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Relativistic Plasma Physics in the vicinity of black holes/2

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Relativistic Plasma Physics in the vicinity of black holes - 2

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The kinetic equation:

$$\frac{\partial \Phi}{\partial x^\alpha}\frac{dx^\alpha}{d\tau} + \frac{\partial \Phi}{\partial p^\alpha}\frac{dp^\alpha}{d\tau} = 0$$

The equation of motion for individual particles:

$$\frac{m dp^\alpha}{d\tau} = -\Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma + qF^{\alpha\beta} p_\beta$$

Distribution function in the seventh-dimensional space $f(x^\alpha, \mathbf{p})$:

$$\Phi(x^\alpha, p^\beta) = f(x^\alpha, \mathbf{p})\delta[(-g_{\alpha\beta}p^\alpha p^\beta)^{1/2} - m]\theta(p^t)$$

taking into account that

$$F^{t\beta}u_\beta = \Gamma(\mathbf{E}\cdot\mathbf{v})/\alpha$$

$$F^{i\beta}u_\beta = \Gamma[\mathbf{E} + \mathbf{v}\times\mathbf{B} - \beta(\mathbf{E}\cdot\mathbf{v})/\alpha]^i$$

we derive:

$$p^\alpha \nabla_\alpha f = -\alpha q[\mathbf{E} + \mathbf{v}\times\mathbf{B} - (\beta/\alpha)(\mathbf{E}\cdot\mathbf{v})]\nabla_p f$$

Transport equations:

$$J^{\alpha}_{;\alpha} = 0$$

$$T^{\alpha\beta}_{;\beta} = qF^{\alpha\beta}J_{\beta}$$

$$M^{\alpha\beta\gamma}_{;\gamma} = q(F^{\alpha}_{\gamma}T^{\gamma\beta} - F^{\beta}_{\gamma}T^{\alpha\gamma})$$

All quantities appearing in these equations may be split onto their components in GCMF's. For example:

$$J^{\alpha} \equiv nu^{\alpha} = e^{\alpha}_{(\nu)}I^{(\nu)}$$

$$I^{(\nu)} \equiv \int p^{(\nu)}\Phi' d\Omega_4' = -n\eta^{(\nu)(t)}$$

$$T^{\alpha\beta} = e^{\alpha}_{(\nu)}e^{\beta}_{(\mu)}\Pi^{(\nu)(\mu)}$$

$$\Pi^{(t)(t)} \equiv mn(W+1) = \int p^{(t)}p^{(t)}\Phi' d\Omega_4'$$

$$\Pi^{(i)(t)} = \Pi^{(t)(i)} \equiv q^{(i)} = \int p^{(i)}p^{(t)}\Phi' d\Omega_4'$$

$$\Pi^{(i)(k)} = \int p^{(i)}p^{(k)}\Phi' d\Omega_4'$$

More definitions:

$$M^{\alpha\beta\gamma} = e^{\alpha}_{(\nu)} e^{\beta}_{(\mu)} e^{\gamma}_{(\eta)} N^{(\nu)(\mu)(\eta)}$$

$$N^{(t)(t)(t)} \equiv m^2 n V = \int p^{(t)} p^{(t)} p^{(t)} \Phi' d\Omega_4'$$

$$\Pi^{(i)(t)(t)} \equiv 2mg^{(i)} = \int p^{(i)} p^{(t)} p^{(t)} \Phi' d\Omega_4'$$

$$\Pi^{(i)(k)(t)} \equiv m\mu^{(i)(k)} = \int p^{(t)} p^{(i)} p^{(k)} \Phi' d\Omega_4'$$

$$N^{(i)(k)(l)} \equiv m\delta^{(i)(k)(l)} = \int p^{(i)} p^{(k)} p^{(l)} \Phi' d\Omega_4'$$

