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SMR 1673/42

AUTUMN COLLEGE ON PLASMA PHYSICS

5 - 30 September 2005

Detection of drift-kinetic vortices as a manifestation of Alfvénic turbulence

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Outline

Keywords: *turbulence, coherent structures, space plasma measurements*

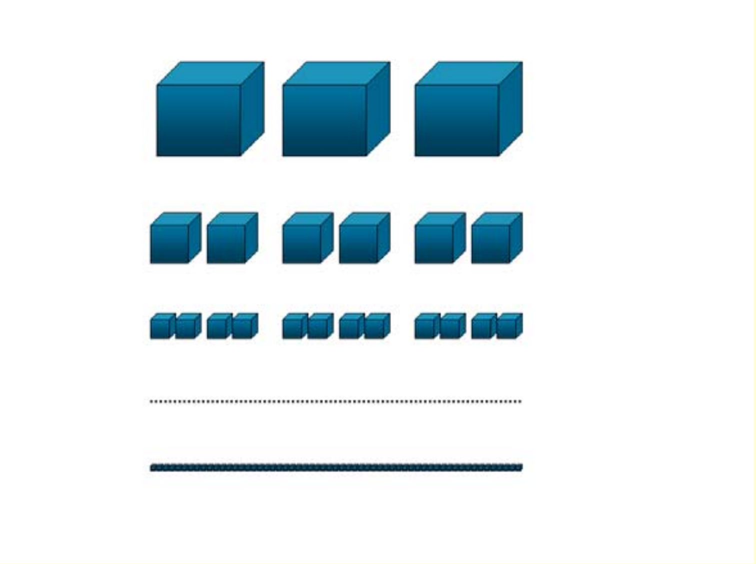
- Phenomenology of turbulence
- Hydrodynamic and MHD turbulence
- MHD turbulence in the solar wind
- Turbulence in the polar cusp
- Drift-wave turbulence and coherent structures
- Analytical solutions, drift-kinetic Alfvén vortices
- In situ detection of drift-kinetic electromagnetic vortices in the cusp

Classical picture of turbulence

Richardson (1922) cascade

•Turbulent (Richardson) cascade:

•Energy goes from larger scales to smaller scales

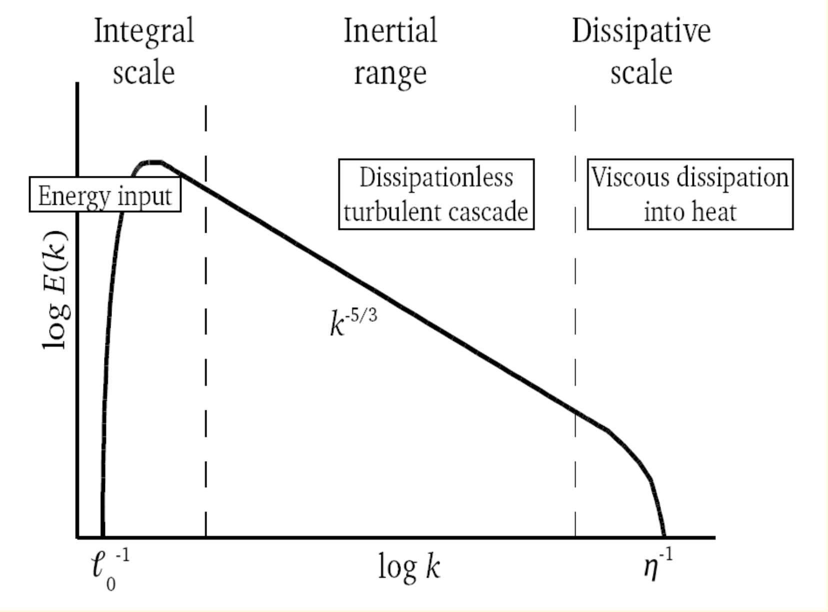


•General classical picture:
•Energy fed at some large scale lo
•Dissipationless transfer towards smaller scales in the inertial range
•Dissipation into heat at the dissipation scale

