The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR 1673/10

AUTUMN COLLEGE ON PLASMA PHYSICS

5 - 30 September 2005

Radiation Reaction and Relativistic Hydrodynamics

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Radiation Reaction and Relativistic Hydrodynamics

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Based on: Berezhiani, Hazeltin, and Mahajan

PRE 69, 056406 (2004)

NON RELATIVISTIC CASE

$$P = \frac{2e^2}{3c^3}\left(\frac{d\mathbf{v}}{dt}\right)^2$$

$$m\dot{\mathbf{v}} = e\mathbf{E} + \frac{e}{c}\mathbf{v}\times\mathbf{B} + \mathbf{f}$$

$$\mathbf{f} = \frac{2e^2}{3c^3}\frac{d^2\mathbf{v}}{dt^2}$$

$$m\dot{\mathbf{v}} = \frac{2e^2}{3c^3}\ddot{\mathbf{v}} \qquad \rightarrow \quad \dot{\mathbf{v}} \propto \exp(3mc^3t/2e^2)$$

IN THE REST FRAME (v=0):

$$\ddot{\mathbf{v}} = \frac{e}{m}\dot{\mathbf{E}} + \frac{e}{mc}\dot{\mathbf{v}}\times\mathbf{B} \approx \frac{e}{m}\dot{\mathbf{E}} + \frac{e^2}{m^2c}\mathbf{E}\times\mathbf{B}$$

$$\mathbf{f} = \frac{2e^3}{3mc^3}\dot{\mathbf{E}} + \frac{2e^4}{3m^2c^4}\mathbf{E}\times\mathbf{B}$$

$$|\mathbf{f}| << |\mathbf{f}_L|$$

$$\lambda >> \frac{e^2}{mc^2} = 2.8\times10^{-13}\,cm, \qquad B << \frac{m^2c^4}{e^3} = 6\times10^{15}\,G$$

$$\lambda >> \frac{\hbar}{mc} = 3.86\times10^{-11}\,cm \qquad (\hbar\omega << mc^2 = 5.1\times10^5\,eV)$$

RELATIVISTIC CASE

$$g^{\alpha\beta}\ (+1,-1,-1,-1)$$
$$ds^2 = dx_\alpha dx^\alpha$$
$$x^\alpha\,(x^0 = ct\,,\ x^i = \mathbf{r}\ )$$
$$u^\alpha = dx^\alpha / ds = (\gamma\,, \gamma \mathbf{v} / c),$$
$$\gamma = \left(1 - \mathbf{v}^2 / c^2\right)^{-1/2}$$
$$p^\alpha = mcu^\alpha,$$
$$u^\alpha u_\alpha = 1, \qquad p^\alpha p_\alpha = m^2 c^2$$

$$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha$$

Lorentz–Abraham–Dirac (LAD) equation :

$$mc\frac{du^\alpha}{ds} = \frac{e}{c}\left(F^{\alpha\beta} + F_{ret}{}^{\alpha\beta}\right)u_\beta$$

$$F_{ret}{}^{\mu\nu} = (F_{ret}{}^{\mu\nu} + F_{adv}{}^{\mu\nu})/2 + (F_{ret}{}^{\mu\nu} - F_{adv}{}^{\mu\nu})/2$$

$$mc\ \frac{du^\alpha}{ds} = \frac{e}{c}F^{\alpha\beta}u_\beta + g^\alpha$$

$$g^\alpha u_\alpha = 0$$

$$g^{\alpha} = \frac{2e^2}{3c}\left(\frac{d^2u^{\alpha}}{ds^2} - u^{\alpha}R\right)$$

$$R = u^{\beta}\frac{d^2u_{\beta}}{ds^2} = -\frac{du^{\beta}}{ds}\frac{du_{\beta}}{ds}$$

$$R = \frac{\gamma^6}{c^4}\left[\left(\dot{\mathbf{v}}\right)^2 - \frac{\left(\mathbf{v}\times\dot{\mathbf{v}}\right)^2}{c^2}\right] > 0$$

