The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR 1673/36

AUTUMN COLLEGE ON PLASMA PHYSICS

5 - 30 September 2005

Generation and Dynamics of Large Scale Flows in Magnetized Plasmas and Rotating Fluids

J.J. Rasmussen

Risoe National Laboratory
Denmark

Generation and Dynamics of Large Scale Flows in Magnetized Plasmas and Rotating Fluids

Jens Juul Rasmussen

Odd Erik Garcia, Volker Naulin, Anders H. Nielsen, Bjarne Stenum

jens.juul.rasmussen@risoe.dk

Association EURATOM - Risø National Laboratory,

Optics and Plasma Research, OPL-128

DK-4000 Roskilde, Denmark

Contents

RISØ

- Global flows, occurrences and definitions
- Flows and turbulence
- Generation of global flow in rotating fluid: rectification of small scale fluctuations: Rossby waves :: Drift waves
- Electromagnetic effects on zonal flow drive in magnetically confined toroidal plasmas
- Conclusions

Jovian Atmosphere

Zonal flows in the Jovian atmosphere with large scale vortical structures:: 2D turbulence, inverse cascade of energy.

Magnetically confined plasma

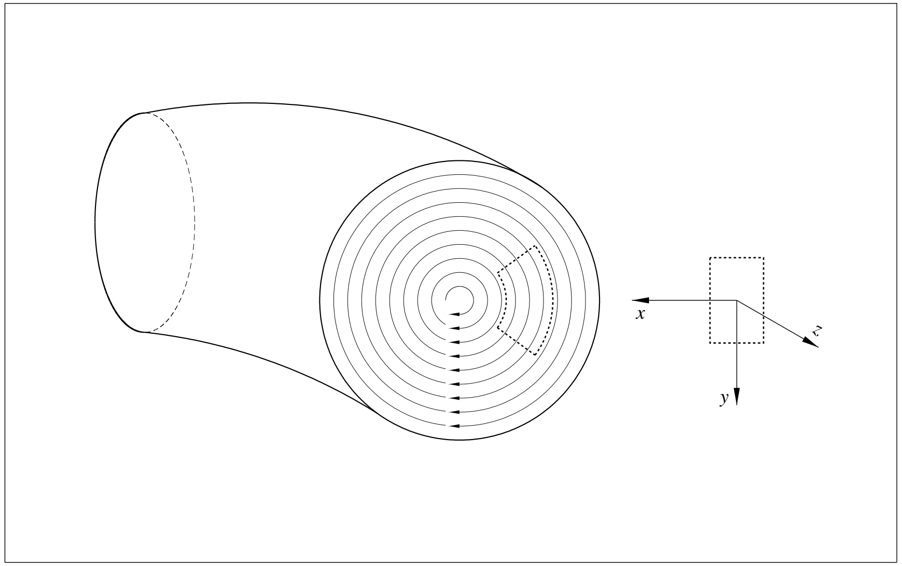


Figure 1

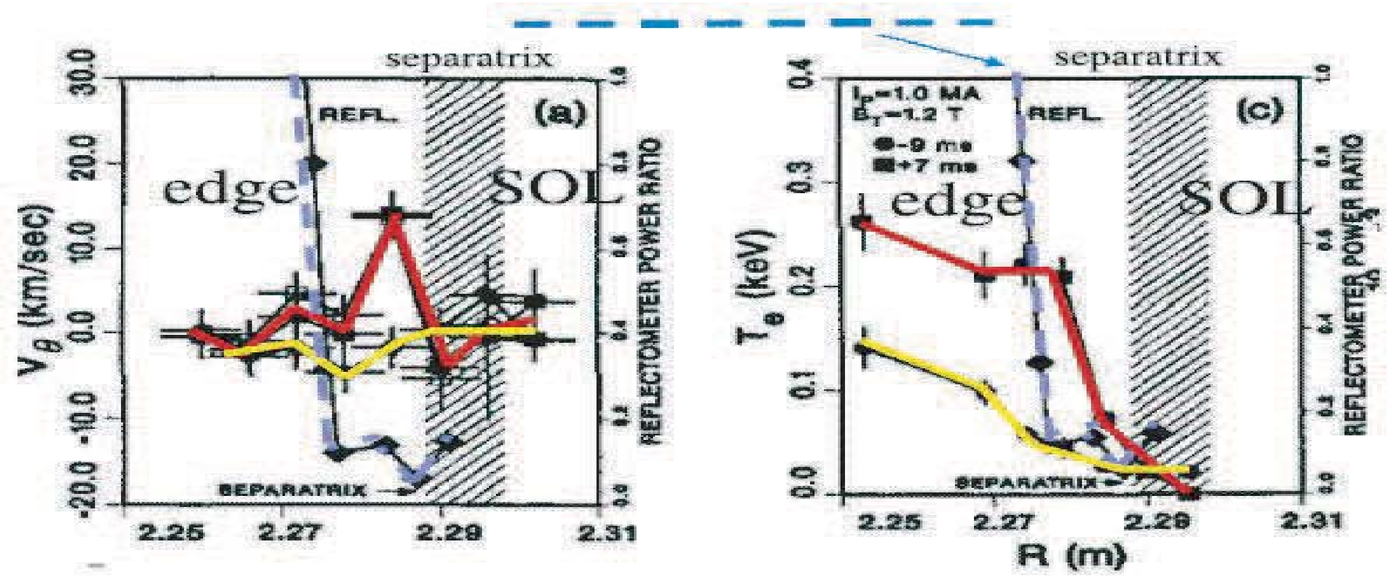
Radial coordinate : x, velocity : u; ∇p along x Poloidal coordinate : y, veloci-ty : v also refered to as zonal direction (geophysics)

Plasma turbulence quasi-2D perpendicular to $\mathbf{B}$

Terminology: Zonal flows: small scale/amplitude flows mainly in the core relates to transport barriers

Global poloidal flows/poloidal flows/poloidal spin-up : large scale/amplitude sheared flows near the edge, instrumental in the L-H transition.

Shear flow and H-mode



DIIID

K H Burrell
PFCF 34, 1859
(1992)

High confinement mode - H-mode - (Wagner et al. PRL 1982; Asdex) is essential for tokamak (stellarator) operation.

Connected with a sheared flow in the edge!

Experimental conditions well established; but still no full theoretical explanation. Various generation mechanisms.

Flows: how do they act?

Sheared flows do influence the turbulent transport:
Radial particle flux: $\Gamma = \langle nu \rangle$
The poloidal flow do not contribute to Γ!

Flows are said to suppress turbulence:
Turbulence shear decorrelation!

(Biglari *et al* Phys. Fluids B **2**, 1 (1990))