Kolmogorov (1941) spectrum

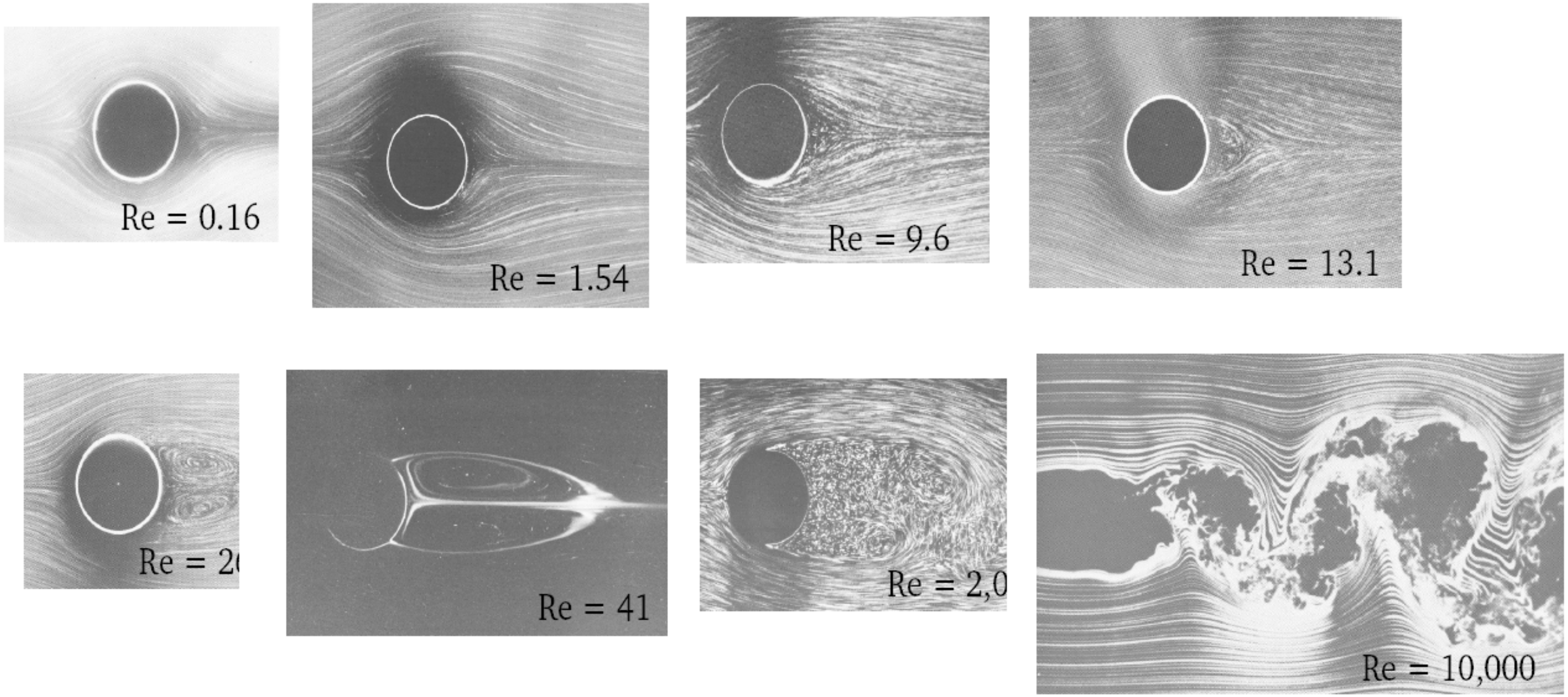
•Statistical and dimensional arguments based on Richardson's cascade picture

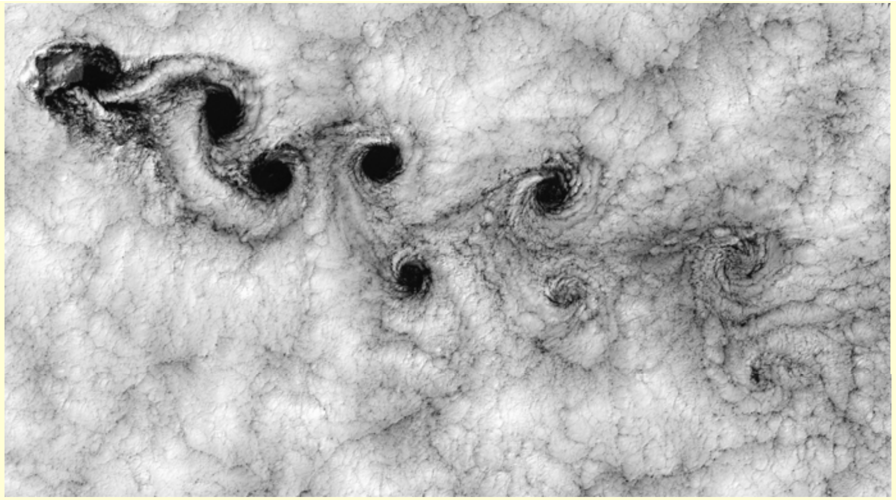
•Assumptions: homogeneous, isotropic, incompressible turbulence



D. Sundkvist, ICTP, 22 Sept 2005 (davids@irfu.se)

Hydrodynamical turbulence





Vortex street

Hydrodynamics vs MHD

Fluid turbulence is contained in the Navier-Stokes equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{v} = -\frac{1}{\rho}\boldsymbol{\nabla}p + \frac{\mu}{\rho}\nabla^2\mathbf{v}.$$

MHD turbulence in the MHD equations

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{v} = -\frac{1}{\rho}\boldsymbol{\nabla}p + \frac{1}{4\pi\rho}(\boldsymbol{\nabla}\times\mathbf{B})\times\mathbf{B} + \nu\nabla^2\mathbf{v}.$$

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla}\times(\mathbf{v}\times\mathbf{B}) + \mu\nabla^2\mathbf{B}.$$

- Magnetic field complicates the dynamics
- In magnetized plasmas, the ambient magnetic field decouples || and ⊥ dynamics

Kraichnan-Iroshnikov (1965): slower energy transfer rate

- quasi-two-dimensional turbulence
- suppresses k-spectrum: not Kolmogorov scaling any more

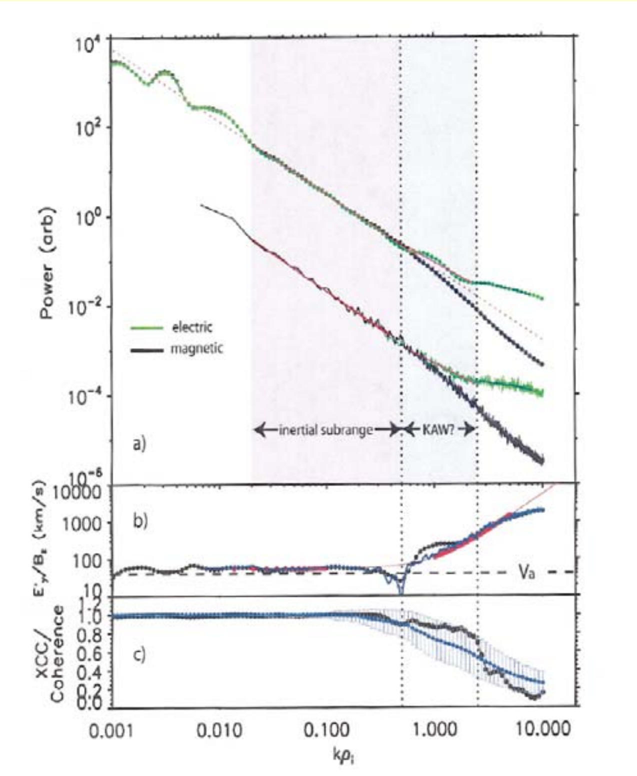
The inverse cascade and coherent structures