$$-\int ds\, g^{\alpha} = \frac{2e^2}{3c}\int dx^{\alpha}R = \Delta P^{\alpha}$$

$$\frac{d\gamma m\mathbf{v}}{dt} = \mathbf{f}_L + \frac{2e^2\gamma^2}{3c^3}\left[\ddot{\mathbf{v}} + \frac{3\gamma^2}{c^2}\left(\mathbf{v}.\dot{\mathbf{v}}\right)\dot{\mathbf{v}} + \frac{\gamma^2}{c^2}\left(\mathbf{v}.\ddot{\mathbf{v}}\right)\mathbf{v} + \frac{3\gamma^4}{c^4}\left(\mathbf{v}.\dot{\mathbf{v}}\right)^2\mathbf{v}\right]$$

$$\frac{d\gamma mc^2}{dt} = \mathbf{f}_L.\mathbf{v} + \frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{2e^2\gamma^4\mathbf{v}.\dot{\mathbf{v}}}{3c^2}\right) - \frac{2e^2c}{3}R$$

"Defects" of LAD equation:

1) LAD is of third order in the time derivative of the particle position so that giving the initial position and velocity of the particle does not determine the solution uniquely.

2) In the absence of an external force, there exist solutions of exponentially increasing velocity (run-away solutions).

3) The fact that the self-force is *induced* by the external force and vanishes when that force vanishes is not reflected in the LAD equation.

Landau-Lifshitz-Rohrlich (LLR) Equation

$$\frac{e}{c}F^{\alpha\beta}u_\beta >> g^\alpha$$

$$mc\frac{du^\alpha}{ds} = \frac{e}{c}F^{\alpha\beta}u_\beta$$

$$\frac{d^2u^\alpha}{ds^2} = \frac{e}{mc^2}\frac{\partial F^{\alpha\beta}}{\partial x^\sigma}u_\beta u^\sigma + \frac{e^2}{m^2c^4}F^{\alpha\beta}F_{\beta\sigma}u^\sigma$$

$$g^\alpha = \frac{2e^3}{3mc^3}\frac{\partial F^{\alpha\beta}}{\partial x^\sigma}u_\beta u^\sigma - \frac{2e^4}{3m^2c^5}F^{\alpha\beta}F_{\sigma\beta}u^\sigma +$$

$$+\frac{2e^4}{3m^2c^5}\left(F_{\beta\gamma}u^\gamma\right)\left(F^{\beta\sigma}u_\sigma\right)u^\alpha$$

$$\mathbf{f} = \frac{2e^3}{3mc^3}\gamma\left[\frac{d\mathbf{E}}{dt} + \frac{1}{c}\mathbf{v}\times\frac{d\mathbf{B}}{dt}\right] +$$

$$+ \frac{2e^4}{3m^2c^4}\left[\mathbf{E}\times\mathbf{B} + \frac{1}{c}\mathbf{B}\times\mathbf{B}\times\mathbf{v} + \frac{1}{c}\mathbf{E}(\mathbf{v}\mathbf{E})\right] -$$

$$- \frac{2e^4}{3m^2c^5}\gamma^2\mathbf{v}\left[\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right)^2 - \frac{1}{c^2}(\mathbf{v}\mathbf{E})^2\right]$$

In Ultra-Relativistic case:

$$\gamma >> 1 \quad (\mathrm{v} \approx c)$$

$$\mathbf{f} \propto \frac{e^2}{m^2c^4}\gamma^2\,\mathbf{f}_L^{\,2}$$

Radatation reaction force can be larger than the Lorentz force (Landau)

$$\mathbf{f} \approx \mathbf{f}_L \qquad \rightarrow \qquad \frac{e^2\mathbf{f}_L\gamma^2}{m^2c^4} \approx 1$$

It does not contradict to the condition:

$$\frac{e^2\mathbf{f}_L\gamma}{m^2c^4} << 1$$

In a laser plasma case the damping force becomes significant above

$$5\times10^{22}W/cm^2 \quad (\lambda = 0.88\mu m)$$

(Zhidkov et al. (2002); Koga (2004))

Damping or Acceleration?