Three-dimensional, $f(x^\alpha, \mathrm{p})$-based definitions:

$$n \equiv \int f' d\Omega_3'$$

$$W \equiv \frac{1}{mn} \int \epsilon' f' d\Omega_3' - 1$$

$$V \equiv (nm^2)^{-1} \int \epsilon^{i2} f' d\Omega_3'$$

$$q^{(i)} \equiv \int p^{(i)} f' d\Omega_3'$$

$$g^{(i)} \equiv \frac{1}{2m} \int \epsilon^i p^{(i)} f' d\Omega_3'$$

$$\Pi^{(i)(k)} \equiv \int \frac{p^{(i)} p^{(k)}}{\epsilon'} f' d\Omega_3'$$

$$\mu^{(i)(k)} \equiv \frac{1}{m} \int p^{(i)} p^{(k)} f' d\Omega_3'$$

$$\eta^{(i)(k)(l)} \equiv \frac{1}{m} \int \frac{p^{(i)} p^{(k)} p^{(l)}}{\epsilon'} f' d\Omega_3'$$

Assumption 1: In the GCMF $\mathbf{E}' = 0$.

$$\mathbf{E}' = \Gamma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - (\Gamma - 1)\frac{(\mathbf{v} \cdot \mathbf{E})}{v^2}\mathbf{v}$$

$$\mathbf{B}' = \Gamma(\mathbf{B} - \mathbf{v} \times \mathbf{E}) - (\Gamma - 1)\frac{(\mathbf{v} \cdot \mathbf{E})}{v^2}\mathbf{v}$$

$$\mathbf{E} = \Gamma(\mathbf{E}' - \mathbf{v} \times \mathbf{B}') - (\Gamma - 1)\frac{(\mathbf{v} \cdot \mathbf{B}')}{v^2}\mathbf{v}$$

$$\mathbf{B} = \Gamma(\mathbf{B}' + \mathbf{v} \times \mathbf{E}') - (\Gamma - 1)\frac{(\mathbf{v} \cdot \mathbf{B}')}{v^2}\mathbf{v}$$

which yields:

$$\mathbf{E} = -\Gamma \mathbf{v} \times \mathbf{B}' = -\mathbf{v} \times \mathbf{B}$$

also we recover connection between LNRF and GCMF components:

$$B^{(i)} = \frac{1}{\Gamma} e^{(i)}_{[k]} B^{[k]}$$

we get ${B'}^2 = B^2 - E^2$ and $[\mathbf{b} \equiv \mathbf{B}/|B|]$

$$|B'| = (|B|/\Gamma)[1 + \Gamma^2 (\mathbf{v} \cdot \mathbf{b})^2]^{1/2}$$

Assumption 2: $\mathbf{q} = \mathbf{g} = 0$.

$$\Pi^{(i)(k)} = P_{\|} b^{(i)} b^{(k)} + P_{\perp}(\eta^{(i)(k)} - b^{(i)} b^{k})$$

$$\mu^{(i)(k)} = \mu_{\|} b^{(i)} b^{(k)} + \mu_{\perp}(\eta^{(i)(k)} - b^{(i)} b^{k})$$

where pressure and modified pressure are defined as:

$$P_{\perp} \equiv \frac{1}{2} \int \frac{{p'_{\perp}}^2}{\epsilon'} f' d\Omega'_3, \qquad P_{\|} \equiv \int \frac{{p'_{\|}}^2}{\epsilon'} f' d\Omega'_3$$

$$\mu_{\perp} \equiv \frac{1}{2m} \int {p'_{\perp}}^2 f' d\Omega'_3, \qquad \mu_{\|} \equiv \frac{1}{m} \int {p'_{\|}}^2 f' d\Omega'_3$$

taking into account that $e^{\alpha}_{(i)} e^{(i)}_{m} = \delta^{\alpha}_{m} + u^{\alpha} u_{m}$ we derive fror the stress-energy tensor:

$$T^{\alpha\beta} = [mn(W+1) + P_{\perp}] u^{\alpha} u^{\beta} + P_{\perp} g^{\alpha\beta}$$

$$+ (P_{\|} - P_{\perp}) \Lambda^{\alpha}_{m} \Lambda^{\beta}_{n} b^{m} b^{n}$$

where

$$\Lambda^{\alpha}_{m} \equiv \frac{\delta^{\alpha}_{m} + u^{\alpha} u_{m}}{[1 + \Gamma^2 (\mathbf{v} \cdot \mathbf{b})^2]^{1/2}}$$

Assumption 3: neglecting $\eta^{(i)(k)(l)}$.

