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AUTUMN COLLEGE ON PLASMA PHYSICS

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Phase Space Vortices or Holes as Key Elements in Plasma Dynamics

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Phase Space Vortices or Holes as Key Elements in Plasma Dynamics *

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1. Introduction
   standard wave concept (SWC) ; anomalous transport in linearly stable plasmas ?

2. Numerical observation of holes
   in linearly two-stream unstable and stable plasmas

3. List of recent observations of hole-like structures
   in space, nonneutral plasmas and particle beams; periodic structures in labs

4. Theory of hole-like structures ("BGK - modes")
   - nonpropagating, solitary and periodic e-holes (immob.ions)
   - traveling, solitary e-holes and further generalizations
   - energy density of hole carrying plasmas

5. Linearity vs nonlinearity
   - a comparison (analytic and numeric) ; nontopological distributions

6. Hole scenario of linearly stable , current carrying pair plasmas
   - damping phase and preparationof a seed hole
   - multiple generation and preparation of holes
   - saturation and fully developed structural turbulence
   - relaxation to a new collisionless equilibrium

7. Extensions and Applications

Extensions	Applications
-finite amplitudes	-the Risoe-experiment
-collisional effects	-the Sendai- experiment
-relativistic corrections	-the Greifswald-Kiel-experiment
-quantum corrections	-holes at FERMI-lab,CERN & BROOKHAVEN
-holes in particle accelerators	

8. Summary and conclusions
   - analogy to vortices in incompressible fluids, ideal MHD plasmas etc

* Lecture for Autumn College on Plasma Physics , ICTP, Trieste, Sept.2005.

Standard wave concept in plasma theory

Small amplitude waves in the infinitesimal amplitude limit can be predicted and described by a linearized version of the governing equations. Only for larger amplitudes nonlinear effects come into play.

always applicable?

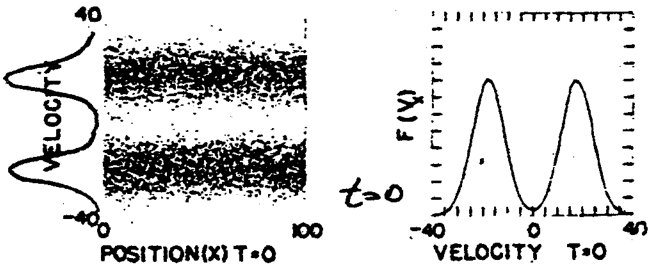


FIG. 1. Initial conditions for two equal counterstreaming beams.

in Refs. 3 and 4 with greater L, showing formation, coalescing, and long-time persistence of Bernstein, Green, and Kruskal (B.G.K.) modes[7] as seen here in Fig. 2. The initial perturbations in all runs are introduced by the random numbers used, in conjunction with a mapping function in the velocity initialization as described in Refs. 3 and 4. Each column in Fig. 2 shows two (x, v_x) phase plots, the first at field-energy-saturation time and the second at the final time of 10 plasma periods, $T_p = 2\pi\omega_p^{-1}$, followed by

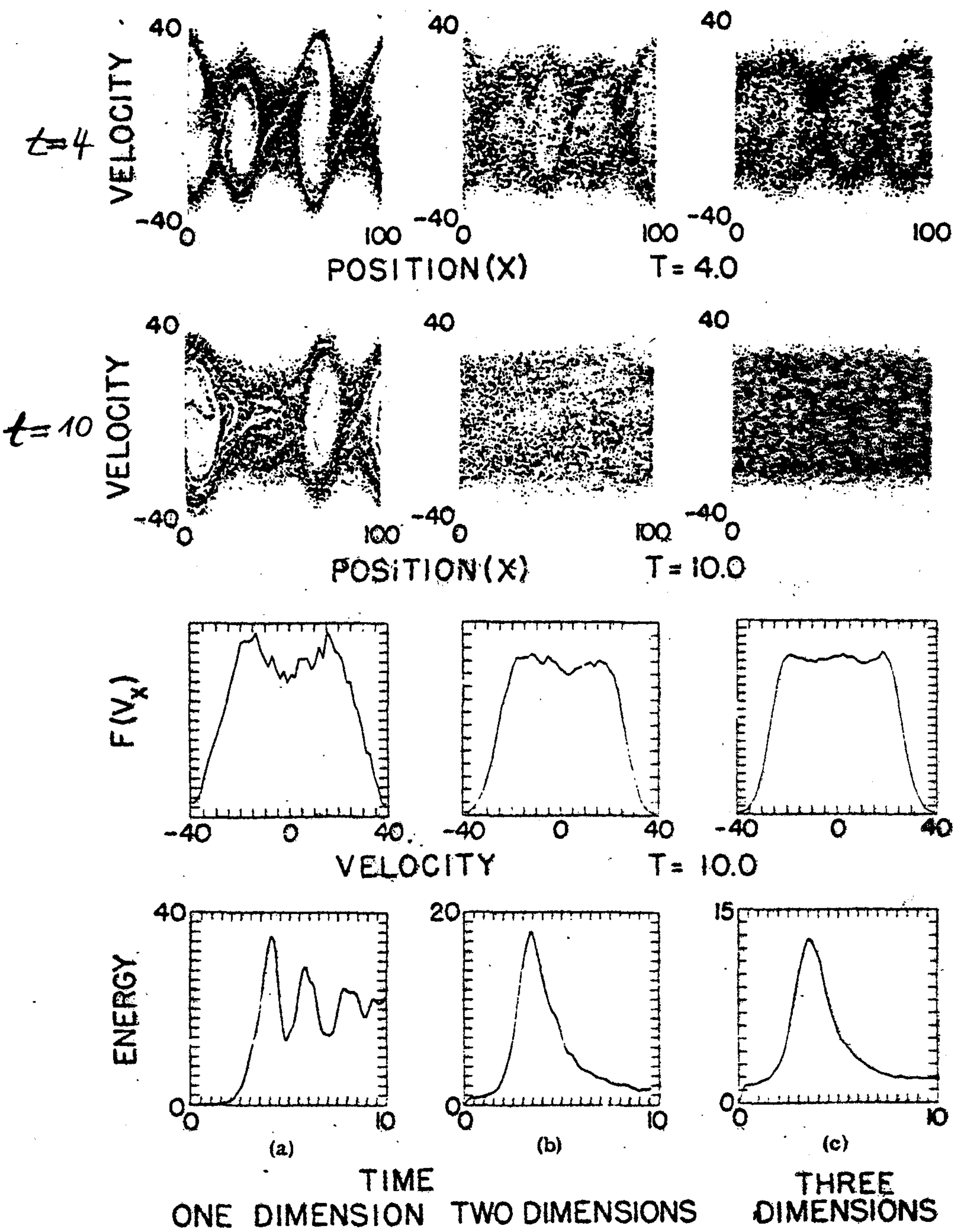
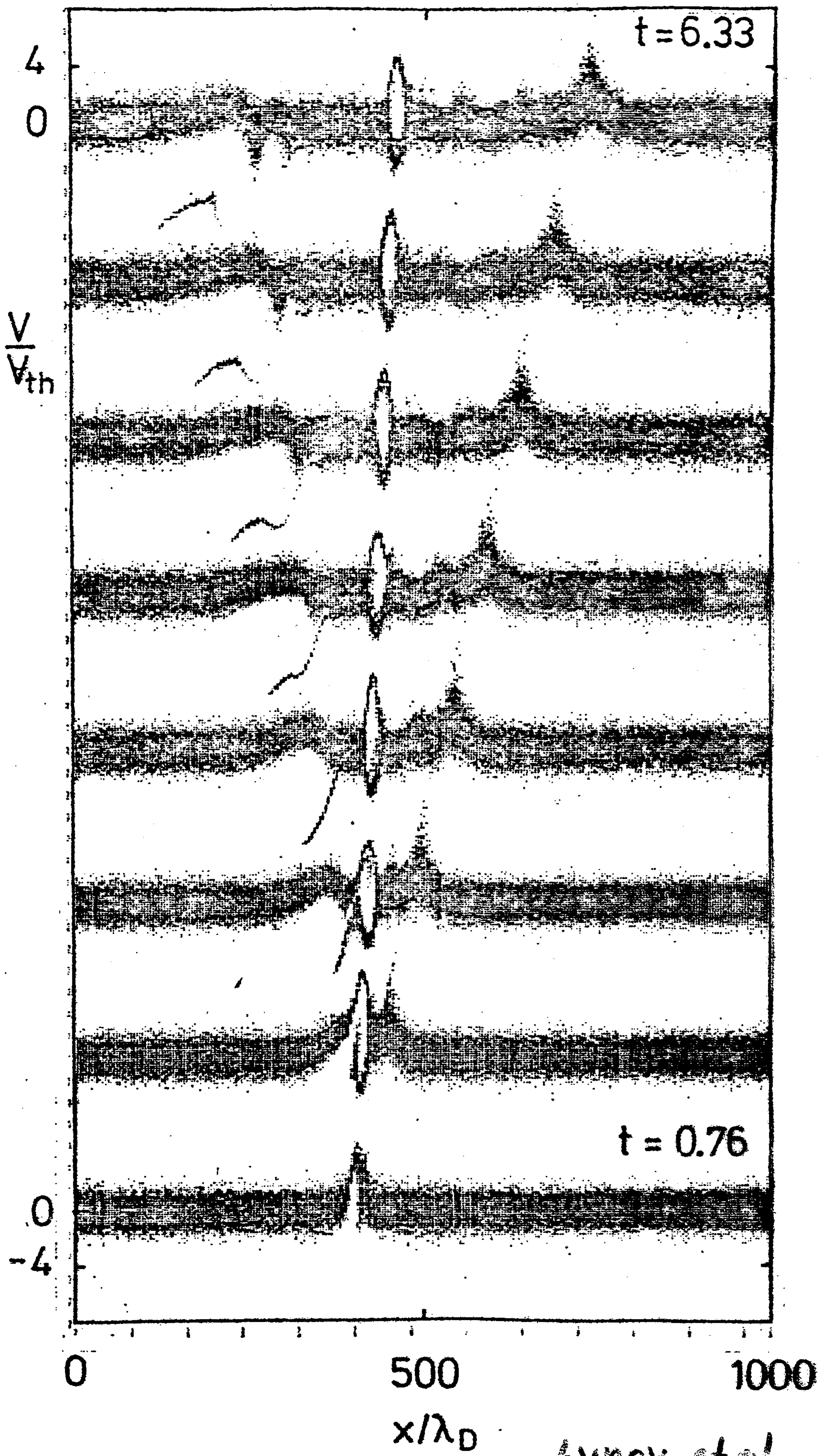


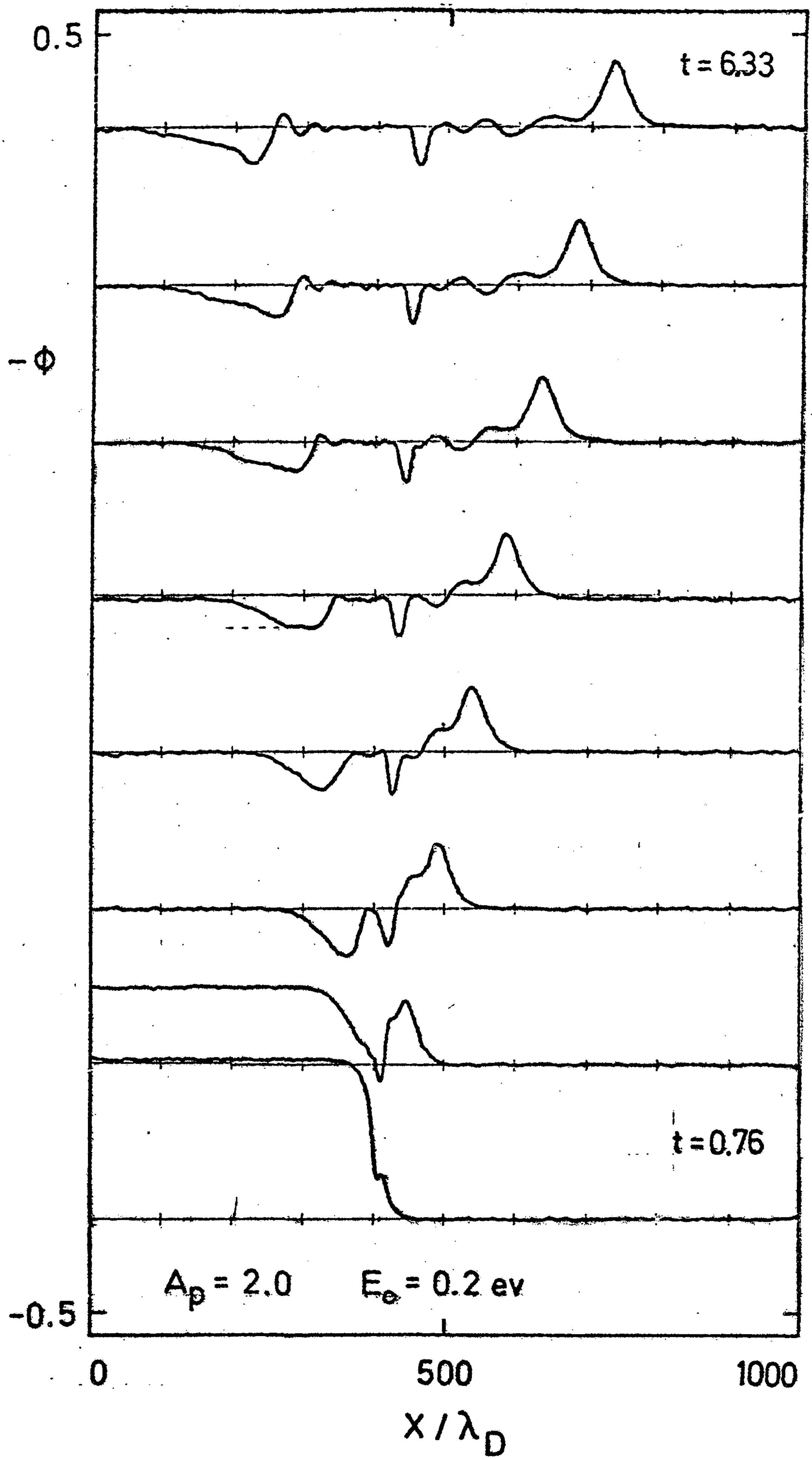
FIG. 2. Results of two equal beam simulations in one, two, and three dimensions.

Morse & Nielson
Phys. Rev. Lett. 23, 1087 (1969)



(a) *Fig. 5*

Lynov et al!
Physica Scripta, Vol. 20
(1979) 328



(b)

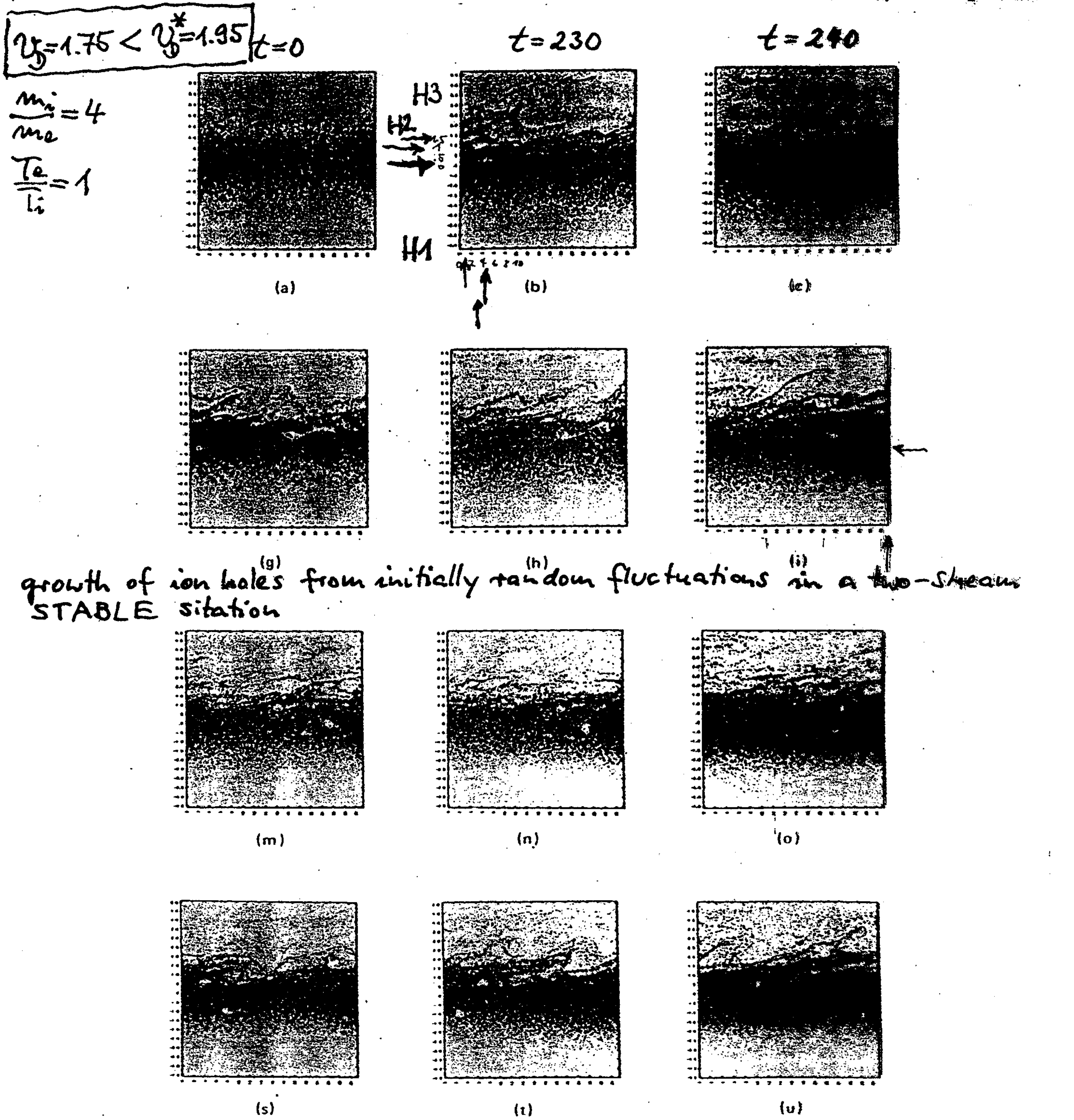
Fig. 5   Lynov et al

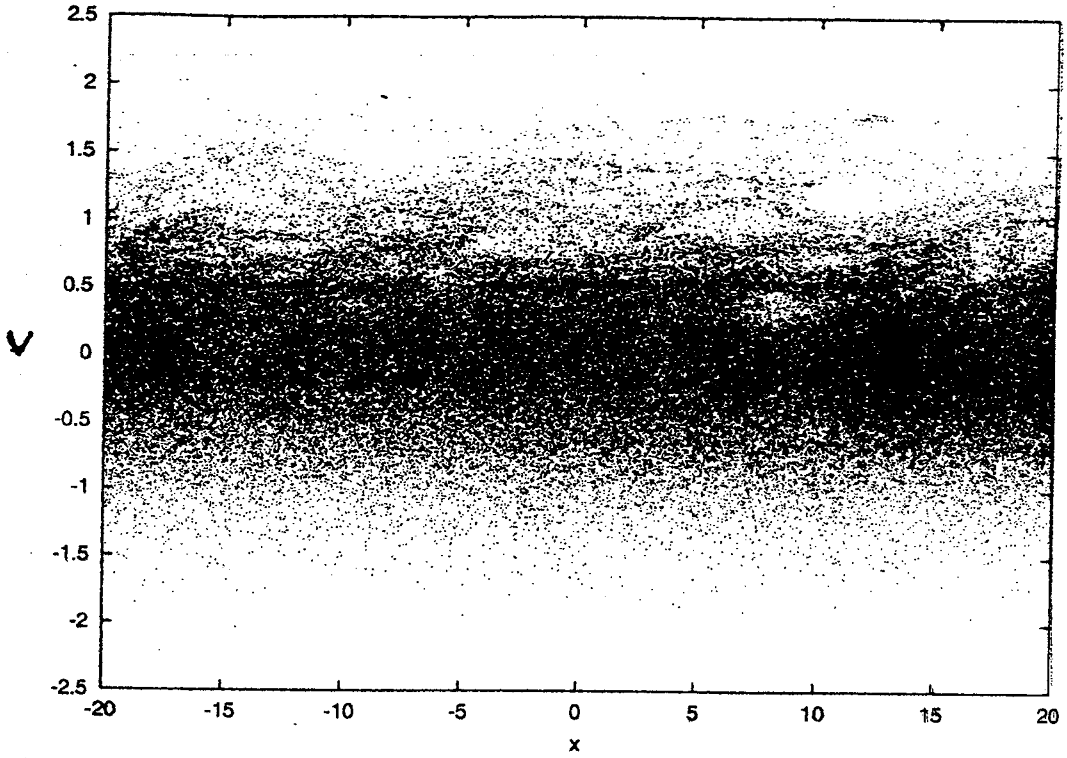
and Δv_m. This also implies that the smaller cells–where the contributions to $P(\overline{\delta f})$ will be due almost entirely to discrete particle fluctuations–will have a (nearly) Gaussian $P(\overline{\delta f})$. The $P(\overline{\delta f})$ measurements discussed in this paper are significantly broadened above the discrete particle level. We stress again the importance of this low discrete particle collisionality in the simulations. Use of too few particles leads to the destruction of the clumps' small-scale velocity structure.