The importance of radiation reaction in determining the interaction of intense ***coherent*** radiation with a free electron has been pointed out by **Sanderson (1965):**

The incident beam loses momentum at a mean rate:

$$I\sigma / c$$

$\sigma=8\pi r^2{}_0/3$ - is the Thomson cross section

I=c W=c<E^2>/4π – is the intensity

$$V_z = \left(\frac{I\sigma}{mc}\right)t$$

"The effect of the damping term $\left(\frac{2e^2}{3c^3}\ddot{V}\right)$ *is to reduce the phase lag between **V** and **E**. Therefore, it produces on acceleration in the z direction"* – **T.W.B. Kibble (1966).**

Gunn and Ostriker (1971)—"*We see that for any initial conditions the radiative losses will ultimately lead to increases in the particle energy, and that for the conditions considered here the energy will continue to increase without limit*"

Here the rate of change of γ is proportional to $\boldsymbol{u}_\perp \cdot \boldsymbol{v}$, simply the projection of the electric field along the motion. In the absence of radiation $\boldsymbol{u}_\perp$ lags $\boldsymbol{v}$ by exactly $\frac{1}{2}\pi$; the field is always perpendicular to the velocity, and $d\gamma/d\eta$ vanishes. The effect of radiation drag, however, is to induce a phase lag in $\boldsymbol{u}_\perp$, as remarked earlier, making $\boldsymbol{u}_\perp \cdot \boldsymbol{v}$ always positive around the orbit. Since, for a plane wave, $\boldsymbol{u}_\perp \cdot \boldsymbol{E} = |\boldsymbol{u}_\perp \times \boldsymbol{B}|$, a similar acceleration is induced in the ζ-motion in the direction of propagation.

Steiger and Woods (1972):

For circular polarization:

$$\mathbf{f} = -\frac{2e^2}{3c^3}\omega^2\gamma^4\mathbf{v}$$

$$\gamma m\frac{\partial \mathbf{v}}{\partial t} = e\mathbf{E} + \frac{e}{c}\mathbf{v}\times\mathbf{B} - \frac{2e^2}{3c^3}\omega^2\gamma^4\mathbf{v}$$

As a consequence of the phase shift, there arises a force $(\mathbf{v}\times\mathbf{B})_z$. This force is the source of the longitudinal acceleration.

Landau and Lifshitz (1959- *The classical theory of fields*)

The radiation force acting on an electron which scatters photons can be derived (in the Thompson regime) not only through energy-momentum considerations but also by averaging the radiation reaction force.

$$\mathbf{v} \to 0$$

$$\overline{\mathbf{f}} = \frac{2e^3}{3mc^3}\overline{\dot{\mathbf{E}}} + \frac{2e^4}{3m^2c^4}\overline{\mathbf{E}\times\mathbf{B}} = \sigma\frac{\overline{E^2}}{4\pi}\mathbf{n}$$

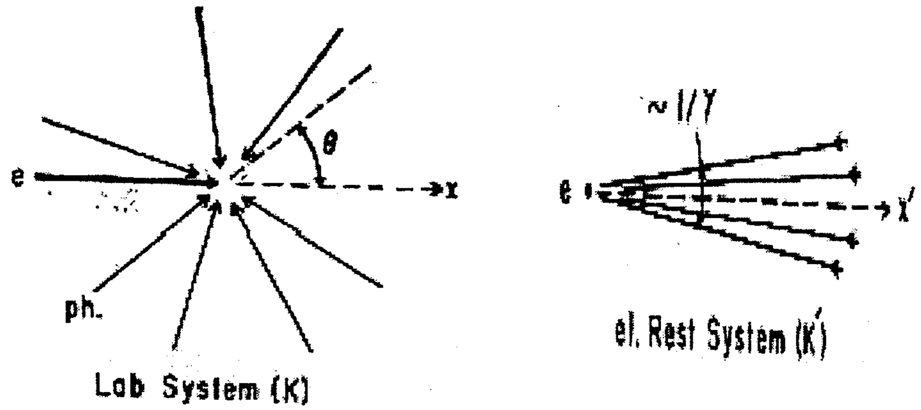
$$\overline{\mathbf{v}}_n = \mathbf{n}(I\sigma / mc)t$$

What happens when it picks up speed?

$$\mathbf{f} = \frac{2e^4}{3m^2c^4}E^2(1-\mathbf{v}_n/c)[\mathbf{n} - \gamma^2(1-\mathbf{v}_n/c)\mathbf{v}/c]$$

