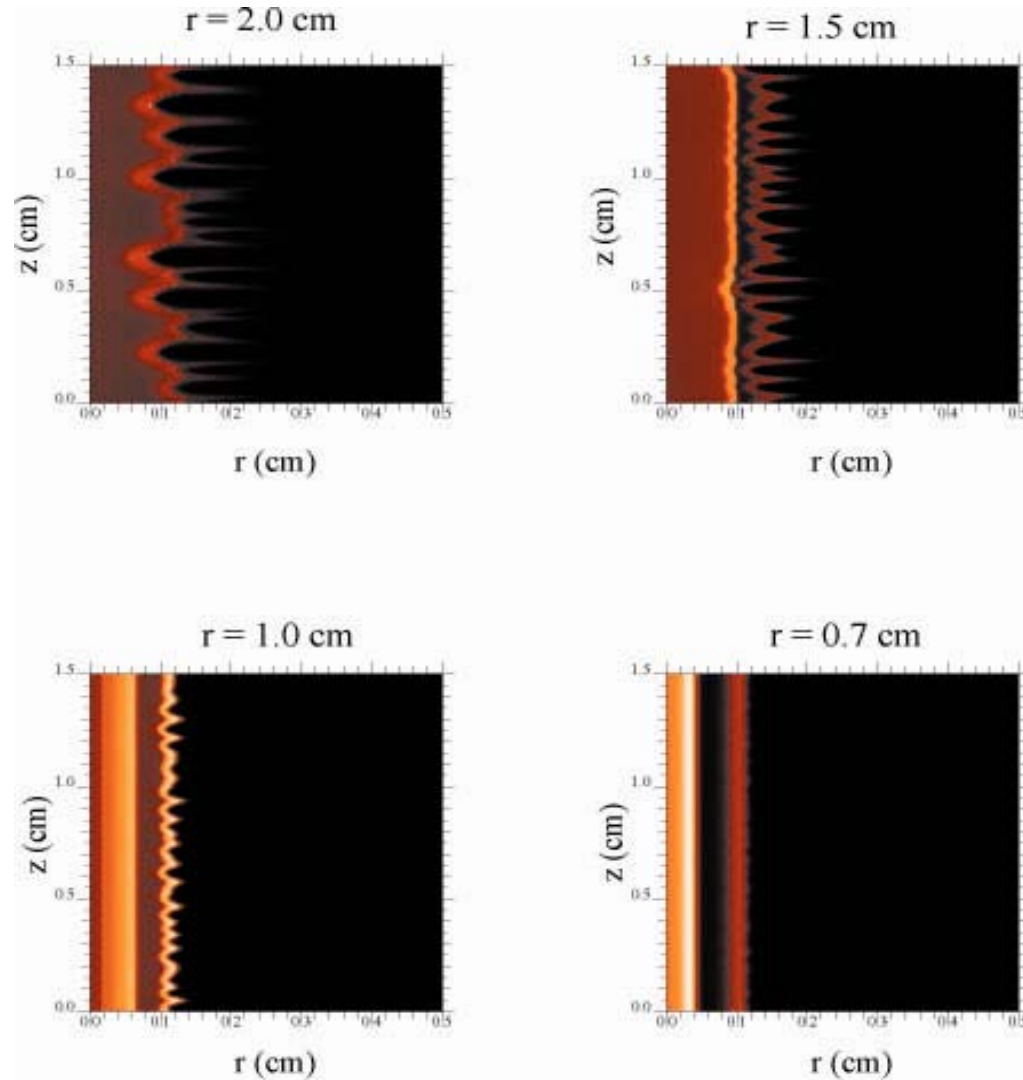
# Ionization charge state (four initial radii)



