

The Abdus Salam International Centre for Theoretical Physics





SMR 1673/48

AUTUMN COLLEGE ON PLASMA PHYSICS

5 - 30 September 2005

Neutrino Plasma Coupling in Dense Astrophysical Plasmas

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ABSTRACT



There is considerable interest in the propagation dynamics of neutrinos in a background dispersive medium, particularly in the search for a mechanism to explain the dynamics of type II supernovæ and solve the solar neutrino problem. Neutrino interactions with matter are usually considered as non selfconsistent single particle processes. We describe neutrino streaming instabilities within supernovæ plasmas, resulting in longitudinal and transverse waves using coupled kinetic equations for both neutrinos and plasma particles including magnetic field effects. The transverse waves have energies in the γ -ray range which suggests that this may be a possible mechanism for γ -ray bursts which are associated with supernovæ. Another interesting result is an asymmetry in the momentum balance imparted by the neutrinos to the core of the exploding star due to a magnetic field effect. This can result in a directed velocity of the resulting neutron star or pulsar and can explain the so called natal kick.



Non-Linear Scattering Instabilities



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Non-Linear Scattering Instabilities





To form a neutron star 3 ×10⁵³ erg must be released (gravitational binding energy of the original star) • light+kinetic energy ~ 10⁵¹ erg • • gravitational radiation < 1% • • neutrinos 99 % •

¶ Electron density @ 100-300 km: $n_{e0} \sim 10^{29}$ - 10^{32} cm⁻³

- ¶ Electron temperature @ 100-300 km: $T_e \sim 0.1 0.5$ MeV
- ¶ Degeneracy parameter $\Theta = T_e/E_F \sim 0.5$ 0.7
- \P Coulomb coupling constant $\Gamma \sim 0.01$ 0.1

¶ v_e luminosity @ neutrinosphere~ 10⁵² - 5×10⁵³ erg/s ¶ v_e intensity @ 100-300 Km ~ 10²⁹ - 10³⁰ W/cm²

¶ Duration of intense v_e burst ~ 5 ms (resulting from p+e \rightarrow n+ v_e)

 \P Duration of ν emission of all flavors ~ 1 - 10 s





Supernova Explosion

• How to turn an implosion into an explosion

- New neutrino physics
- $-\lambda_{mfp}$ for ev collisions ~ 10¹⁶ cm in collapsed star
- $\begin{array}{l} \lambda_{mfp} \text{ for collective} \\ \text{ plasma-neutrino} \\ \text{ coupling } \sim 100 \text{m} \end{array}$
- How?
 - New non-linear force neutrino ponderomotive force
 - For intense neutrino flux collective effects important
 - Absorbs 1% of neutrino energy

 \Rightarrow sufficient to explode star





Single particle dynamics governed by Hamiltonian (Bethe, '87):

$$H_{eff} = \sqrt{\mathbf{p}_{v}^{2}c^{2} + m_{v}^{2}c^{4}} + 2G_{F}n_{e}(\mathbf{r}, t)$$

 G_F - Fermi constant n_e - electron density

$$\mathbf{F}_{pond} = -\sqrt{2}G_F \nabla n_v(\mathbf{r},t)$$

Force on a single electron due to neutrino distribution

Ponderomotive force^{*} due to neutrinos pushes electrons to regions of lower neutrino density

* ponderomotive force derived from semi-classical (L.O.Silva et al, '98) or quantum formalism (Semikoz, '87) ¶ Effective potential due to weakinteraction with background electrons¶ Repulsive potential

$$\mathbf{F} = -\sqrt{2}G_F \nabla n_e(\mathbf{r},t)$$

Force on a single neutrino due to electron density modulations

Neutrinos bunch in regions of lower electron density



- The interaction can be easily represented by neutrino refractive index.
- The dispersion relation: $(E_v V)^2 p_v^2 c^2 m_v^2 c^4 = 0$ (Bethe, 1986)

E is the neutrino energy, p the momentum, m_v the neutrino mass.

The potential energy

$$V = \sqrt{2}G_F n_e$$

 G_F is the Fermi coupling constant, n_e the electron density

 \Rightarrow Refractive index

$$N_{\nu} = \left(\frac{ck_{\nu}}{\omega_{\nu}}\right)^2 = \left(\frac{cp_{\nu}}{E_{\nu}}\right)^2$$

$$N_{\nu} \cong 1 - \frac{2\sqrt{2}G_F}{\hbar k_{\nu}c} n_e$$

Note: cut-off density

 \mathcal{E}_{v} neutrino energy

$$n_{ec} > \frac{\mathcal{E}_{v}}{2\sqrt{2}G_{F}}$$

Electron neutrinos are refracted away from regions of dense plasma - similar to photons.





For intense neutrino beams, we can introduce the concept of the Ponderomotive force to describe the coupling to the plasma. This can then be obtained from the 2nd order term in the refractive index.

