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Relativistically Intense Waves in Plasmas: Nonlinear Dynamics and Applications

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Relativistically Intense Waves in Plasmas: nonlinear dynamics and applications

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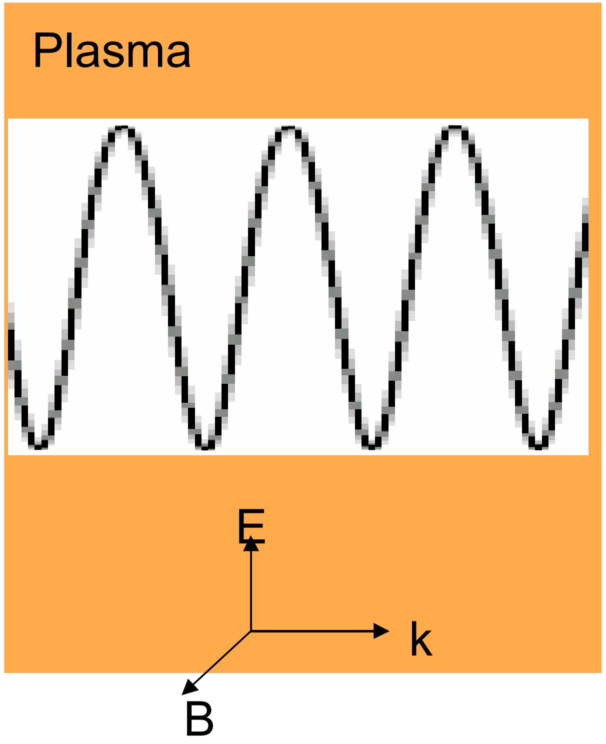
- **Physics of linear propagation of Electromagnetic Waves in a Plasma**



- *infinite homogeneous unmagnetized plasma*
- *plane wave propagation*

Wave equation

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial \vec{J}}{\partial t}$$

$$\omega^2 = \omega_{pe}^2 + c^2k^2$$

Dispersion relation:

$$\boxed{\epsilon = 1 - \frac{\omega_{pe}^2}{\omega^2}}$$

Dielectric constant

$$\vec{J} = -en\vec{v}$$

Conduction current

$$m\frac{d\vec{v}}{dt} = -e\vec{E} \Rightarrow \boxed{\vec{v} = -i\frac{e\vec{E}}{m\omega}}$$

Shielding current

If $\omega < \omega_{pe}$ then no propagation

PHYSICAL CONSEQUENCES OF NONLINEARITY

• relativistic increase of electron mass - lowers effective ω_p

- hence permits propagation into overdense plasmas

- finite plane wave with intensity variation can self-focus

- $$\frac{c}{(1-\omega_{peff}^2/\omega^2)^{1/2}} < \frac{c}{(1-\omega_{pe}^2/\omega^2)^{1/2}}$$

USEFUL IN LASER FUSION SCHEMES e.g. FAST IGNITION

• coupling to plasma waves through v x B forces

- excitation of large amplitude space charge fields moving at nearly speed of light

USEFUL FOR PLASMA BASED PARTICLE ACCELERATORS

• other areas of application include radiation around pulsars, table top terawatt laser experiments etc.

Q: What is the nature of the nonlinear propagation of relativistically intense electromagnetic / electrostatic waves in plasmas?

Relativistic: $v_{jitter} = \frac{eE}{m\omega} \sim c$

When can this happen? Laser power > 10^{18} W/cm^2

at wavelengths ~ 10^{-4} cm

Where can this happen?

- ***Radiation around pulsars***
- ***Laser fusion applications - petawatt lasers***
- ***Plasma based accelerators – large electrostatic fields***

In general a difficult question to handle, but simple model approaches (depending on the application) permit looking at some exact nonlinear solutions in the form of solitary pulses, nonlinear plane waves etc.

Our main focus will be on discussing the properties and applications of a few classes of one dimensional nonlinear traveling wave solutions of in cold collisionless plasmas

Problem has a long history – going back to the fifties but is still a very active field and is driven both by applications and the innate challenge of the subject.

OUTLINE

- Akhiezer-Polovin Model Solutions:
  - Purely transverse waves
  - Pure longitudinal waves
  - Mixed modes
  - Numerical investigations
- Special class of coupled electromagnetic and plasma waves
  - group velocity issues
  - connection to non-propagating solitons
  - new class of solutions
- Effect of ion dynamics & finite temperature
- Stability issues
- Summary including open problems and future directions

PHYSICAL MODEL

- Finite amplitude EM wave such that $v_{osc} / c = eE/m\omega c \sim 1$
- 1D approximation – variations along direction of propagation
- since $v_{osc} >> v_{the}$ electron fluid can be considered to be cold
- assume ions to be stationary - restricts $eE/m\omega \sim 0.1\, c$
- Maxwell equations + electron fluid equations

Relativistic Cold Plasma Equations

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t}, \tag{1a}$$

$$\nabla \cdot \boldsymbol{E} = -4\pi e(n - n_0), \tag{1b}$$

$$\nabla \times \boldsymbol{B} = -\frac{4\pi e}{c} n\boldsymbol{v} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t}, \tag{1c}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{1d}$$

$$\frac{\partial \boldsymbol{p}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{p} = -e\left(\boldsymbol{E} + \frac{\boldsymbol{v} \times \boldsymbol{B}}{c}\right), \tag{1e}$$

$$\boldsymbol{p} = m\boldsymbol{v}\left(1 - \frac{v^2}{c^2}\right)^{-1/2}, \tag{1f}$$

(Akhiezer & Polovin, Sov.Phys. JETP **3** (1956) 696)

- Seek special one dimensional solutions that are functions of

$$\xi = (z - ut)$$

Traveling wave ansatz

- Normalize

$$\beta = \frac{u}{c} \quad ; \quad \vec{\rho} = \frac{\mathbf{p}}{mc} \quad ; \quad \xi = \frac{\omega_{pe}}{c}(z - ut)$$

$$\boxed{\text{waves stationary in a frame moving with speed } u}$$

$$\frac{\partial}{\partial t} = -\omega_{pe}\beta\frac{\partial}{\partial \xi} \; ; \; \frac{\partial}{\partial z} = \frac{\omega_{pe}}{c}\frac{\partial}{\partial \xi}$$

- changes PDEs to ODEs – easy to integrate some and reduce the number of variables

Equations in the wave frame

$$\hat{z} \times \frac{\partial \vec{E}_\perp}{\partial \xi} = \beta \frac{\partial \vec{B}_\perp}{\partial \xi}$$

$$\boxed{\vec{B}_\perp = \frac{1}{\beta}(\hat{z} \times \vec{E}_\perp) + \vec{B}_0}$$

$$\hat{z} \times \frac{\partial \vec{B}_\perp}{\partial \xi} = -\beta \frac{\partial \vec{E}_\perp}{\partial \xi} + \frac{4\pi}{c} en\vec{v}$$

$$\hat{z} \cdot \frac{\partial \vec{B}}{\partial \xi} = 0$$

$$\boxed{n = \frac{n_0 u}{(u - \hat{z} \cdot \vec{v})}}$$

$$\hat{z} \cdot \frac{\partial \vec{E}_\perp}{\partial \xi} = 4\pi e(n - n_0)$$

$$(\hat{z} \cdot \vec{v} - u)\frac{\partial \vec{p}}{\partial \xi} = e\vec{E} + \frac{e}{c}(\vec{v} \times \vec{B})$$

Traveling wave equations

$$\frac{d^2\rho_x}{d\xi^2} + \frac{1}{(\beta^2 - 1)} \frac{\beta\rho_x}{\beta(1+\rho^2)^{1/2} - \rho_z} = 0$$

$$\frac{d^2\rho_y}{d\xi^2} + \frac{1}{(\beta^2 - 1)} \frac{\beta\rho_y}{\beta(1+\rho^2)^{1/2} - \rho_z} = 0$$

$$\frac{d^2}{d\xi^2}[\beta\rho_z - (1+\rho^2)^{1/2}] + \frac{\rho_z}{\beta(1+\rho^2)^{1/2} - \rho_z} = 0$$

β > 1 necessary to get bounded solutions for first two equations

Some exact nonlinear solutions

- *Pure transverse waves:* $\rho_z = 0$

$$\frac{d^2}{d\xi^2}[\beta\cancel{\rho_z} - (1+\rho^2)^{1/2}] + \cancel{\frac{\rho_z}{\beta(1+\rho^2)^{1/2} - \rho_z}} = 0$$

$\Rightarrow$ $\rho^2 = constant$

$$\frac{d^2\rho_x}{d\xi^2} + \frac{1}{(\beta^2-1)}\frac{\beta\rho_x}{\beta(1+\rho^2)^{1/2} - \cancel{\rho_z}} = 0$$

$$\frac{d^2\rho_y}{d\xi^2} + \frac{1}{(\beta^2-1)}\frac{\beta\rho_y}{\beta(1+\rho^2)^{1/2} - \cancel{\rho_z}} = 0$$

Solutions

$$\boxed{\begin{aligned} \rho_x &= \rho cos\omega(t - \frac{z}{u}) \\ \rho_y &= \rho sin\omega(t - \frac{z}{u}) \end{aligned}}$$

$$\omega = \frac{\omega_{pe}}{(1+\rho^2)^{1/4}} \frac{\beta}{(\beta^2 - 1)^{1/2}}$$

Phase velocity $u = \beta c = c\epsilon^{-1/2}$

Dielectric const $\epsilon = 1 - \frac{\omega_{pe}^2}{\omega^2}(1+\rho^2)^{-1/2}$

Substituting for ρ in the Maxwell's eqnns, one can show:

$$\boxed{\begin{aligned} E_x &= \left(\frac{mc\omega\rho}{e}\right) sin\left(\frac{\omega}{u}\xi\right) \\ E_y &= \left(\frac{mc\omega\rho}{e}\right) cos\left(\frac{\omega}{u}\xi\right) \end{aligned}}$$

Circularly polarized waves

$$\rho^2 = (e^2 E_0^2/m^2c^2\omega^2) \;\; ; \;\; E_0^2 = E_x^2 + E_y^2$$

$$\epsilon = 1 - \frac{\omega_{pe}^2}{\omega^2}\left(1 + \frac{e^2E_0^2}{m^2c^2\omega^2}\right)^{-1/2}$$

Propagation into over dense plasmas provided

$$\omega_{pe}\left(1 + \frac{e^2E_0^2}{m^2c^2\omega^2}\right)^{-1/4} < \omega < \omega_{pe}$$

Two important results:

- Only circularly polarized waves can propagate as pure transverse waves in a plasma

- Relativistic effects permit propagation of electromagnetic waves into overdense plasmas i.e. for $\omega < \omega_p$

provided $$\omega_p\left(1+\frac{e^2E_0^2}{m^2c^2\omega^2}\right)^{-1/2} < \omega$$

Physical reason: lowering of effective plasma frequency and consequent weakening of screening effect

$eE/m\omega c \sim 1$ ⇨ Power density reqmt $10^{18}\ \mathrm{W/cm^2}$

for $\omega = 2.6 \times 10^{15}$ (Ruby laser)

- **Pure longitudinal waves**

$$\rho_x = \rho_y = 0 \quad ; \quad \rho_z = \rho$$

$$\frac{d^2}{d\xi^2}[\beta\rho_z - (1+\rho^2)^{1/2}] + \frac{\rho_z}{\beta(1+\rho^2)^{1/2} - \rho_z} = 0$$

Can be reduced to :

$$\frac{d^2}{d\xi^2}\frac{1-\beta v_z}{\sqrt{1-v_z^2}} = \omega_{pe}^2 \frac{\beta^2 v_z}{\beta - v_z}$$

Equation is integrable

Periodic solutions expressible in terms of elliptic functions

Frequency:

$$\omega = \omega_{pe}\frac{\pi}{\sqrt{2}}\left[\int_0^{\alpha_m}\frac{d\alpha}{(1-\alpha^2)^{5/4}}\left(\frac{(1-\alpha^2)^{1/2}}{(1-\alpha_m^2)^{1/2}}-1\right)^{-1/2}\right]^{-1}$$

$$eE = \sqrt{2}m\omega_{pe}c\left(\frac{1}{(1-\alpha_m^2)^{1/2}}-\frac{1}{(1-\alpha^2)^{1/2}}\right)^{-1/2}$$

$$\boxed{-\alpha_m \le v_z \le \alpha_m}$$

Frequency depends on the maximum amplitude of the velocity.

Two limits

- $\alpha_m << 1$

$$\omega = \omega_{pe}(1 - \frac{3}{16}\alpha_m^2) \ \ ; \ \ \alpha_m = (eE_{max}/m\omega_{pe}c)$$

- $\alpha_m \rightarrow 1$

$$\omega \approx 2^{-3/2}\pi\omega_{pe}c(1-\alpha_m^2)^{1/4}$$

$$eE_{max} = \sqrt{2}m\omega_{pe}c(1-\alpha_m^2)^{-1/4}$$

$$\omega \approx \frac{\pi}{2}\left(\frac{\omega_{pe}^2 mc}{eE_{max}}\right)$$

Pure longitudinal waves with frequencies less than the plasma frequency can propagate provided the associated electric field Is high enough

Mixed modes

Analytic solutions in general not possible – look at numerical solutions or some interesting limits

- $\beta \longrightarrow \infty$

Almost transverse wave $\boxed{\rho_z \longrightarrow 0}$ Why?

$$(\cancel{\hat{z}\cdot}\vec{v} - u)\frac{\partial \vec{p}}{\partial \xi} = \underline{e\vec{E} + \frac{e}{c}(\vec{v}\times\vec{B})}$$

nearly cancel in the z direction

Take $\rho_y = 0$ (plane polarized wave)

$$\frac{d^2\rho_x}{d\xi^2} + \frac{1}{(\beta^2 - \cancel{1})}\frac{\beta\rho_x}{\beta(1+\rho^2)^{1/2} - \cancel{\rho_z}} = 0$$

$$\frac{d^2}{d\xi^2}[\beta\rho_z - \cancel{(1+\rho^2)^{1/2}}] + \frac{\rho_z}{\beta(1+\rho^2)^{1/2} - \cancel{\rho_z}} = 0$$

Reduced equations

$$\frac{d^2\rho_x}{d\xi^2} + \frac{1}{\beta^2}\frac{\rho_x}{(1+\rho^2)^{1/2}} = 0$$

$$\frac{d^2\rho_z}{d\xi^2} + \frac{1}{\beta^2}\frac{\rho_z}{(1+\rho^2)^{1/2}} = 0$$

Two constants of motion:

$$H = \frac{1}{2}(\dot{\rho_x}^2 + \dot{\rho_z}^2) + \frac{1}{\beta^2}\sqrt{1+\rho^2}$$

$$M = \rho_y\dot{\rho}_x - \rho_x\dot{\rho}_y$$

Equns. exactly integrable – give periodic solutions – amplitudes oscillate between two limits set by two constants of motion

- $\beta \to 1$

$$\text{Let } \eta = \xi(\beta^2 - 1)^{-1/2}$$

$$\frac{d^2 \rho_x}{d\eta^2} + \frac{\rho_x}{(1+\rho^2)^{1/2} - \rho_z} = 0$$

$$\frac{d^2 \rho_y}{d\eta^2} + \frac{\rho_y}{(1+\rho^2)^{1/2} - \rho_z} = 0$$

$$\frac{d^2}{d\eta^2}[\rho_z - (1+\rho^2)^{1/2}] + \underbrace{\frac{(\beta^2 - 1)\rho_z}{(1+\rho^2)^{1/2} - \rho_z}}_{\text{neglect}} = 0$$

$$\Longrightarrow \quad (1+\rho^2)^{1/2} - \rho_z = C^2$$

Solution: (for small amplitudes)

$$\rho_x = A_x cos(\omega\eta) \;\; ; \;\; \rho_y = A_y sin(\omega\eta)$$

$$\rho_z = \frac{A_x^2 - A_y^2}{4\sqrt{1+\frac{1}{2}(A_x^2 + A_y^2)}} cos(2\omega\eta)$$

$$\omega = [1 + \frac{1}{2}(A_x^2 + A_y^2)]^{-1/4}$$

$$A_x^2 + A_y^2 = 2(C^4 - 1)$$

- Frequency of ρ_z wice that of ρ_x
- Amplitude of ρ_x arger to ρ_z

Numerical solutions for coupled longitudinal-transverse oscillations

(Kaw and Dawson, PF 13 (1970) 472)

- **β = 10**

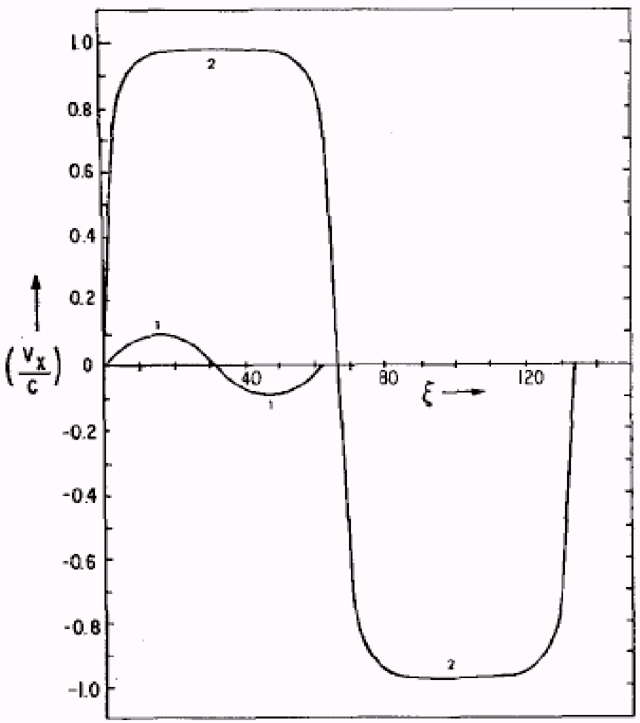


FIG. 1. Plot of v_x/c vs $\xi = (\omega_p/c)(z - ut)$ for $\beta = u/c = 10$. For curve 1, the amplitude $\rho_0 = 0.1$, whereas for curve 2, $\rho_0 = 3.0$.