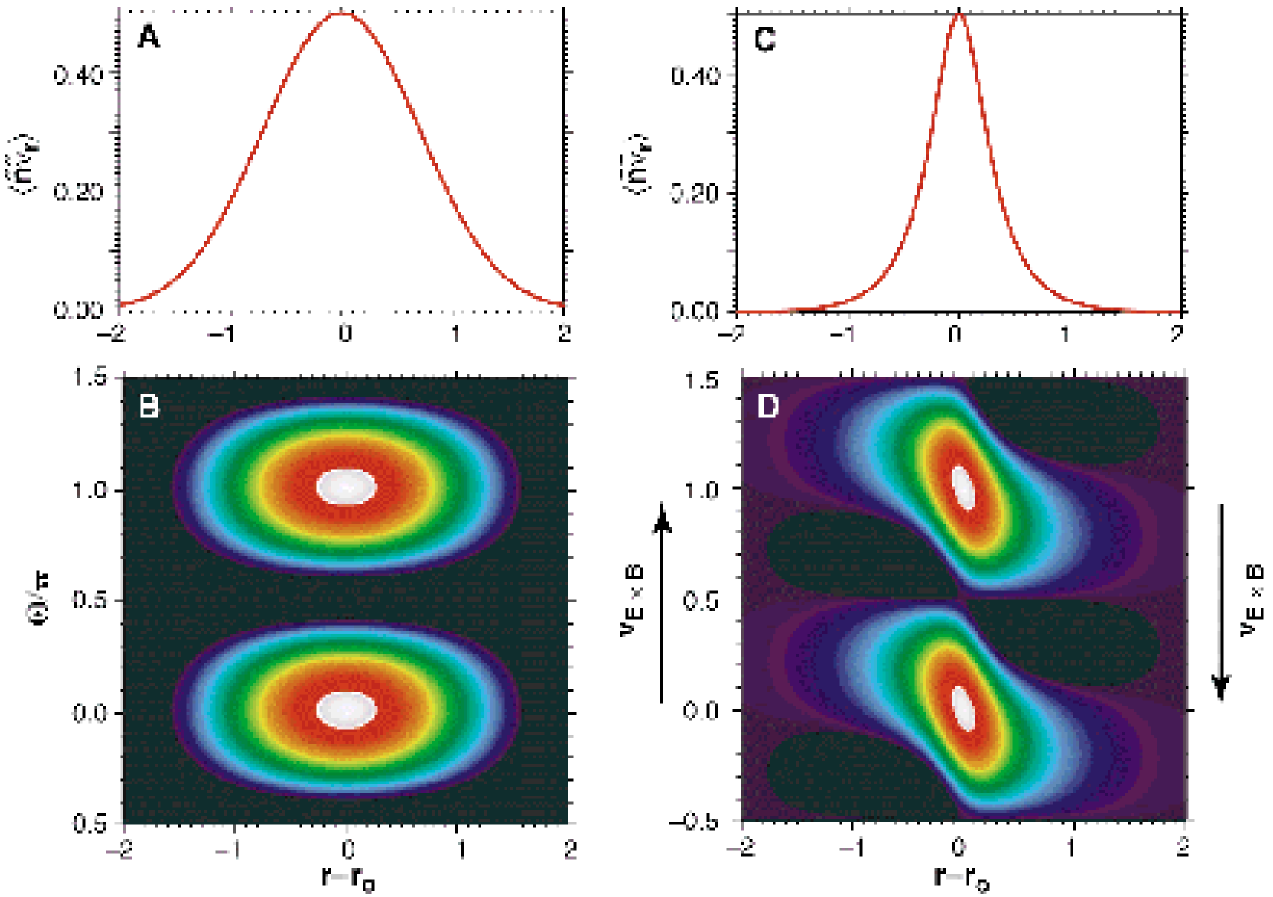
$\omega_{shear} > \gamma_{inst}$

Flows generated by out of the turbulence due to inverse cascade, $\rightarrow$ turbulent fluctuation energy will
Condensate in flow energy.
does not contribute to Γ.

Monitoring of energy transfer processes is crucial for understanding the dynamics.

Review: Diamond *et al* PPCF **47**, R35 (2005)

Shear: Vortex tilting.



K.H. Burrell Phys. Plasma **4**, 1499 (1997)

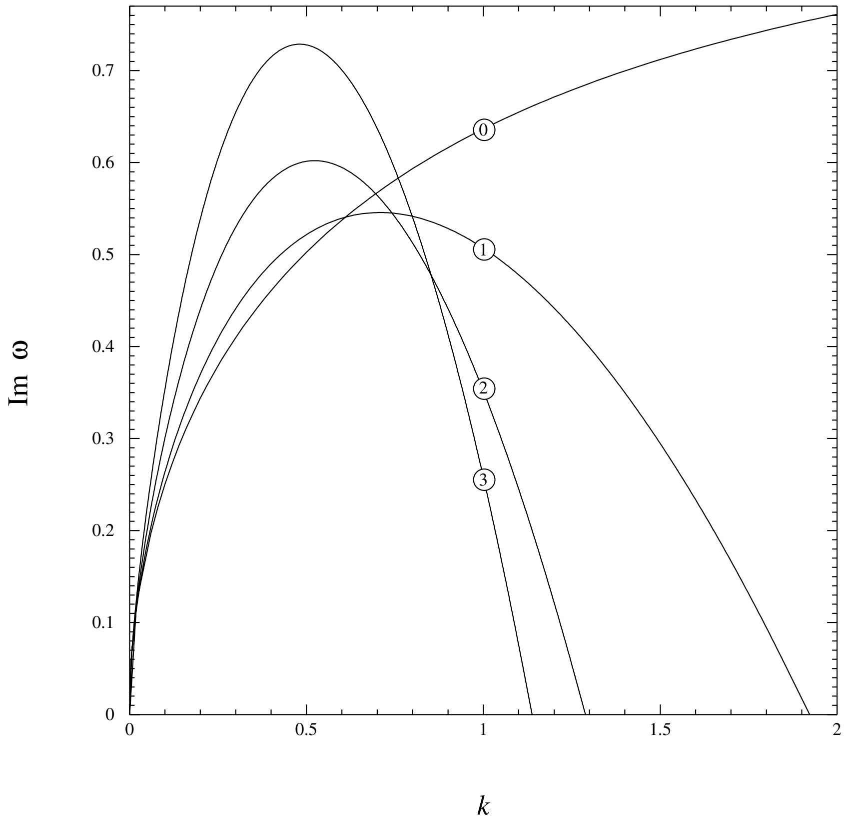
Elongated structures suffer shear enhanced dissipation: $\exp(-\mu t^3/\Delta_t)$, Δ_t “tilting time”.

Garcia and Bian, Phys. Plasma **12**, 014503 (2005)

Shear flow stabilization?

Influence of a background shear flow $V(x)\hat{y}$ on the classical Rayleigh-Taylor (interchange) instability::

Benilov *et al* Phys. Fluids **14** 1674 (2002)



Numerical solution of Taylor-Goldstein eq.: $V(x) = V_0 \tanh x$, $V_0 = 0,\ 0.5,\ 1.0,\ 2.0$

Stability for $2\pi/L_y > k_c$

Interchange turbulence

RISØ

$$\frac{\partial n}{\partial t} + \{\phi, n\} + \mathcal{K}(n + T - \phi) = \nu \nabla^2 n \,,$$

$$\frac{\partial T}{\partial t} + \{\phi, T\} + \frac{2}{3}\mathcal{K}(n + \frac{7}{2}T - \phi) = \kappa \nabla^2 T \,,$$

$$\frac{\partial \omega}{\partial t} + \{\phi, \omega\} + \mathcal{K}(n + T) = \mu \nabla^2 \omega \,.$$

$\{f, g\} = (\partial f/\partial x)(\partial g/\partial y) - (\partial f/\partial x)(\partial g/\partial y)$