Definition
$$F_{POND} = \frac{N-1}{2} \nabla \xi$$
 [Landau & Lifshitz, 1960]

where ξ is the energy density of the neutrino beam.

$$N = 1 - \frac{2\sqrt{2}G_F}{\varepsilon_v} n_e \qquad \Rightarrow \quad F_{\text{Pond}} = -\frac{\sqrt{2}G_F n_e}{\varepsilon_v} \nabla \xi$$

 n_{ν} is the neutrino number density.

$$\mathbf{F}_{\text{Pond}} \equiv -\sqrt{2}G_F n_e \nabla n_v$$



Force on one electron due to electron neutrino collisions f_{coll}

$$f_{coll} = \boldsymbol{\sigma}_{v_e} \boldsymbol{\xi} \qquad \boldsymbol{\sigma}_{v_e} = \left(\frac{G_F k_B T_e}{2\pi \hbar^2 c^2}\right)^2$$

 $\sigma_{_{\! V\!e}}$ is the neutrino-electron cross-section

Total collisional force on all electrons is

$$F_{\text{coll}} = n_e f_{coll} = n_e \sigma_{v_e} \xi$$
$$\frac{F_{\text{Pond}}}{F_{\text{coll}}} = \frac{\sqrt{2\pi \hbar^3 c^3}}{G_F k_B^2 T^2} \frac{|k_{Mod}|}{k_v}$$

 $|\mathbf{k}_{mod}|$ is the modulation wavenumber.

For a 0.5 MeV plasma
$$\frac{F_{Pond}}{F_{coll}} \approx 10^{10}$$

 $\sigma_{ve} \Rightarrow$ collisional mean free path of 10¹⁶ cm.



Kinetic equation for neutrinos

(describing neutrino number density conservation /collisionless neutrinos)

$$\frac{\partial f_{v}}{\partial t} + \mathbf{v}_{v} \cdot \frac{\partial f_{v}}{\partial \mathbf{r}} - \sqrt{2}G_{F} \left(\nabla n_{e} + \frac{1}{c^{2}} \frac{\partial \mathbf{J}_{e}}{\partial t} - \frac{\mathbf{v}_{v}}{c^{2}} \times \nabla \times \mathbf{J}_{e} \right) \cdot \frac{\partial f_{v}}{\partial \mathbf{p}_{v}} = 0$$

Electron density oscillations driven by neutrino pond. force (collisionless plasma)

$$\frac{\partial f_e}{\partial t} + \mathbf{v}_e \cdot \frac{\partial f_e}{\partial \mathbf{r}} - \sqrt{2}G_F \left(\nabla n_v + \frac{1}{c^2} \frac{\partial \mathbf{J}_v}{\partial t} - \frac{\mathbf{v}_e}{c^2} \times \nabla \times \mathbf{J}_v \right) \cdot \frac{\partial f_e}{\partial \mathbf{p}_v} - e \left(\mathbf{E} + \mathbf{v}_e \times \mathbf{B} \right) \cdot \frac{\partial f_e}{\partial \mathbf{p}_e} = 0$$

Dispersion relation for electrostatic plasma waves

$$1 + \chi_e(\omega_L, \mathbf{k}_L) + \chi_v(\omega_L, \mathbf{k}_L) = 0$$

Electron susceptibility

Neutrino susceptibility

 $\partial \hat{f}_{...0}$

$$\chi_{\nu}(\omega_{L},\mathbf{k}_{L}) = -2G_{F}^{2} \frac{k_{L}^{3}n_{e0}n_{\nu0}}{m_{e}\omega_{pe0}^{2}} \left(1 - \frac{\omega_{L}^{2}}{c^{2}k_{L}^{2}}\right)^{2} \chi_{e} \int d\mathbf{p}_{\nu} \frac{\mathbf{k}_{L} \cdot \frac{\mathbf{v}_{\nu}}{\partial \mathbf{p}_{\nu}}}{\omega_{L} - \mathbf{k}_{L} \cdot \mathbf{v}_{\nu}}$$



Geometry of neutrino emission

Neutrino distribution in the neutrinosphere $\equiv \mathbf{f}_{v0} (\mathbf{k}_v)$





Monoenergetic neutrino beam

$$f_{v0} = n_{v0} \delta(\mathbf{p}_v - \mathbf{p}_{v0})$$

Dispersion Relation

$$\omega_L^2 = \omega_{pe0}^2 + \left(\frac{m_\nu^2 c^4 \cos^2 \theta}{E_{\nu 0}^2} + \sin^2 \theta\right) \frac{\aleph k_L^4 c^4}{\left(\omega_L - k_L c \cos \theta \frac{p_{\nu 0} c}{E_{\nu 0}}\right)^2} \qquad \theta \equiv \mathbf{k}_L^{\Lambda} \mathbf{p}_{\nu 0}$$

¶ If $m_v \rightarrow 0$ direct forward scattering is absent ¶ Similar analysis of two-stream instability:

• maximum growth rate for $k_L v_{v0\parallel} = k c \cos \theta \approx \omega_{pe0}$

•
$$\omega = \omega_{\text{pe0}} + \delta = k_{\text{L}} v_{v0\parallel} + \delta$$

$$\gamma_{\max} = \frac{\sqrt{3}}{2} \omega_{pe0} \left(\frac{\tan^2 \theta}{\sin^2 \theta} \aleph \right)^{1/3} \propto G_F^2$$

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Weak Beam ($\delta / \omega_{pe0} \ll 1$) Growth rate Strong Beam ($\delta / \omega_{pe0} \gg 1$) $\gamma_{max} \propto G_F^{1/2}$

Single v-electron scattering
$$\propto {
m G_F}^2$$

Collective plasma process much stronger than single particle processes



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Neutrino beam with arbitrary momentum distribution

$$f_{v0} = n_{v0}\hat{f}(p_{vx})\delta(p_{vy})\delta(p_{vz})$$

Neutrino susceptibility

$$\chi_{\nu}(\omega_{L}, \mathbf{k}_{L}) \propto -\frac{n_{\nu0}}{(\omega_{L} - k_{L} \cos \theta)^{2}} \int dp_{\nu x} \frac{f_{\nu0}}{|p_{\nu x}|}$$
From monoenergetic beam to arbitrary neutrino energy distribution
$$\frac{1}{E_{\nu0}} = \frac{\lambda_{\nu0}}{2\pi\hbar c} \rightarrow \frac{<\lambda_{\nu}>}{2\pi\hbar c}$$