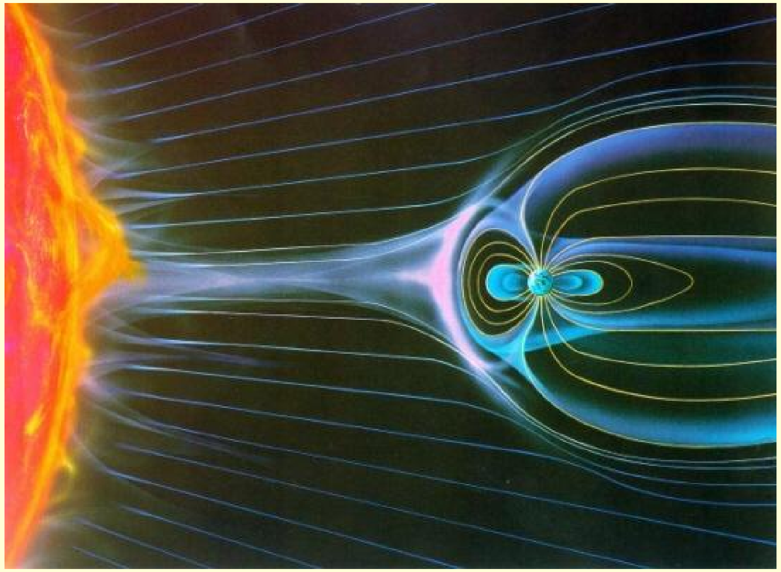
- Direct cascade: k_{in} --> large wavenumbers (smaller scales)
- Inverse cascade: k_{in} --> small wavenumbers (larger scales)

- General picture: inverse cascade towards larger scales – self-organization into large scale coherent structures
- Details: Nonlinearities of vector type cause waves to self-interact with subsequent trapping into vortices

MHD turbulence in the solar wind

- Energy injection – solar wind (supersonic)
- $\beta \approx 5$, $v_{ph} = v_A$ (Alfvénic)
- Taylor's hypothesis (Doppler shift): $\omega = kv_{sw}$

