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Electron Holes, Ion Holes and Double Layers

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Electrostatic Phase Space Structures in Theory and Experiment

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Contents:

Abstract:

The paper reviews the state-of-the-art in the observation and analytical description of localized electrostatic phase space structures. These structures occur on the Debye length scale and introduce a kind of intermittency in the dynamics of externally driven collisionless plasmas. Holes, the one group of structures investigated, are nonlinear saturated states of two-stream instabilities in which saturation is provided by particle trapping. They are ring-shaped vortices in phase space and are macroscopically manifest in local density depressions. Double layers, on the other hand, are narrow monotonic potential transitions and connect differently biased plasmas, resembling in some sense phase transitions. The controlling function of these nonlinearly excited d.c. states in the dynamical evolution of bounded plasmas exhibiting transient phenomena is discussed.

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1. Introduction

Electron holes (EH), ion holes (IH), and double layers (DL), the objective of this report, constitute a subclass of nonlinear electrostatic modes, often referred to as BGK modes [1]. Generally speaking, they are saturated states of two-stream unstable collisionless plasmas in which saturation is provided by particle trapping. Representing states far away from thermodynamic equilibrium, these structures are found under laboratory conditions in current carrying and voltage driven plasmas as well as in plasmas driven by beam particle injection or by wave launching. Experimental evidence is furthermore provided by observations in the auroral zone of the ionosphere.

Although some of them have been known for a long time [2], dating back until 1929 when Langmuir analyzing his experiments, inferred on the existence of DLs, renewed interest came up only recently, mainly due to the improved access to these entities in laboratory and numerical experiments. Also new analytical material is available supplementing the experimental data.

Trapping, of course, implies that these structures are not amenable to macroscopic descriptions like MHD or other fluid descriptions. It is the Vlasov picture which has to be invoked.

In the language of modern dynamics these structures appear to be attractors and are generated rather independently of the details of initial and boundary conditions. Provided that the excitation mechanism is sufficiently strong, they will come up inevitably and can last sufficiently long to affect the characteristic properties of a plasma, e.g. its dynamical evolution.

The situation is in some sense analogous to conventional fluid dynamics, where the Bénard cells, having received a great deal of attention, play a similar role. It is well known that a horizontally layered fluid, heated from below, becomes structured by the appearance of convection rolls, when the Rayleigh number exceeds a certain threshold value. This analogy can be strengthened by noticing that in certain approximations both media behave like incompressible two-dimensional fluids. The Vlasov description for one space dimension takes place in the two-dimensional phase space. Due to Liouville's theorem the two-dimensional phase space fluid is incompressible, and it is an easy matter to cast the equations governing the motion of an electron phase space fluid, the Vlasov equation and Poisson's equation, into the equations for a two-dimensional ordinary fluid.

This report mainly refers to stationary isolated structures, that is to say, to steady-state electrostatic excitations, the field energy densities of which are localized in space. Time varying electrostatic modes with a larger spatial extent, such as periodic ones, have been studied in some detail in refs. [3, 4]. These studies include an investigation of the temporal evolution of an initial plasma state towards a periodic equilibrium and show within the adiabatic approximation the existence of 'preferred' BGK states. The transient part of the plasma behaviour is (except in section 7) omitted in this review. Moreover, periodic equilibria are known to be prone to the coalescence instability [5, 6]. This implies that ultimately a plasma develops that is modified by isolated, narrow Debye-scale structures akin to the intermittence structures in fluid dynamics. This kind of plasma being governed by "preferred" BGK states is the concern of this report.

In the following, I shall discuss some of the topics of this specific field whose growing attractiveness is reflected by two symposia that took place in 1982 and 1984 at Roskilde and at Innsbruck, respectively. The proceedings of these symposia [7, 8] are sources of further information as well as several review articles [9, 10, 11].

Section 2 is devoted to an experiment which played a key role in the development of the field of holes. An analysis of EHs, IHs and DLs is represented in the subsequent sections. The evidence of these structures in computer simulations and laboratory experiments is reported in section 6, whereas the dynamical transient properties are addressed in section 7.

2. The Risø experiment on electron holes

2.1. The laboratory experiment

As mentioned, big progress in the understanding of electrostatic structures has been achieved experimentally at the Risø-Institute in Denmark [12, 13, 14]. Figure 1 shows the schematic set-up of this experiment. It consists of a single-ended Q-machine in which a collisionless plasma is produced by surface ionization on a hot tantalum cathode. Radial confinement is obtained by a homogeneous magnetic field ($\omega_{ce} \gg \omega_{pe}$). A surrounding cylindrical brass tube with a slit acts as a wave guide. A short negative voltage pulse on the left part forces the enclosed electrons to leave and to enter the target plasma where they constitute an effective, two-stream like perturbation. The plasma response to this excitation was measured by axially movable Langmuir probes. Figure 2 shows the time dependence of the *negative* electrostatic potential recorded at various positions. Two distinct structures of opposite polarity are seen. A fast moving negative potential pulse and a positive, slower potential pulse. The latter structure being associated with a density depression is the aforementioned EH. It is remarkably stable. The fast pulse, on the other hand, decreases in amplitude and spreads. It could be identified as a Gould–Trivelpiece soliton which is governed for small amplitudes by a Korteweg–de Vries (KdV) equation. The width of the soliton is broader than that of the hole despite its larger amplitude. This is somewhat surprising because the hole should be broader if it belonged to the class of KdV solitons, too. Whereas the velocity of the soliton is several times larger than the electron thermal velocity v_{te}, and corresponds to the Gould–Trivelpiece mode, the hole velocity is of the order of v_{te} only or even less. Argued from a linear basis, it is in the velocity range where one would expect strong electron Landau damping and, hence, a complete suppression of the hole structure. This does, however, not correspond to the real observation. Note also a slight asymmetry of the hole increasing with time. Furthermore, it is experimentally found that the hole but not the soliton vanishes when the pressure is increased. In addition, in cases where two holes are

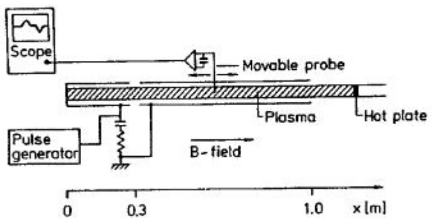

Fig. 1. Schematic set-up of the Risø experiment. After [13].

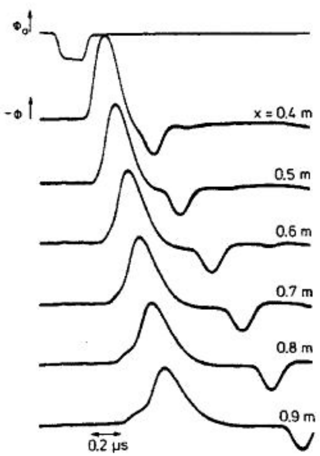

Fig. 2. Time-dependence of $-\phi$ at equidistant positions. After [13].

created, the coalescence of both to a single hole could be seen [14]. These features prove experimentally the non-soliton property of EHs and indicate that the hole is something else.

2.2. The numerical experiment

Unfortunately the device did not allow measurements of the electron distribution function f_e to get further information. However, the authors could perform a particle simulation which was adapted to their laboratory experiment. The numerical results are presented in the next two figures.

In figs. 3 and 4, three structures are seen: The fast moving soliton, the hole, which is almost standing in this frame, and a new structure (later on being referred to as SEADL) moving to the left. As seen by the phase space pattern the soliton accelerates and decelerates the whole electron fluid and is, hence, a macroscopic phenomenon. But the other two structures, as one easily recognizes, are of different type. For both, the distribution remarkably deviates from Maxwellian. The hole has a vortex-like structure in phase space (remember the analogy to fluid dynamics mentioned in the introduction) being characterized by a deficit of slow (deeply trapped) electrons within the structure. The third structure represents a monotonic potential transition (DL) and is associated with a two-stream-like distribution at maximum potential, the two branches of which unite on the low potential side. The latter structure is the most unstable one and shows the onset of a transition into a hole.

Before discussing the EH properties analytically, it should be noted that this sort of trapping vortices has been observed many times in computer runs simulating two-stream unstable situations [15, 16, etc.].

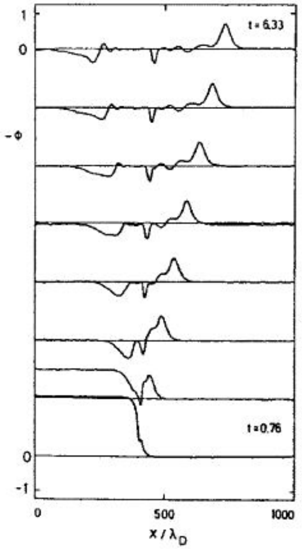

Fig. 3. The spatial dependence of $-\phi$ at equidistant time steps. After [13].

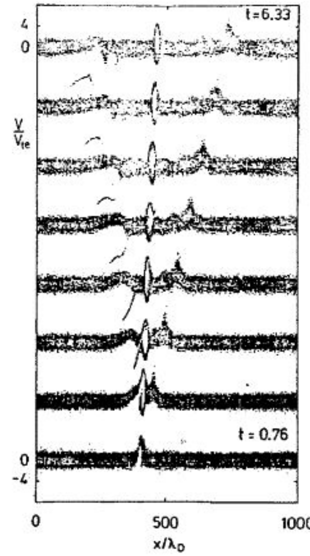

Fig. 4. The electron phase space corresponding to fig. 3. After [13].