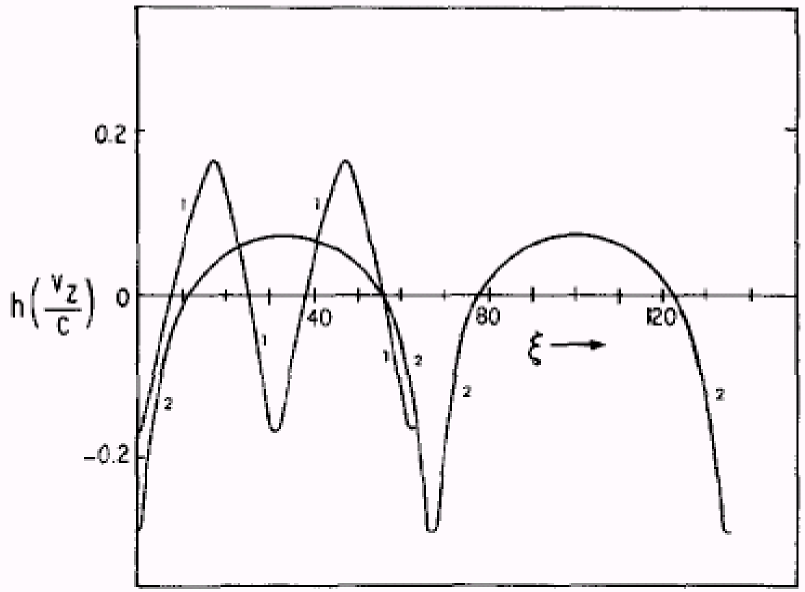
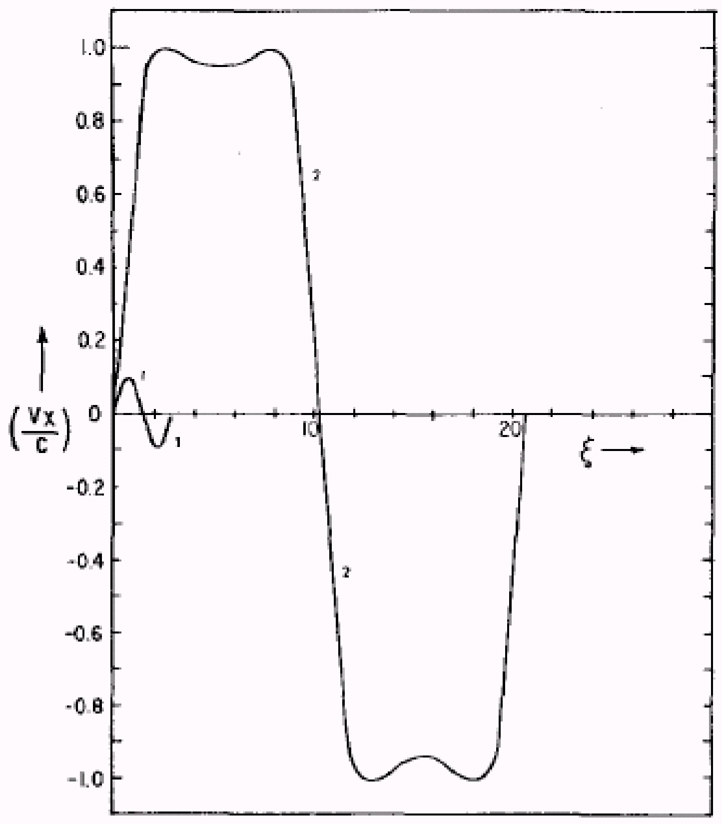


FIG. 2. Plot of $h(v_z/c)$ vs ξ for $\beta = 10$. For curve 1, $\rho_0 = 0.1$, $h = 500.0$ and for curve 2, $\rho_0 = 3.0$, $h = 1.0$.

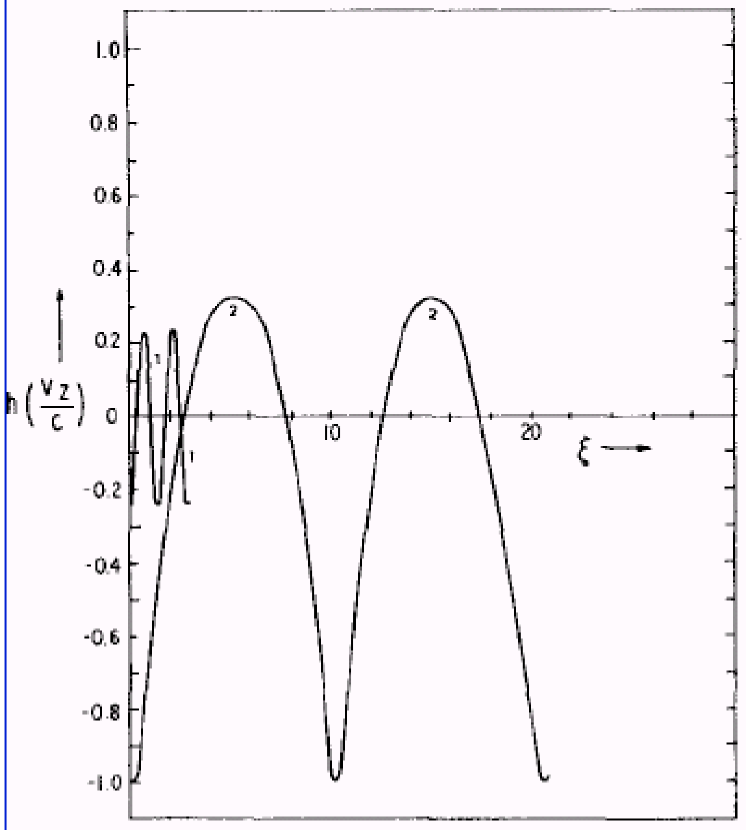
Curve 1 : small amplitude
Curve 2: large amplitude

- amplitudes
- frequencies
- shapes

- $\beta = 1.1$



Plot of (v_x/c) vs ξ for $\beta = 1.1$. curve 1: $\rho_0 = 0.1$; curve 2: $\rho_0 = 10$.



Plot of $h(v_z/c)$ vs ξ for $\beta = 1.1$. curve 1: $\rho_0 = 0.1$, $h = 100$; curve 2: $\rho_0 = 10$, $h = 1$.

Some physics issues:

- when does the nearly transverse wave approximation breakdown?

Upper limit to generation of electrostatic field in plasmas

from $$\nabla \cdot \mathbf{E} = 4\pi e n_0;$$

$$\Rightarrow \quad E = 2 e n_0 \lambda$$

Beyond this limit approximation breaks down even though $\beta \gg 1$

Validity condition: $$\boxed{\frac{\omega}{\omega_p}\left(\frac{eE_0}{m\omega_p c}\right) \lesssim 1.}$$

• **what happens at very large amplitudes?**

For much of the period v_x tends to c and hence square wave shape

The corresponding electric field has a saw-tooth shape

Max and Perkins, **(Phys. Rev. Letts. 27 (1971) 1342) obtained analytic expressions for the numerical results of Kaw and Dawson**

$$\rho_x = \rho_0 - \frac{(\zeta - \frac{1}{4}P)^2}{2(\beta^2 - 1)}, \quad 0 \leqslant \zeta \leqslant \tfrac{1}{2}P,$$

$$= -\rho_0 + \frac{(\zeta - \frac{3}{4}P)^2}{2(\beta^2 - 1)}, \quad \tfrac{1}{2}P \leqslant \zeta \leqslant P;$$

Series of linked parabolas

$$R = A(1 - \eta^2) - \tfrac{2}{3}\eta^2 + \tfrac{1}{3}(1 - \eta^2)\ln(1 - \eta^2);$$

$$1 - \eta \gg (\beta^{-2} + \rho_0^{-2})^{1/2}$$

Boundary layer problem

$$R(\eta) = R_0 + (2 + R_0)(1 - \eta) - \tfrac{1}{2}R_0(1 - \eta)\ln\left(\frac{16(1 - \eta)^2}{R_0^2\beta^{-2} + \rho_0^{-2}}\right)$$

$$1 \gg 1 - \eta \geqslant 0$$

$$\eta \equiv 4\zeta/P - 1 \text{ and } R \equiv \beta\rho_z/\rho_0$$

Max and Perkins 1971

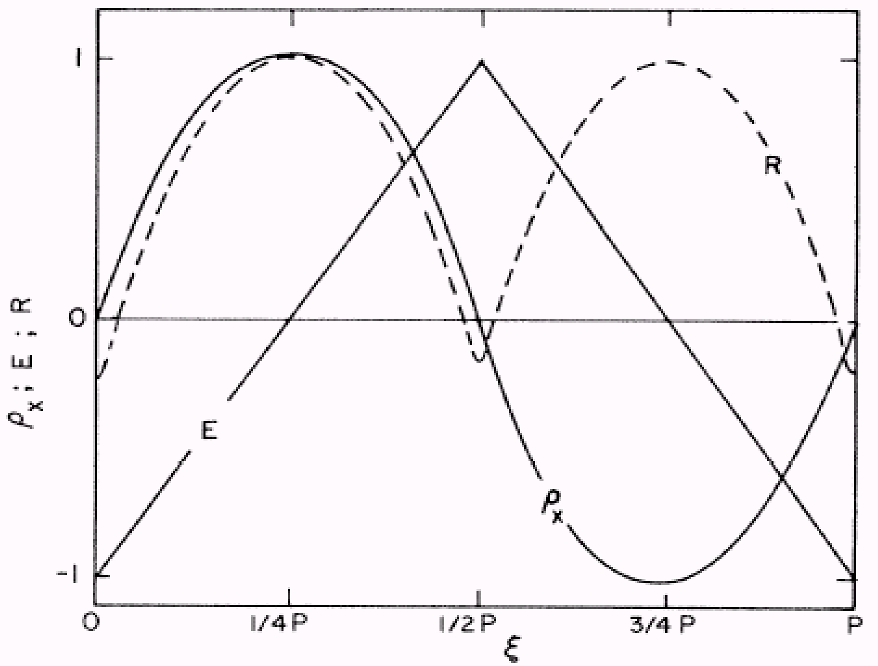


FIG. 1. Strong linearly polarized electromagnetic waves. The curves show transverse ρ_x and longitudinal R components of electron momentum [see Eq. (6)] as well as the transverse electric field. All quantities have been normalized to their maximum value. The period is P.

Transmission in an overdense inhomogeneous plasma

(**Max and Perkins, 1971**)

Propagation condition in the strongly relativistic regime:

$$\frac{1}{\beta^2}=\frac{c^2k^2}{\omega^2}=1-\frac{\pi}{2}\frac{\omega_p{}^2}{\omega}\frac{mc}{eE}>0$$

For a weakly inhomogeneous plasma use a WKB analysis for constancy of energy flux for waves propagating in a reflectionless plasma

$$\boxed{\tfrac{1}{2}kL=\tfrac{1}{2}(\omega/c\beta)L=\tfrac{1}{2}\omega L\omega^4\nu_i{}^2/c\omega_p{}^4>1.}\qquad \nu_i=eE_i/mc\omega$$

Physical mechanism of indefinite wave transmission:

As wave penetrates into higher density there is a WKB amplification of the electric fields which decreases the effective plasma frequency faster than its increase due to density changes.

Linearly polarized solutions – a more general overview

$$X = \sqrt{(\beta^2 - 1)}\rho_x \quad ; \quad Z = \beta\rho_z - \sqrt{(1 + \rho^2)}$$

$$\ddot{X} + \frac{\beta}{\sqrt{\beta^2 - 1 + X^2 + Z^2}} X = 0$$

$$\ddot{Z} + \frac{\beta}{\sqrt{\beta^2 - 1 + X^2 + Z^2}} Z + 1 = 0$$

$$H = \frac{\dot{X}^2}{2} + \frac{\dot{Z}^2}{2} + \underline{\beta\sqrt{\beta^2 - 1 + X^2 + Z^2} + Z}$$

Analogy: 'particle orbits' in a potential → nonlinear solutions

For $\beta > 1$, H is positive definite in the entire $X - Z$ plane and will give bound solutions.

Equilibrium point : $$X = 0\ ;\ Z = -1\ ;\ H_{min} = \beta^2 - 1$$

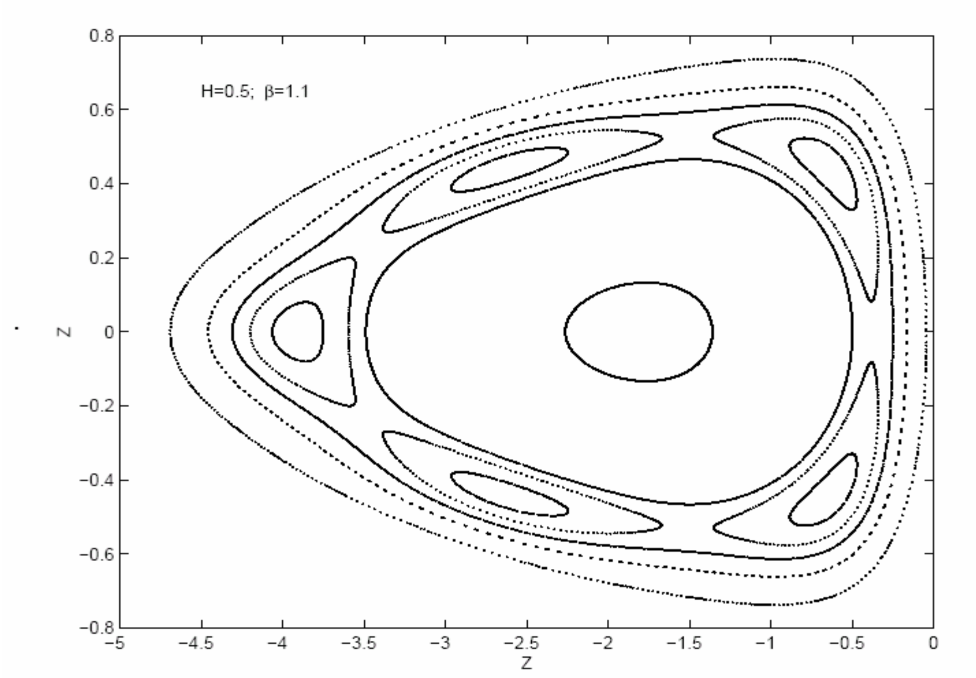
Linear Solutions :