We see that the effect of this force is always to accelerate the particle in the direction, **n**, of the Poiting vector! (Essen)

Motion Through a " Photon Gas"



$$\tan(\theta') = \sin(\theta) / \gamma(\cos(\theta) - \mathrm{v} / c)$$

$$\gamma >> 1$$

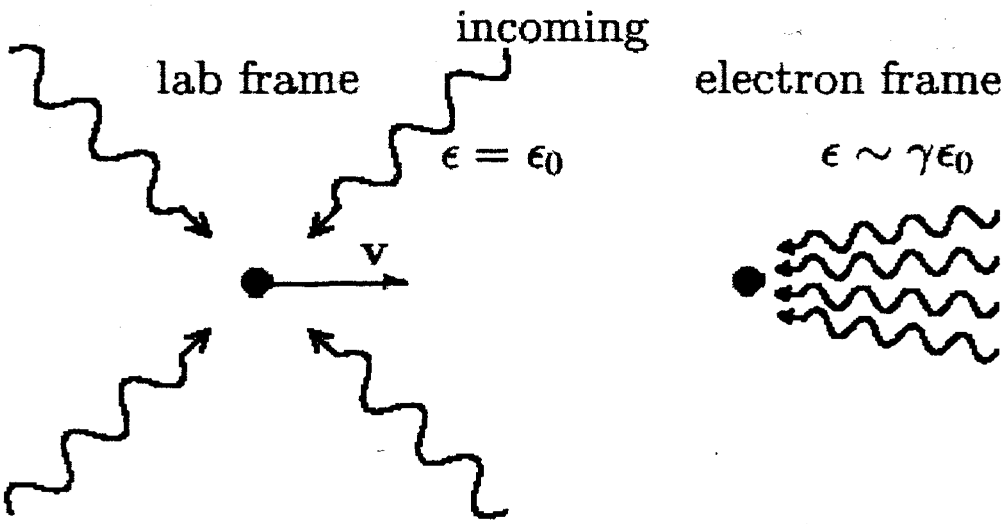
$$\tan(\theta') \rightarrow \quad -\frac{\cot(\theta)}{\gamma}$$

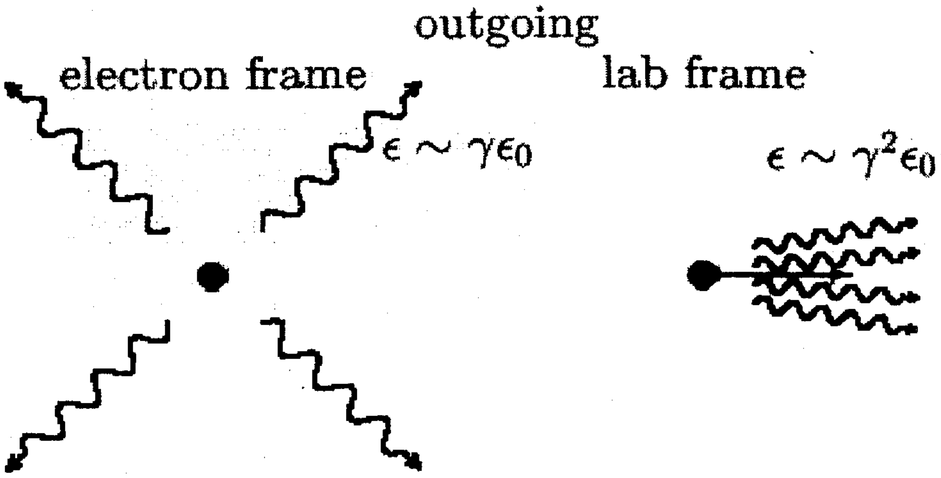
$$\omega_r' = \omega_r$$

$$k^{\alpha} u_{\alpha} = inv$$

$$\omega'\left(1 - \frac{v}{c}\cos\theta'\right) = \omega\left(1 - \frac{v}{c}\cos\theta\right)$$

$$\omega' \approx 4\gamma^2\omega \qquad (\gamma >> 1)$$





$$\frac{d\sigma}{d\Omega} =$$

RECOIL

$$mc\frac{du^{\alpha}}{ds}=\frac{\sigma}{c}\left(\overline{T}^{\alpha\beta}u_{\beta}-u^{\alpha}u_{\beta}u_{\delta}\overline{T}^{\beta\delta}\right)$$

$$\overline{T}^{\alpha\beta}=\begin{pmatrix}\overline{W}, & \overline{\mathsf{S}}/c\\ \overline{\mathsf{S}}/c, & \sigma_{i,k}\end{pmatrix}$$