IV. SIMULATION RESULTS

A. Random starts and the development of hole intermittency

We made a series of runs with random starts (see Sec. III) and various electron drift velocities below the linear stability boundary of $3.9v_{th,i}$. [redacted] duced initially in the particular runs discussed [redacted] Sec. IV A. Results for a representative run with [redacted] are shown in Figs. 1–4. Figure 1 shows the [redacted] ion distribution functions at $\omega_{p_e}t = 0$ [redacted] tial ion phase space is shown in [redacted] 3, such a plasma is nonlinearly [redacted] lop and considerable distortion [redacted] tion and flattening of the [redacted] place (see Fig. 1). This [redacted] function appears to be the [redacted] of the instability. As discussed in Ref. 3, [redacted] been observed for drift velocities in the [redacted]. This range contains drifts above [redacted] the linear stability boundary of $v_D =$ [redacted]





repetition and confirmation of Berman et al (random start)

3.Some recent observations (a list)

a)Holes in nonneutral plasmas and particle beams

J.D Moody and D.F.Driscoll Phys.Plasmas 2,4482 (1995)	holes (rarefaction pulses) in magnetized pure electron plasma column
W.Bertsche et al Phys.Rev.Lett.91,265003-1(03)	excitation of holes by sweeping an applied voltage downward in frequency (chirping)
P.L.Colestock and L.K Spentzouris Tamura Symp.,Austin(1994)	notches in distribution fct-measured by WCM (wall current monitor)at FERMI lab
S.Koscielniak et al Phys.Rev.ST-AB 4,044201(01)	periodic holes on coasting proton beam at CERN synchrotron
M.Blaskiewcz et al Phys.Rev.ST-AB 7,044402(04)	humps on bunched beams at RHIC - Brookhavn

b)Periodic structures at V_th

C.Franck et al Phys.Plasmas 8,4271 (01)	periodic ion humps in Kiel double plasma- sudden transition to C_s wave train
D.S.Montgomery et al Phys.Rev.Lett.87,155001(01)	slow electron acoustic mode generation by stim.laser scattering in Los Alamos lab

c)Space observations

R.E.Ergun et al Phys.Rev.Lett.87,045003-1(01) L.Anderson et al Phys.Plasmas 9,3600(02)	DLs and holes in downward current region of aurora
R.E.Ergun et al Phys.Plasmas 9,3685,95(02)	oblique DLs and holes in upward current region of aurora by FAST satellite
C.Cattell et al Nonlin.Proc.in Geophysics 10,13(03)	large amplitude holes in near earth`s magnetosphere,magnetopause and bow shock by POLAR and CLUSTER

--> $\boxed{\text{Holes and Double Layers are ubiquitous structures in driven plasmas}}$

Experimental result:

1. $\exists$ of a solitary $\phi(x) \geq 0$

2. vortex structure in phase space

3. $v_{EH} \leq v_{th} < v_{KdV}$

4. density dip at center

5. $\Delta_{EH} < \Delta_{KdV}$ (smaller halfwidth)

6. EH disappears if p_n is increased

7. Inelastic scattering (e.g. coalescence)

How to explain these results analytically especially the fact that $v_{EH} \leq v_{th}$?
(Landau damping?!) v_0

4. Theory

4a Construction of a standing, solitary electron hole by the potential method

$$\boxed{\begin{aligned} \left[v\partial_x + \phi'(x)\partial_v\right] f_e(x,v) &= 0 \\ \phi''(x) &= \int dv\, f_e(x,v) - 1 \end{aligned}}$$

VLASOV

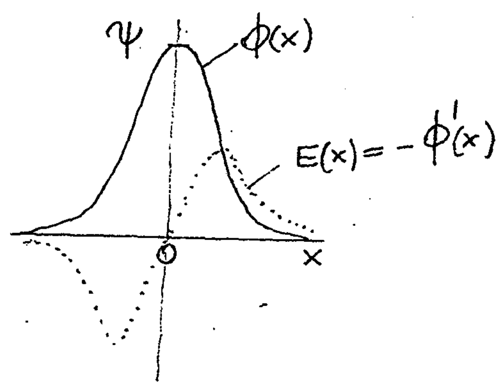
POISSON

- dimensionless
- stationary
- immobile ions

unperturbed plasma in thermal equilibrium

$$\Rightarrow \boxed{f_0(v) = \frac{1}{\sqrt{2\pi}} \exp[-v^2/2]} \quad \text{Maxwellian} \quad \Rightarrow \boxed{n_0 = \int f_0(v)\,dv = 1}$$

our goal : construction of a localized, self-consistent perturbation imbedded in a thermal plasma



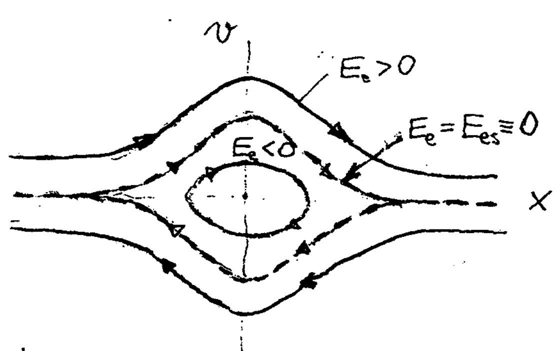
$\phi(x)$ bell shaped $\quad \boxed{0 \le \phi \le \psi}$

$E(x)$ bipolar

use is made by the

POTENTIAL METHOD (PM)

H.S. Plasma Phys. 14, 905 (1972)



phase space portrait of single particle dynamics

PM: **i)** solve Vlasov eq. with the appropriate boundary conditions

ii) determine $\phi(x)$ self-consistently by solving POISSON's eq.

$$\boxed{E_e := \frac{v^2}{2} - \phi > -\psi}$$

single particle energy a constant of motion

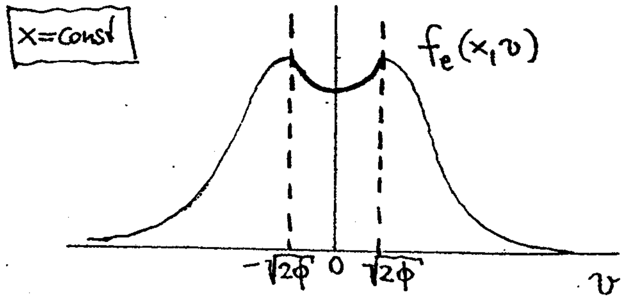
Separatrix : $\boxed{E_{es} := \frac{v_s^2(x)}{2} - \phi(x) \overset{!}{=} 0}$ separates free ($E_e > 0$) from trapped ($E_e < 0$) particles

$$\boxed{v_s(x) = \pm\sqrt{2\phi(x)}}$$

any fct $f_e(E_e)$ satisfies VLASOV eq. **step i)**

E_e: a constant of motion

appropriate ansatz for $f_e(E_e)$



$$\boxed{f_e(x,v) = \frac{1}{\sqrt{2\pi}} \begin{cases} \exp\left[-\left(\frac{v^2}{2} - \phi\right)\right] & E_e > 0 \\ \exp\left[-\beta\left(\frac{v^2}{2} - \phi\right)\right] & E_e \leq 0 \end{cases}}$$

notice : continuity of f_e

$|x| \to \infty \quad \phi \to 0 \quad (v_s(x) \to 0)$

$$\boxed{f_e(x,v) \to f_0(v)}$$

$f_0(v)$: an even fct in v

β a yet undetermined constant

$\boxed{\beta = 0}$ plateau-like trapped part. distr.

$\boxed{\beta < 0}$ (see figure) describes a deficit (notch) of electrons trapped in the potential hill ($-\phi$ potential well)

$\boxed{\beta = 1}$ ordinary Maxwellian in v ("isothermal electrons")

density :

$$n_e = \int dv\, f_e(x,v) = \sqrt{\frac{2}{\pi}} \left\{ \underbrace{\int_0^{v_s} dv\, e^{-\beta\left[\frac{v^2}{2} - \phi\right]}}_{n_{et}} + \underbrace{\int_{v_s}^{\infty} dv\, e^{-\left[\frac{v^2}{2} - \phi\right]}}_{n_{ef}} \right\}$$

$$n_{ef} = e^{\phi} \frac{2}{\sqrt{\pi}} \int_{v_s/\sqrt{2}}^{\infty} dx\, e^{-x^2} \equiv e^{\phi} \Bigg[\underbrace{\frac{2}{\sqrt{\pi}} \int_0^{\infty} dx\, e^{-x^2}}_{1} - \underbrace{\frac{2}{\sqrt{\pi}} \int_0^{v_s/\sqrt{2}} dx\, e^{-x^2}}_{\mathrm{erf}(v_s/\sqrt{2}) \equiv \mathrm{erf}(\sqrt{\phi})} \Bigg]$$

$$= e^{\phi}\left[1 - \mathrm{erf}\sqrt{\phi}\right] \equiv e^{\phi}\, \mathrm{erfc}\sqrt{\phi}$$

$$= 1 - \frac{2}{\sqrt{\pi}}\sqrt{\phi} + \phi - \frac{4}{3\sqrt{\pi}}\phi^{3/2} + \frac{\phi^2}{2} + \dots$$

in the small amplitude limit $\psi \ll 1$ ($\Rightarrow$ weak e-hole)

notice: half power expansion in ϕ !

$$n_{et} = \frac{1}{\sqrt{|\beta|}} \begin{cases} e^{\beta\phi}\,\mathrm{erf}\sqrt{\beta\phi} & \beta \geq 0 \\ \frac{2}{\sqrt{\pi}} W(\sqrt{-\beta\phi}) & \beta < 0 \end{cases}$$

$$\boxed{\begin{array}{l} W(x) = e^{-x^2}\int_0^x dt\, e^{t^2} \\ \text{DAWSON integral} \end{array}}$$

$$= + \frac{2\sqrt{\phi}}{\sqrt{\pi}} + \frac{4}{3}\frac{\beta}{\sqrt{\pi}}\phi^{3/2} + O(\phi^{5/2})$$

↳ cancellation
because of continuity of f_e at separatrix

$\psi \ll 1$

$$\boxed{n_e(\phi) = 1 + \phi - \frac{4}{3} b\, \phi^{3/2} + \frac{\phi^2}{2} + ..}$$

$$\boxed{b := \frac{1-\beta}{\sqrt{\pi}}}$$

isothermal limit: $\beta \to 1$
$\Rightarrow$ integer power expansion

equation of state: $(m_e \to 0)$

$$0 \approx \phi'(x) - \frac{1}{n_e(x)} \underbrace{P_e'(x)}_{P_e'(\phi)\phi'(x)} \Rightarrow \boxed{P_e'(\phi) = n_e(\phi)}$$

$$P_e(\phi) = 1 + \int_0^\phi n_e(\phi)\, d\phi \Rightarrow$$

$$T_e(\phi) := \frac{P_e(\phi)}{n_e(\phi)} = 1 + \frac{4}{3} b\, \phi^{3/2} + \ldots$$

step ii)

self-consistent determination of $\phi(x)$

$$\boxed{\phi''(x) = n_e(\phi) - 1 =: -V'(\phi)}$$

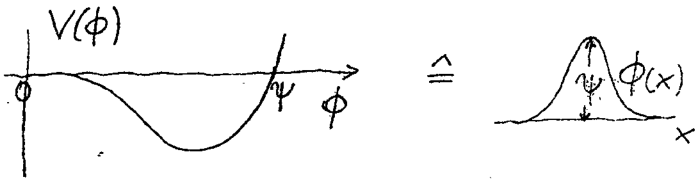
times $\phi'(x)$ and integration

$$\boxed{\frac{\phi'(x)^2}{2} + V(\phi) = 0}$$

$\hat{=}$ "classical energy" $\frac{v^2}{2} + V(x)$

$$\boxed{\begin{array}{l} x \hat{=} \phi \\ t \hat{=} x \\ v = \dot{x} \hat{=} \phi'(x) \end{array}}$$

$\boxed{V(\phi) \text{ "classical potential"}}$ $\boxed{V(0) \overset{!}{=} 0}$



two necessary conditions:

1. $V(\phi) < 0$ in $0 < \phi < \psi$
2. $\boxed{V(\psi) \overset{!}{=} 0}$

$$-V(\phi) = \int_0^\phi n_e(\phi)\, d\phi - \phi = \phi + \frac{\phi^2}{2} - \frac{8}{15} b\, \phi^{5/2} + \ldots - \phi = \frac{\phi^2}{2}\left(1 - \frac{16}{15} b\sqrt{\phi}\right)$$

second condition : $\boxed{V(\psi)=0}$

later termed: NDR
Nonlinear Dispersion Relation

$$\boxed{\frac{16}{15} b\sqrt{\psi} = 1}$$

existence condition :

$$b = \frac{1-\beta}{\sqrt{\pi}} = \frac{15}{16\sqrt{\psi}} \gg 1$$

$\Rightarrow -\beta$ is a large positive number

or $\boxed{\beta \text{ is strongly negative and determined by } \psi}$ (notch in f_e!)

terminology: $\Rightarrow$ $\boxed{\text{electron hole}}$

$$\boxed{-V(\phi) = \frac{\phi^2}{2}\left(1-\sqrt{\frac{\phi}{\psi}}\right)} > 0 \quad \Leftarrow$$

in "classical energy"

$$\phi'(x) = \pm\sqrt{-2V(\phi)}$$

$$\boxed{\frac{d\phi}{\sqrt{-2V(\phi)}} = \pm dx} \quad \Rightarrow \quad x = x(\phi) \quad \text{inversion} \quad \boxed{\phi(x) = \psi \operatorname{sech}^4(x/4)}$$

F. Bauer, H.S., Physica D 54, 235 (1992)

- solution is determined completely (its shape and velocity)
- a standing, solitary e-hole requires a specific trapped electron parameter β $\Rightarrow$ $\boxed{\text{only a one-parameter } (\psi) \text{ family of solutions remains}}$
- the method of construction is called $\boxed{\text{"POTENTIAL METHOD"}}$ (notice its advantage against Bernstein-Greene-Kruskal method)
- no solution would exist for isothermal electrons ($\beta = 1$)

1^{st} conclusion : $\boxed{\text{Only by a rigorous analysis of resonant (trapped) particles a solution could be achieved but not within an ordinary (integer) expansion scheme!}}$

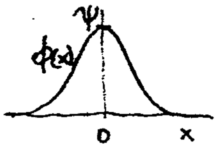
$\boxed{\text{since } \beta < 0 \text{: distribution has a NONTOPOLOGICAL character } (|\partial_v f_1| \gtrsim |\partial_v f_0|)}$

$\boxed{\begin{array}{l}\text{nonlinear term in } n_e(\phi):\ b\phi^{3/2} \simeq O(\phi) \\ \Rightarrow \text{non negligible at any values of } \psi \text{ (including infin.)}\end{array}}$

next generalizations : • periodic wave structure
• propagating wave structure

4b Periodic chain of standing e-holes

previous solution:



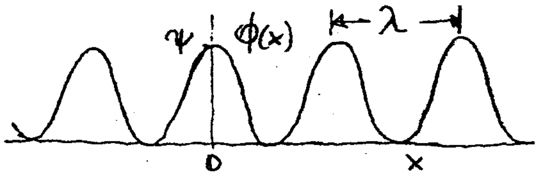
$\phi', \phi'', \dots \to 0$

as $|x| \to \infty$ or

$\boxed{\phi \to 0}$

$\boxed{\phi = 0}$ is the spatial position of no trapped electrons

now: wanted: wave train periodic in x



$\boxed{\text{wavenumber } k = 2\pi/\lambda}$

new situation

at $\boxed{\phi = 0}$ $\phi' = 0$ but $\boxed{\phi'' > 0}$

claim: this can be achieved by changing the normalization of f_e:

step i)
$$\boxed{f_e(x,v) = \frac{(1 + k_0^2 \psi/2)}{\sqrt{2\pi}} \begin{cases} \exp\left[-\left(\frac{v^2}{2} - \phi\right)\right] & E_e > 0 \\ \exp\left[-\beta\left(\frac{v^2}{2} - \phi\right)\right] & E_e \leq 0 \end{cases}}$$

3 parameters: $\psi, \beta, k_0 \neq k$ in generality

proof:

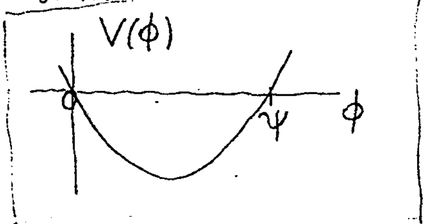
$$n_e(\phi) = \left(1 + \frac{k_0^2 \psi}{2}\right)\left[1 + \phi - \frac{4}{3} b \phi^{3/2} + \dots\right]$$
$$\simeq 1 + \frac{k_0^2 \psi}{2} + \phi - \frac{4}{3} b \phi^{3/2} + O(\phi^2)$$

$\Rightarrow$ $\boxed{\text{at } \phi = 0 \quad \phi'' = n_e(0) - 1 = \frac{k_0^2 \psi}{2} > 0}$ qed

$\Rightarrow$ step ii)
$$-V(\phi) = \int_0^\phi n_e(\phi) d\phi - \phi = \frac{k_0^2 \psi \phi}{2} + \frac{\phi^2}{2}\left(1 - \frac{16}{15} b \sqrt{\phi}\right) + \dots$$

$\boxed{V(0) = 0}$

wanted:



NDR: $\boxed{V(\psi) = 0}$ $\Longrightarrow$ $\boxed{\frac{16}{15} b \sqrt{\psi} = 1 + k_0^2}$ NDR

hence: $\boxed{\beta = \beta(\psi, k_0)}$ $\Rightarrow$ a larger depression in f_e when $k_0 \neq 0$

insertion of NDR in $V(\phi)$:

replace 1 by $\frac{16}{15} b \sqrt{\psi} - k_0^2$ in V

$$\boxed{-V(\phi) = \frac{k_0^2}{2} \phi(\psi - \phi) + \frac{8b}{15} \phi^2 \left[\sqrt{\psi} - \sqrt{\phi}\right]}$$

$$= \frac{k_0^2}{2}\left[\phi(\psi - \phi) + \frac{S}{\sqrt{\psi}} \phi^2 (\sqrt{\psi} - \sqrt{\phi})\right]$$

$\boxed{k_0 = 0}$ old expression $\hat{=}$ $\boxed{S = \infty}$

$$\boxed{S := \frac{16 b \sqrt{\psi}}{15 k_0^2} \overset{NDR}{=} \frac{1 + k_0^2}{k_0^2}}$$

steepening or distortion parameter

region of existence: $\boxed{1 \leq S \leq \infty}$

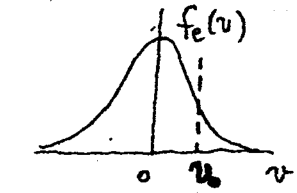
$\boxed{\text{no harmonic wave } \boxed{S = 0}}$!