They have also been found in the simulation of plasmas interacting with launched large amplitude waves as, for example, in the Raman [17] and Brillouin [18] back scattering or in the lower hybrid heating [19]. We, therefore, may state that trapping vortices or holes are common structures in plasmas driven by distinct excitation mechanisms.

3. Theory of electron holes

3.1. *The analytical description*

Theoretically [20, 21], electron holes sharing these properties have been explained as equilibrium solutions of the Vlasov–Maxwell equations, as will be shown next. These equations reduce in the elctrostatic, one-dimensional limit to

$$[v\,\partial_x + \phi'(x)\,\partial_v] f_e(x,v) = 0\,, \tag{1}$$

$$\phi''(x) = \int \mathrm{d}v\, f_e(x,v) - 1\,. \tag{2}$$

Equation (1) is the electron Vlasov equation, and eq. (2) is Poisson's equation. They are formulated in the wave frame $(x - v_0 t \rightarrow x)$ where the structure becomes stationary assuming that it propagates with a velocity v_0 in the laboratory frame. Ions are treated as a constant neutralizing background. The spatial coordinate x, the velocity v, and the electrostatic potential ϕ are normalized by the Debye length λ_D, the electron thermal velocity v_{te}, and T_e/e, respectively, where T_e is the electron thermal energy in the unperturbed medium which is assumed to be in thermal equilibrium. The latter implies the boundary conditions

$$\phi,\ \phi' \rightarrow 0\,, \qquad f_e(x,v) \rightarrow c \exp[-\tfrac{1}{2}(v + v_0)^2] \tag{3}$$

as $|x| \rightarrow \infty$ $(c = (2\pi)^{-1/2})$.

The method of solution used here differs from the original BGK approach [1] which suffers from the fact that non-well-behaved distributions are involved [22]. BGK distributions, as it is well known, can become negative or show a singular behaviour in velocity space, the mildest form of singularities being discontinuities in f or in its derivatives in v. One can live with such jumps if there exist neighbouring smooth solutions [22]. It seems, however, not meaningful to justify discontinuous distributions within the Vlasov description. The reason is that the latter itself is an approximation and demands that the collision term in the superimposed kinetic diffusion equation, whatever its detailed form in the presence of strong electric fields will be, is negligible. Steep gradients in the velocity space causes an enhanced diffusion in this generalized description, smoothing the one-particle distribution function. This smoothing process reduces the strength of the collision term so that ultimately a collision-free Vlasov state may be approached which is characterized by smooth continuous distributions.

The so-called 'alternative method' applied here appears to be the proper method to describe this kind of 'preferred BGK states' because it allows from the outset the incorporation of such 'physically acceptable' distributions. It was used by the author in deriving finite amplitude ion wave solutions [23] and consists in solving (1) in terms of the constants of motion where use is made of the global form of ϕ (which is bell-shaped here), and inserting this solution into (2) which is solved then for ϕ [20–23].

A solution of (1) consistent with the boundary condition (3) is given by

$$f_e(x,v) = \begin{cases} c \exp[-\tfrac{1}{2}(\sigma\sqrt{v^2 - 2\phi} + v_0)^2] & E_e > 0\,, \\ c \exp[-\tfrac{1}{2}\beta(v^2 - 2\phi) - \tfrac{1}{2}v_0^2] & E_e \le 0\,, \end{cases} \tag{4}$$

where $E_e = v^2/2 - \phi$, and $\sigma = \operatorname{sgn} v$ are the constants of motion of free electrons $(E_e > 0)$. Trapped electrons are represented by the second line in (4), for which $E_e \le 0$. The distribution function (4) at a fixed position within the structure is illustrated qualitatively in fig. 5. Note that the distribution function is continuous everywhere, especially at the border of trapped particles which is given by $|v| = \sqrt{2\phi}$ (dashed line). The population of trapped particles is reduced when β, the trapped particle parameter, turns out to be negative. Hence, f_e given by (4) is of vortex-type when β is sufficiently negative. The phase space trajectories are shown qualitatively in fig. 6 for a bell-shaped electric potential.

Integration of (4) yields the electron density n_e which depends on the parameters v_0 and β [23]. It can be written as [23, 11]

$$n_e = \exp(-v_0^2/2)[F(v_0^2/2, \phi) + T_-(\beta,\phi)]\,,$$

where $F(T_-)$ denotes the contribution of the free (trapped) electrons. These functions are given by

$$F(u,\phi) = I(\phi) + K(u,\phi)\,, \qquad T_-(\beta,\phi) = \frac{2}{\sqrt{\pi|\beta|}} W(\sqrt{-\beta\phi})\,, \qquad \beta \le 0\,,$$

where $I(\phi) = \exp\phi(1 - \operatorname{erf}\sqrt{\phi})$,

$$K(u,\phi) = \frac{2}{\sqrt{\pi}} \int_0^{\pi/2} \mathrm{d}\bar{\phi}\, \sqrt{u} \cos\bar{\phi} \exp[-\phi\tan^2\bar{\phi} + u\cos^2\bar{\phi}]\operatorname{erf}(\sqrt{u}\cos\bar{\phi})\,,$$

and W is Dawson's integral,

$$W(x) = \exp(-x^2)\int_0^x \mathrm{d}t \exp(t^2)\,,$$

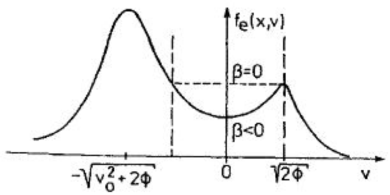

Fig. 5. The electron distribution function in velocity space.

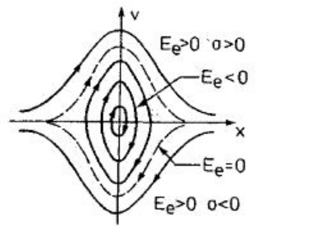

Fig. 6. The particle trajectories in phase space in the vicinity of an EH.

Poisson's equation (2) then becomes

$$\phi''(x) = n_e(\phi;v_0,\beta) - 1 \equiv -\partial V/\partial\phi\,. \tag{5}$$

In (5) we have introduced the 'classical potential' V (Sagdeev potential) which depends on ϕ, v_0, and β.
It becomes (see also ref. [30])

$$V(\phi;v_0,\beta) = \phi - \exp(-v_0^2/2)[P_-(\beta,\phi) - 1 + H(v_0^2/2,0,\phi)]\,,$$

where

$$P_-(\beta,\phi) = I(\phi) + 2\sqrt{\phi/\pi}(1-\beta^{-1}) + \beta^{-1}2(\pi|\beta|)^{-1/2}W(\sqrt{-\beta\phi})\,,$$

$$H(u,0,\phi) \equiv \int_0^{\phi} K(u,\bar\phi)\,\mathrm{d}\bar\phi$$

$$= \frac{2}{\sqrt{\pi}}\int_0^{\pi/2} \mathrm{d}\bar\phi\,\sqrt{u}\cos\bar\phi\,\exp(u\cos^2\bar\phi)\,\mathrm{erf}(\sqrt{u}\cos\bar\phi)\,\frac{1}{\tan^2\bar\phi}\,[1-\exp(-\phi\tan^2\bar\phi)]\,.$$

The integrated form of (5) reads

$$\tfrac{1}{2}\phi'(x)^2 + V(\phi;v_0,\beta) = 0\,, \tag{6}$$

where $V(0;v_0,\beta)=0$ is assumed. There is a unique correspondence between the electric potential ϕ and the 'classical potential' V, as illustrated in fig. 7, for a bell-shaped potential having a maximum value ψ (amplitude).

Two conditions are necessary for the existence of a solution:

$$\text{(i)}\quad V(\phi;v_0,\beta)<0 \quad\text{in}\quad 0<\phi<\psi\,, \tag{7a}$$

$$\text{(ii)}\quad V(\psi;v_0,\beta)=0\,. \tag{7b}$$

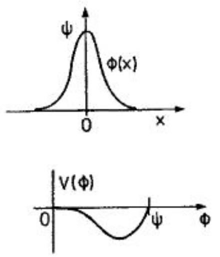

Fig. 7. The correspondence between electric and 'classical potential'.

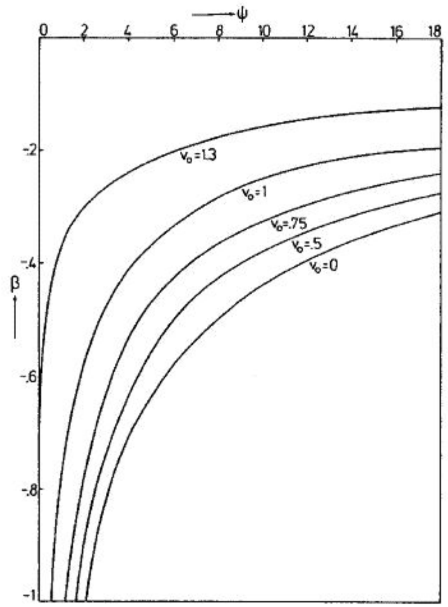

Fig. 8. The range of existence of EH solutions in the parameter space. After [21].

The second condition is called the *nonlinear dispersion relation* because it implicitly determines the hole velocity v_0 in terms of ψ and β.