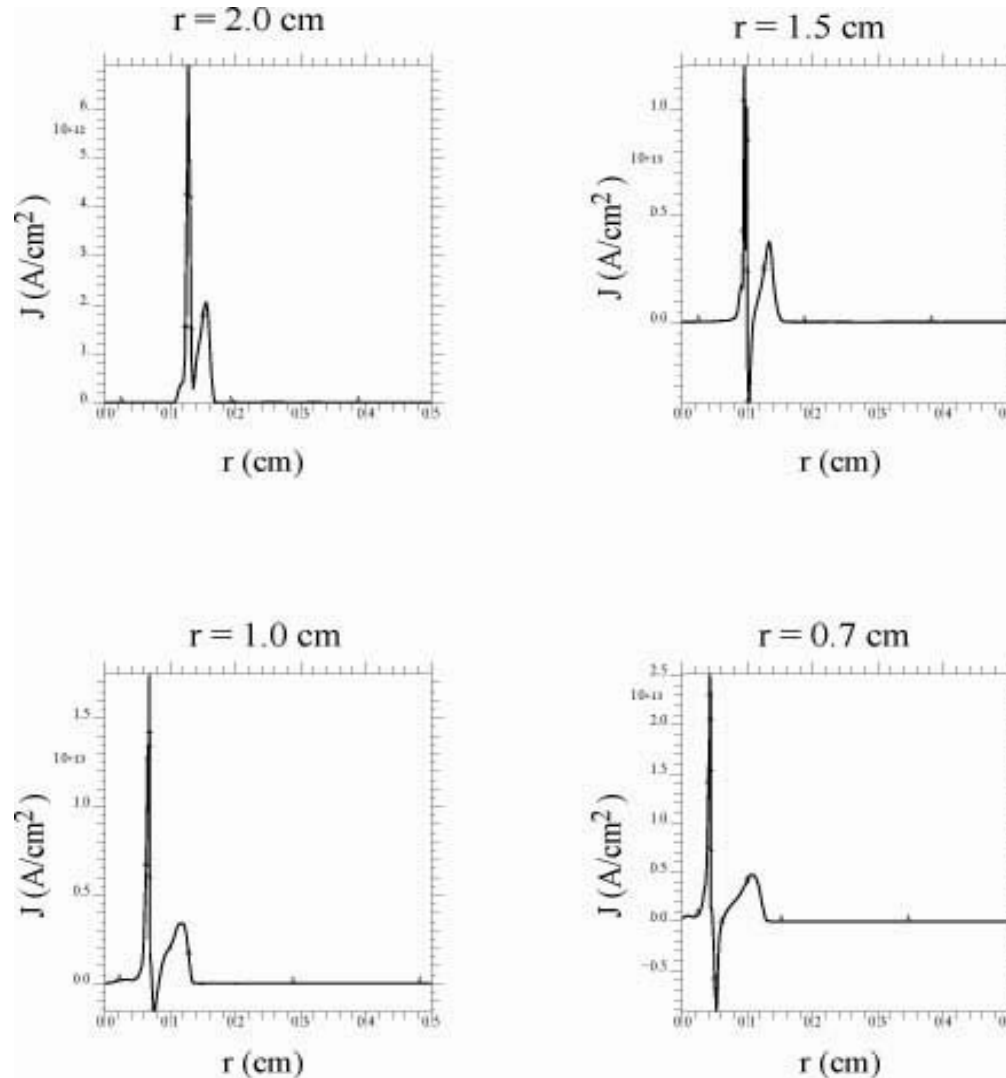
# Profiles of charge state



# Axial current (four initial radii)



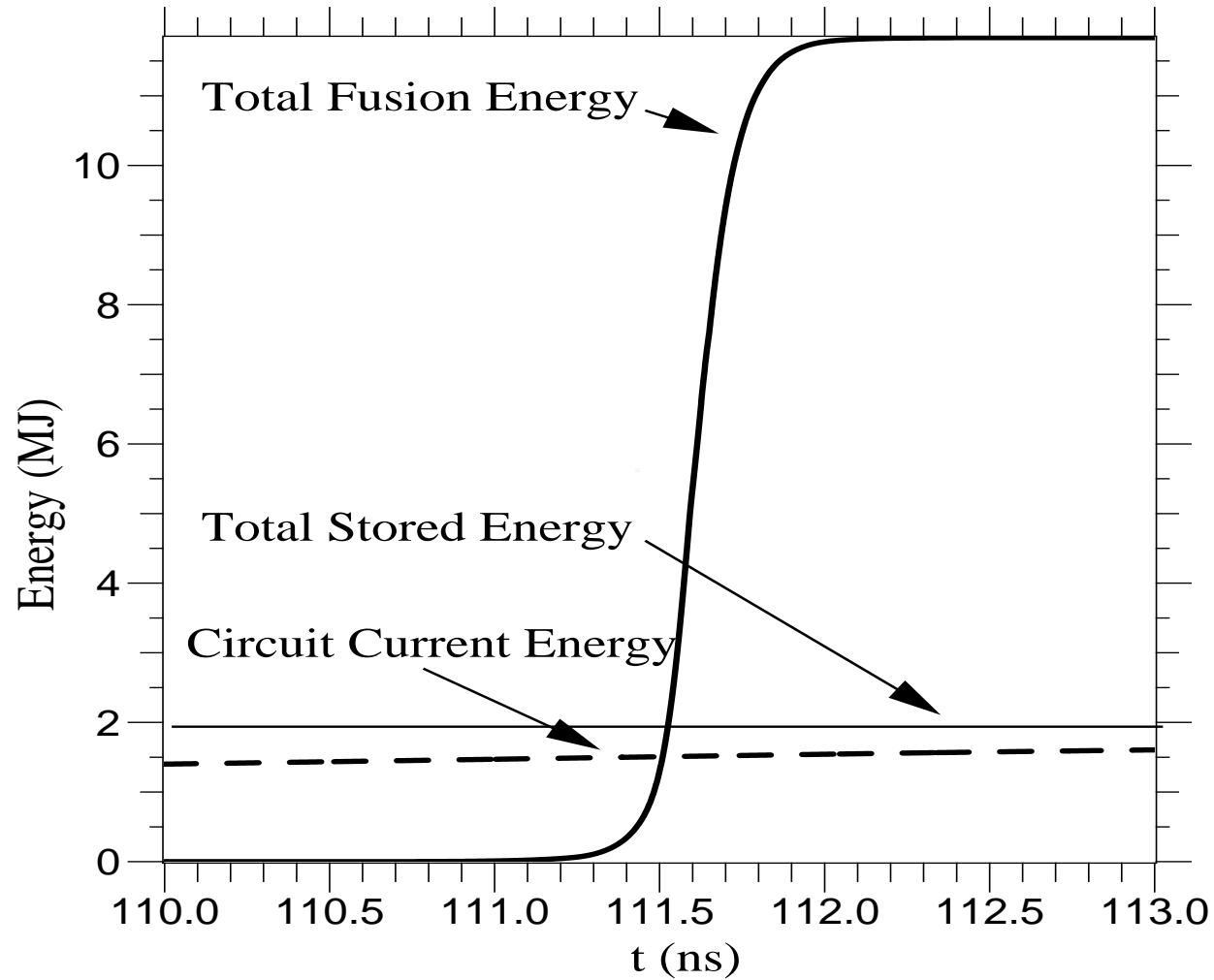
# Profiles of axial currents



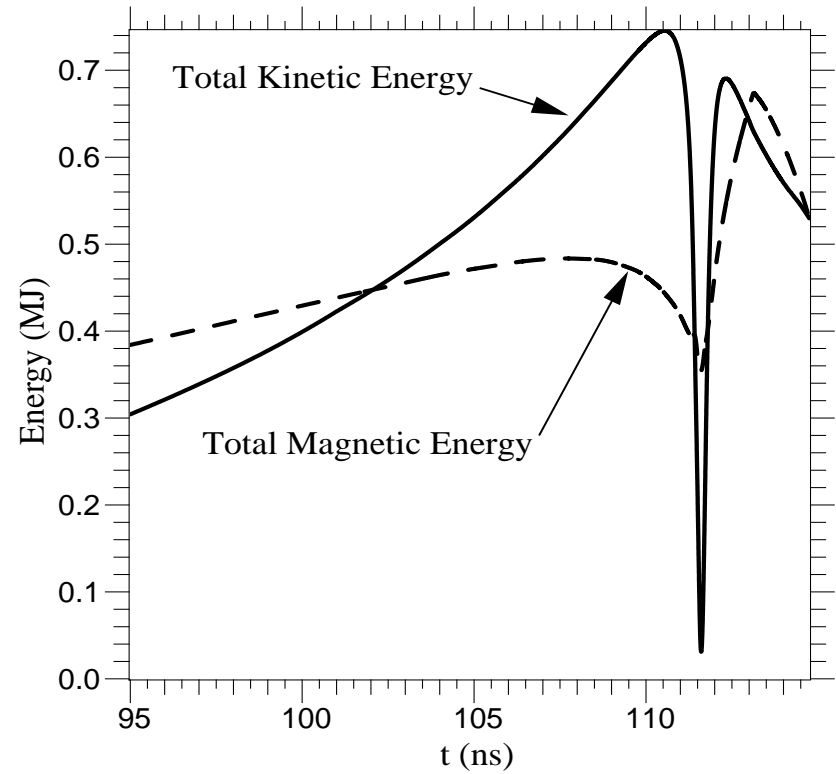