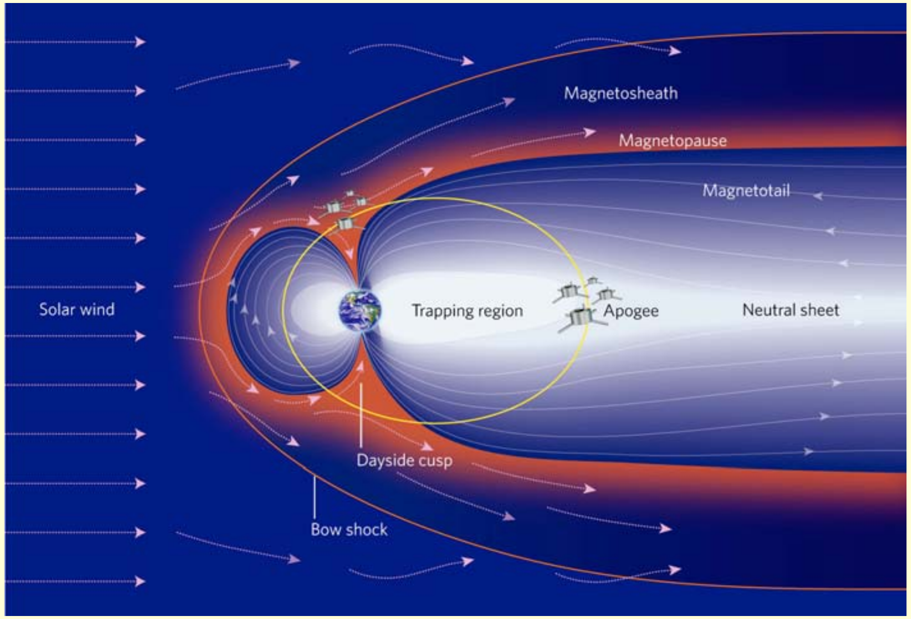


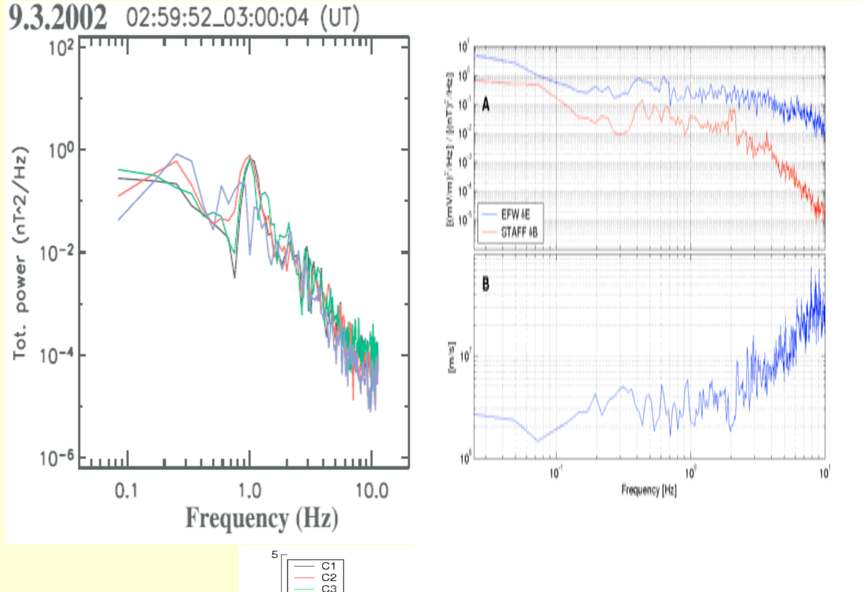


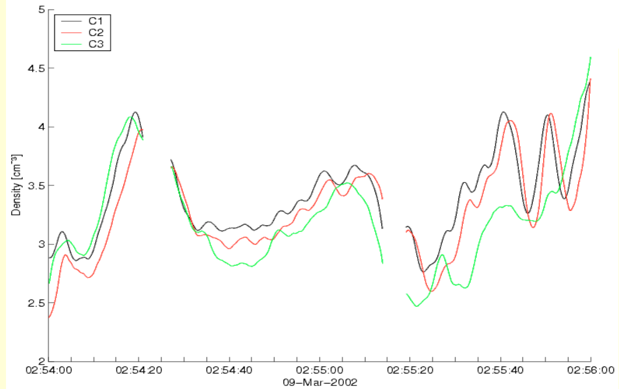
- Kolmogorov spectrum with index = -5/3 (fluid like)
- Close to "dissipation scale", Alfvén waves --> Kinetic Alfvén waves
- Kinetic Alfvén waves --> Heating?

Bale et al. *Measurement of the Electric Fluctuation Spectrum of Magnetohydrodynamic Turbulence, PRL, 94, 215002-1 (2005)*

Turbulence in the polar cusp







Goldstein *Turbulence on a small scale, Nature, 436, 782 (2005).*

- High-altitude cusp
  - $m_e/m_i << \beta << 1$
  - Strongly inhomogeneous
  - Kinetic Alfvén waves and ion cyclotron waves close to dissipation scale
  - MHD inadequate ---> Drift-wave Alfvénic turbulence

Nykyri et al. *Ion cyclotron waves in high altitude cusp: CLUSTER observations at varying spacecraft separations, GRL, 30, 2263 (2003).*

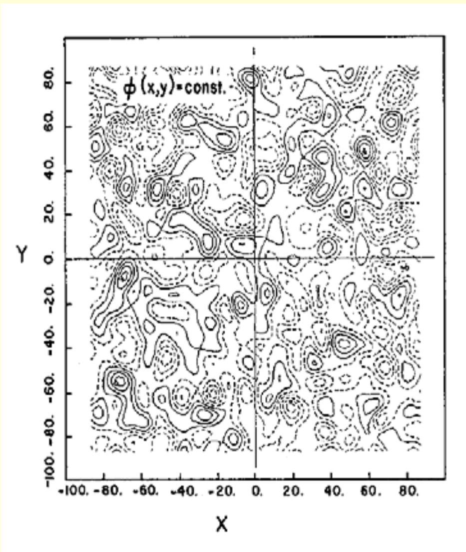
Sundkvist et al. *Wave characteristics near proton gyrofrequency in high-altitude cusp, Ann. Geophys., 23, 983-995 (2005).*

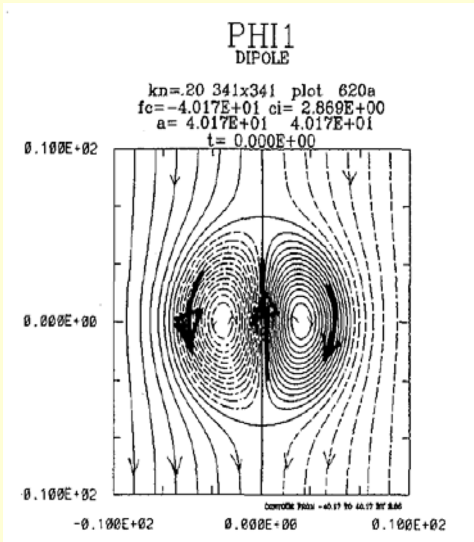
Drift-wave turbulence and coherent structures

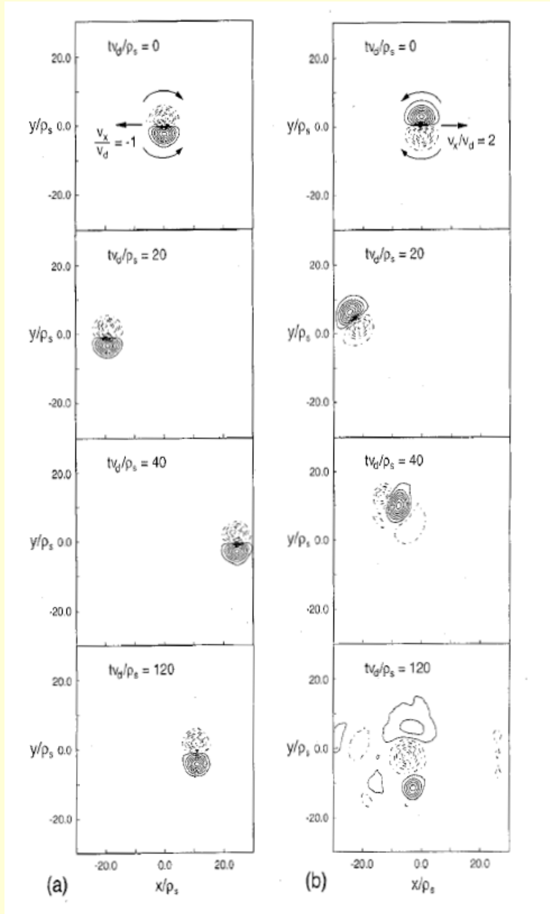
A model equation for low-frequency drift-wave plasma turbulence is the Charney-Hasegawa-Mima (CHM) equation

$$\frac{\partial}{\partial t}\left(\nabla^2\phi - \phi\right) - \left[\nabla\phi \times \hat{z}\cdot\nabla\right]\left[\nabla^2\phi - \ln\left(\frac{n_0}{\omega_{ci}}\right)\right] = 0$$