An exploitation of the range in the parameter space [21] for which genuine EH solutions exist, is shown in fig. 8, the range of existence lying between the curves $v_0 = 1.3$ and $v_0 = 0$. Obviously, EHs need a negative β for their existence. This as well as the finite value of v_0, $0 < v_0 < 1.3$, is in accordance with the experimental observation. There is apparently no limitation in the hole amplitude ψ.

3.2. The limit of small amplitudes

In the small amplitude limit, $\psi \ll 1$, the whole structure can be resolved analytically. The electron density becomes

$$n_e(\phi;v_0,\beta) = 1 - \tfrac{1}{2}Z_r'(v_0/\sqrt{2})\phi - \tfrac{4}{3}b\phi^{3/2} + \cdots\,, \tag{8}$$

where Z_r is the real part of the plasma dispersion function ($Z(z) = \pi^{-1}\int_{\cup}\mathrm{d}t\,[\exp(-t^2)/(t-z)]$), and b is given by the expression

$$b = \pi^{-1/2}\exp(-v_0^2/2)(1-\beta-v_0^2)\,. \tag{9}$$

The nonlinear dispersion relation (7b) becomes

$$-\tfrac{1}{2}Z_r'(v_0/\sqrt{2}) = \tfrac{16}{15}b\psi^{1/2}\,, \tag{10}$$

which can be solved for v_0:

$$v_0 = 1.305(1 - \tfrac{16}{15}b\psi^{1/2} + \cdots)\,. \tag{11}$$

This shows that the EH is a nonlinear descendant of the *slow electron acoustic mode*, which in the long wavelength limit is defined by the linear dispersion relation $\omega = 1.305kv_{te}$ [20, 24]. The "energy law" (6) with V given by

$$V(\phi; v_0, \beta) = -\tfrac{8}{15}b\phi^2(\sqrt{\psi} - \sqrt{\phi})\,, \tag{12}$$

can be integrated to yield

$$\phi(x) = \psi\,\mathrm{sech}^4\left(\left(\frac{b\sqrt{\psi}}{15}\right)^{1/2} x\right). \tag{13}$$

The condition (7a) implies $b > 0$ from which follows $\beta < -0.71$. Hence, β must be sufficiently negative. The potential form (13) deviates from that of the KdV soliton which is given by

$$\phi(x) = \psi\,\mathrm{sech}^2(\sqrt{\tfrac{1}{6}\psi}\,x)\,, \tag{14}$$

in two respects: in the power of the sech and in the scaling of the argument. It follows that the EH's width, $\Delta_{EH} \sim \psi^{-1/4}$, is less than that of the KdV soliton, $\Delta_{KdV} \sim \psi^{-1/2}$, for comparable amplitudes, in agreement with the observation.

The evolution equation, to which (13) is a stationary solution, reads

$$\phi_t + 1.305(1 - 2b\sqrt{\phi})\phi_x - 1.305\phi_{xxx} = 0\,. \tag{15}$$

This modified KdV equation has been derived earlier for small amplitude ion acoustic solitons in the case of nonisothermal electrons [25]. It possesses a finite set of conservation laws only and does not belong to the class of integrable differential equations [26]. Hence, the coalescence of two EHs propagating with comparable speed such that the interaction time exceeds the bounce time of electrons in the superimposed potential well [27], seems to be a natural event in the class of solutions of (15).

The electron density is diminished at the center, $n_e(\psi) = 1 - \frac{4}{15}b\psi^{3/2}$, and it is clear that collisions with neutral particles introduced by an increase of the gas pressure will fill up the trapped electron region. The existence condition, $\beta < -0.71$, is then no longer met, and the hole – but not the soliton – will disappear.

It is worth mentioning that this second acoustic branch representing modes of low phase velocity in comparison with the faster Gould–Trivelpiece mode, was noticed by Stix [24] in a linear analysis, and by the author [20] in the nonlinear analysis of EHs. This mode has its pendant in the ion dynamics, as we shall see, where the corresponding mode was recognized in connection with finite amplitude ion acoustic waves in ref. [23]. The relation of this slow electron acoustic mode to the second mode observed by Krapchev and Ram [4] below the electron plasma frequency has not yet been worked out, but it seems

that the latter refers to the same branch. The observation by these authors on the basis of adiabatic theory, namely, that a critical strength for the electrostatic potential is needed for the existence of this trapped particle mode, leads to the conclusion that nonadiabatic, fast driving processes like the one in the Risø experiment are necessary to excite low amplitude EHs.

The third structure in figs. 3 and 4, which is identified as a DL based on the slow electron acoustic branch [28, 29] moves with a velocity given by $v_0 = 1.305(1 - \frac{8}{3}b\psi^{1/2})$ which is smaller than that of the corresponding EH. Thus, it appears that even details of the experimentally observed structures can be explained analytically.

3.3. *Electron holes in bounded plasmas*

If the radial boundedness of the plasma is taken into account [20], the nonlinear dispersion relation (10) is essentially modified by a extra term $K_\perp^2$ on the left-hand side, where $K_\perp = 2.4/R$ and R is the normalized plasma (cylinder) radius. This implies that R has to satisfy $R > 5$ for an EH to exist, a requirement which is satisfied in the experiment.

Recently, a fuller investigation of the existence of EHs in the finite amplitude region in radially bounded plasma has been made by Lynov et al. [30]. Including also waterbag distributions, they essentially arrive at the same results, in cases where a comparison is possible. Their general results show that for non-vanishing $K_\perp$ there is a limit for the allowed amplitude, which tends to infinity as $K_\perp \to 0$. For $K_\perp > 0.5$ only extremely low amplitude EHs can exist. As expected, there is also an upper limit for the speed $v_0 \le v_{0MAX}(K_\perp)$.

The range of existence in the full parameter space is indicated in fig. 9 (shaded area). The hole width Δ_{EH} is plotted versus the amplitude in case of Maxwellian-like distributions. For larger amplitudes, the hole width is seen to increase with increasing amplitude, in contrast to its small amplitude behaviour. The EHs extracted from a particle simulation (circles) are found to lie entirely in the area admitted by the theory. For an individual hole, however, a discrepancy up to 40% between the observed width and the calculated one was sometimes also found, indicating that other distributions than the modified Maxwellians may be of interest too (see e.g. ref. [31]).

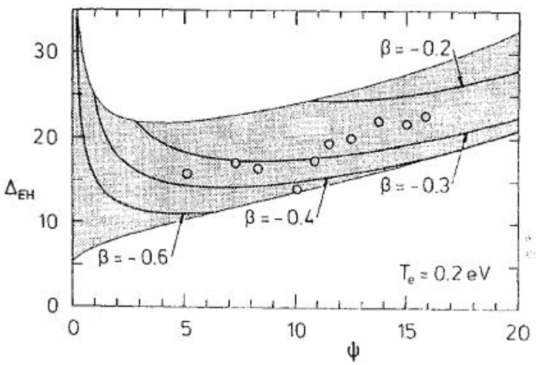

Fig. 9. The existence range of EH. The upper limiting curve is given by $v_0 = v_{0MAX}$, the lower one by $v_0 = 0$. After [30].

3.4. *Two selected properties of electron holes*

Despite the non-soliton property of holes it is instructive to associate 'particle' properties with an EH [32, 27]. For this reason, we generalize the electron density (8) in the small amplitude limit:

$$n_e = 1 + \lambda^{-2}\phi + \int_t \mathrm{d}v\, \bar{f} + \mathrm{O}(\phi^2)\,, \tag{16}$$

where

$$\lambda^{-2} = -\mathrm{P}\int \frac{1}{v}\, f_0'(v)\,\mathrm{d}v\,, \qquad \bar{f} \equiv f_t(\sqrt{2\phi - v^2}) - f_0(v) + \frac{1}{v}\, f_0'(v)\phi\,.$$

P means principle value, f_t is the trapped electron distribution, and f_0 stands for the unperturbed free electron distribution; $\bar{f}$ is, as we know, negative because of the lack of trapped particles. The integration in (16) has to be taken over the trapped range. One immediately checks that (16) reduces to (8) if the distributions (4) are inserted.

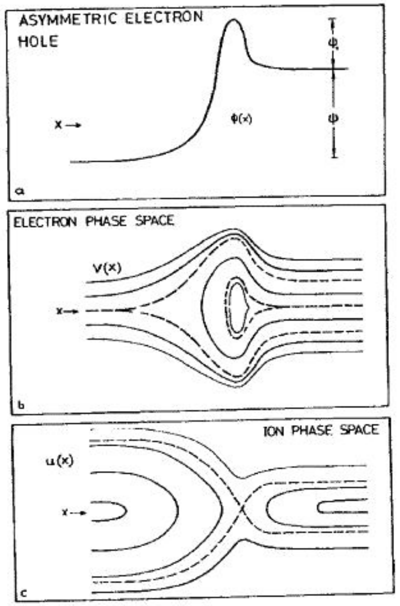

Fig. 10. Three diagrams characterizing an asymmetric EH. After [33].