$\hat{=}$ $\boxed{\text{cnoidal waves}}$ inclusively solitary wave $\boxed{S = \infty}$

$\beta = \beta(\beta, \psi)$

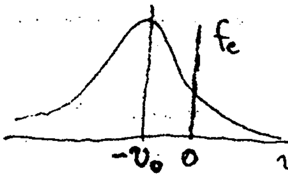
4c Propagating electron hole equilibria ($m_i = \infty$)

next goal: construction of stationary e-holes propagating in laboratory space with phase velocity v_0



lab frame $\quad \boxed{\begin{array}{l} x' = x - v_0 t \\ v' = v - v_0 \end{array}} \xrightarrow[\text{transf.}]{\text{Galilei}} \boxed{\begin{array}{c} x \\ v \end{array}}$ (drop prime)

lab frame $\qquad$ wave frame



wave frame $\quad \boxed{f_0(v) \sim e^{-v^2/2}} \Longrightarrow \boxed{f_0(v) \sim e^{-\frac{1}{2}(v+v_0)^2}}$

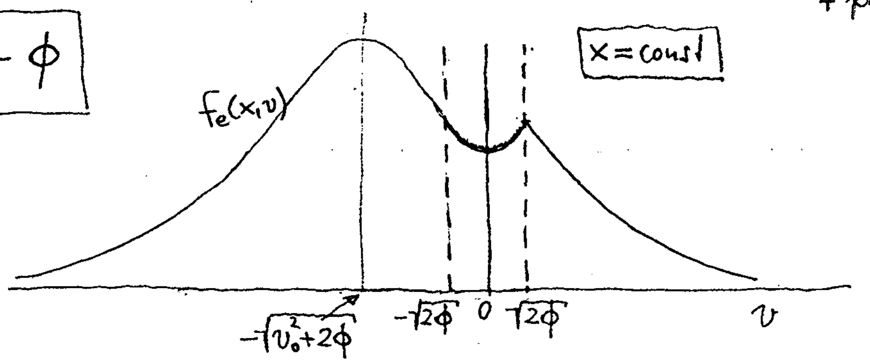
Maxwellian $\qquad$ shifted Maxwellian

PM: i) appropriate ansatz for f_e

$$\boxed{f_e(x,v) = \frac{(1 + k_0^2 \psi/2)}{\sqrt{2\pi}} \begin{cases} \exp\left[-\frac{1}{2}\left(\sigma\sqrt{v^2 - 2\phi} + v_0\right)^2\right] & E_e > 0 \\ \exp\left[-\frac{\beta}{2}(v^2 - 2\phi) - \frac{v_0^2}{2}\right] & E_e \leq 0 \end{cases}}$$

4 parameters: ψ, β, k_0, v_0

$\boxed{E_e := \frac{v^2}{2} - \phi}$



properties:

- $\boxed{\phi \to 0}$ shifted Maxwellian : $\Big(\sigma \underbrace{\underbrace{\sqrt{v^2}}_{|v|}}_{v} + v_0\Big) = (v + v_0)$; $\sigma = \operatorname{sgn} v$

- $f_e(\cdot; v)$ no longer an even function in v

- f_e is now depending on $\boxed{\text{2 constants of motion}}$

  1. $\boxed{E_e = \frac{v^2}{2} - \phi}$ $\forall$ particles

  2. $\boxed{\sigma = \operatorname{sgn} v}$ for untrapped or free particles ($E_e > 0$)

- as before, f_e is continuous everywhere

- only the untrapped part of f_e is predetermined by the requirement of a thermal plasma (at $\phi = 0$); for f_{et} we need some flexibility ($\to \beta$)

ii) velocity integration ($k_0 = 0$) solitary wave limit

$$\boxed{n_e(\phi) = 1 - \frac{1}{2} Z_r'(v_0/\sqrt{2})\,\phi - \frac{4}{3} b(\beta, v_0)\,\phi^{3/2} + \ldots}$$

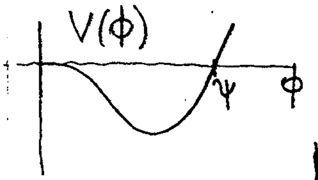
$$\boxed{b(\beta, v_0) := \frac{1}{\sqrt{\pi}}\left(1 - \beta - v_0^2\right) \exp(-v_0^2/2)}$$

$v_0 = 0 \qquad b(\beta, 0) = \frac{1-\beta}{\sqrt{\pi}}$ as before

$$\boxed{Z_r(x) := P\int dt\, \frac{\exp(-t^2)}{t - x} \cdot \frac{1}{\sqrt{\pi}}}$$

real part of PLASMA DISPERSION FUNCTION

$$\boxed{-V(\phi) = \int_0^\phi n_e(\phi)\, d\phi \overset{-\phi}{=} \frac{\phi^2}{2}\left[-\frac{1}{2} Z_r'(v_0/\sqrt{2}) - \frac{16}{15} b(\beta, v_0)\sqrt{\phi}\right]}$$

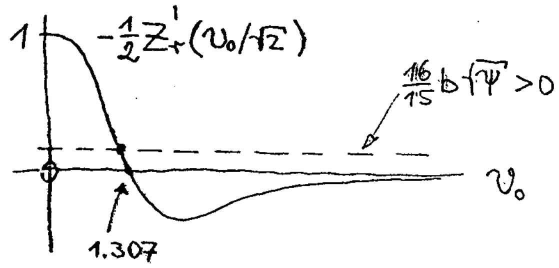


$$\boxed{V(\psi) \overset{!}{=} 0} \qquad \boxed{-\frac{1}{2} Z_r'(v_0/\sqrt{2}) = \frac{16}{15} b(\beta, v_0)\sqrt{\psi}} \quad \text{NDR}$$

NDR in $V(\phi)$:

$$\boxed{-V(\phi) = \frac{8}{15} \underbrace{b(\beta, v_0)}_{>0} \phi^2 \left[\sqrt{\psi} - \sqrt{\phi}\right]}$$

provided that



$\rightarrow$ bell-shaped solution exists with $\boxed{v_0 \lesssim 1.307}$ (thermal range!)

$\rightarrow$ $b(\beta, 1.3) = \frac{1}{\sqrt{\pi}}\left(1 - \beta - 1.3^2\right)\exp(-1.3^2/2) > 0$

$\curvearrowright \boxed{-\beta > 0.7}$ $\curvearrowright$ negative β represents notch in f_e at v_0

$$\rightarrow \boxed{\phi(x) = \psi\, \mathrm{sech}^4\left(\sqrt{\frac{b(\beta, v_0)\sqrt{\psi}}{15}}\, x\right)} \quad \text{non-KdV type}$$

$$\boxed{\Delta_{EH} \sim \frac{1}{\psi^{1/4}}} < \boxed{\Delta_{KdV} \sim \frac{1}{\sqrt{\psi}}} \qquad \text{qed}$$

Lit: H.S Physica Scripta 20, 336 (1979)

$k_0 \neq 0$

$$\boxed{-\frac{1}{2} Z_r'\left(v_0/\sqrt{2}\right) = \frac{16}{15} b(\beta, v_0)\sqrt{\Psi} - k_0^2}$$

NDR

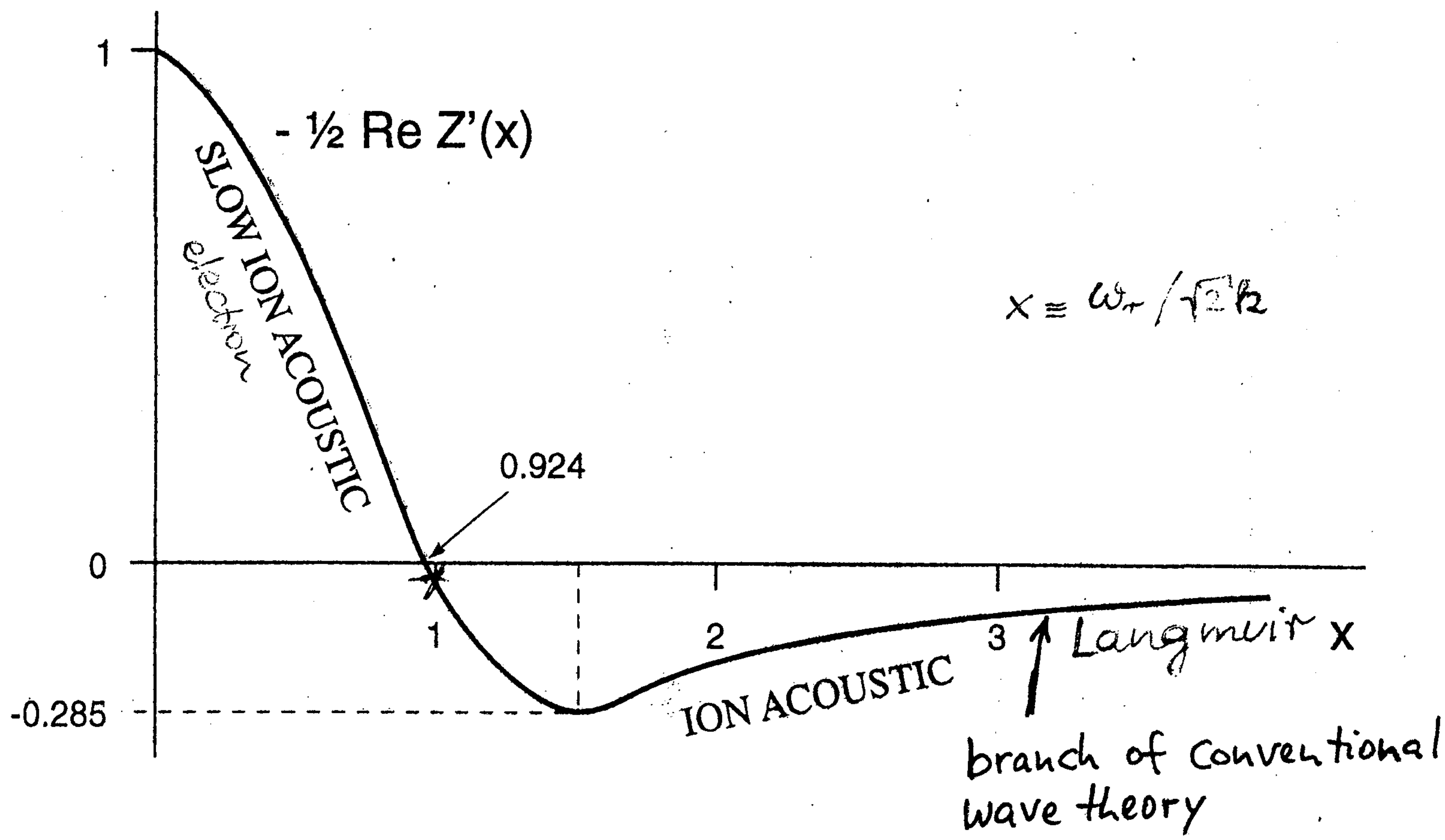


Fig. 2

The potential method (I)

generalization : v_D drift between electrons & ions
m: finite ion mass

Lit. HS Phys. Plasmas Vol. 7, 4831 (2000)

We choose f_e and f_i as plausible functions of the constants of motion:

$$f_e(x,v) = \frac{1+K}{\sqrt{2\pi}} \begin{cases} \exp\left[-\frac{1}{2}\left(\sigma_e\sqrt{2E_e} - \tilde{v}_D\right)^2\right], & E_e > 0, \\ \exp\left(-\tilde{v}_D^2/2 - \beta E_e\right), & E_e \leq 0, \end{cases}$$

$$E_e = \frac{v^2}{2} - \phi.$$

$$f_i(x,u) = \frac{1+A}{\sqrt{2\pi}} \begin{cases} \exp\left[-\frac{1}{2}\left(\sigma_i\sqrt{2E_i} + u_0\right)^2\right], & E_i > 0, \\ \exp\left(-u_0^2/2 - \alpha E_i\right), & E_i \leq 0, \end{cases}$$

$$E_i = \frac{u^2}{2} + \theta(\Phi - \psi).$$

$\tilde{v}_D = v_D - u_0$

$\theta = T_e / T_i$

Nonlinear destabilization of current-carrying plasmas by negative energy holes or zero structure:

$$\boxed{\begin{aligned}-V(\phi) &= \frac{k_0^2}{2}\phi(\psi-\phi) + \frac{8}{15}b(\beta,\tilde{v}_D)\phi^2[\sqrt{\psi}-\sqrt{\phi}] \\ &+\frac{4}{15}b(\alpha,u_0)\theta^{3/2}\{2[\psi^{5/2}-(\psi-\phi)^{5/2}]-\phi\sqrt{\psi}(5\psi-3\phi)\}\end{aligned}}$$

A. Luque, H.S., J.M. Grießmeier, Phys. Plas. 9 (2002) 4841
A. Luque, H.S., Phys. Reports 415 (2005) 261

$$0 \le \phi \le \psi \ll 1$$

NDR:

$$\boxed{k_0^2 - \frac{1}{2}Z_r'\left(\frac{\tilde{v}_D}{\sqrt{2}}\right) - \frac{\theta}{2}Z_r'\left(\frac{u_0}{\sqrt{2}}\right) = \frac{16}{15}\left[\underbrace{b(\beta,\tilde{v}_D)}_{B_e} + \frac{3}{2}\underbrace{b(\alpha,u_0)\theta^{3/2}}_{B_i}\right]\underline{\sqrt{\psi}}}$$

total energy density:

$$\boxed{w = \frac{1}{4L}\int_{-L}^{+L}dx\left[\int_{-\infty}^{+\infty}dv\,v^2 f_e(x,v) + \frac{1}{\theta}\int_{-\infty}^{+\infty}du\,u^2 f_i(x,u) + \phi'(x)^2\right]}$$

$\psi = 0$ $\boxed{w_H = \frac{1}{2}(1+v_D^2+\frac{1}{\theta})}$ $\psi \neq 0$ $\boxed{w_S \equiv w_H + \Delta w}$

def. $\Delta w < 0$ negative energy hole

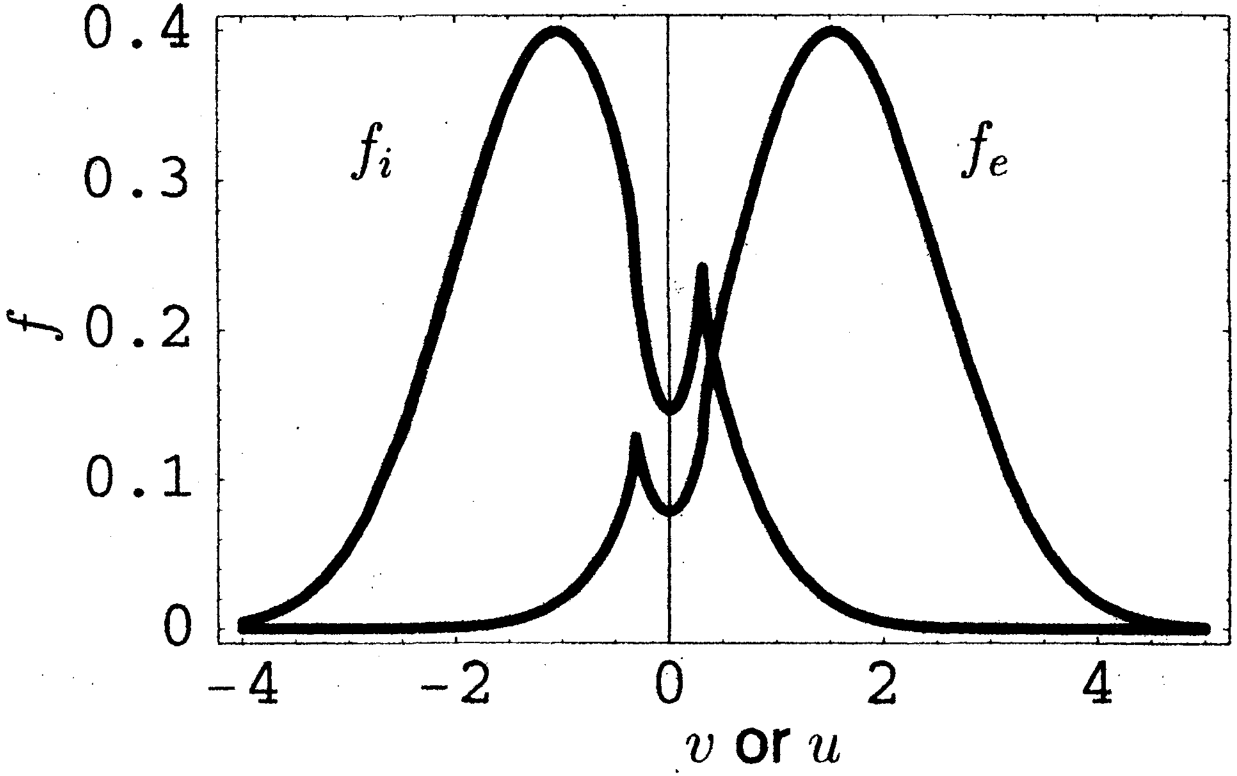
$$\boxed{\begin{aligned}\Delta w &= \frac{1}{2}\left[\psi + \left(\frac{k_0^2\psi}{2}-\epsilon\right)\left(1+v_D^2\right) + \frac{A-\epsilon}{\theta}\right] \\ &+\frac{v_0^2}{4L}\int_{-L}^{+L}dx\left[-\frac{1}{2}Z_r'\left(\frac{\tilde{v}_D}{\sqrt{2}}\right)\phi - \frac{4}{3}b(\beta,\tilde{v}_D)\phi^{3/2}\right] \\ &+\frac{u_0^2}{4L\theta}\int_{-L}^{+L}dx\left[-\frac{\theta}{2}Z_r'\left(\frac{u_0}{\sqrt{2}}\right)(\psi-\phi) - \frac{4}{3}b(\alpha,u_0)\theta^{3/2}(\psi-\phi)^{3/2}\right]\end{aligned}}$$

J.-M. Grießmeier, H.S., Phys. Plasmas 9, 2462 (2002)
J.M. Grießmeier, A. Luque, H.S. Phys. Plasm 9, 3816 (2002)

$$A = \frac{\theta}{2}Z_r'\left(\frac{u_0}{\sqrt{2}}\right)\psi + \frac{4}{3}b(\alpha,u_0)(\theta\psi)^{3/2}$$