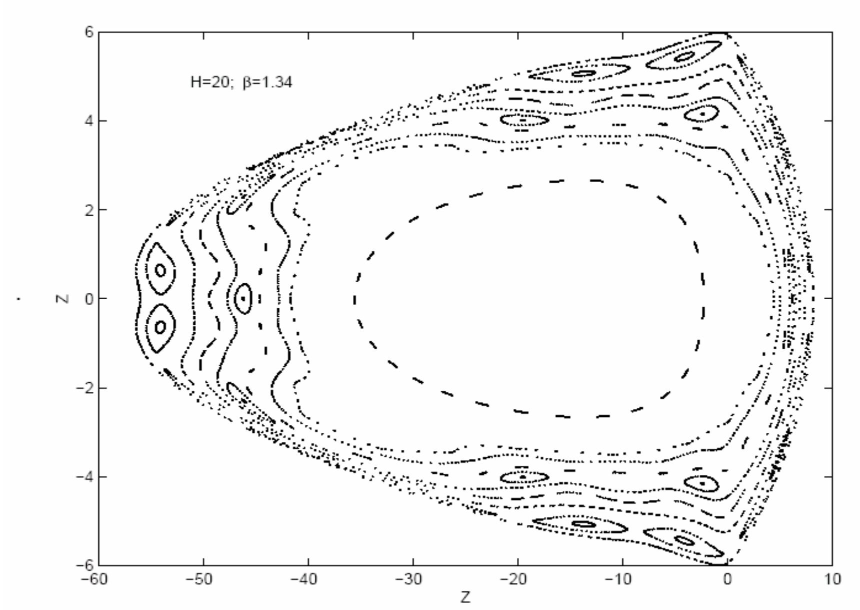
perturb around equilibrium $$X = \tilde{x}\ \ ;\ Z = -1 + \tilde{z}$$

$$\ddot{\tilde{x}} + \tilde{x} = 0 \quad \Rightarrow \quad \boxed{\omega^2 = \omega_{pe}^2 + c^2 k^2}$$

$$\ddot{\tilde{z}} + \left(1 - \frac{1}{\beta^2}\right)\tilde{z} = 0 \quad \Rightarrow \quad \boxed{\omega^2 = \omega_{pe}^2}$$

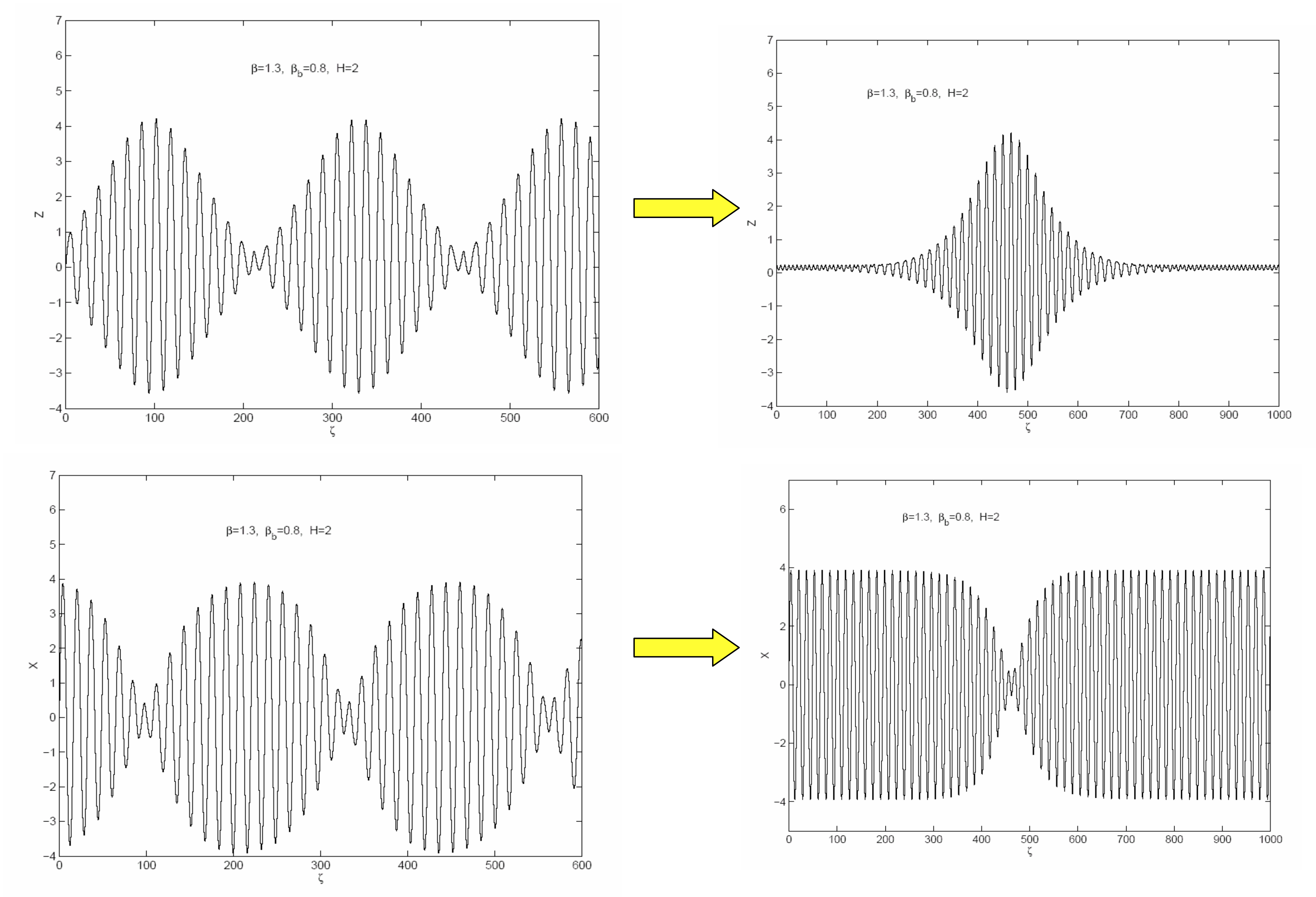
Typical Poincare plot in the plane X=0; X´ **> 0**





<u>Types of orbits:</u>

- Fixed points – periodic orbits – ratio of frequencies is a rational number
- Quasi-periodic orbits – ratio irrational
- Islands surrounding fixed points – amplitude modulated waves
- Separatrix orbit - solitons – modulation period infinite
- Stochastic orbits – aperiodic solutions

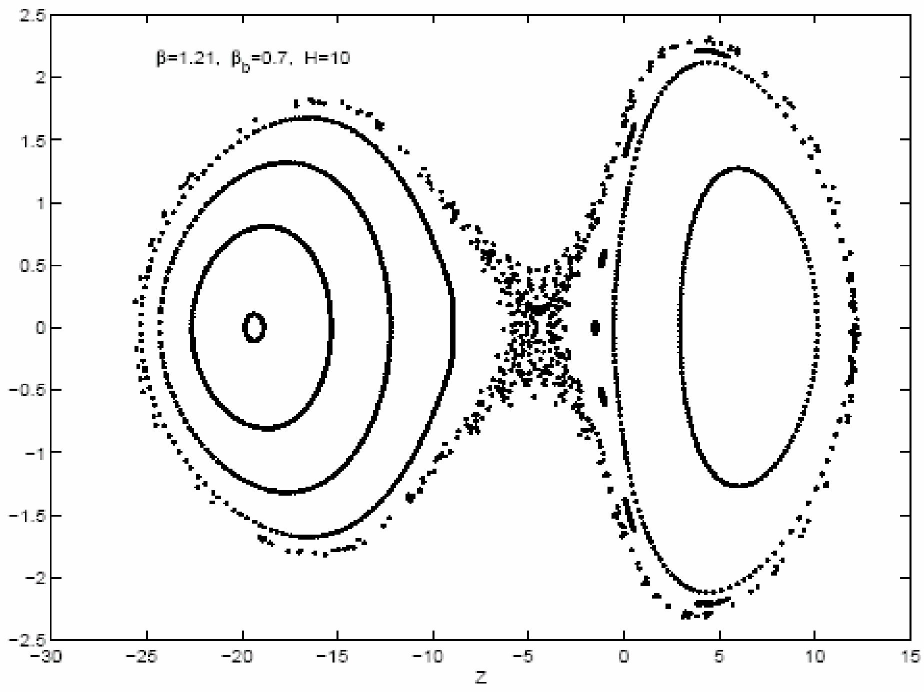


β=1.3, β_b=0.8, H=2
Z
ζ
β=1.3, β_b=0.8, H=2
Z
ζ
β=1.3, β_b=0.8, H=2
X
ζ
β=1.3, β_b=0.8, H=2
X
ζ

- Initially no stochastic orbits seen for this problem

- `Square root potential' – a **challenge to integrability**

- B Grammaticos et al, Phys. Lett. A 124 (1987) 65
  - showed it was not integrable
- Numerical evidence of stochastic orbits found by F.J. Romeiras et al



Relativistically intense standing waves

(Marburger and Tooper, PRL 35 (1975) 1001)

New class of exact solutions: good model for total reflection

Standing waves in an overdense plasma

$$E_t(z,t) = F_t(z) f_t(t)$$ ansatz

$$\boxed{e\vec{A}/mc^2 = F(\omega z/c)(-\hat{x}\cos\omega t + \hat{y}\sin\omega t)}$$ circular polarization

$p_z = 0$ $\quad \vec{p} = -e\vec{A}/c$

The electron density is bunched in the longitudinal direction because of ponderomotive force effects and are balanced by generation of static longitudinal fields. (ions are assumed to be immobile)

$$\Phi(z) = (mc^2/e)\varphi(\omega z/c)$$

Field equations

$$\varphi' = -d(1+F^2)^{1/2}/d\zeta\,,$$

$$-\varphi'' = (\omega_p{}^2 - \omega_{p0}{}^2)/\omega^2\,,$$

$$F'' + F = (\omega_p{}^2/\omega^2)F/(1+F^2)^{1/2}\,,$$

where $\zeta = \omega z/c$, $\omega_p{}^2 = 4\pi n e^2/m$, $\omega_{p0}{}^2 = 4\pi n_0 e^2/m$ and primes denote $d/d\zeta$.

$$H = P^2/2M(F) + V(F)$$

One dimensional nonlinear oscillator

$$V(F) = \tfrac{1}{2}F^2 - \alpha(1+F^2)^{1/2}$$

$$P = M(F)F'$$

$$M(F) = (1+F^2)^{-1}$$

$$\alpha = \omega_{p0}{}^2/\omega^2$$

Overdense plasma $\alpha > 1$

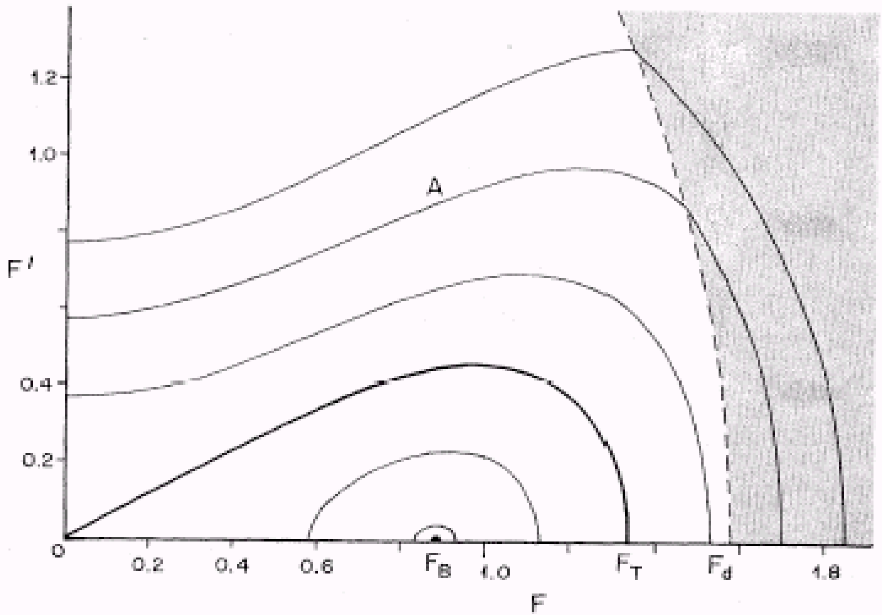


FIG. 1. Fields B (F') versus transverse E (F) in units $m\omega c/e$ for a quarter period when $\omega_{p0}{}^2/\omega^2 = \frac{4}{3}$. The depletion region where there are no electrons is shaded. Curve A corresponds to Fig. 3.

three kinds of motion for F when $\alpha > 1$: (1) oscillations about zero when the maximum field F_m exceeds $F_T \equiv 2(\alpha^2 - \alpha)^{1/2}$; (2) oscillations about a bias field $F_B = (\alpha^2 - 1)^{1/2}$ when $F_m < F_T$; (3) motion with infinite period along the separatrix $F_m = F_T$.

Summary:

- Looked at nonlinear propagation of electromagnetic and electrostatic waves in a cold collisionless plasma

- Large class of traveling wave solutions to the Akhiezer-Polovin equns.

- Pure transverse, pure longitudinal, mixed modes etc.

- Physics of penetration into overdense plasmas – both homogeneous and inhomogeneous – relativistic effect

- Standing wave solutions

- Most solutions are superluminal

- Next lecture – special class of coupled light and plasma wave solutions

Modulated nonlinear waves

An interesting class of exact one dimensional nonlinear traveling wave solutions consisting of

$\Downarrow$

Envelope solitons of light waves where the modulation envelope travels as a large amplitude plasma wave

Step beyond the cold plasma traveling wave
Envelope and phase propagate with different speeds

Circularly polarized waves
They couple to plasma waves only through amplitude modulation

MODEL EQUATIONS

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = 0$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)(\gamma u) = \frac{\partial \phi}{\partial x} - \frac{1}{2\gamma}\frac{\partial}{\partial x}A^2$$

$$u_\perp = \frac{\vec{A}}{\gamma}$$

$$\gamma = 1/\sqrt{1 - u_\perp^2 - u^2}$$

$$\Downarrow$$

$$\boxed{\gamma^2 = \frac{1 + A^2}{1 - u^2}}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n - 1$$

$$\frac{\partial^2 \vec{A}}{\partial t^2} - \frac{\partial^2 \vec{A}}{\partial x^2} = \frac{n}{\gamma}\vec{A}$$

(Akhiezer & Polovin, Sov.Phys. JETP **3** (1956) 696)

- $\xi = x - \beta t \;;\; \tau = t$ - transformation to traveling wave variables

- ansatz – circular polarization + temporal modulation

$$\boxed{\vec{A} = R(\xi)[\hat{y} + i\hat{z}]exp\{i\theta(\xi) - i\lambda\tau\} + c.c.}$$

β = normalized group velocity

$\lambda = \omega(1 - \beta^2)$ related to phase speed

Circular polarization prevents harmonic generation

- For plasma oscillations (which form the envelope) $\dfrac{\partial}{\partial \tau} = 0$

- Integrating the continuity equation: $n(\beta - u) = \beta$

- Integrating the parallel momentum equation: $\gamma(1 - \beta u) - \phi = 1$

- Substitute for n and u in wave equation and Poisson's equation

COUPLED LIGHT WAVE – PLASMA WAVE EQNS

$$R'' + \frac{R}{1-\beta^2}\left[\left(\lambda^2 - \frac{M^2}{R^4}\right)\frac{1}{1-\beta^2} - \frac{\beta}{\beta-u}\,\frac{1-\beta u}{1+\phi}\right] = 0$$

$$\phi'' = u/(\beta - u)$$

$$u = \frac{\beta(1+R^2) - (1+\phi)[(1+\phi)^2 - (1-\beta^2)(1+R^2)]^{1/2}}{(1+\phi)^2 + \beta^2(1+R^2)}$$

$$M = R^2\{(1-\beta^2)\theta' - \lambda\beta\}$$

CONSTANT OF MOTION

$$K = \frac{R'^2}{2} - \frac{\phi'^2}{2(1-\beta^2)} + V(R,\phi)$$

$$V(R,\phi) = \frac{\lambda^2}{(1-\beta^2)^2}\frac{R^2}{2} + \frac{M^2}{2R^2(1-\beta^2)^2} - \frac{\phi}{1-\beta^2}$$

$$- \frac{\beta}{(1-\beta^2)^2}[\beta(1+\phi) - \{(1+\phi)^2 - (1-\beta^2)(1+R^2)\}^{1/2}]$$

A Hamiltonian problem of coupled anharmonic oscillators (one of them with a negative mass), with two degrees of freedom

In general, RICH variety of solutions including periodic, quasi-periodic, chaotic and solitons.