For distributions with equal neutrino density n_{v0} and equal de Broglie wavelength $<\lambda_v>$, growth rates are identical



Role of electron-ion collisions in the instability (hydro)

BGK model of collisions $\chi_e(\omega_L, \mathbf{k}_L) = -\frac{\omega_{pe}^2}{\omega_r(\omega_r + i\nu_L)}$ $V_s \approx V_{ei}$ New dispersion relation **Electron-ion** $\omega_L(\omega_L + i\nu_{ei}) = \omega_{pe0}^2 + \left(\frac{m_\nu^2 c^4 \cos^2 \theta}{E_{\nu 0}^2} + \sin^2 \theta\right) \frac{\aleph k_L^4 c^4}{\left(\omega_L - k_L c \cos \theta \frac{p_{\nu 0} c}{E_{\nu 0}}\right)^2}$ collision frequency Similar analysis as before leads to $\gamma_{\max} = \frac{\sqrt{2}}{2} \omega_{pe0} \left(\frac{\tan^2 \theta}{\sin^2 \theta} \aleph \frac{\omega_{pe0}}{v_{ei}} \right)^{1/2} \propto G_F \quad VS \qquad \gamma_{\max} \propto G_F^{2/3}$

(without collisions)

Instability threshold is $\propto G_F^2$

since it is proportional to (Damping electrons) x (Damping neutrinos)

(with collisions)



Instability regimes: hydrodynamic vs kinetic





Estimates of the Instability Growth Rates

 $n_{e0} = 10^{29} \text{ cm}^{-3}$ $L_{\nu} = 10^{52} \text{ erg/s}$ $R_{m} = 300 \text{ Km}$ $< E_{\nu} > = 10 \text{ MeV}$

> Growth distance ~ 1 m (without collisions)

Growth distance ~ 300 m (with collisions)

- 6 km for 20 e-foldings -

Mean free path for neutrino electron single scattering \sim 10^{11} km





Saturation Mechanism





Preliminary Results



¶ Preliminary results indicate strong heating up to 0.5 MeV;

I Further analysis is necessary to include relativistic corrections on electron Landau damping - present model overestimates eLD;

¶ Initial v_e burst (~ ms) can heat the plasma efficiently;

I Detailed quasi-linear theory for v's and e's will give signatures of v-driven instabilities and more accurate results \rightarrow information to be included in supernovae code

I Stimulated "Compton" scattering must also be considered



• For transverse plasmon neutrino interactions the kinetic equations are:-

where

$$\frac{\partial f_{v}}{\partial t} + \underline{v}_{v} \frac{\partial f_{v}}{\partial \underline{r}} + \underline{F}_{v} \frac{\partial f_{v}}{\partial \underline{p}_{v}} = 0 \qquad \qquad \frac{\partial f_{e}}{\partial t} + \underline{v}_{e} \frac{\partial f_{e}}{\partial \underline{r}} + \underline{F}_{e} \frac{\partial f_{e}}{\partial \underline{p}_{e}} = 0$$

$$\underline{F}_{v} = -\sqrt{2}G_{F} \left(\underline{\nabla}n_{e} + \frac{1}{c^{2}} \frac{\partial J_{e}}{\partial t} - \frac{1}{c^{2}} \underline{v}_{v} \times \nabla \times J_{e} \right)$$

$$\underline{F}_{e} = -e(\underline{E} + \underline{v}_{e} \times \underline{B}) - \sqrt{2}G_{F} \left(\nabla n_{v} + \frac{1}{c^{2}} \frac{\partial J_{v}}{\partial t} - \frac{\underline{V}_{e}}{c^{2}} \times \nabla \times J_{v} \right)$$

$$= e_{F} = -e(\underline{E} + \underline{v}_{e} \times \underline{B}) - \sqrt{2}G_{F} \left(\nabla n_{v} + \frac{1}{c^{2}} \frac{\partial J_{v}}{\partial t} - \frac{\underline{V}_{e}}{c^{2}} \times \nabla \times J_{v} \right)$$

note that we can introduce the boson fields E_{ν} and B_{ν} given by

$$E_{v} = -\underline{\nabla}n_{e} - \frac{1}{c^{2}}\frac{\partial J_{e}}{\partial t} \qquad \qquad B_{v} = \underline{\nabla} \times \underline{J}_{e}$$

• The dispersion relation for transverse plasmons in the collisionless limit is

$$\varepsilon_{t} + \chi_{v} = 0$$

$$\tau e \qquad \chi_{v} = -2G_{F}^{2}Ak_{t} \frac{n_{e0}n_{v0}}{m_{e}\omega_{p_{e0}}^{2}} \left(1 - \frac{\omega^{2}}{c^{2}k^{2}}\right)\chi_{e}\int \frac{d\underline{p}_{v}k}{\omega - \underline{k} \cdot v_{v}} \frac{\partial f_{v0}}{\partial p_{v}}$$

$$A = 2\frac{\omega_{p}^{2}}{\omega} \left(\frac{\partial}{\partial \omega}\omega^{2}\varepsilon^{t}\right)^{-1}$$

where



Neutrino heating is necessary for a strong explosion

The shock exits the surface of the proto-neutron star and begins to stall approximately 100 milliseconds after the bounce.