With (16) Poisson's equation becomes

$$-\phi'' + \lambda^{-2}\phi \approx -\int_t \bar{f}\,\mathrm{d}v \equiv \rho_{\mathrm{EH}}\,. \tag{17}$$

The second term in (17) can be interpreted as a shielding term. It is produced by free electrons and becomes most effective for large distances ($\phi \to 0$), where the charge density ρ_{EH} vanishes. In view of $\bar{f} < 0$, ρ_{EH} is positive. It is at maximum in the core of the hole. Defining charge, mass, momentum, and energy of the hole:

$$(Q,M,P,T) \equiv \int_{-\infty}^{+\infty} \mathrm{d}x \int_t \mathrm{d}v\,(-1,1,v,v^2/2 + \phi)\bar{f}\,, \tag{18}$$

we get $Q = -M > 0$. An EH can thus be interpreted as a cloud of positively charged particles embedded in an electron fluid which acts as a dielectric medium. A d.c. electric field in one direction gives rise to a movement of the electrons into the opposite direction carrying with them the positively charged cloud. This 'wrong' direction for the motion of the cloud is compensated by assigning a negative mass to it.

The possibility of asymmetric EHs, fig. 10, which can be excited in plasmas with different asymptotic states, e.g. in a double plasma device, should be pointed out [33]. A slight asymmetry was already noticed in the EH produced in the laboratory experiment, fig. 2, where the asymmetry is caused by reflection of electrons.

3.5. *Some remarks on the stability of phase space structures*

A few remarks on the stability of phase space structures in general close this section. There is numerical evidence [16] that EHs are stabe in one dimension with respect to electrostatic perturbations, but dissolve if perturbations in the transverse direction are admitted. Actualy, the stability behaviour of structures like that covered in this review is still unsolved from the theoretical point of view. One reason is that the linear eigenvalue problem obtained from a normal mode analysis is highly nonlocal [5, 6, 29, 34–37] and has not yet been rigorously solved.

It can be summarized as follows: Assume that $\phi_1(x)$ represents the electrostatic perturbation such that the total electrostatic potential becomes

$$\phi(x,t) = \phi_0(x) + \phi_1(x)\exp\mathrm{i}(ky - \omega t) + \mathrm{c.c.}\,,$$

where $\phi_0(x)$ is the equilibrium potential now (without loss of generality, the perturbations are assumed to depend on the perpendicular variable y only). Denoting the unperturbed equilibrium distribution by

$$F_{0e}(x,\boldsymbol{v}) = (2\pi)^{-1/2} f_e(x,v_x)\exp(E_\perp)$$

where f_e is given by eq. (4), and defining G_e by

$$G_e(x,\boldsymbol{v}) \equiv \left(\frac{\partial}{\partial E_\parallel} + \frac{kv_y}{\hat{\omega}}\,\frac{\partial}{\partial E_\perp}\right)F_{0e}\,,$$

where

$$E_\parallel = v_x^2/2 - \phi_0(x)\,, \quad E_\perp = v_y^2/2 \quad \text{and} \quad \hat{\omega} = \omega - kv_y\,,$$

one finds [36, 37, 29] for the linearized spectral problem

$$K(x,\omega,k)\phi_1(x) \equiv [\partial_x^2 - k^2 + V''(\phi_0)]\phi_1(x) + \mathrm{i}\int \mathrm{d}^2v\;\hat{\omega}G_e \exp[\mathrm{i}\hat{\omega}\tau(x,\sigma)]$$

$$\times \int_{x_0}^{x} \mathrm{d}x'\, \frac{\exp[-\mathrm{i}\hat{\omega}\tau(x',\sigma)]}{v_x(x',E_\parallel,\sigma)}\,\phi_1(x') = 0\,.$$

Herein, $v_x(x,E_\parallel,\sigma) \equiv \sigma\sqrt{2[E_\parallel + \phi_0]}$ is the parallel velocity expressed by the constants of motions, and $\tau(x,\sigma) \equiv \int_{x_0}^{x} \mathrm{d}x'\, v_x^{-1}(x',E_\parallel,\sigma)$ is the transit time, i.e., the time a particle needs to move from x_0 to x. $V(\phi_0)$ is, as before, the classical potential. This equation involves tedious integrations along the unperturbed orbits and, as a consequence, has not yet been solved, neither analytically nor numerically.

Other stability methods are not applicable either. Energy principles or, more generally, Lyapunov stability theorems are not available for this type of kinetic equilibria. There do exist necessary and sufficient stability criteria [38–43] which, however, demand equilibrium distributions that depend on the isotropic energy, $E = E_\parallel + E_\perp$, and are strictly monotonically decreasing in this variable, two conditions which apparently are not met here.

Some progress has been made in ref. [44]. By extension of the work of Schindler et al. [43], the aperiodic nature of eventually existing unstable perturbations could be shown within the class of square integrable functions. However, the question whether there are indeed unstable perturbations, could be approved in the so-called fluid limit only [5, 34–37, 44], the justification of which is, however, lacking. This latter approximation consists in a truncation of an infinite series of differential operators into which the nonlocal spectral operator K can be converted.

A second difficulty in the treatment of the stability problem is the lack of self-adjointness of K for the case of propagating structures and of perturbations which are turned on adiabatically. For more details, see ref. [44]. Attention should also be drawn to a series of papers published by Lewis, Seyler and Symon [45–47].

Further efforts are definitely needed to decide explicitly the stability of localized phase space structures.

4. The ion hole and related phase space structures

4.1. Analytical properties of ion holes

In view of the previous sections we can easily infer on the existence of IHs [48, 21].

The Vlasov–Poisson system

$$\left[\partial_t + \boldsymbol{v}\cdot\nabla + \frac{q_s}{m_s}\boldsymbol{E}\cdot\partial_{\boldsymbol{v}}\right] f_s = 0\,, \qquad s = \mathrm{e,i}\,, \tag{19}$$

$$\nabla\cdot\boldsymbol{E} = 4\pi\sum_{s=\mathrm{i,e}} q_s \int \mathrm{d}^3v\, f_s\,, \qquad \boldsymbol{E} = -\nabla\phi\,, \tag{20}$$

is seen to be essentially invariant with respect to the transformation $(\phi, -e) \rightarrow (-\phi, +e)$. Hence, a hump in ϕ describing an EH goes over to a negative dip representing an IH, the characteristic features of which are shown in fig. 11. There is not a one-to-one correspondence between both holes, because the second, 'passive' species is treated differently in both cases. For an EH the ion density was assumed to be constant, for an IH we may generally assume that n_e is of Boltmann-type, $n_e = \exp(\phi)$. This introduces some characteristic changes in the properties of IHs. The most remarkable ones are

$$\text{(i)} \quad T_e/T_i > 3.5\,, \tag{21}$$

$$\text{(ii)} \quad |e\phi_{\min}|/T_e \le 1\,, \tag{22}$$

which must be valid for ion holes to exist. Condition (i) says that the electrons must be sufficiently hot which is the same requirement as for ion acoustic waves. The second condition implies that the depth of the potential energy is limited by the electron thermal energy.

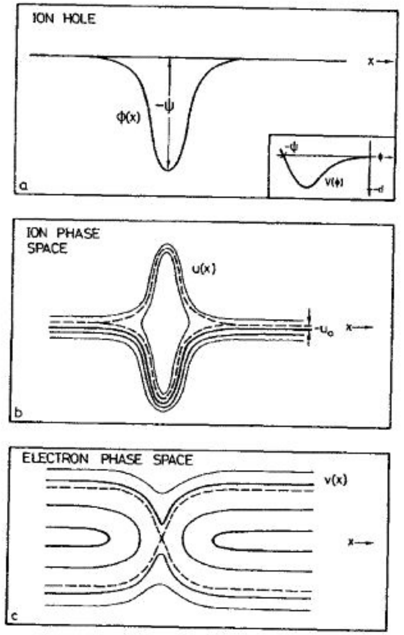

Fig. 11. Three diagrams characterizing an IH. After [11].

Figure 12 shows the existence diagram for IHs. We have the same situation as before, i.e., IHs exist only when the trapped ion parameter is sufficiently negative. The ion distribution is of vortex-type. The IH velocity u_0, which this time is normalized by the ion temperature v_{ti}, is of order unity and, hence, very small. IHs are more or less standing structures.

For small amplitudes we get [48]

$$u_0 = 1.305\left[1 + \frac{T_i}{T_e} - \tfrac{16}{15} b\psi^{1/2}\right], \tag{23}$$

where b follows from (9) if v_0, β, are replaced by u_0, α. Equation (23) tells us that IHs are based on the *slow* ion acoustic branch [23, 24, 48].

Again an asymmetric version of a hole exists [11, 33, 49, 50] shown in fig. 13 together with the corresponding phase space plots.

The quasi-particle interpretation of an IH can be summarized by

$$-\phi'' + (1 + \lambda^{-2})\phi = \int_t \tilde{f}\,\mathrm{d}v \equiv \rho_{\mathrm{IH}} < 0\,, \tag{24}$$

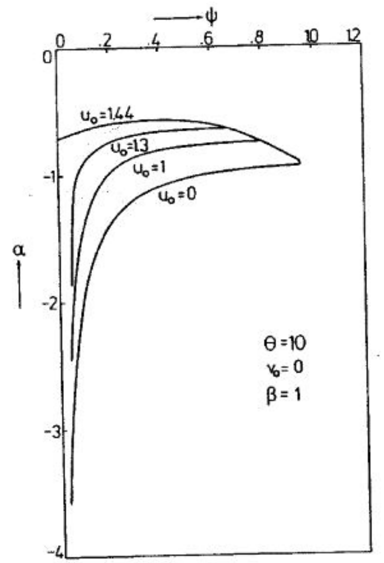

Fig. 12. The existence diagram of ion holes for $\theta = T_e/T_i = 10$; α represents the trapped ion parameter, and $e\phi_{\min}/T_e = -\psi$. After [21].