Note $\beta < 1$; (group speed less than c)

Some analytic limits

- Small amplitude modulation: weak density response

$$u \longrightarrow 0$$

$$u = \frac{\beta(1+R^2) - (1+\phi)[(1+\phi)^2 - (1-\beta^2)(1+R^2)]^{1/2}}{(1+\phi)^2 + \beta^2(1+R^2)}$$

$$\Rightarrow \quad \phi \approx (1+R^2)^{1/2} - 1$$

One variable eliminated – substitute in eqn. for R and expand

$$R'' + \frac{R}{1-\beta^2}\left[\left(\lambda^2 - \cancel{\frac{M^2}{R^4}}\right)\frac{1}{1-\beta^2} - \frac{\beta}{\beta - \cancel{u}}\,\frac{1-\beta\cancel{u}}{1+\phi}\right] = 0$$

Nonlinear Schrodinger equation:

$$R'' + \frac{R}{1-\beta^2}\left[\frac{\lambda^2}{1-\beta^2} - 1 + \frac{R^2}{2}\right] = 0$$

$$\boxed{A_y = R_m \operatorname{sech}\left(\frac{(1-\beta^2-\lambda^2)^{1/2}}{1-\beta^2}(x-\beta t)\right) \times \cos\left[\frac{\lambda\beta}{1-\beta^2}\left(x - \frac{t}{\beta}\right)\right],}$$

$$R_m = 4\left(\frac{(1-\beta^2)(1-\beta^2-\lambda^2)}{4\lambda^2 - (1-\beta^2)(3+\beta^2)}\right)^{1/2}$$

- Single peak in R and ϕ
- Balance of electrostatic and ponderomotive forces on the electron fluid
- Continous spectrum in λ and exists for all values of β.

(Kozlov, Litvak, Suvarov JETP 49, (1979) 75; Kaw, Sen, Katsoules, PRL 68 (1992) 3172)

$$A_y = R_m \,\mathrm{sech}\left[\frac{(1-\beta^2-\lambda^2)^{1/2}}{1-\beta^2}(x-\beta t)\right]$$
$$\times\cos\left[\frac{\lambda\beta}{1-\beta^2}\left(x-\frac{t}{\beta}\right)\right],$$

- Isolated light pulse
- Frequency $\omega = \lambda/(1-\beta^2)$ • wave number $k = \lambda\beta/(1-\beta^2)$
- phase velocity $1/\beta$ • group velocity β
- envelope scale length $(1-\beta^2)/(1-\beta^2-\lambda^2)^{1/2}$ real when $\lambda^2 < 1-\beta^2$.

Doppler shifted wave frequency in the frame of the group speed

$$\bar{\omega} = (\omega - k\beta)/(1-\beta^2)^{1/2} = \lambda/(1-\beta^2)^{1/2}$$

Physical meaning of condition → $\bar{\omega}/\omega_p < 1$

Light wave finds itself in an over-dense plasma and hence gets trapped

Trapping scale length: $(1-\beta^2)^{1/2} c/(\omega_p^2 - \bar{\omega}^2)^{1/2}$

- $\beta = 0$ **limit** **(Esirkepov et al, JETP Lett 68 (1999) 36)**

u = 0 ; $\phi = (1 + R^2)^{1/2} - 1$

Coupled equations reduce exactly to the following single equation

$$\boxed{\frac{R''}{1+R^2} + R\left[\lambda^2 - \frac{RR'^2}{(1+R^2)^2} - \frac{1}{\sqrt{1+R^2}}\right] = 0.}$$

Exact solution:

$$\boxed{R = \frac{2\sqrt{1-\lambda^2}\cosh(\sqrt{1-\lambda^2}\,\xi)}{\cosh^2(\sqrt{1-\lambda^2}\,\xi) + \lambda^2 - 1}}$$

- Single peak, continous spectrum
- physical constraint of <u>n remaining positive</u> gives limit $\boxed{\lambda = \sqrt{2/3}.}$

GENERAL NUMERICAL SOLUTION OF THE COUPLED SYSTEM

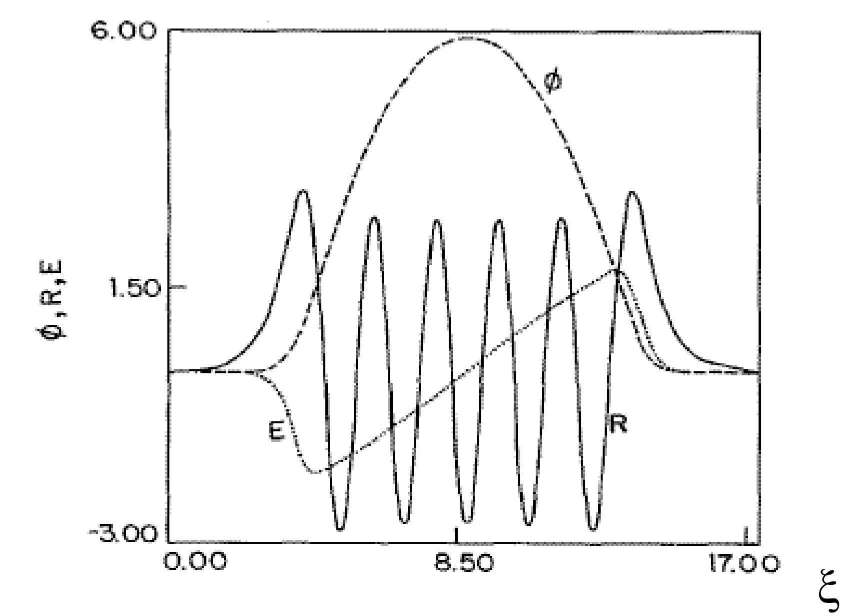
$$R'' + \frac{R}{1-\beta^2}\left[\left(\lambda^2 - \frac{M^2}{R^4}\right)\frac{1}{1-\beta^2} - \frac{\beta}{\beta - u}\,\frac{1-\beta u}{1+\phi}\right] = 0$$

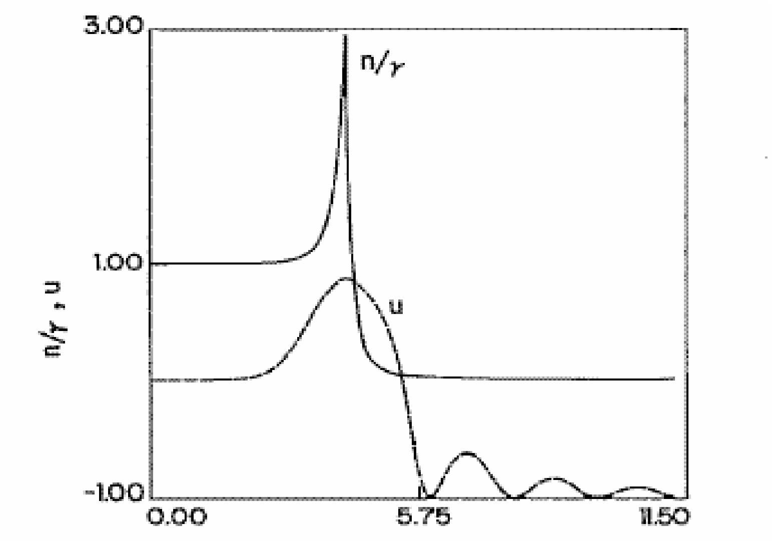
$$\phi'' = u/(\beta - u)$$

$$u = \frac{\beta(1+R^2) - (1+\phi)[(1+\phi)^2 - (1-\beta^2)(1+R^2)]^{1/2}}{(1+\phi)^2 + \beta^2(1+R^2)}$$

Nonlinear eigenvalue problem – pulse like solutions exist for only certain discrete values of λ and β

- **Another class of solitons – with multi-peaks in R.**





- electrostatic forces dominate inside and electrons driven to the edges
- laser amplitude **R** has **multiple peaks** in the near empty cavity
- solutions occur for **discrete values** of λ for a given value of β

(Kozlov, Litvak, Suvarov JETP 49, (1979) 75; Kaw, Sen, Katsoules, PRL 68 (1992) 3172)

- **Approximate analytic for large amplitudes**

ϕ_{max} is large

the longitudinal velocity $u \to -1$

$n \to \beta(1+\beta)^{-1}$

$n/\gamma \to 0$

Light wave propagates in an essentially plasma free region

$$K_{eff} = \frac{R'^2}{2} + \frac{\lambda^2}{(1-\beta^2)^2}\frac{R^2}{2} - \frac{\phi'^2}{2(1-\beta)^2} - \frac{\phi}{(1-\beta^2)(1+\beta)} + \frac{\beta}{(1+\beta)(1-\beta^2)} + \cdots$$

$$R = R_0 \sin[\lambda\xi/(1-\beta^2)]$$

$\phi = \phi_{max} - \xi^2/2(1+\beta)$ (parabolic) $\xi_{max} \simeq [2(1+\beta)\phi_{max}]^{1/2}$

Fit a standing wave pattern in the empty cavity

$$p\pi(1-\beta^2)/\lambda \simeq \xi_{\max}.$$

Approximate analytic relation

$$\boxed{R_0^2 = \frac{2(1-\beta)}{\lambda^2}\left[\frac{\pi^2 p^2}{2\lambda^2}(1-\beta^2)(1-\beta)-\beta\right]}$$

Physical interpretation

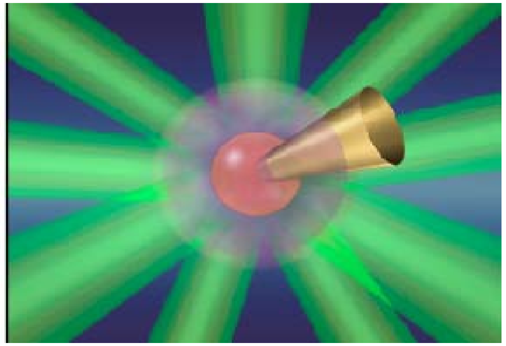
The soliton pulse is a light wave that is trapped in a plasma wave that it creates itself. The front of the pulse generates a plasma wave as a wake field which is then reabsorbed by the tail of the pulse

The density piles up at the edge and has a very sharp profile

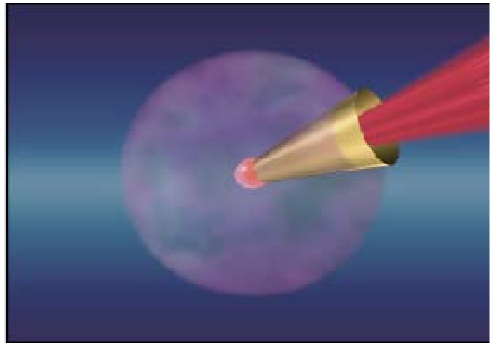
Fast Ignition Scheme in Laser Fusion

- Two separate lasers are used – one for compression and one for generating an ignition spark
- Delivery of high energy into the core plasma – can a solitonic pulse help?

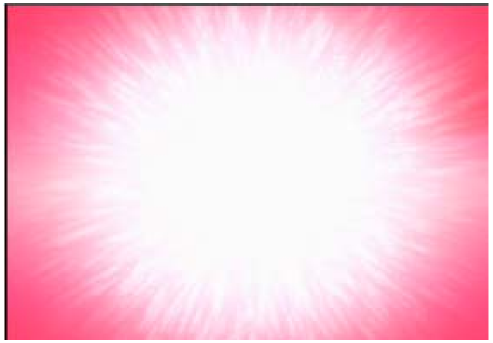
- Low group velocity solitons – relevant for this application. Do they exist?



Implosion



Heating



Ignition and burn

ILE, Osaka

- **Plasma accelerator – basic gain mechanism**

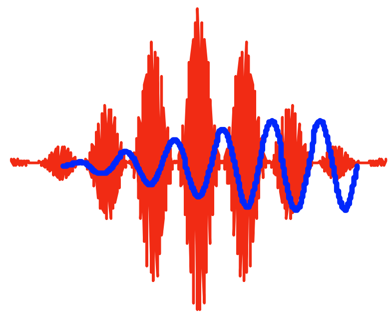
A particle moving close to the velocity of the wave sees a DC electric field and gets accelerated – very much like a surfer riding a huge ocean wave

•How does one excite a plasma wave?

- Beat Wave Scheme
  - Beat frequency of two laser beams matches with the plasma frequency.
- Wake field Scheme
  - Laser beam
  - Particle beam
  - Plasma is exposed to a large perturbation. It responds by ringing at the plasma frequency

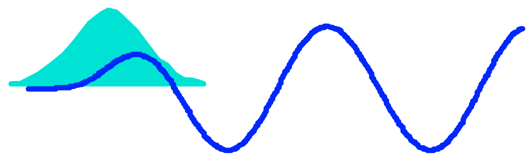
- Plasma Beat Wave Accelerator(PBWA)

  Two-frequencies, i.e., a train of pulses



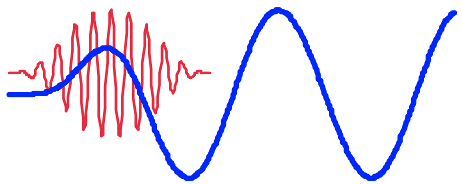
- Plasma Wake Field Accelerator(PWFA)

  A high energy electron bunch



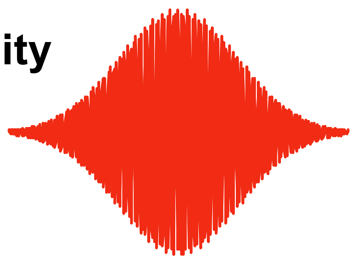
- Laser Wake Field Accelerator(LWFA)

  A single short-pulse of photons

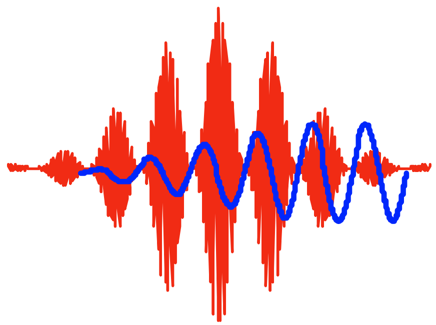


- Self Modulated Laser Wake Field Accelerator(SMLWFA)

  Raman forward scattering instability



evolves to



Group and phase velocities

$$\vec{A} = R(\xi)[\hat{y} + i\hat{z}]exp\{i\theta(\xi) - i\lambda\tau\} + c.c.$$

- The envelope $R = (A_x^2 + A_y^2)^{1/2}$ exhibits a modulation propagating at the group speed β

- Phase speed:

  - For small amplitudes R is always positive and the phase speed can be shown from the exponential factor to be **1/β**

$\Rightarrow$ $\boxed{V_p V_g = 1}$

  - For large amplitudes R is oscillatory and one needs to determine the phase speed directly from the z and t dependence of Rcos(θ - $\lambda\tau$)

- Group speed of nonlinear pulses is an important issue for certain applications

What is the group velocity of these nonlinear pulses?