Distribution functions

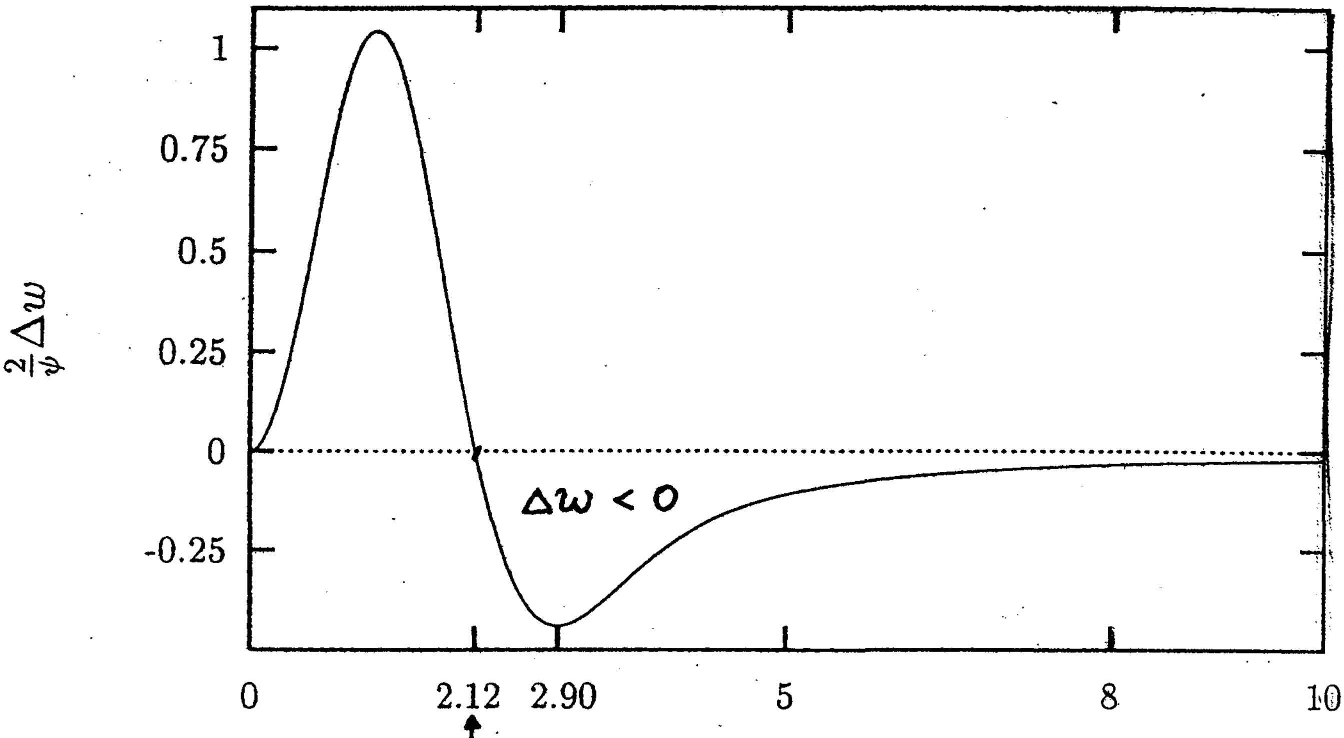
a typical solution of f_e, f_i for solitary e-hole in the negative energy range



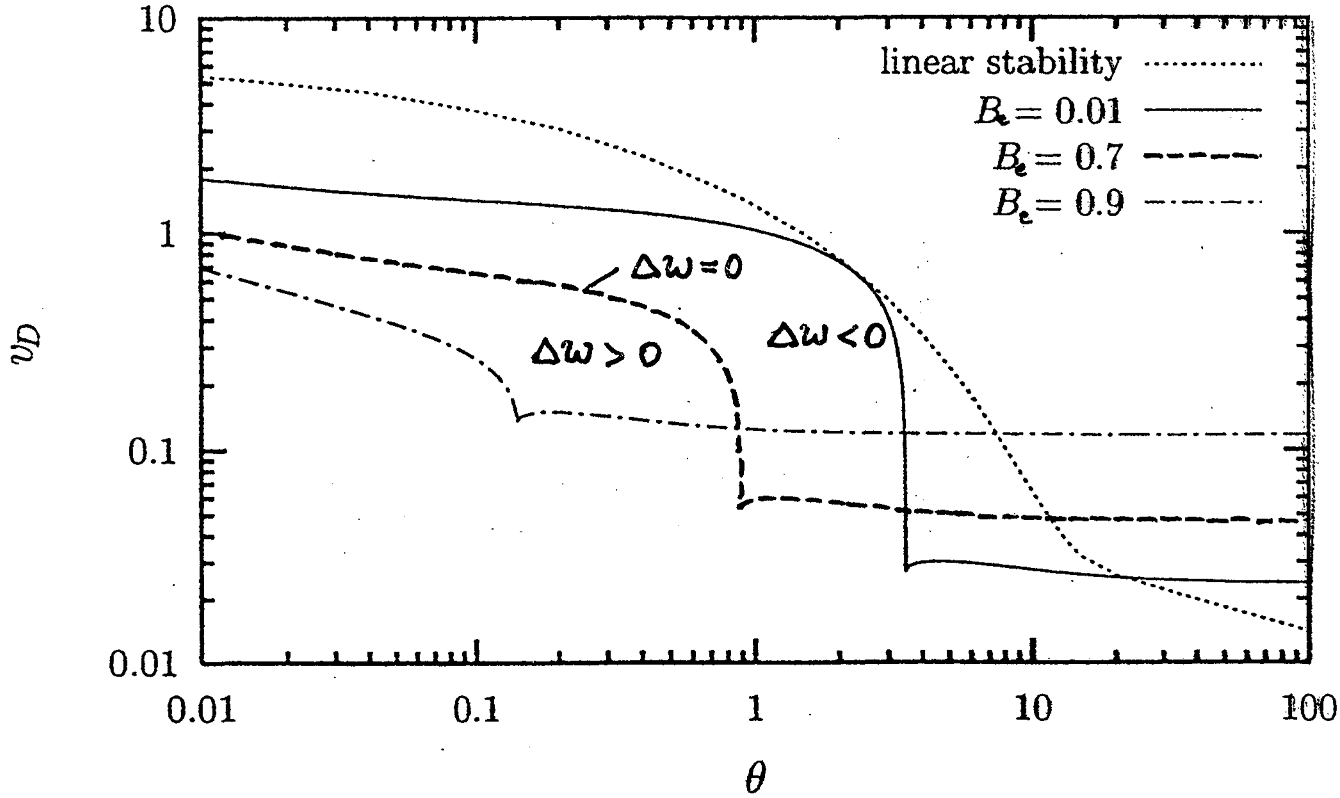
Note the different normalization for v and u.

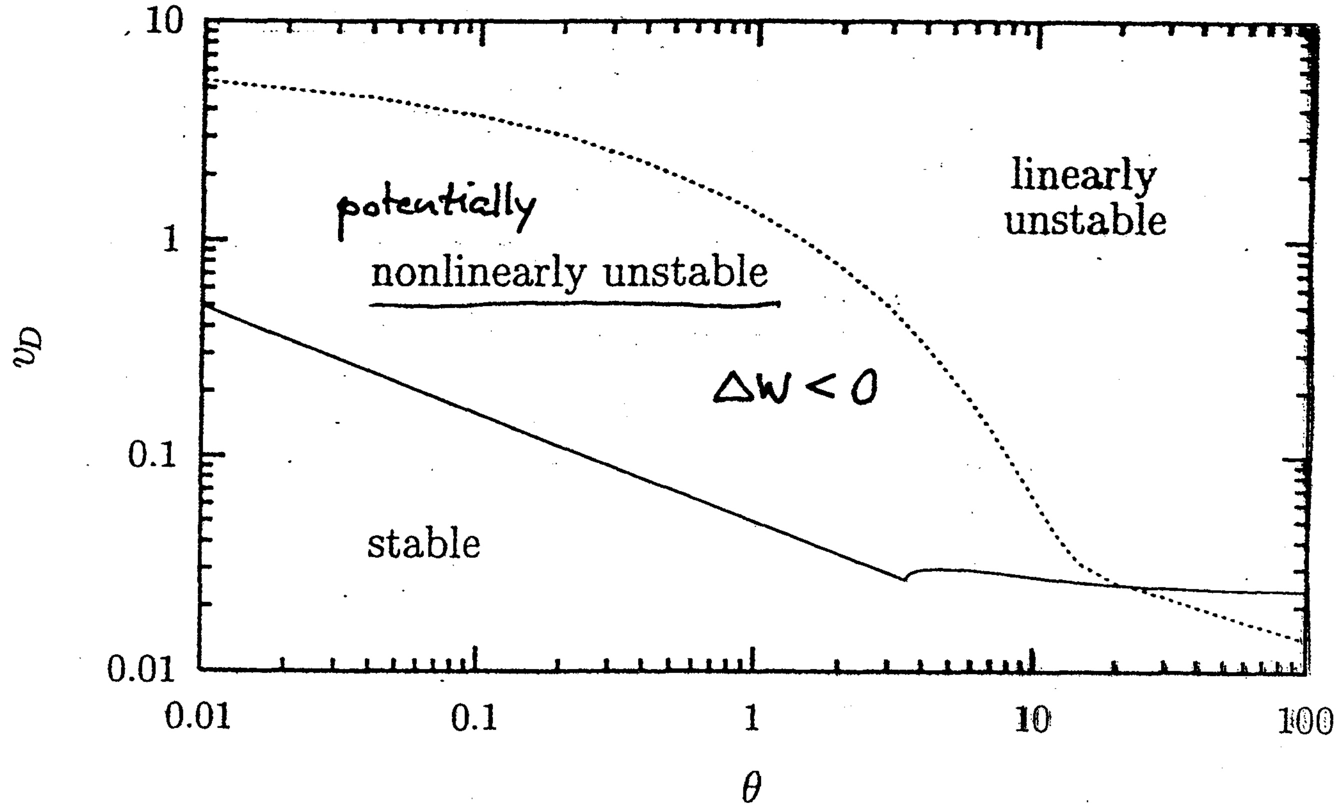
solitary e-hole $\quad B_i = 0$

$$\Delta w = \frac{\psi}{2}\left[1 + \frac{1}{2}Z_r'\left(\frac{u_0}{\sqrt{2}}\right)(1 - u_0^2)\right] \qquad u_0 = \sqrt{\frac{\theta}{\delta}}\, v_0$$



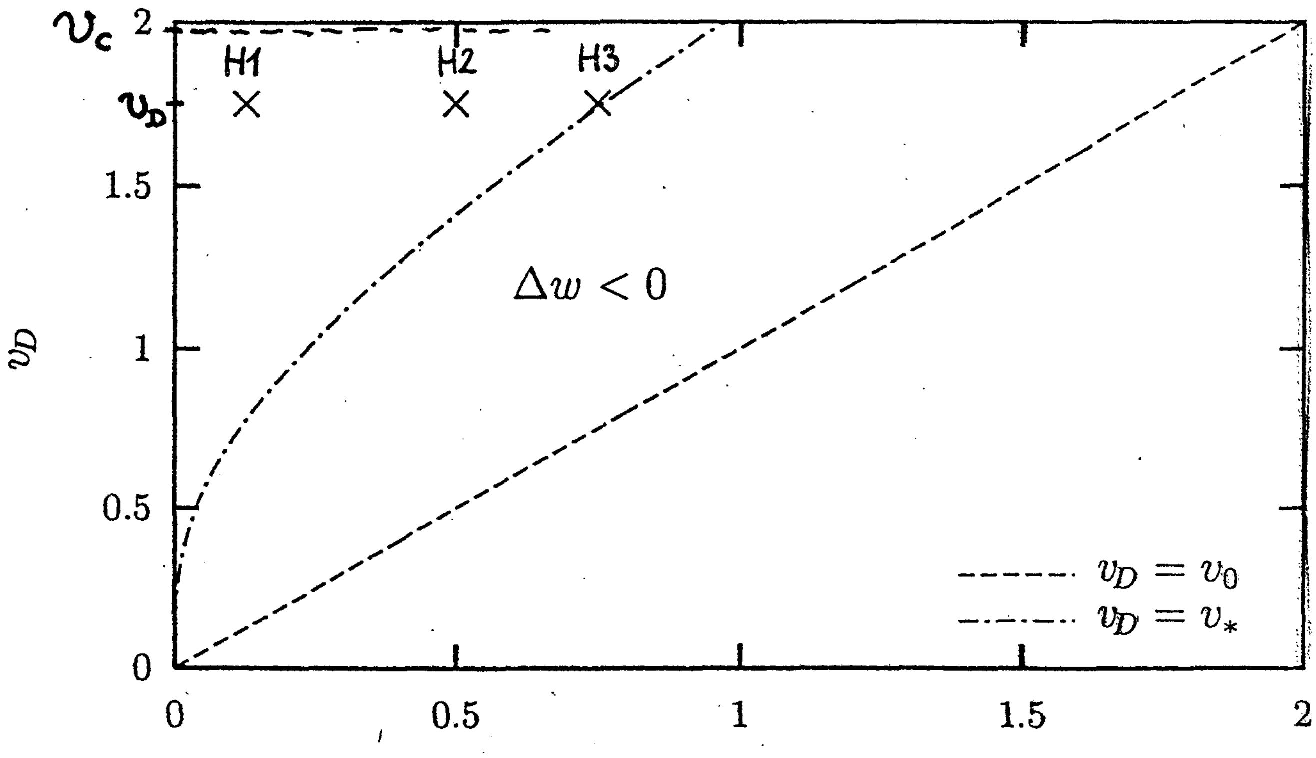
$\Delta w = 0 \quad u_0 = 2.12$ ion mobility important



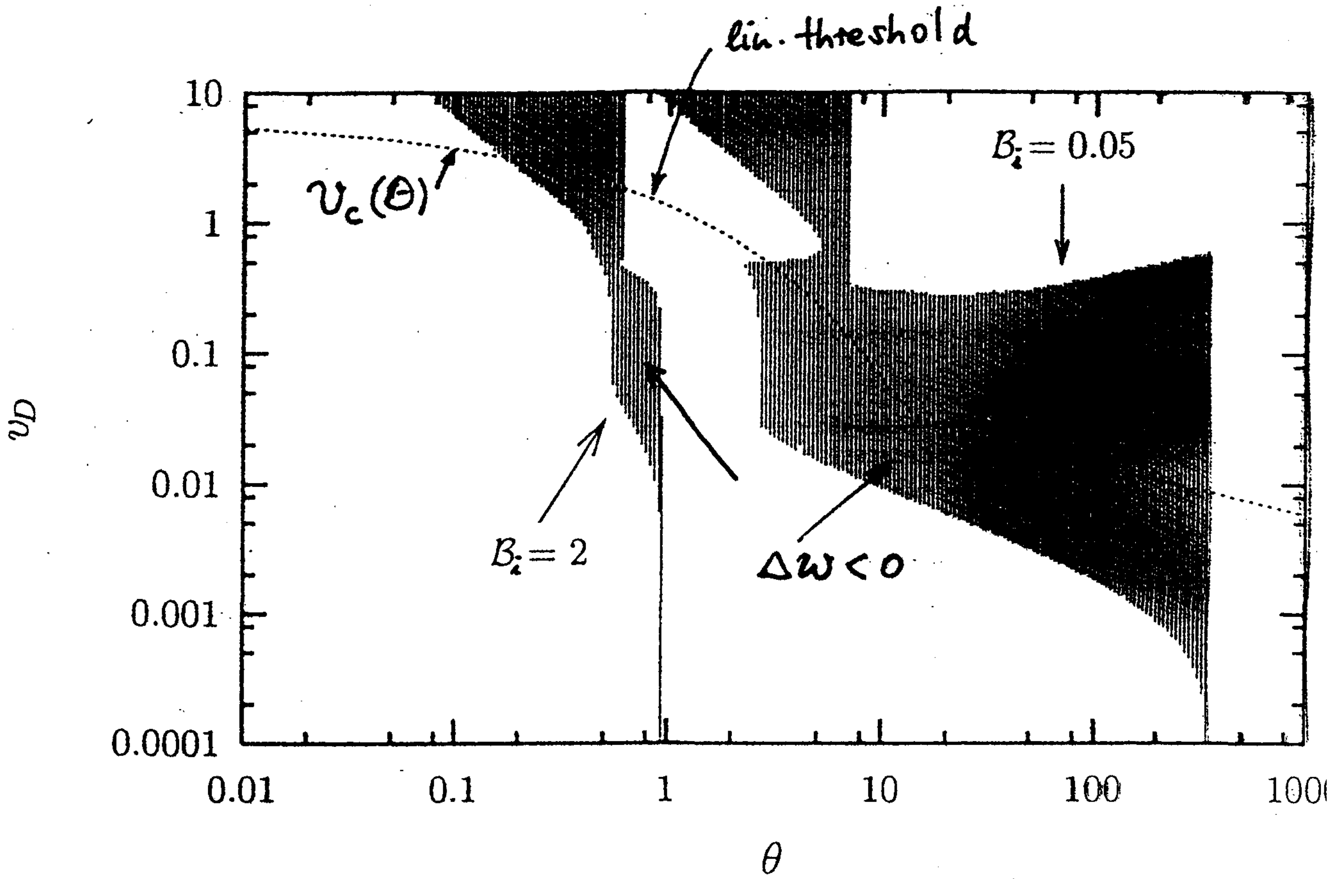


solitary i-hole

$$\Delta w = \frac{\psi}{2}\left[1 + \frac{1}{2} Z_r'\left(\frac{\tilde{v}_D}{\sqrt{2}}\right)(1 + v_D^2 - v_0^2)\right] \qquad B_e = 0$$



for H1, H2, H3 see Berman et al (1985);



Conclusion: for given $\Theta = T_e/T_i$ and $v_D < v_c(\Theta)$ (linear stability) a trapped ion parameter $B_i \equiv \frac{3}{2} b(\alpha, u_0)\Theta^{3/2}\psi^{1/2}$ can always be found such that the ion hole has negative energy or zero $\Delta W \leq 0$.

5. Linearity versus nonlinearity

another special solution of Vlasov–Poisson system:

harmonic solution (e-branch) $\theta \to 0$

$$\boxed{k^2 - \tfrac{1}{2} Z_r'(\omega_r / \sqrt{2}\, k) = 0} \quad \text{(NDR)}$$

$$\boxed{-V(\phi) = \tfrac{k^2}{2}\phi(\psi - \phi)} \quad \text{(V)}$$

in contrast: Landau theory

$$\boxed{k^2 - \tfrac{1}{2} Z'(\omega / \sqrt{2}\, k) = 0} \quad \text{(LDR)}$$

$$\omega := \omega_r + i\gamma$$

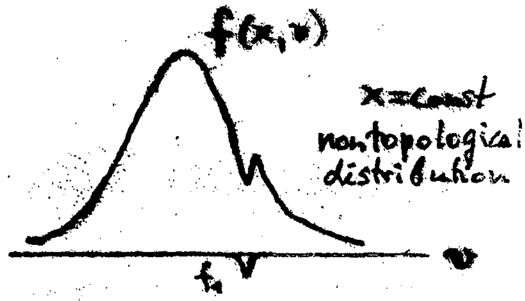
in thermal range $v_{ph} = \omega_r/k \approx 1$

$\gamma < 0$: strong Landau damping

$\Rightarrow$

nonexistence of undamped modes in thermal velocity range within Landau theory

resolution of discrepancy:



- linearization in Vlasov eq. means

  $\boxed{E\,\partial_v f_1}$ negligible w.r.t. $\boxed{E\,\partial_v f_0}$

  $0(\varepsilon^2)$ $\qquad$ $0(\varepsilon)$

- e–hole equilibria, however, demand

  $|E\,\partial_v f_1|$ comparable with $|E\,\partial_v f_0|$

  $0(\varepsilon)$ $\qquad$ $0(\varepsilon)$

$\Rightarrow$

the smallness of f_1 not necessarily implies the smallness of $\partial_v f_1$!