Isotropic case:

$$\overline{S}=0,\quad \sigma_{i,k}=\delta_{ik}\overline{W}/3$$

$$\frac{dm\gamma\mathsf{v}}{dt}=-\frac{4}{3c}\sigma\overline{W}\gamma^{2}\mathsf{v}$$

Anisotropic case:

$$\overline{T}^{\alpha\beta}(T^{00}=T^{01}=T^{10}=T^{11}=\overline{W})$$

$$\frac{dm\gamma\mathsf{v}}{dt}=\sigma\overline{W}\,\frac{1-\mathsf{v}/c}{1+\mathsf{v}/c}$$

The relativistic plasma – a "Compton Rocket" effect (**O'DELL** – (1980))

Let $I_\nu(\hat{k})$ be the spectral intensity (power/frequency/area/solid angle) of the incident radiation in the direction $\hat{k}$, and let $\phi(p)$ be the phase-space number density (number/configuration volume/momentum volume) of electrons with momentum p. The expected rate of energy transfer to the plasma per electron (or positron) is then

$$\frac{dE}{dt} = -\int\left\{\frac{I_\nu(\hat{k})}{h\nu}(1-\beta\cdot\hat{k})\delta\left(\nu' - \frac{1-\beta\cdot\hat{k}}{1-\beta\cdot\hat{k}'}\nu\right)[(h\nu')-(h\nu)]\frac{d\sigma}{d\Omega'}d\nu' d\Omega' d\nu d\Omega\right\}\phi(p)d^3p \Big/ \int\phi(p)d^3p$$

$$= -\int\left\{I(\hat{k})(1-\beta\cdot\hat{k})\left[\left(\frac{1-\beta\cdot\hat{k}}{1-\beta\cdot\hat{k}'}\right)-1\right]d\sigma d\Omega\right\}\frac{\phi(p)d^3p}{n}, \quad (3)$$

where $I(\hat{k}) = \int I_\nu(\hat{k})d\nu$ is the incident intensity, and $n = \int\phi(p)d^3p$ is the electron (and positron) number density. Correspondingly, the expected rate of momentum transfer to the plasma per electron (or positron) is

$$\frac{dp}{dt} = -\int\left\{\frac{I_\nu(\hat{k})}{h\nu}(1-\beta\cdot\hat{k})\delta\left(\nu' - \frac{1-\beta\cdot\hat{k}}{1-\beta\cdot\hat{k}'}\nu\right)\left[\left(\frac{h\nu'}{c}\hat{k}'\right)-\left(\frac{h\nu}{c}\hat{k}\right)\right]\frac{d\sigma}{d\Omega'}d\nu' d\Omega' d\nu d\Omega\right\}\phi(p)d^3p \Big/ \int\phi(p)d^3p$$

$$= -\frac{1}{c}\int\left\{I(\hat{k})(1-\beta\cdot\hat{k})\left[\left(\frac{1-\beta\cdot\hat{k}}{1-\beta\cdot\hat{k}'}\right)\hat{k}'-\hat{k}\right]d\sigma d\Omega\right\}\frac{\phi(p)d^3p}{n}. \quad (4)$$

$$I(\hat{k}) = F(r)\delta(\hat{k}-\hat{r}),$$

$$\frac{dE}{dt} = -\sigma_T F(r)\int\{(1-\beta\cdot\hat{r})[\gamma^2(1-\beta\cdot\hat{r})-1]\}\frac{\phi(p)d^3p}{n},$$

$$\frac{dp}{dt} = -\frac{\sigma_T F(r)}{c}\hat{r}\int\{(1-\beta\cdot\hat{r})[\gamma^2(1-\beta\cdot\hat{r})-\gamma^2(1-\beta\cdot\hat{r})^2-1]\}\frac{\phi(p)d^3p}{n}$$