For the third-order tensor we get:

$$M^{\alpha\beta\gamma} = m[(mnV + 3\mu_\perp)u^\alpha u^\beta u^\gamma + \frac{\mu_\parallel - \mu_\perp}{|h|^2}$$

$$\times(u^\alpha h^\beta h^\gamma + u^\beta h^\gamma h^\alpha + u^\gamma h^\alpha h^\beta)$$

$$+\mu_\perp(u^\alpha g^{\beta\gamma} + u^\beta g^{\gamma\alpha} + u^\gamma g^{\alpha\beta})]$$

Conclusion: hydrodynamical description of the relativistic collisionless plasma involves the set of macroscopic variables: n, W, V, P, μ.

Conservation equations:

Continuity:

$$\partial_t \rho + \nabla(\rho \Gamma [\alpha \mathbf{v} - \beta]) = 0$$

Energy:

$$\rho u^\alpha \left(W + \frac{P_\perp}{\rho} \right)_{,\alpha} - u^\beta P_{\perp,\beta} + (P_\parallel - P_\perp) u^\alpha \left[ln \frac{|h|}{n} \right]_{,\alpha} = 0$$

Momentum:

$$(\delta^\alpha_\beta + u^\alpha u_\beta) T^{\beta\gamma}{}_{;\gamma} = 0$$

Energy momentum:

$$u_\beta M^{\alpha\beta\gamma}_{;\gamma} = q(F^{\alpha\beta} T^\gamma_\beta - F^\gamma_\beta T^{\alpha\beta}) u_\beta$$

Connections between V, $\mu_{\|}$, and $\mu_{\perp}$:

$$g_{\alpha\beta}M^{\alpha\beta\gamma}_{;\gamma} = 0$$

$$u_{\alpha}u_{\beta}M^{\alpha\beta\gamma}_{;\gamma} = 0$$

$$h_{\alpha}h\beta M^{\alpha\beta\gamma}_{;\gamma} = 0$$

leads to:

$$V = (2\mu_{\perp} + \mu_{\|})/\rho + const$$

$$u^{\alpha}\mu_{\|,\alpha} - (\mu_{\|}/n)u^{\alpha}n_{,\alpha} + 2\mu_{\|}u^{\alpha}[ln(|h|/n)]_{,\alpha} = 0$$

$$u^{\alpha}\mu_{\perp,\alpha} - (2\mu_{\perp}/n)u^{\alpha}n_{,\alpha} - \mu_{\perp}u^{\alpha}[ln(|h|/n)]_{,\alpha} = 0$$

combining last two equations and eliminating $|h|$ terms we get for modified pressures:

$$\frac{\mu_{\perp}^{2}\mu_{\|}}{n^{5}} = const$$

Nonrelativistic ($\epsilon' \simeq m$)case

For enthalpy $\sigma \equiv [P_\perp + \rho(W+1)]/n$ we get:

$$\sigma = m + \frac{2P_\perp}{\rho} + \frac{P_\parallel}{2\rho}$$

and two equations of state:

$$\frac{P_\perp}{n|h|} = const$$

$$\frac{P_\parallel |h|^2}{n^3} = const$$

These are well-known CGL equatons of state!