The CHM-equation admit both wave and coherent (vortex) solutions and is appropriate to describe low-frequency drift-wave turbulence







Horton & Hasegawa *Quasi-two-dimensional dynamics of plasmas and fluids, Chaos, 4, 227-251 (1994)*

Generalization of CHM-equation

Can be derived from two-fluid equations:

$$\partial_t \mathbf{v}_j + (\mathbf{v}_j \cdot \boldsymbol{\nabla})\mathbf{v}_j = \frac{q}{m_j}(\mathbf{E} + v_j \times \mathbf{B}) - \frac{1}{m_j n}\boldsymbol{\nabla} p_j$$

For the parameters in the cusp we can assume

$$\mathbf{B} = B_0 \hat{z} + \delta \mathbf{B}_\perp$$

$$\mathbf{E}_\perp = -\boldsymbol{\nabla}_\perp \phi$$

Momentum fluid equations lead to the drift velocities

$$\mathbf{v}_{i,\perp} \approx \frac{c}{B_0}\hat{z} \times \boldsymbol{\nabla}_\perp \phi + \frac{cT_i}{eB_0 n_i}\hat{z} \times \boldsymbol{\nabla} n_i + \frac{c}{B_0 \omega_{ci}}(\partial_t + \mathbf{v}_i \cdot \boldsymbol{\nabla})\mathbf{E}_\perp$$

$$\mathbf{v}_e \approx \frac{c}{B_0}\hat{z} \times \boldsymbol{\nabla}_\perp \phi - \frac{cT_e}{eB_0 n_e}\hat{z} \times \boldsymbol{\nabla} n_e + v_{ez}\left(\hat{z} + \frac{\mathbf{B}_\perp}{B_0}\right)$$

which together with quasi-neutrality and the momentum and continuity equations gives the system

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_E \cdot \boldsymbol{\nabla}\right) p_i = \frac{d}{dt} p_i = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(n - \frac{en_0}{m_i \omega_{ci}^2}\nabla_\perp^2 \varphi\right) - \frac{e}{m_i^2 \omega_{ci}^3}\boldsymbol{\nabla} \cdot \{p_i, \boldsymbol{\nabla}_\perp \varphi\} = 0$$

$$\{A, B\} = \frac{\partial A}{\partial x}\frac{\partial B}{\partial y} - \frac{\partial A}{\partial y}\frac{\partial B}{\partial x}$$

$$ne\hat{z} \cdot \boldsymbol{\nabla}\phi + \frac{ne}{c}\frac{\partial A_z}{\partial t} - \frac{m_e c}{4\pi e}\frac{\mathrm{d}}{\mathrm{d}t}\nabla_\perp^2 A_z = \hat{z} \cdot \boldsymbol{\nabla} p_e$$

$$\frac{\partial n}{\partial t} + \mathbf{v}_\mathbf{E} \cdot \boldsymbol{\nabla} n + \frac{c}{4\pi e}\hat{z} \cdot \boldsymbol{\nabla}\nabla_\perp^2 A_z = 0$$

This system generalizes and reduce to CHM for A_z =0, p = 0. The system describes both drift-Alfvén waves and vortices

Vortex solutions

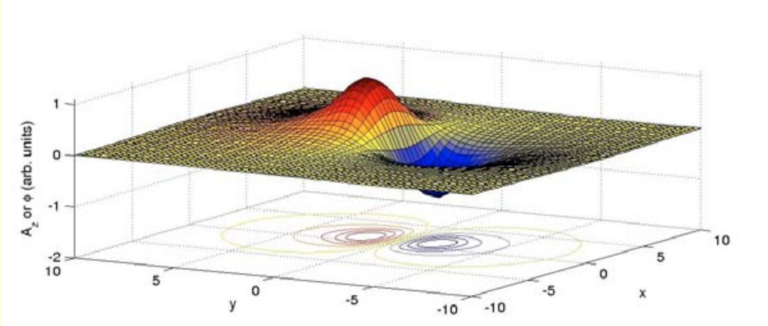
Solutions exist and can be found by using the Larichev-Reznik ansatz:

Assume function of x and $\eta = y + \alpha z - ut$

Transformed system have solution

$$\phi = [ur + c_1 J_1(k_1 r) + c_2 J_1(k_2 r)] \cos\theta \qquad (r \le r_0)$$

$$\phi = c_3 K_1(sr) \cos\theta \qquad (r \ge r_0)$$



Dipole vortex

In addition vortex chains (cat-eyes) and monopole vortices can exist

Questions: Do such coherent vortices exist in magnetoplasmas, and if they do, what are their charcteristics?

Shukla, P. K., Yu, M. Y. & Varma, R. K., Phys. Lett. A 109, 322-324 (1985).
Petviashvili, V. I. & Pokhotelov, O. A., JETP Lett. 42, 54-56 (1985).
Shukla, P. K., Yu, M. Y. & Stenflo, L., Phys. Rev. A 34, 3478-3480 (1986).
Liu, J. & Horton, W., J. Plasma Phys. 36, 1-24 (1986).
Kaladze et al. Plasma Phys. and Cont. Fusion 29, 589 (1987)
Chmyrev, V. M. et al., Physica Scripta 38, 841-854 (1988).