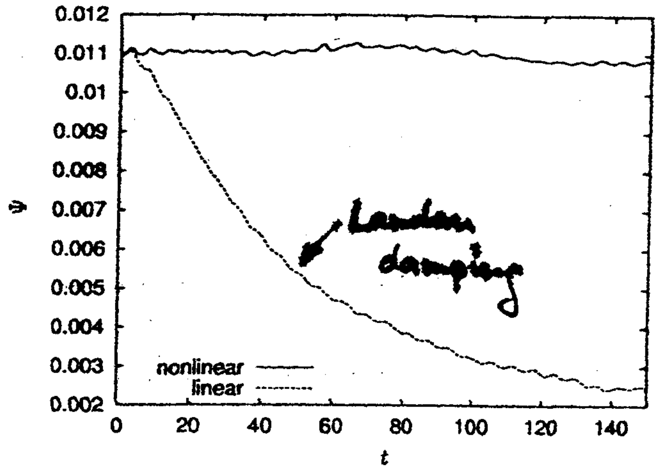
e.g. $\partial_v(f_0 + f_1) \approx 0$ at resonant velocity

$\Rightarrow$ for e–hole equilibria: $E\,\partial_v f_1$ is <u>not</u> negligible no matter how small: $|E|, |f_1|$ are.

➔

Phase space vortices require a solution of the full Vlasov–Poisson system even in the infinitesimal amplitude limit; these modes are lost in the process of linearization

e-hole



NL $E(\partial_v f_0 + \partial_v f_1)$

$f = f_0 + f_1$

$O(1)$ $O(\varepsilon)$

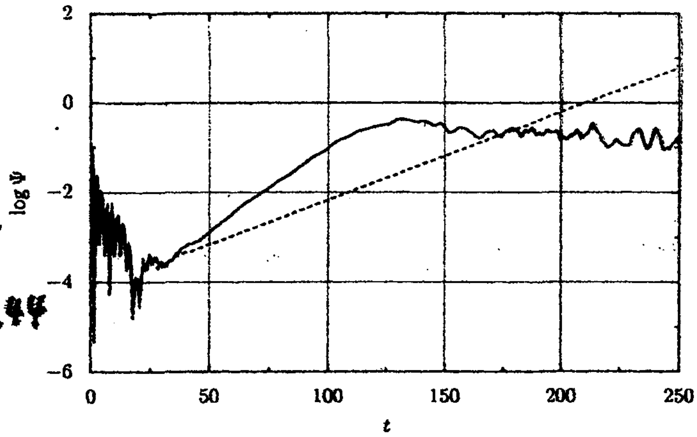
L $E(\partial_v f_0)$

Figure 15. Evolution of the amplitude in linear and nonlinear runs of the Vlasov code with the same initial conditions corresponding to an electron hole. Note how the linear run damps out the structure where it remains stationary in the nonlinear run.

two-stream instability

two topol. dist.

$v_D = 2 > v_D^* = 1.44$



L

NL

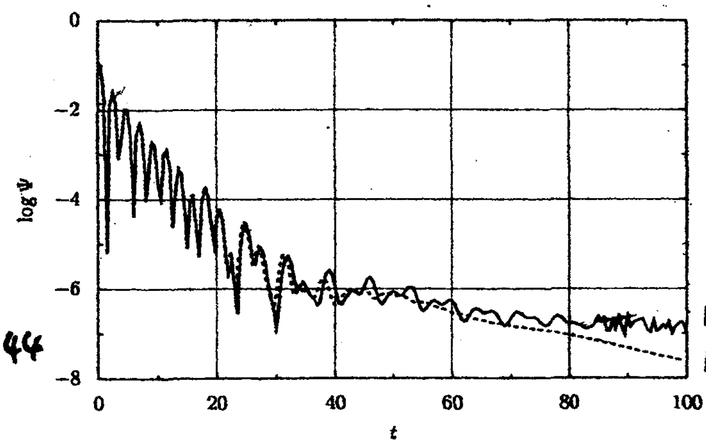
see Fig. 18

Figure 16. Evolution of the amplitude of the electrostatic potential in the linearly unstable regime of the two-stream instability, $\delta = 1/100$, $\theta = 1$, $v_D = 2.0$. The solid line represents the evolution of the fully nonlinear equation while the dashed one pictures the linear evolution. The simulation was performed with $M = 2000$, $N = 5$.

Nonlinear Landau Damping

t=0 topolog. distinct

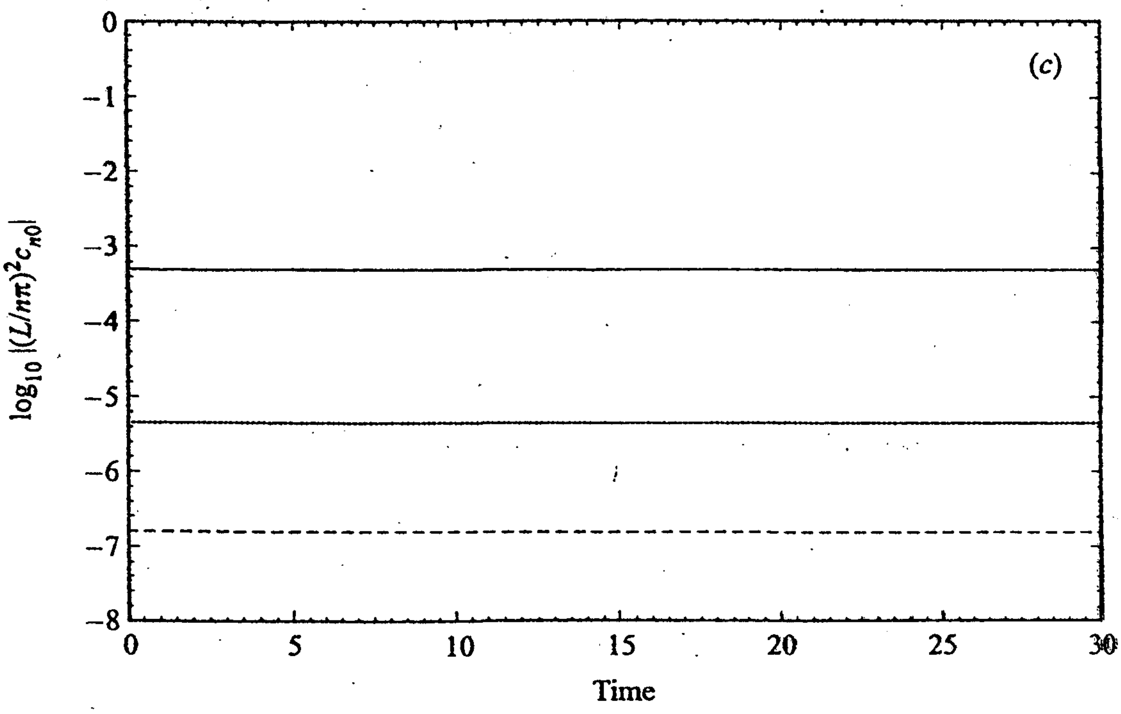
$v_D = 1 < v_D^* = 1.44$



NL phase space hole

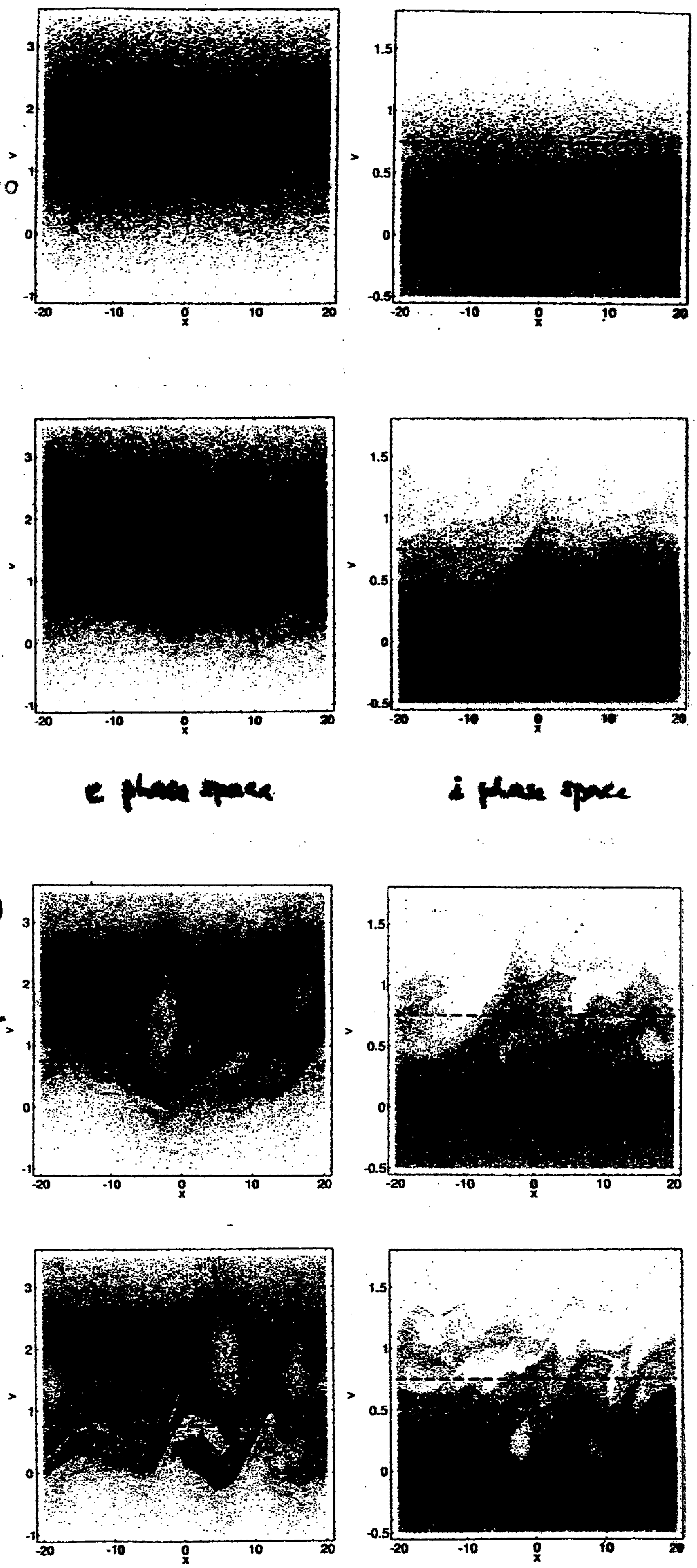
L

Figure 17. Same as in Fig. 16 but with $v_D = 1.0$, this is, in the linearly stable regime.



numerical proof
of existence, stationarity and stability
of e-holes in 1-D VLASOV-POISSON system

Fig. 2c of $\boxed{\text{Korn, Schamel J. Plasma Phys. 56, 307 (1996)}}$ $m_i = \infty$



PIC (Berman et al)

$\delta = 1/4$

$\theta = 1$

$v_D = 1.75 < v_D^* = 1.96$

lin. stable system !

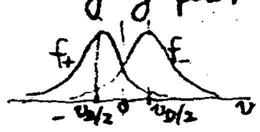
$\Delta W_i = 0$ for ion holes at

$v^* = 0.75$

$(\beta = 0)$

Hole scenario of linearly stable current carrying pair plasmas

6. Nonlinear instability and saturation of linearly stable current-carrying pair plasmas *



+ noise ($f_1/f_0 = 0.08$)

$\boxed{v_D = 2 < v_D^* = 2.6}$ lin. stable

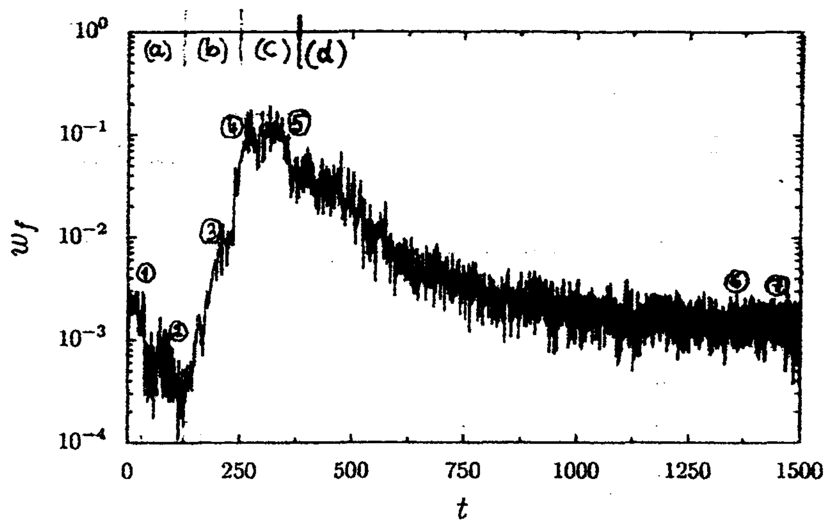
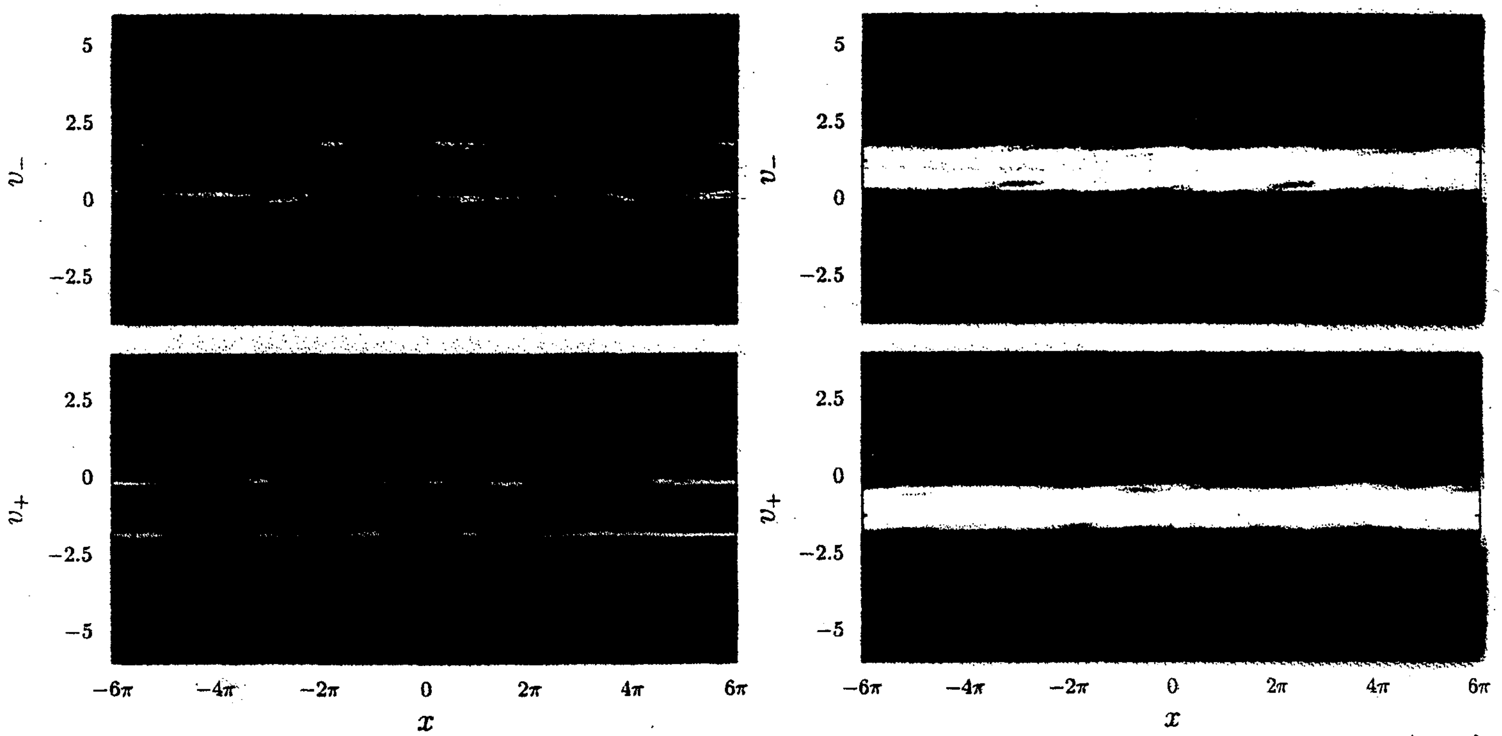
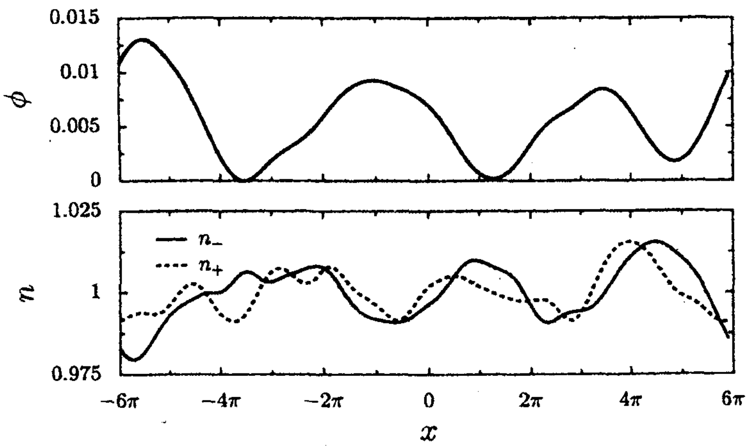


FIG. : Evolution of the field energy $w_f = \int (\partial_x \phi)^2 \, dx$. ~~Three~~ Four phases can be distinguished: (a) an initial transient damping, (b) nonlinear instability and (c) saturation and (d) decay towards a new equilibrium.
seed hole — multiple generation of holes — anomalous structural diffusion

A. Luque, H.S., B. Eliasson and P.K. Shukla (submitted)



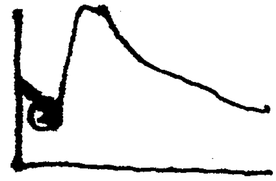
(a) damping and preparation of seed hole