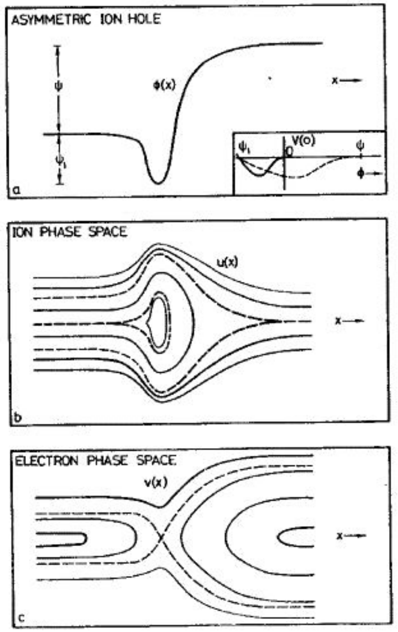

Fig. 13. Three diagrams characterizing an asymmetric IH. After [11].

where $\tilde{f}$ and λ are defined in analogy to (16). The unity stems from the linear term in the Taylor expansion of $\exp(\phi)$. An IH can be understood as a negatively charged cloud embedded in an ion fluid, having a negative charge and a negative mass: $Q = M = \int \rho_{\mathrm{IH}}\,\mathrm{d}x < 0$.

4.2. *Two related systems*

Two other important collisionless systems should be mentioned which develop similar structures. The first one are stellar systems, the second one particle accelerators and storage rings.

The distribution of stars is governed by the Vlasov equation in which the gravitational field $\boldsymbol{E}$ follows self-consistently from Poisson's equations

$$[\partial_t + \boldsymbol{v}\cdot\nabla + \boldsymbol{E}\cdot\partial_v]f = 0\,, \tag{25}$$

$$\nabla\cdot\boldsymbol{E} = -4\pi G\int \mathrm{d}^3v\, f\,. \tag{26}$$

G being the universal gravitation constant. Comparing (25), (26), with (19), (20), resp. (24), for ion holes, we conclude that due to the minus sign in (26), we get a bunch of trapped particles rather than a deficiency. Hence, an IH is the pendant to a galaxy where only trapped particles are present (no shielding!). The Coulomb repulsion of ions in the former corresponds to the gravitational attraction of stars in the latter; the two-stream instability is replaced by the Jeans instability [51].

The circulating particles in a storage ring or particle accelerator represent another Vlasov system [52],

$$\partial_t f + v\,\partial_x f + \frac{F}{m}\,\partial_v f = 0\,, \tag{27}$$

where x is the longitudinal coordinate along the tube, $v(\equiv v_x)$ is related to the radius, and F is the electric force determined self-consistently. The effective mass of a particle is denoted by m. It is determined by the radial variation of the guide magnetic field and turns out to be proportional to $\mathrm{d}\omega/\mathrm{d}E$, where ω is the circulation frequency, and E the relativistic longitudinal energy. The latter expression can be positive or negative depending on whether E is below or above a critical value E_c, called the transition energy. When m is positive ($E < E_c$), a hole can be observed in the charged particle beam [53]. It is caused by two-stream instabilities. If, on the other hand, m is negative ($E > E_c$), the beam is observed to clump, the density becomes modulated forming clusters. The underlying instability is called the negative-mass-instability. The repulsive Coulomb force acts on particles with negative mass in the same manner as the attractive force acts on positive masses. Hence, to a positive-mass system with a deficiency of particles there corresponds a negative-mass system with an excess of particles. This duality principle is sometimes called mass-conjugation-theorem [54]. Saturn's rings and holes in the rings constitute another dual system [51].

5. Double layers

5.1. *General remarks on double layers*

The by far most important and most known structure we are discussing in this review, is the DL for which numerous original and review papers have been presented dealing with experimental [2, 55–94],

Fig. 14. The electrical and classical potential of a DL. The dotted line represents the dipole-like total charge density.

theoretical [9, 11, 28, 29, 33, 48, 95–110], and numerical [111–120] investigations. A DL is defined as a monotonic transition of the electric potential connecting smoothly two differently biased plasmas. This is achieved by a dipole-like charge distribution, as shown in fig. 14, where the electrical and classical potential are drawn together with the total charge density, $\delta n := n_i - n_e$, for a single ionized plasma. According to Poisson's equation, $\phi'' = -\delta n$, a positively charged layer gives rise to a region of negative curvature of ϕ and vice versa, and hence, two oppositely charged layers are needed to build-up the DL structure. Again, trapped particles must be involved, as can be seen by a simple counter argument. Namely, if only streaming (i.e. nonreflected) particles would be present, the spatial constancy of each current, $n_j u_j = \text{const.}$, $j = i,e$, would imply that the required asymptotic charge neutrality cannot hold simultaneously on both sides of the DL structure. Due to the different acceleration each species experiences in the DL, the densities are affected differently: The density of ions (electrons) injected from the high (low) potential side decreases (increases) with decreasing potential. Therefore, if the densities are equal on one side, they have to differ on the other side, and charge neutrality cannot be established there.

Theoretically, the search for DL solutions (resp. electrostatic shock solutions, see later) was lead astray for more than two decades by the exclusive use of the BGK method. Even Knorr and Goertz [104], who presented the first working picture of the so-called strong DL, had to pay a price for the application of the BGK method insofar as their solution is an approximative, iterative one, only.

The more straightforward way of getting DL solutions having smooth distributions, is again the 'alternative method' which was used by Bujarbarua and the author [95] in connection with strong DLs, and by Perkins and Sun [105] with respect to a specific kind of SIADLs (see later).

Finally, in the description of theoretical approaches, also the work of Block [9, 96–98] being based on macroscopic equations, should not be forgotten, in which many of the results obtained later by kinetic theories have been anticipated. His review articles as well as that of Torvén [55, 57], Levine and Crawford [65] and Sato [58] assimilate the state-of-the-art of DL generation and observation in the seventies, and contain material about DLs that includes magnetic and geometry effects, laboratory specialities, such as wall and ionization effects as well as observations of fluctuations. We will come back to that in a more detailed description of the experiments.

5.2. The strong double layer

As in the discussion of solitons or solitary wave solutions, one has to specify on which branch of normal modes DL solutions are looked for. DLs based on different branches, turn out to differ in the phase space topology. A DL in its simplest form, ignoring magnetic field [108–110], or geometry [120] effects, and taking into account a minimum set of distributions only, is given by one of the three structures shown in fig. 15.

The first column represents the proper DL called the *strong or beam-type DL*. Its main characteristic

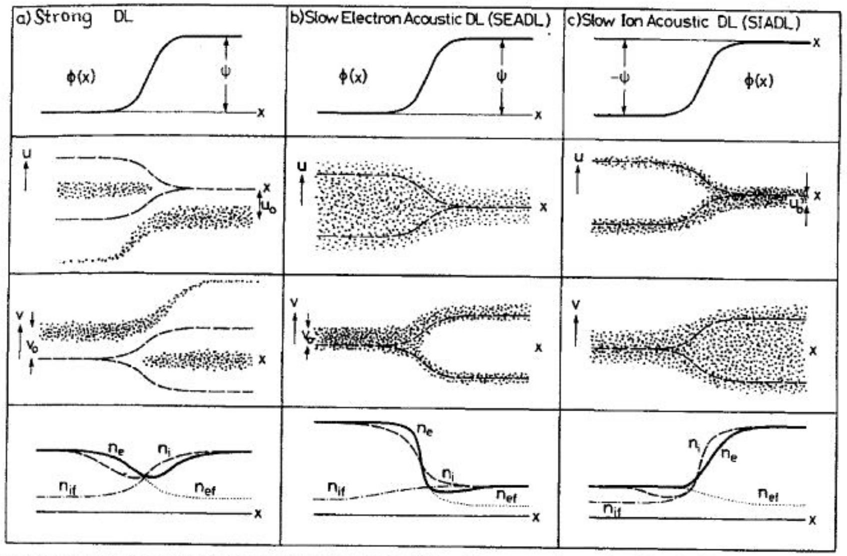

Fig. 15. Three types of DLs. The second (third) row represents the ion (electron) phase space, whereas the various densities are plotted in the fourth row. After [29].

features [9, 11, 33, 95, 104] are listed as follows:

– there are four distributions involved which are separated in phase space;

– each drifting species has to enter the DL region with finite velocity v_0 (resp. u_0) called the *Bohm-criterion* for DLs: $v_0 \geq v_{0c}(|u_0| \geq |u_{0c}|)$ for electrons (ions), where v_{0c} (u_{0c}) is of order unity;

– the trapped species described by the trapped particle parameters α and β must be sufficiently present: $(\alpha, \beta) > (\alpha_c, \beta_c)$ which are $O(1)$;

– it exists in the finite amplitude regime only, $\psi > \psi_c = O(1)$, (i.e. there is no corresponding linear mode);

– the Langmuir 'condition' [96], which relates the electron and the ion current in a definite way $(j_e/j_i = \sqrt{m_i/m_e})$, is *not* a necessary requisite but seems to be preferred by stability arguments;

– this type of DL is usually subject to various two-stream instabilities yielding to hf (lf) turbulence on the high (low) potential side which superimposes, usually in a nondestructive manner, the coherent DL structure.

These items should now be commented on in some more detail.