The initial electron neutrino pulse of 5x10⁵³ ergs/second is followed by an "accretion" pulse of all flavours of neutrinos.



This accretion pulse of <u>-0.1 0.0 0.1 0.2 0.3</u> neutrinos deposits energy behind the stalled shock, increasing the matter pressure sufficiently to drive the shock completely through the mantle of the star.







- Neutrino spectra and time history of the fluxes probe details of the core collapse dynamics and evolution.
- Neutrinos provide heating for "delayed" explosion mechanism.
- Sufficiently detailed and accurate simulations provide information on convection models and neutrino mass and oscillations.



Neutrino Landau Damping

General dispersion relation describes not only the neutrino fluid instability but also the neutrino kinetic instability

¶ EPW wavevector $\mathbf{k}_{L} = \mathbf{k}_{L\parallel}$ defines parallel direction ¶ neutrino momentum $\mathbf{p}_{\nu} = \mathbf{p}_{\nu\parallel} + \mathbf{p}_{n\perp}$ ¶ arbitrary neutrino distribution function $\mathbf{f}_{\nu 0}$ ¶ Landau's prescription in the evaluation of χ_{ν}

For a Fermi-Dirac neutrino distribution

$$\gamma_{\text{Landau}} \approx -\frac{k_L c}{2} \pi \frac{\mathbf{G}_{\mathbf{F}}^2 n_{e0} n_{v0}}{m_e c^2 k_B T_v} \frac{\text{Li}_2(-\exp E_F / T_v)}{\text{Li}_3(-\exp E_F / T_v)}$$



Contribution from the pole



Neutrino Landau damping leads to damping of EPWs by energy transfer to the neutrinos **Important for the neutron star cooling process**



Plasma cooling by Neutrino Landau damping

Neutrinos drain energy from the plasma by damping plasma waves

- unlike the usual neutron star cooling plasma process the number of neutrinos is conserved -

 $Q_{epw} - \text{ energy loss rate} \qquad Q_{EPW} = \int \frac{d\mathbf{k}}{(2\pi)^3} \gamma_{vLandau}(\mathbf{k}) W_{EPW}$ $W_{epw} - \text{ spectral energy density of EPWs} - Bose distribution$ $Q_{EPW} = -\frac{3\pi}{4} \frac{W_{EPW}}{k_D^3} c \left(\frac{\omega_{pe0}}{c}\right)^4 \frac{G_F^2 n_{e0} n_{v0}}{m_e c^2 E_{v0}} \frac{Li_2(-\exp\mu/T)}{Li_3(-\exp\mu/T)} \left(\frac{3}{4} + \frac{1}{4\beta_{th}^4} - \frac{1}{\beta_{th}^2} - \ln\beta_{th}\right)$ $B_{th} - \text{typical thermal velocity}$ $\frac{\beta_{th}}{2\pi/k_d} - \text{Debye Length}$

Stronger than mechanism proposed by Tsytovich (1961) For a broad range of parameters more important than usual plasma cooling process



1) Neutrino beam plasma instability can result in photon production.



In supernovæ the frequency of the photons is in the MeV energy range - *i.e.* γ -rays.

2) The neutrino heated plasma can also produce electron-positron pairs. If the rate of production is greater than the the rate of annihilation then the resulting structure is a relativistic electron/positron fireball.

<u>γ-Ray Bursts (GRBs)</u>

A few percent of the neutrino energy must be converted to γ -rays to explain the GRBs which are thought to be associated with supernovæ (1).



Conclusions

- General description of neutrino formed scattering instabilities into longitudinal and transverse plasmons.
- Neutrino Landau damping.
- Quasi-linear theory developed
- Possibility of neutrino generation of γ-rays in supernova plasmas



Outline

Intense fluxes of neutrinos in Astrophysics

- Neutrino dynamics in dense plasmas (making the bridge with HEP)
- **Plasma Instabilities driven by neutrinos**
- Supernovae, neutron stars and ν driven plasma instabilities
- Gamma-ray bursters: open questions e⁺e⁻ 3D electromagnetic beam plasma instability Consequences on GRBs and relativistic shocks Conclusions and future directions



Neutrinos are the most enigmatic particles in the Universe

Associated with some of the long standing problems in astrophysics

Solar neutrino deficit Gamma ray bursters (GRBs) Formation of structure in the Universe Supernovae II (SNe II) Stellar/Neutron Star core cooling Dark Matter

Intensities in excess of 10³⁰ W/cm² and luminosities up to 10⁵² erg/s



LeptonsElectron eMuon μ Tau τ LeptonsElectron neutrino v_e Muon neutrino v_{μ} Tau neutrino v_{τ}

An electron beam propagating through a plasma generates plasma waves, which perturb and eventually break up the electron beam



Electroweak theory unifies electromagnetic force and weak force

A similar scenario should also be observed for intense neutrino bursts



Length scales

← Compton Scale HEP

$\begin{array}{l} \text{Hydro Scale} \rightarrow \\ \text{Shocks} \end{array}$

Plasma scale

 $\lambda_{\rm D}, \lambda_{\rm p}, r_{\rm L}$

>> 14 orders of magnitude

Can intense neutrino winds drive collective and kinetic mechanisms at the *plasma scale* ?