$$= +\frac{\sigma_T F(r)}{c}\hat{r}\int\{(1-\beta\cdot\hat{r})[\gamma^2(1-\beta\cdot\hat{r})(-\beta\cdot\hat{r})+1]\}\frac{\phi(p)d^3p}{n}.$$

$$\frac{dE}{dt} = -\sigma_T F(r)[\tfrac{4}{3}\langle(\gamma\beta)^2\rangle],$$

$$\frac{dp}{dt} = +\frac{\sigma_T F(r)}{c}\hat{r}[\tfrac{2}{3}\langle(\gamma\beta)^2\rangle+1],$$

$$\frac{f_r\,(\text{hot})}{f_r\,(\text{cold})} = \left[\frac{2}{3}\langle(\gamma\beta)^2\rangle+1\right] = \left[\frac{2}{3}\frac{\langle p^2\rangle}{(mc)^2}+1\right],$$

Phinney –(1982); Sikora and Wilson-(1981)

$$\frac{dp^{\mu}}{d\tau} = -\sigma\left(u_{\alpha} T^{\alpha\mu} + [u_{\alpha} T^{\alpha\beta} u_{\beta}]\, u^{\mu}\right).$$

The equations derived by O'Dell are merely a special case: a plane-parallel (or point source) radiation field propagating in the e_1 direction is described by a stress–energy tensor with all components zero except $T^{00} = T^{01} = T^{10} = T^{11} = F$, the flux. Then putting $u_{\mu} = (-\gamma_r, \gamma_r \beta_r \cos\theta, u_2, u_3)$ into equation (2), we have

$$\frac{dp^0}{dt} = \frac{1}{\gamma_r}\frac{dp^0}{d\tau} = -\sigma F\left[-(1-\beta_r\cos\theta) + \gamma_r^2(1-\beta_r\cos\theta)^2\right]$$

$$\frac{dp^1}{dt} = \frac{1}{\gamma_r}\frac{dp^1}{d\tau} = \sigma F\left[(1-\beta_r\cos\theta) - \gamma_r^2(1-\beta_r\cos\theta)^2\beta_r\cos\theta\right]$$

which are O'Dell's equations (6) and (7), before averaging over particles

$$\left\langle\frac{dp^{0'}}{dt'}\right\rangle \equiv \left\langle\frac{dE'}{dt'}\right\rangle = -\sigma F'\frac{4}{3}\langle\gamma_r^2\beta_r^2\rangle$$

$$\left\langle\frac{dp^{1'}}{dt'}\right\rangle = \sigma F'\left(1 + \frac{2}{3}\langle\gamma_r^2\beta_r^2\rangle\right),$$

BHM - 2004

$$T^{\alpha\beta} = \frac{1}{4\pi}\left[- F^{\alpha\gamma} F^{\beta}_{\ \gamma} + \frac{1}{4} g^{\alpha\beta} F_{\gamma\delta} F^{\gamma\delta} \right]$$

$$\left\{ \begin{array}{l} \frac{\sigma}{c}\left(T^{\alpha\beta} u_\beta - T^{\beta\gamma} u^\alpha u_\beta u_\gamma \right) \equiv \\ - \frac{2e^4}{3m^2c^5}\left(F^{\alpha\beta} F_{\gamma\beta} u^\gamma - F_{\beta\gamma} F^{\beta\delta} u^\alpha u^\gamma u_\delta \right) \end{array} \right\}$$

$$\boxed{ \begin{array}{l} g^\alpha = \frac{2e^3}{3mc^3} \frac{\partial F^{\alpha\beta}}{\partial x^\gamma} u_\beta u^\gamma + \\ \quad + \frac{\sigma}{c}\left(T^{\alpha\beta} u_\beta - T^{\beta\gamma} u^\alpha u_\beta u_\gamma \right) \end{array} }$$

Exact(!) - in LLR app.