Previous single-spacecraft observations of coherent structures

- Chmyrev et al (1988) suggested that they observed drift-kinetic vortices in the plasma sheet boundary layer
- To be sure one needs to separate temporal from spatial phenomena

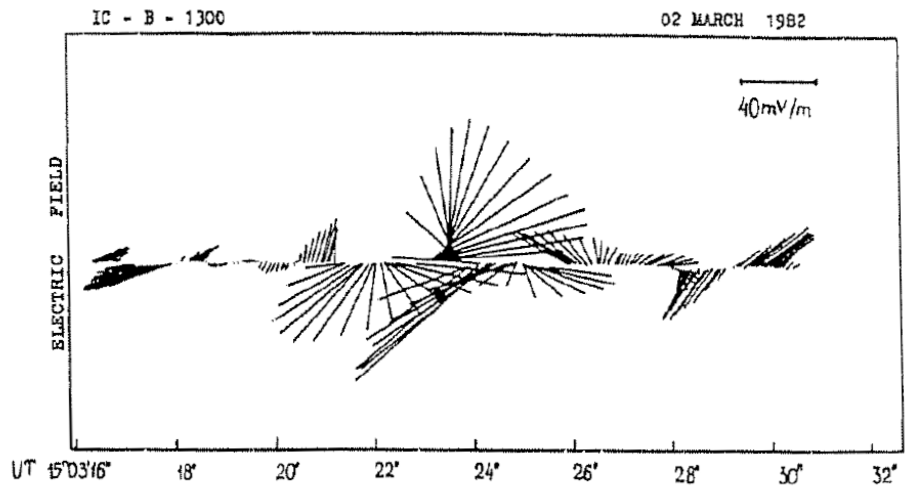
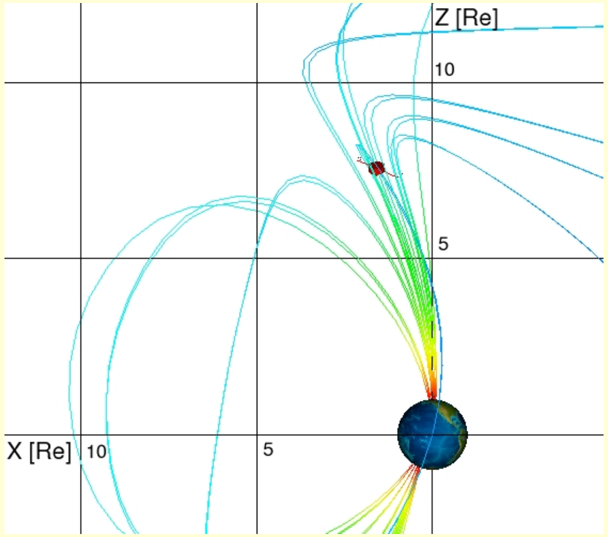


Fig. 5. Variations of the horizontal component of the electric field vector in the frequency range 0.12–0.8 Hz along the satellite orbit at the second interval in Fig. 1.

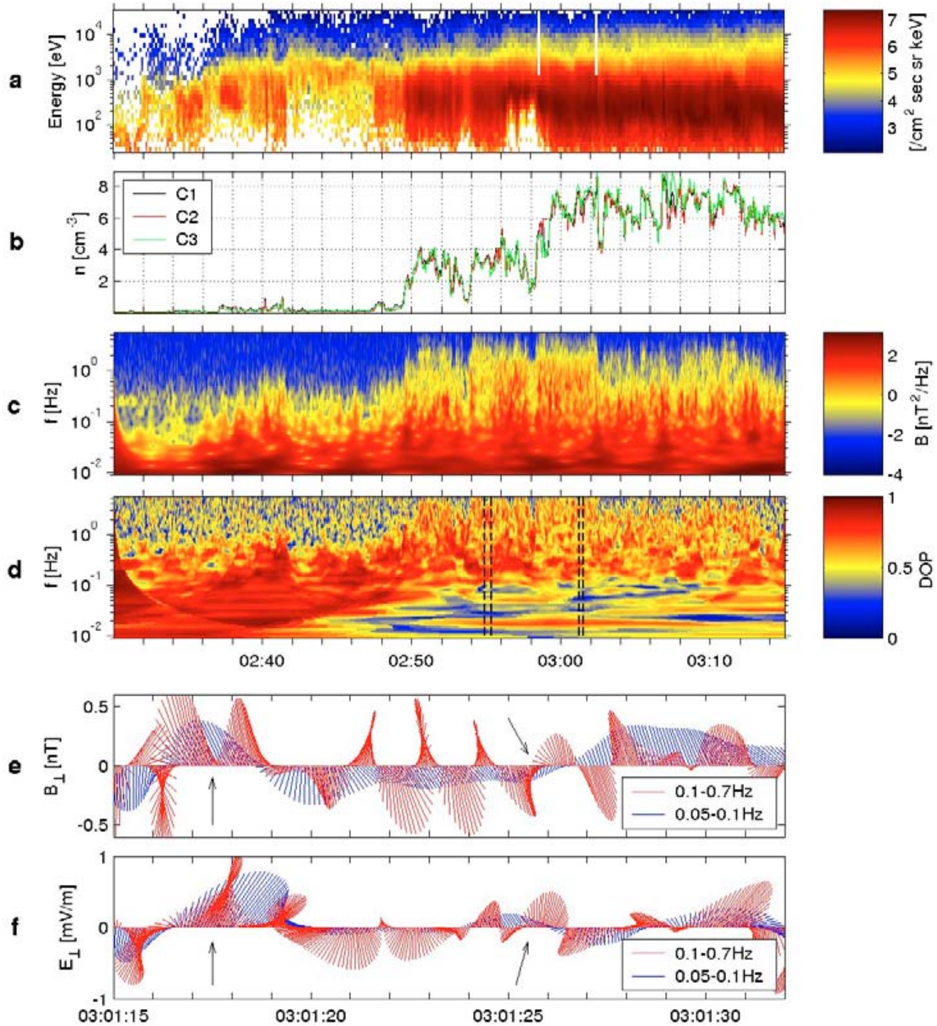
Chmyrev et al. *Alfvén vortices and related phenomena in the Ionosphere and the Magnetosphere, Physica Scripta, 38, 841-854 (1988).*