Bingham, Bethe, Dawson, Su (1994)





Flavor conversion - electron neutrinos convert into another v flavor

Equivalent to mode conversion of waves in inhomogeneous plasmas

$$\frac{d^2 \psi_i}{dx^2} + k_i^2 \psi_i = 0 \qquad k_i^2 = \frac{E_i^2 - m_i^2 c^4 - V_{eff i}}{c^2 \hbar^2} \qquad i = 1, 2, 3 \text{ (each v flavor)}$$

Mode conversion when $k_1 = k_2$, $E_1 = E_2$

$$\frac{d^2 \psi_1}{dx^2} + k_1^2 \psi_1 = \lambda_1 \psi_2 \qquad \qquad \lambda_i = \frac{1}{2} \frac{\Delta m^2 c}{\hbar^2} \frac{E_i}{p_i} \sin 2\theta$$

$$\frac{d^2\psi_2}{dx^2} + k_2^2\psi_2 = \lambda_2\psi_1$$

Fully analytical MSW conversion probabilities derived in unmagnetized plasma and magnetized plasma

(Bingham et al., PLA 97, 2002)



Semi-classical effective v-e interaction Lagrangian

$$L_{\text{int}} = -\frac{G_F}{\sqrt{2}} (1 + C_V) (n_e - \mathbf{J}_e \cdot \mathbf{v}_v)$$

Semi-classical v Hamiltonian

$$H_{eff} = \sqrt{\left(\mathbf{P}_{v} - \sqrt{2}G_{F} \frac{\mathbf{J}_{e}(\mathbf{r}, t)}{c^{2}}\right)^{2}c^{2} + m_{v}^{2}c^{4}} + \sqrt{2}G_{F}n_{e}(\mathbf{r}, t)$$

 $P_{v} = p_{v} + \sqrt{2}G_{F} \frac{J_{e}(\mathbf{r}, t)}{c^{2}}$ Neutrino Canonical Momentum

Equivalent to interaction of charged particle with an e.m. field

v Charge $\sqrt{2}G_{F}$ Lorentz Gauge **4-Potential** $\left(n_e, \frac{\mathbf{J}_e}{c}\right)$ $\nabla \cdot \frac{\mathbf{J}_{e}(\mathbf{r},t)}{c} + \frac{1}{c} \frac{\partial n_{e}}{\partial t} = 0$





(Silva et al, PRE 1998, PRD 1998)



Neutrino repels nearby electrons - Dressed neutrino with equivalent charge

$$\mathbf{F}_{v} = -\sqrt{2} G_{F} \left(\nabla n_{e}(\mathbf{r},t) + \frac{1}{c^{2}} \frac{\partial \mathbf{J}_{e}(\mathbf{r},t)}{\partial t} - \frac{\mathbf{v}_{v}}{c} \times \nabla \times \frac{\mathbf{J}_{e}(\mathbf{r},t)}{c} \right)$$

Fourier Transform + electrostatic waves

$$\mathbf{F} = -i\sqrt{2} G_{F} \mathbf{k} \left(1 - \frac{\omega^{2}}{k^{2}c^{2}} \right) n_{e}(\omega,\mathbf{k}) = -\frac{\sqrt{2} G_{F} \mathbf{k}^{2}}{4\pi e} \left(1 - \frac{\omega^{2}}{k^{2}c^{2}} \right) \mathbf{E}(\omega,\mathbf{k}) = e_{v}(\omega,\mathbf{k}) \mathbf{E}(\omega,\mathbf{k})$$

neutrino induced charge

$$e_{v}(\omega,\mathbf{k}) = -\frac{\sqrt{2} G_{F} \mathbf{k}^{2}}{4\pi e} \left(1 - \frac{\omega^{2}}{k^{2}c^{2}} \right) \approx -\frac{\sqrt{2} G_{F} \mathbf{k}_{D}^{2}}{4\pi e} = -\frac{\sqrt{8} \pi e}{k_{B} T_{e}} G_{F} n_{e0}$$

(Nieves and Pal, '94)

(Mendonca et al, PLA 1997)



Kinetic equation for neutrinos

(describing neutrino number density conservation / collisionless neutrinos)

$$\frac{\partial f_{v}}{\partial t} + \mathbf{v}_{v} \cdot \frac{\partial f_{v}}{\partial \mathbf{r}} - \sqrt{2}G_{F}\left(\nabla n_{e}(\mathbf{r}, t) + \frac{1}{c^{2}}\frac{\partial \mathbf{J}_{e}(\mathbf{r}, t)}{\partial t} - \frac{\mathbf{v}_{v}}{c} \times \nabla \times \frac{\mathbf{J}_{e}(\mathbf{r}, t)}{c}\right) \cdot \frac{\partial f_{v}}{\partial \mathbf{p}_{v}} = 0$$

Kinetic equation for electrons driven by neutrino pond. force (collisionless plasma)

$$\frac{\partial f_e}{\partial t} + \mathbf{v}_e \cdot \frac{\partial f_e}{\partial \mathbf{r}} - \sqrt{2} G_F \left(\nabla n_v(\mathbf{r}, t) + \frac{1}{c^2} \frac{\partial \mathbf{J}_v(\mathbf{r}, t)}{\partial t} - \frac{\mathbf{v}_e}{c} \times \nabla \times \frac{\mathbf{J}_v(\mathbf{r}, t)}{c} \right) \cdot \frac{\partial f_e}{\partial \mathbf{p}_e} - e \left(\mathbf{E} + \frac{\mathbf{v}_e}{c} \times \mathbf{B} \right) \cdot \frac{\partial f_e}{\partial \mathbf{p}_e} = 0$$

Maxwell's Equations



Two stream instability Neutrinos driving electron plasma waves v₀ ~ c Anomalous heating in SNe II