Vorticity $\omega = \nabla^2 \phi$

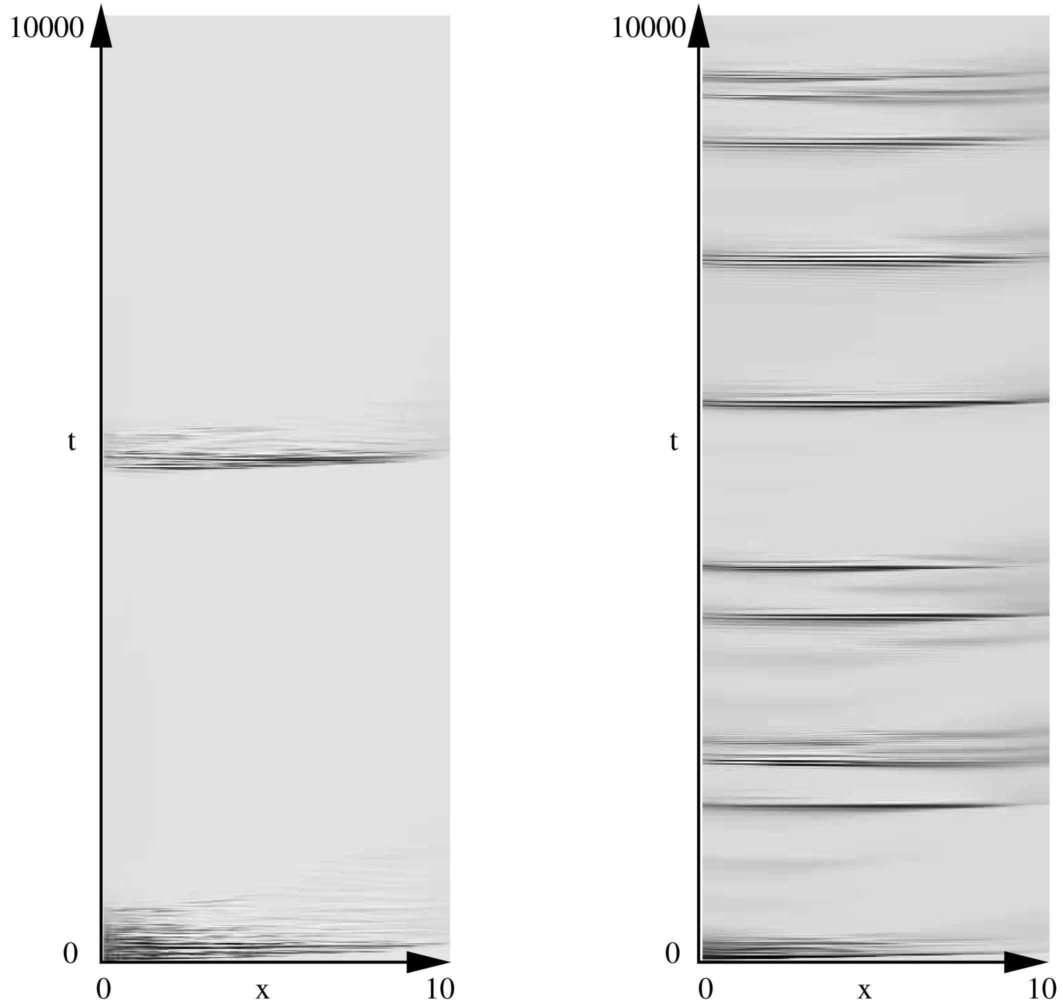
The curvature operator

$$\mathcal{K} = -\nabla \cdot \frac{\mathbf{B} \times \nabla}{B^2}$$

Solved numerically on a slab domain, outboard midplane: periodic in y and bounded in x; $\alpha = L_y/L_x$

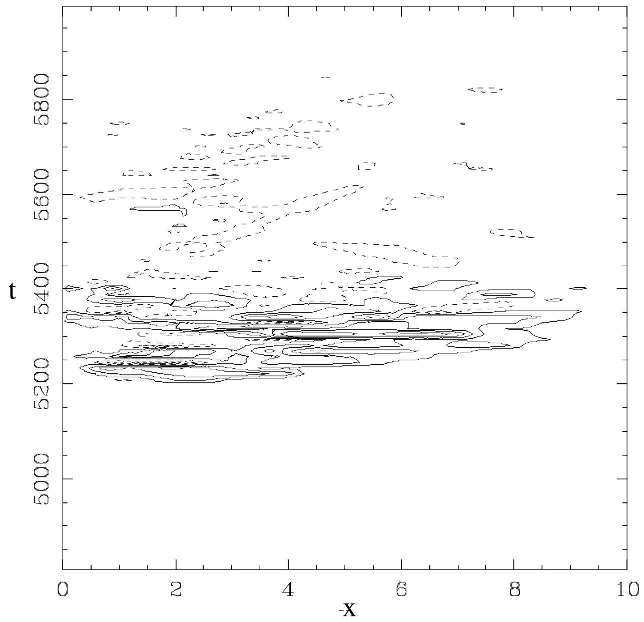
Naulin et al. PRL **81**, 4148 (1998); Phys. Plasma **10**, 1075 (2003)

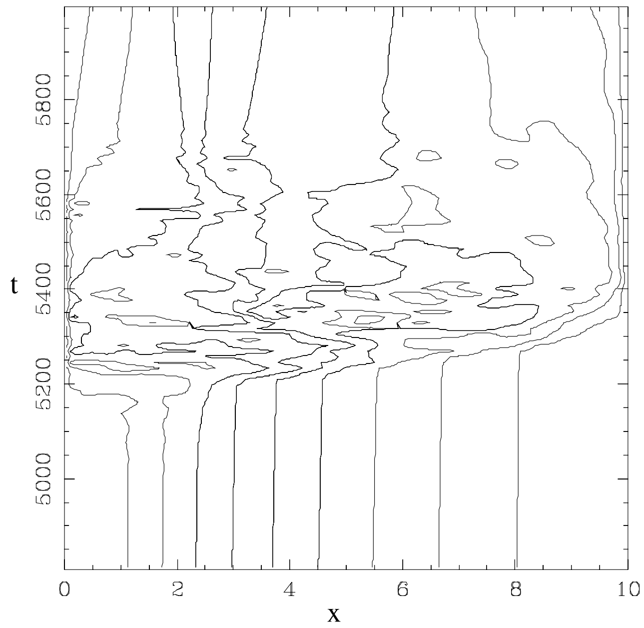
Heat flux

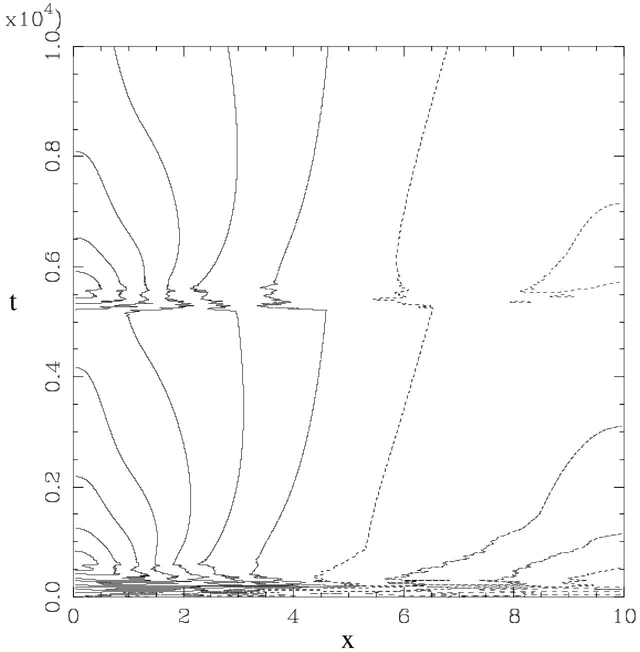


Poloidally averaged heat flux $\Gamma_T = \langle uT \rangle$; $\alpha = L_y/L_x = 1$; $\nu = \kappa = \mu = 10^{-3}$ (left) and 10^{-2} (right)