- For the small amplitude NLS soliton one can obtain an analytic nonlinear dispersion relation

$$1-\beta^2 = \frac{-(R_m^2 - 4 + 4R_m^2\omega^2) + [(R_m^2 - 4 + 4R_m^2\omega^2)^2 + 64R_m^2\omega^2]^{1/2}}{8\omega^2}$$

- For a large amplitude plane wave (infinite train) a similar relation gives

$$V_{g\infty} = \beta[1 - (\gamma_\infty - 1)/2\Omega^2\gamma_\infty(\gamma_\infty + 1)]$$

(Mori et al, 1991)

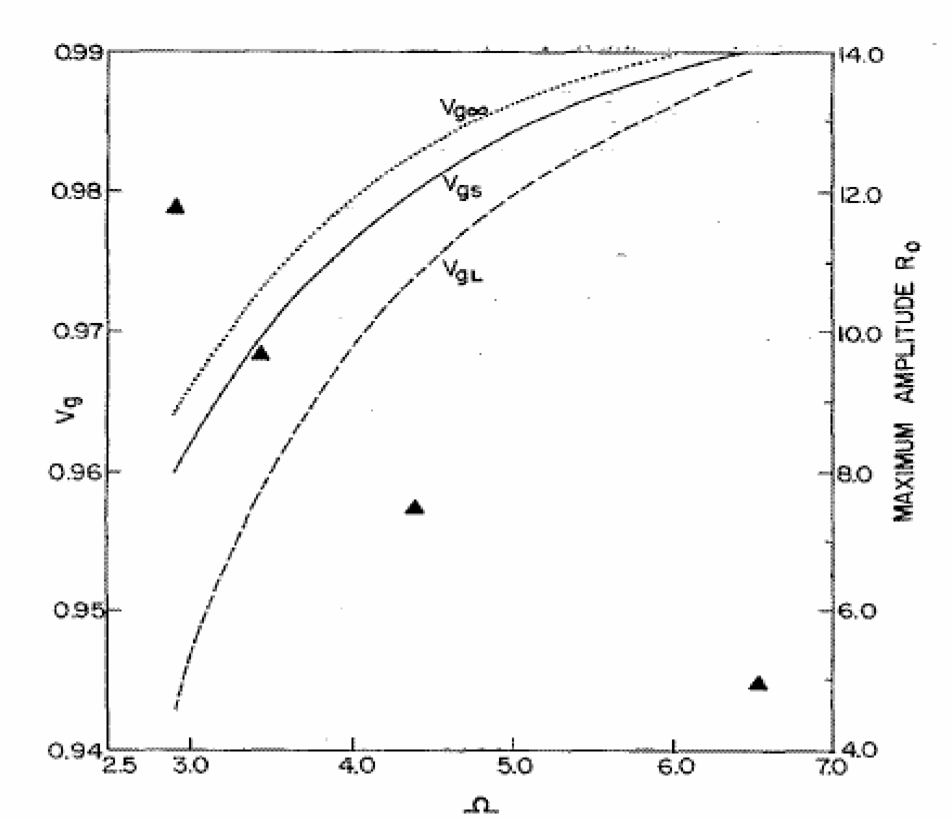
- For the large amplitude and high p solitons an approximate relation is

$$R_0^2 = \frac{2(1-\beta)}{\lambda^2}\left[\frac{\pi^2 p^2}{2\lambda^2}(1-\beta^2)(1-\beta) - \beta\right]$$

- For a detailed study a numerical investigation has been made

• $\beta \rightarrow 1$ limit, underdense plasmas

(Kaw, Sen, Katsoules, PRL 68 (1992) 3172)



• Nonlinearity makes electrons heavier – weakens plasma dielectric effect
  - nonlinear group velocities faster than linear
• Solitons travel slower than plane waves – coupling to plasma acts as a drag – difference in speeds upto 25%
• Particle and photon acceleration potential

Photon acceleration:

- large change in n/γ within a pulse

- can get a very large frequency multiplication factor by using these pulses as photon accelerators

$$\boxed{\frac{\omega_f}{\omega_i} = \frac{1+\beta(1-f_{\min}/f_{\max})^{1/2}}{1-\beta(1-f_{\min}/f_{\max})^{1/2}}}$$

$$\longrightarrow (1+\beta)/(1-\beta) \qquad \sim 20$$

For a pulse with $f_{\min}/f_{\max} \longrightarrow 0$

Summary of results so far

- Two classes of solitons – single peak and multi-peak
- Single peak – exact solution for β **= 0**, continous spectrum in **1** $\geq \lambda \geq \lambda_{\mathbf{c}}$

  - small amplitude NLS soliton at all β
- Multi-peak (in R) - numerical solutions

  - discrete spectrum mainly explored in the $\beta \to 1$ limit

• WKB analysis by Kuehl and Zhang **(Kuehl & Zhang, PRE 48(1993) 1316)**

a simple `small amplitude' model provides useful insight

$$\boxed{\begin{aligned} & R'' + R(\phi - \lambda) = 0 \\ & \phi'' + \phi = R^2 \end{aligned}}$$

Importance of charge separation – dispersive term ϕ''

QUESTIONS?

- Do single peak solitons exist for **arbitrary amplitudes** and **finite** β ?

IMPORTANT FOR ENERGY TRANSPORT INTO OVERDENSE PLASMAS

- What is the nature of the spectrum of the single peak solitons if they exist ?
- What is the nature of the transition between β **= 0** and **finite** β ?
- **Spectrum of multi-peak** solitons for small values of β**?**

Some of these questions have been examined through numerical explorations of the full set of coupled 1d equations

RESULTS

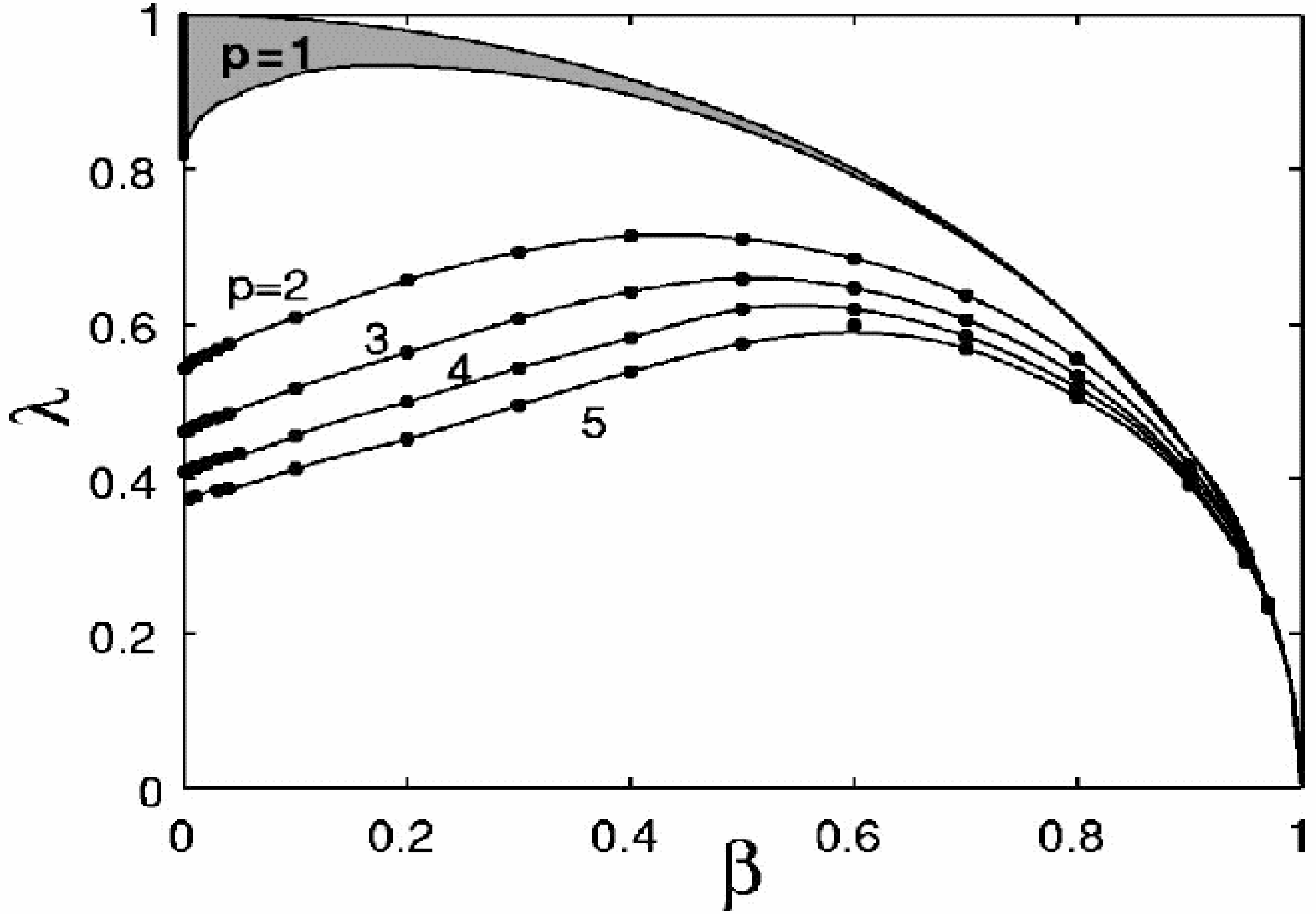
• Single peak solitons **exist** for finite β and spectrum appears **continous** over a finite range of λ.

- in contrast to earlier theoretical claims in
  Farina & Bulanov, PRL 86 (2001)5289

• Transition from $\beta = 0$ to β finite appears smooth.

• Multi-peak in R solitons have a discrete spectrum at small β and their nature similar to solutions at high β - approximate analytic relation obtained.

CONSOLIDATED SPECTRUM



SINGLE PEAK SOLITON

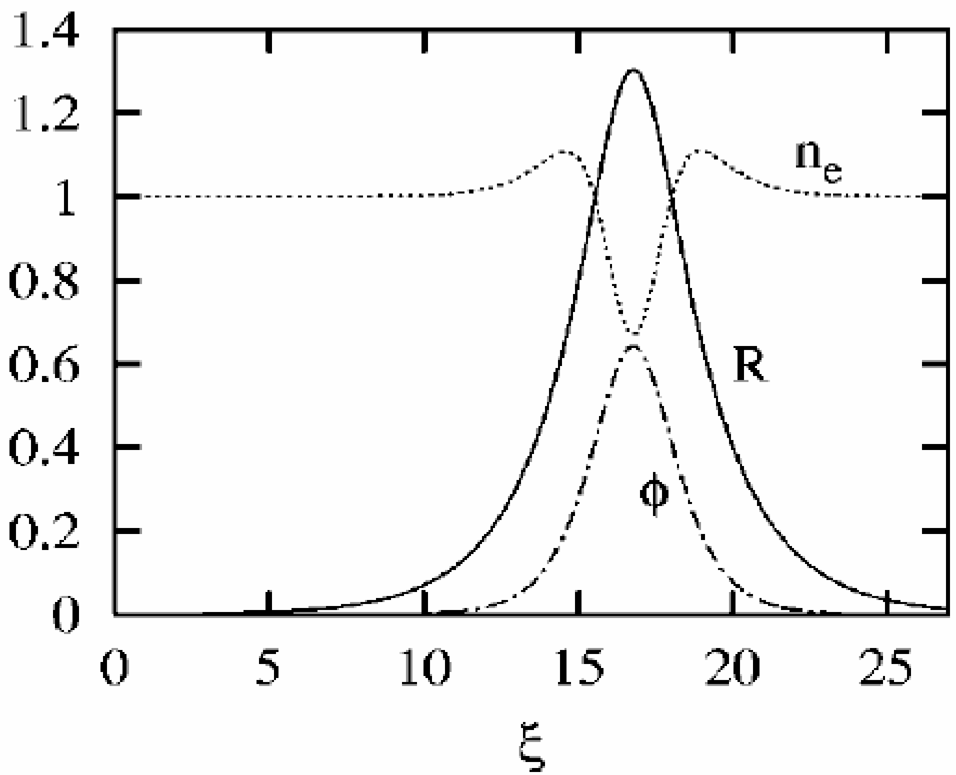


FIG. 2. Single peak soliton for $\beta = 0.01$, $\lambda = 0.87$. R (solid line) is the vector potential, ϕ (dashed–dotted) is the scalar potential, n (dotted) is the electron density.

NATURE OF SPECTRUM FOR SINGLE PEAK SOLITONS

- Discrete vs continuous – a **subtle mathematical issue**
- For $\beta \rightarrow \mathbf{1}$ limit (underdense plasmas)

  - quasi- neutral limit – continuous spectrum (Kuehl & Zhang, PRE 48(1993) 1316)
  - inclusion of ϕ'' soliton not isolated but has small wake
  - small amplitude isolated with discrete spectrum (Dimant et al PoP 4 (1997) 1489)
  - since eigenvalue small any weak physical effect like nonstationarity, external perturbations etc. can give a width – practically a continuous spect

- Our numerical results show a continuous spectrum with ϕ'' retained but above caveat may apply
- However our width is quite large - $\Delta \lambda / \Delta \sim 25\%$
- A corresponding analysis for small β limit lacking – **OPEN PROBLEM**

MULTIPEAK (IN R) SOLITON

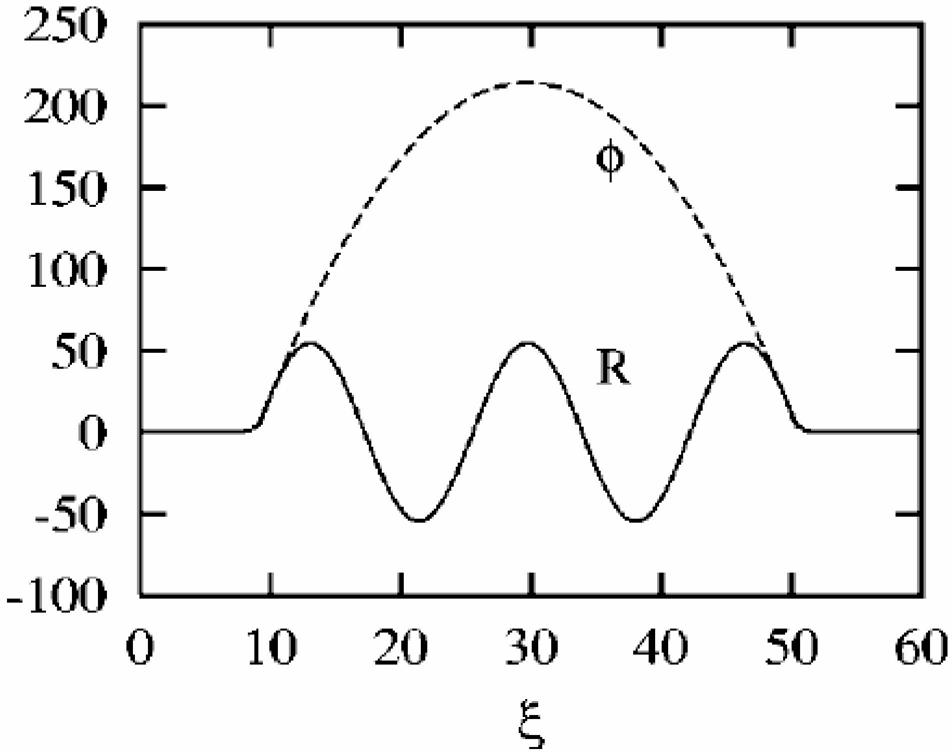


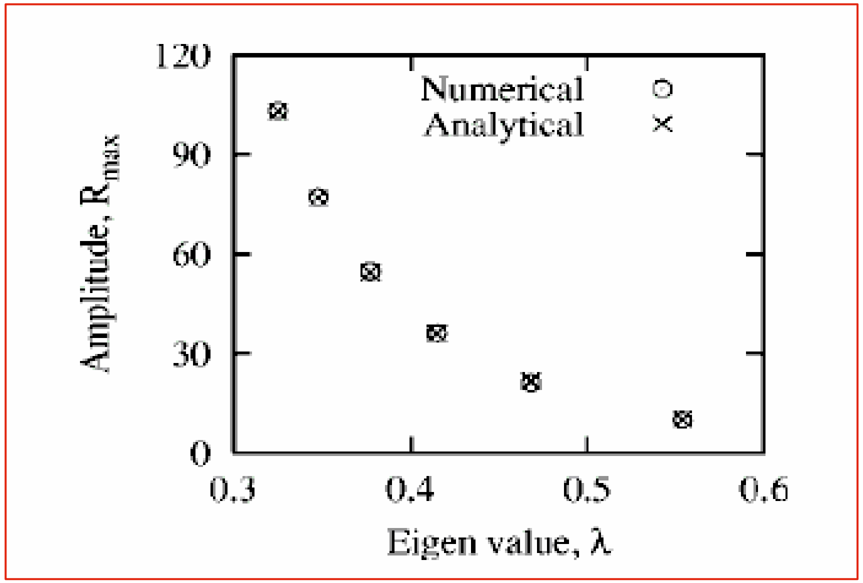
FIG. 4. Multipeak solution for $\beta = 0.01$, $\lambda = 0.37719$, $p = 5$.