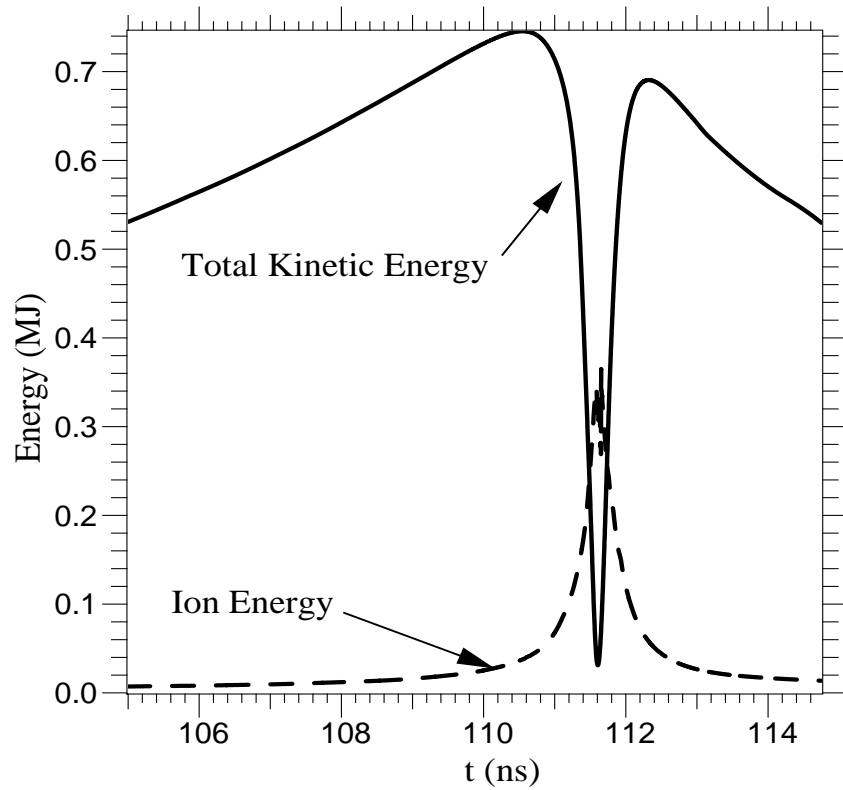
# Large Energy Production

- Xe liner is used
- Both the masses of the liner and the target are optimized
- Optimized parameters of the Z-facility are used
- Initial radius of 0.7cm is used
- Perturbation level of 0.1% is used
- 12 MJ of Energy is produced with a stored energy of 2 MJ.
- Real breakeven is possible with existing technology