Burst structure

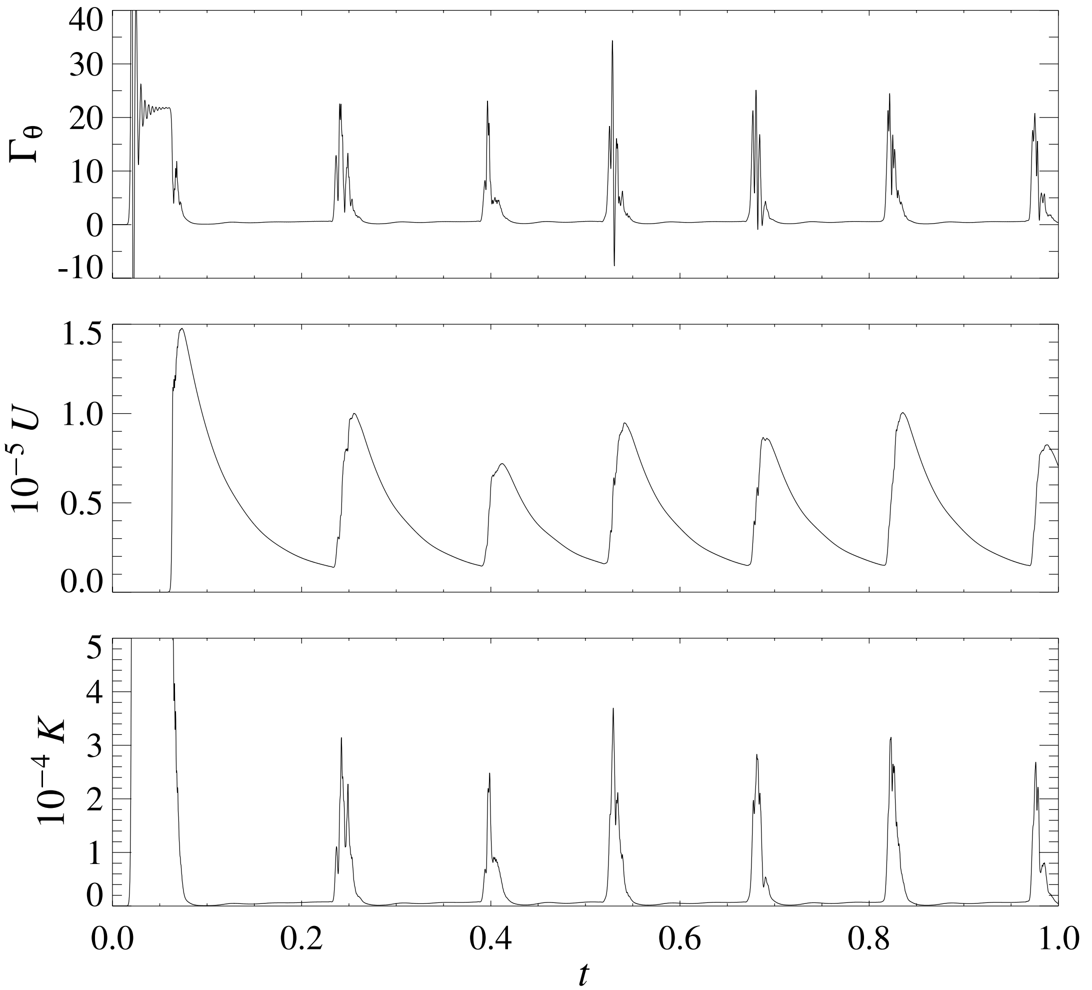






Spatio-temporal evolution of a burst event, heat flux, Γ_T, temperature gradient, poloidal flow

Bursting and energy exchange



Γ_θ flux; U, K energy in the flow, fluctuations

Garcia and Bian PRE **68**, 047301 (2003)

Flows: Reynolds stress

RISØ

Momentum equation/vorticity equation:

$$\frac{\partial \omega}{\partial t} + \{\phi, \omega\} + \mathcal{K}(n + T) = \mu \nabla^2 \omega \, .$$

Flows: Reynolds stress

$$\frac{\partial \omega}{\partial t} + \{\phi, \omega\} + \mathcal{K}(n+T) = \mu \nabla^2 \omega \,.$$

Reynolds decomposition (Reynolds (1894)):

$$\omega = \Omega + \widetilde{\omega}, \;\; \phi = \Phi + \widetilde{\phi}, \;\; \mathbf{v} = \mathbf{V} + \widetilde{\mathbf{v}}$$

$$\Omega = \langle \omega \rangle \equiv \frac{1}{L_y} \int_0^{L_y} \omega dy$$

Zonal velocity $V = \langle v \rangle$; $U = 0$

Flows: Reynolds stress

$$\frac{\partial \omega}{\partial t} + \{\phi, \omega\} + \mathcal{K}(n + T) = \mu \nabla^2 \omega \,.$$

$$\Omega = \langle \omega \rangle \equiv \frac{1}{L_y} \int_0^{L_y} \omega dy$$

Zonal velocity $V = \langle v \rangle$; $U = 0$

Flow evolution:

$$\frac{\partial V}{\partial t} = -\frac{\partial}{\partial x} \langle uv \rangle + \mu \frac{\partial^2}{\partial x^2} V$$

Flows: Reynolds stress

RISØ

$$\frac{\partial \omega}{\partial t} + \{\phi, \omega\} + \mathcal{K}(n+T) = \mu \nabla^2 \omega \,.$$

$$\Omega = \langle \omega \rangle \equiv \frac{1}{L_y} \int_0^{L_y} \omega dy$$

Zonal velocity $V = \langle v \rangle$; $U = 0$

$$\frac{\partial V}{\partial t} = -\frac{\partial}{\partial x} \langle uv \rangle + \mu \frac{\partial^2}{\partial x^2} V$$

Quasilinear approximation: Contribution from the k'te wave-component:

$$\partial_x \langle uv \rangle = -2k \partial_x (|\psi_k|^2 \partial_x \theta_k)$$

θ_k is the phase of ψ_k.

Flow generation for $\partial_x \theta_k \neq 0$ Radial propagation

Diamond and Kim, Phys. Fluids B **3**, 1626 (1991)

Zonal flow in rotating fluid

RISØ

Homogenization of potential vorticity (PV) in geophysical flows

P. Rhines *The Sea* (1977); (1979) Ann. Rev. Fluid Mech. **11**, 401 (1979)

$$\frac{D\Pi}{Dt} = \frac{D}{Dt}\left(\frac{\omega + f}{H(r)}\right) = 0$$

$D/Dt \equiv \partial/\partial t + \mathbf{v} \cdot \nabla \mathbf{v}$, ω is the relative vorticity of a fluid element, f is background vorticity, $H(r)$ is the depth of the fluid layer.

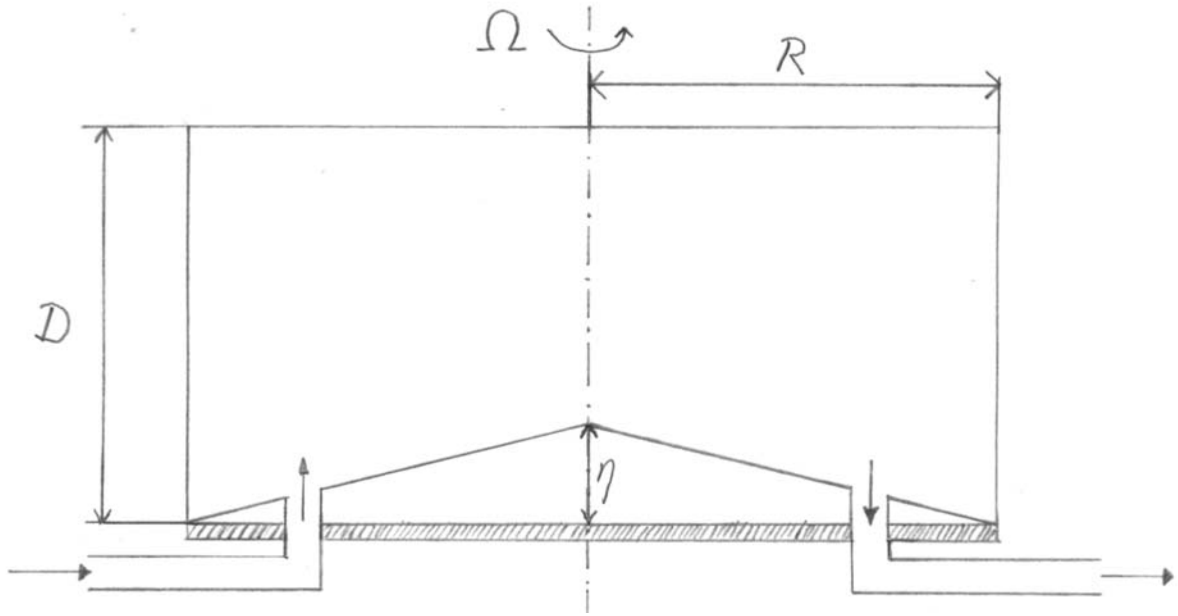
Movement towards deeper regions stretch the vortices and enhance ω; towards shallower regions compress the vortices and decrease ω.

Mixing of $\Pi \rightarrow$ low relative vorticity over shallow regions and higher relative vorticity over deeper regions.

Plasma case: Ion vorticity equation (cold ions):

$$\frac{D\Pi_i}{Dt} = \frac{D}{Dt}\left(\frac{\omega + \omega_{ci}}{n(r)}\right) = 0$$

Rotating fluid experiment



Experimental setup, rotating tank with a rigid lid. R = 19.4 cm, D = 20 cm, $\eta = 5$ cm, rotation rate 12 rpm.