When the pressure is isotropic:

$$P \sim n^{5/3}$$

Ultrarelativistic ($\epsilon' \simeq |p|$)case

For the enthalpy:

$$\sigma = (3P_{\perp} + P_{||})/\rho$$

if $P_{||} \gg \rho \gg P_{\perp}$, then:

$$\frac{P_{||}|h|}{n^2} = const$$

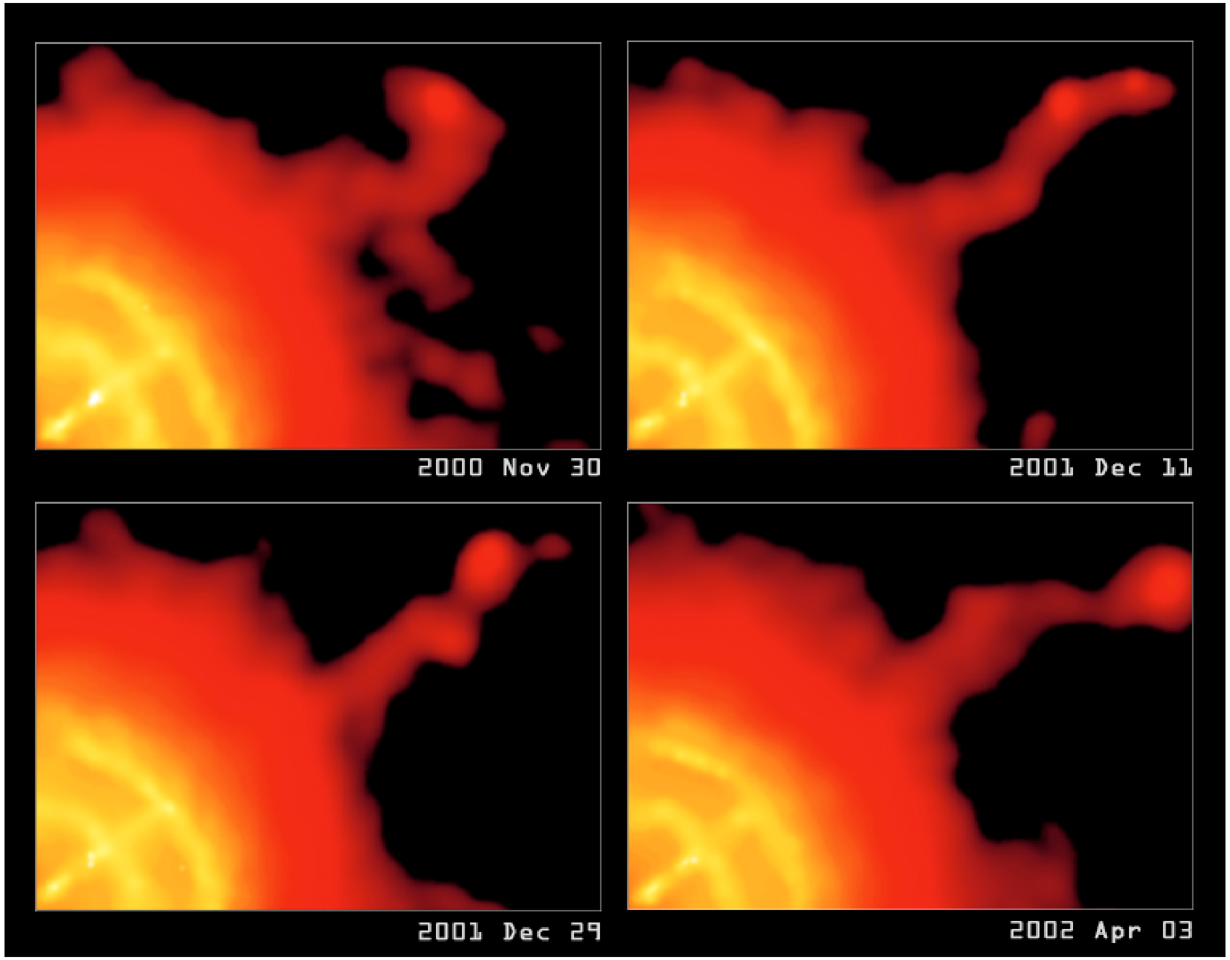
if $P_{||} \ll \rho \ll P_{\perp}$, then:

$$\frac{P_{\perp}^2}{n^2|h|} = const$$

When the pressure is isotropic:

$$P \sim n^{4/3}$$

Possible application: pulsar wind/jet



- *The Chandra images in this montage show the erratic variability of a jet of high energy particles that is associated with the Vela pulsar, a rotating neutron star. Possibly firehose instability…*