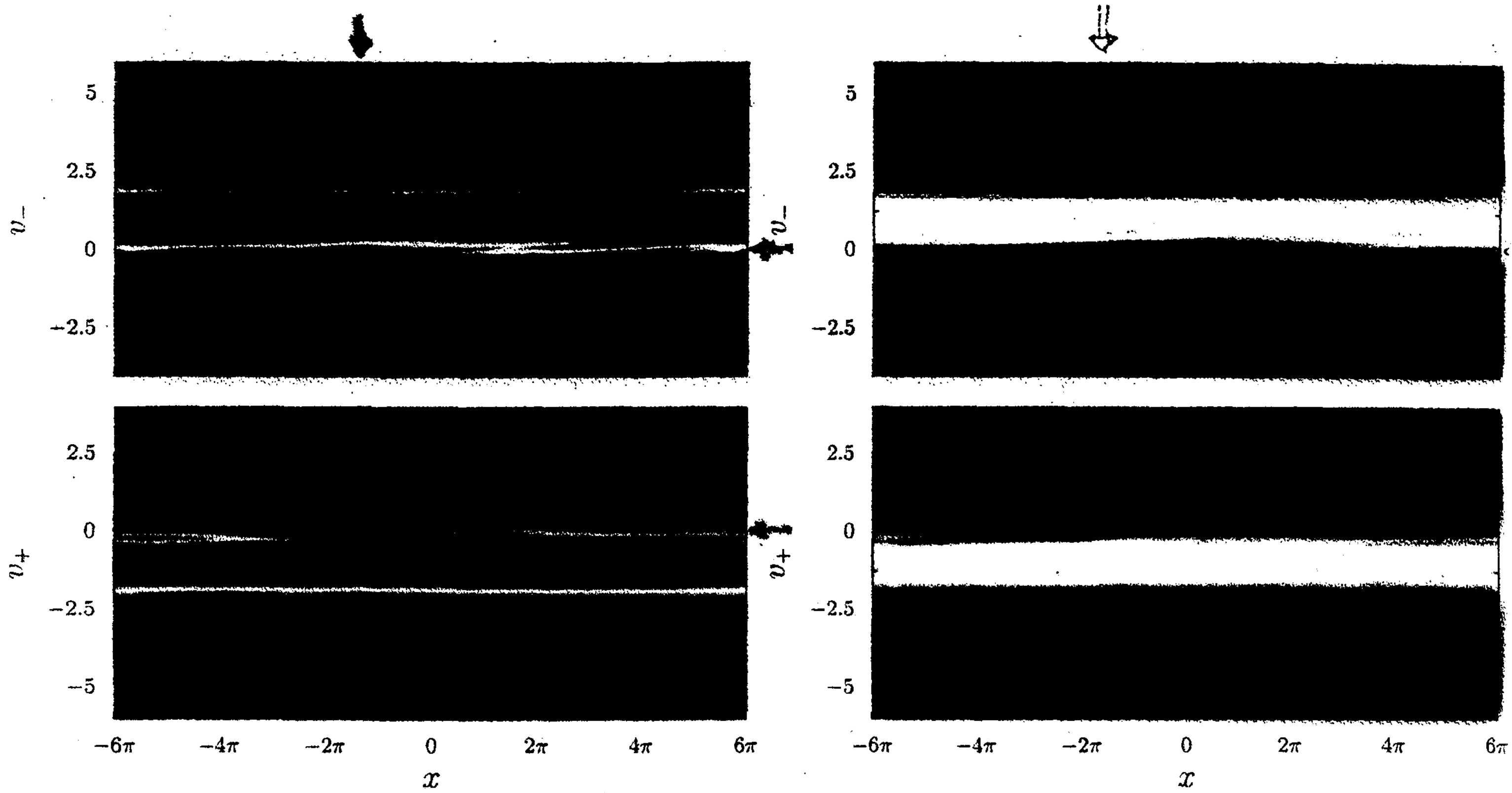


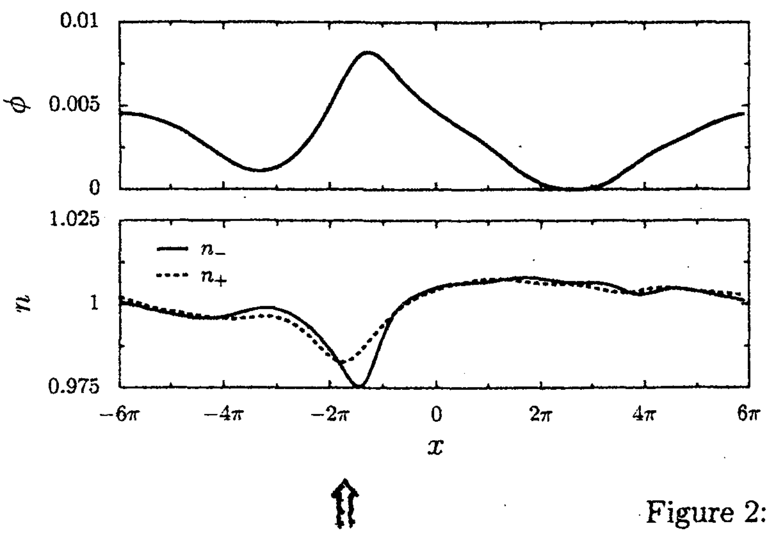
snapshot 1 in initial damping phase (a)

⇒ a lot of coherency due to ~~Landau~~ nontopological fluctuations

Figure 1: $t = 37.5$





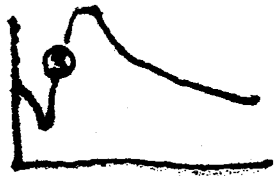


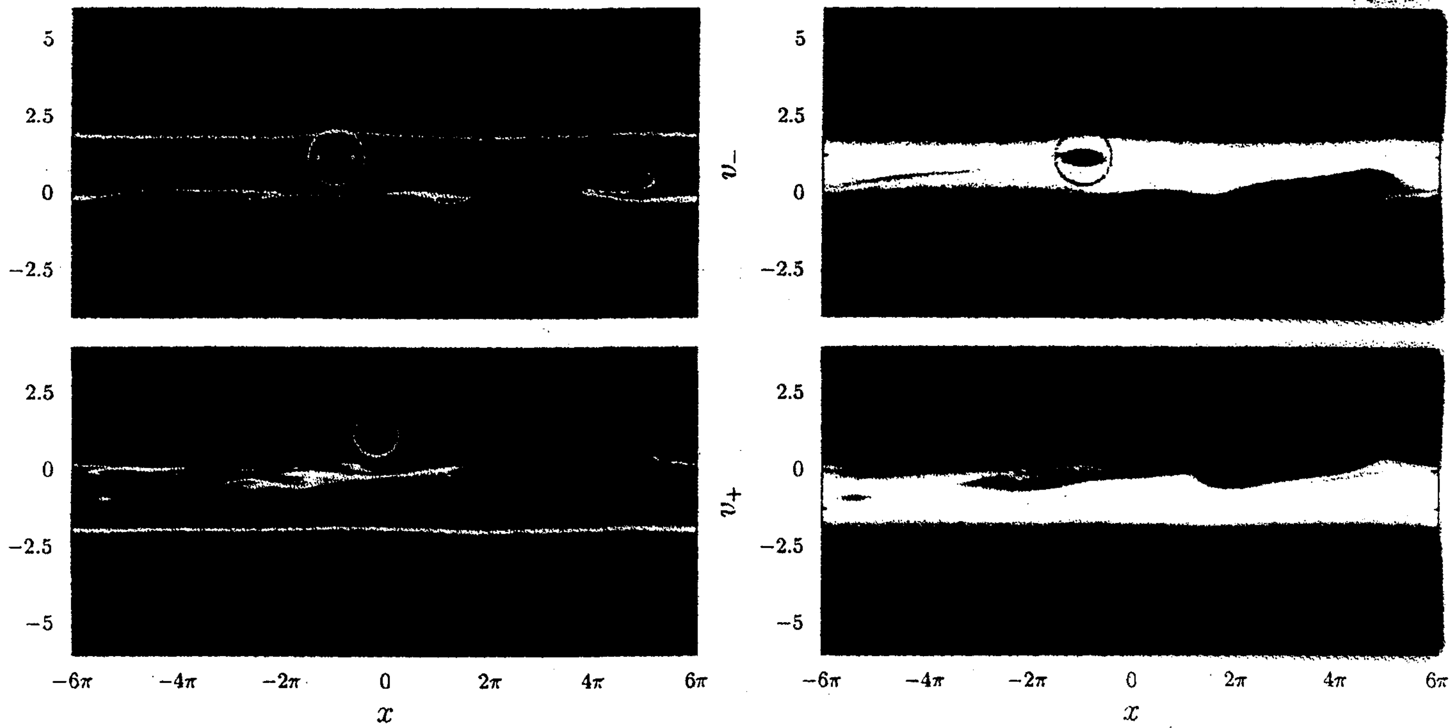
seed hole

Figure 2: $t = 120$

snapshot ② at minimum W_f

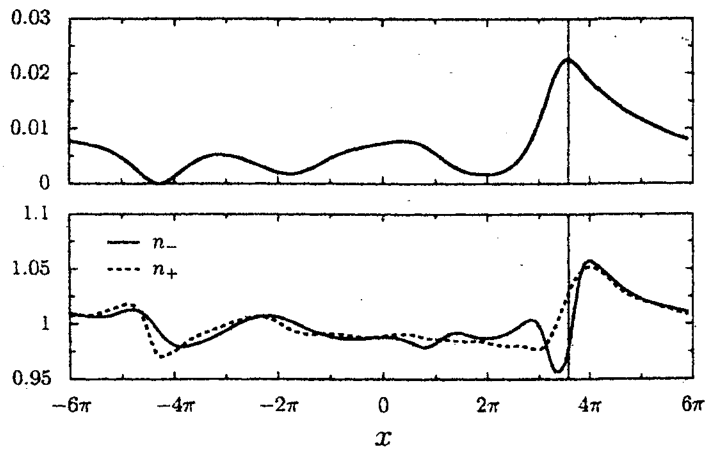
- a distinct seed hole in f_- is clearly visible
- most activity at $v_\mp \approx 0$
- a negative energy – hole





(b) multiple generation and growth of holes

⇑



snapshot ③ in phase (b) : of −hole

- $\boxed{\text{acceleration and nonlinear growth}}$
- creation of a bipolar structure ($x \approx 4\pi$)

  - asymmetric −hole experiences a net $E < 0$

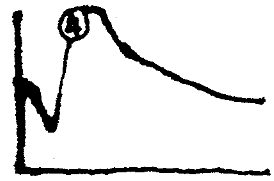
$$\rightsquigarrow \underbrace{M}_{<0} \ddot{x} = Q \underbrace{E}_{<0} \qquad \rightsquigarrow \ddot{x} > 0$$

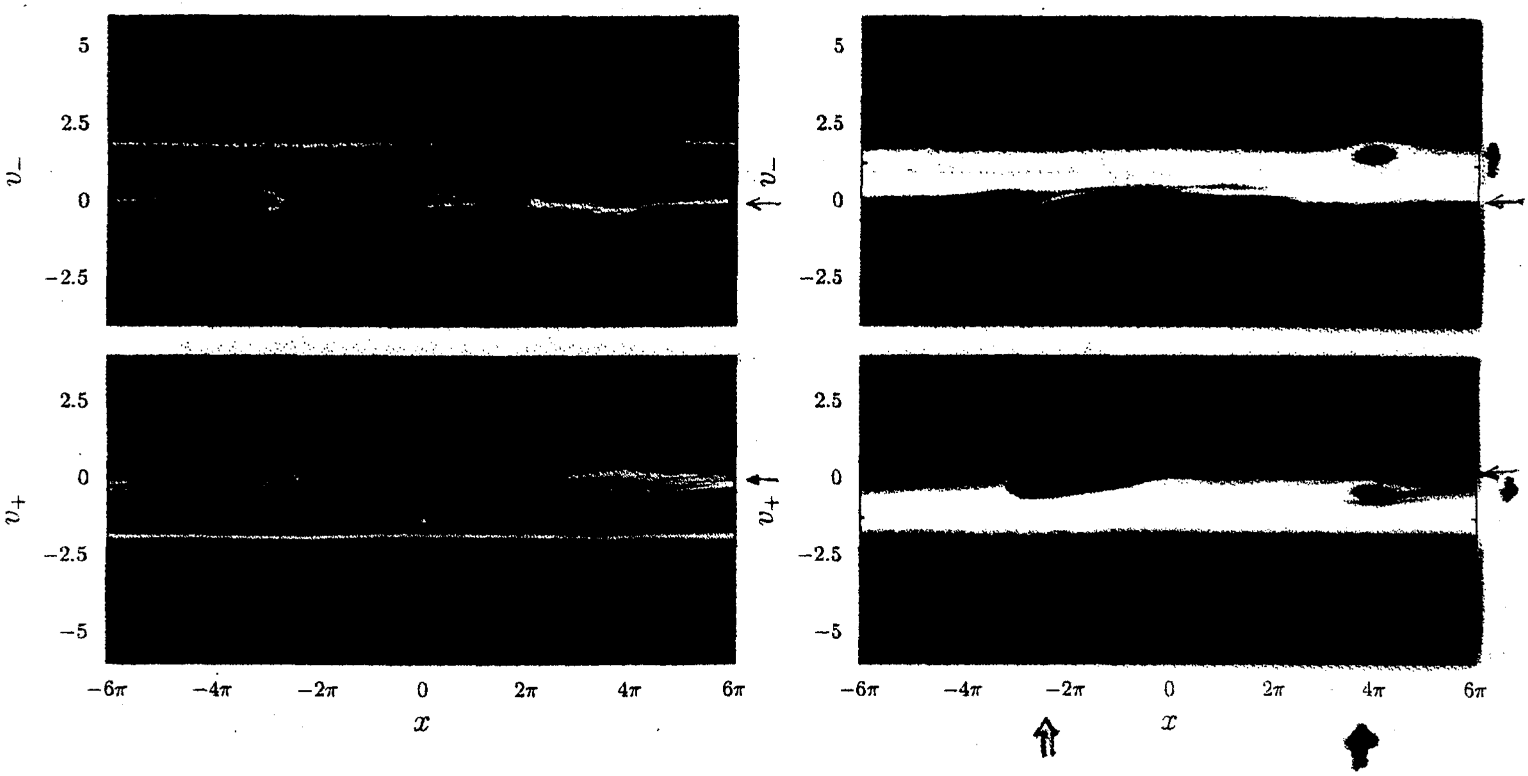
Figure 3: $t = 187.5$

interpretation of −hole :
a cloud of positively charged particles ($Q > 0$) embedded in a −fluid which acts as a dielectric medium

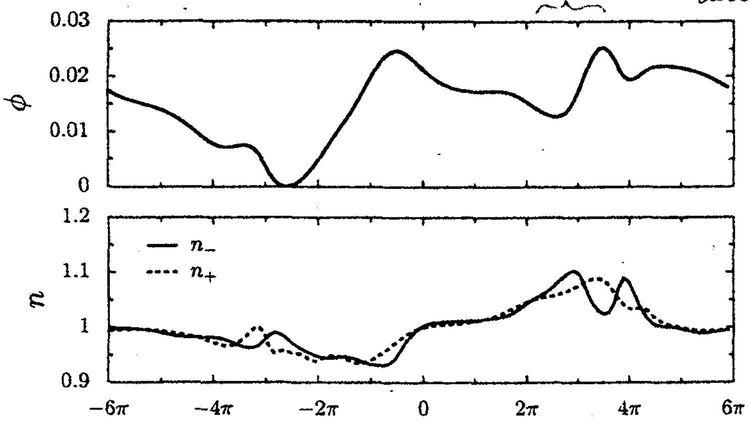
$$(Q, M) := \int dx \int dv \, (\underbrace{q_-}_{<0}, \underbrace{m_-}_{>0}) \underbrace{\tilde{f}_{-t}}_{<0} \quad \text{due to lack of particles} \qquad \rightsquigarrow \boxed{Q > 0 \,,\, M < 0}$$

Eliasson & Shukla
Phys. Rev. Lett. 92 (2004) [illegible] : −hole is attracted (repelled) by maximum (minimum) of n_+





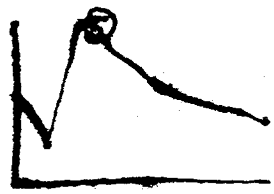
old - hole (symmetric ∧ E ≳ 0) deceleration, decay

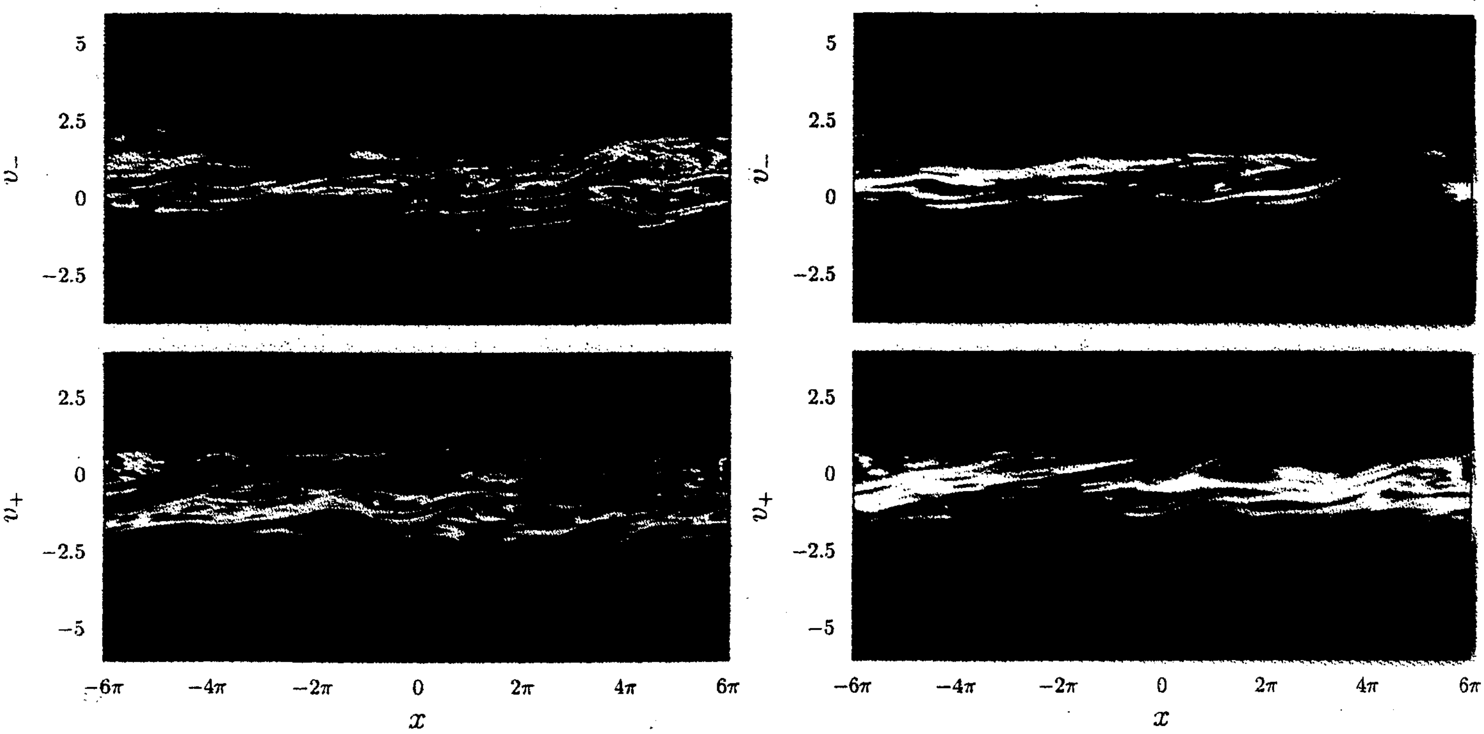


snapshot (4)

- self-generation of new pair of seed holes near $v_\pm \approx 0$ ($x \simeq -2\pi$)
- multiplication of structures
  ↑ exp. growth of W_f
- splitting of old bipolar structure into a −hole (experiences acceleration) and a +hole (deceleration) ($x \approx 4\pi$)