NATURE OF MULTI-PEAK SOLITONS

$$R_{\max} = \pi p / 2\lambda^2$$ Approximate analytic relation

Can be understood physically by fitting in a standing wave pattern inside the nearly empty cavity with a parabolic profile of ϕ and using K = 0 to obtain the maximum amplitude of ϕ .

$$p\,\pi/2\lambda = w \approx \sqrt{2\,\phi_{\max}}.$$



(Poornakala, Das, Sen and Kaw, PoP 9 (2002) 1820)

Other issues:

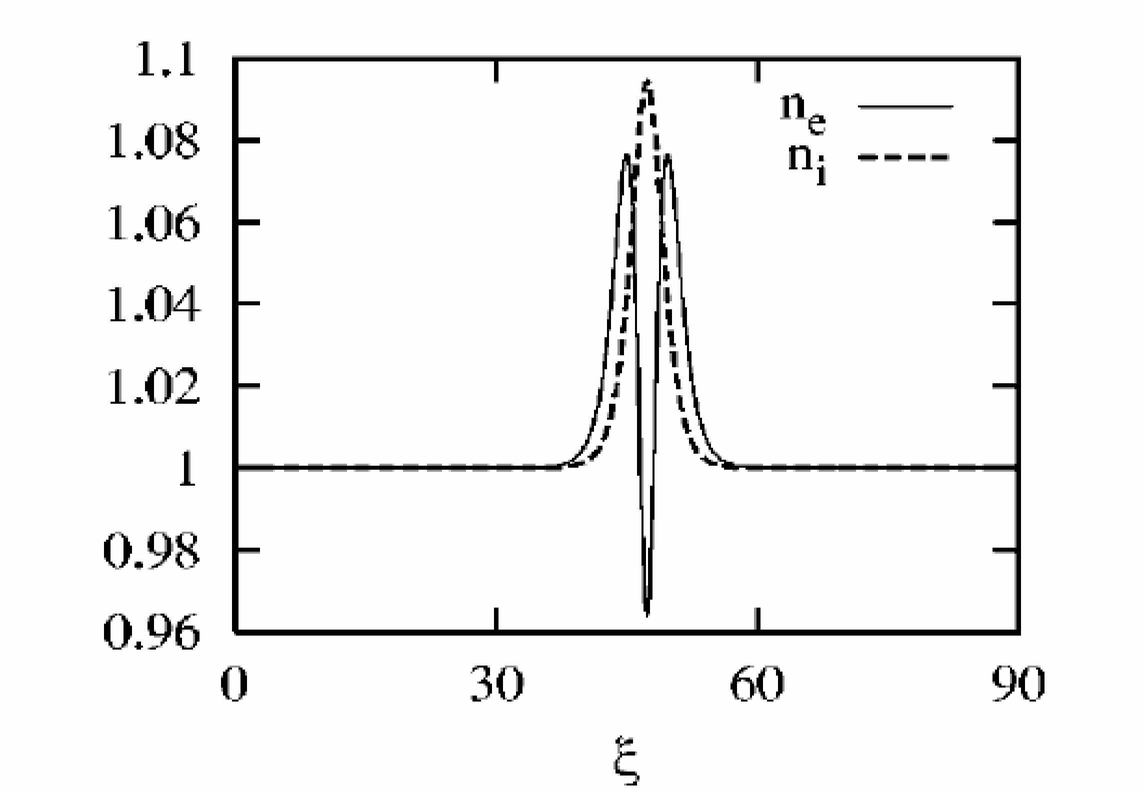
- Inclusion of ion dynamics
- Finite temperature effects
- Stability
- Propagation in inhomogeneous media
- Higher dimensions
- Experimental evidence?

INCLUSION OF ION DYNAMICS

- Single peak solitons of large amplitude continue to exist at low β

  but smooth connection to standing solitons is broken and they exist

  only in the range $\boxed{\sqrt{m_e/m_i} < \beta < 1}$



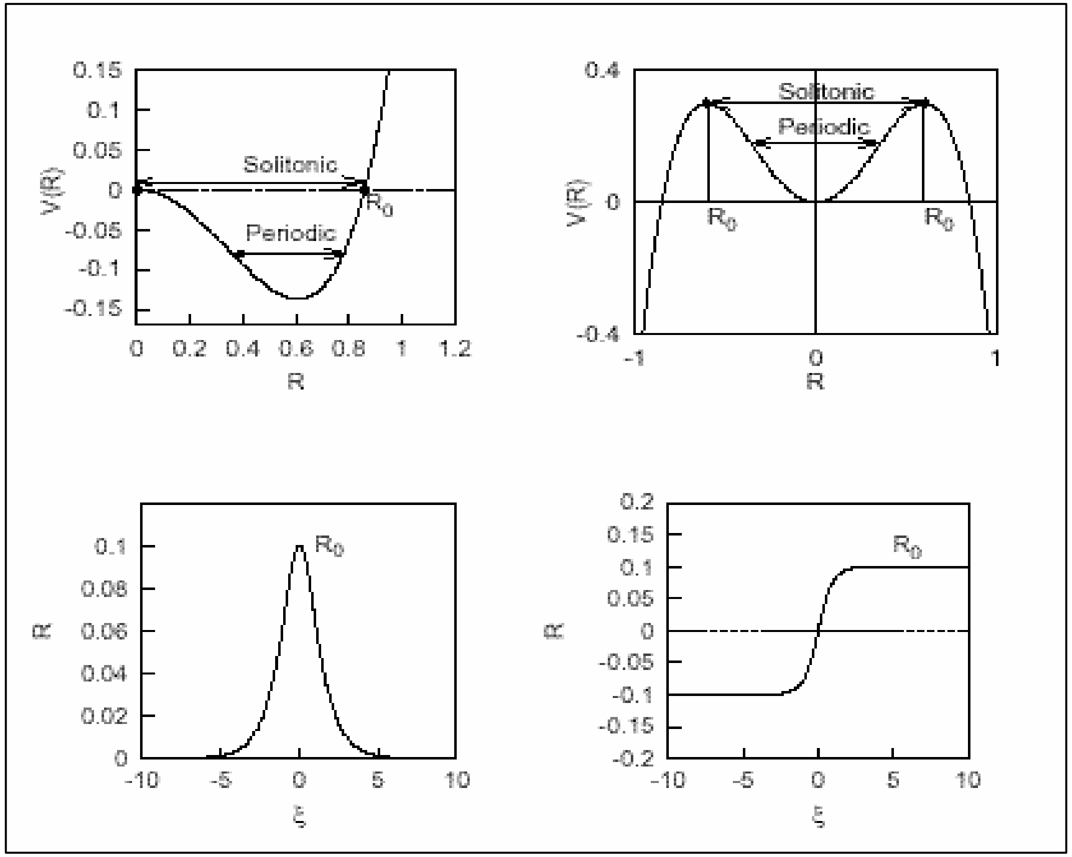
$\beta = 0.05$
$\lambda = 0.93$
$\phi_{max} = 0.4121$
$R_{max} = 0.996$

Non-quasi-neutral

DARK SOLITONS

In the spectral gap $0 < \beta < \sqrt{m_e/m_i}$ dark solitons can exist

(Farina & Bulanov, Plasma Phys. Rept 27 (2001) 641)



SOLITONS IN A WARM PLASMA

- Cold plasma model inadequate when group velocity of pulse becomes comparable to electron and ion thermal velocities

- Both ion dynamics and temperature effects important close to ion acoustic speeds

- Kinetic effects can also play an important role – relativistic version

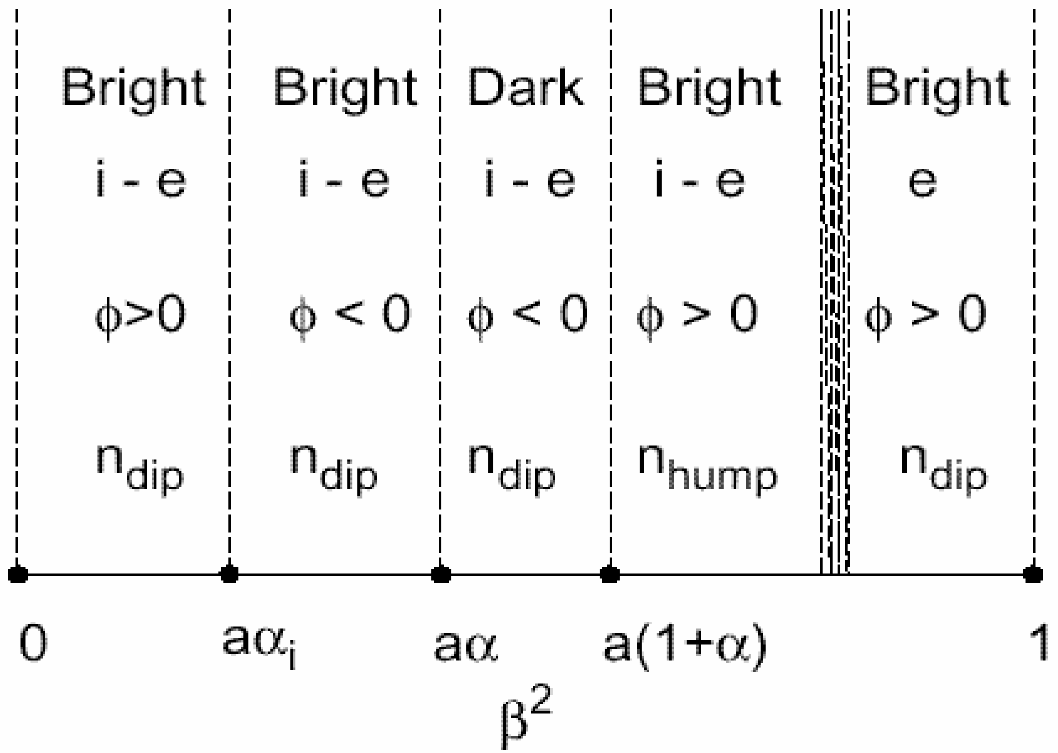
An approximate approach

- Fluid model – nonrelativistic pressure terms
- Modest amplitude waves – weakly relativistic – expansion to third order in amplitude
- Nearly quasi-neutral dynamics
- Examined various limits in terms of group velocity and the various relevant thermal and collective velocities
- Looked for existence of both bright and dark solitons
- Finite temperature decreases amplitude of standing solitons or narrows their width for the same amplitude
- Introduces additional bands of existence region for bright solitons in comparison to cold case
- Interesting interplay of temperature and ion dynamics – leading to different soliton structures – particularly density profiles.

TABLE I. Symbols used: $a = m_e/m_i$, $\alpha_{e,i} = T_{e,i}/m_e c^2$, $\alpha = \Gamma_e \alpha_e + \Gamma_i \alpha_i$, $\Gamma_e = 1$, $\Gamma_i =$ ion adiabaticity parameter, $\beta =$ normalized propagation speed.

Propagation speed β	Soliton variety	Potential ϕ at center	Density n at center	Plasma species involved
$0 \leqslant \beta^2 < a\alpha_i$	bright	positive	evacuation	ions and electrons
$a\alpha_i < \beta^2 < a\alpha$	bright	negative	evacuation	ions and electrons
$a\alpha < \beta^2 < a(1+\alpha)$	dark	negative	evacuation	ions and electrons
$a(1+\alpha) < \beta^2$	bright	positive	accumulation	ions and electrons
$a(1+\alpha) \ll \beta^2$	bright	positive	evacuation	only electrons

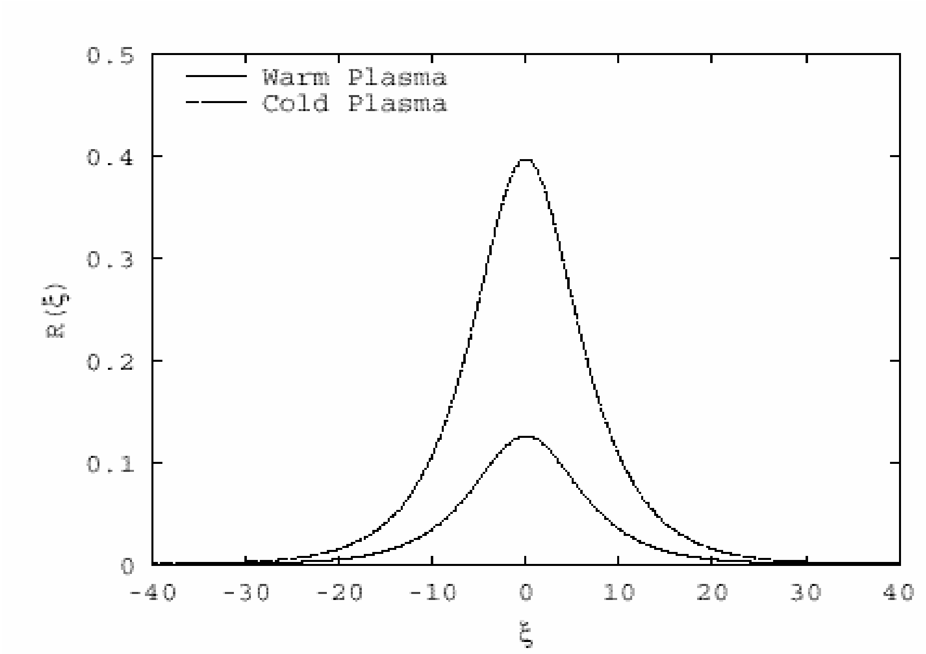
SOLITON SPECTRA IN WARM PLASMA

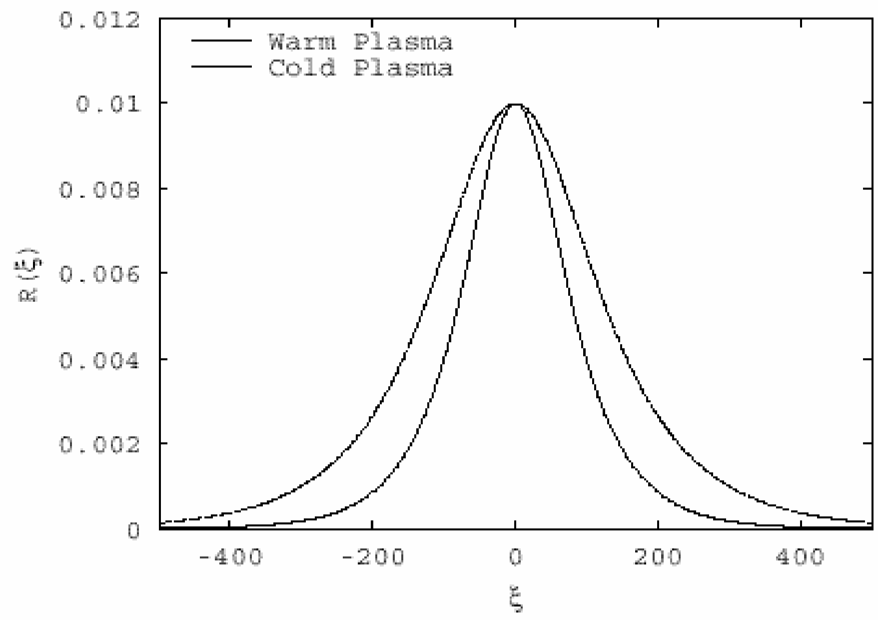


$$a = m_e/m_i,\ \alpha_{e,i} = T_{e,i}/m_e c^2,\ \alpha = \Gamma_e \alpha_e + \Gamma_i \alpha_i$$

Bright Soliton for $\beta^2 < a\alpha$

$$R(\xi) = 2\sqrt{\frac{\alpha(1-\lambda^2-\beta^2)}{(1-\beta^2)}}\,\mathrm{sech}\left(\frac{\sqrt{(1-\lambda^2-\beta^2)}}{(1-\beta^2)}\xi\right)$$





$$\frac{n}{\gamma_e} = 1 - \frac{R^2}{2\alpha} - \frac{R^2}{2}$$

Physics of density depletion

- **When light intensity is maximum ponderomotive force of radiation pushes out the electrons – ions follow due to quasi-neutrality**

$\beta^2 < a\alpha_i,$

- ions Boltzmann – produce dip if $\phi > 0$

$$\boxed{\phi = -\frac{R^2}{2a\alpha}(\beta^2 - a\alpha_i)}$$

$\beta^2 > a\alpha_i,$

- $\phi < 0$; ion thermal effects not important but ion inertia is important. Ions accelerate in the wave frame as they approach the minimum of the potential – conservation of ion flux requires a dip in the ion density at maximum R

$$\boxed{\beta^2 > a(1+\alpha)}$$

$$n = 1 + \frac{a}{\beta^2}\frac{R^2}{2}$$

$$\phi = \frac{R^2}{2}\left(1 + \frac{a\alpha_e}{\beta^2}\right)$$

$$R(\xi) = 2\beta\sqrt{\frac{(1-\lambda^2-\beta^2)}{(1-\beta^2)(\beta^2-a)}}\operatorname{sech}\left(\frac{\sqrt{(1-\lambda^2-\beta^2)}}{(1-\beta^2)}\xi\right)$$