Collisionless damping of electron plasma waves Neutrino Landau damping Anomalous cooling of neutron stars

Electroweak Weibel instability Generation of quasi-static B field Primordial B and structure in early Universe



Usual perturbation theory over kinetic equations + Poisson's equation

$$n_{e} = n_{0} + n_{e1} \qquad f_{e} = f_{e0}(\mathbf{p}_{e}) + f_{e1}$$

$$\mathbf{v}_{e} = \mathbf{v}_{1} \qquad f_{v} = f_{v0}(\mathbf{p}_{v}) + f_{v1}$$

$$\mathbf{v}_{v} = \mathbf{v}_{v0} + \mathbf{v}_{v1} \qquad \mathbf{E} = \mathbf{E}_{1}$$

Dispersion relation for electrostatic plasma waves

$$1 + \chi_e(\omega_L, \mathbf{k}_L) + \chi_v(\omega_L, \mathbf{k}_L) = 0$$

Electron susceptibility Neutrino susceptibility

$$\chi_{\nu}(\omega_{L},\mathbf{k}_{L}) = -2 G_{F}^{2} \frac{k_{L}^{3} n_{e0} n_{\nu 0}}{m_{e} \omega_{pe0}^{2}} \left(1 - \frac{\omega_{L}^{2}}{c^{2} k_{L}^{2}}\right)^{2} \chi_{e} \int d\mathbf{p}_{\nu} \frac{\mathbf{k}_{L} \cdot \frac{\partial J_{\nu 0}}{\partial \mathbf{p}_{\nu}}}{\omega_{L} - \mathbf{k}_{L} \cdot \mathbf{v}_{\nu}}$$

(Silva et al, PRL 1999)

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Monoenergetic neutrino beam & slab geometry & cold plasma





Instability analysis



Supernovae II



To form a neutron star > 3 × 10⁵³ erg must be released (gravitational binding energy of the original star) • light+kinetic energy ~ 10⁵¹ erg • • gravitational radiation < 1% • • neutrinos 99 % •

¶ Electron density @ 100-300 km: $n_{e0} \sim 10^{29}$ - 10^{32} cm⁻³

¶ Electron temperature @ 100-300 km: $T_e \sim 0.1 - 0.5$ MeV

- ¶ Degeneracy parameter $\Theta = T_e/E_F \sim 0.5 0.7$
- \P Coulomb coupling constant $\Gamma \sim 0.01$ 0.1

¶ v_e luminosity @ neutrinosphere ~ 10^{52} - 5×10^{53} erg/s ¶ v_e intensity @ 100-300 Km ~ 10^{29} - 10^{30} W/cm² ¶ Duration of intense v_e burst ~ 5 ms

(resulting from $p + e \rightarrow n + v_e$)

 \P Duration of v emission of all flavors $\, \sim 1$ - 10 s







Estimates of the Instability Growth Rates





$$\left(\frac{\Delta E_{\nu}}{10^{50} \text{ erg}}\right) \approx 1.2 \times 10^{-1} \left(\frac{R}{500 \text{ Km}}\right)^3 \left(\frac{T}{2 \text{ MeV}}\right) \times \left\{ 0.145 \left(\frac{n}{10^{30} \text{ cm}^{-3}}\right) + \left(\frac{T}{2 \text{ MeV}}\right)^3 \right\}$$

Neutrino heating to re-energize stalled shock

$$\left(\frac{\Delta E_{\nu}}{10^{50} \text{erg}}\right) \approx 1 - 0.1$$

(Silva et al, PoP 2000)







Neutrino Landau Damping I



(Silva et al, PLA 2000)



Neutrino Landau damping reflects contribution from the pole in neutrino susceptibility

$$\chi_{\nu}(\omega_{L},\mathbf{k}_{L}) \propto \int d\mathbf{p}_{\nu} \frac{\mathbf{k}_{L} \cdot \left(\partial \hat{f}_{\nu 0} / \partial \mathbf{p}_{\nu}\right)}{\omega_{L} - \mathbf{k}_{L} \cdot \mathbf{v}_{\nu}} \longrightarrow \int d\mathbf{p}_{\perp} |\mathbf{p}_{\perp}| \left\{ P \int \frac{\left(\partial \hat{f}_{\nu 0} / \partial \mathbf{p}_{\parallel}\right)}{\mathbf{p}_{\parallel} - \mathbf{p}_{\parallel 0}} d\mathbf{p}_{\parallel} + \left(\pi \left(\partial \hat{f}_{\nu 0} / \partial \mathbf{p}_{\parallel}\right)\right)\right) \right\}$$

EPW wavevector $\mathbf{k}_{L} = \mathbf{k}_{L||}$ defines parallel direction neutrino momentum $\mathbf{p}_{n} = \mathbf{p}_{v||} + \mathbf{p}_{v\perp}$ arbitrary neutrino distribution function f_{v0} Landau's prescription in the evaluation of χ_{v}

For a Fermi-Dirac neutrino distribution

$$\gamma_{\text{Landau}} \approx -\frac{k_L c}{2} \pi \frac{\mathbf{G_F^2} n_{e0} n_{v0}}{m_e c^2 k_B T_v} \left(1 - \frac{\omega_L^2}{c^2 k_L^2}\right)^2 \frac{\text{Li}_2(-\exp E_F / T_v)}{\text{Li}_3(-\exp E_F / T_v)}$$