Kinetic Equation

$$p^{\alpha}\frac{\partial f}{\partial x^{\alpha}}+\frac{\partial}{\partial p^{\alpha}}\left(\frac{e}{c}F^{\alpha\beta}p_{\beta}+mcg^{\alpha}\right)f=$$

$$=\text{Col.}\ (\text{Beliaev, Budker})$$

$$p^{\alpha}\frac{\partial f}{\partial x^{\alpha}}+\frac{\partial}{\partial p^{\alpha}}\left(\frac{e}{c}F^{\alpha\beta}p_{\beta}\right)f=$$

$$=\text{Col.}\ (B,B)+\text{“Self-Interaction”}$$

→ (Kuz'menkov - 1972)

$$\Gamma^{\alpha}=c\int\frac{d^3p}{p_0}\,p^{\alpha}f$$

De Groot "Rel. Kin. Theory"

The Energy - Momentum

$$T^{\alpha\beta}=c\int\frac{d^3p}{p_0}\,p^{\alpha}p^{\beta}f$$

The Entrophy 4-flow

$$S^{\alpha}=-c\int\frac{d^3p}{p_0}\,p^{\alpha}f\left[\ln(2\pi\hbar)^3f-1\right]$$

$$M^{\alpha\beta\gamma}=c\int\frac{d^3p}{p_0}\,p^{\alpha}p^{\beta}p^{\gamma}f$$

$$\int \frac{d^3p}{p_0} \cdots = 2\int d^4p \ldots \delta\left(p^{02} - (p^2 + m^2c^2)\right)\Theta(p^0)$$

$$\frac{\partial \Gamma^\alpha}{\partial x^\alpha} = 0$$

$$\frac{\partial T^{\beta\alpha}}{\partial x^\beta} - \frac{e}{c} F^{\alpha\beta}\Gamma_\beta = F^\alpha_{rad}$$

where

$$F^\alpha_{rad} = \frac{2e^3}{3m^2c^4}\frac{\partial F^{\alpha\beta}}{\partial x^\gamma} T^\gamma_\beta +$$

$$+ \frac{\sigma}{c}\overline{T}^{\alpha\beta}\Gamma_\beta - \frac{\sigma}{m^2c^3}\overline{T}_{\beta\gamma} M^{\beta\gamma\alpha}$$

$$\frac{\partial}{\partial x} : M - e\, F : T = e^3 \frac{\partial F}{\partial x} : M +$$

$$+ \sigma\left(\overline{T} : T\right) - \sigma\left(\overline{T} : \overline{Z}\right)$$

$$\overline{Z} = \int p \cdot p \cdot p \cdot p\, f$$

MAXWELLIAN PLASMA

The local Max. distribution

$$f(p) = (2\pi\hbar)^{-1} \exp\left(\mu - p^\alpha u_\alpha / T\right)$$

μ – the chemical potential

u^α – the hydrod. four velocity : $u^\alpha = (\gamma c, \gamma \vec{u})$

$\gamma = (1 - u^2/c^2)^{-1/2}$ $\qquad (u^\alpha u_\alpha = c^2)$

$$\Gamma^\alpha = n u^\alpha$$

$$n = n_R = \left[4\pi T\, m^2 c / (2\pi\hbar)^3\right] K_2(mc^2/T)\, e^{\frac{\mu}{T}}$$

$$\boxed{T^{\alpha\beta} = \frac{1}{c^2} W_{ent.}\, u^\alpha u^\beta - g^{\alpha\beta} p}$$

$W_{ent.} = mc^2 n \dfrac{K_3}{K_2}$ $\qquad p = nT$

$$M^{\alpha\beta\gamma} = A_1 u^\alpha u^\beta u^\gamma + A_2\, g^{\{\alpha\beta,}\, u^{\gamma\}}$$

$A_1 = m^2 n\left(1 + 6 K_3 / K_2 Z\right)$

$A_2 = - c^2 m^2 n \left(K_3 / Z K_2\right)$ $\qquad Z = \dfrac{mc^2}{T}$

$$\overline{T}_{\alpha\beta} M^{\alpha\beta\gamma} = A_1 \overline{T}^{\alpha\beta} u_\alpha u_\beta u^\gamma + 2A_2 \overline{T}^{\alpha\beta} u_\alpha$$

$$\boxed{\frac{\partial T^{\alpha\beta}}{\partial x^\beta} - \frac{e}{c} F^{\alpha\beta} n u_\beta = F^\alpha_{rad}}$$

$$F^\alpha_{rad} = \frac{2e^3}{3m^2c^4} \frac{\partial F^{\alpha\beta}}{\partial x^\gamma} T^\gamma_\beta +$$