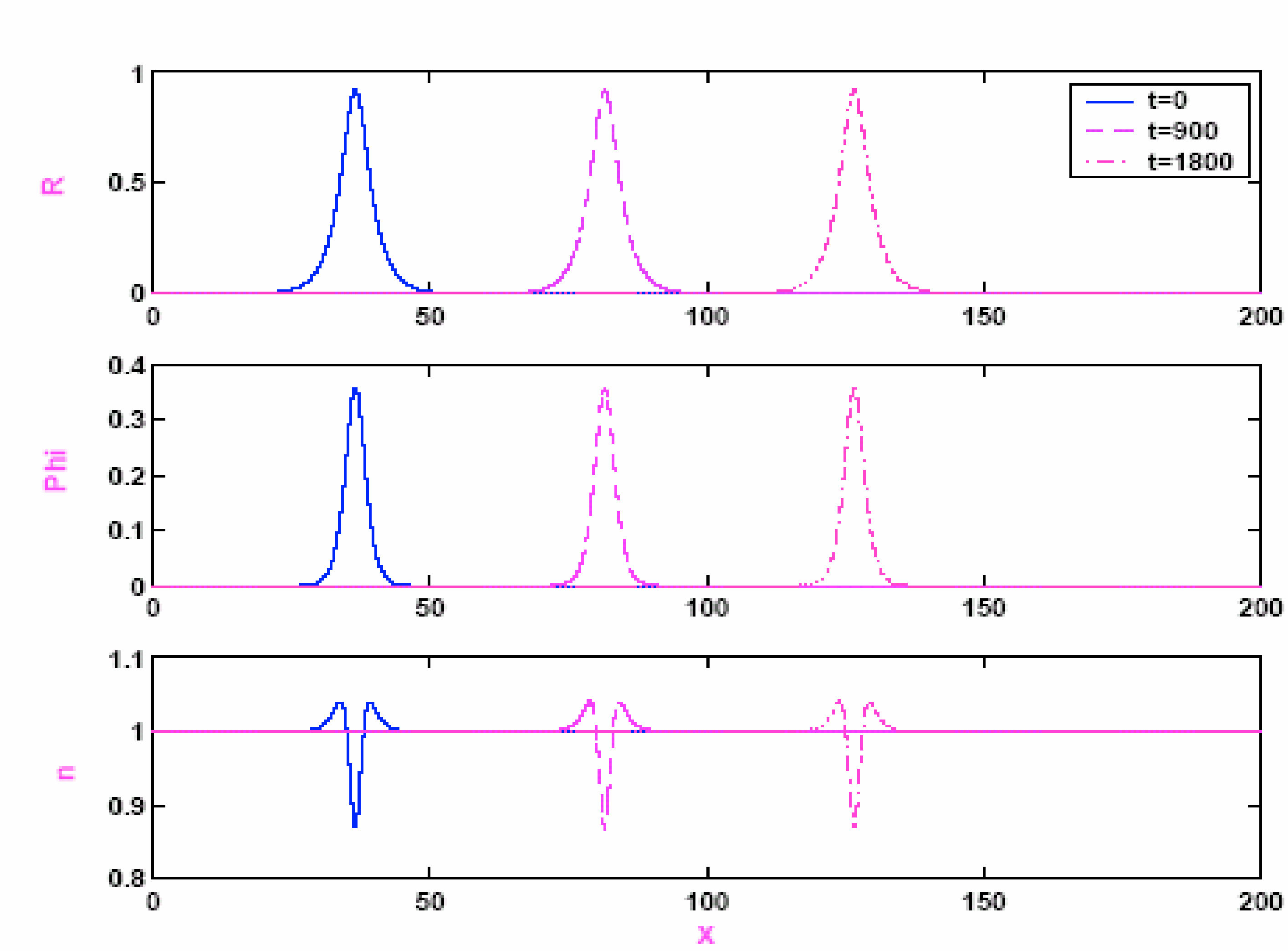
Ion inertia effects dominant

Density bunching because $\phi>0$ – ions decelerate

Constant ion flux condition requires rise in density – electrons follow suit

Stability and accessibility issues:

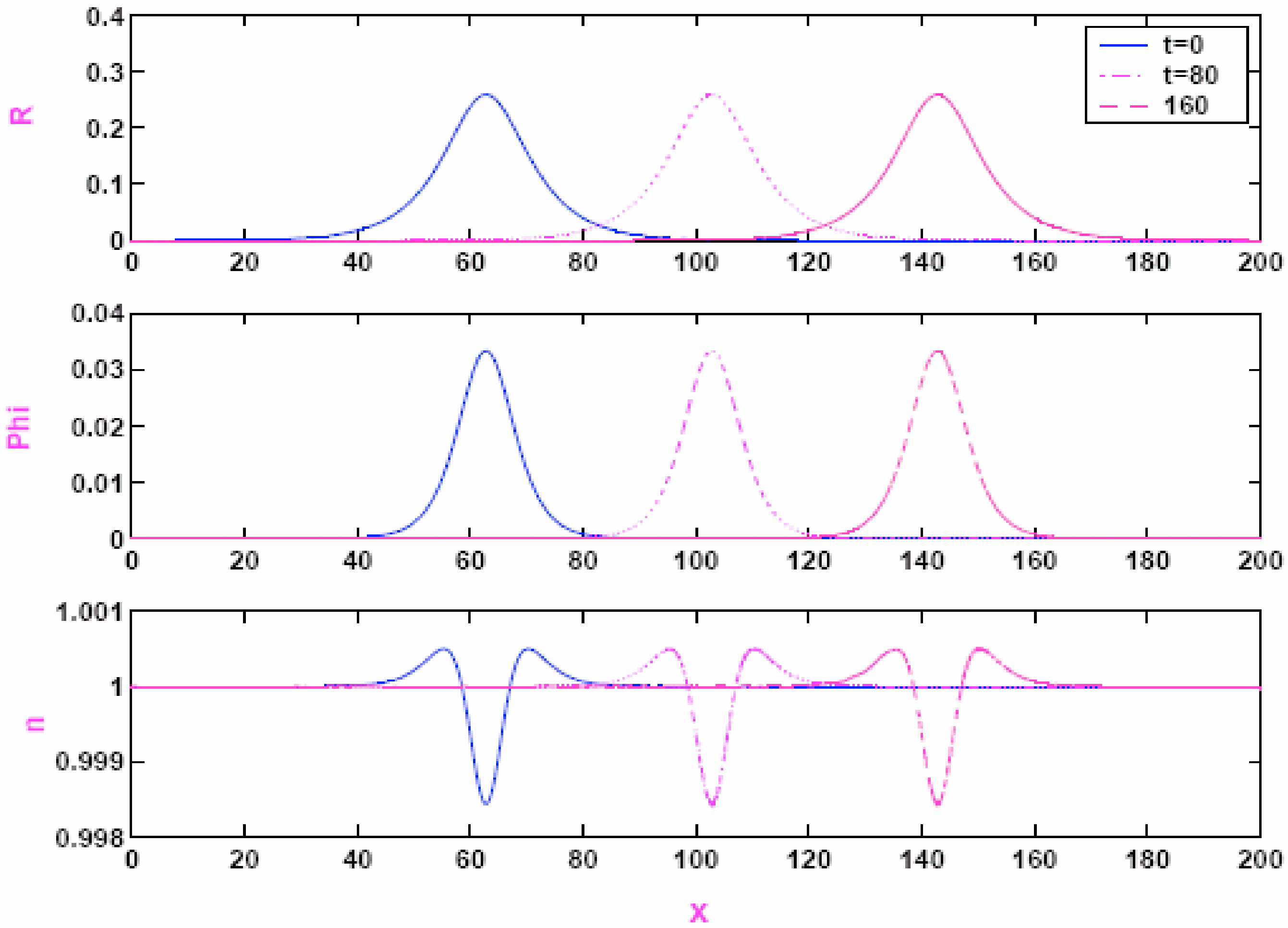
- Numerical solution of original time-space dependent cold fluid equations
- Use traveling wave soliton solutions as initial conditions
- Use arbitrary pulses (e.g. Gaussian) to see development of pulses
- Launch two solitary pulses and study collisions
- launch pulse in an inhomogenous density profile and watch reflection



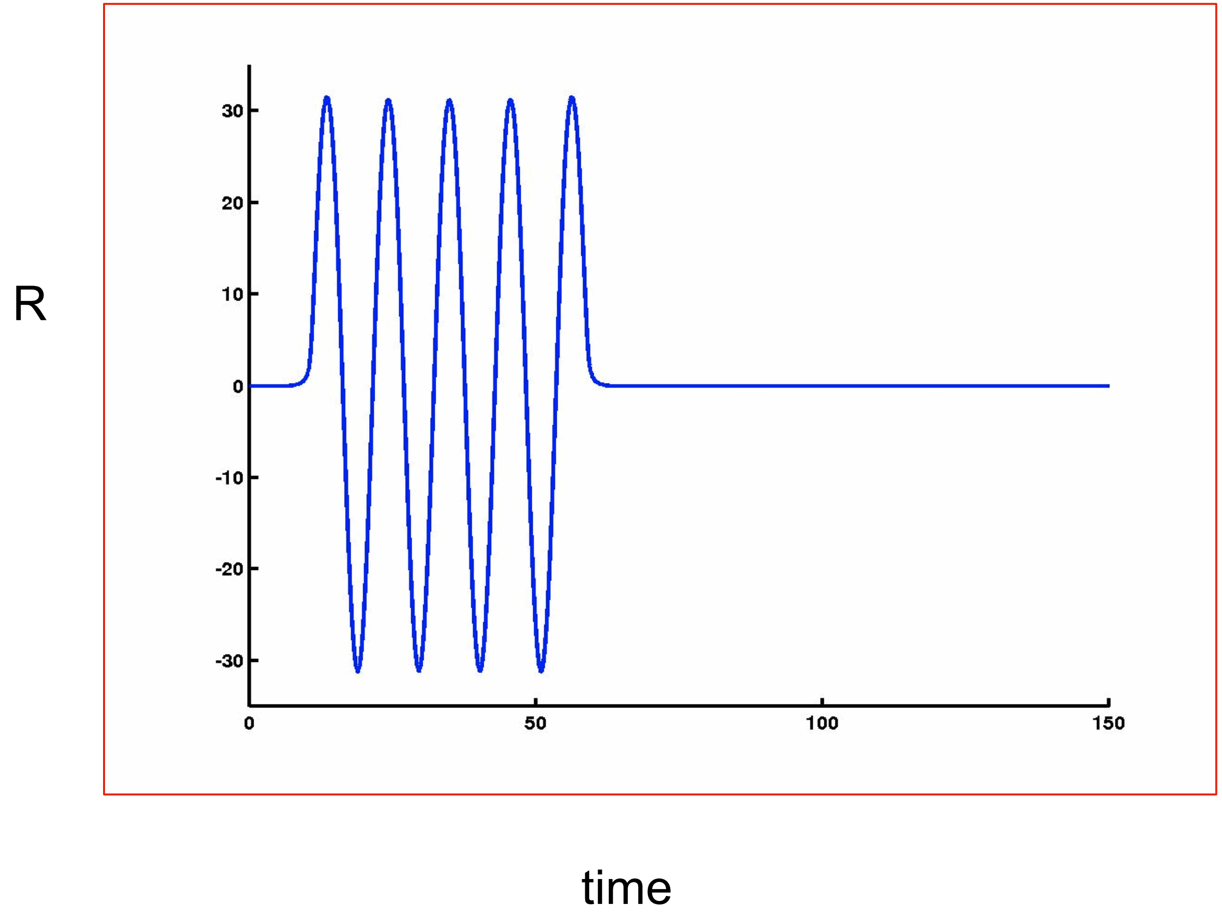
β = 0.05

Single hump low velocity pulse lasts a long time

Intermediate $\beta = 0.5$

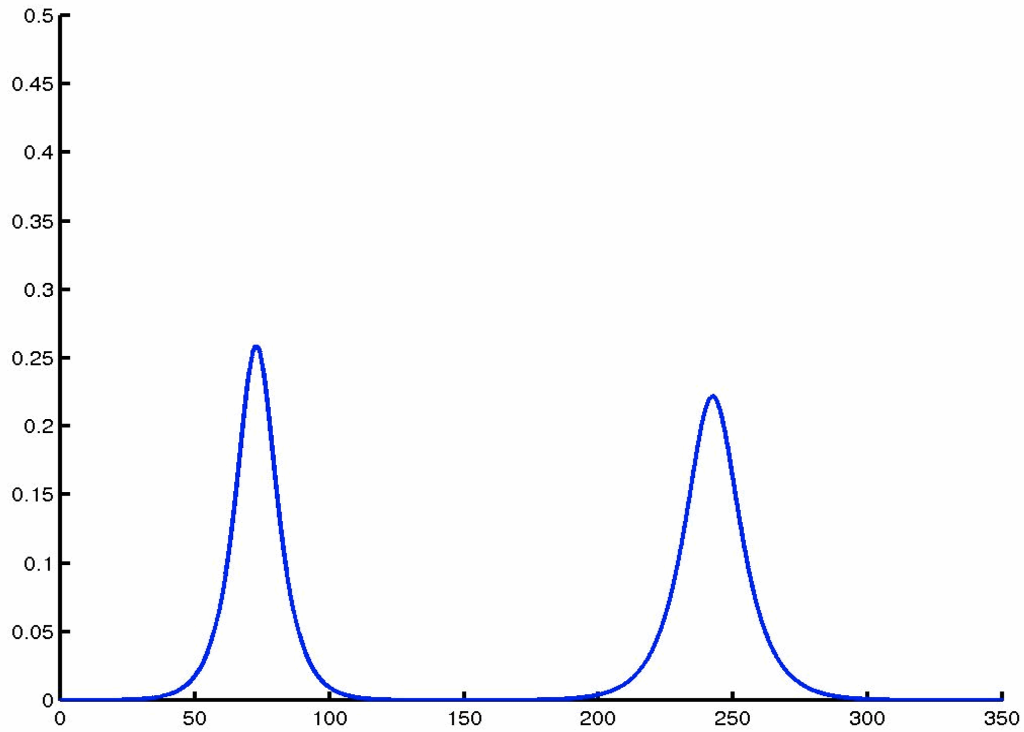


Multi-peak soliton



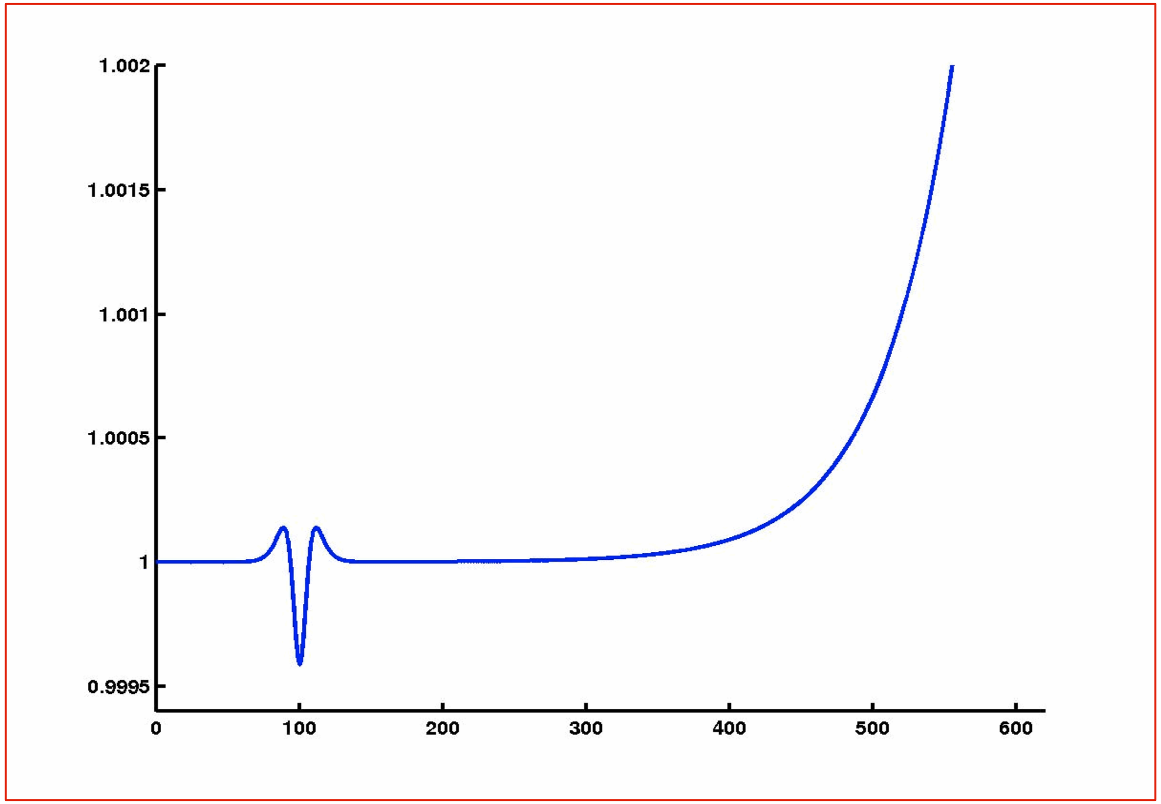
Collision between low amplitude single hump pulses