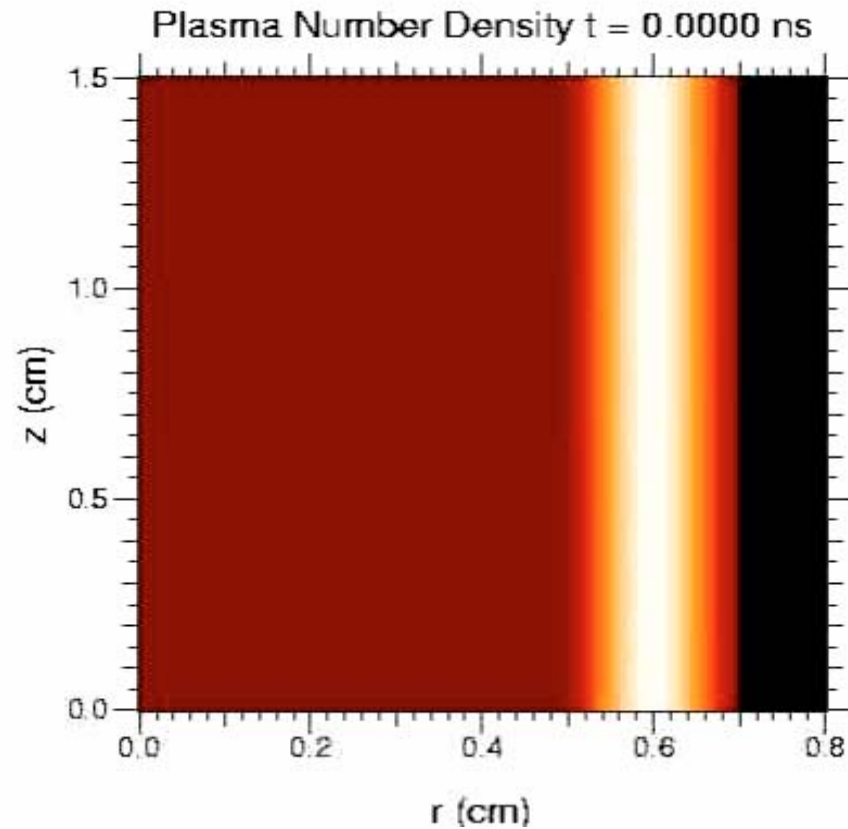
# Breakeven



# Energies

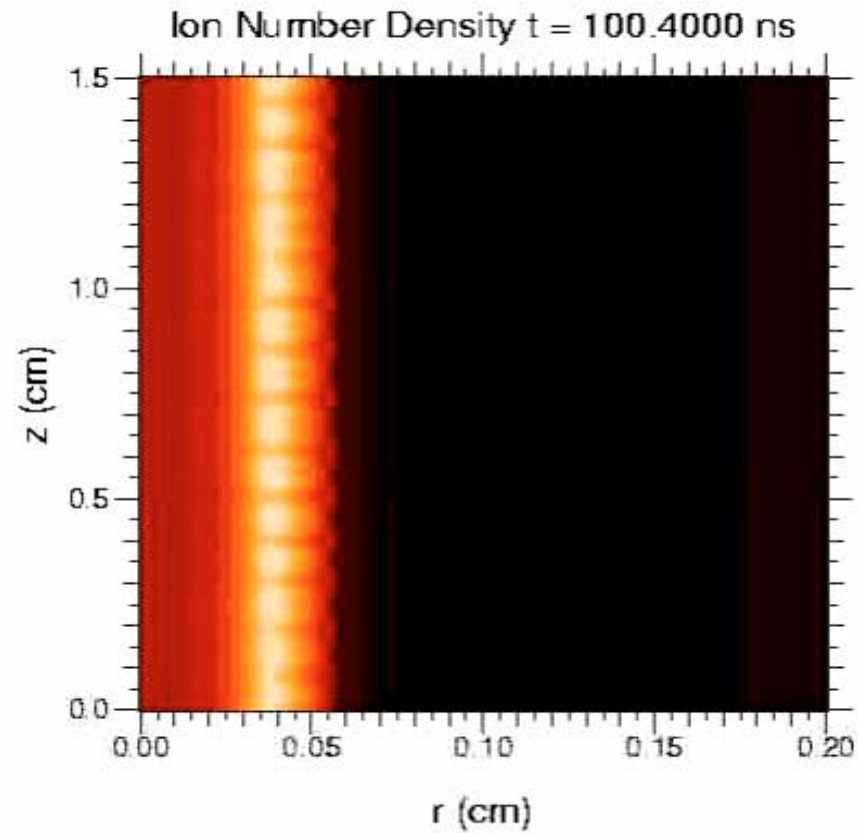