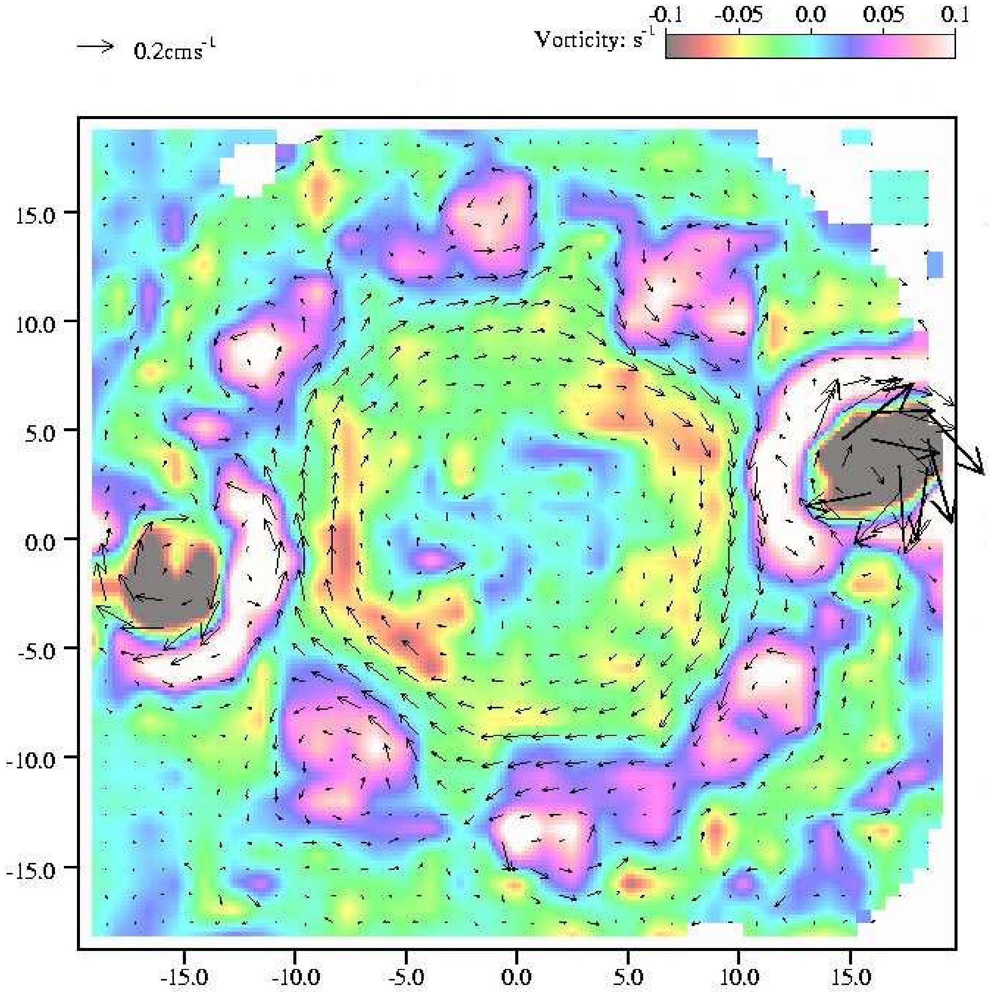
$\Pi = \omega + \kappa r$ (expansion $H(r) = 1 - \kappa r,\ \kappa < 0$)

Mixing: periodically pumping water in and out of two holes (diameter $2\,cm$). Forcing period: T_F ($T_F = 6.6\,s$) Diagnostics: particle tracking: instantaneous velocity field.

Expect $\omega < 0$ around the center: Anticyclonic flow.

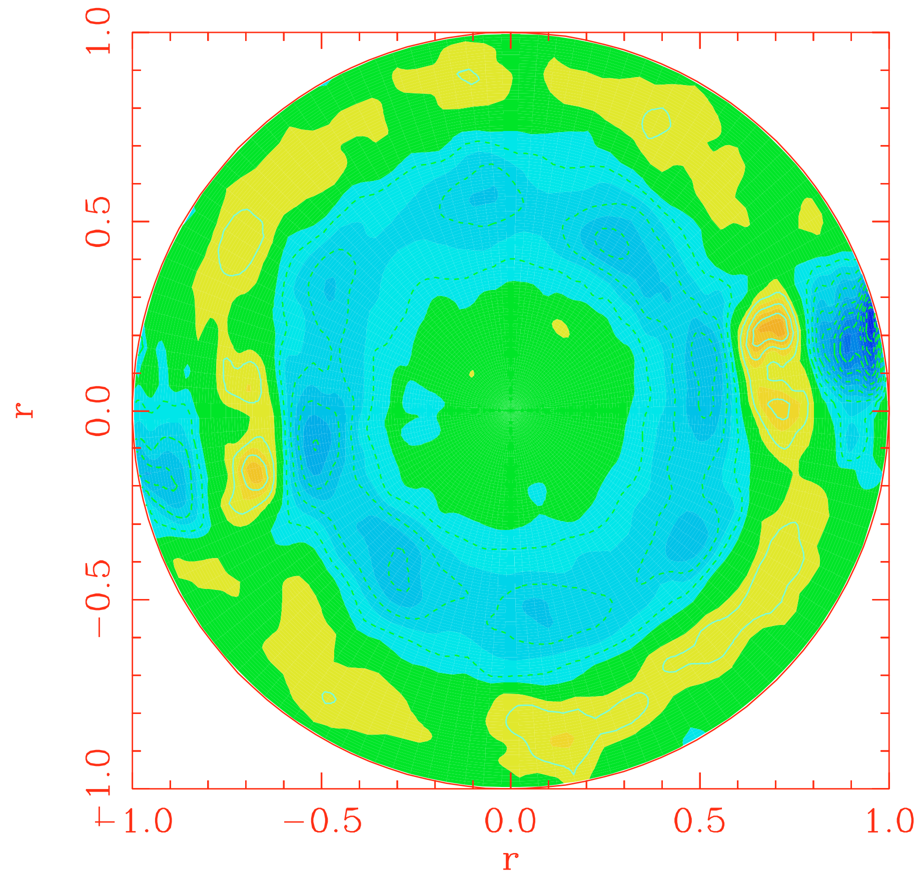
Rasmussen *et al*, Physica Scripta, in press (2005)

Vorticity field



Velocity field shown by arrows and vorticity contours averaged over 10 forcing periods. An anticyclonic circulation is observed

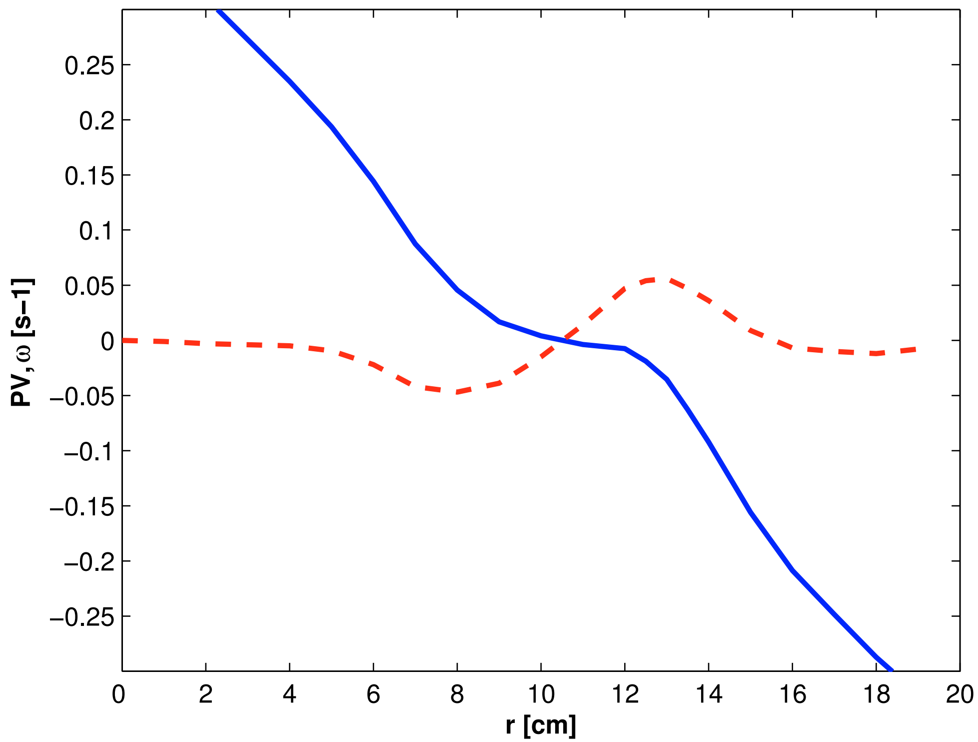
Azimuthal velocity



The azimuthal velocity averaged over 20 forcing periods. Blue designates negative velocity, i.e. anti-cyclonic motion and red positive velocity