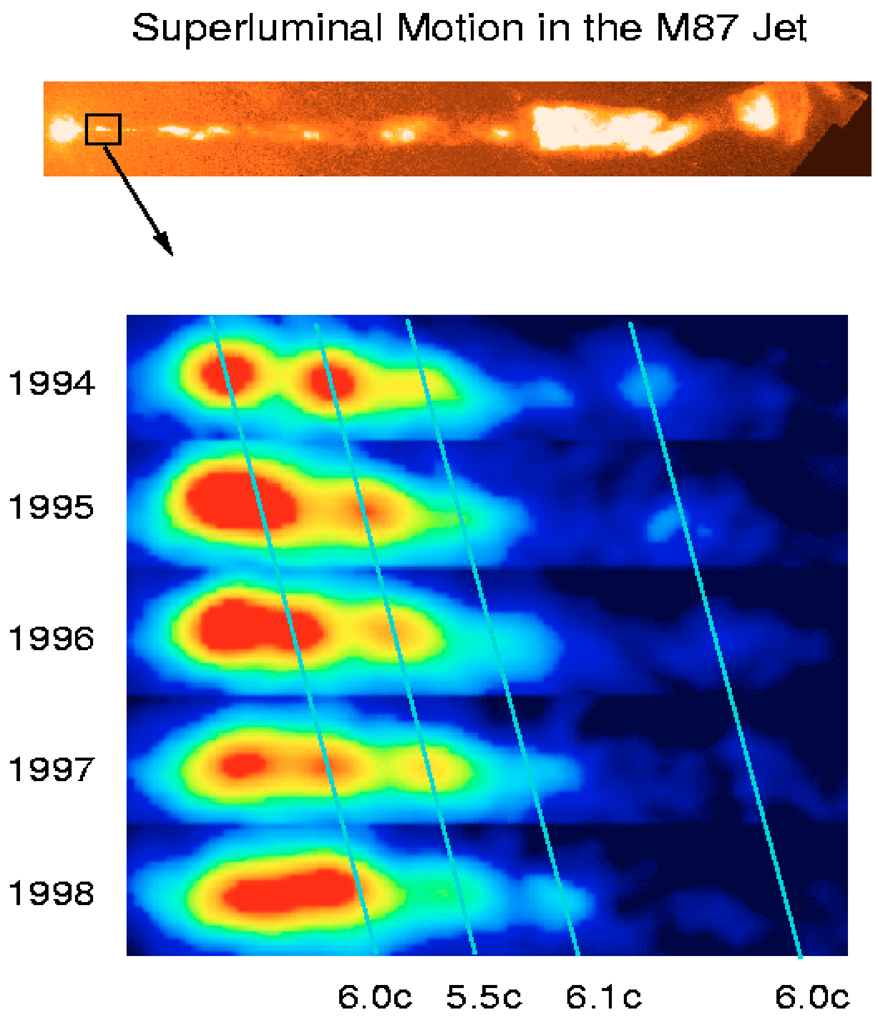
In relativistic pulsar winds pressure anisotropy due to synchrotron radiation losses is established on characteristic time scales:

$$t_0 = \frac{3m^3c^5}{2e^4B^2}$$

When $t > t_0$ influence of radiation is negligible and this kind of MHD might be adequate.

Tsikarishvili, Rogava & Tsiklauri D. G.: **Relativistic Hot Stellar Winds With Anisotropic Pressure** Ap.J. **439**, 822 (1995).

Extragalactic relativistic jets



- *The region where jet is born/launched is close to the central black hole and GR corrections can be important.*
- *Rogava & Khujadze, Gen. Rel. Grav. 29, 345 (1997).*