# Ion Density (full run)

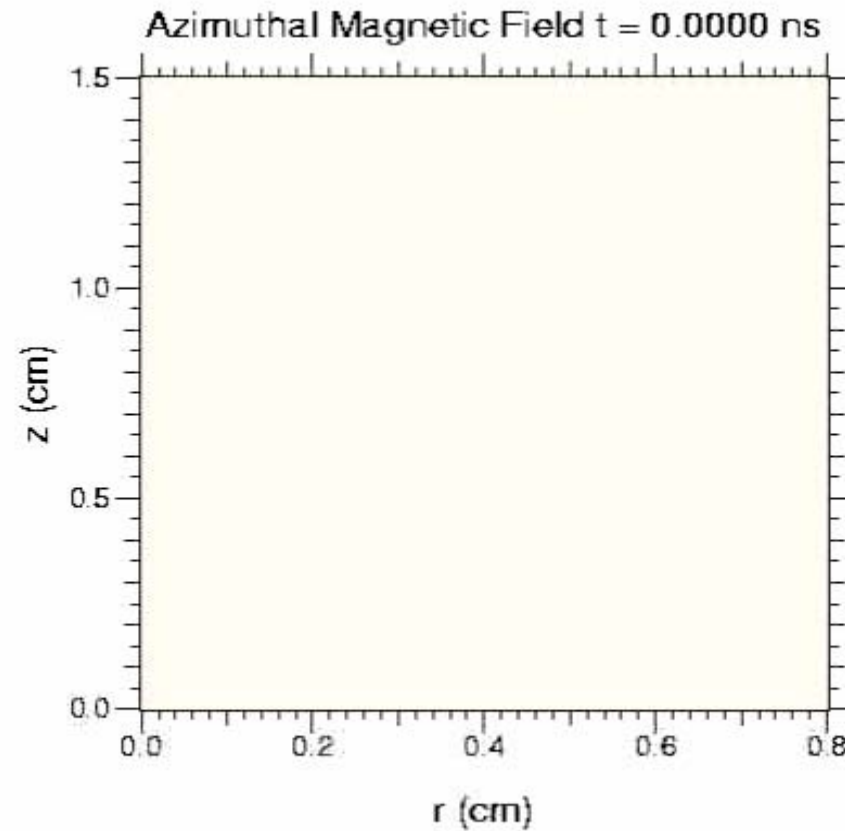




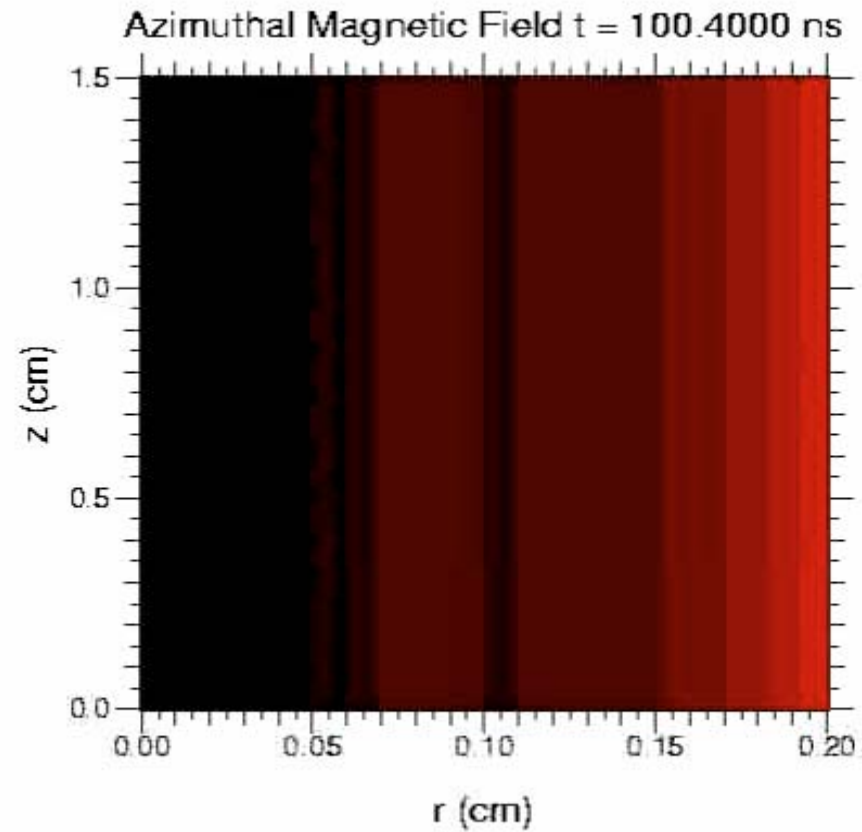
# Ion Density (near peak)