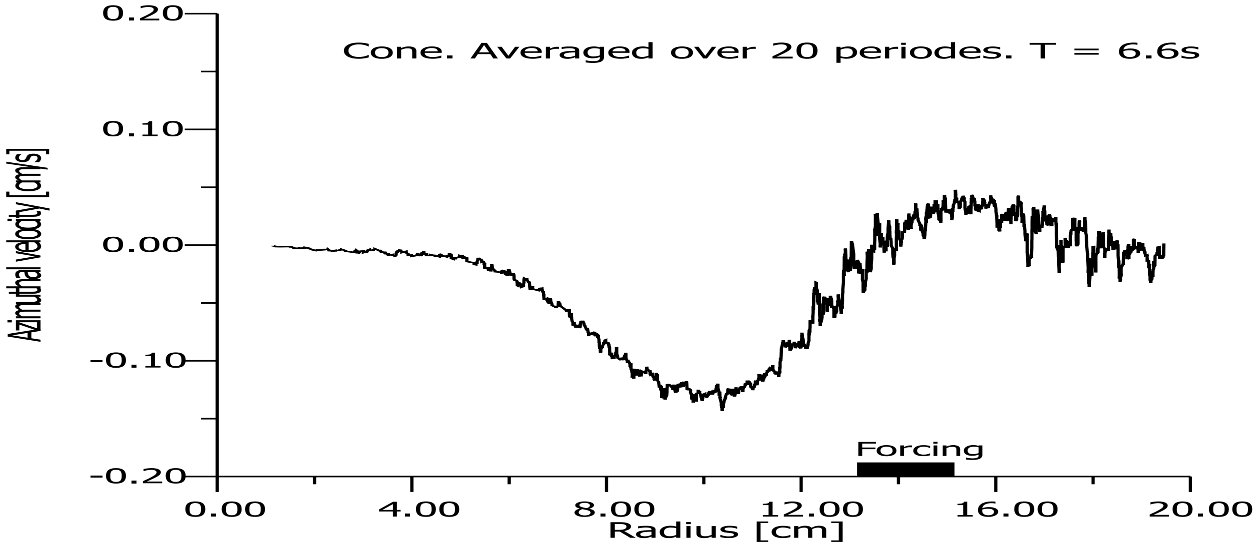
Flow generated by rectification of symmetrically forced small scale structures. Rachet effect: bottom topography is symmetry breaking.

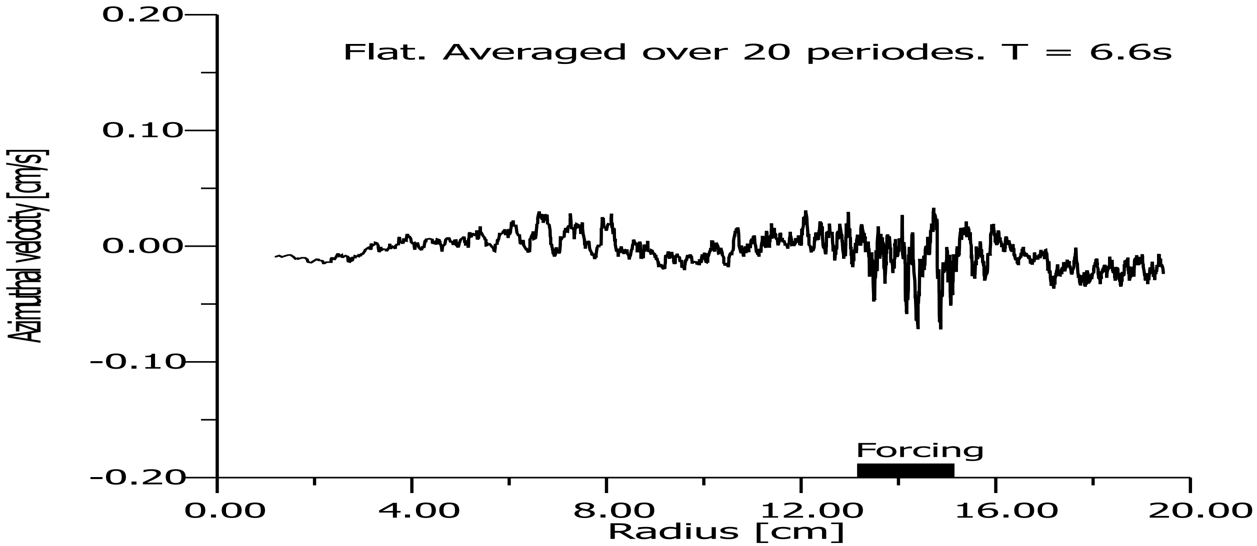
Potential vorticity



Azimuthally averaged vorticity and potential vorticity. Local flattening of potential vorticity.

Averaged azimuthal velocity





Numerical results

The forced quasi-geostrophic vorticity equation on a disk with no-slip boundary conditions at the walls.

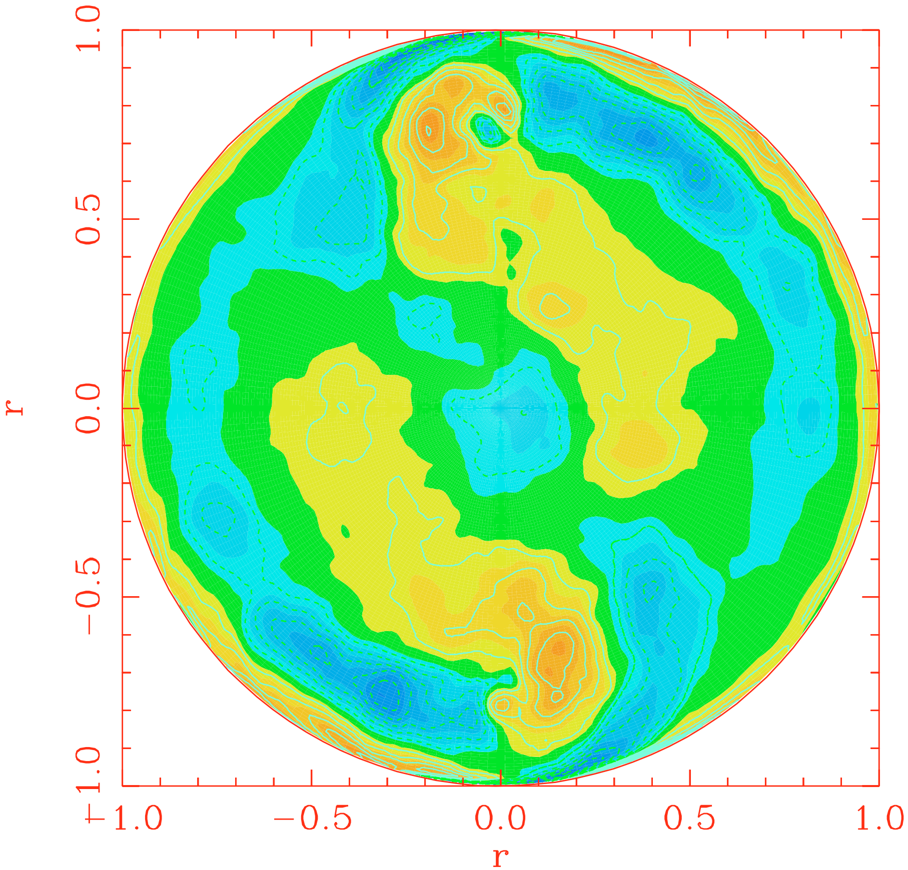
$$\frac{\partial \omega}{\partial t} + \frac{1}{r}[\phi, \omega] - \frac{\kappa}{r}\frac{\partial \phi}{\partial \theta} = -\nu\omega + \frac{1}{Re}\nabla^2\omega + F \ , \quad (1)$$

Length is scaled as R, time as f^{-1}, and κ by f/R. $\nu = \sqrt{E}$, Ekman number $E = \mu/D^2\Omega$ with a spin down time $\tau_E \approx 90\, s$.