Neutrinos drain energy from the plasma by damping plasma waves

unlike the usual neutron star cooling *plasma process* the number of neutrinos is conserved -

$$\mathbf{Q}_{\mathsf{epw}}$$
 energy loss rate - $Q_{EPW} = \int \frac{d\mathbf{k}}{(2\pi)^3} \gamma_{vLandau}(\mathbf{k}) W_{EPW}$

W_{epw} - spectral energy density of EPWs - Bose distribution

$$Q_{EPW} = -\frac{3\pi}{4} \frac{W_{EPW}}{k_D^3} c \left(\frac{\omega_{pe0}}{c}\right)^4 \frac{G_F^2 n_{e0} n_{\nu 0}}{m_e c^2 E_{\nu 0}} \frac{Li_2 (-\exp\mu/T)}{Li_3 (-\exp\mu/T)} \left(\frac{3}{4} + \frac{1}{4\beta_{th}^4} - \frac{1}{\beta_{th}^2} - \ln\beta_{th}\right)$$

 β_{th} - typical thermal velocity 2 π/k_d - Debye Length

Typical turbulence cooling times $\approx 10^{-4}$ Gyr Neutron star cooling time scale $\approx 1-10$ Gyr



Free energy in particles (e, i, e⁺) transferred to the fields (quasi-static B field)

Fundamental plasma instability laser-plasma interactions shock formation magnetic field generation in GRBs

Signatures: B field + filamentation + collisionless drag

Free energy of neutrinos/anisotropy in neutrino distribution transferred to electromagnetic field



Usual perturbation theory over kinetic equations + Faraday's and Ampere's law

Cold plasma

$$(\omega^{2} - k^{2}c^{2})(1 - \omega\Delta_{\nu}\phi(\hat{f}_{\nu0})) = \omega_{pe0}^{2} \qquad \mathbf{k} = k\mathbf{e}_{z}$$

$$\phi(\hat{f}_{\nu0}) = \int d\mathbf{p}_{\nu} \frac{v_{\nu\perp}}{\omega - kv_{\nuz}} \cos^{2}\theta \left\{ \frac{\hat{\mathcal{J}}_{\nu0}}{\hat{\mathcal{P}}_{\nu\perp}} + \frac{k}{\omega} v_{\nu\perp} \frac{\hat{\mathcal{J}}_{\nu0}}{\hat{\mathcal{P}}_{\nuz}} - \frac{k}{\omega} v_{\nuz} \frac{\hat{\mathcal{J}}_{\nu0}}{\hat{\mathcal{P}}_{\nu\perp}} \right\}$$

Monoenergetic v beam ($m_v = 0$)

$$\hat{f}_{v0} = \hat{f}_{v0} (\mathbf{p}_{v\perp}, p_{vz})$$

$$\left(\omega^{2}-k^{2}c^{2}\left(1+\Delta_{v}\frac{k^{2}c^{2}}{\omega^{2}}\beta_{vx0}^{2}\right)=\omega_{pe0}^{2}\qquad \omega\approx i\gamma_{\text{Weibel}} \& |\gamma_{\text{Weibel}}|<<|k|$$

$$\gamma_{\text{Weibel}} = \beta_{vx0} \frac{k^2 c^2}{\sqrt{k^2 c^2 + \omega_{pe0}^2}} \Delta_v^{1/2} \quad \propto \quad \mathbf{G}_{\mathbf{F}}$$

(Silva et al, PFCF 2000)



Gamma Ray Bursters

- Short intense bursts of a few MeV γ-rays with x-ray to IR afterglow
- Total energy 10⁵¹-10⁵⁴ erg (with beaming of radiation \downarrow)
- Nonthermal GRB spectrum
- Duration a fraction of s to 100's of s

GRBs involve 3 stages:

- Central engine (?) produces relativistic outflow
- This energy is relativistically transferred from the source to optically thin regions

• The relativistic ejecta is slowed down and the shocks that form convert the kinetic energy to internal energy of accelerated particles, which in turn emit the observed gamma-rays ($\gamma > 100$, B-field close to equipartition)



External shocks arise due to the interaction of the relativistic matter with the interstellar medium

Internal shocks arise from the collisions of plasma shells: faster shells catch up with slower ones and collide





To explain present observations near equipartition B-fields have to be present

Necessary to generate B-field such that: $|B|^{2}/\varepsilon_{\text{plasma shells}} \sim 10^{-5} - 10^{-3}$

Weibel instability can be the mechanism to generate such fields (Medvedev and Loeb, 2000)

To definitely address this issue: 3D PIC simulations



3D PIC simulations of the e⁺e⁻ Weibel instability



Simulation details

200 x 200 x 100 cells (20 x 20 x 10 c^{3}/ω_{p}^{3} volume) **or** 256 x 256 x 100 cells (25.6 x 25.6 x 10 c^{3}/ω_{p}^{3} volume) 16 particles per species per cell >100 million particles total Periodic system

CRAY T3E 900 - NERSC (64 nodes) epp cluster (40 nodes)

PIC codes OSIRIS (R. G. Hemker, UCLA,2000) PARSEC (J. Tonge, UCLA, 2002)



B-field evolution



(Silva et al, submitted ApJLett 2002)



Mass density evolution (yv/c = 0.6)





Magnetic field energy density (yv/c = 0.6)





- Isosurfaces (Green regions of lower values, Yellow regions of higher values) of the magnitude of the magnetic field
- •Isosurfaces drawn at a) 0.1, b) 0.025, c) 0.01 and d) 0.006



Energy evolution ($\gamma v/c = 0.6$)