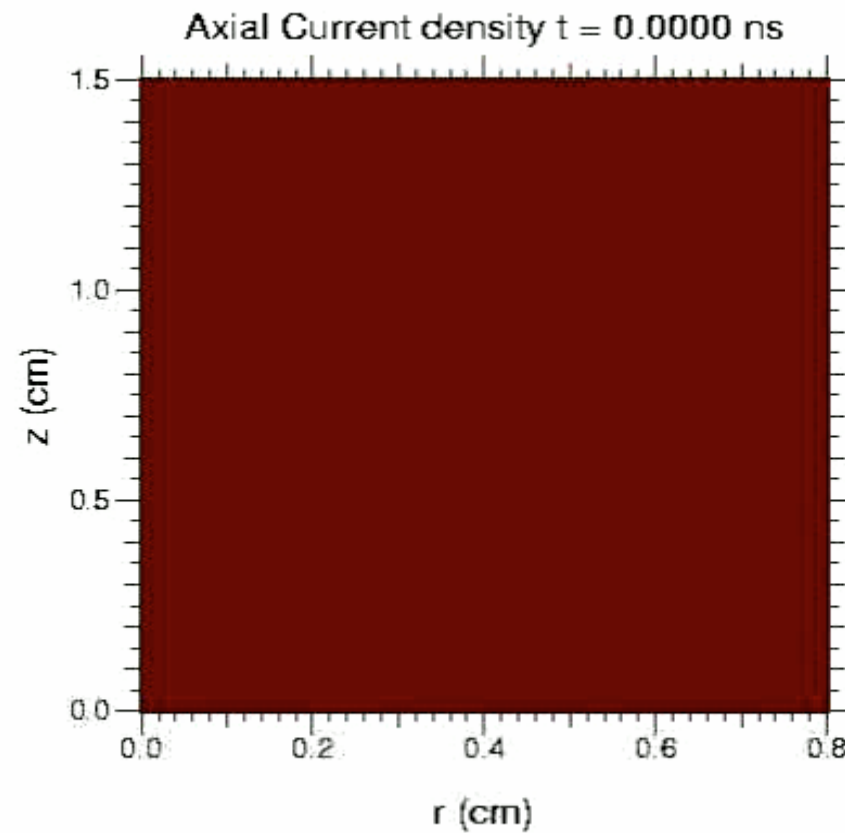
# Magnetic Field (Full run)



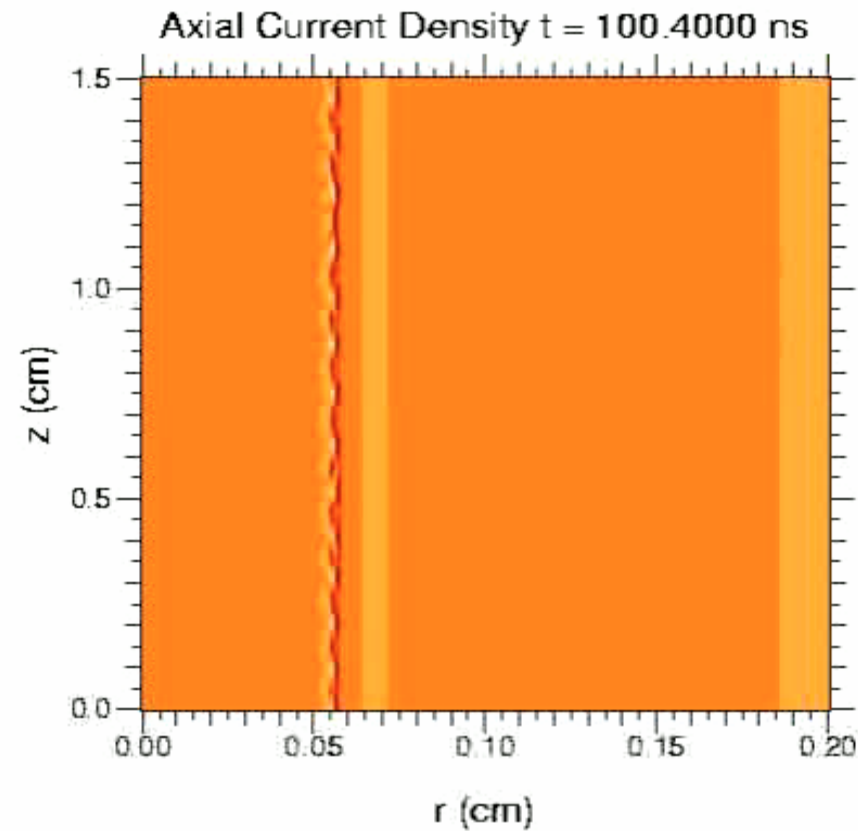
# Magnetic field (near peak)