In deriving these results, modified Maxwellians like that in eq. (4) were used [95]. A representative distribution for the electrons is shown in fig. 16 in the quasi-phase space (i.e., x is replaced by ϕ which in the DL case is a one-to-one relation). One notices the trapped electron component at the high-potential side symmetrically orientated around $v = 0$, and the well-separated drifting component of electrons which enter the DL region at the low potential side $\phi = 0$ with finite drift velocity. In this case ($\theta = f = 1$, $\alpha = \beta = 1$, $|u_0| = v_0 = 3.59$), the normalized ion distribution $f_i(u, \phi)$ is simply obtained by reversing the

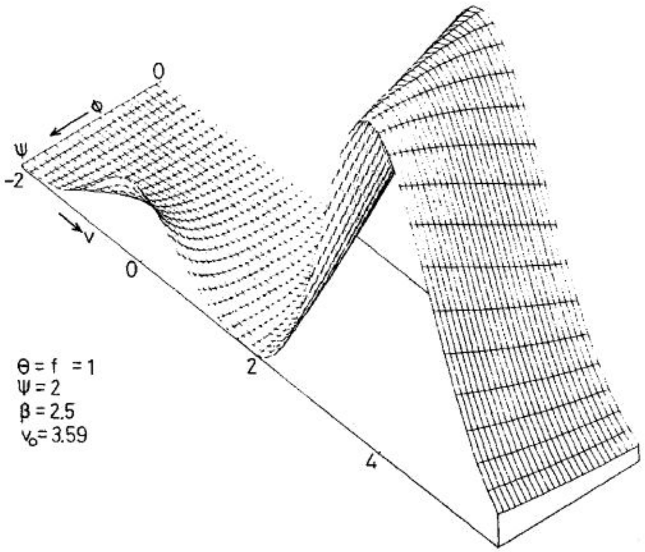

Fig. 16. The electron distribution of a strong DL. After [95].

sign of the velocity $u = -v$, and by replacing ϕ by $\psi - \phi$, where ψ is the DL amplitude. $\theta = T_e/T_i$ is the temperature ratio of the drifting particles at the entrance, and f (not to be mixed up with a distribution function) reflects the ratio of the current densities: $j_e/j_i = (m_i/m_e)^{1/2}f$. In the present case, the Langmuir condition is assumed ($f = 1$) leading to the equality of the normalized drift velocities $|u_0| = v_0 = 3.59$ which exceed unity. This latter property is termed Bohm criterion, since a similar condition holds true for the ions entering a negative wall sheath. It is worthwhile to note that the most general expressions for the two Bohm criteria are given by $V''(0) < 0$ for the electrons, and by $V''(\psi) < 0$ for the ions, respectively, where $V(\phi)$ is the classical potential. (The Langmuir condition can be derived macroscopically in the following manner: Neglecting temperature and trapped particle effects, and assuming that the initial drift is negligible, one gets, using energy conservation, $e\phi_0 = m_e n_e v_{De}^2/2 = m_i n_i v_{Di}^2/2$, where ϕ_0 is the potential jump, and v_{Dj} (j = i,e) is the drift velocity after acceleration. If the densities are equal, it follows $j_e/j_i = en_e v_{De}/en_i v_{Di} = (m_i/m_e)^{1/2}$). These investigations emphasize in some sense the preferential role played by the Langmuir condition since presumably a minimum amount of fluctuations is involved. Although the general stability of DLs is an unsolved problem, as discussed at the end of section 3, it seems that a minimization of the larger of both drift velocities can lead to the lowest fluctuation level. Since the drift velocity is dependent on the amplitude ψ there might be the possibility that by lowering ψ the drift velocities will fall below the threshold of two-stream instabilities [104]. However, this lowering of ψ has its limits (see item 4), and it is not yet clear whether truthful strong DL solutions can be found that are stable at least in the asymptotic, homogeneous region. The investigations of ref. [99] indicate instability, an observation which seems to be in accord with the experiments. We terminate the analytical

description of the strong DL by mentioning that it is this type of DL which is usually excited in a strongly driven plasma, and which gives rise to the observed strong particle acceleration.

We now turn to the second and third type of DLs.

5.3. *The double layers based on the slow acoustic branches*

The second column in fig. 15 shows the so-called Slow Electron Acoustic Double Layer (SEADL) [106, 29]. It is based, as the name says, on the slow electron acoustic mode and does have a small amplitude limit. Its most striking feature is the "tuning fork" configuration of the particle distribution in the electron phase space. Favourable conditions for its existence are hot ions (including $T_i = \infty$ which represents immobile ions [29]), and a less dense plasma on the high potential side. As seen immediately, a SEADL is a descendant of EHs. Since both demand the same prevailing conditions, it is not surprising that they can be generated simultaneously in a plasma (see section 2 and figs. 3 and 4).

Like in the IH case a simple exchange of the species leads to the Slow Ion Acoustic Double Layer (SIADL), third column in fig. 15. The tuning fork configuration is now found in the ion phase space. There is an upper limit of the amplitude $\psi \lesssim \psi_c$. It is this DL structure which can exist under current-free conditions [105]. As in the IH case, the electrons have to be sufficiently hot.

The analytical description of a SEADL can be summarized as follows [106, 29]. A similar argumentation holds for SIADLs. For small amplitudes the densities are given by a half-power expansion in ϕ (see eq. (8)). Poisson's equation then reads

$$\phi'' = A\phi + B_1\phi^{3/2} + B_2[\psi^{3/2} - (\psi - \phi)^{3/2}] + C_1\phi^2 + C_2\phi\psi + O(\phi^{5/2}) \equiv -\partial V/\partial\phi \,, \tag{28}$$

where the coefficients A, B_1, etc. generally depend on α, β, u_0, v_0 and θ. (Note that the $O(\phi^{1/2})$ term is missing in (28) due to the assumed continuity of the distribution function at the separatrix.) By integration, the classical potential becomes

$$-V(\phi) = \tfrac{1}{2}A\phi^2 + \tfrac{2}{5}B_1\phi^{5/2} + B_2\{\phi\psi^{3/2} - \tfrac{2}{5}[\psi^{5/2} - (\psi - \phi)^{5/2}]\} + \tfrac{1}{3}C_1\phi^3 + \tfrac{1}{2}C_2\phi^2\psi \,. \tag{29}$$

In the present case the two conditions, $V(\psi) = 0$, and $V'(\psi) = 0$, expressing the vanishing of the electric field and of the charge density on the high potential side, respectively, have to be satisfied simultaneously.

$$0 = A + \tfrac{2}{5}[2B_1 + 3B_2]\psi^{1/2} + [\tfrac{2}{3}C_1 + C_2]\psi \,. \tag{30a}$$

$$0 = A + (B_1 + B_2)\psi^{1/2} + (C_1 + C_2)\psi \,, \tag{30b}$$

which can be resolved for B_1 and A, resulting in

$$A = -2B_2\psi^{1/2} + \tfrac{2}{3}C_1\psi - C_2\psi \,, \tag{31a}$$

$$B_1 = B_2 - \tfrac{5}{3}C_1\psi^{1/2} \,. \tag{31b}$$

An evaluation of the nonlinear dispersion relation (31a) again shows that the slow electron acoustic branch comes into play. Inserting (31a,b) into (29) yields

$$-V(\phi) = B_2\{\psi^{1/2}\phi(\psi - \phi) - \tfrac{2}{5}[\psi^{5/2} - \phi^{5/2} - (\psi - \phi)^{5/2}]\} + \tfrac{1}{3}C_1\phi^2[\psi^{1/2} - \phi^{1/2}]^2 \,, \tag{32}$$

which represents the most general expression for the classical potential of a small amplitude SEADL. A further simplification is obtained, if $|B_2| \ll |C_1|\psi^{1/2}$ holds, in which case one has

$$-V(\phi) = \tfrac{1}{3}C_1\phi^2[\psi^{1/2} - \phi^{1/2}]^2 . \tag{33}$$

Performing the last step in deriving DL solutions, namely solving the 'energy equation' (6), one gets

$$\phi(x) = \tfrac{1}{4}\psi(1 + \tanh \kappa x)^2 , \qquad \kappa = (C_1\psi/24)^{1/2} , \tag{34}$$

an expression which was first obtained by Kim [106].

5.4. *The double layer based on the ordinary ion acoustic branch*

Torvén [121], and more recently Goswami and Bujarbarua [122], investigated the possibility of DL solutions on the ordinary ion acoustic branch. In this case a macroscopic description suffices, and the half-power terms are missing in eq. (28). Including the next order, ϕ^3 in that equation, one gets DL solutions of the following kind:

$$-V(\phi) = \tfrac{1}{4}B\phi^2(\psi - \phi)^2 , \tag{35}$$

$$\phi(x) = \tfrac{1}{2}\psi(1 - \tanh \kappa x) , \qquad \kappa = \sqrt{\tfrac{1}{8}B\psi} , \tag{36}$$

provided that the constant B is positive. This latter condition is not satisfied if the electrons simply obey an isothermal electron equation of state ($n_e \sim \exp \phi$), in which case only ion acoustic solitons exist. The existence of Ion Acoustic DLs (IADL) requires, therefore, deviations from isothermality as it is, e.g., given by a two-temperature electron distribution function. Both compressive and rarefactive DLs can be obtained in the latter case, dependent on the temperature and density ratio of hot and cold electrons. A compressive (rarefactive) DL is defined by a decrease (increase) of the density and of the potential in the direction of propagation and is generally subsonic (supersonic). The corresponding solitons [123] in contrast, require reversed conditions and are both supersonic.