$$+ \frac{\sigma n}{c}\left[(1+2G/z)\overline{T}^{\alpha\beta} u_\beta - c^{-2}(1+6G/z)\overline{T}^{\beta\gamma} u_\beta u_\gamma u^\alpha\right]$$

with $G(z) = K_3/K_2$

$$S^\alpha = n S u^\alpha \quad \rightarrow \quad S = \ln\left(K_2 \exp\left[zG/pz^2\right]\right) + C_1$$

$$u^\alpha \frac{\partial S}{\partial x^\alpha} = \frac{z}{nmc^2} u_\alpha F^\alpha_{rad}$$

Cold Plasma ($T \to 0$)

$$mc \frac{dU^\alpha}{ds} = \frac{e}{c} F^{\alpha\beta} U_\beta + g^\alpha$$

$$\frac{d}{ds} = (\gamma/c)\left(\frac{\partial}{\partial t} + \vec{u}\cdot\vec{\nabla}\right)$$

Thus, as expected, the cold plasma fluid eq. has a form similar to the one for particle motion

Relativistic Plasma ($T \gg mc^2$)

$$G \simeq 4/z = 4\frac{T}{mc^2} \gg 1 \quad (G - \text{"effective mass"})$$

$$U^\beta \frac{\partial}{\partial x^\beta}\left(4\frac{T}{c^2}U^\alpha\right) - \frac{1}{n}\frac{\partial p}{\partial x_\alpha} = \frac{e}{c}F^{\alpha\beta}U_\beta + \frac{\sigma T}{\pi e c}\frac{\partial F^{\alpha\gamma}}{\partial x^\beta}U^\beta U_\gamma +$$

$$+ \frac{\sigma T}{ec^2}J^\alpha + 8\sigma\frac{T^2}{m^2c^4}\left\{\mathcal{T}^{\alpha\beta}U_\beta - 3\mathcal{T}^{\beta\gamma}U_\beta U_\gamma U^\alpha/c^2\right\}$$

$$\left(J^\alpha = \sum e\Gamma^\alpha\right\} \qquad \frac{\partial F^{\alpha\beta}}{\partial x^\beta} = -(4\pi/c)J^\alpha$$

For the ultrarelativistic case it could dominate the flow dynamics

Electron–Positron Plasma

One fluid description: $T_\pm = T_0$, $n_\pm = n_0/2$, $u^\alpha_\pm = u_0^\alpha$ — i.e. the flow with large spatiotemporal scales

$$\frac{\partial}{\partial x^\beta}\left(T^{\alpha\beta} + \overline{T}^{\alpha\beta}\right) = F^\alpha$$

$$F^\alpha = \frac{\sigma n}{c}\left[\left(1+2G/z\right)\overline{T}^{\alpha\beta}u_\beta - c^{-2}\left(1+6G/z\right)\overline{T}^{\beta\gamma}u_\beta u_\gamma u^\alpha\right]$$

$$F^0_{rest} = -4\sigma n G z^{-1}\,\overline{T}^{00}_{rest}$$

$$F^i_{rest} = \sigma n\left(1+2Gz^{-1}\right)\overline{T}^{i0}_{rest}$$

$$\left\{\begin{array}{l} \langle \gamma'^2 - 1\rangle = \langle \gamma'^2\beta'^2\rangle = 3Gz^{-1} \\ \langle \ldots \rangle \text{ - denotes averaging, } \beta' = v'/c \\ J_0' = \overline{T}^{00}_{rest}, \quad J_1' = \overline{T}^{30}_{rest} \end{array}\right\}$$

$$F^0_{rest} = -\sigma n \frac{4}{3} J_0' \langle \gamma'^2\beta'^2\rangle$$

$$F^z_{rest} = \sigma n J_1'\left(1+\frac{2}{3}\langle\gamma'^2\beta'^2\rangle\right)$$

In agreement with Phinney, O'Dell, Sikora, ...

"Compton" scattering regime

$\hbar\omega \sim mc^2$

Blumenthal (1974)

Madau, Thompson (2002)

Sikora ... (1996)

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The radiative drag force is derived by resorting to a phenomenological, "test-particle approach"

Radiation-Reaction Force ???

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