The forcing is modeled by localized vorticity sources with alternating positive and negative vorticity:
$F = A_0[G(x, y; r_1)\sin(\sigma_F t) + G(x, y; r_2)\sin(\sigma_F t + \pi)]$, $G(x, y, r_{1,2})$ localized at the positions of the two holes.

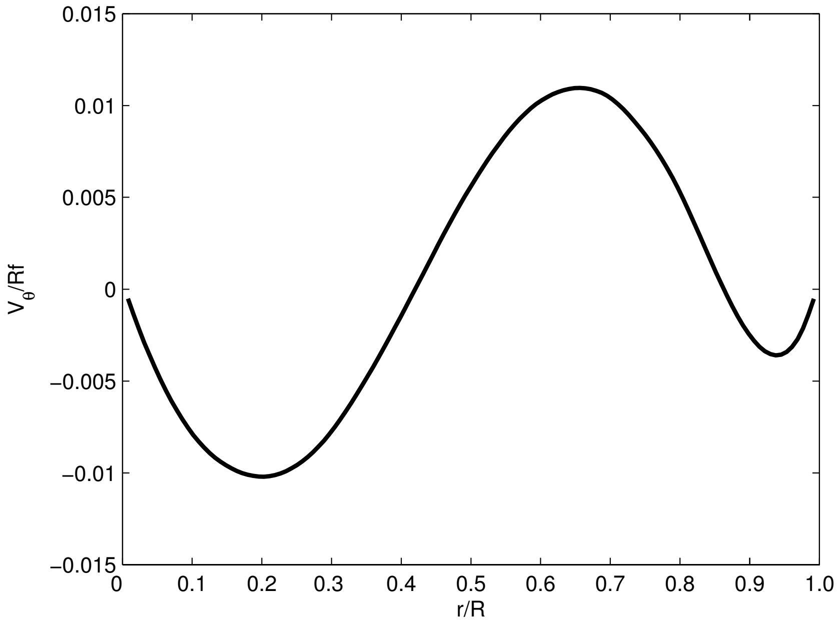
For the experimental condition the scaled values of $\kappa = -0.256$ and $E = 4.55 \times 10^{-4}$. While $Re \approx 80.000$ and volume viscosity is negligible.

Vorticity field



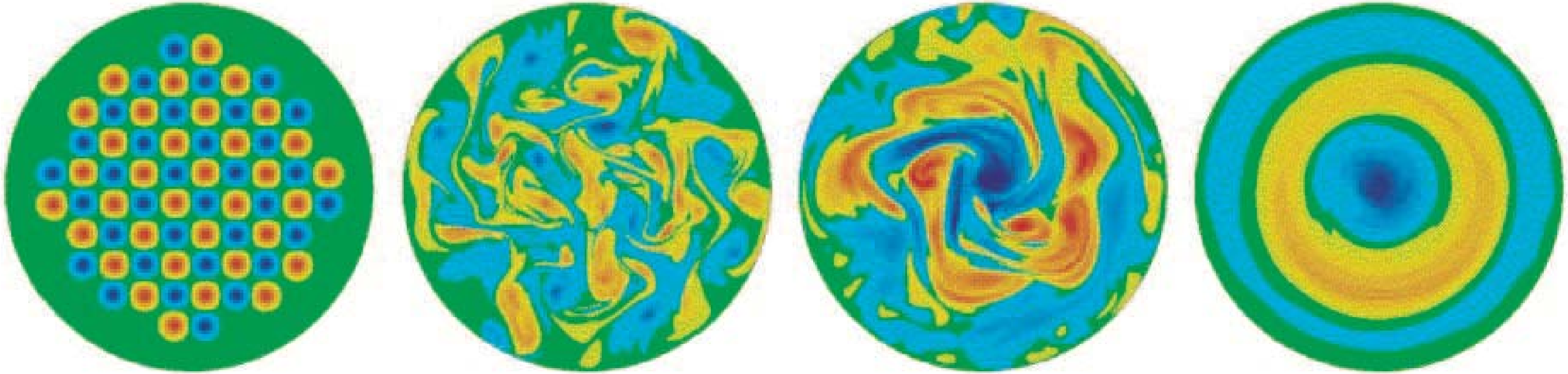
Numerical solution for the same parameters as in the experiment. Vorticity field averaged over 20 forcing periods for the case of a conical bottom.
Red: positive vorticity and blue: negative.

Averaged azimuthal velocity



Numerical solution for the same parameters as in the experiment. Averaged azimuthal velocity.

Zonal bands



Decaying turbulence.

The number of bands and their width depends on κ and turbulence level

The Rhines scale length (Rhines, JFM **69** 417 (1975)), k_κ^{-1} determines the cross over from nonlinear behavior to wavelike behavior: balancing $[\phi, \omega]$ with $\kappa \partial\phi/\partial\theta$.

$k_\kappa = \sqrt{\kappa/2U_rms}$

Similar for drift waves in plasmas (Naulin, New J. Phys. **4** 28 (2002)).

Flow in toroidal hot plasmas

RISØ

Drift-Alfvén turbulence in 3D flux tube geometry,

(Naulin Phys. Plasma **10**, 4016 (2003), Naulin *et al* Phys. Plasma **12**, 052515 (2005))

Vorticity equation:

$$\frac{\partial \omega}{\partial t} = -\mathbf{v_E} \cdot \nabla_\perp \omega + K(n) + \nabla_\| J + \nu_\omega \nabla_\perp^2 \omega,$$

Curvature operator: $K = -\omega_B(\sin s\, \partial_x + \cos s\, \partial_y)$, and

$$\nabla_\| J = \frac{\partial J}{\partial s} - \{\hat{\beta} A_\|, J\} = \frac{\partial J}{\partial s} + \hat{\beta}\{A_\|, \nabla_\perp^2 A_\|\}$$

$\nabla \times \widetilde{\mathbf{B}} = J\mathbf{b}$ and $\widetilde{\mathbf{B}} = \nabla A_\| \times \mathbf{b} \rightarrow J_\| = -\nabla_\perp^2 A_\|$

$\hat{\beta} = (2\mu_0 p_e/B^2)(qR/L_\perp)^2$.

Flow in toroidal hot plasmas

RISØ

Mean poloidal flow, V_0, generation, averaging over a flux-surface:

$$\partial_t V_0 = -\partial_x \langle uv \rangle + \hat{\beta}\partial_x \langle \widetilde{B}_x \widetilde{B}_y \rangle + \langle K(n) \rangle + \nu_\Omega \partial_{xx} V_0.$$

$u = -\partial_y \phi$, $v = \partial_x \phi$ and $\widetilde{B}_x = \partial_y A_{||}$ and $\widetilde{B}_y = -\partial_x A_{||}$

$Re \equiv \langle uv \rangle$: Reynolds stress; $Ma \equiv \langle \widetilde{B}_x \widetilde{B}_y \rangle$ Maxwell stress.