I close this section with two remarks. Firstly, DLs are, in general, as abundant as solitons, since by appropriate modifications a soliton solution can be cast into a DL solution [124]. Secondly, the DLs based on the slow ion branch are a manifestation of an old idea of Sagdeev and others [125], concerning the necessity of reflected ions and represent the first self-consistent solutions of what was called earlier a laminar collisionless electrostatic shock [126].

6. Experiments on holes and double layers

Numerical simulations and laboratory experiments performed in the last decade have revealed many aspects about the existence and the possible generation mechanism of phase space structures. Only a few of them shall be mentioned here. Since we have already treated in some length EHs we will concentrate here on IHs and DLs.

6.1. *Experiments on symmetric and asymmetric ion holes*

One of the first numerical observations of phase space structures which are related to (asymmetric) IHs and SIADLs was made by Sakanaka [127], simulating the interaction of an ion beam injected into a plasma, a process which was studied further in refs. [128–130]. These simulations reveal that the electric field at the ion beam front is distributed such that the leading beam particles will be accelerated whereas the succeeding particles will be slowed down. This gives rise to a region exhibiting a tuning-fork configuration corresponding to a SIADL, followed by a quiet heated region where the beam has merged into the background plasma, and by a region with one or more IHs which accomplish the merging of he two-stream unstable beam. Experimentally, IHs have first been observed by Pécseli et al. [129] in a double-plasma device by increasing suddenly the energy of an already existing ion beam. This modification in handling the ion beam seems not to be essential as a wave pattern similar to that of Sakanaka's numerical solution arises. Experiments on similar lines have been carried out in refs. [130, 131]. Concerning the interpretation of this a different opinion was expressed by Pierre et al. [131] who argue that the structure in the rear of the shock may also be due to a slow wave pattern, i.e., a nonlinear solitary slow wave in the *stable* region of the ion–ion two-system situation.

IHs and its asymmetric version were, furthermore, seen in simulations treating current-carrying plasmas [132–136]. An electron drift with velocity as low as $v_d \approx 0.5 v_{te}$ [134, 135], the lower limit depending on details of the numerical condition, gives rise via the ion acoustic two-stream instability to the local excitation of an IH which due to electron reflection on the negative potential dip becomes asymmetric. Details of this scenario were described in ref. [33]. An example of this type of simulations is given in fig. 17a,b, showing a plane asymmetric IH in two dimensions (fig. 17a), the growth of which was, in this case, triggered by an imposed density depression, and a plane SIADL (fig. 17b), respectively. Both turn out to be weakly transversally unstable [29, 36, 37]. In the laboratory, asymmetric IHs and SIADLs have been generated by Fujita et al. [67], when an applied positive potential was switched-off suddenly, by Chan et al. [137], when electrons were abruptly injected, and by Sekar and Saxena [68], when a steadily injected e-beam was modulated by a negative step potential.

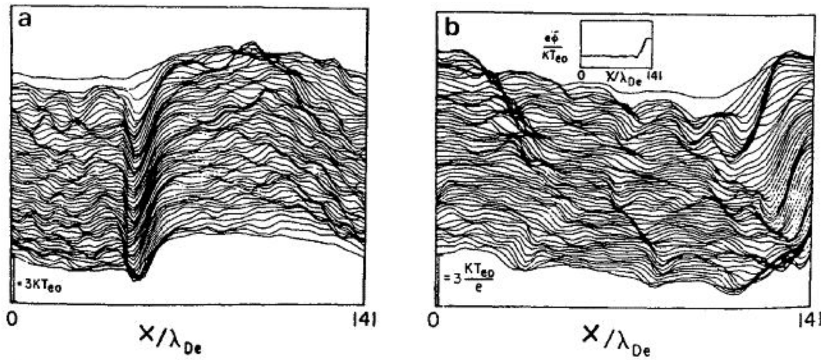

Fig. 17a,b. The electrostatic potential in two dimensions obtained in simulations of current-carrying plasmas. Figure 17a shows an asymmetric IH and fig. 17b a SIADL. After [132].

6.2. Experiments on non-propagating double layers – Current disruption by electron reflection

An important observation made in these simulations and real experiments are the strong current reduction at the moment when the asymmetric IH becomes excited, and the possible transition of the latter into a SIADL (see fig. 17b) or even into a strong DL [136]. This observation seems to play a key role in the understanding of more involved laboratory experiments in which a negative resistance and an associated current disruption are found. I should mention here and investigation of Lutsenko et al. [60, 138] in a straight high current, low pressure discharge, who found (i) the formation of a DL at a point where the plasma density is depressed, (ii) the generation of intense beams of electrons and ions in the space-charge region, (iii) the appearance of a high resistance which was not 'anomalous', and (iv) intense microwave emission at the instant when the high-current beam is formed. The authors suspect that this 'volume-mechanism' limiting the current may as well be the cause for the current disruptions observed in high-current toroidal discharges or in a plasma focus. The intimate relation between the current disruption and the negative potential dip (NPD) in the formation process of DLs will be investigated further after the following brief survey on DL experiments.

DLs – most of them will be strong DLs although a serious investigation discriminating the different types of DLs has not yet been performed – can be found under various circumstances.

DLs, to start with, appear in front of the hot cathode in a low pressure discharge tube like that of Lutsenko et al. [60, 138] when the electron emission is sufficient to maintain the discharge current. Nonpropagating DLs in the axially uniform positive column have been found this way by several authors [71–74, 65]. In these experiments extremely large discharge currents were involved in which the electron drift velocity was close to the electron thermal velocity. DLs are also formed in discharge tubes with varying cross section at the place of an abrupt change of the tube diameter [59, 75–77], in which case the continuity of the current through the discharge tube is maintained by the DL acceleration of the particles.

Stationary striations occurring in discharge tubes [78] are a a modified form of DLs in which ionization and particle losses to the wall play a crucial role.

The injection of an electron or an ion beam into a plasma in double and triple devices is a further possibility to excite DLs. In a modified double plasma device, Wong and co-workers [79, 61] found DLs which were generated by ionization due to the beam. If the beam was sufficiently strong, the DL was propagating with a speed close to the ion acoustic speed. It disappeared when it arrived at the opposite end, and it reappeared at the place where the beam was injected giving rise to a repetitive phenomenon, to be discussed in more detail in the next section. In the case of a triple-plasma machine [80–82] where two plasmas at different bias were produced independently, an electron beam was injected from the low potential side into the target plasma, creating a strong DL. The generation of a DL due to the injection of an ion beam along a converging magnetic field (see also later) was reported in refs. [83–85].

A DL can further be generated by applying a large positive potential to the anode or to a metal electrode immersed in a weakly ionized plasma [63, 64]. When the potential exceeds the ionization potential of the gas, an additional discharge occurs around the electrode, and the sheath detaches and converts into a DL providing the transition between the main plasma and the newly generated plasma. These experiments, among many others, are especially appropriate to prove the co-existence of DLs and of fluctuations. Torvén and Lindberg [64] measured Langmuir oscillations in the high potential region which have a phase velocity typically 10–20% smaller than the electron beam velocity. The spatial increase compares favourably with the linear growth rate of the beam–plasma interaction. Low frequency fluctuations ($\omega < \omega_{pi}$) with a peak at the center of the DL have also been observed. Their origin is not yet clear, but it seems that they are partly caused by a motion back and forth of the DL with an amplitude of the order of the layer thickness.

DLs, moreover, are created at the boundary of two independently produced plasmas if a potential difference is applied between their sources [86, 87]. In this case, DL amplitudes up to $\psi = e\phi_{max}/T_e = 2.5 \times 10^3$ could be found. The DL position was changeable by the variation of the density ratio of the two plasmas or by varying the strength of the applied potential.

An interesting observation has been made by Hollenstein et al. [62] as well as by Chan and Hershkowitz [88]. Using a long triple plasma device, multiple, staircase-like DLs were observed when the bias voltage on the high potential boundary grid was sufficiently high. The electron drift velocity was not much below the electron thermal speed, and an ion hole scenario like that sketched above, supplemented perhaps by ionization and fluctuation effects, may be applicable. These experiments are of special interest because they yield information about the intermittence type small scale structures observed in the aurora ionosphere [90, 91].

A survey on experimental DLs would not be complete without saying something about the U-shaped DLs, structures that are related to the auroral DLs just mentioned. U (or V)-shaped DLs are three-dimensional potential structures in which the equipotential lines are U-shaped. They are formed when the axial current, flowing along magnetic field lines, has a finite spatial extent in the perpendicular (radial) direction. Experimental studies have been conducted by refs. [92–94, 83–85]. An interesting feature of such a DL configuration is a density duct at the low potential side (note that in one dimension, such a density behaviour is found for SIADL, fig. 15). The radial electric field in the region where the density drop takes place, gives rise to $E_r \times B$ plasma rotation which excites electrostatic ion cyclotron waves showing several azimuthal mode numbers. If, in addition, the magnetic field is converging like the earth's magnetic field, the U-shaped profile is more pronounced, and the axial potential drop is found to appear around the position of the magnetic mirror.

Our concern for the last part of this report is to learn more about the DL formation process itself, that is to say, about the intimate relation between the current disruption and the negative potential dip (NPD), and about a transient phenomenon related to this.