Multi-satellite detection of DKA vortices

Cusp crossing 2002-03-09



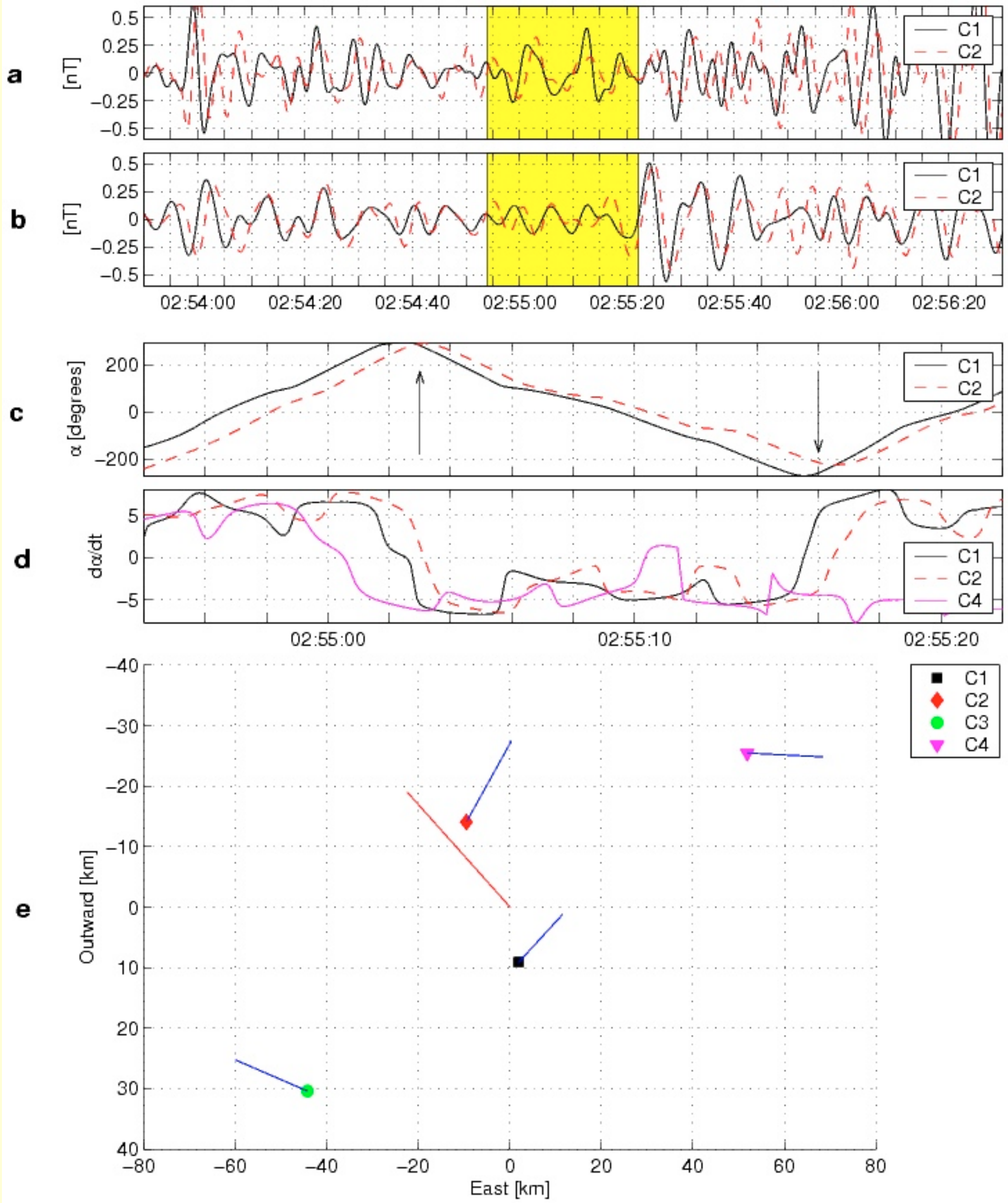
- Northern outbound crossing
- Northward IMF -> high-latitude reconnection
- Turbulent boundary layer formed
- Hodograms along trajectory suggest vortex motion
- Cluster --> Multi-point measurements



Sundkvist, Krasnoselskikh, Shukla et al. *In Situ Multi-Satellite Detection of Coherent Vortices as a Manifestation of Alfvenic Turbulence, Nature, 436, 825-828 (2005).*