Figure 4: $t = 221.25$





(c) saturation and fully developed turbulence

snapshot 5 in phase (c):

chaotic, structured state

dominant process $\boxed{\text{anom., struct. diffus}}$

$\boxed{\textbf{colliding and merging of holes}}$

effective scattering

anomalous transport

heating and friction

see $\langle v \rangle$, v_{th}^2

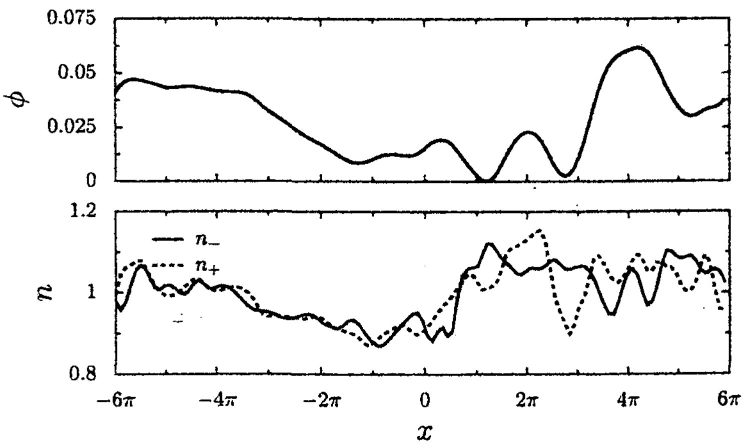
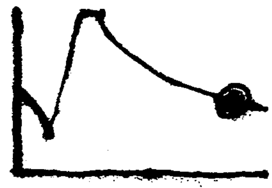
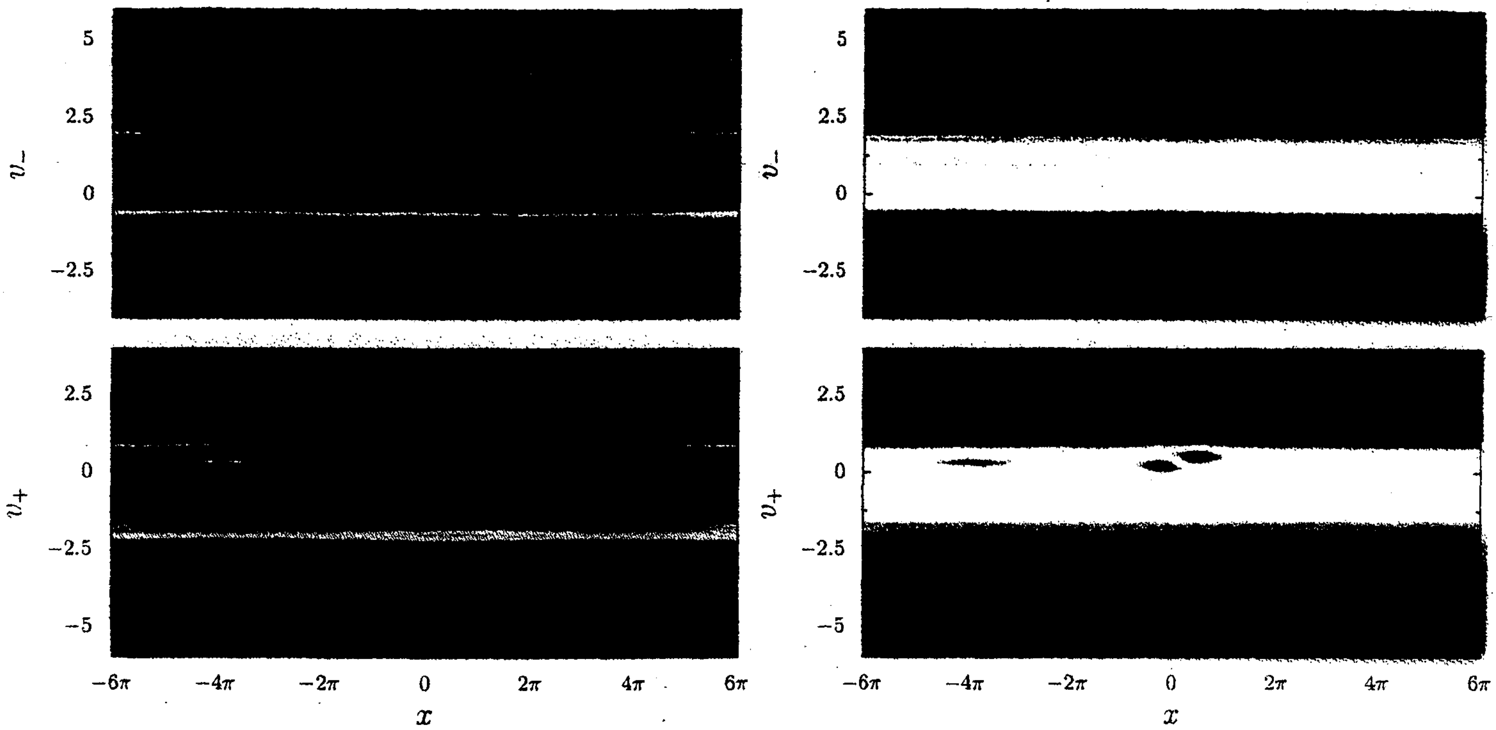


Figure 5: $t = 375$

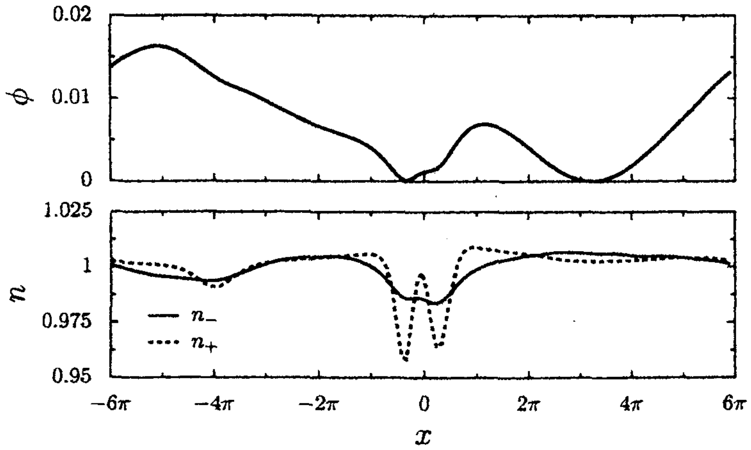
conductivity

$$\sigma_{inh} = \sigma_{hom}\left[1 - \frac{1}{2LE_0}\int_{-L}^{L} dx\, n_0\, \partial_x \phi\right]$$





(d) relaxation to a new collisionless equilibrium



snapshot 6

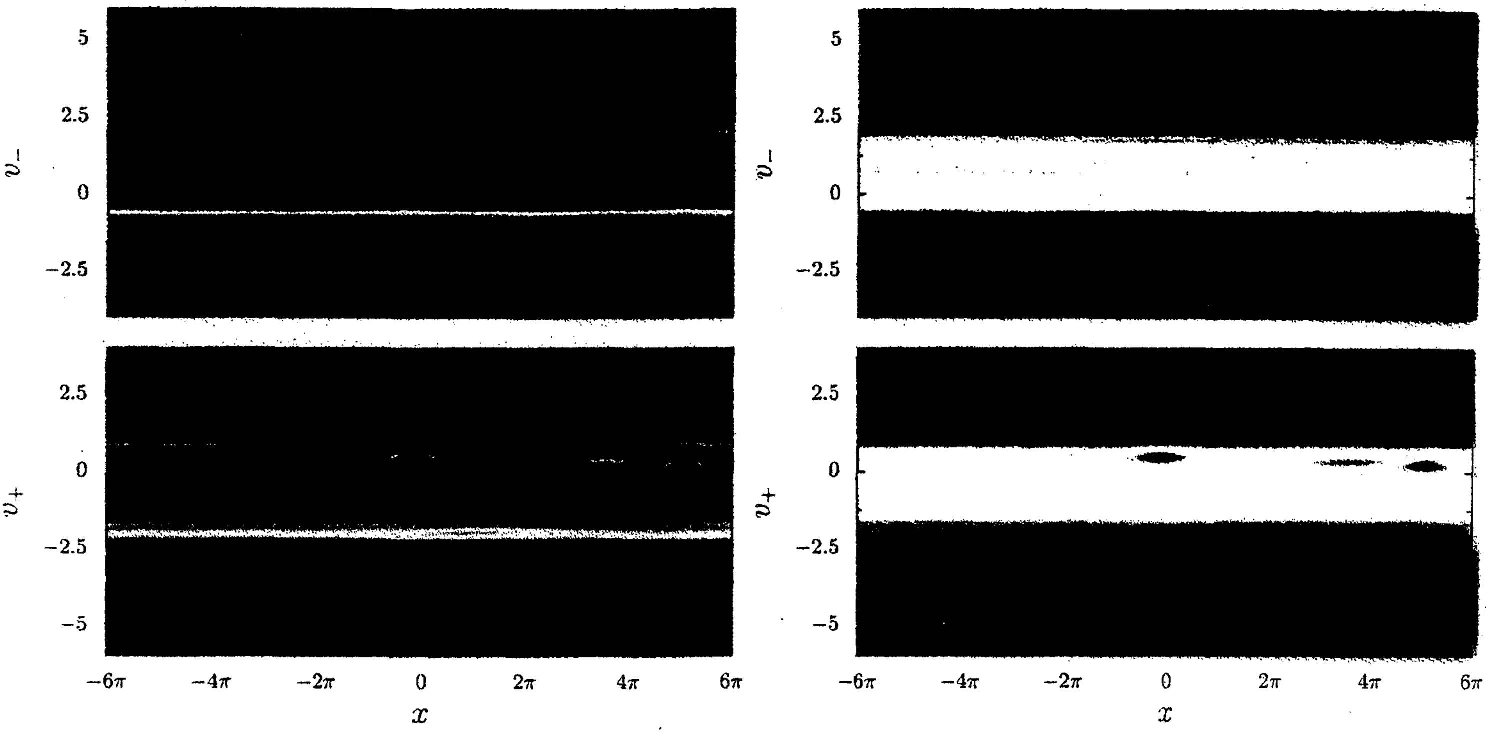
almost asymptotic state

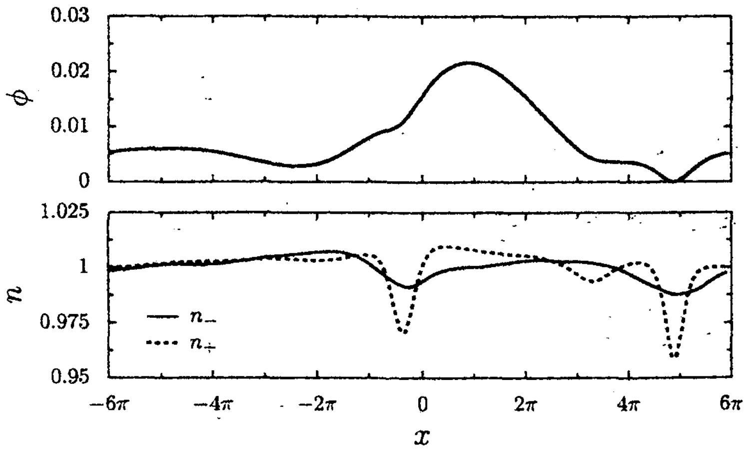
3 + holes survive

(which will later coalesce into one big + hole)

both distributions flat-topped
(see extra figure)

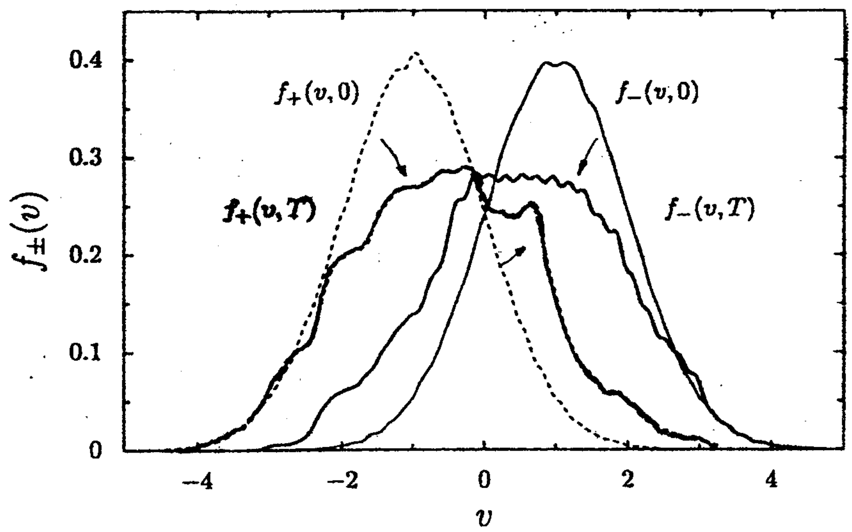
Figure 6: $t = 1350$





snapshot 7
like 6
still 3 +holes

Figure 7: $t = 1425$



flat-topped

FIG. : Velocity distribution functions at $t = 0$ and at $t = T = 1500$.

with hole in f_+ at $t=1500$

→ asymptotic state $t \to \infty$: non-Maxwellian + superimposed hole (structured dissipationless equilibrium)

Conclusion :

- although NO linear instability → a transient, structural dynamics governed by HOLES is seen
- NONLINEAR generation of holes basically as ENERGY-NEUTRAL modes
- anomalous structural diffusion in the COARSE-GRAINED distribution provides the dissipation mechanism
- relaxation to a new collisionless equilibrium with FLAT-TOPPED distributions (rather than Maxwellians) + some LONG-LIVING phase space holes

7. Extensions

– finite amplitudes
$0 \le \phi \le \Psi$, Ψ arbitrary

$$n_e(\phi) = e^{-v_0^2/2} \Big[\underbrace{F(v_0^2/2, \phi)}_{\text{free}} + \underbrace{T_\pm(\beta, \phi)}_{\text{trapped}} \Big]$$

Lit.
Schamel (1971, 72, 82, 86
S. Bujarbarua, H. S.
J. Plasma Phys 25, 515 (81

e-hole ($m_i = \infty$) : $\beta < 0$, Ψ unlimited
i-hole : $\alpha < 0$, $\Psi \le 1$ (exp. observation!)

– collisional effects
holes in weakly dissipative plasmas (numerical)
v_D = const
- ∃ of dissipative hole equilibrium
- enhancement of resistivity → of anomalous transport
- importance of ion mobility

Lit.
J. Korn, H. S. (1996
J. Plasma Phys 54, 307; 385
A. Luque, H.S. (2005)
Phys. Rep. 415, 261

– relativistic corrections
lab frame ; Maxwellian → Jüttner, Synge distr. ($\phi = 0$)
$0 \le \phi \le \Psi$ (arbitrary) ;

$$f(p, \phi) = \begin{cases} \text{eq. (14) of Eliasson, Shukla I} \\ \text{eq. (13) " " " II} \end{cases}$$

- $\beta < 0$
- increase of amplitude and width if v_{th}/c increases

Lit.
B. Eliasson, P.K. Shukla
I Phys. Lett. A340, 237 (05)
II Phys. Plasmas (2005 to appear

– quantum corrections
Wigner (1932) pseudo-distribution satisfying von Neumann eq.
(alternate form of Quantum Mechanics for collisionless many particle system)

$$\varepsilon := \frac{\hbar}{m_e v_{th} \lambda_D} = \frac{\lambda_{dB}}{\lambda_D}$$

- tunneling of particles counteracts hole formation
- no holes if $\varepsilon \simeq O(1)$

$\varepsilon \ll 1$ analytical
$\varepsilon \simeq O(1)$ numerical

Lit.
analytical:
A. Luque, H.S. and R. Fedele
Phys. Lett. A 324, 185 (2004
A. Luque, H.S.
Phys. Reports 415, 261 (05
numerical:
N.D. Suh, M.R. Feix, P. Bertrand
J. Chem. Phys. 94, 403 (1991)

– holes (humps) in particle accelerators (synchrotrons)

holes in coasting beams
humps
- analogy to hole theory in Vlasov-Poisson plasmas : $\beta < 0$
  → FERMI lab, CERN

humps in bunched beams
holes
- ∃ of hot spots on bunch
  → RHIC Brookhaven

Lit.
H. Schamel (1997) Phys. Rev. Lett. 79, 2811
H.S., R. Fedele (2000) Phys. Plasmas 7, 3421
M. Blaskiewicz, J. Wei, A. Luque, H.S. Phys. Rev. ST Accel. Beams 7, 044402 (04)
A. Luque, H. Schamel Phys. Reports 415, 261 (2005)

Applications

– Risø experiment (1979)
single ended Q-machine with slit; excitation by neg. voltage pulse
→ 2 structures of opposite polarity
a fast KdV soliton $\phi < 0$
(based on Gould-Trivelpiece mode)
a slow e-hole $\phi > 0$
(first exp. observation)

Lit.
J.P. Lynov, P. Michelsen, H.L. Pécseli, J.J. Rasmussen, K. Saéki, V.A. Turikov
Phys. Scripta 20, 328 (1979)

– Sendai experiment (2004)
fullerene pair plasma
C_{60}^{+} by impact ionization
C_{60}^{-} " electron attachment
observation of
ion plasma wave
intermediate frequency wave (so-called)
(slow) ion acoustic wave

Lit.
W. Oohara, R. Hatakeyama
– Phys. Rev. Lett. 91, 205005 (03)
– Trieste Conference (2004) (unpublished ?)
theoretical explanation
H.S., A. Luque paper III
New J. Phys. 7, 69 (2005)

– Greifswald-Kiel experiment (2001)
double plasma device
periodic injection of ion bunches
δn fluctuation measurements
observation: spontaneous acceleration of periodic ion structure from
v_{thi} (periodic ion hump structure) to
c_s (periodic ion acoustic wave)

Lit.
C. Franck, T. Klinger, A. Piel, H.S.
Phys. Plasmas 8, 4271 (2001)
(inclusively theoretical explanation)

– holes at FERMI lab (1994), CERN (2001) & BROOKHAVEN (2004)

Lit.