Torvén et al. [70] experimenting with a triple plasma device, found that the spontaneous current

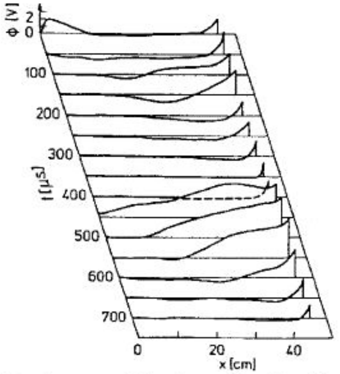

Fig. 18. Space-time history of the plasma potential in a plasma diode with positive applied potential. After [142].

disruption caused by electron reflection on a NPD, is the triggering mechanism for a DL. (We note in parenthesis that in a recent linear turbulent heating experiment of Inuzuka et al. [139], strong DLs have been recorded coincident with a NPD and a low value of the heating current.) However, as described also by other authors [140–144], the NPD need not be an IH. It was stretched in this experiment and, thus it was more quasi-neutral-like. This controlling function of the NPD could be studied in more detail, if in addition to the applied voltage U_0 an inductance L was introduced in the external circuit of the device. If U_0 was above a critical value, periodic current disruptions correlated with DLs were seen. The inductive over-voltage produced by the disrupted current was several hundreds of volts over the plasma ($T_e \approx 8$ eV, $n_e \approx 10^{16}\ \mathrm{m}^{-3}$), and was found to be concentrated at the DL that maintained as long as the circuit was able to produce the over-voltage. The energy stored initially as magnetic energy in the inductance, was transferred to particle energy in the DL. In this series of experiments the DL was periodically switched on and off, and was more or less standing. There exist, however, situations as already mentioned, in which the DL is found to be in motion giving rise to a periodic phenomenon called potential relaxation oscillations, the concern of our last paragraph.

7. Potential relaxation oscillations (PRO)

7.1. *Propagating double layers in finite length plasmas*

The oscillatory phenomenon to be discussed here has been observed in many experiments [142, 145–153], in which the finite length of the system is of crucial importance. A typical arrangement is a plasma diode (single-ended Q-machine) consisting of a grounded plasma source and a positively biased collector plate terminating the plasma. If the applied voltage is sufficiently large, low frequency oscillations of typically 1–10 kHz are seen. Figure 18 shows the space-time behaviour of the electrostatic potential of Iizuka et al. [142], the evolution of which being initiated by an expanding plasma. One readily recognizes two main phases within one cycle of about 400 μs. The first one is characterized by a strong propagating DL which is accompanied by a broad NPD on its low potential side, the second one by a fast increase of ϕ in the whole column shortly after the DL has reached the anode. The collected target current is sawtooth-like in time with the decaying phase during the presence of the DL. The oscillation period is correlated with the transit time of the DL which moves with approximately 2–3 times the ion sound speed. This propagation velocity is determined by the speed of the expanding plasma on the low potential side, enabling the DL to satisfy the two Bohm criteria [99]. The second, more rapid phase, is due to an instability of the electron-rich sheath which is formed at the anode after the arrival of the DL. The electrons in this sheath and in the column are quickly lost, and the resultant positive space charge gives rise to an increase of the space potential because the ions cannot respond on this fast time scale.

Burger [154] who investigated this phenomenon in connection with thermionic converters as well as Braithwaite and Allen [155] argue that there exists a second d.c. state which is adapted by the system after a rearrangement of the electrons, occurring on the fast electron time scale during which the ions "are virtually frozen".

The formation process of the DL itself could be resolved numerically by refs. [152, 156] in a plasma diode simulation in which the particles are emitted with equal temperatures. It is found that the DL is preceded by an EH which propagates with v_{te} through the plasma and which provides after its arrival at the collector the change in phase space topology necessary for the build-up of the DL. The EH, the original vortex-like structure of which is shifted in phase space and is subsequently cut by the anode, leaves behind low energy electrons on the anode side and accelerated electrons which on the cathode side join continuously with fresh low velocity electrons provided by the source. The former constitute the trapped electrons, the latter the free streaming (untrapped) electrons in the DL. The ions, on the other hand, are accelerated by the EH hump potential in such a way that a high energy component moving towards the cathode survives, forming the free ions in the DL configuration. The trapped low velocity ions are produced by the source.

In other words, in this situation the EH takes over the role of the IH to 'trigger' a strong DL in a bounded plasma by a mechanism which is quite different from the IH generation mechanism (see also [157, 158]). The NPD is then merely a result of the ambipolar expansion of the plasma provided by the source into the tenuous plasma giving rise to a small, time-varying modification of the DL profile. The expanding electrons diffuse ahead of the ions and produce the NPD acting as a barrier (virtual cathode) for low energy electrons.

The oscillation frequency of the PRO turns out to be independent of the applied voltage [152]. Thus this phenomenon bears many characteristic features of the constant frequency oscillations noticed by Enriques et al. [146] and investigated by Allen and co-workers [147]. They were observed always when the d.c. applied voltage was roughly equal to that required to produce d.c. current saturation. The d.c. current–voltage characteristics in the presence of oscillations has a negative slope corresponding to a differential negative resistivity provided that the electron density is high enough. Further references in which a correlation between DL generation and negative resistance was pointed out, are given by [65, 159–162]. Knorr [163] finally discussed the negative resistance and the associated hysteresis effects in DL carrying plasmas with regard to Thom's (cusp) catastrophy theory [164].

7.2. *Two related phenomena*

Two comparisons should be made before closing this section. Current relaxation oscillations of similar type have been reported in solid state physics and in electro-negative gas discharges.

The first ones are the well-known Gunn oscillations [165] in semiconductors having two conduction bands with different energy minima (e.g. n-type GaAs). These oscillations are characterized by a repetitive propagation of an electric field pulse (DL) seen during the phase of current limitation and associated negative resistance. This pulse is maintained by electrons in the lower (upper) conduction band with a high (low) mobility in analogy to the free (trapped) species. If the applied d.c. field exceeds a threshold value, electrons may tunnel into the higher conduction band where, supported by collisions, the conduction drift velocity is reduced and therewith the current.

A second analogue observation has been made by Sabadil [166]. He got periodic current oscillations in the positive column of an oxygen discharge similar to the Gunn oscillations. The dissociation of O_2 by electron attachment into O^- and O leads to heavy negative ions which play the role of conduction electrons having the larger effective mass. This dissociation process requires a minimum electron energy of 4.4 eV which must be delivered by the external field. If the concentration of these negative ions N_- exceeds that of electrons N_e by a certain amount ($N_-/N_e \approx 20$), a dipole space charge layer propagating periodically from the cathode to the anode is seen, the so-called T-layer. The frequency of the oscillation decreases monotonically with increasing distance of the electrodes. T-layers could also be observed in CO_2 discharges [167].

8. Conclusions

After having gone through this article the reader will have got the impression that DLs and the related phase space structures are far from being understood completely, as there is still a gap between theory and experiment. This impression is quite correct. The origin of the difficulties in getting a better understanding lies in the apparent complexity of the problem. It is not only the variety of experimental conditions and peculiarities which renders the problem difficult. A basic difficulty is also given by the very kinetic nature of the phenomena. Distribution functions, even if one neglects magnetic field or geometry effects, may be much more complicated in reality than the ones used in the theoretical treatments, e.g., through the presence of additional streams, secondary electrons, neutral particles, deviations from asymptotic Maxwellians, etc. The kinetic nature of these objects implies, furthermore, that an access via macroscopic quantities (current, potential, density, temperature, resistivity, etc.) in both experiment and theory will not suffice to describe and to understand these sophisticated phenomena. At least the microscopic distribution functions and the fluctuation spectrum have to be taken into account in a more complete description.

Fortunately, the nature seems to be less annoying as it could be. In the last section, e.g., we have seen that several features of the potential relaxation instability can be obtained in rather simple particle simulations. This raises, of course, hopes that by appropriate modifications or relaxations of idealized conditions, e.g., the inclusion of effects of the external circuit, more involved features can be understood at least qualitatively. Another point is, that obviously the plasma does not use all of its degrees of freedom but restricts its evolution to a subspace of collective motions. A vortex in phase space is already a very restricted kind of ordered motion, and the plasma, if it is excited sufficiently strong, has the tendency to approach very quickly to such a state. Thus it seems, coming back to the introduction, that these structures possess the property of attracting deviating time-dependent solutions very similar to the approach to self-similar states of time-dependent solutions in other areas of nonlinear physics, e.g. in fluid dynamics. From this it may follow that the true time evolution of a given dynamical system can be divided into separate time intervals, each of which being governed by one or more excited nonlinear steady-state kind of structures, even if the whole system changes appreciably for a long time. The regions connecting these preferred intervals in which the dynamical evolution is more violent are, of course, out of the scope of a steady-state treatment and, therefore, globally valid descriptions are hardly to get.

In this sense the phase space structures discussed in this review are a manifestation of this reduced albeit nonlinear dynamical behaviour. It seems to be beneficial to consider them as basic elements in the description of more complex driven plasmas far away from thermodynamic equilibrium. It could be an interesting task for future investigations to find out to what extent such a reduced program will work.

In view of the novel results obtained in recent years by the common efforts of experimentalists and theoreticians, one may also think of other phenomena to be attacked on similar lines. An example are the current disruptions observed in high-current low-pressure discharges, such as tokamaks, z-pinches or plasma foci.

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