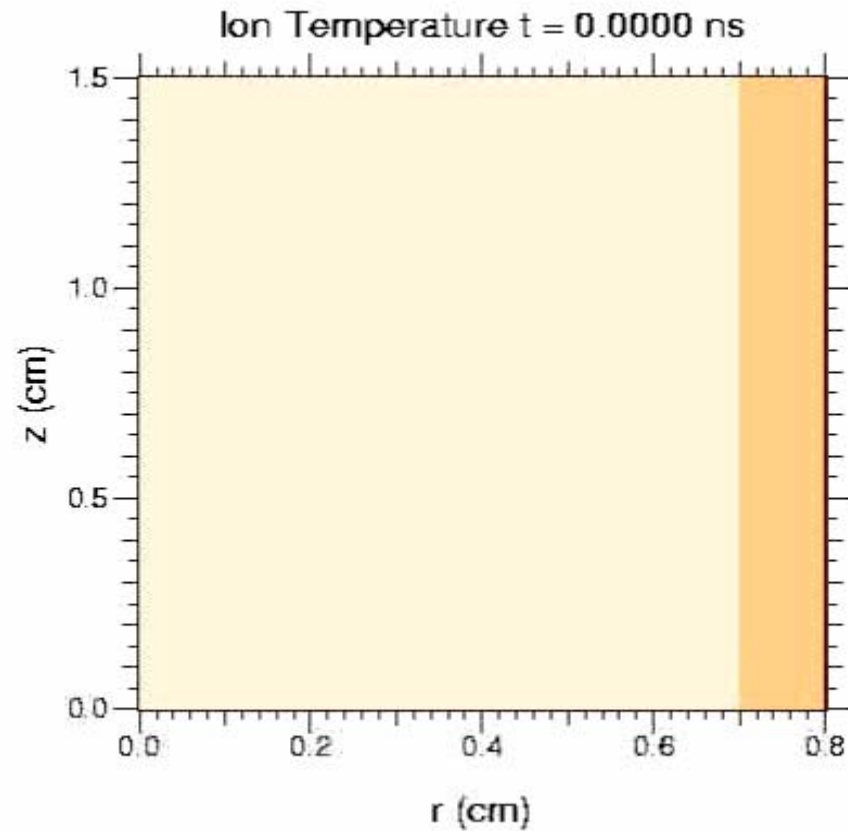
# Axial current (full run)



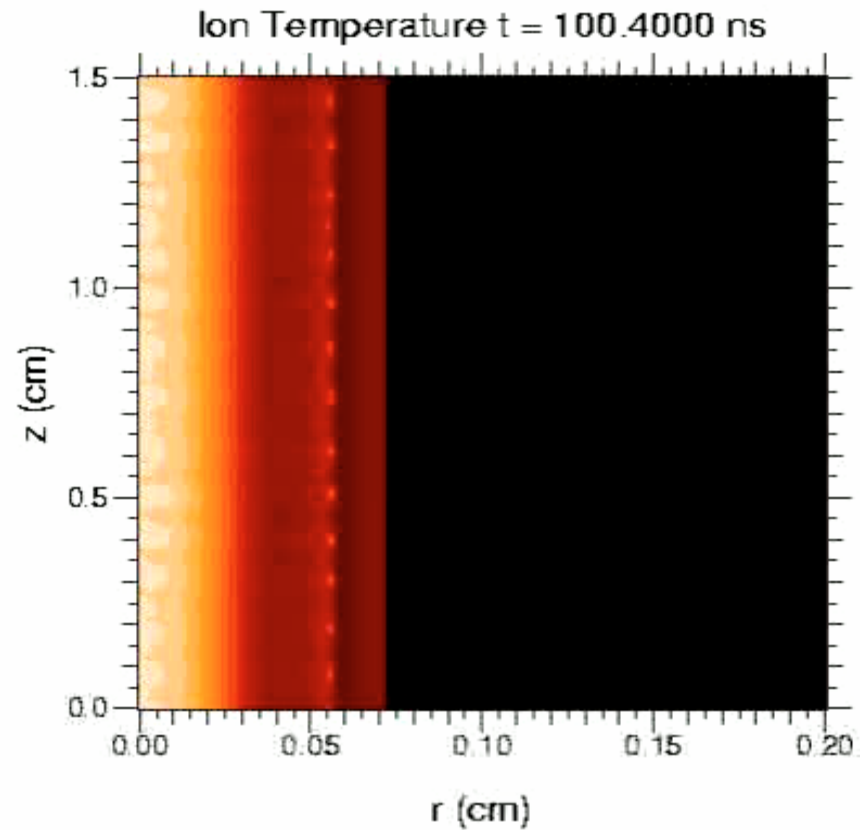
# Axial current (near peak)



# Plasma ion temperature (full run)



# Plasma ion temperature (Near peak)



# Conclusions

- R-T instability can be controlled.
- Pinch current is amplified.
- Current rise time is reduced.
- Breakeven fusion (i.e., nuclear energy larger than stored energy) is possible.
- Reactor design is not yet considered.