B-field spectral energy density



Particle Kinetic Energy



Electron-positron Weibel instability II



3D Simulation

200 x 200 x 100 cells (20 x 20 x 10 c³/ω_p³ volume) 8 particles per species per cell, 64 million particles total

Computer

Simulations were run on 64 nodes of the Cray-T3E 900 at NER SC





In gamma ray bursters, Weibel instability can explain near equipartition B-fields

Weibel instability also crucial to understand pulsar winds, and relativistic shock formation

Challenge: relativistic collisionless shocks e⁻-e⁺/i (theory) and three-dimensional PIC simulations of relativistic shocks



In different astrophysical conditions involving intense neutrino fluxes, neutrino driven plasma instabilities are likely to occur

> Anomalous heating in SNe II Plasma cooling by neutrino Landau damping in neutron stars Electroweak Weibel instability in the early universe

Challenge: reduced description of ν driven anomalous processes to make connection with supernovae numerical models



Neutrino surfing electron plasma waves



$$\left|\Delta E_{\nu}\right|_{\max} \approx \left|\mathbf{F}\right| L_{dp} \approx 8\sqrt{2}G_{F}\varepsilon n_{e0}$$

$$\begin{split} \gamma_{\phi} &= 10 \\ \varepsilon &= 10^{-2} \\ n_{e0} &= 10^{32} \, cm^{-3} \\ L_{dp} &= \lambda_{p} \gamma_{\phi}^{2} \approx 3 \times 10^{-2} \, cm \\ dE_{\nu} \, / \, dL \approx 8 \sqrt{2} G_{F} \varepsilon n_{e0} \, / (\lambda_{p} \gamma_{\phi}^{2}) \approx 200 \, eV \, / \, cm \end{split}$$



Equivalent to physical picture for RFS of photons (Mori, '98)



Plasma waves driven by electrons, photons, and neutrinos

Electron beam

$$\left(\partial_t^2 + \omega_{pe0}^2\right) \delta n_e = -\omega_{pe0}^2 n_{e-beam}$$

Photons

Neutrinos

$$\left(\partial_t^2 + \omega_{pe0}^2\right)\delta n_e = \frac{\omega_{pe0}^2}{2m_e}\nabla^2 \int \frac{d\mathbf{k}}{(2\pi)^3}\hbar \frac{N_{\gamma}}{\omega_{\mathbf{k}}}$$

$$\left(\partial_t^2 + \omega_{pe0}^2\right) \delta n_e = \frac{\sqrt{2n_{e0}G_F}}{m_e} \nabla^2 n_v$$

 δn_e Perturbed electron plasma density

Ponderomotive force

physics/9807049, physics/9807050

Kinetic/fluid equations for electron beam, photons, neutrinos coupled with electron density perturbations due to PW
 Self-consistent picture of collective e,γ,v-plasma interactions



Super-Kamiokande

 Japanese Super-Kamiokande experiment – a large spherical "swimming pool" filled with ultra-pure water which is buried 1000 metres below ground!



In November, 2001, one of these PMTs imploded and the resulting shockwave caused about 60% of the other PMTs to implode also. The "shock" in the tank was so large that it was recorded on one of Japan's earthquake monitoring stations 8. km



• Super-Kamiokande obtained this neutrino image of the Sun!



Neutrino from the Sun

Solar Neutrinos

The p-p chain

$$4p + 2e^- \rightarrow He^4 + 2v_e + 2\gamma + 26.7 \text{MeV}$$

3% of the energy is carried away by neutrinos One neutrino is created for each ≈13 MeV of thermal energy The "Solar Constant", S (Flux of solar radiation at Earth) is

Neutrino flux at Earth, φ_{v} ,



 $S = 1.37 \times 10^{6} \text{ erg/cm}^{2} \text{s}$

$$\varphi_v = S / 13 \text{ MeV} \approx 6.7 \times 10^{10} \text{ neutrinos/cm}^2 \text{s}$$

These are all electron neutrinos (because the p-p chain involves electrons).

PROBLEM: Only about one-thirds of this flux of neutrinos is actually observed.

SOLUTION: <u>The MSW Effect</u>

Neutrinos interact with the matter in the Sun and "oscillate" into one of the other neutrino "flavours" – Neutrino matter oscillations – electron neutrinos get converted to muon or tau neutrinos and these could not be detected by the early neutrino detectors!



Big Bang Neutrinos

- The "Big Bang" Model of cosmology predicts that neutrinos should exist in great numbers – these are called <u>relic</u> <u>neutrinos</u>.
- During the Lepton era of the universe neutrinos and electrons (plus anti particles) dominate:
 - $\sim 10^{86}$ neutrinos in the universe
 - Current density $n_v \sim 220 \text{ cm}^{-3}$ for <u>each</u> flavour!
- Neutrinos have a profound effect on the Hubble expansion:
 - Dark matter
 - Dark energy
 - Galaxy formation
 - Magnetic field generation

in the early universe



Supernovæ II Neutrinos

- A massive star exhausts its fusion fuel supply relatively quickly.
- The core implodes under the force of gravity.
- This implosion is so strong it forces electrons and protons to combine and form neutrons in a matter of seconds a city sized superdense mass of neutrons is created.
- The process involves the weak interaction called "electron capture" $p^+ + e^- \rightarrow n + v_a$

A black hole will form unless the neutron degeneracy pressure can resist further implosion of the core. Core collapse stops at the "proto-neutron star" stage – when the core has a
$$\sim 10$$
 km radius.

• **Problem**: How to reverse the implosion and create an explosion?