MHD-limit (high β) it can be shown that $A_{||} \approx \phi/\sqrt{\hat{\beta}}$:

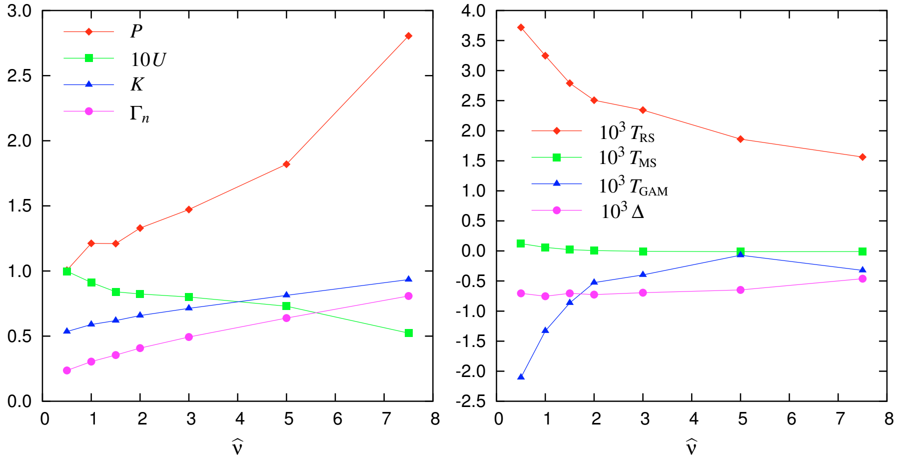
thus Ma cancels Re

Energy exchange:

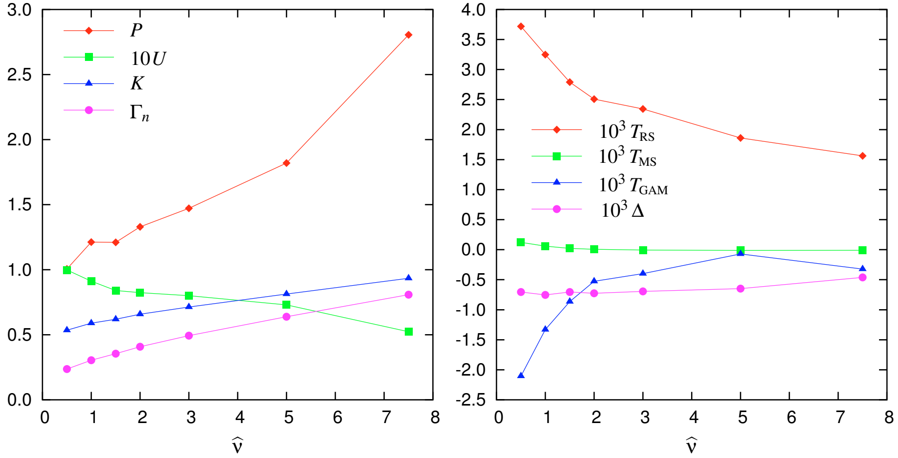
$$\frac{\partial U}{\partial t} = \int \langle uv \rangle \frac{\partial V_0}{\partial x} dx - \hat{\beta} \int \langle \widetilde{B}_x \widetilde{B}_y \rangle \frac{\partial V_0}{\partial x} dx - \omega_B \int \langle n \sin s \rangle V_0 dx - \nu_\omega \int (\partial_x V_0)^2 dx.$$

Maxwell vs Reynolds stress

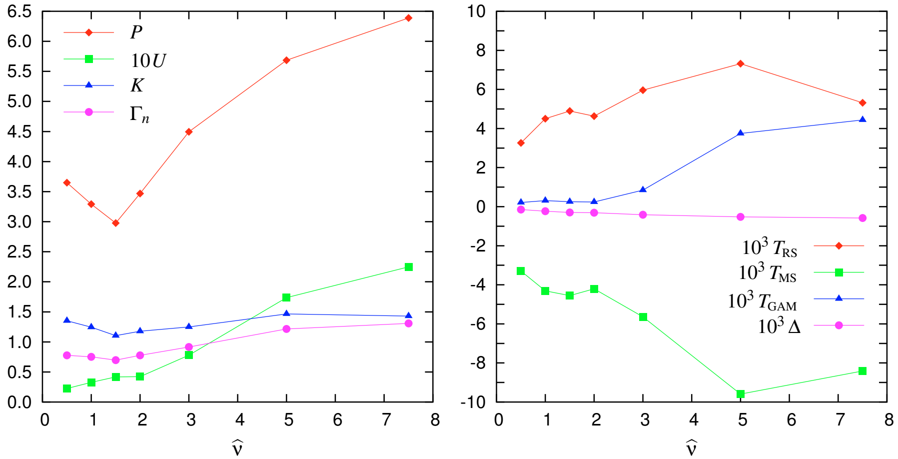
Small β



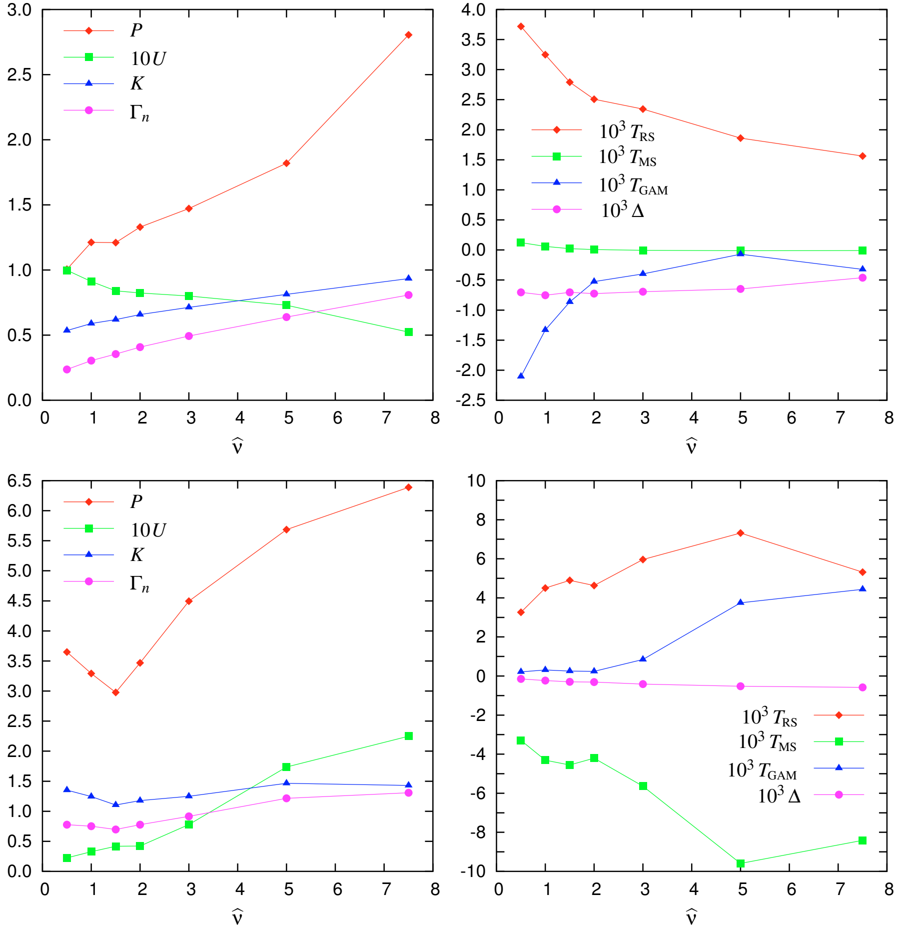
Maxwell vs Reynolds stress



Large β



Maxwell vs Reynolds stress



At high β-values Ma cancels Re Vianello *et al* Nucl. Fus. **45**, 763 (2005)

Gams: Geodesic acoustic modes are always important, geometry effect!

Conclusions

- Poloidal flows indeed limit the turbulent flux
- Self-generated flows take energy out of fluctuations, they do not quench fluctuations
- Flows are generated by Re-stress in anisotropic turbulence
- In hot plasmas finite β effects tends to cancel the zonal flows: the Ma-stress counteracts the Re-stress, here the GAMs become important
- Externally added shear flows may control turbulence, redistribution, but not really stabilize. Finite system size and viscous effects are important
- Flow generation in “simple” systems are well understood. However, effects of real toroidal geometry and shaping are still not explored.