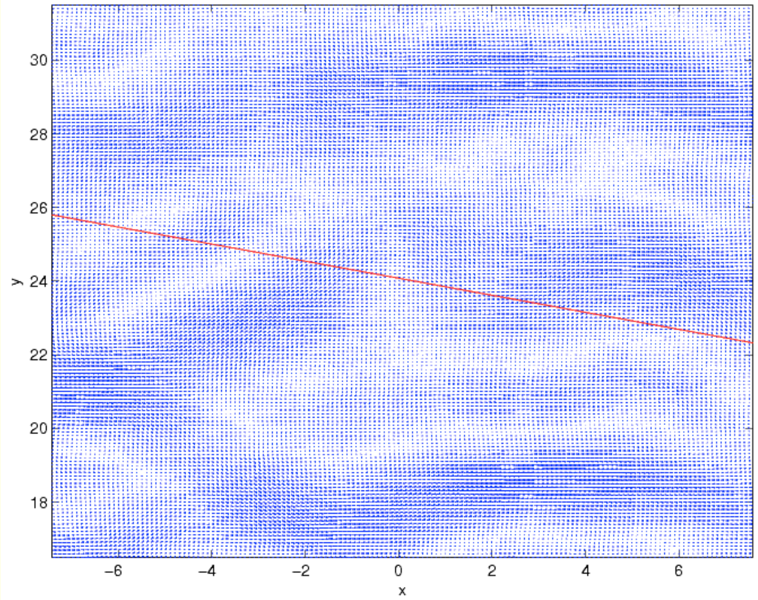
Detection of DKA vortices – Multipoint timeseries analysis

- New point of view: look for coherent structures by examining time-series from several spacecraft simultaneously
- Rotation of field vectors spatially coherent
- In accord with vortex convected past the spacecraft
- Vortex radius approximately 2-6 ρ_p



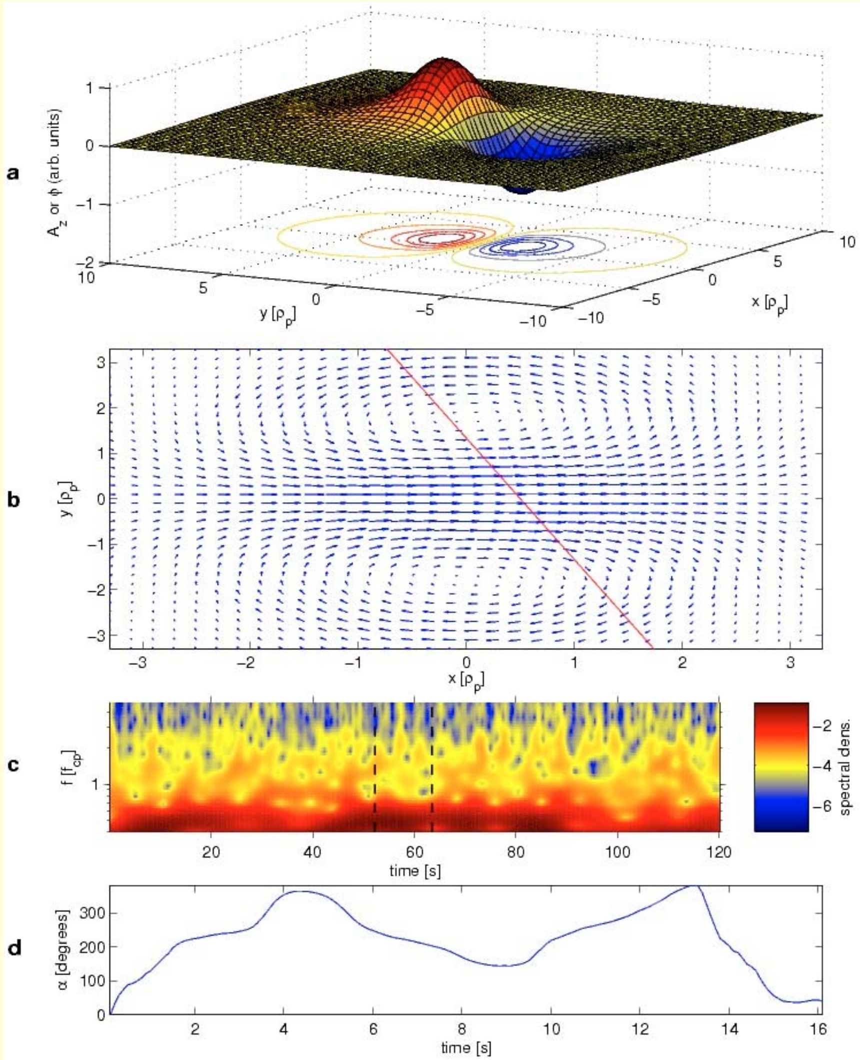
Sundkvist et al. *In Situ Multi-Satellite Detection of Coherent Vortices as a Manifestation of Alfvenic Turbulence, Nature, 436, 825-828 (2005).*

Detection of DKA vortices – Comparison with theory



Sundkvist et al. *Manuscript in preparation, J. Geophy. Res., (2005).*

- Model of vortex turbulence in cusp: isolated DKA vortices
- In accord with observations
    - spectra
    - rotation of field vectors



Sundkvist et al. *In Situ Multi-Satellite Detection of Coherent Vortices as a Manifestation of Alfvenic Turbulence, Nature, 436, 825-828 (2005).*

- Importance of findings:
  - Drift-wave Alfvénic turbulence exhibit coherent structures
  - Cross-field anomalous transport
  - New way of transporting mass and energy through boundary layers

Summary

- High-altitude cusp boundary layer:
- low beta (kinetic regime) drift-wave Alfvénic turbulence
- Appropiate equations are generalizations of Charney-Hasegawa-Mima equation
- Strong drift-turbulence: drift-waves and coherent vortices (owing to self-interaction among kinetic Alfvén waves via vector nonlinearities)
- Analytical solutions exist: Drift-Kinetic electromagnetic Alfvén vortices (DKA vortices)
- Multi-point measurements by Cluster four spacecraft confirms existence of small(large) scale coherent structures (vortices)
- Observations of coherent structures can be described as DKA vortices and vortex chains
- Cross-field transport