FERMI lab Main Ring (coasting beam)
externally applied sinusoidal voltage kick
measurement of beam transfer fct ($f(p)$)
by wall current pick up (WCM)

depletion zones in momentum distrib.
at lowest measurable signal level

P.L. Colestock, L.K. ~~Spentzouris~~
(Tamura Symp. 1994)

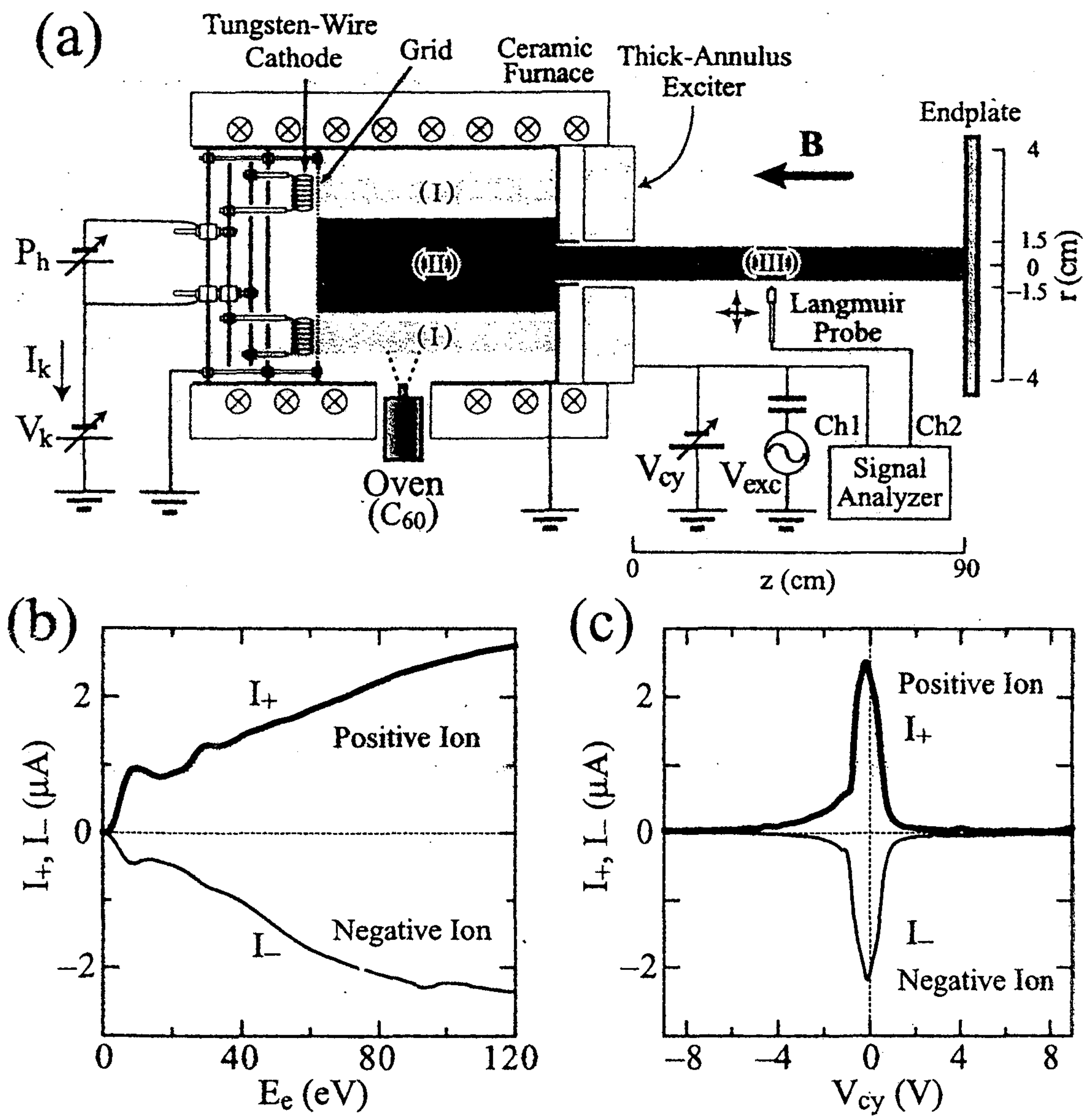
CERN (proton storage ring)
periodic holes introduced artificially

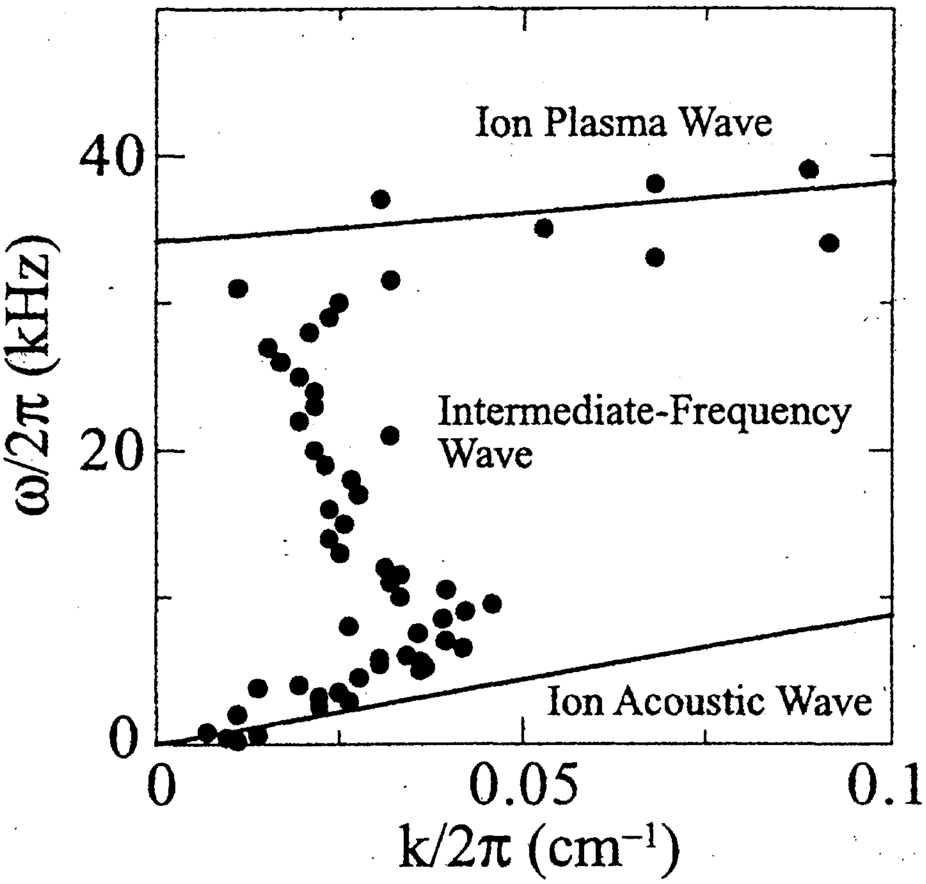
S. Koscielniak, S. Hancock, M. Lindroos
Phys. Rev. ST-AB, 4, 044201 (2001)

BROOKHAVEN RHIC
bunched beam
WCM measurements of
humps on bunched beam

M. Blaskiewicz, J. Wei, A. Luque, H.S.
Phys. Rev. ST-AB, 7, 044402 (2004)

The experiment of HATAKEYAMA & OOHARA
(Tohoku Univ., Sendai, Japan)




Ion Plasma Wave
Intermediate-Frequency
Wave
Ion Acoustic Wave
40
20
0
ω/2π (kHz)
0
0.05
0.1
k/2π (cm⁻¹)

3. The theory of generalized hole and double layer equilibria in fullerene pair plasmas ×

- stationary solutions of VLASOV - POISSON system
- demonstration of POTENTIAL method by e-hole solution

⟹ in e-i plasma with immobile ions

$\Rightarrow$ NDR $\rightarrow$ phase velocity v_0

$V(\phi) \rightarrow$ spectral composition of $\phi(x)$

- generalization to $m_i < \infty$, nonzero current between e, i

Lit. H.S. Phys. Plasmas 7, 4831 (2000)

fullerene pair plasma :

$$\boxed{\delta := m_{C_{60}^-} / m_{C_{60}^+} = 1 \quad , \quad \Theta := T_{C_{60}^-} / T_{C_{60}^+} = 1 \,, \quad \begin{matrix} v_D = 0 \\ \alpha = \beta \end{matrix}} \tag{1}$$

NDR
$$\boxed{-\frac{1}{2} Z_r'(v_0/\sqrt{2}) = -k_0^2/2 + \underbrace{\frac{4}{3} b(\beta, v_0)\sqrt{\psi}}_{=: B}} \tag{2}$$

$V(\phi)$
$$\boxed{\begin{aligned} -V(\phi) = \frac{k_0^2}{2}\phi(\psi-\phi) - \frac{4}{3} b(\beta, v_0)\Big\{\sqrt{\psi}\,\phi(\psi-\phi) \\ + \frac{2}{5}\Big[(\psi-\phi)^{5/2} - (\psi^{5/2} - \phi^{5/2})\Big]\Big\} \end{aligned}} \tag{3}$$

hole symmetry around $\psi/2$

$$b(\beta, v_0) = \frac{1}{\sqrt{\pi}}(1 - \beta - v_0^2)\exp(-v_0^2/2) \tag{4}$$

$$\boxed{\frac{\phi'(x)^2}{2} + V(\phi) = 0} \tag{5}$$

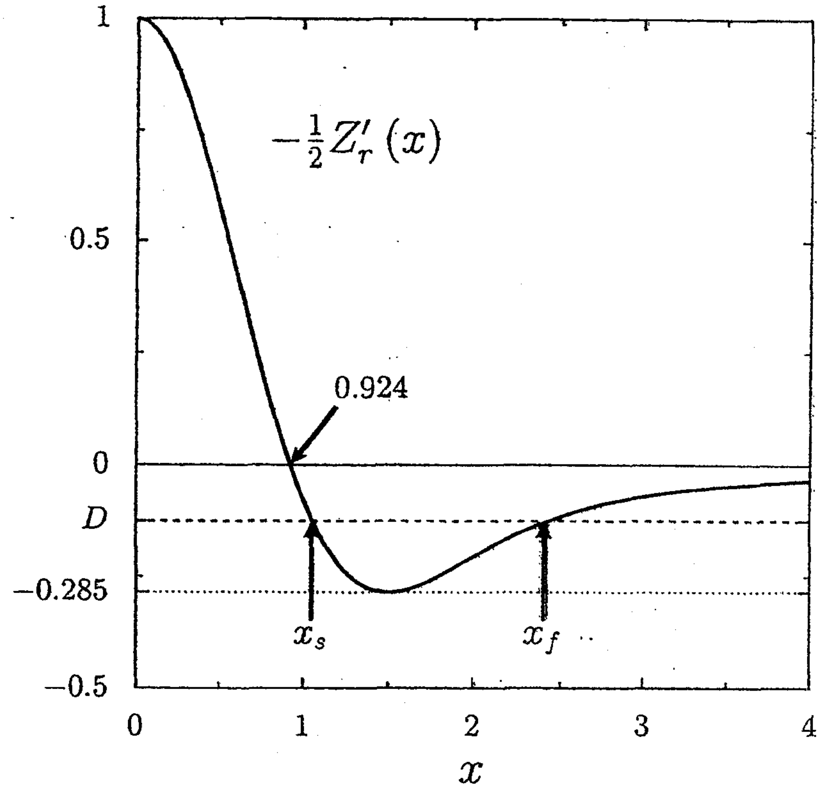
quadrature $\rightarrow \phi(x)$

× H. Schamel, A. Luque, Kinetic theory of periodic hole and double layer equilibria in pair plasmas, New J. Phys 7 (2005) 69

$\boxed{\text{Harmonic wave}}$ $\quad b(\beta, v_0) = 0 = B$

$$V(\phi) = \frac{k^2}{2}\phi(\psi - \phi) \rightsquigarrow \phi(x) = \frac{\psi}{2}\left[1 + \cos(kx)\right], \quad k_0 = k$$

$$\boxed{-\frac{1}{2} Z_r'(v_0/\sqrt{2}) = -k^2/2}$$



$\boxed{k^2 \ll 1}$

$\omega_s = 1.307\, k\,(1 + k^2/2)$

slow acoustic mode

$\omega_f = \sqrt{2}\,(1 + 3k^2/4)$

plasma wave

two branches exist which join at

$$\left.\begin{array}{l} x = \dfrac{\omega}{\sqrt{2}\,k} = 1.5 \\ k^2/2 = -0.285 \end{array}\right\} \quad k_{max} = 0.755, \quad \frac{\omega(k_{max})}{\sqrt{2}} = 1.133$$

(turning point)

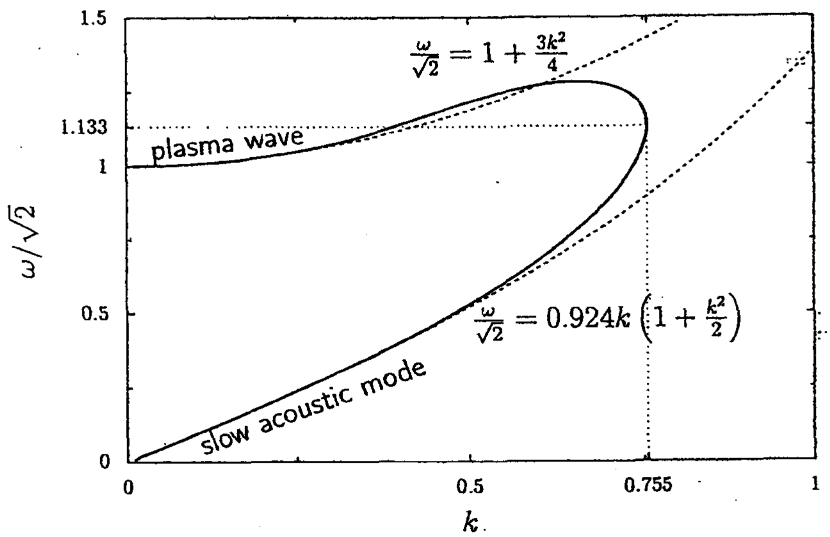


FIG. The NDR for a harmonic wave structure with $\omega := kv_0$ (solid line). The long wavelength limit, $k \ll 1$, is drawn by dashed lines showing the fast plasma wave and the slow acoustic mode.

Harmonic wave
Continuation

status of trapping:

$$b(\beta, v_0) = \frac{1}{\sqrt{\pi}} \underbrace{(1-\beta-v_0^2)}_{\stackrel{!}{=}0 \text{ for slow wave}} \underbrace{e^{-v_0^2/2}}_{\approx 0 \text{ for fast wave}} \stackrel{!}{=} 0$$

↷ no need for special trapping scenario

$$\curvearrowright \; -\beta = \underbrace{v_0^2}_{(1.307)^2} - 1 = 0.708 \quad \curvearrowright \boxed{\beta < 0}$$

⇒ only a notch in the trapped particle region of f provides the proper existence conditions for the slow acoustic mode (being hence NONLINEAR in character)

generalized periodic waves

$$\boxed{-\frac{1}{2} Z_r'\left(\frac{v_0}{\sqrt{2}}\right) = -\frac{k_0^2}{2} + B}$$

↷ turning point condition

$$\frac{k_0^2}{2} = 0.285 + B$$

$$k_{max} = \sqrt{0.57 + 2B}$$

$$\omega(k_{max}) = \underbrace{\sqrt{2}\, 1.5}_{2.12} k_{max}$$

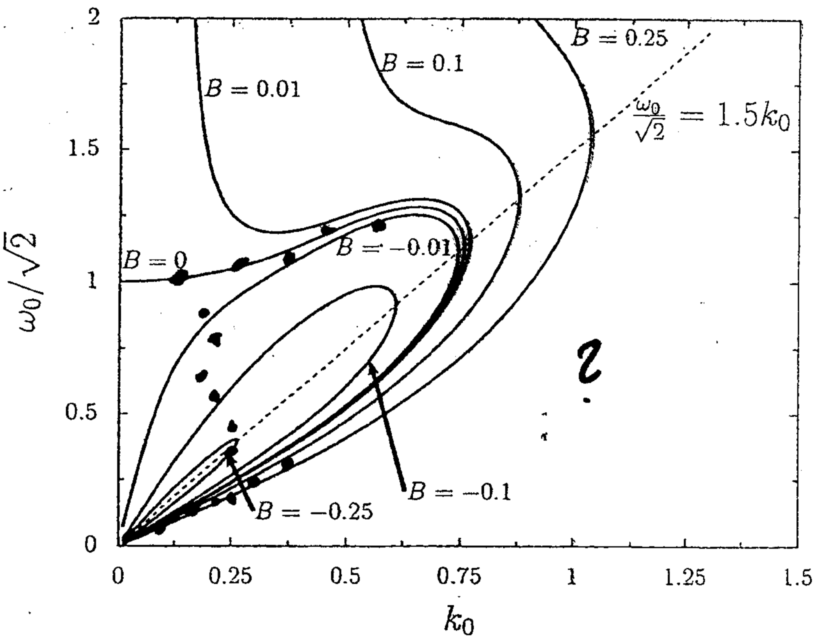
$$\curvearrowright \boxed{\frac{\omega(k_{max})}{\sqrt{2}\, k_{max}} = 1.5}$$

trapping condition

$$\boxed{-\beta = \frac{3}{4}\sqrt{\frac{\pi}{\psi}}\, e^{v_0^2/2} B + v_0^2 - 1}$$

given v_0 and ψ

increase of B gives a larger depression in f



plot of dispersion diagram for periodic waves (cnoidal waves)

$$-V(\phi) = \frac{k_0^2}{2}\phi(\psi - \phi) - B\{ \quad \}$$

nonharmonicity

Vergleich: 2-D, inkompr., ideale Flüssigkeit ↔ 1-D VP
Comparison: 2D, incompr., ideal fluid ↔ 1D Vlasov-Poisson

	2-D, inkompr., id. Fluid	1-D Vlasov-Plasma
	$\nabla\cdot\underline{u} = 0$ $(\partial_t + \underline{u}\cdot\nabla)\underline{u} = -\frac{1}{\rho}\nabla p$	$\partial_t f + v\partial_x f + \partial_x\phi\,\partial_v f = 0$ $\partial_x^2\phi = \int dv\, f - 1$
	$\underline{u} = \nabla H \times \hat{z} = (\partial_y H, -\partial_x H, 0)$ $H(x,y,t)$ "Stromfkt" stream fct $f := (\nabla\times\underline{u})\cdot\hat{z} = -\nabla^2 H$ Wirbelstärke vorticity	$v \to y$ $H := \frac{y^2}{2} - \phi(x,t)$ Einteilchenenergie single particle Hamiltonian
	$\boxed{\partial_t f + [f, H] = 0}$ convective cell eq.	Hamiltonsche Form d. Bewegungsgleichg Hamiltonian form of equation of motion Poissonklammer Poisson bracket x, y – kan. Variable
Selbst-konsistenz	$-\nabla^2 H = f(x,y,t)$	$-\nabla^2 H = \int dy\, f(x,y,t) - 2$

Gemeinsamkeiten: common ground

- Hamiltonsche Struktur der Wirbeldynamik (vortex dynamics) im Phasenraum (phase space)
- Filamentierung (eddies)
- "gefangene" und "freie" ("trapped" & "free") Fluidelemente

convective cell eq.

- pure electron plasma in Penning-Malmberg trap
- strongly magn., cold e-i plasma

Unterschiede: differences

(1) bei (VP) separiert H
↑ ellipt. Gleichg ist degeneriert (1-D Quelle von H)

(2) $f \geq 0$ in (VP)

(3) andere Selbstkonsistenz

Hamiltonian nature of (coupled, driven) nonlinear wave equations describing dynamics of vortices ($\perp$) & eddy currents ($\parallel$)

⇒ drift wave turbulence
MHD turbulence
geostrophic flow etc.

8 SUMMARY AND CONCLUSIONS

- No doubt, linear wave theories are deficient for low amplitude plasma dynamics (turbulence) if trapping of particles or fluid elements is involved

⟹ FAILURE OF STANDARD WAVE CONCEPT

-Like in fluid dynamics nonlinear stationary solutions of governing eqs play a central role (secondary, tertiary solutions,...)

-In the classical two-stream problem this is manifest in a nonlinear instability even if $V_d < V_d^*$
provided that fluctuations are present that violate linear criterion

-Seed holes of zero/negative energy are spontaneously created which experience acceleration and growth

⟹ NEW PARADIGM FOR PLASMA STABILITY

-In weakly dissipative plasmas a similar scenario holds provided that initial fluctuation exceeds a threshold (which disappears with collision frequency)

⟹ ANOMALOUS TRANSPORT WITH ENHANCED RESISTIVITY BY PRESENCE OF SELF-GENERATED HOLES

-Pair plasmas are best suited to investigate and understand the underlying dynamics