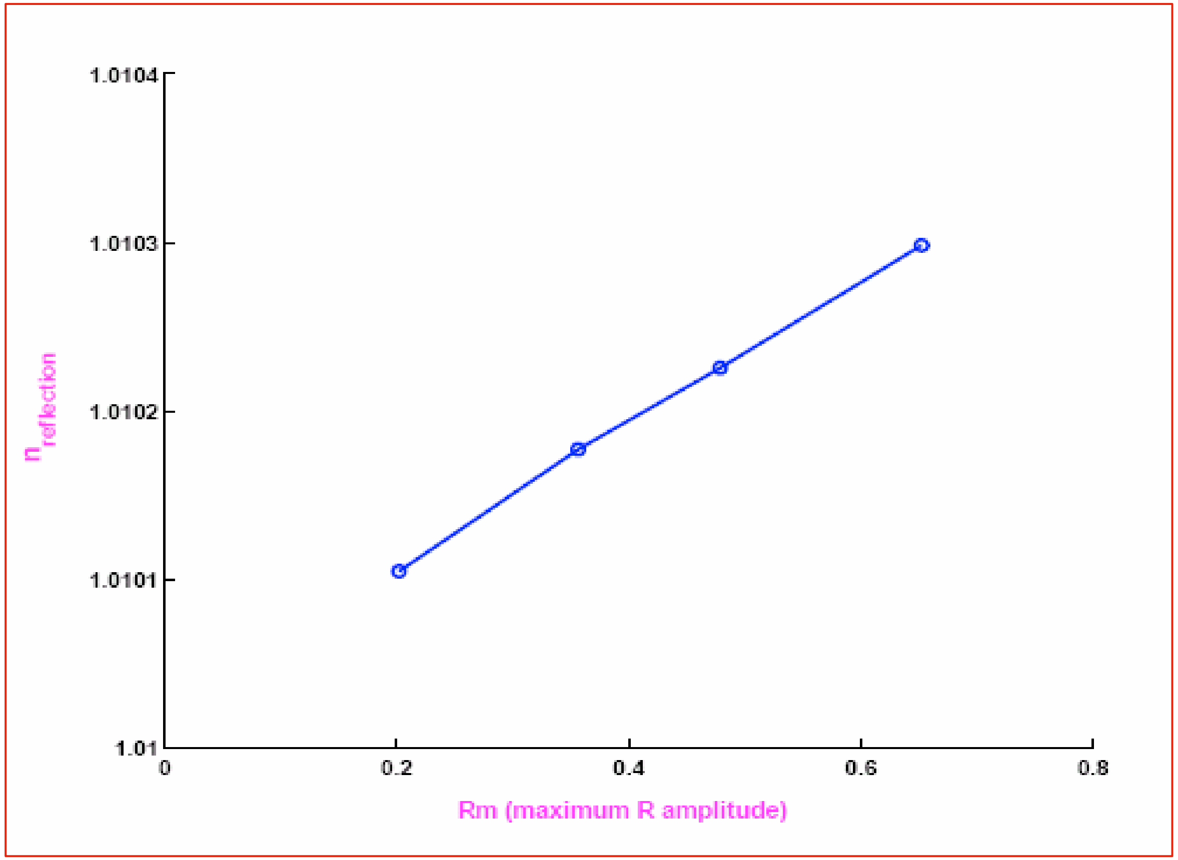
R



Propagation up a density gradient

n

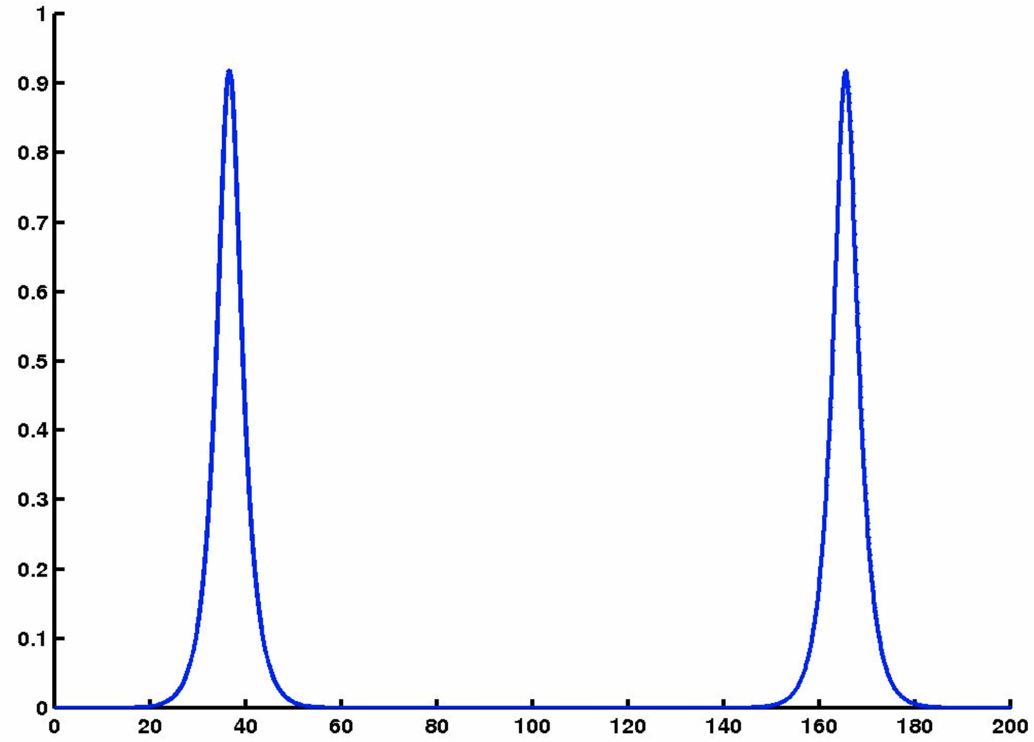




1.0104
1.0103
1.0102
1.0101
1.01
0
0.2
0.4
0.6
0.8
n_reflection
Rm (maximum R amplitude)

Collision between large amplitude pulses

R



Experimental Observation of Solitons

(M. Borghesi et al, PRL 88 (2002) 135002)

- 10^{19} watt/cm^2 laser pulse in an underdense plasma
- long lived macroscopic bubbles with space charges
- proton beam probe diagnostic
- remnant of a cloud of relativistic solitons
- Supported by simulation and analytic studies
- elongated structures

EXPERIMENT

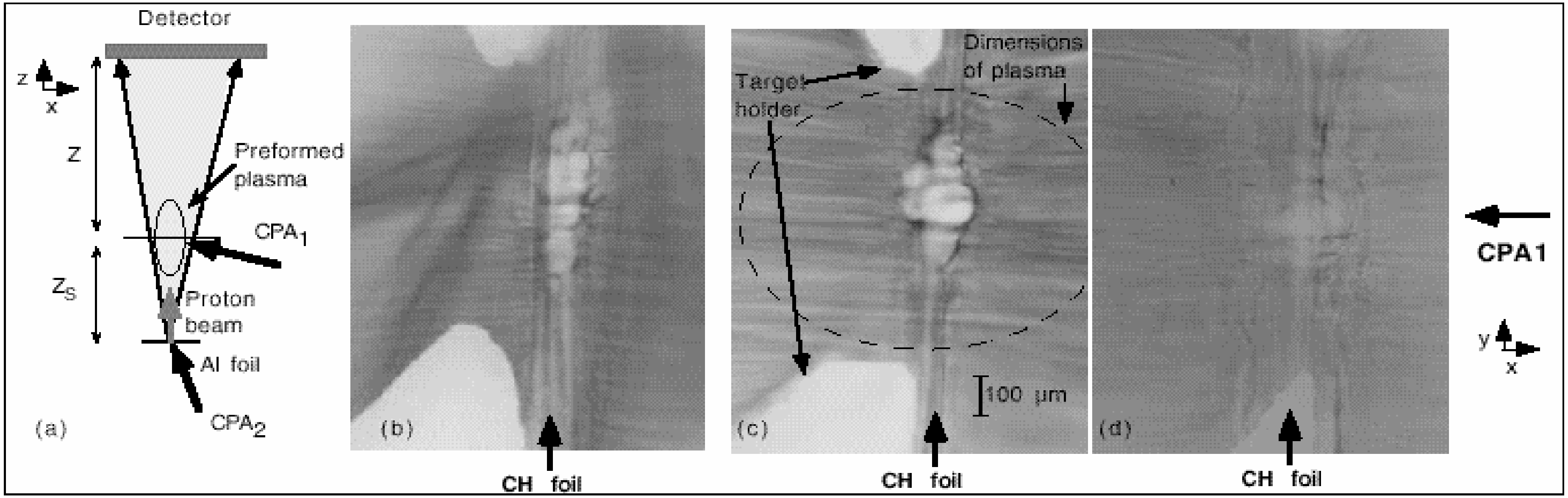


FIG. 1. (a) Experimental arrangement. (b), (c), (d) Proton images of the preformed plasma taken with 6–7 MeV protons, respectively: (b) 25 ps; (c) 45 ps; (d) 95 ps after the CPA_1 interaction. The scale refers to dimensions in the object plane. The dashed line indicates the dimensions of the preformed plasma defined by $n \approx 0.01 n_{cr}$ (at $\lambda = 1\ \mu$m).

PIC Simulation

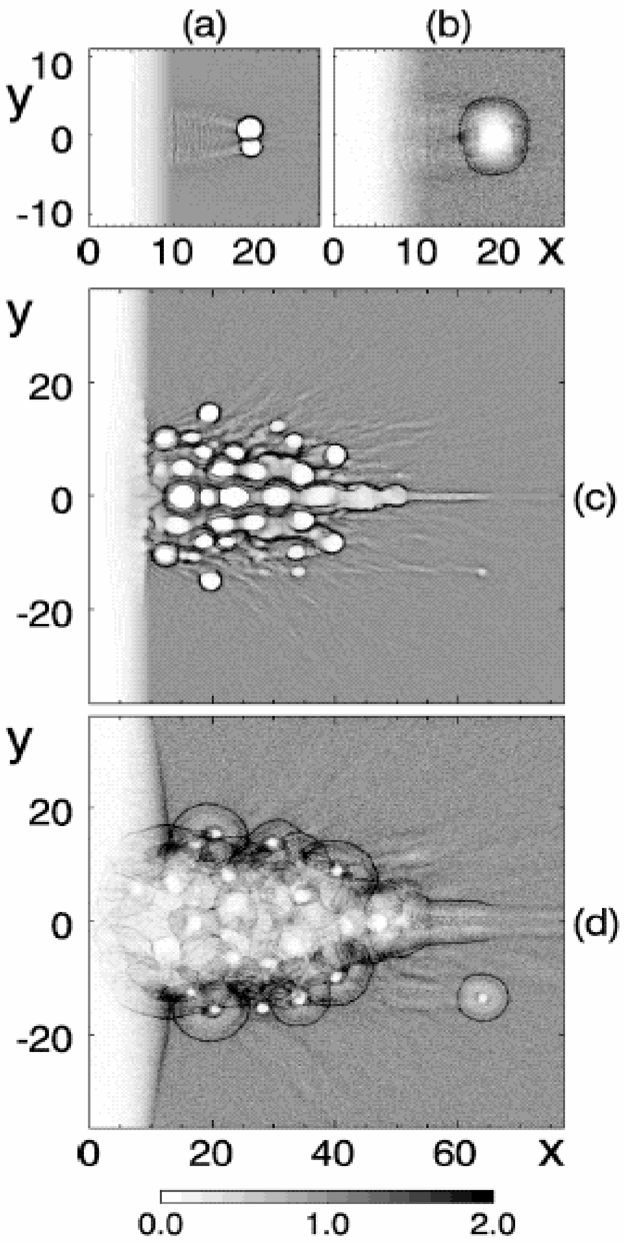


FIG. 2. Ion density distributions of two merging postsolitons during (a) and after (b) the merging process, as obtained from 2D PIC simulations. Ion density distribution in a cloud of merging postsolitons at $t = 140$ (c) and at $t = 550$ (d).

SUMMARY AND CONCLUSIONS

- reviewed a special class of exact 1d nonlinear solutions consisting of coupled light pulse and plasma wave solitons

- two classes – single peak in ϕ and R;
  single peak in ϕ and multi-peak in R

- spectra in the cold plasma model –some unresolved issues

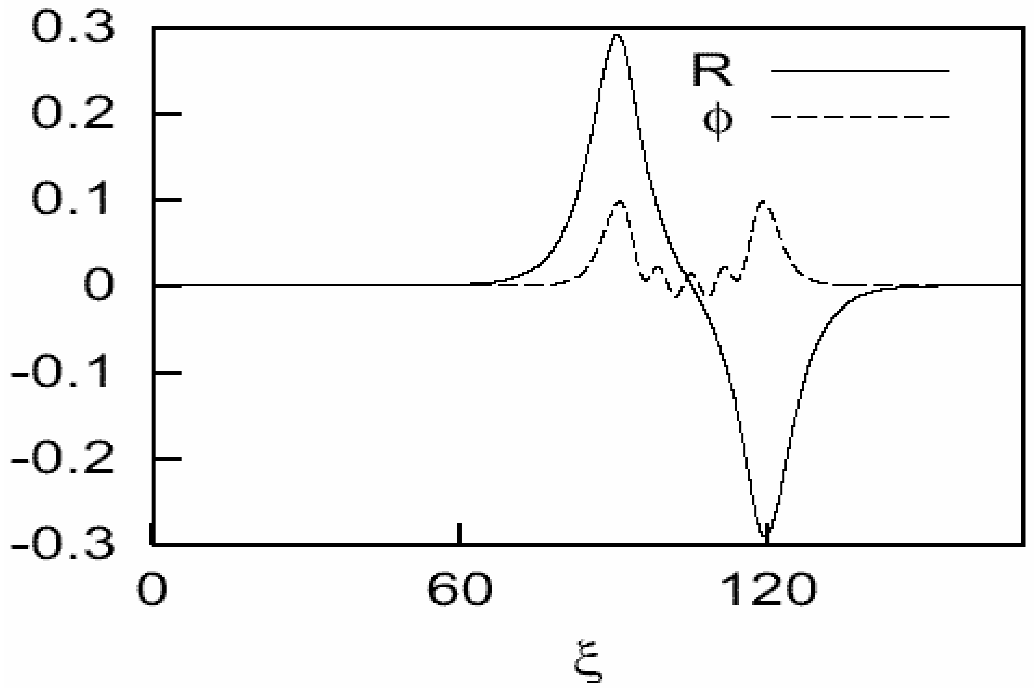
- ion dynamics + finite temperature effects – small amplitude

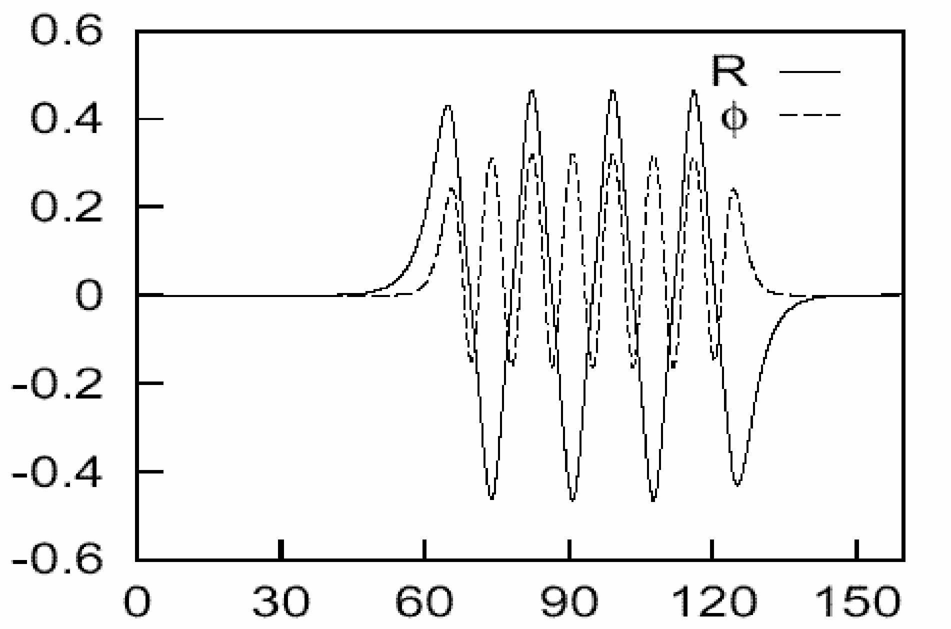
- Future directions:

  - higher dimensional effects – PIC simulation
  - kinetic effects – Landau damping, particle trapping
  - stability and accessibility conditions
  - new class of solutions with interesting structures in ϕ

A NEW CLASS OF SOLITONS – MULTIPEAK IN ϕ

Back to the cold plasma model with no ion dynamics





Multipeak in ϕ solitons

Density resembles a grating